various possible nonmesonic effects in the calculations of the matrix elements and have concluded that they could probably not account for a deviation of this magnitude. Recent theoretical calculations by Fujita *et al.*,[8] indicate that meson exchange effects between nucleons could reduce the strength of the G-T interaction in the beta decay of complex nuclei relative to that for a free nucleon by an amount comparable to the above deviation and may be responsible for a considerable part of this discrepancy.

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Analysis of Photonuclear Cross Sections*

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The problem of obtaining a photonuclear cross section from a yield curve measured with a bremsstrahlung beam is discussed. This paper constitutes part of a larger report on the same subject containing numbers enabling cross-section analysis in the energy range 2 Mev to 1 Bev.

INTRODUCTION

CROSS sections of photon-induced nuclear processes are usually studied with the aid of a bremsstrahlung beam. Such radiation contains photons of all energies from zero to the kinetic energy of the initiating electrons, and so the desired cross section can seldom be measured directly. Instead, it must be deduced from an integral (bremsstrahlung) yield curve. The present report deals with the process of making this analysis.

Let a sample and a suitable monitor be simultaneously irradiated by a bremsstrahlung beam of maximum energy χ. If $N(\chi,k)$ is the number of photons of energy k (per unit range of k) which enter the sample per unit of monitor response, $\sigma(k)$ is the desired photo cross section in cm^2 per nucleus, and η_s is the number of nuclei of the appropriate type per cm^2 of sample, then, the number of reactions which occur per unit of monitor response, $\alpha(\chi)$, is given by the following integral:

$$\alpha(\chi)=\eta_s\int_0^\infty N(\chi,k)\sigma(k)dk. \qquad (1)$$

If the measurements are repeated for a series of values of χ then a series of points on the bremsstrahlung yield curve, $\alpha(\chi)$, are obtained. Each point is a measurement of the relative response of the monitor and the sample to the photon beam. Hence, if the response of the monitor is known, the cross section for the reaction can be deduced.

Equation (1) holds only if the sample is uniformly thick over the lateral extent of the beam. Otherwise, η_s represents the average thickness of the sample and due to the angular dependence of the radiation becomes a function of χ. The upper limit to the integral in Eq. (1) follows since by definition $N(\chi,k)\equiv 0$ for $k>\chi$.

Sometimes an energy dependent experimental bias will make it impossible to obtain the number of reactions per unit of monitor response directly from the measurements. Then, Eq. (1) must be modified by replacing $\sigma(k)$ by $G(k)\sigma(k)$, [where $G(k)$ represents the experimental bias] and by suitably redefining $\alpha(\chi)$. The solution of Eq. (1) would then yield a value for $G(k)\sigma(k)$.

Various methods for obtaining a practical solution of Eq. (1) have been proposed in the literature.[1–6] The method to be discussed here is not essentially different from some which have been proposed,[1–4] but has the advantage of providing a clear insight into the problem and of setting forth the relation between the solutions which are obtained and the actual cross section. In addition, the present procedure minimizes computational labor and eliminates the propagation of computational errors from one value of $\sigma(k)$ to the next. This

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latter advantage follows because the method is not an iterative one.

Tables of numbers have been prepared which can be used to obtain solutions of Eq. (1) in the energy range 2 Mev to 1 Bev. These tables are too lengthy for inclusion in this report, but copies may be obtained from the Physics Research Laboratory, University of Illinois or from the authors. The tables were calculated with the aid of the digital computer Illiac, which is at the University of Illinois.

A preliminary report of this work has been presented previously.[7]

THE RADIATION SPECTRUM

The radiation spectrum, $N(\chi,k)$, which appears in Eq. (1) is determined by the bremsstrahlung cross section so far as its gross features are concerned, but there are many modifying factors. Here we take the spectrum to be the product of three factors: first, a function which has the same shape as the bremsstrahlung cross section; second, a photon transmission function which describes the distortion caused by photon absorption in the radiator, sample, and other material in the beam; and third, a function which normalizes the spectrum to unit monitor response.

$$N(\chi,k)=[\Phi(\chi,k)/k]f_s(k)/F(\chi). \tag{2}$$

The bracketed term in Eq. (2) is the function which is proportional to the bremsstrahlung cross section, and when written in the form shown makes the dominant $1/k$ dependence of the bremsstrahlung explicit. $\Phi(\chi,k)$ is thus a function proportional to the intensity spectrum. The monitor response function, $F(\chi)$, is the function which normalizes the spectrum to unit monitor response, and $f_s(k)$ is the photon transmission function referred to above.

Actually, $\Phi(\chi,k)$ should be replaced by a more complicated function bearing a rather tenuous relation to the bremsstrahlung cross section. This follows from the fact that the shape of the bremsstrahlung cross section is somewhat angle dependent.[8] Hence, the spectrum incident on the sample is influenced by the mean angle and solid angle which it subtends to the radiator and also by the thickness of the radiator since multiple scattering of the radiating electrons usually dominates the angular distribution. In addition, energy losses in the radiator will change the shape of the spectrum.

The inclusion of an angular dependence to the shape of the radiation spectrum is a considerable complication and very little work has yet been done on the problem. The spectrum for zero angle has been studied[9,10] but this is a very restrictive case—one that is seldom approached in practice. For present purposes the angle would have to be considerably smaller than mc^2/χ, the intrinsic angle of bremsstrahlung. If the angle is mc^2/χ or larger the spectrum will have a shape close to that of the integrated over angles bremsstrahlung cross section due to the strong influence of multiple scattering.

Here we have taken $\Phi(\chi,k)$ to be proportional to the integrated over angles cross section as given by Schiff.[8] We have therefore also neglected the effects of energy losses suffered by the electrons in the radiator. This is a good approximation for most purposes.

THE MONITOR RESPONSE FUNCTION

In order to calculate the monitor response function, $F(\chi)$, one must write down an expression for the radiation spectrum which is incident on the monitor per unit of monitor response. This spectrum will be designated by $N_m(\chi,k)$. It will differ from $N(\chi,k)$ in two respects: first, $f_s(k)$ must be replaced by a similar function, $f_m(k)$, which is valid for the monitor arrangement; second, a correction must be made if the sample and the monitor do not subtend the identical solid angle to the radiator. The monitor spectrum can be written as follows:

$$N_m(\chi,k)=\frac{\omega_m(\chi)}{\omega_s(\chi)}\left[\frac{\Phi(\chi,k)}{k}\right]\frac{f_m(k)}{F(\chi)}. \tag{3}$$

The ratio $\omega_m(\chi)/\omega_s(\chi)$ in Eq. (3) is a ratio of effective solid angles. It is the ratio of the angular distribution of the radiation integrated over the monitor solid angle to that integrated over the sample solid angle.

We now define $R(\chi)$ to be the number of units of monitor response produced per unit of radiant energy incident on the monitor. Then, it follows from Eq. (3) and from the definition of $N_m(\chi,k)$ that the monitor response function is:

$$F(\chi)=\frac{\omega_m(\chi)}{\omega_s(\chi)}R(\chi)\int_0^\infty f_m(k)\Phi(\chi,k)dk. \tag{4}$$

For the simple case of a calorimeter monitor where the unit of monitor response is taken to be equal to one unit of energy, $R(\chi)$ is independent of χ and has value one. In general, $R(\chi)$ will depend on the shape of the radiation spectrum and hence on the photon transmission function $f_m(k)$. Thus, a monitor calibration made for a given experimental arrangement may require significant change when used in a different arrangement.

THE REDUCED YIELD CURVE

The solution of Eq. (1) can now be considered since the radiation spectrum, $N(\chi,k)$ has been completely defined through Eqs. (2) and (4). However, $N(\chi,k)$ is geometry dependent and so any numbers which are evaluated would not have general application. For this reason Eq. (1) will be transformed to a reduced, or geometry independent, form.

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We define a reduced yield curve, $Y(\chi)$, equal to $F(\chi)\alpha(\chi)$ and a reduced cross section, $\Omega(k)$, equal to $\eta_s f_s(k)\sigma(k)/k$. The reduced cross section has dimensions (energy)$^{-1}$ and represents the number of energy units absorbed by the sample per unit of energy entering the solid angle subtended by the sample. The desired reduced form of Eq. (1) is

$$Y(\chi)=\int_0^\infty \Phi(\chi,k)\Omega(k)dk. \tag{5}$$

The values of $\Omega(k)$ which are obtained by solving Eq. (5) can easily be transformed to values of $\sigma(k)$ through use of the definition of $\Omega(k)$.

METHOD OF CROSS-SECTION ANALYSIS

Equation (5) can be solved, in principle, by forming appropriate combinations of integrals and differentials of $Y(\chi)$. This procedure has been carried out by Spencer[5] who approximated $\Phi(\chi,k)$ by a function of comparatively simple form. However, one measures $Y(\chi)$ only for a limited number of values of χ and the functional form of $Y(\chi)$ is not known. Hence, even in principle, one can only obtain average values for the cross section from an experiment. The method to be presented here accepts this limitation from the beginning but fully sets forth the relationship between the average values for $\Omega(k)$ which are obtained and the true values.

Suppose that $Y(\chi_i)$ is one of a set of values for $Y(\chi)$ which is known from experiment. We assume that the measurements were made at equally spaced values of χ so that $\chi_i-\chi_{i-1}=\Delta$ for all values of i. This last assumption is not necessary but it is convenient. The interval Δ will be referred to as the bin width. We further assume that $Y(\chi_i)=0$ for all values of $i<a$

Now consider the result of taking a linear combination, $C(\chi_m,\Delta)$, of the measured values.

$$C(\chi_m,\Delta)=\sum_{i=a}^{m} B(\chi_m,\Delta,\chi_i)Y(\chi_i). \tag{6}$$

The linear combination is specified by a series of numbers $B(\chi_m,\Delta,\chi_i)$ which we call B-numbers. As the notation indicates, the B-numbers are functions of χ_m, χ_i, and Δ. Note however that χ_m and χ_i must always differ by an integral number of bin widths.

The relationship between $C(\chi_m,\Delta)$ and the reduced cross section is obtained by substituting into Eq. (6) values for the $Y(\chi_i)$ obtained through use of Eq. (5). The result is

$$C(\chi_m,\Delta)=\int_0^\infty T(\chi_m,\Delta,k)\Omega(k)dk, \tag{7}$$

where

$$T(\chi_m,\Delta,k)=\sum_{i=a}^{m} B(\chi_m,\Delta,\chi_i)\Phi(\chi_i,k). \tag{7a}$$

The new function $T(\chi_m,\Delta,k)$ will be called a weighting function and its determination is crucial to the development of solutions for $\Omega(k)$ and to the investigation of the validity of these solutions. The weighting function is composed of a linear combination of bremsstrahlung spectra and so it automatically satisfies the following conditions: first, it is identically zero for values of k greater than χ_m; second, over the energy range $\chi_m>k>\chi_m-\Delta$ it has the same shape as the high-energy end of a bremsstrahlung spectrum; third, it is automatically partitioned into bins of width Δ.

The basic problem is to choose the B-numbers in such a way that the weighting function has a desirable form. That is, so that $C(\chi_m,\Delta)$ is simply related to the cross section.

The B-numbers were chosen in such a way that the weighting function has area Δ, and is essentially different from zero only when k is within a bin or two of χ_m. In that case, it is meaningful to make use of the centroid energy of $T(\chi_m,\Delta,k)$ which we denote by k_m^Δ. Then Eq. (7) can be approximated by the following:

$$C(\chi_m,\Delta)=\Delta\Omega(k_m^\Delta). \tag{8}$$

One can expect Eq. (8) to be a good approximation whenever the curvature of $\Omega(k)$ is small for values of k near χ_m. It should be noted that Eq. (8) is the first and only approximation that the method of solution requires.

The method of solution is now complete since Eq. (8) combined with Eq. (6) yields a value for the cross section in terms of the measurements, and Eq. (8) combined with Eq. (7) gives the relation between the solution and the actual cross section.

There is a set of B-numbers for each energy at which a cross section is desired. Sets of these numbers, and their associated weighting functions, for the energy range 2 Mev to 1 Bev have been calculated and are available on request. They were computed from Eq. (7a) coupled with the following restrictive conditions:

$$T(\chi_m,\Delta,k)=1 \quad \text{for} \quad k=\chi_m-\Delta/2, \tag{9a}$$

$$T(\chi_m,\Delta,k)=0 \quad \text{for} \quad k=\chi_i-\Delta/2 \quad (i\neq m, b, c\cdots), \tag{9b}$$

$$B(\chi_m,\Delta,\chi_i)=0 \quad \text{for} \quad i=b, c,\cdots. \tag{9c}$$

The exact choice χ_b, $\chi_c,\cdots$ varied but they were chosen to give an effective change in the bin size for values of k not too close to χ_m. Through the use of Eq. (9c) a lot of very small B-numbers are eliminated from the tables while the solutions for $\Omega(k)$ are not significantly affected.

Once a set of B-numbers has been computed they may be used in Eq. (7a) to determine the associated weighting function. A typical example of such a weighting function is shown in Fig. 1. It can be seen that the long "tail" of the function remains very small down to the lowest energies. The area contained under the

function in the range 5 to 25 Mev is only 0.1% of the area contained between 49 and 50 Mev. The bin sizes are 3 Mev up to 39 Mev and 1 Mev from 39 to 50 Mev.

To determine the accuracy of the method in reproducing any specified cross section one can calculate values of $\Omega(k_m^{\Delta})$ according to Eqs. (7) and (8) and the result can be compared to the assumed cross section. Examples of such tests are shown in Figs. 2 and 3. The solid curves represent the assumed cross section in each case while the points represent the values which would result from a yield curve analysis (if all errors were due to the method). The method is seen to be extremely accurate except very close to threshold, or when the cross section has appreciable curvature over a bin width. In such cases, one can improve the solution replacing Eq. (8) by some more appropriate approximation.

In the approximation of this method, a value for the integrated cross section is obtained by adding up

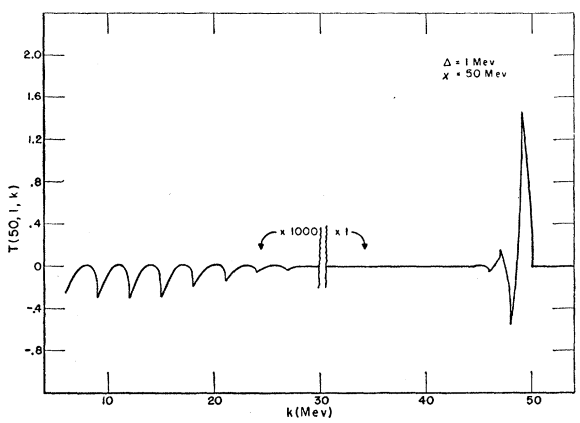

FIG. 1. Plot of a weighting function, $T(\chi_m,\Delta,k)$ $\chi_m=50$ Mev and $\Delta=1$ Mev.

the values obtained for the cross section (computed every Δ Mev) and multiplying by the bin width Δ. The accuracy of this procedure can be investigated by forming the corresponding weighting function and it is found to be very good.

STATISTICAL ERRORS

In practice there will always be some statistical uncertainty associated with the measured values of the reduced yield. The resulting statistical uncertainty on a calculated cross-section value can easily be obtained from Eqs. (6) and (8) coupled with the usual rules of statistics. The exact expressions will not be given here. The statistical problem is not too serious as long as the cross section increases monotonically with energy, but it can become very serious as soon as the cross section begins to decrease. This can be traced to the fact that the cross-section solutions depend strongly on first, second, and third differences of the yield curve.

Two rather simple approximate expressions can be

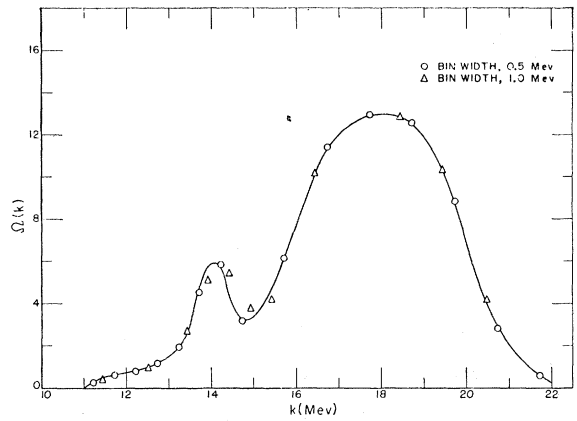

FIG. 2. A test of the method of analysis. The solid curve represents a hypothetical cross section. The points represent solutions which would be obtained from a yield curve analysis if all errors were due to the method of analysis.

given for the statistical uncertainties to be expected. If x_i represents the fractional standard deviation in the yield measurement $Y(\chi_i)$, then the uncertainty in $\Omega(k)$ and $\int_0^{\chi_m}\Omega(k)dk$ are given by

$$\epsilon[\Omega(k_m^{\Delta})]\approx 2(x_m/\Delta)B(\chi_m,\Delta,\chi_m)Y(\chi_{m-1}), \tag{10}$$

$$\epsilon\left[\int_0^{\chi_m}\Omega(k)dk\right]\approx x_m B(\chi_m,\Delta,\chi_m)Y(\chi_m). \tag{10a}$$

Equation (10) gives the uncertainty in the values for the reduced cross section, and Eq. (10a) gives the same for its integral. These expressions show that the statistical uncertainty rises linearly with the yield curve for both cases. In addition, the uncertainty on the cross section increases as the bin width is decreased. This corresponds to the fact that the resolution of the method is proportional to the bin width. The bin size chosen for an analysis should be small enough to resolve the relevant structure of the cross section, but

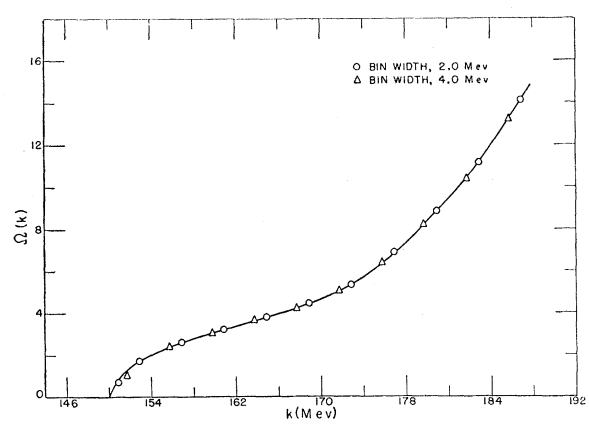

FIG. 3. A test of the method of analysis. The solid curve represents a hypothetical cross section. The points represent solutions which would be obtained from a yield curve analysis if all errors were due to the method of analysis.

it should be kept as large as possible in order to get the greatest statistical accuracy.

Equations (10) and (10a) can be combined to yield an expression for the fractional error on a cross-section value in terms of the fractional error on its integral:

$$\frac{\epsilon[\Omega(k_m{}^\Delta)]}{\Omega(k_m{}^\Delta)}\approx 2\left[\int^{\chi_m-\Delta}\Omega(k)dk\Big/[\Omega(k_m{}^\Delta)\Delta]\right]\times\left[\epsilon\int_0^{\chi_m-\Delta}\Omega(k)dk\Big/\int_0^{\chi_m-\Delta}\Omega(k)dk\right]. \quad (11)$$

Equation (11) demonstrates the fact that the fractional error on a cross section can be very much larger than the corresponding error on the integral.

DATA SMOOTHING

In many experiments it is extremely difficult to achieve sufficient statistical accuracy in the cross-section solutions. This forces one to consider the possibility of smoothing. Any smoothing procedure will introduce errors of a systematic nature into the data being smoothed. Small systematic errors in the yield values can lead to large errors in the cross-section solutions so it is not advisable to smooth the yield curve data. Rather, the analysis should be done first.

APPLICATION TO OTHER PROBLEMS

The general form of the problem considered here is a very common one in experimental physics. Typical examples are the equation connecting the pulse-height distribution from a radiation detector to the incident spectrum of radiation, or the equation connecting the energy spectrum of particles from a thick target to the spectrum which would come from a thin target. Mathematically, these problems are the same as the present one, but the particular form of weighting function chosen here may not be suitable. Once a suitable system for specifying the weighting function has been worked out, however, B-numbers can be calculated and the remainder of the problem is the same as that considered here.

MODIFICATION OF THE RADIATION SPECTRUM

In Eq. (1) a relation between the cross section, the radiation spectrum, and the yield curve was given. The subsequent development of the analysis method was based on the assumption that the radiation spectrum was known. We now wish to determine how the analysis will be affected if the spectrum is changed from $N(\chi,k)$ to $N'(\chi,k)$. In place of Eq. (1) we write

$$\alpha'(\chi)=\eta_s\int_0^\infty N'(\chi,k)\sigma(k)dk. \quad (12)$$

The only kind of spectrum modification which interests us here is one which involves the intrinsic shape of the spectrum and so we get an expression for $N'(\chi,k)$ through the use of a function $\Phi'(\chi,k)$ instead of $\Phi(\chi,k)$. Then, by analogy with Eq. (2), we write

$$N'(\chi,k)=\left[\frac{\Phi'(\chi,k)}{k}\right]\frac{f_s(k)}{F'(\chi)}. \quad (13)$$

Note that Eq. (13) contains a new monitor response function, $F'(\chi)$, which by definition normalizes the new spectrum to unit monitor response.

The function $\Phi'(\chi,k)$ can be constructed from $\Phi(\chi,k)$ in the following way:

$$\Phi'(\chi,k)=\int_k^\chi S(\chi,y)\Phi(y,k)dy. \quad (14)$$

This just expresses the fact that $\Phi'(\chi,k)$ can be constructed by making a linear combination of various $\Phi(\chi,k)$. $S(\chi,y)$ can be called a spectrum generating function. In general it will be a smooth function but it may contain one or more delta functions as well.

It can be shown that the new monitor response function is related to the old one [Eq. (4)] by the same spectrum generating function as follows:

$$F'(\chi)=\int_0^\chi S(\chi,y)F(y)dy. \quad (15)$$

We now transform Eq. (12) to the reduced form in analogy with Eq. (5), but we do the reduction as if we were unaware of the spectrum change. That is, we multiply Eq. (12) by $F(\chi)$ to get a reduced yield curve $Y'(\chi)$. The result is:

$$Y'(\chi)=\left[\frac{F(\chi)}{\int_0^\chi S(\chi,y)F(y)dy}\right]\int_0^\chi S(\chi,y)Y(y)dy. \quad (16)$$

This equation gives a general expression for the effect of a change in the radiation spectrum. The function $S(\chi,y)$ provides a more powerful way of assessing the effects of a spectrum change than an examination of the spectra themselves. The integrals in Eq. (16) are of the same form as the integral in Eq. (1) and so Eq. (16) can be solved for $Y(y)$ by the method which has been discussed. Here, however, our purpose is to draw some rather qualitative conclusions from a study of Eq. (16).

For spectrum changes which result from energy straggling in the radiator, energy spread in the electrons striking the radiator, or other deviations from $\Phi(\chi,k)$ which are mostly confined to the high-energy end of the spectrum, the primary features of $S(\chi,y)$ are the following: (1) $S(\chi,y)$ is large only near $y=\chi$. (2) $S(\chi,y)$ is nearly a function of $\chi-y$ only. [The normalization of S is unimportant since it cancels out in Eq. (16).]

If the foregoing conditions hold, it is meaningful to

make use of the centroid energy of $S(\chi,y)$ which we denote by $(\chi-\delta)$ where δ is a constant small compared to χ. Then, Eq. (16) can be rewritten in the following appropriate form:

$$Y'(\chi)=[F(\chi)/F(\chi-\delta]Y(\chi-\delta). \qquad (17)$$

It is immediately seen from this equation that two major effects are produced by a spectrum change. These are: an energy shift δ; and a renormalization which comes from the monitor. As an example, consider the spectrum change produced by energy losses in a 10-mil platinum radiator to which the sample subtends an angle of 10 degrees. It can be shown[11] that for 16-Mev electrons, and a monitor response function which is essentially linear, that the energy shift, δ, is only about 150 kev and the renormalization factor is about 1%. It should be noted that near the threshold of a reaction the approximate expression given by Eq. (17) is not valid. Also, if the cross section has a width comparable to δ then Eq. (17) is not valid and, in fact, the effect of the spectrum change can be very great.[12]

As another example of a spectrum generating function consider Φ' and Φ to differ only at their low-energy extreme. In that case $S(\chi,y)$ consists of a delta function situated at $y=\chi$ plus a smooth function, say $W(\chi,y)$, which is only large near $y=0$. If $W(\chi,y)$ is negligible at all energies above the threshold of the cross section then the only difference between Y' and Y lies in the monitor renormalization.

Both the effect described above, and the energy shift described by Eq. (17) are illustrated in the example of Fig. 4. From the hypothetical cross section given by the solid curve a yield curve was constructed using for Φ' the zero degree bremsstrahlung cross section as given by Schiff[8] (but with a screening constant of 191 instead of the recommended value of 111). This spectrum has been much used for cross-section analysis in the past.[1–3]

[11] A. S. Penfold (unpublished calculations, 1955).
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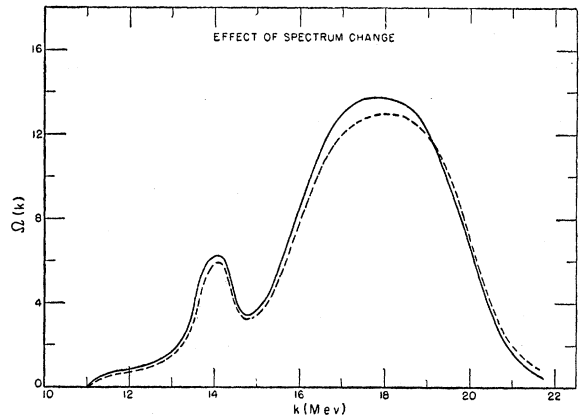

FIG. 4. A test of the effect of a spectrum change. The solid curve represents a hypothetical cross section. The dashed curve shows the result of constructing a yield curve with the spectrum Φ' (see text) and analyzing this curve with numbers based on the spectrum Φ (see text).

The dashed curve shows the solution of this yield curve obtained with B-numbers calculated for the integrated over angles bremsstrahlung cross section with a screening constant of 111.[8] The monitor assumed for this example was a thimble ion chamber imbedded in a Lucite atmosphere.[2]

Several other more simple types of spectrum changes were tried with results which bear out the qualitative statements which have been made. In particular, the chief result of a spectrum change which is primarily confined to the high-energy limit of the spectrum is a small energy shift in the resulting cross section.

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