GPT Simulations:
Grigory's Cryounit (Nb_3Sn unit) at UITF

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Outline

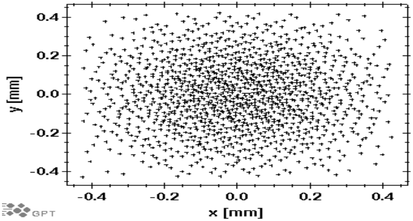
- Used the optimized parameter for UITF (Courtesy- Alicia Hofler, 03/18/2016, UITF Internal Review)
- 2-7-cell cavities are replaced by Grigory's cryounit (5-5 cell rf cavities)
- 1D field profiles for the cavities
- Initial distributions
- Buncher cavity voltage calculation
- Calibration and optimization of amplitude and crest phases of rf cavities
- Energy gain
- Beam Envelope
- End distribution

Initial distribution

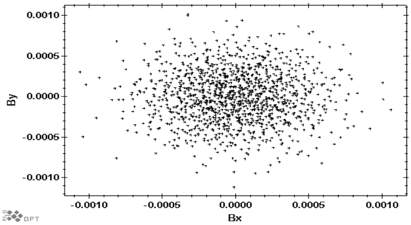
- 200 kV D.C. gun
- 1 nA beam current at 750 MHz (with space charge3Dmesh, q=-1.33333×10^{-18} C)
- Thermal electron energy E_{th}: 0.025 eV (approx. $k_B T$ at room temperature (20°C))
- Thermal emittance 0.061 mm-mrad, Gaussian transverse momentum distributions
- 21.3 ps bunch length, Gaussian temporal distribution
- 213 μm beam size, Gaussian spatial distributions
- 2500 macro-particles

Initial distribution

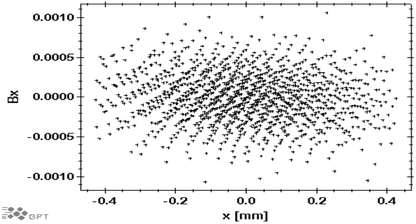
spatial distribution



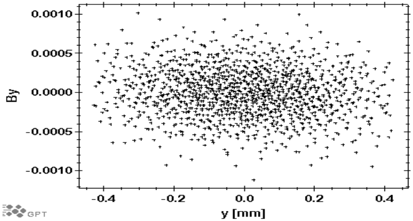
momentum distribution



horizontal phase space



vertical phase space



Buncher Voltage Calculation

The voltage V_{bun} required to form a longitudinal waist a distance L downstream of the buncher is:

$$V_{bun} = \frac{\lambda_{RF}}{2\pi eL} mc^2 \gamma(\gamma^2 - 1) = \frac{\lambda_{RF} mc^2 \gamma^3 \beta^2}{2\pi L}$$

For electron beam, $E_0 = mc^2 = 0.511$ MeV, and for 750 MHz buncher, $\lambda_{RF} = c/f = 3 \times 10^8 / 750 \times 10^6 = 0.4$ m

$$V_{bun} = \frac{\lambda_{RF} mc^2 \gamma^3 \beta^2}{2\pi eL} = \frac{0.4 \times 0.511 \times 10^6 V \gamma^3 \beta^2}{2\pi L} = 32531.30 V \frac{\gamma^3 \beta^2}{L}$$

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[1] *** A. Chao

Calculation of bunching voltage

$$V_{bun} = 32531.30\mathrm{V}\frac{\gamma^3\beta^2}{L}$$

Keeping the buncher gradient same, For @ 350 keV beam, $\beta = 0.8048$, $\gamma = 1.6849$ and we got L = 3.00 m from the simulation. So

$$V_{bun} = 32531.30\mathrm{V}\frac{1.6849^3 \times 0.8048^2}{3.0} = 32531.30\mathrm{V} \times 1.033$$
$$= 33604.8\mathrm{V} \simeq 34\mathrm{kV}$$

For @ 200 keV beam, $\beta = 0.6952$, $\gamma = 1.3912$ and L = 1.25 m. So

$$V_{bun} = 32531.30\mathrm{V}\frac{1.3911^3 \times 0.6952^2}{1.25} = 32531.30\mathrm{V} \times 1.041$$
$$= 33865.5\mathrm{V} \simeq 34\mathrm{kV}$$

Calculation of bunching voltage

Changing the phase focus at a distance of 3 m downstream from the buncher for 200 keV beam, the required voltage is:

$$V_{bun} = 32531.30\text{V}\frac{\gamma^3\beta^2}{L} = 32531.30\text{V} \times \frac{1.3911^3 \times 0.6952^2}{3}$$
$$= 14108.301 \text{ V}$$

Calculation of bunching voltage

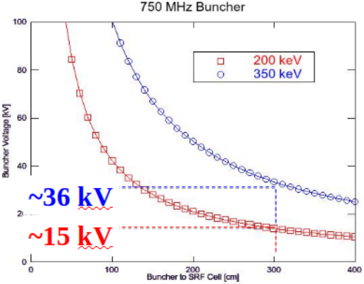
UITF Overview and Introduction, 3/10/2016:

keV region 750 MHz Buncher @ 200 and 350 keV

The LERF 750 MHz buncher will initially be used at UITF.

The voltage V_b required to form a longitudinal waist a distance L downstream of the buncher is used (Handbook of Accelerator Physics & Eng. P. 554) to estimate required RF power < 1 kW.

$V_b = (\lambda_{RF}/2\pi L)\, m_e c^2 \gamma\, (\gamma^2-1)$ where $\lambda_{RF} = c$ / 750 MHz



Courtesy T. Powers

Vc	E (V/m)	P (kW)
2.36E+05	2.80E+06	9702
2.28E+05	2.70E+06	9021
2.19E+05	2.60E+06	8366
2.11E+05	2.50E+06	7734
2.02E+05	2.40E+06	7128
1.94E+05	2.30E+06	6546
1.85E+05	2.20E+06	5990
1.77E+05	2.10E+06	5457
1.69E+05	2.00E+06	4950
1.60E+05	1.90E+06	4467
1.52E+05	1.80E+06	4010
1.43E+05	1.70E+06	3576
1.35E+05	1.60E+06	3168
1.26E+05	1.50E+06	2784
1.18E+05	1.40E+06	2426

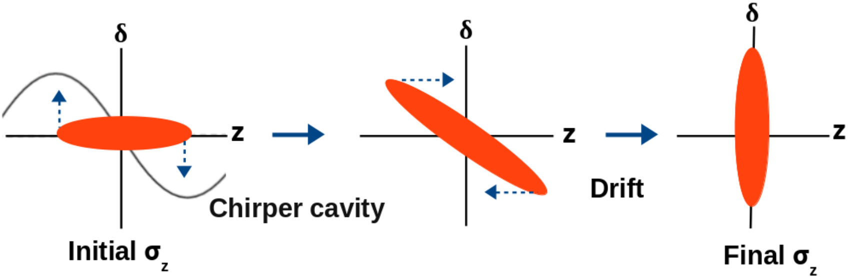
Facility

Office of Science
U.S. DEPARTMENT OF ENERGY

Courtesy M. Poelker

Bunch Compression

Ballistic compression is carried out by chirping an electron bunch in a chirper cavity at zero crossing, followed by a drift where slow electrons at the head move back with respect to the centroid and fast electrons at the tail catch up with the centroid.



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[2] D. Nguyen, J. Lewellen, L. Duffy, Bunch Compression, USPAS June 16-20, 2014

Bunch Compression Transfer Matrix

M_{56} matrix element converts particles initial energy deviation to its final z position with respect to the centroid position.

$$\begin{pmatrix} z_f \\ \delta_f \end{pmatrix} = \begin{pmatrix} M_{55} & M_{56} \\ M_{65} & M_{66} \end{pmatrix} \cdot \begin{pmatrix} z_i \\ \delta_i \end{pmatrix} = \begin{pmatrix} 1 & M_{56} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} z_i \\ \delta_i \end{pmatrix} \tag{1}$$

After time T, the distance between two particles having velocity difference Δv, $\Delta z = -\Delta v T$.

$$\begin{aligned} \frac{\Delta p}{p} &= \gamma^2 \frac{\Delta \beta}{\beta} = \gamma^2 \frac{\Delta v}{v} \\ \Delta v &= \frac{v}{\gamma^2} \frac{\Delta p}{p} \end{aligned} \tag{2}$$

$$\begin{aligned} \Delta z &= -\Delta v . T = -\frac{vT}{\gamma^2} \frac{\Delta p}{p} = -\frac{L}{\gamma^2} \frac{\Delta p}{p} = M_{56} \frac{\Delta p}{p} \\ M_{56} &= -\frac{L}{\gamma^2} \end{aligned} \tag{3}$$

Bunch Compression

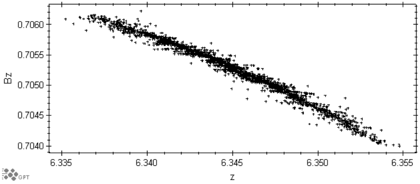
Final position of an electron

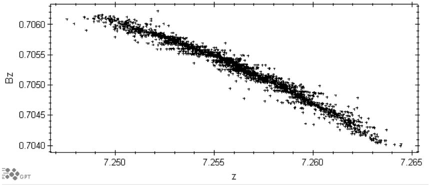
$$z_f = z_i + M_{56}\delta_i \tag{4}$$

Change in bunch length

$$\Delta z_f = \Delta z_i + M_{56}\Delta\delta_i \tag{5}$$

Bunch Compression at arbitrary phase



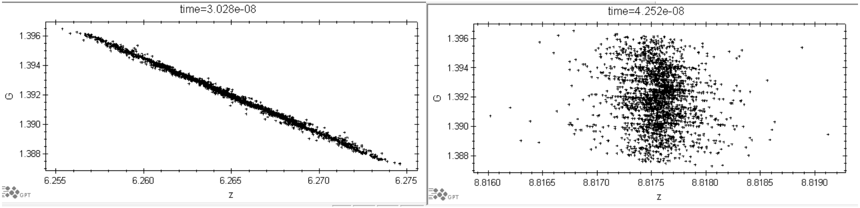


here $\Delta z_i = 0.020$m, $\beta = 0.7050$, $\gamma = 1.41$, L $= 0.91$ m,
$\Delta\delta_i = \Delta\beta/\beta = 0.00245/0.7050 = 0.0034752$.

$$\begin{aligned} \Delta z_f &= \Delta z_i + M_{56}\Delta\delta_i = 0.020 - 0.91 \times 0.0034752 \\ \Delta z_f &= 0.020 - 0.00159 = 0.0168m \end{aligned} \quad (6)$$

which is very close to the measured value 0.016 m.

Bunch Compression Factor, $\phi_s = 0^\circ$ (-22.0°)



$$V_b = 32531.30\,V\frac{\gamma^3\beta^2}{L} = 32531.30\,V \times \frac{1.3914^3 \times 0.6953^2}{2.5525} = 16597.25\,V$$

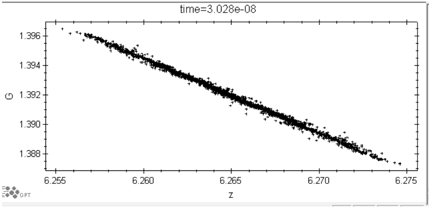
Compression factor:

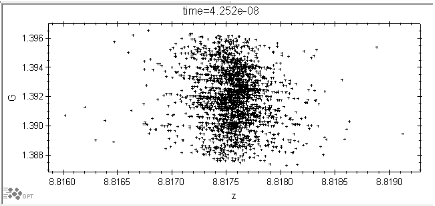
$$\frac{\Delta z_f}{\Delta z_i} = 1 + M_{56}h_1; \quad h_1 = \frac{eV_{rf}k_{rf}}{E_{1c}}\sin\phi_{rf}$$

$$\frac{\Delta z_f}{\Delta z_i} = 1 - \frac{LeV_{rf}k_{rf}}{E_{1c}}\sin\phi_{rf} = 1 - \frac{L \times 2\pi eV_{rf}}{\gamma\lambda_{rf}mc^2}\sin\phi_{rf} \tag{7}$$

$$= 1 - \frac{2.5525 \times 2\pi \times 16597.25}{1.3914 \times 0.4 \times 0.511 \times 10^6} = 1 - 0.92 = 0.08$$

Bunch Compression Factor, $\phi = 0^\circ$ (-22.0°)





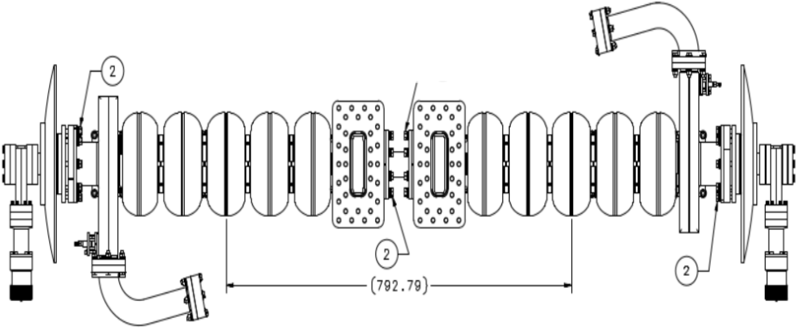
Compression factor:

$$\begin{aligned}\frac{\Delta z_f}{\Delta z_i} &= 1 - \frac{LeV_{rf}k_{rf}}{E_{1c}} \sin\phi_{rf} = 1 - \frac{L \times 2\pi eV_{rf}}{\gamma\lambda_{rf} mc^2} \sin\phi_{rf} \\ &= 1 - \frac{2.5525 \times 2\pi \times 16597.25}{1.3914 \times 0.4 \times 0.511 \times 10^6} = 1 - 0.92 = 0.08\end{aligned} \tag{8}$$

From the plots

$$\frac{\Delta z_f}{\Delta z_i} = \frac{0.0018}{0.02} = 0.09 \tag{9}$$

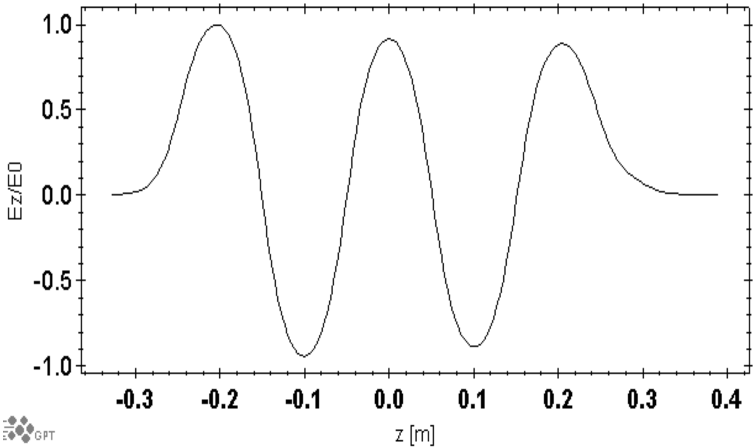
Grigory's Cryounit- Nb_3Sn unit



[3]

[3] JL0041172—C75 same as Grigory's Nb_3Sn pair.pdf

1-D Field map for the Grigory's cryounit



Calibration and Optimized Energy after the cavities

Energy before first cavity (after the buncher), $E_1 = 1.9766196175688e{+}05$ eV, Energy after first cavity= E_2 , Energy after second cavity $= E_3$, phase crest for the first cavity $= \Phi_2$, phase crest for first cavity $= \Phi_2$, phase crest for the second cavity $= \Phi_3$,

Case	E_{peak} MV/m	E_{acc} MV/m	V_c MV	Φ_2 °	E_2 eV	Φ_3 °	E_3 eV
1	8.51871	4.04	2	33	1.163053238405e+06	-64.2	3.1085230961689e+06
2	12.77806	6.06	3	52	1.8684010049981e+06	-14.3	4.8479851836237e+06
3	17.03742	8.08	4	68	2.3838510784815e+06	9	6.3882215130879e+06
4	21.29678	10.10	5	79.4	2.7066152107197e+06	19.8	7.7178121435109e+06
5	25.55614	12.12	6	92.6	2.8105844050187e+06	29.6	8.8311281754053e+06
6	29.81549	14.14	7	100.8	2.6946156408175e+06	31.9	9.7306937112609e+06

here we have

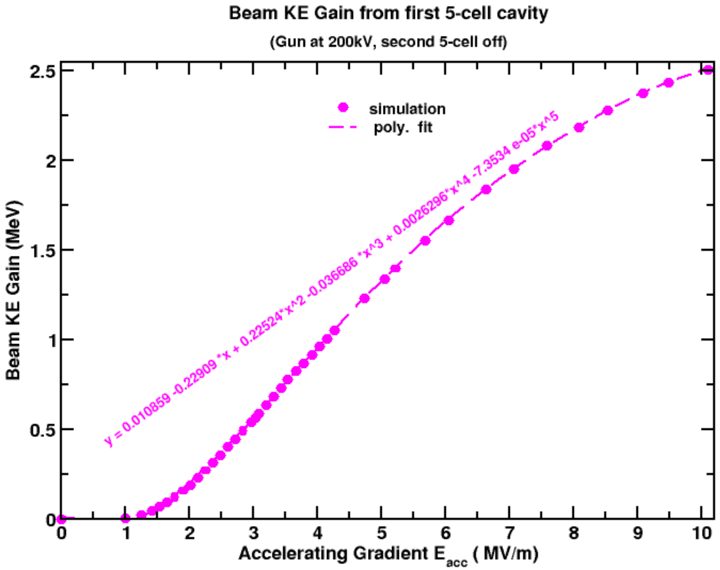
$$\frac{E_{peak}}{E_{acc}} = 2.1086$$

Gradient Calibration of the first 5-cell cavity

Energy before first cavity $E_1 = 0.197662$ MeV, Energy after first cavity$=$ E_2 , Energy phase crest for the first cavity $= \Phi_2$,

S.N.	E_{acc} MV/m	Φ_2 °	E_2 MeV	$\approx$ Gain (MeV)
1	0.0000	—	0.197662	0.000
2	1.000	-4.5	0.2037	0.0061
3	1.250	-3.6	0.2245	0.0269
4	1.422750	-2.8	0.248020	0.0504
5	1.5413125	-0.7	0.269172	0.0715
6	1.7784375	3.5	0.323180	0.1255
7	1.897000	5.1	0.356617	0.1590
8	2.0155625	6.8	0.391260	0.1936
9	2.134125	7.5	0.429466	0.2318
10	2.371250	11.1	0.512198	0.3145
11	2.608375	13.8	0.601002	0.4033
12	2.7269375	16.6	0.646780	0.4491
13	2.9640625	19.6	0.740051	0.5423
14	3.030000	19.9	0.766140	0.5685
15	3.31975	25.9	0.881655	0.6840
16	3.4383125	27.1	0.928842	0.7312
17	3.7940000	29.0	1.069602	0.8720
18	3.9125625	31.2	1.116504	0.9188
19	4.040000	33.0	1.163053	0.9654
20	4.1496875	34.7	1.207486	1.0098
21	4.268250	35.1	1.253500	1.0558
22	4.6239375	40.1	1.384393	1.1867

Gradient Calibration of the first 5-cell cavity

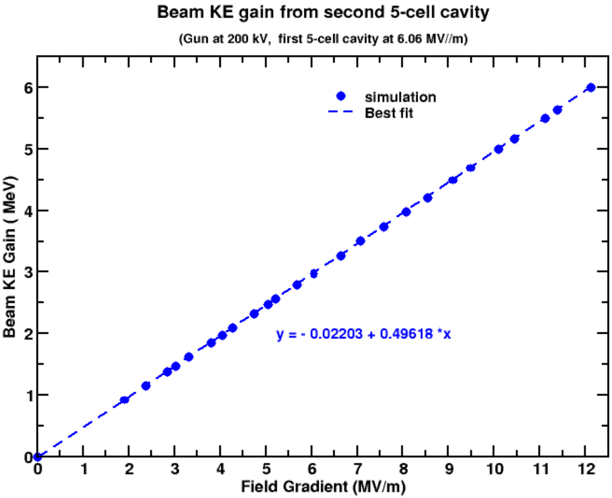


Gradient Calibration of the Second 5-cell cavity

Energy before second cavity $E_2 = 1.868401$ MeV (Gset=6.06 MV/m),
Energy after second cavity= E_3 , phase crest for the first cavity = Φ_3,

S.N.	E_{acc} MV/m	Φ_3 °	E_3 MeV	≈ Gain (MeV)
1	1.897	-17.8	2.789877	0.9215
2	2.37125	-16.6	3.022931	1.1545
3	2.8455	-16.4	3.256478	1.3881
4	3.03	-15.7	3.347549	1.4791
5	3.31975	-15.9	3.490251	1.6218
6	3.794	-15.5	3.724511	1.8561
7	4.04	-15.4	3.846202	1.9778
8	4.26825	-15.3	3.959131	2.0907
9	4.7425	-15.2	4.193976	2.3256
10	5.05	-15.1	4.346458	2.4780
11	5.21675	-15.1	4.429178	2.5608
12	5.691	-14.8	4.664541	2.7961
13	6.06	-14.3	4.847985	2.9796
14	6.6395	-14.3	5.135971	3.2676
15	7.07	-14.4	5.350122	3.5117
16	7.588	-14.3	5.607390	3.7390
17	8.08	-13.7	5.852428	3.9840
18	8.5365	-14.0	6.079754	4.2114
19	9.09	-13.8	6.355287	4.4869
20	9.485	-13.4	6.552086	4.6837
21	10.10	-13.4	6.858266	4.9900
22	10.4335	-13.1	7.024664	5.1563
23	11.11	-13.2	7.361873	5.4935
24	11.382	-13.3	7.496916	5.6285
25	12.12	-13.1	7.865505	5.9971

Gradient Calibration of the Second 5-cell cavity

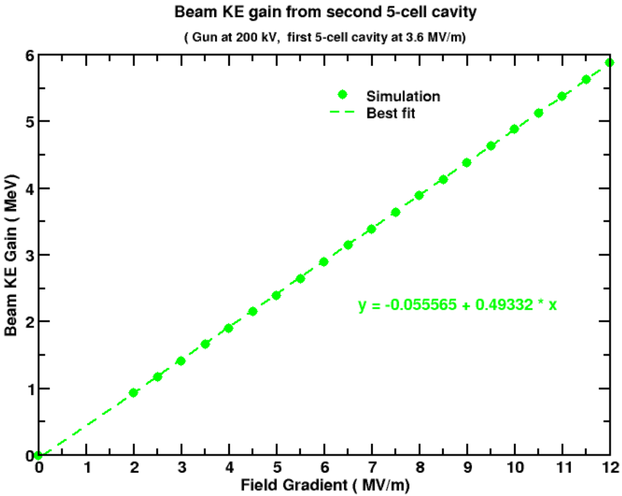


Gradient Calibration of the Second 5-cell cavity

Energy before the second first cavity E_2 =0.993230 MeV (Gset=3.6 MV/m), Energy after second cavity= E_3 , GPT phase crest for the first cavity $= \Phi_3$,

S.N.	E_{acc} MV/m	Φ_3 °	E_3 MeV	≈ Gain (MeV)
1	2.0	-89.5	1.923780	0.9306
2	2.5	-88.1	2.165810	1.1726
3	3.0	- 87.0	2.409544	1.4163
4	3.5	-85.8	2.654252	1.6610
5	4.0	-85.1	2.899852	1.9066
6	4.5	-83.8	3.146296	2.1531
7	5.0	-83.4	3.393450	2.4002
8	5.5	-83.0	3.640595	2.6474
9	6.0	-82.8	3.888178	2.8950
10	6.5	-82.6	4.136220	3.1430
11	7.0	-81.9	4.384297	3.3911
12	7.5	-81.8	4.632416	3.6392
13	8.0	-80.7	4.880777	3.8876
14	8.5	-80.6	5.129014	4.1358
15	9.0	-80.6	5.377768	4.3845
16	9.5	-80.5	5.626814	4.6336
17	10.0	-80.3	5.875495	4.8822
18	10.5	-80.0	6.123960	5.1307
19	11.0	-79.8	6.372304	5.3790
20	11.5	-79.6	6.621875	5.6286
21	12.0	-79.5	6.870773	5.8780

Gradient Calibration of the second 5-cell cavity

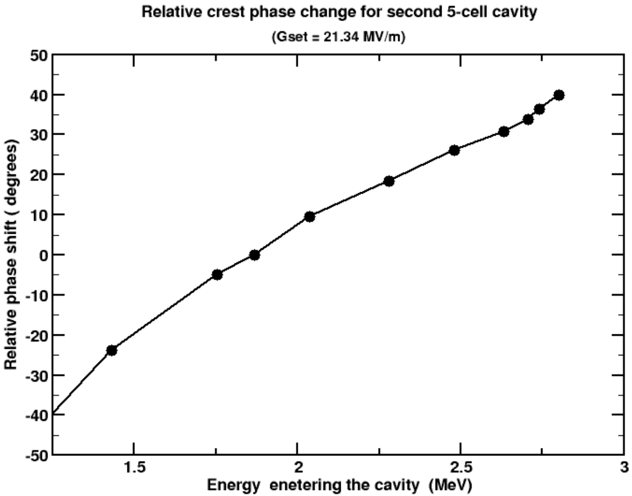


Relative phase change with energy entering to the second 5-cell cavity, Gset = 10.10 MV/m

Energy before first cavity $E_1 = 0.197662$ MeV, Energy phase crest for the second cavity $= \Phi_3$,

S.N.	E_2 MeV	GPT phase Φ_3	E_3 MeV	$\approx$ Gain (MeV)	relative phase shift $\delta\phi$
1	0.356617	-273.0	5.024127	4.6685	-259.4
2	0.512198	-187.6	5.335405	4.8232	-174.0
3	0.693210	-132.6	5.582306	4.8890	-119.0
4	0.766140	-116.8	5.671907	4.9058	-103.2
5	0.881655	-94.1	5.808283	4.9266	-80.5
6	1.069602	-71.9	6.020152	4.9506	-58.3
7	1.165664	-60.8	6.125682	4.9600	-47.2
8	1.429708	-37.5	6.409343	4.9796	-23.9
9	1.753846	-18.5	6.750150	4.9963	-4.9
10	1.868401	-13.6	6.868402	5.0000	0.0
11	2.037898	-4.1	7.043802	5.0060	9.5
12	2.280690	4.9	7.293050	5.0124	18.5
13	2.480022	12.5	7.497852	5.0178	26.1
14	2.633396	17.2	7.653972	5.0206	30.8
15	2.706615	20.2	7.727941	5.0213	33.8
16	2.739804	22.9	7.761832	5.0220	36.5
17	2.800836	26.4	7.823840	5.0230	40
18	2.810503	29.7	7.833798	5.0232	43.3

Relative phase change with energy entering to the second 5-cell cavity, Gset = 10.10 MV/m



Energy gain with Phase Off-Crest for the first 5-cell Cavities for 10.10 MV/m

Energy before first cavity $E_1 = 0.197662$ MeV

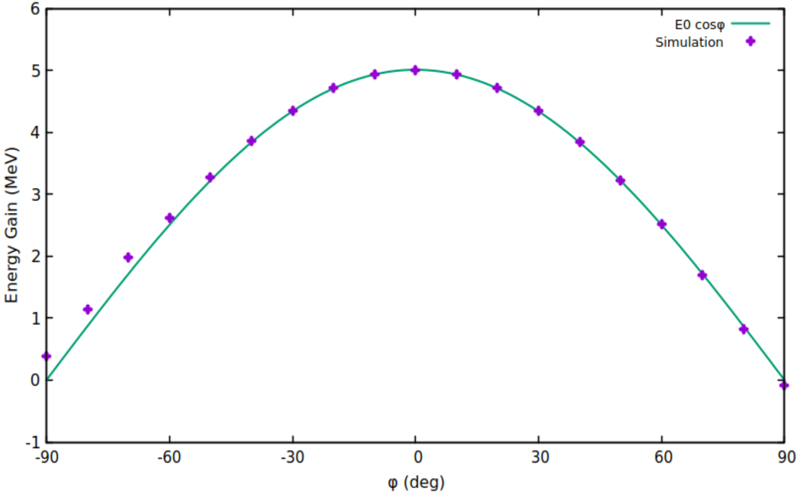
Phase Off-crest (degrees)	E_2 MeV	$\approx$ Gain MeV
-30	1.805643	1.61
-20	2.346064	2.15
-10	2.614502	2.42
0	2.706615	2.51
10	2.657247	2.46
20	2.491512	2.29
30	2.213993	2.01
40	1.822634	1.62
50	1.313846	1.12

Energy gain with Phase Off-Crest for the second 5-cell Cavities for 10.10 MV/m

Energy before the second (after the first), $E_2 = 2.706615$ MeV

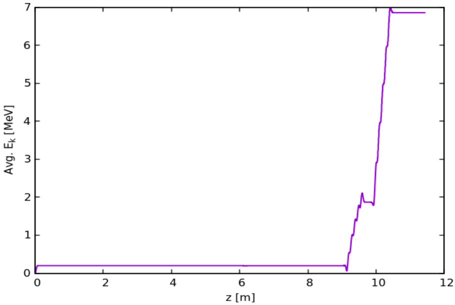
Phase Off-crest (degrees)	E_3 MeV	$\approx$ Gain MeV
-90	3.105390	0.39
-80	3.849218	1.14
-70	4.601437	1.98
-60	5.325640	2.62
-50	5.993095	3.28
-40	6.578034	3.87
-30	7.058910	4.35
-20	7.417340	4.71
-10	7.639765	4.93
0	7.717812	5.01
10	7.646217	5.01
20	7.426170	4.72
30	7.061253	4.35
40	6.561058	3.85
50	5.939284	3.23
60	5.213374	2.51
70	4.401732	1.70
80	3.527260	0.82
90	2.615386	-0.09

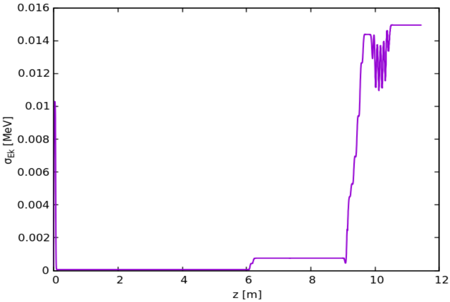
Energy gain with Phase Off-Crest for the second cavity for 10.10 MV/m



Energy Gain, first 5-cell 6.06 MV/m, second 5-cell 10.10 MV/m

As the particle transits the cavity, the variation in the fields lead to a ripple in the energy gain as the function of positions.

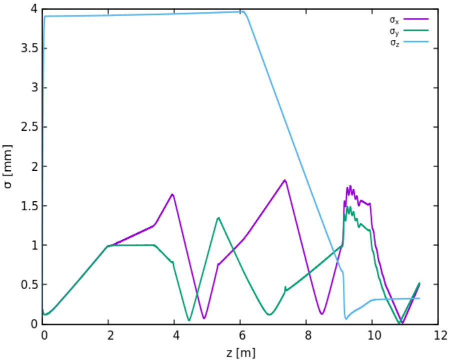




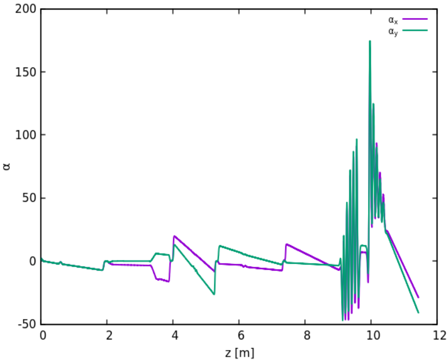
The average beam KE is 6.858 MeV and the energy spread (stde) of the beam passing through the second cavity is about 0.014 MeV (0.22 %).

Beam Envelope

Standard Deviation

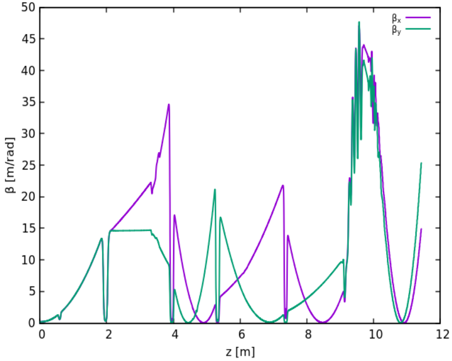


Courant-Snyder Alpha

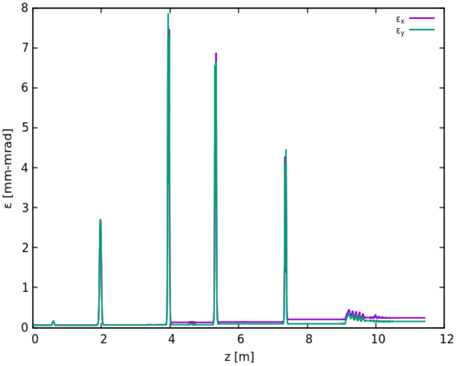


Beam Envelope

Courant-Snyder Beta

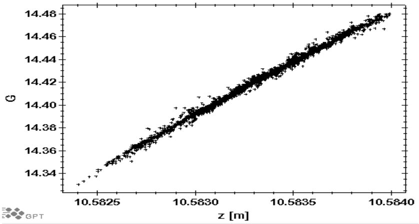


normalized emittance, $\epsilon_x = 0.2429$, $\epsilon_y = 0.1544$

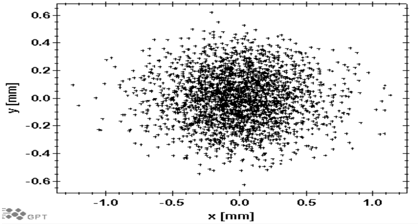


Final distribution

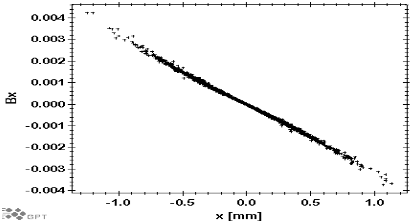
Energy distribution, Mean KE=6.858 MeV



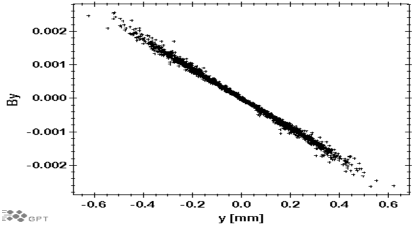
spatial distribution



horizontal phase space



vertical phase space



Summary

1. Through GPT modelling, the Grigory's cryo-unit is calibrated and optimized in UITF
2. For 200 keV beam, the beam goes cleanly through the unit.
3. The average electron beam kinetic energy is 6.858 MeV with sigma energy spread around 14 keV.
4. The beam is not fully relativistic for the first 5-cell cavity, there is no full energy gain out from the cavity. The energy gain is not linear with field inside the cavity
5. The beam is relativistic for the second 5-cell cavity, there is approximately full energy gain out from the cavity. The energy gain is linear with field inside the cavity

6 GPT modelling can be done based on the desired beam requirements after the Cryounit

Thank You !

Backup 1

If N identical cells have a phase advance of π between them (at fixxed time), then the accelerating voltage is

$$V_{acc} = NE_0L.TS$$

where synchronism factor is

$$S(\beta) = \begin{cases} \frac{1}{N}\left[1 + \sum_{m=1}^{(N-1)/2}(-1)^m \; 2\cos(m\pi\beta_G/\beta)\right] \text{; for odd N} \\ \\ \frac{2}{N}\left[1 + \sum_{m=0}^{N/2-1}(-1)^m \; 2\sin\left((m+\frac{1}{2})\pi\beta_G/\beta\right)\right] \text{; for even N} \end{cases}$$