Cycle Design
&
Carnot Analysis

By

VenkataRao Ganni

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Tentative Schedule

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Applications of Cryogenics

Particle Accelerators use magnets and RF cavities

At room temperature the iron core saturates at about 2 T, where as the magnets built with super conductors can be designed for large magnetic fields like 10 T and more and are compact

Similarly the room temperature RF cavities are built for less than 500 MHz. Higher frequency designs typically require low temperature environment for efficient operation

For a given energy, the accelerators designed with superconductors require:

- **Lower capital cost**
  - **Since it requires fewer number of magnets and/or RF cavities**
  - **Less length of the accelerator**
- **Lower operating cost**

There fore for large accelerators, superconducting structures at cryogenic temperatures are a proven and cost effective

All large particle accelerators need Cryogenics

Applications of Cryogenics

Particle Accelerators use magnets and RF cavities

At room temperature the iron core saturates at about 2 T, where as the magnets built with super conductors can be designed for large magnetic fields like 10 T and more and are compact

High frequency (~100 MHz to 3000 MHz) RF cavity designs *typically* use low temperature environment for efficient and high quality beam operation although there are exception like room temperature RF used from AM radio, under 1 MHz, to 11.4 GHz (From Jay Benesch)

For a given energy, the accelerators designed with superconductors require:

- **Lower capital cost**
  - **Since it requires fewer number of magnets and/or RF cavities**
  - **Less length of the accelerator**
- **Lower operating cost**

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All large particle accelerators need Cryogenics

Introduction

- **Helium refrigeration and liquefaction systems are an extension of the traditional household refrigeration systems**
- **Let's begin with the question,**

  "What is a refrigeration system?"

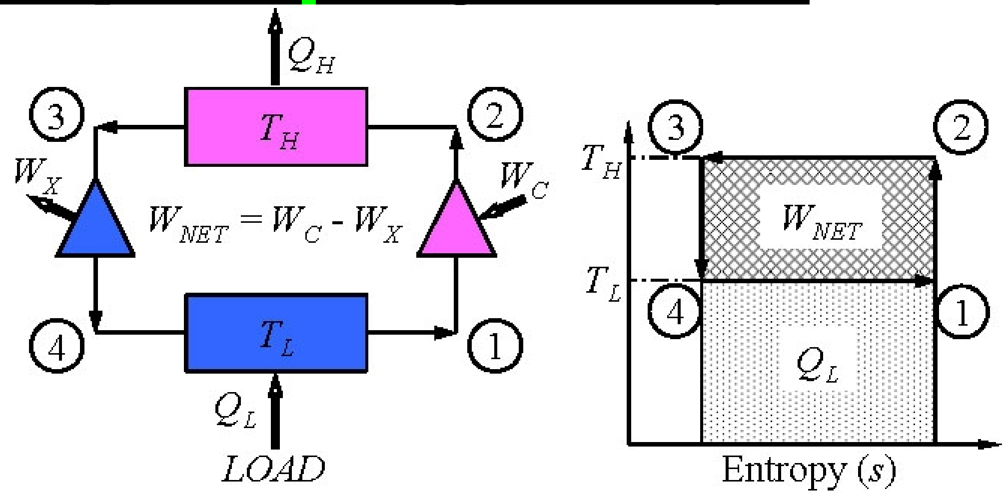
- **A refrigeration system *<u>transfers</u>* heat energy from *<u>low temperature to high temperature</u>*.**

- **Normally, the term refrigeration is used for absorbing heat energy at a constant temperature, but this does not have to be the case**

- **Let's look at an ideal vapor compression cycle, operating between two constant temperature reservoirs…**

Introduction (cont.)

Carnot Vapor Compression Refrigeration Cycle



#1 to #2: Compressor	• **Fluid is compressed isentropically (requiring W_C)**
#2 to #3: Condenser	• **Heat, Q_H, is rejected isothermally (at T_H)**
#3 to #4: Expander	• **Fluid is expanded isentropically (extracting W_X)**
#4 to #1: Evaporator	• **Heat, Q_L, is absorbed isothermally (at T_L)**

Net input work: $W_{CARNOT} = W_C - W_X = Q_H - Q_L = (T_H - T_L)\cdot\Delta S$

Cooling provided: $Q_L = T_L\cdot\Delta S$

Coefficient of Performance: $COP = Q_L / W_{CARNOT} = (T_H/T_L - 1)^{-1}$

Inverse COP: $COP_{INV} = 1 / COP = T_H/T_L - 1$

Introduction (cont.)

- The coefficient of performance (COP) is the ratio of the cooling provided to the required input power
- Carnot vapor compression cycle, the refrigerator operating between -10 and +50 °C

$$COP = \{(273+50) / (273-10) - 1\}^{-1} = 4.4\ W/W$$

- The Carnot work (W_{CARNOT}) for 4.4 kW of cooling is 1 kW

  Note: This is not a violation of the first law of thermo since a *refrigerator* is *transferring energy* from one temperature to another and *not converting it*

- Thermodynamic efficiency is the ratio of the ideal input power to the actual required input power

Introduction (cont.)

- **The Carnot cycle is an 'ideal cycle' in the sense that it does not have any 'irreversibilities' (i.e., 'lost work')**

  for a given path from state 1 to 2, with no irreversibilities, the heat transfer is

$$Q = \int_1^2 T \cdot dS$$

Note: This is a statement of the 2nd law of thermodynamics

Introduction (cont.)

- The Carnot cycle is an 'ideal cycle' in the sense that it does not have any 'irreversibilities' (i.e., 'lost work').

- *The term 'idealized cycle' will be relegated to a practical system* that one can visualize using *ideal components*

- The Carnot cycle has the maximum *COP* (or the minimum inverse *COP*) for the process of *transferring heat energy between two thermal reservoirs*

- This distinction gives the *'Carnot cycle' the recognized qualification* for 'efficiency' comparisons of other cycles performing the same function.

Introduction (cont.)

- **For general process cycles an *exergy* (or 'reversible work') analysis is performed to determine the minimum required work input or maximum obtainable work output for an *ideal process***

Note: (Mass) specific physical exergy is defined as,

$$\varepsilon = h - T_0 \cdot s$$

where, ***h*** **is the enthalpy [J/g]**

T_0 **is the reference, or 'zero' availability, temperature (say, 300 K environment)**

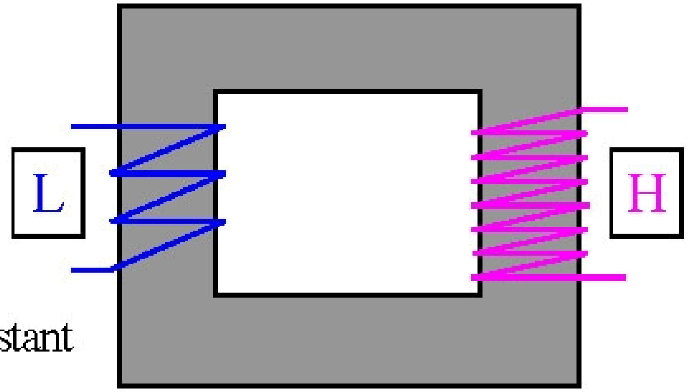
s **is the entropy [J/g-K]**

Introduction (cont.)

Ideal Electrical Transformer

$$\frac{V_H}{V_L} = \frac{I_L}{I_H}$$

$$P = V_H \cdot I_H = V_L \cdot I_L = \text{constant}$$



Ideal 'Thermal' Transformer

$$\frac{T_H}{T_L} = \frac{Q_H}{Q_L}$$

$$S = \frac{Q_H}{T_H} = \frac{Q_L}{T_L} = \text{constant}$$

- **Why is *any* input energy required to *transfer* heat energy from a cold to a hot temperature reservoir?**
- **A thermal transformer that permits the heat energy transfer from cold temperature to hot temperature, with no input work *does not exist*.**
- **This is quite unlike an ideal electrical transformer, which will permit the transfer between voltage and current with no additional input power.**
- **This 'transmission' (or transfer) limitation of heat energy between temperatures implies that there is a *'quality'* for *heat energy*.**
- **The *source* *and* *sink* temperatures sets this limit on the conversion *'quality'* *for the* *heat energy*.**

Introduction (cont.)

- **The minimum work required (or maximum work output), known as the *reversible work*, is *independent of the path* (from state point 1 to 2) *and the working fluid*.**
  - **This is a very important statement!**
- **In other words,**
  - **The selection of the process path (cycle) and the working fluid are based upon the desired working fluid properties (i.e., saturation temperature and pressure, latent heat, density, specific heat, viscosity, thermal conductivity, etc.) for the available practical components**
- **These selections are coupled,**
  - ***But do not determine the reversible work!***

Introduction (cont.)

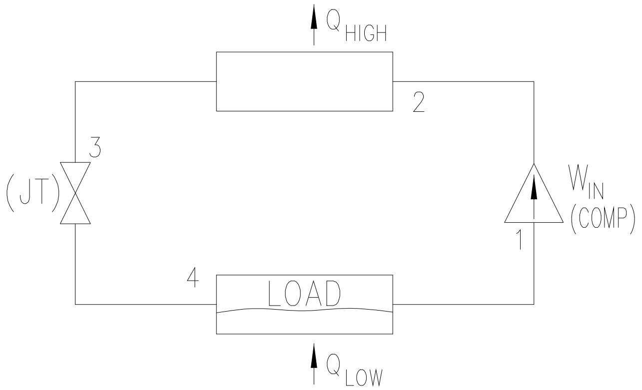
System Performance & Efficiency

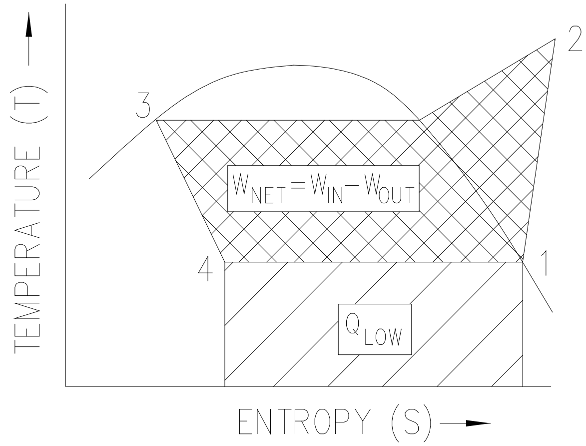
- For ideal systems the conversion from mechanical to electrical energy (or visa-versa) can be 100%.
- Approximately 3kW of thermal energy is required to produce 1kW of mechanical energy.
- This thermodynamic limitation is expressed by the 2nd Law of Thermodynamics and embodies the concept that the thermal energy has a 'quality' (or 'availability')
- For refrigeration, the input energy required is due to the loss in 'availability' (or decrease in 'quality') of the thermal energy as it is transferred from a low temperature (load) to a high temperature (environment).

Introduction (cont.)

Vapor compression process

e.g.: Typical Freon refrigerator



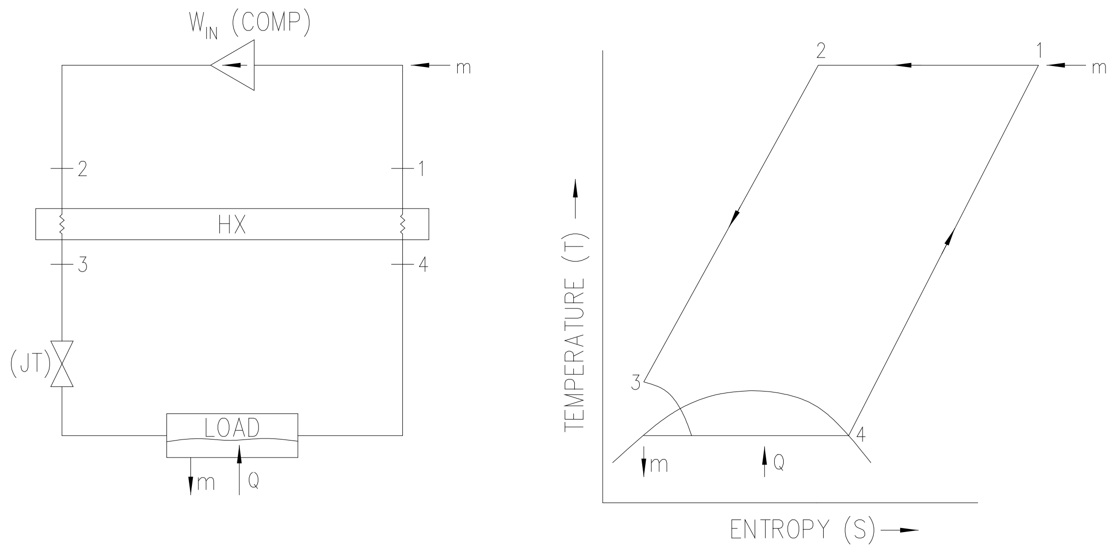


This process typically requires 1 kW of input power for ~3 kW of cooling load, so the efficiency as compared to the Carnot cycle, otherwise known as the exergetic efficiency is,

$$\text{exergetic efficiency} = \frac{W_{Carnot}}{W_{actual}} = \frac{Q_{LOW} \cdot COP_{INV}}{W_{actual}} = \frac{3 \cdot (0.23)}{1} = 0.68$$

Introduction (cont.)

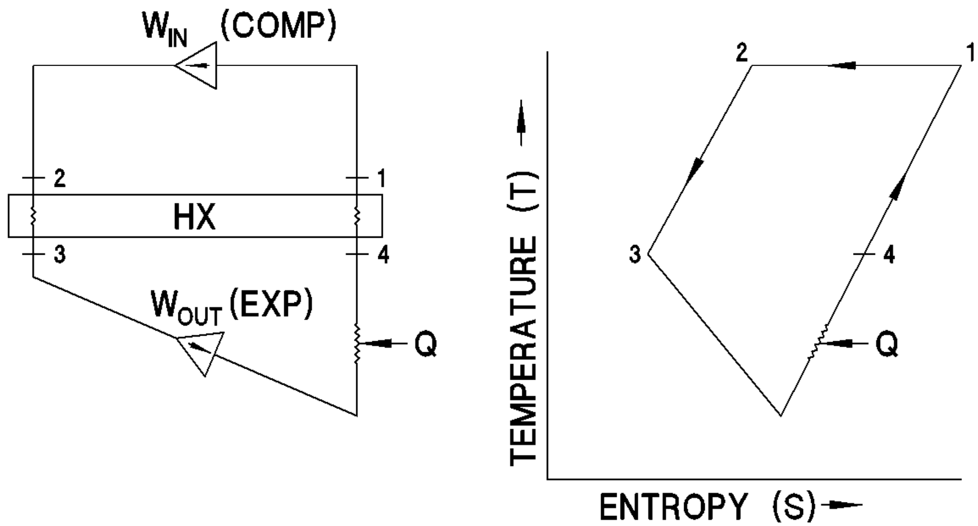
- **Hampson process**



- Uses a heat exchanger (HX) between the compressor and the load for heat energy exchange between the supply and return streams.
- Process supports lower temperature load operations more efficiently than the vapor compression process.

Introduction (cont.)

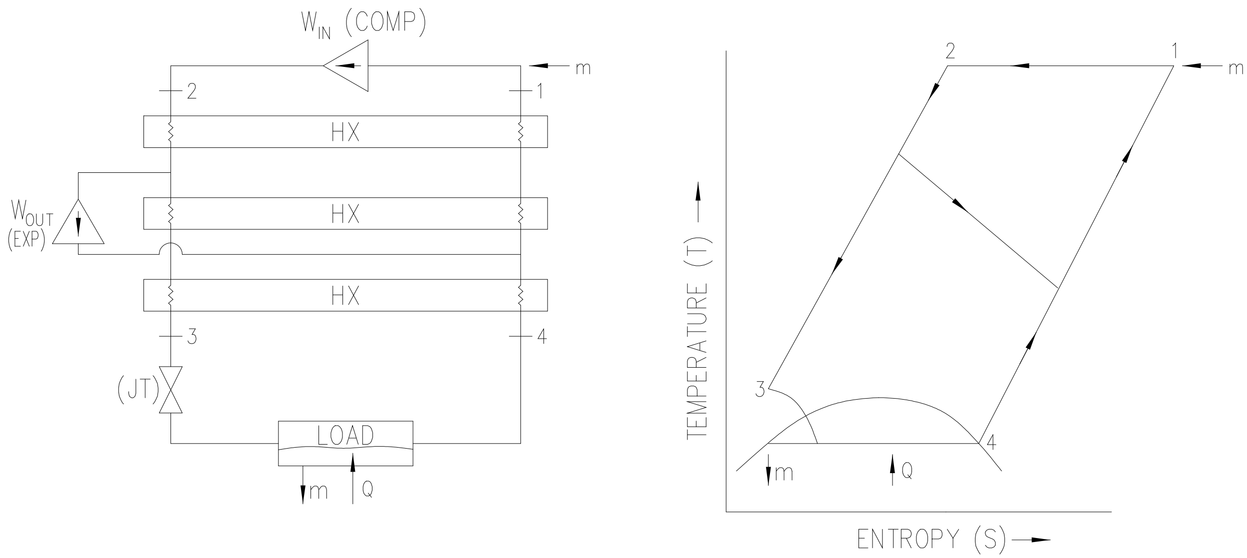
- **Modified Brayton process**



- Uses a heat exchanger (HX) and an expander between the compressor and the load
- Process supports lower temperature load operations more efficiently than the Hampson process

Introduction (cont.)

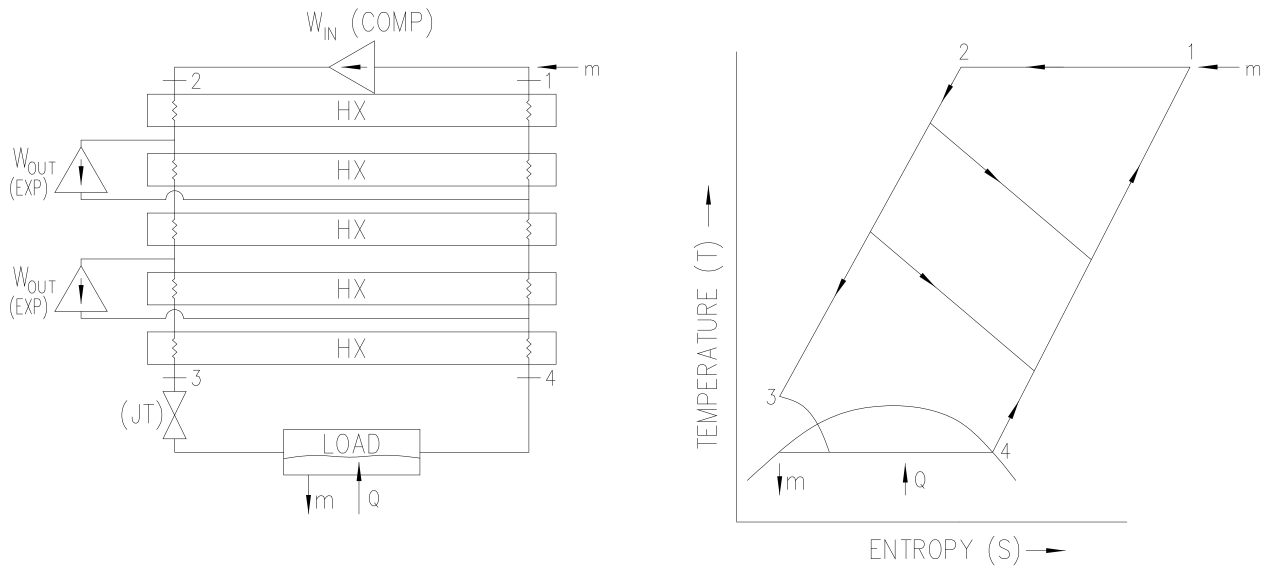
- **Claude process**



- Additional heat exchangers and an expander are used between the compressor and the load.
- Supports lower temperature load operations more efficiently than the Hampson process.

Introduction (cont.)

- **Collins helium liquefaction process**



- Process developed by Sam Collins [1] at MIT and is an extension of the Claude cycle.
- Supports lower temperature load operations more efficiently than the Claude cycle.
- The widely used helium liquefiers originally known as CTI-1400's were based on this process.

Introduction (cont.)

Summary - Key Ideas

- **Coefficient of performance and thermodynamic efficiency**
- **Carnot cycle (as a reversible cycle operating between two constant temperature reseviors).**
- **Quality of thermal energy**
- **Reversible work and fluid/process path independence**
- **Exergy analysis as a means to determine the reversible work for an arbitrary reversible process**
- **Present day cryogenic processes cycles are an extension of basic cycles modified to achieve better efficiency**

Carnot Helium Refrigeration and Liquefaction Systems

Before proceeding to the Carnot helium refrigerator and liquefier it is instructive to revisit the introduction to the 2nd Law of Thermodynamics.

Clausius (In)equality (the 2nd Law of Thermodynamics)

$$\frac{\Delta Q_L}{T_L} = \frac{\Delta Q_H}{T_H}$$

'quality' or 'availability'

This equation is a statement of thermal energy quality equivalence

Eg:

$$\frac{300W}{300K} = \frac{4W}{4K} = \frac{2W}{2K}$$

Carnot Helium Refrigeration and Liquefaction Systems (Cont)

Q_L = 1W at T_L = 4.22 K is equivalent in quality as

Q_H = 70 W at T_H = 300K

Ambient condition (i.e., 300K and 1 atm)
is the 'zero-grade' energy state

exergy is $$\varepsilon = h - T_0 \cdot s$$

$$W_{REV} = W_{ideal} = \sum_i (\dot{m}_i \cdot \varepsilon_i)_{IN} - \sum_j (\dot{m}_j \cdot \varepsilon_j)_{OUT}$$

<u>Carnot Refrigeration System:</u> **Min. input power for a given rate of *<u>thermal energy transfer between two thermal reservoirs.</u>***

The <u>work input</u> for the <u>Carnot system</u> expressed as:

$$W_{carnot} = T_0 \cdot \Delta S - \Delta H$$

<u>This is a very powerful equation</u>

- **The terms are as follows:**
- $T_0 \cdot \Delta S$ **is the heat rejected to the environment**
  or, the input power to an isothermal compressor
- ΔH **is the heat absorbed or the ideal refrigeration**
  or, the ideal work output from an ideal expander
- W_{carnot} **is the ideal net input work required**
  which is the difference between (a) and (b) above

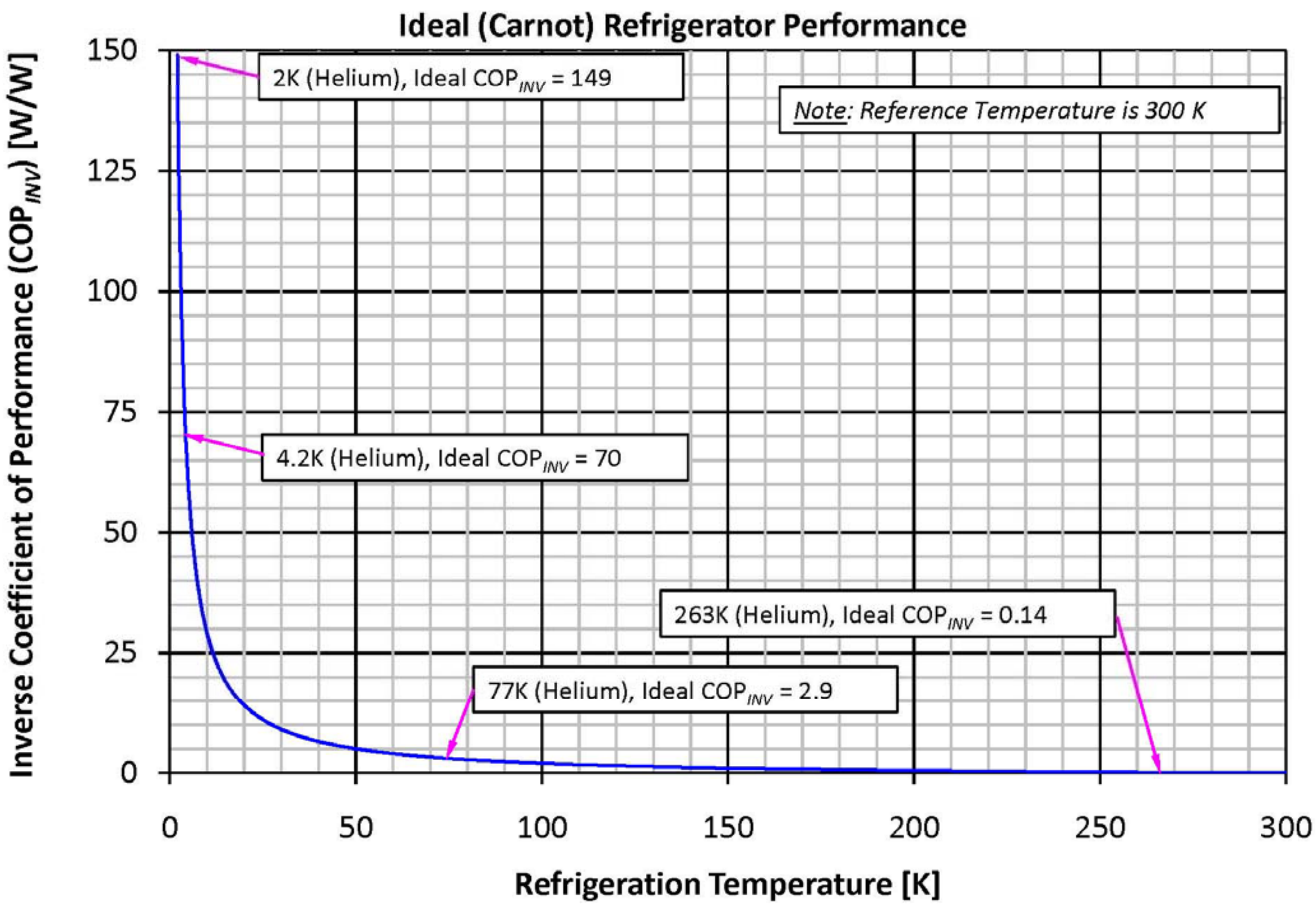
Carnot Helium Refrigeration and Liquefaction Systems (Cont)

- **A refrigerator *transfers* heat energy from a low temperature reservoir to a higher temperature reservoir.**

- **Most helium refrigerators transfer heat energy from approximately 4.22K to ambient 300K.**

- **A liquefier is different from a refrigerator since we cool high temperature fluid to a low temperature, which then leaves the cycle (at a low temperature). The heat energy removed is constantly varying (decreasing as it is being cooled), although it is rejected at the same (high or ambient) temperature reservoir.**

Carnot Helium Refrigeration and Liquefaction Systems (Cont)



	T_load (K)	Pcarnot W / W
A / C Sys.	263.0	0.14
Methane	111.7	1.69
Oxygen	90.2	2.33
Argon	87.3	2.44
Nitrogen	77.3	2.88
Neon	27.1	10.07
Hydrogen	20.3	13.79
Helium	4.2	70.09
Helium @Lambeda	2.2	137.25
Helium @ 2.0 K	2.0	149.00

Carnot Helium Refrigerator

By definition, a refrigerator *transfers* heat energy from a low temperature reservoir to a higher temperature reservoir.

- **The Carnot work for a refrigerator is as follows:**

$$w_{carnot} = T_0 \cdot \Delta s - \Delta h = \text{specific carnot work}$$

$$COP_{INV} = \frac{W_{carnot}}{Q_L} = \frac{T_0 \cdot \Delta S - \Delta H}{\Delta H} = \frac{\dot{m} \cdot (T_0 \cdot \Delta s - \Delta h)}{\dot{m} \cdot (\Delta h)} = \frac{T_0 \cdot \Delta s - \Delta h}{\Delta h}$$

with Q_L equal to the cooling provided

Carnot Helium Refrigerator (cont)

helium refrigerator operating between 300K ambient and the 4.22K

$$w_C = T_0 \cdot \Delta s = (300)\cdot(4.833) = 1449.9 \ \text{[W/(g/s)]}$$

$$w_X = \Delta h = 20.42 \ \text{[W/(g/s)] (or 1.4\% of } w_C\text{)}$$

$$w_{Carnot} = w_C - w_X = 1429.5 \ \text{[W/(g/s)] (or 98.6\% of } w_C\text{)}$$

$$COP_{INV} = \frac{W_{carnot}}{Q_L} = \frac{\dot{m}\cdot(T_0\cdot\Delta s - \Delta h)}{\dot{m}\cdot(\Delta h)} = \frac{(300)\cdot(4.833) - 20.42}{20.42} \cong 70\left[\frac{W}{W}\right]$$

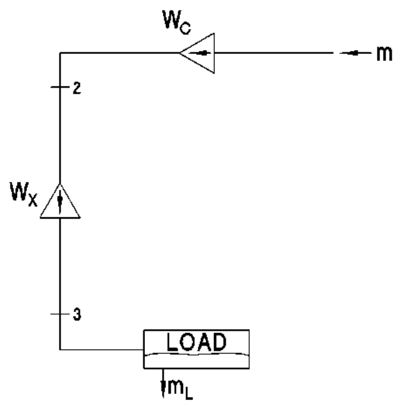
If the expander work is not recovered

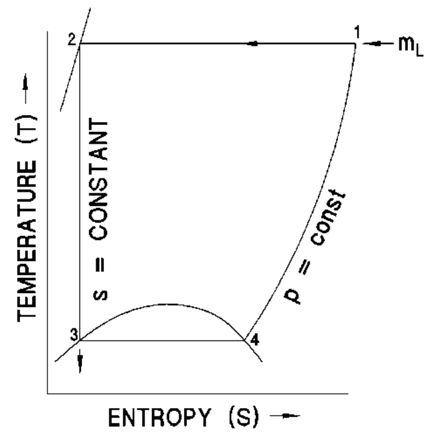
$$COP_{INV} = \frac{\dot{m}\cdot(T_0\cdot\Delta s)}{\dot{m}\cdot(\Delta h)} = \frac{(300)\cdot(4.833)}{20.42} \cong 71\left[\frac{W}{W}\right]$$

<u>Note</u>: Non expander work recovered refrig. systems, start with a 1.4% efficiency penalty

Carnot Helium Refrigeration and Liquefaction Systems (Cont)

Carnot Helium Liquefier





$$w_C = T_0 \cdot \Delta s = (300) \cdot (27.96) = 8387 \text{ [W/(g/s)]}$$

$$w_X = \Delta h = 1564 \text{ [W/(g/s)] (or 18.6\% of } w_C\text{)}$$

$$w_{Carnot} = w_C - w_X = 6823 \text{ [W/(g/s)] (or 81.4\% of } w_C\text{)}$$

In *non expander work recovered* *liquefaction systems*, they start with 18.6% efficiency penalty.

Performance Comparisons of Helium ***Refrigerators*** and ***Liquefiers***

$$\frac{\text{Carnot work required for liquefaction [W/(g/s)]}}{\text{Carnot work required for refrigeration [W/W]}} = \frac{W_{carnot}}{COP_{INV}} = \frac{6823\ \text{[W/(g/s)]}}{70\ \text{[W/W]}} \cong 100\ \text{W/(g/s)}$$

That is, the Carnot work required for approximately 100 W of refrigeration is equivalent (on an equal Carnot work basis) as the Carnot work required to liquefy 1 g/s at 1 atm saturation condition.

Performance Comparisons of Helium *Refrigerators* and *Liquefiers* (Cont.)

***If the expander output work is not recovered*,**

$$\frac{\text{Ideal Power required for liquefaction [W/(g/s)]}}{\text{Ideal Power required for refrigeration [W/W]}} = \frac{8387\ \text{[W/(g/s)]}}{71\ \text{[W/W]}} \cong 120\ \text{W/(g/s)}$$

That is, the Carnot work required for approximately 120 W of refrigeration is equivalent (on an equal Carnot work basis) as the Carnot work required to liquefy 1 g/s at 1 atm saturation condition If the expander output work is not recovered.

Performance Comparisons of Helium Refrigerators and Liquefiers (w & w/o Expander work recovery)

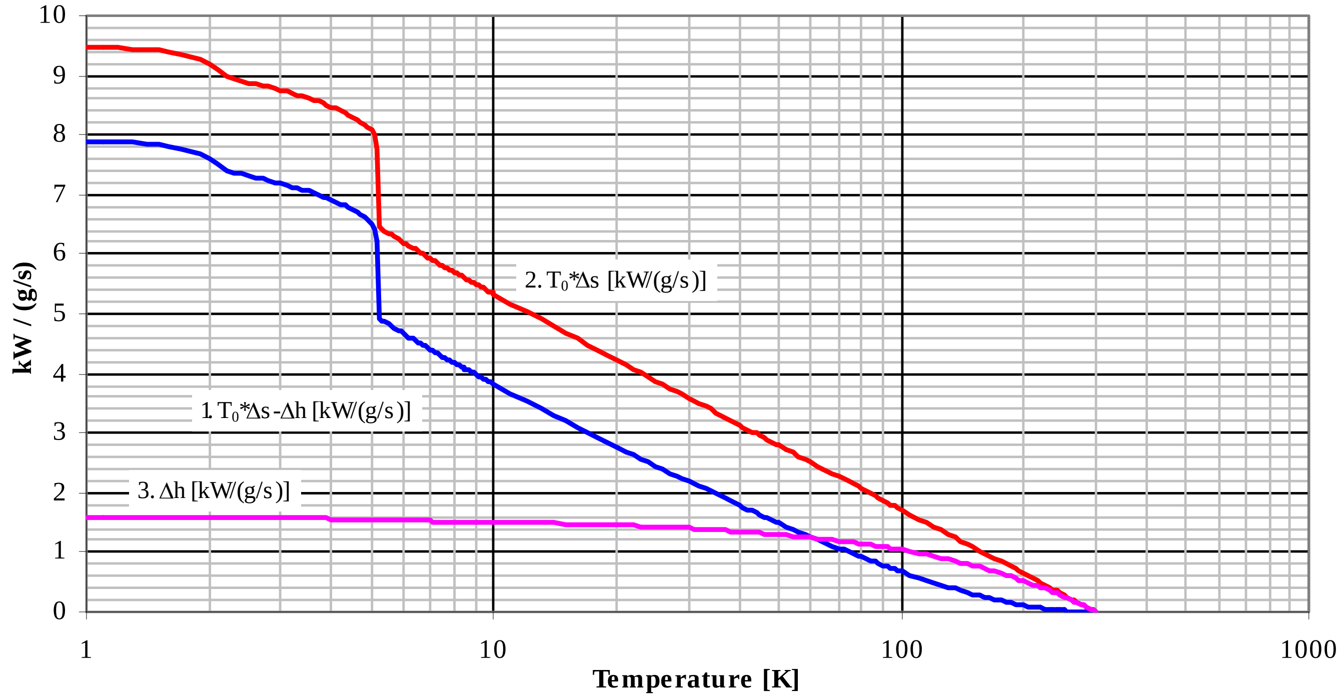
$$\frac{\text{Ideal Power required for liquefaction [W/(g/s)]}}{\text{Ideal Power required for refrigeration [W/W]}} = \frac{8387\ \text{[W/(g/s)]}}{71\ \text{[W/W]}} \cong 120\ \text{W/(g/s)}$$

$$\frac{\text{Ideal Power required for refrigeration [W/(g/s)]}}{\text{Ideal Power required for liquefaction [W/(g/s)]}} = \frac{\left(\frac{w_{Carnot}}{w_C}\right)_l}{\left(\frac{w_{Carnot}}{w_C}\right)_r} = \frac{\left(\frac{6823}{8387}\right)_l}{\left(\frac{1429.5}{1449.9}\right)_r} = \frac{81.4}{98.6} \cong 82.5\%$$

A refrigeration cycle having 30% of Carnot efficiency is expected achieve 25% in liquefaction mode

Carnot Helium Refrigeration and Liquefaction Systems (Cont)

- **Carnot work required for a given liquefaction load**

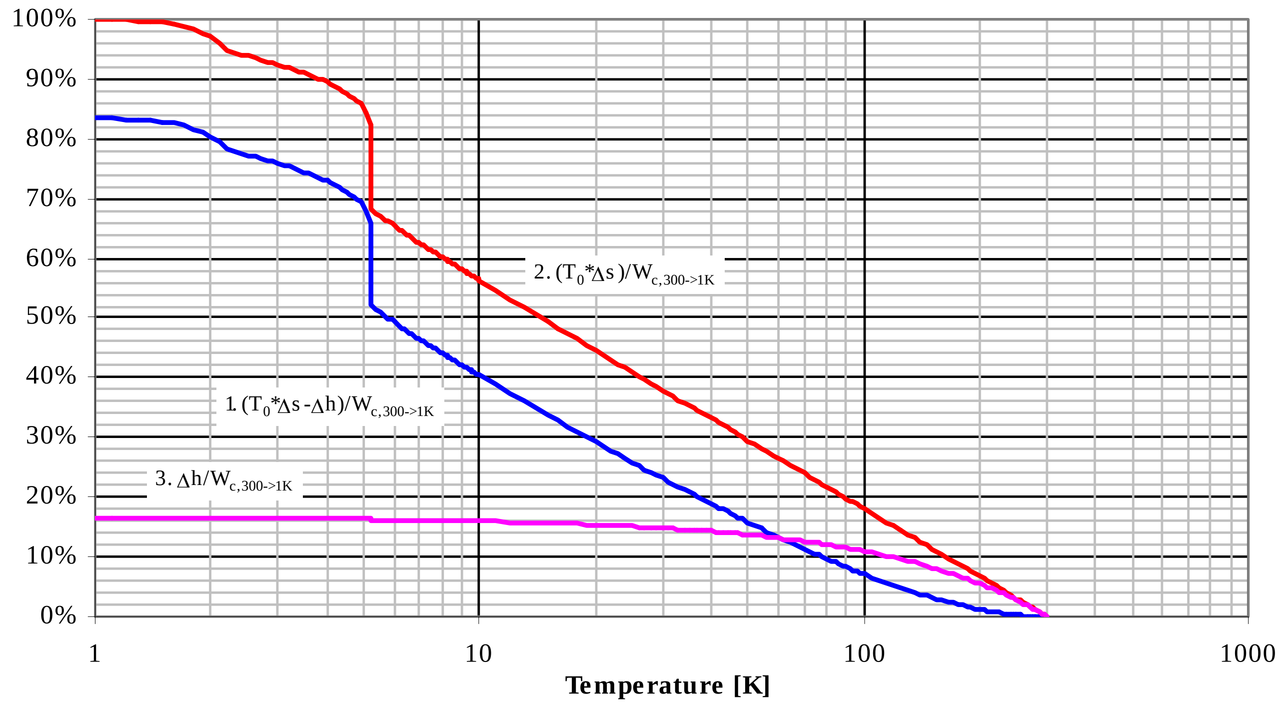


Carnot work [1. $(T_0 \cdot \Delta s - \Delta h)$] required to cool helium from 1 atm & 300K to the specified final temperature

isothermal compressor work [2. $(T_0 \cdot \Delta s)$] and the expander output work [3. (Δh)]

Carnot Helium Refrigeration and Liquefaction Systems (Cont)

Carnot work required for a given liquefaction load



Ratio (in %) of Carnot work, isothermal compressor work and expander output to a reference value for the isothermal compressor work (of 300 to 1K)

Carnot Helium Refrigeration and Liquefaction Systems (Cont)

Carnot work required for liquefaction load for a given temperature range

Temperature Range (K)		$T_0 * \Delta s$ [W/ (g/s)]	%		Δh [W/ (g/s)]	%	$T_0 * \Delta s - \Delta h$ [W/ (g/s)]	%
300 - 80		2058	**24.5%**		1143	**73.0%**	915	**13.4%**
80 - 4.22		6329	**75.5%**		421	**27.0%**	5908	**86.6%**
300 - 4.22		8387	**100.0%**		1569	**100.0%**	6823	**100.0%**

Carnot Helium Refrigeration and Liquefaction Systems (Cont)

Carnot work for different fluids

Fluid	Tsat,0 [K]	Liquefaction (W/(g/s))	Refrigeration (W/W)
Helium	4.22	6823	70
Hydrogen	20.28	12573	13.8
Neon	27.09	1336	10.1
Nitrogen	77.31	770	2.9
Argon	87.28	477	2.4
Oxygen	90.19	635	2.3
Methane	111.69	1092	1.7

Summary

In this chapter the Carnot work (or the minimum input work) required for the refrigeration and liquefaction is explained.

the effects of non recovered expander work (generally the case for most of the helium systems) on the refrigeration and liquefaction processes.

In practice all the systems are compared to the ***true reversible Carnot work*** (W_{carnot}) **, which includes the expander out put work**

3. Ideal Helium Refrigeration Systems and Carnot Step

In this chapter we look at reversible cycles using an *ideal gas* and *perfect components*.

- **This will provide the *basis* for analyzing real systems.**

- **A system design based on an ideal system and constructed with real components should result in an efficient system.**

Carnot Step

- **Typically in helium (refrigerator) systems, there are multiples of certain similar non-simple process steps; e.g., warm screw compressor stages, expansion stages in a cold box, etc. to accomplish a given process.**

- **The *Carnot Step* is defined *(by the author)* as the arrangement (or "spacing") of a *given number* of the same type of process steps which yield the minimum irreversibility.**

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

Carnot Step (Cont.)

- **This optimal arrangement of process steps is applicable to ideal and real processes and will typically yield the minimum energy expenditure (for that process and selected components)**

- **It is important to note that *Carnot Step* is not necessarily a reversible 'step', since it depends on whether the process and/or components are reversible**

- **A typical helium system consists of:**

  (1) load, (2) cold box and (3) compressor

- **Clearly, an efficient system depends upon an efficient design of each part of the system**

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

The Load:

- **Every attempt should be made at the load level (temperature) to minimize the entropy increase of the helium, recovering the returned load flow exergy (refrigeration) while satisfying the load requirement.**
- **The losses introduced from a distribution system become a load as well, requiring the cold box and compressor system to be larger (i.e., greater capital cost), thereby incurring additional operational cost.**
- ***Example*: For a thermal shield between 300K and 4K with equal conductance on both sides, the idealized choice for the shield temperate to minimize the total reversible input power (i.e., the load Carnot Step) is found by equating the temperature ratios and is 35K**

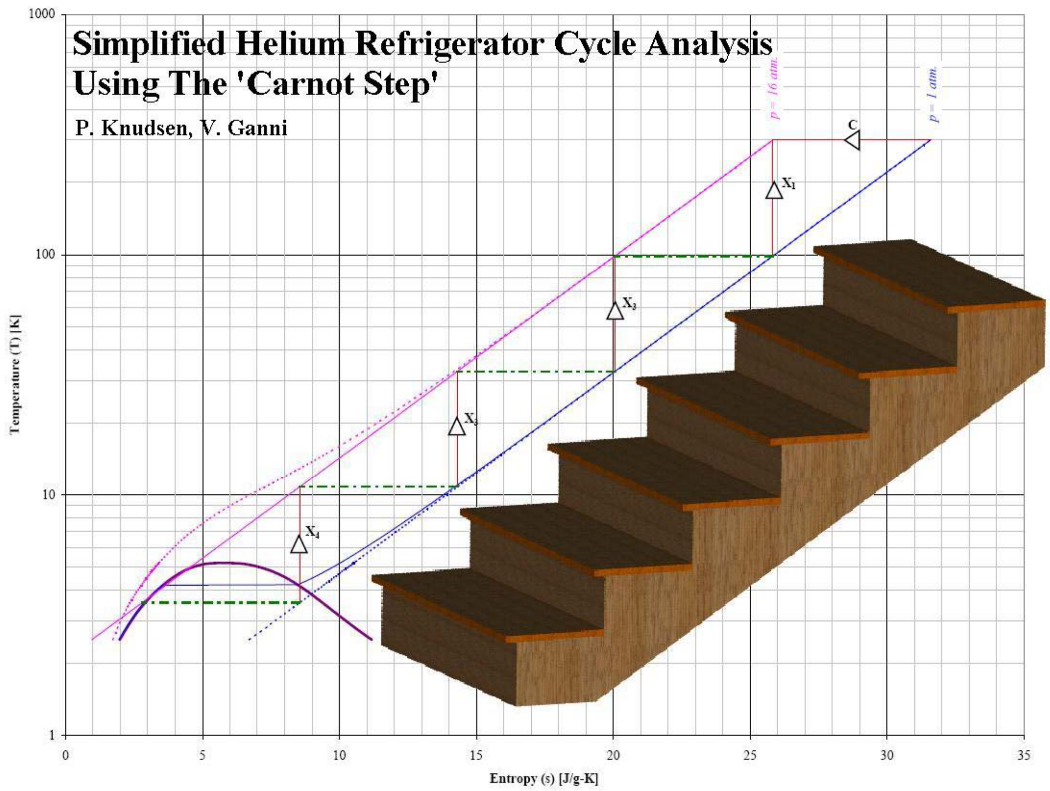
Idealized Helium Systems and Carnot Step (Cont.)

The Cold Box:

The cold box bridges the temperature difference from the load to ambient conditions, transferring the entropy increase at the load to the compressors.

The cold box has no input power and can only utilize the availability (i.e., exergy) supplied to it by the compressor(s). Obviously, it is critically important for the cold box to utilize the supplied exergy with a minimum of 'losses'.

The cold box provides a process path analogous to transferring a load from a deep basement floor (4.2K) to the ground floor (300K) by walking up the stairs. So, given the 'height' between the 'floors' (4.2K to 300K), we would like to know the *<u>minimum number and optimal spacing of the steps</u>* that will yield a minimum irreversibility.



The Cold Box:

- ***The expanders provide cooling (refrigeration) by extracting work. So, the number of cold box (expansion or refrigeration) steps is same as the number of expanders in the process.***
- **The next chapter (4) will address the optimal *Carnot Step* 'spacing' for a given number of steps that yields the minimum irreversibility (or minimum compressor input work).**
- **Therefore, the cold box Carnot Steps spacing (distribution) provides a means for evaluating a given cold box system design.**

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

The Compressor System:

- ***The compressor system* *uses the input energy (usually electrical) to increase the availability (e.g., exergy) of the helium gas being supplied to the cold box*.**
- **For a multistage polytropic compression process, an *equal pressure ratio* among each of the equal efficiency stages yields the minimum actual input work for a given mass flow rate.**
- **Since *isothermal compression* requires the minimum ideal work, it is used to determine the *compressor Carnot Step(s)***
- **Therefore, the compressor Carnot step provides a means for evaluating a given compressor system design (efficiency).**

The Availability to the Cold Box:

- **The availability (exergy) supplied to the cold box is the isothermal work input to the compressor system.**
- **It is also equal to the load Carnot work plus the cold box and load losses (i.e., irreversibility, or lost work).**
- **For a simple two-stream system, using the ideal gas assumption, the specific isothermal work is:**

$$w_{C,\,iso} = \Delta\varepsilon_C = T_i \cdot \Delta s - \Delta h = T_i \cdot \phi \cdot C_p \cdot \ln(P_r)$$

where, for an ideal gas, $\Delta h = 0$ **and,** $\Delta s = \phi \cdot C_p \cdot ln(P_r)$

So, $\Delta s \propto ln(P_r)$

The Availability to the Cold Box:

- **From this it can be seen that the *availability (exergy) supplied to the cold box increases proportionally to the mass flow rate and the logarithm of the pressure ratio***

- **The required decrease in entropy (i.e., increase in exergy) can be achieved by increasing the mass flow rate or increasing the pressure ratio**

- **The analysis that follow are primarily centered on the cold box since the load is specific to a given application and the compressor performance is easily compared to the isothermal compression work**

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

Ideal Helium Refrigeration Systems Using the Ideal Gas:

- **The Carnot (reversible) work required for the refrigerator is,**

$$\Delta\varepsilon_{load} = w_{C,\ ideal} = (T_0 \cdot \Delta s - \Delta h) = [(300)\cdot(4.833) - 20.42] = 1429.5 \text{ W/(g/s)}$$

- **The inverse coefficient of performance is,**

$$COP_{INV} = \frac{w_{C,iso}}{(q_{load} / \dot{m}_{load})} = \frac{w_{C,iso}}{\Delta h} = \frac{[(300)\cdot(4.833) - 20.42]}{20.42} = 70 \text{ W/W}$$

So, then it requires 70 W of ideal (isothermal) input work for every 1 W of load

- **The pressure ratio for an isothermal compressor, *<u>operating with an ideal gas</u>* is,**

$$P_r = e^{\left[w_{C,\ ideal} / (\phi \cdot C_p \cdot T_0)\right]} = 9.91 \quad \text{with, } R = \phi \cdot C_p = 2.077 \text{ J/g-K}$$

<u>Note</u>: if real fluid properties were used, $P_r = 10.57$

Ideal Helium Refrigeration Systems:

- For an ideal gas, an ideal helium refrigerator should be constructed with the compressor operating between a 1 atm suction and ~10 atm discharge pressure
- In this process the gas is cooled to the load temperature at ~10 atm using the return gas and expanded ***isothermally*** (a limitation of the ideal gas assumption since an ideal gas only has a single phase) while adsorbing the refrigeration load
- So, there is only a ***single Carnot step*** (or single expansion step) for the ***ideal gas helium refrigerator***

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

Ideal Helium Liquefier Using the Ideal Gas:

The Carnot work required for the liquefier is,

$$w_{C,iso} = T_0 \cdot \Delta s = (300) \cdot (27.96) = 8387 \text{ [W/(g/s)]}$$

$$w_{X,ideal} = \Delta h \qquad = 1564 \text{ [W/(g/s)] (or 18.6\% of } w_{C,iso})$$

$$w_{C,ideal} = w_{C,iso} - w_{X,ideal} \qquad = 6823 \text{ [W/(g/s)] (or 81.4\% of } w_{C,iso})$$

<u>Note</u>: ***the expander output work is used to reduce the isothermal compression work; for real systems, this is not practical***

- ***So, then it requires 6.823 kW of ideal (net isothermal) input work for every 1 g/s of liquefaction load***

- **As for practical systems, the expander output work is not recovered , so that the pressure ratio for an isothermal compressor, *<u>operating with an ideal gas</u>* is,**

$$P_r = e^{\left[w_{C,iso} / \left(\phi \cdot C_p \cdot T_0 \right) \right]} = 700,000 \quad \text{with, } R = \phi \cdot C_p = 2.077 \text{ J/g-K}$$

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

Idealized Helium Liquefier:

- For an ideal gas, an ideal helium liquefier should be constructed with the compressor operating between a 1 atm suction and a (approximately) 700,000 atm discharge pressure

- It is not a practical option

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

Idealized Helium Liquefier (Cont.):

- **This ideal process is obviously not a very practical liquefier, but it does present the fact that heat energy is being transferred at a temperature that continuously varies**

- **In other words, the liquefaction load is really a refrigeration 'load' whose load temperature begins at 300K and continuously decreases as it is cooled, finally ending at the (specified) load supply temperature (typically 4.2K)**

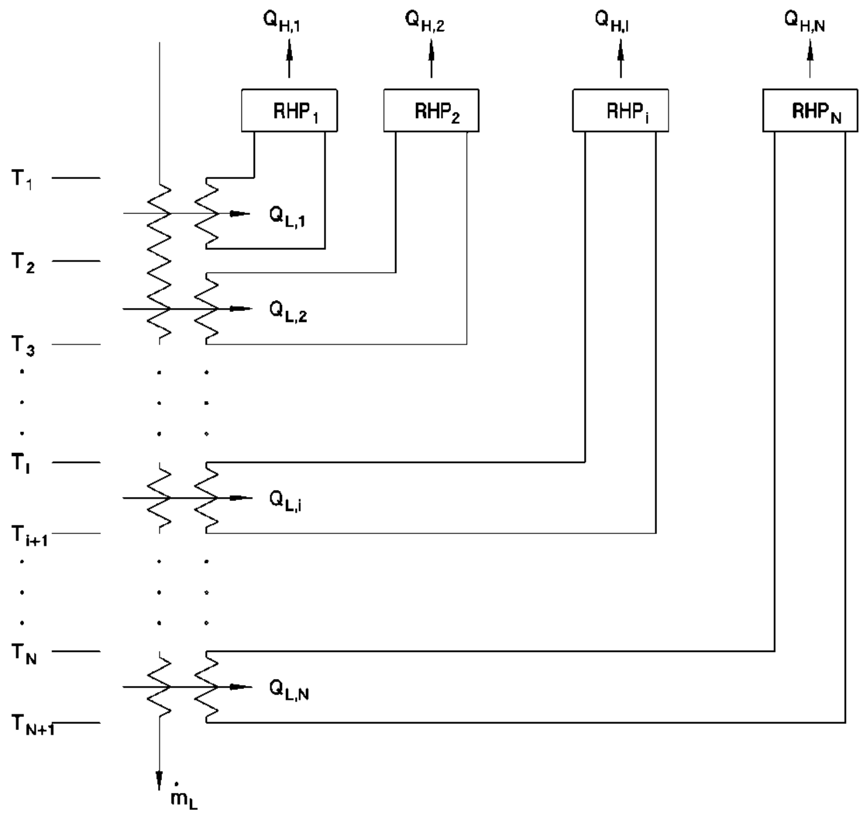


Figure 3.4.1: Ideal liquefier process

Idealized Helium Liquefier (Cont.):

- **Since the pressure ratio required by the Carnot liquefier is impractical, a Claude type of liquefier shown in Figure 3.4.2 is adopted**
- **These 'two' cycles (Figures 3.4.1 and 3.4.2) shown require the same input work if the expander output work in Figure 3.4.2 is recovered**
- **Figure 3.4.3 is a TS diagram representation of the ideal Claude liquefier (ICL).**
- **The ICL requires the same input work as the Carnot liquefier (in Chapter 2) if the ideal gas assumption is valid and the expander output work is recovered (ref. Appendix A)**
  - **Remember that the ICL shown has perfect HX's (infinite NTU's) and isentropic expanders**
- **Figure 3.4.3 reveals some information that is not as apparent in Figure 3.4.2.** ***That is, the ideal temperature step, or 'Carnot Step', established by the expanders***

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

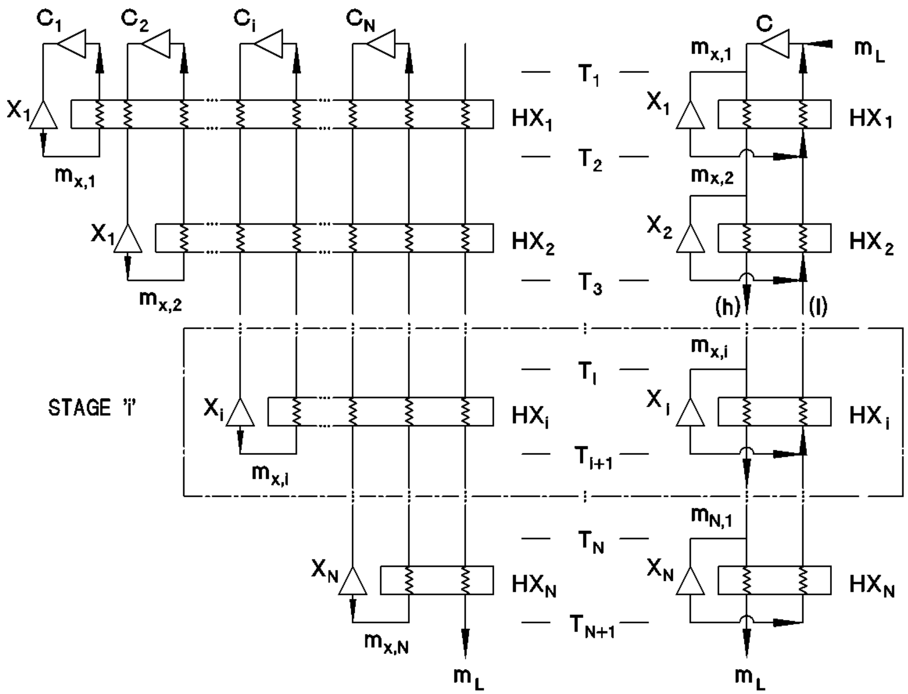


Figure 3.4.2: Ideal Claude Liquefier (ICL)

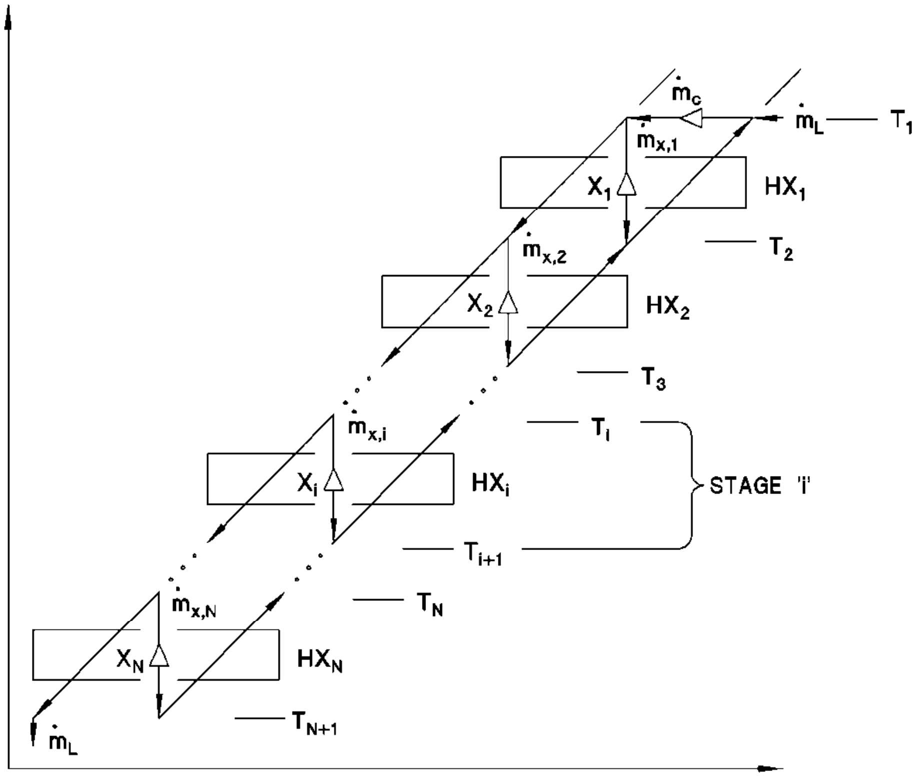


Figure 3.4.3: TS Diagram for the Ideal Claude Liquefier (ICL)

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

- **From Figure 3.4.3 and using ideal gas and isentropic process relations,**

$$T_r = P_r^{\phi} = const.$$

$$T_{r,T} = \left(\frac{T_1}{T_{N+1}}\right) = (T_r)^N$$

$$N = \frac{\ell n(T_{r,T})}{\phi \cdot \ell n(P_r)} = \frac{\ell n(T_{r,T})}{\ell n(T_r)}$$

Where, T_r – expander temperature ratio (***Carnot Step***) $= T_i / T_{i+1}$

P_r – pressure ratio across expanders $= p_h / p_l$

T_1 – ambient temperature [K]

T_{N+1} – load temperature [K]

$T_{r,T}$ – total temperature ratio (ambient to load temperature) $= T_1 / T_{N+1}$

N – total number of (ideal) expansion stages

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

- So, *the Carnot step is the same for each expander stage* (i.e., T_r is the same for each stage) *and equal to the expander temperature ratio (which is set by the pressure ratio*
- As an example, for a 300K to 4.2K liquefier (e.g., T_1 = 300K, T_{N+1} = 4.2K) with an expander pressure ratio of 16 (e.g., P_r = 16), the total temperature ratio is, $T_{r,T}$ = 300 / 4.2 = 71 and the temperature ratio for each expander stage is, $T_r = (16)^{0.4} = 3.03$
- So, the (ideal) number of expander stages required for the ICL is, $N = ln\,(71) / ln\,(3.03) = 3.85 \approx 4$
- Referring to Figure 3.4.3, each expander flow is the same and equal to the liquefaction flow
- As we will see in Chapter 4, this is only true if there is a single perfect HX for each stage (or step); otherwise the expander flow is greater than this

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

- **In the ideal Claude liquefier, the expander flow, temperature ratio and (isothermal) compressor input work (since the pressure ratio for each stage is equal) *are the same for each stage, or Carnot step*.**

- **However, the Carnot work for each stage is *not* the same since the recovered expander output, which is used to reduce the compressor input power, is not the same for each Carnot step**

- **So, there are *multiple Carnot steps* for the *ideal Claude helium liquefier***

Ideal Helium Refrigeration Systems and Carnot Step (Cont.)

Summary

- The *'Carnot Step'* is one of a given number of similar process steps that yield the minimum irreversibility.

- Assuming an ideal gas, the ideal ***refrigerator requires only a single Carnot step*** whereas the ***ideal Claude liquefier requires a number of Carnot steps***, each with the same temperature ratio and expander mass flow

4. The Theory Behind Cycle Design

In this chapter we look at *idealized helium system(s)* and *possible practical systems* that one can visualize.

The *idealized* system *may or may not be a reversible system* depending on the process and the fluid that is used.

A system design based on an idealized system and constructed with real components should result in an efficient (or even an optimum) system.

An analogy to carrying (transferring) a load up from a deep basement floor (4.22K) to the ground floor (300K), the system design must select a process path (or a cycle if performed continuously).

This path can be depicted on a TS or a Te diagram and in our analogy, can be inclined as steps or vertical as an elevator. Generally a 'straight line of approach' path is preferred, except as necessary to accommodate imposed constraints of local deviations.

Dewar Process

Before discussing the design of refrigeration systems, it is very important to understand the load(s) and their effect on the system design.

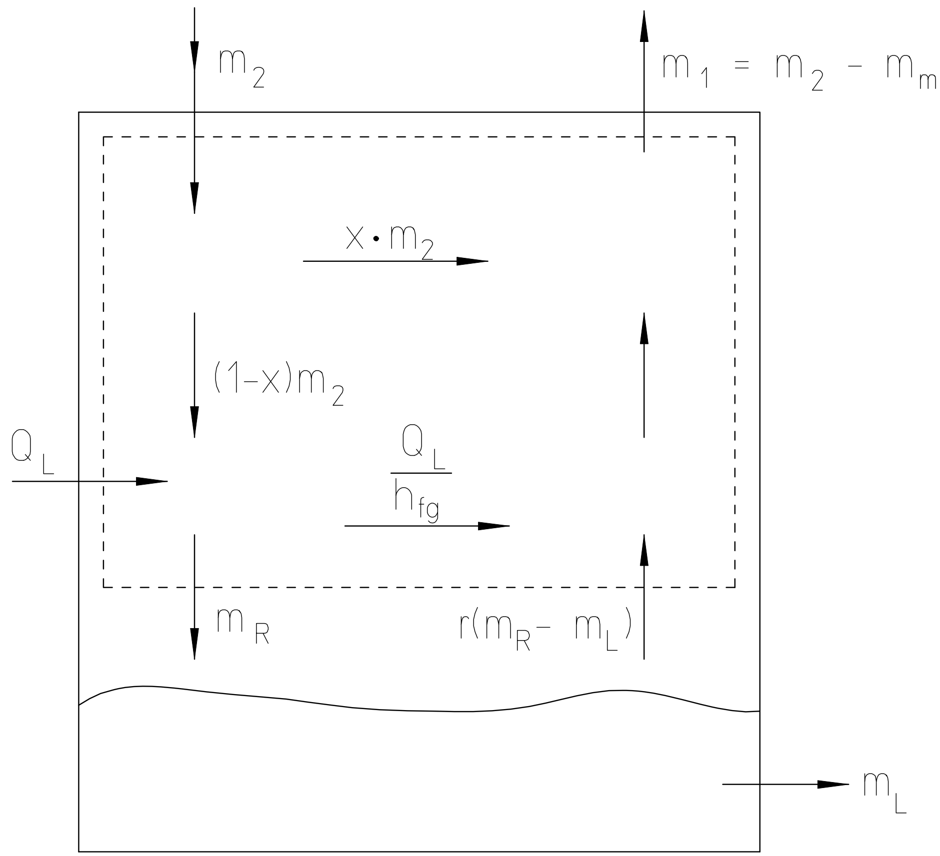
The load and the distribution systems are analyzed first (to minimize entropy generation; see sec 3.1.1) and must be understood before proceeding.

Since the loads and distribution system are project specific, only typical loads can be analyzed here.

Consider a simple system with loads interacting with a helium dewar, as shown in Figure 4.1.1.

The Theory Behind Cycle Design (Cont.)

Dewar Process



The Theory Behind Cycle Design (Cont.)

Dewar Process

X = Quality of m_2 $\qquad r = \frac{\rho_g}{\rho_f}$

$m_R = (1-X)\, m_2 - \frac{Q_L}{h_{fg}}$ ……(1) $\qquad m_m = (1-r)\, m_R + r\, m_L$ ……(2)

Case 1: $\quad m_L = 0 \quad m_R = \frac{m_m}{(1-r)} \rightarrow m_m = (1-r)\left[(1-x)m_2 - \frac{Q_L}{h_{fg}}\right]$

Case 2: $\quad m_R = m_L \rightarrow m_R = m_m \rightarrow m_m = m_R = m_L = \left[(1-x)m_2 - \frac{Q_L}{h_{fg}}\right]$

For Dewar Boil off (heat leak into Dewar) test: $m_2 = m_L = 0$

Flow leaving the Dewar $\quad = -m_m = \left[(1-r)\frac{Q_L}{h_{fg}}\right]$

Or Dewar Heat leak $\quad Q_L = \left[\frac{-m_m}{(1-r)} h_{fg}\right]$

Helium Dewar Process

It is very important for helium systems to account for the mass of the displaced vapor from the Dewar during the filling process (i.e., the ratio of vapor to liquid density is ~ 1 / 7.4 at 1 atm.).

It is important to note that the rate of rise is greater than the makeup rate for the no withdrawal case.

As such, proper accounting is required in helium system liquefaction measurements since the rate of rise ($\dot{m}_R$) compared to the makeup helium ($\dot{m}_m$) can account for a 15% to 35% higher rate of production (depending on whether the dewar pressure is 1.0 or 1.6 atm) as given by equation (2), if the liquid withdrawal is zero.

This effect is not significant for other fluids with (typically) small vapor to liquid density ratios (e.g., 1 / 175, for 1 atm. nitrogen).

Real Gas Helium Refrigeration Systems

Case-1: **Consider the *idealized refrigerator* explained in section 3.3 using a real gas, an isothermal compressor, a single ideal heat exchanger (effectiveness = 1) and a single ideal expander (isentropic efficiency = 1) at the cold end.**

Real Gas Helium Refrigeration Systems

For this configuration shown in Figure 4.3.1, with a given load temperature, there is a unique solution for the high pressure supply to the cold box.

For a load at the 1 atm. saturation condition, the high pressure supply must be approximately 70 atm if the real gas is helium.

This is ***not*** a reversible process since the heat exchanger has a non-zero cold end ΔT equal to the difference between the expander inlet temperature (~7.7K) and the load temperature (~4.2K).

However, the usefulness of this cycle is in presenting the effect of the *real gas* and the minimum number, size and location of the components similar to the ideal system described in sec. 3.3.

The Theory Behind Cycle Design (Cont.)

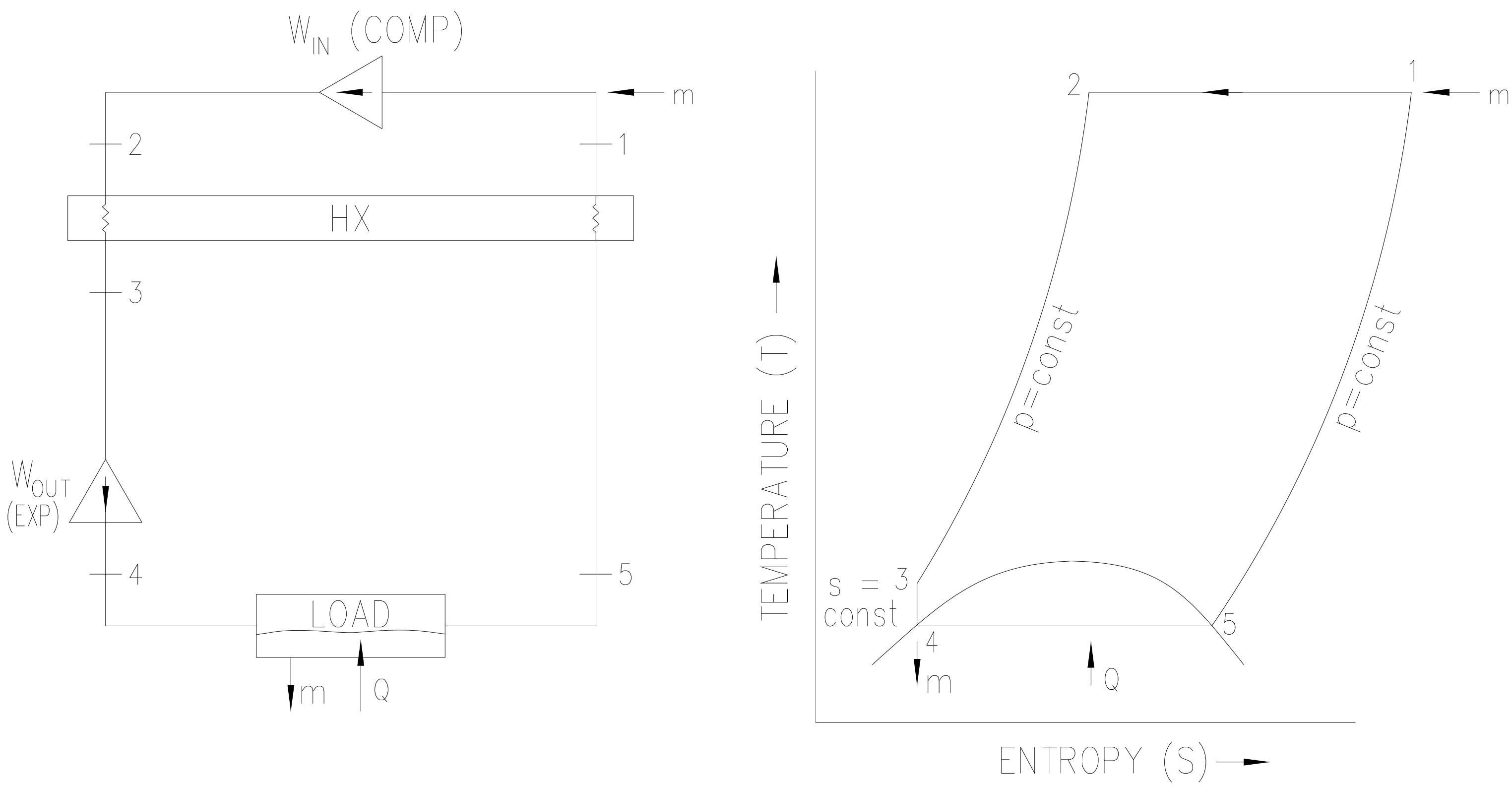


Figure 4.3.1: Idealized helium refrigeration system operating with real gas

The Theory Behind Cycle Design (Cont.)

For helium, the isothermal compressor work for this cycle is,

$$w_{C,\, iso} \cong R \cdot T_0 \cdot \ln(P_r) = (2.077) \cdot (300) \cdot \ln(70) = 2647 \text{ W/(g/s)}$$

$$w_{C,\, iso} \cong T_0 \cdot \Delta S \qquad = 300 \cdot (31.41 - 22.59) \quad = 2646 \text{ W/(g/s)}$$

$$COP_{INV} = w_{C,\, iso} / \Delta h = (2646 - 20.42) / 20.42 = 128.6 \text{ W/W}$$

As shown above, this cycle requires 1.8 times more input power (a Carnot efficiency of 55%) than the idealized cycle described in section 3.3. The efficiency loss in the above cycle is due to the non-ideal, real fluid properties.

Real Gas Helium Refrigeration Systems (Cont.)

Case-2: The above Case presents the influences and the importance of the real fluid properties in refrigeration process.

This is mainly due the real fluid transition from gas to liquid. Consider the following *idealized refrigeration process* to understand the real fluid property's influence.

In the system studied in Figure 4.3.2., the process assumes that at any given temperature there is a

constant entropy difference

between the supply and the return flows.

The Theory Behind Cycle Design (Cont.)

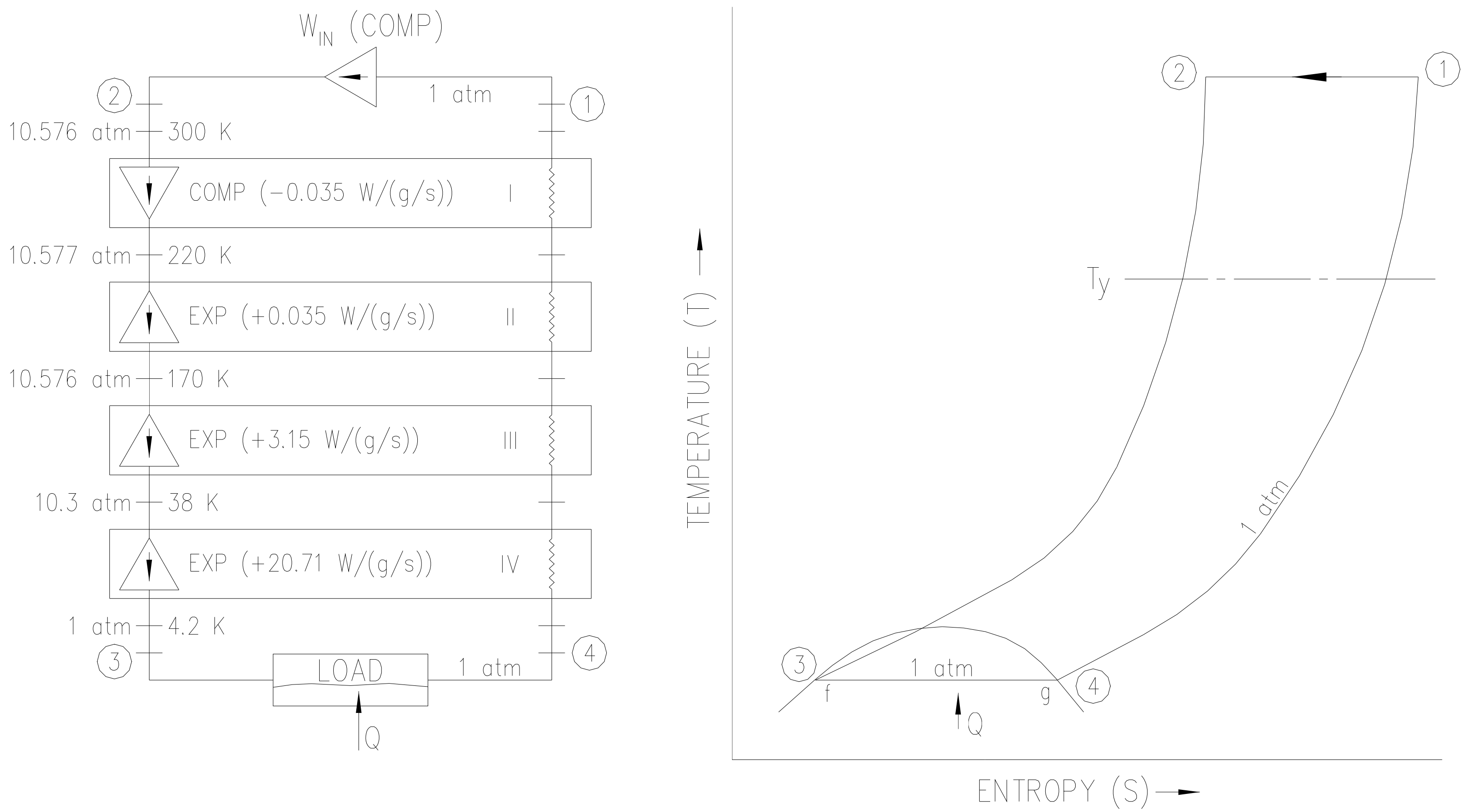
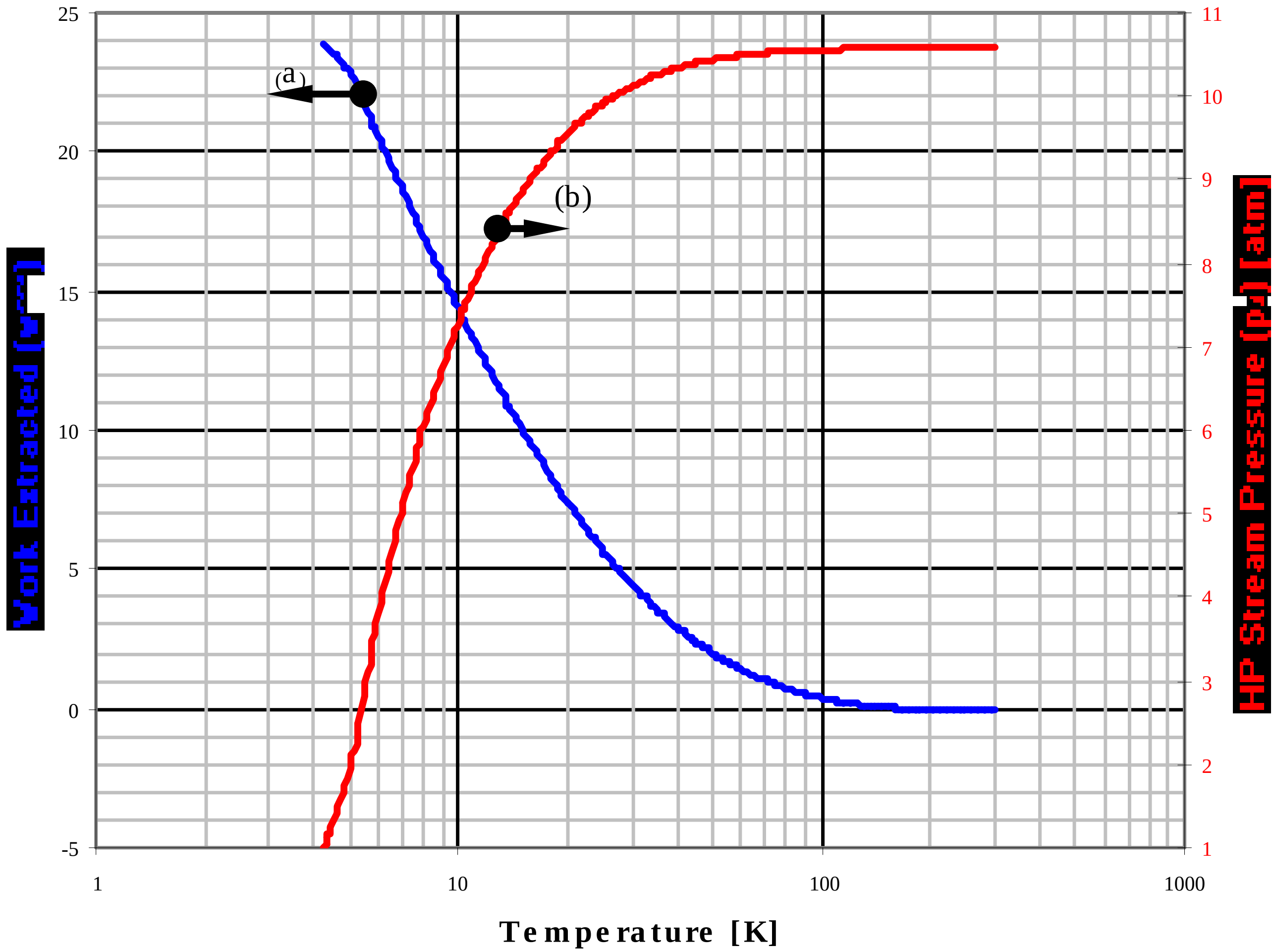


Figure 4.3.2: Idealized helium refrigeration system operating with real gas and ***constant entropy difference***

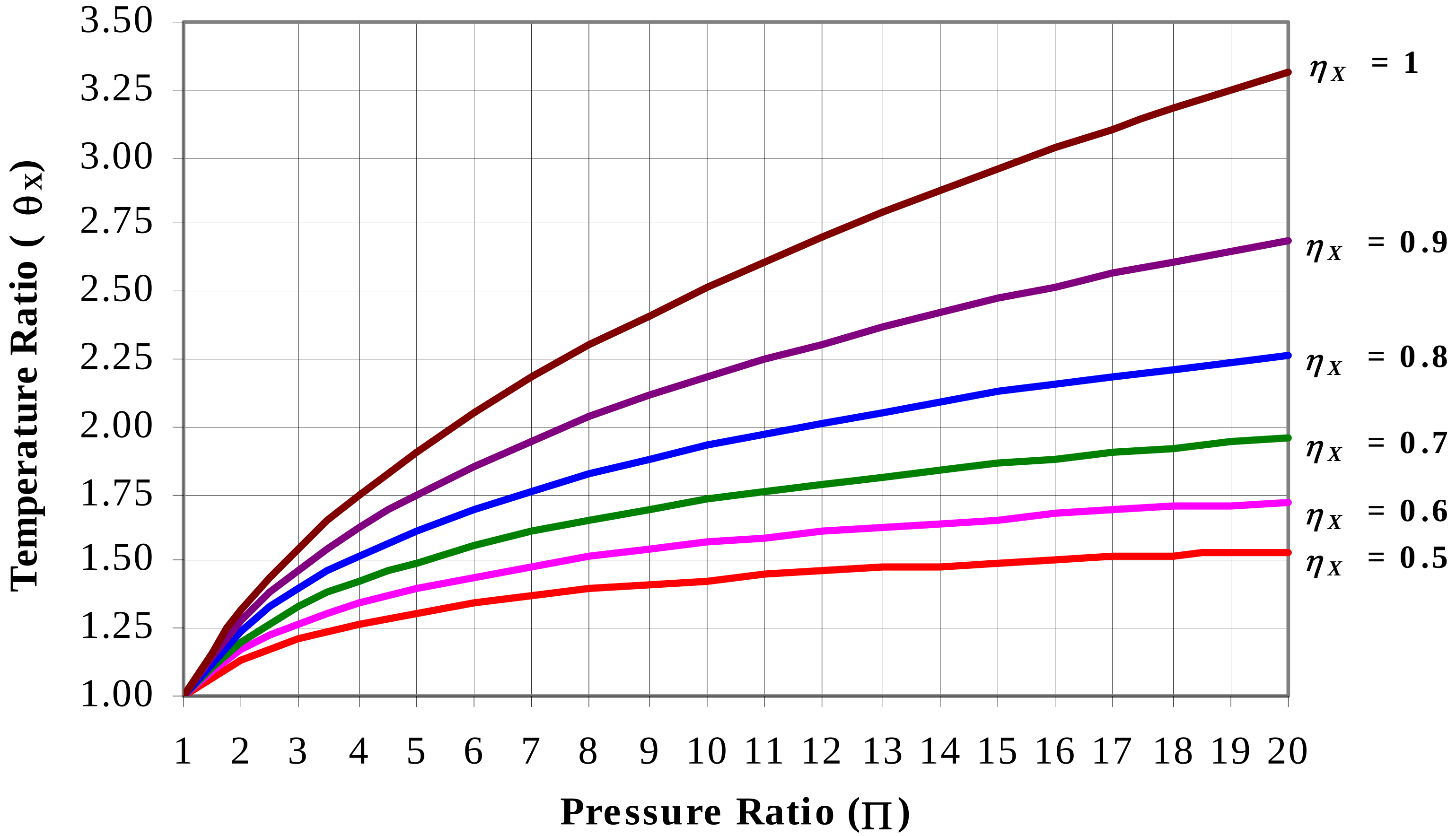
The Theory Behind Cycle Design (Cont.)

Idealized helium refrigeration system (cont.)



The Theory Behind Cycle Design (Cont.)

Expander pressure ratio vs. temperature ratio for selected efficiencies



The Theory Behind Cycle Design (Cont.)

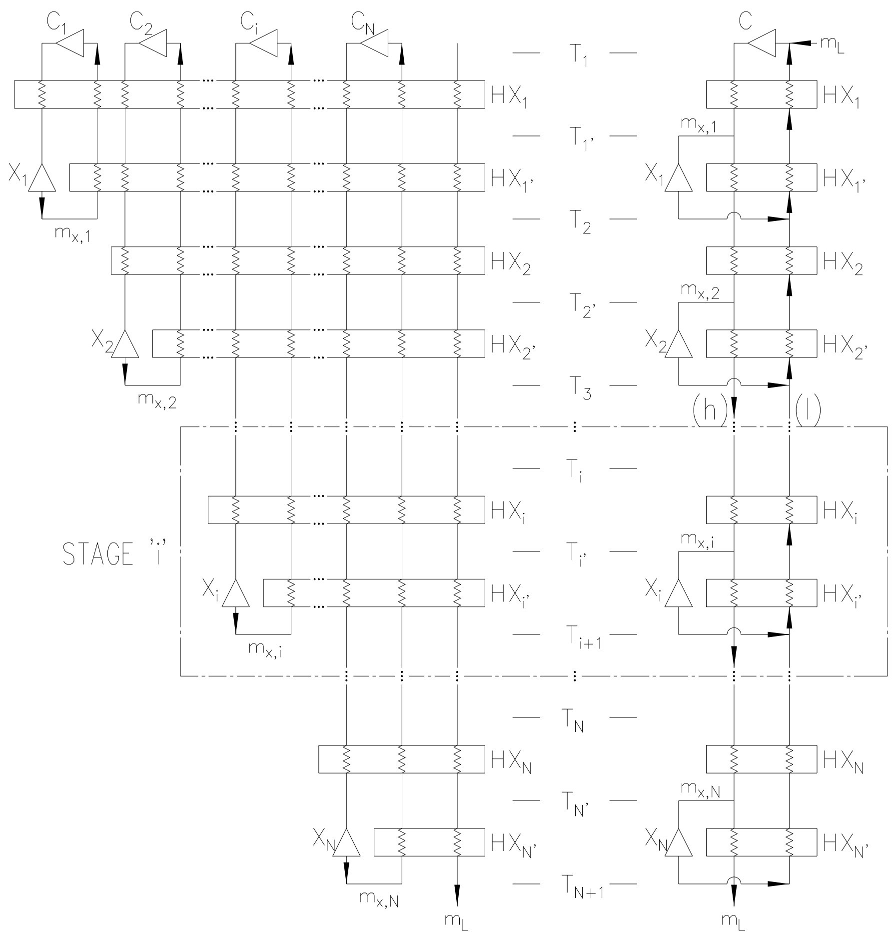


Figure 4.4.2: Claude liquefier with additional HX per stage

The Theory Behind Cycle Design (Cont.)

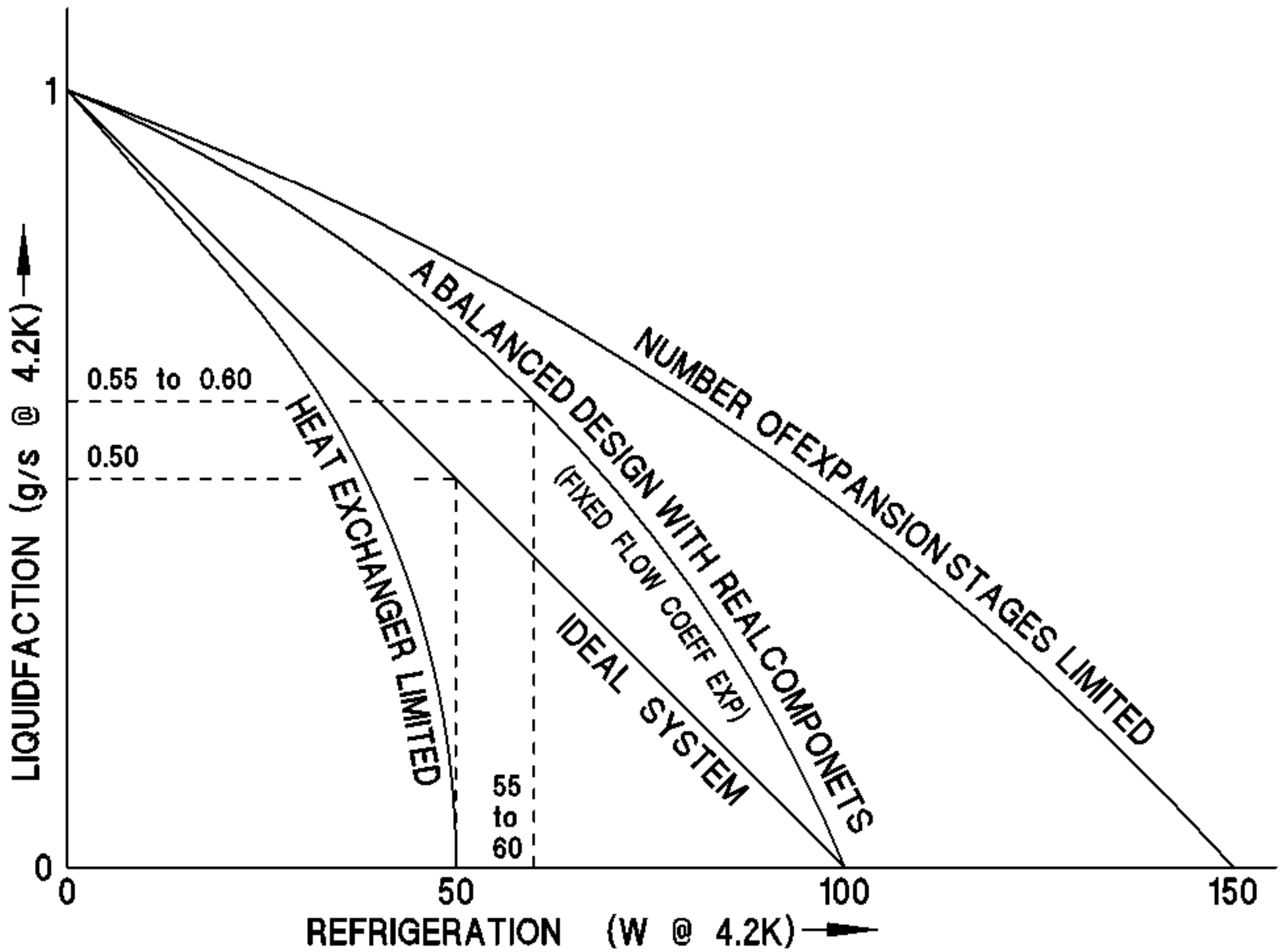


Figure 4.8.1: **The Effect of Components on System Load Capacity**

Summary

The use of the *Carnot step* for cold box design.

For a given number of expansion stages (with equally efficient expanders), these Carnot steps (the stage temperature ratios) are *theoretically* the same for both refrigerator and liquefier and result in minimizing the compressor flow and therefore, the input power.

This is indirectly saying that the ideal placement of the expanders with respect to temperature for both refrigerator and liquefier are approximately (disregarding real gas effects) the same, but the flow requirement through the expanders may not be the same.

***Note:* The importance and ramifications of the Carnot step was recognized by the author in the mid 80's. Since then it has been taught to colleagues and utilized in new system designs, as well as improving the operation of existing systems.**