Computing $\hat{q}$ on a quenched SU(3) lattice

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Outline

- ❑ Jet quenching and importance of transport coefficient $\hat{q}$ in heavy-ion collisions
- ❑ Defining $\hat{q}$ for cold nuclear matter (EIC) and hot QGP (RHIC & LHC)

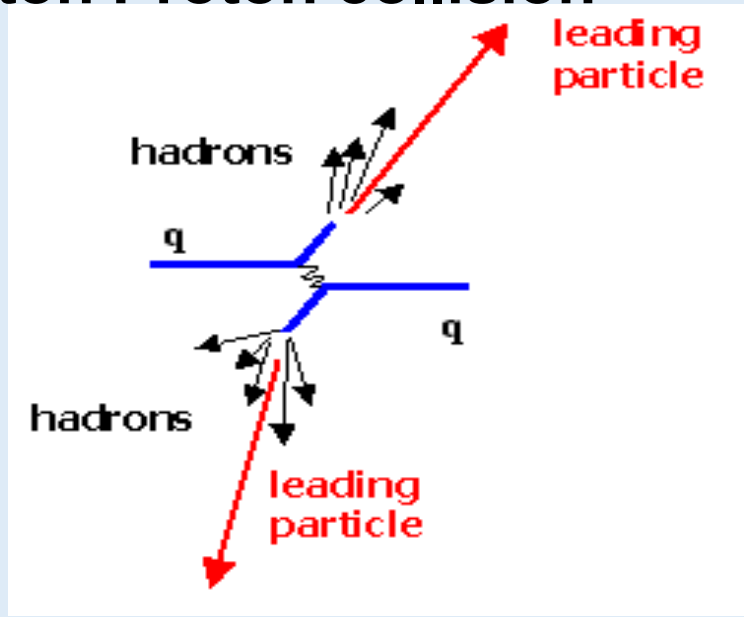
- ❑ Formulating $\hat{q}$ for hot QGP using Lattice gauge theory

1) Previous study done on a quenched SU(2) lattice
2) Extending calculations to a quenched SU(3) lattice

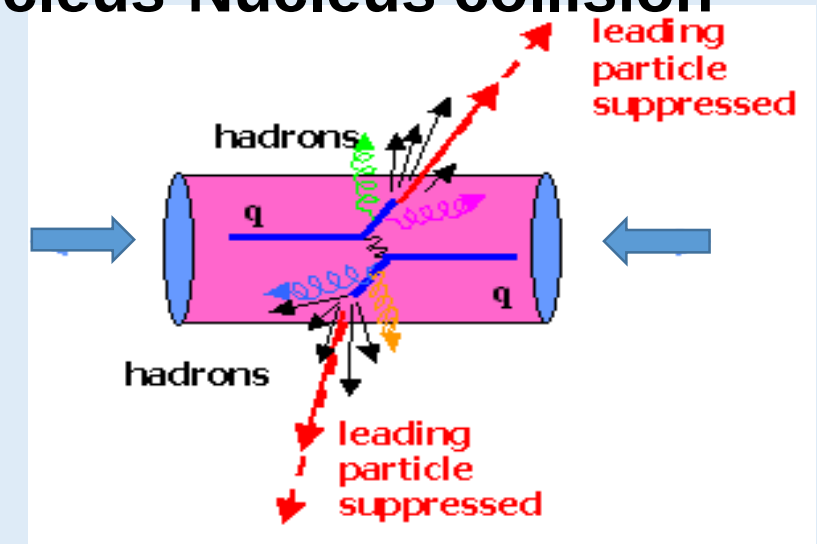
- ❑ Estimates of $\hat{q}$ on a quenched QGP plasma

Leading hadron suppression in Quark-Gluon Plasma

Proton Proton collision

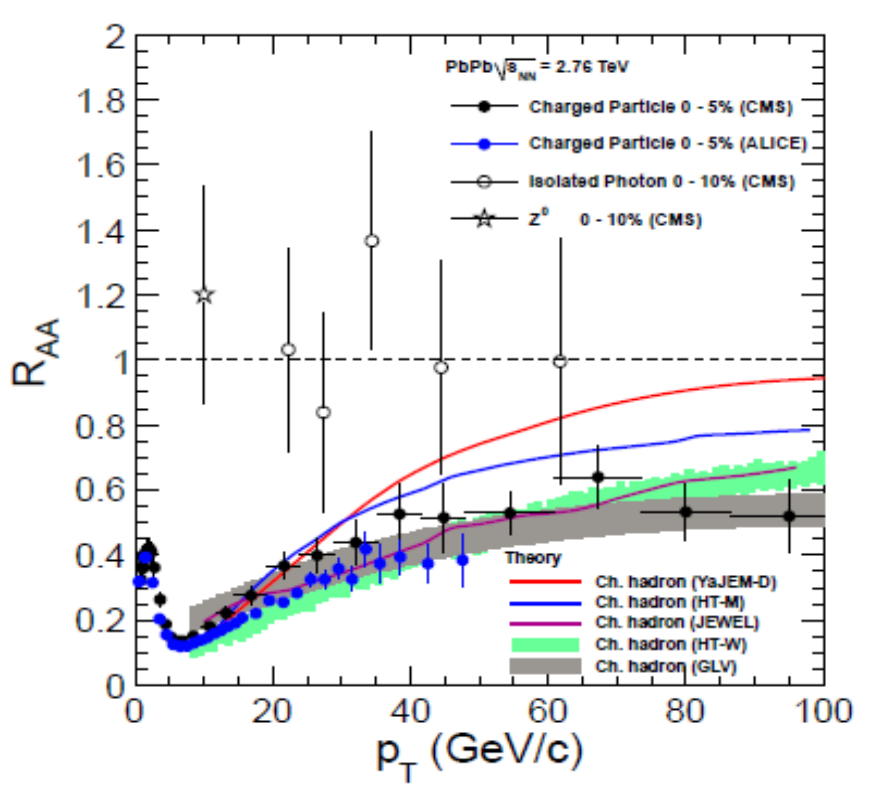


Nucleus-Nucleus collision



Nuclear Modification Factor (R_{AA}):

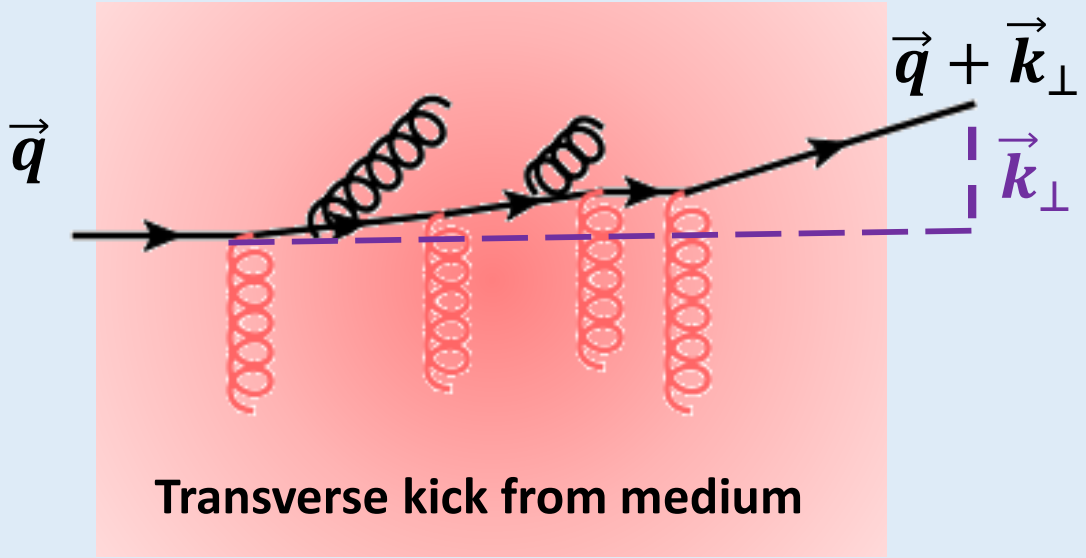
$$R_{AA} \equiv \frac{d^2N^{AA}/dydp_\perp}{d^2N^{pp}/dydp_\perp \times \langle N_{coll}^{AA}\rangle}$$



Mueller et al., Ann. Rev. Nucl. Part. Sci. 62, 361 (2012)

Transport coefficient $\hat{q}$ and leading hadron suppression

Leading parton going through medium



Transport coefficient $\hat{q}$: Average transverse momentum change per unit length

$$\hat{q}(\vec{r}, t) = \frac{< k_\perp^2 >}{L}$$

$\hat{q}$ is Input parameter to full model calculation

Transport coefficient $\hat{q}$ and leading hadron suppression

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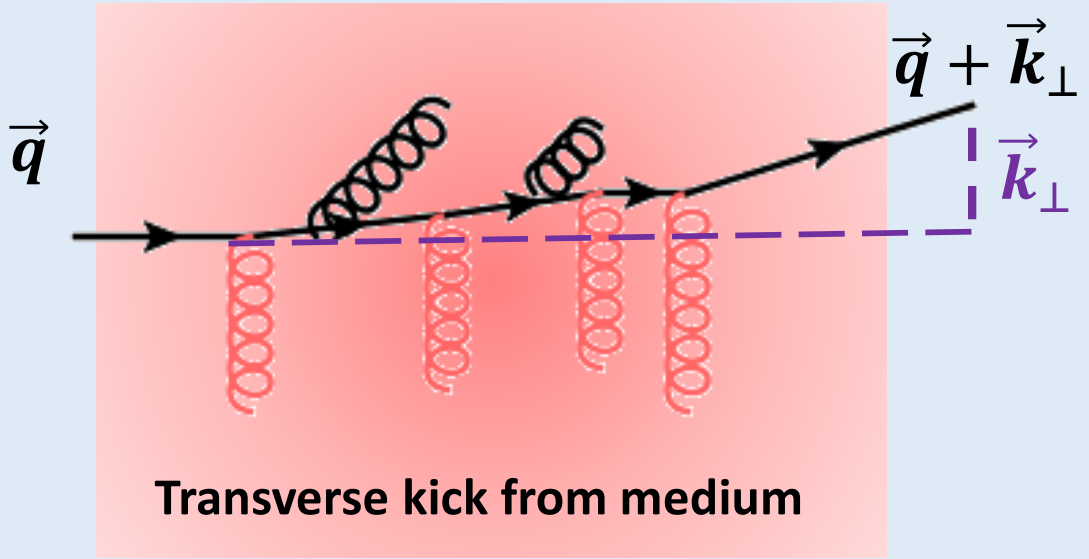
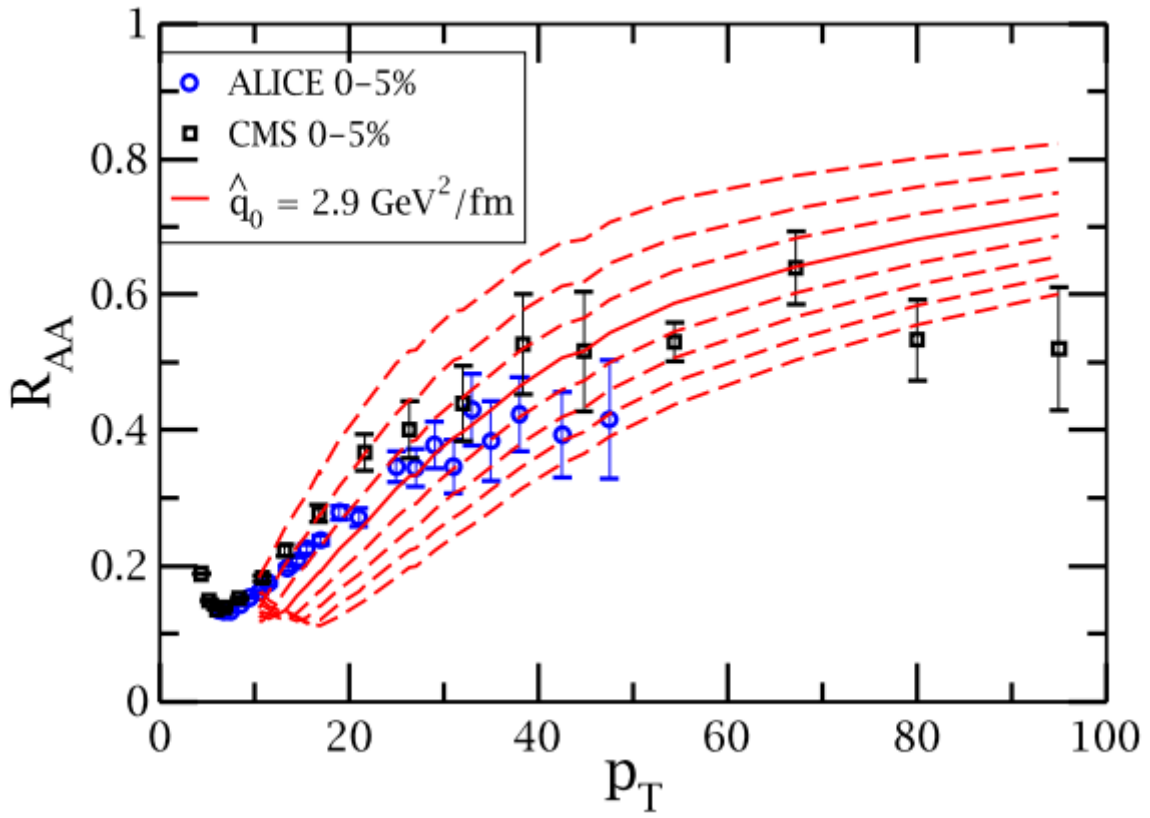


JET Collaboration(Burke et al. 2014)



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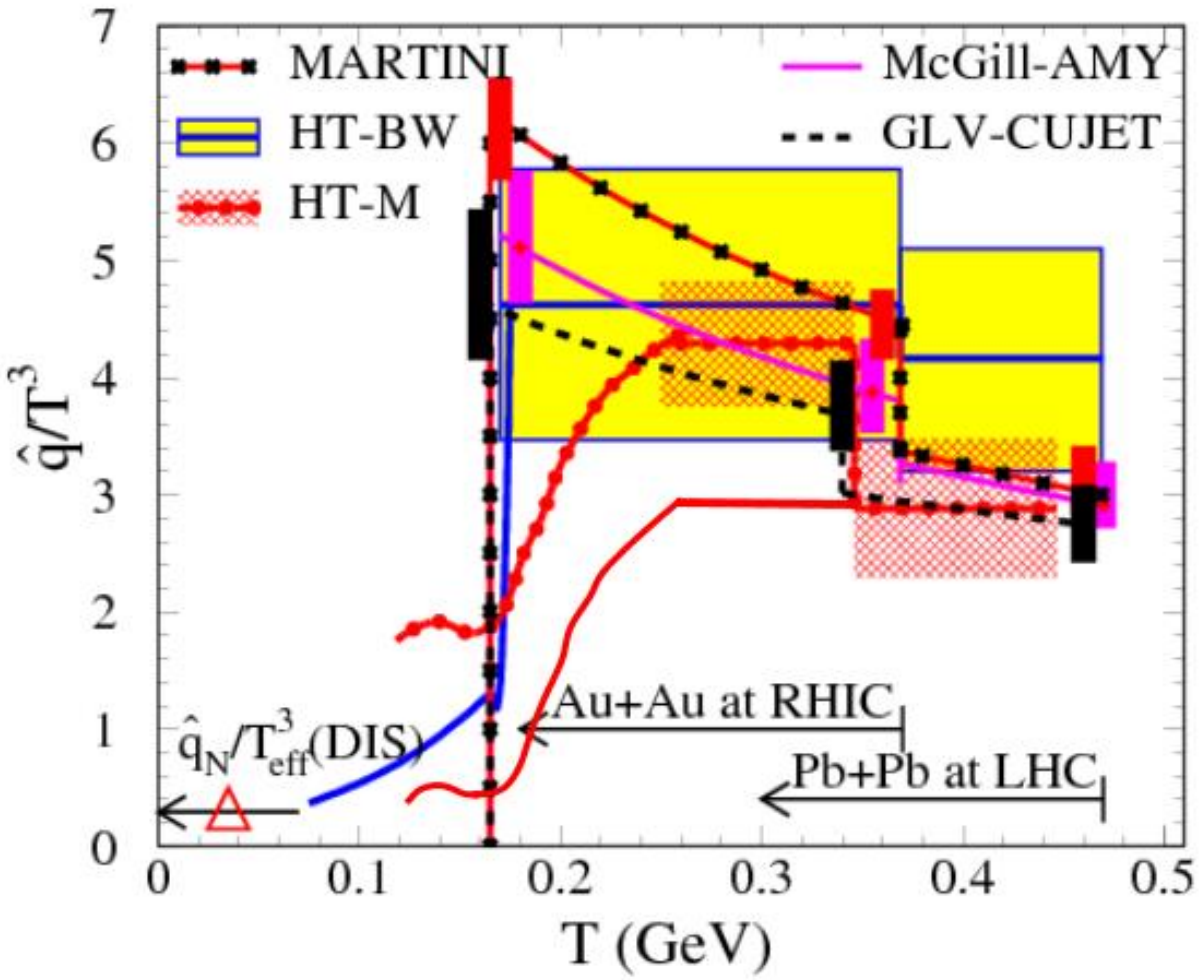
$\hat{q}$ is Input parameter to full model calculation

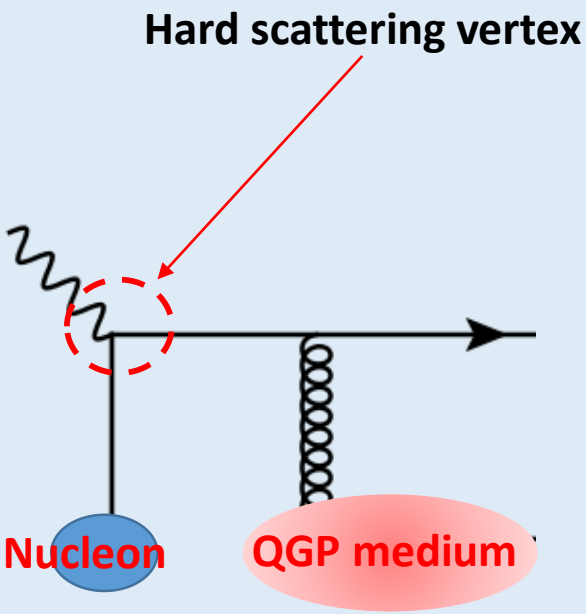
Transport coefficient $\hat{q}$ for hot QGP

Based on fit to the experimental data

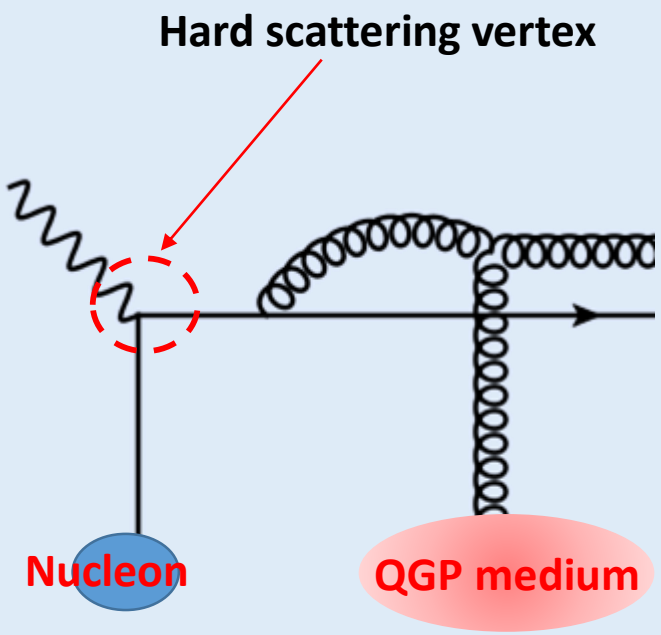
JET Collaboration(Burke et al. 2014)



Jet propagation through QGP medium



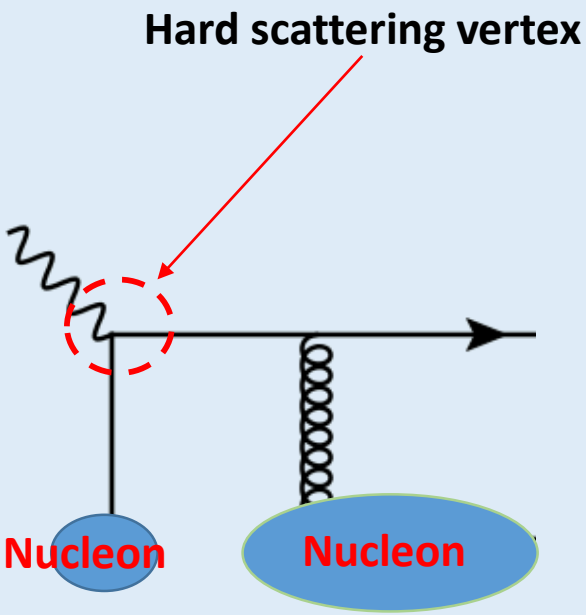
Glauber-gluon exchange



Collinear gluon emission
Glauber-gluon exchange
from the medium

One can use Feynman diagram techniques to compute $\hat{q}$ from these diagrams.

Jet modification in DIS or EIC experiments



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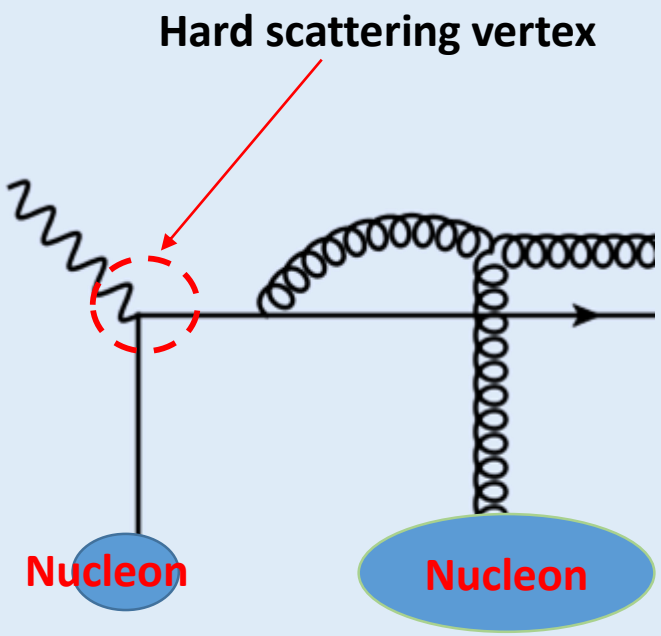


Collinear gluon emission
Glauber-gluon exchange
from Nucleon

One can use Feynman diagram techniques to compute $\hat{q}$ from these diagrams.

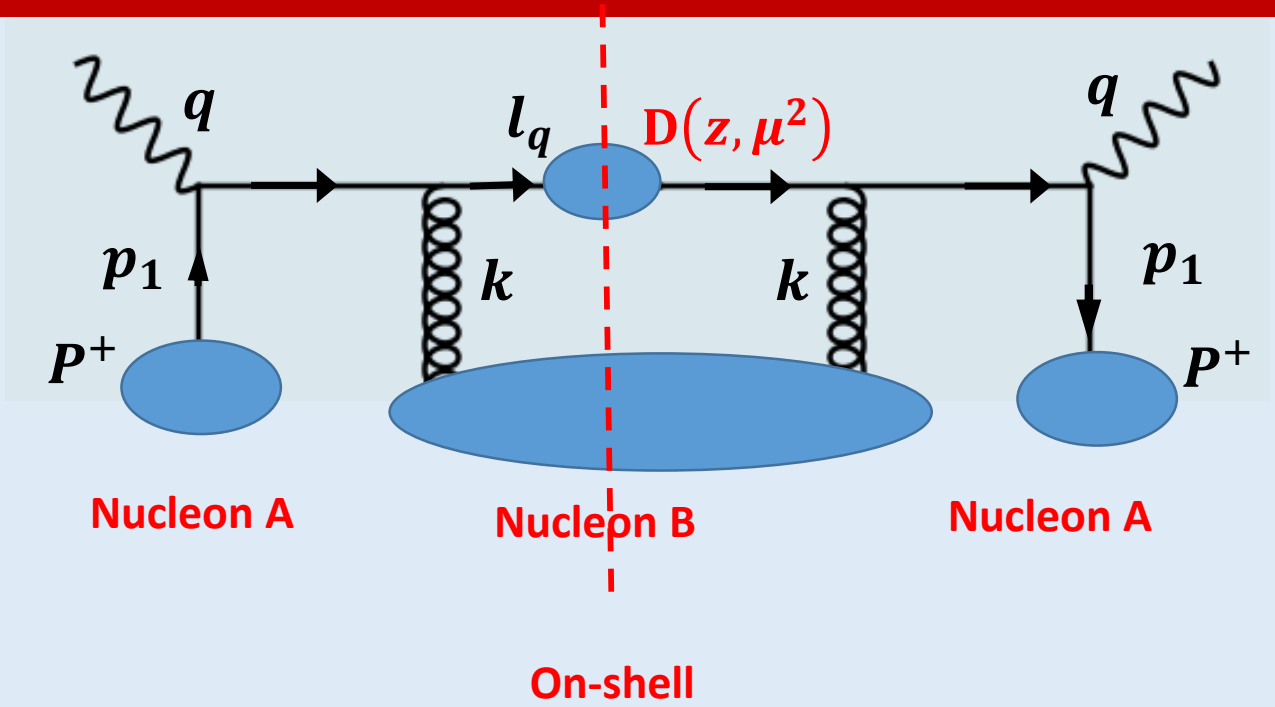
$\hat{q}$ for cold nuclear matter and hot QGP

In light-cone coordinate: (Breit frame)

Photon: $q^2 = (q^0)^2 - (\vec{q})^2 < 0; q = Q\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), -ve\ z\ dir$

Glauber gluon: $k \sim Q(\lambda^2, \lambda^2, \lambda)$; Q=Hard Scale; λ <<1

Parton p_1: $p_1 \sim Q(1, \lambda^2, \lambda)$



Jet Quenching Parameter

$$\hat{q}(\vec{r}, t) = \frac{< l_{q\perp}^2 >}{L}$$

$$\vec{l}_{q\perp} = \vec{k}_\perp$$

$$\hat{q} = \frac{4\pi^2 \alpha_s}{N_c} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} d^2 k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp . \vec{y}_\perp}$$

$$\text{x} \langle M | F_\perp^{+\mu}(y^-, y_\perp) F_{\perp\mu}^{+}(0) | M \rangle$$

- $\hat{q}$ is the one transport coefficient which can be identically defined in cold nuclear matter and hot QGP
- |M> is thermal state, weighted by boltzman factor
- |M> is nuclear state at zero temperature

Radiated-gluon Scattering from nucleus

(In light-cone coordinate)

Glauber Gluons

$q^2 = (q^0)^2 - (\vec{q})^2 < 0$

$q = \left(\frac{-Q}{\sqrt{2}}, \frac{Q}{\sqrt{2}}, 0\right)$

$p_1 \sim Q(1, \lambda^2, \lambda)$

$l_q \sim Q(\lambda^2, 1, \lambda)$

$k \sim Q(\lambda^2, \lambda^2, \lambda)$

$p_2 \sim Q(1, \lambda^2, \lambda)$

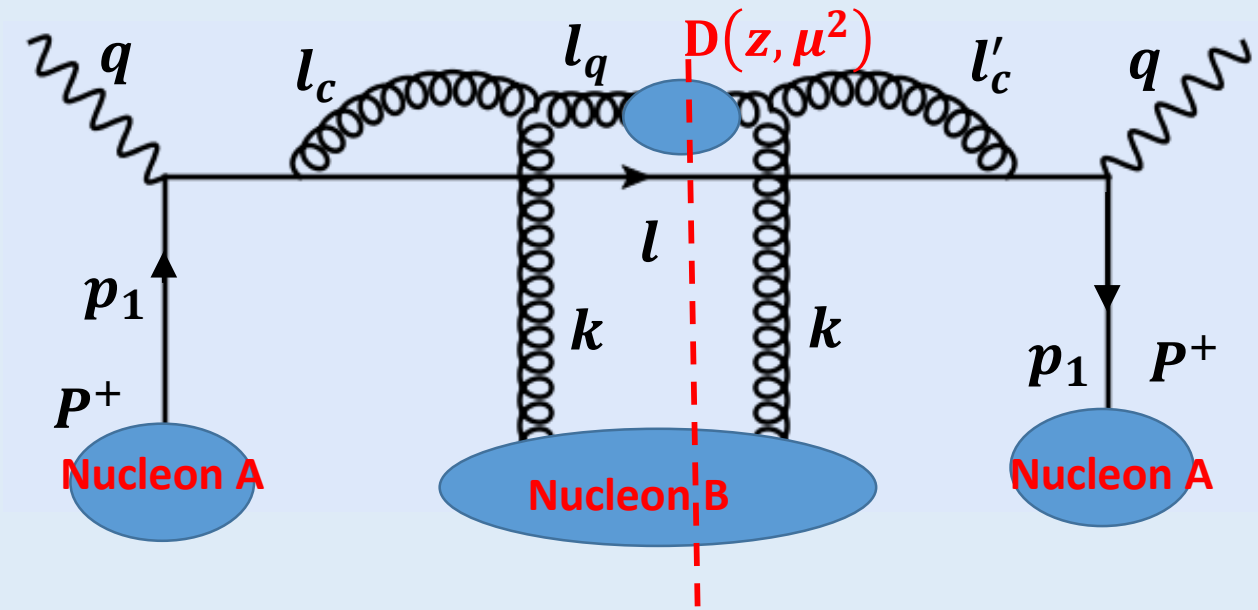
$l_p \sim Q(1, \lambda^2, \lambda)$

Jet Quenching Parameter

$$\hat{q}(\vec{r}, t) = \frac{< l_{q\perp}^2 >}{L}$$

In regime

$\vec{l}_{q\perp}, \vec{l}_\perp \gg \vec{k}_\perp$



Radiated-gluon Scattering from the QGP

(In light-cone coordinate)

$q^2 = (q^0)^2 - (\vec{q})^2 < 0$

$q = \left(\frac{-Q}{\sqrt{2}}, \frac{Q}{\sqrt{2}}, 0\right)$

$p_1 \sim Q(1, \lambda^2, \lambda)$

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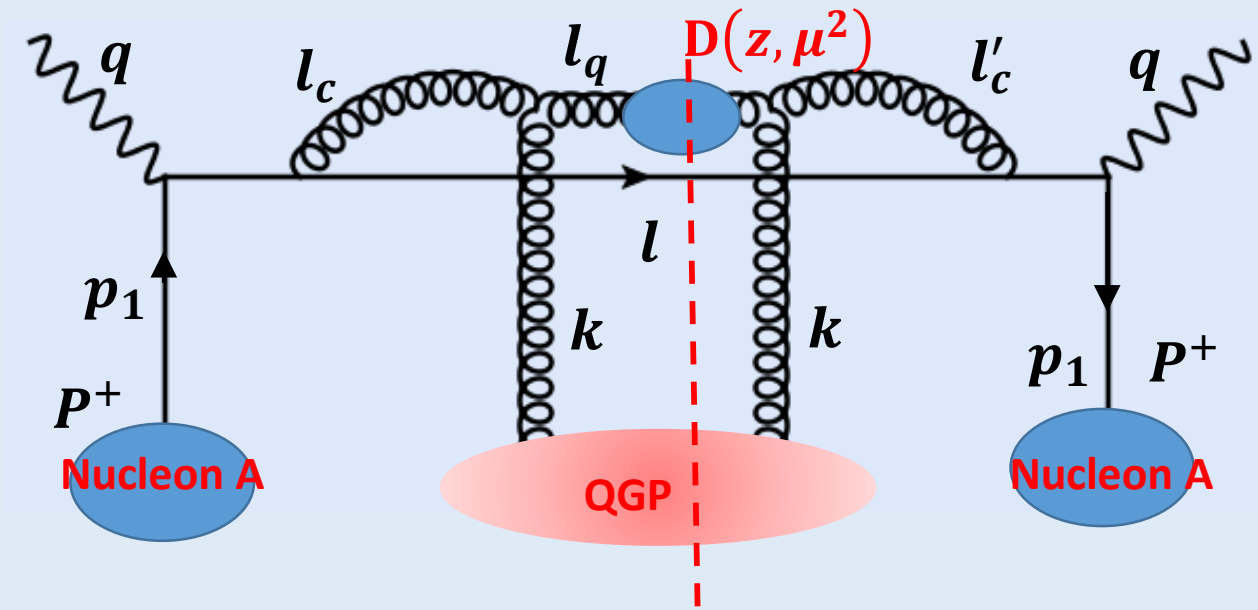
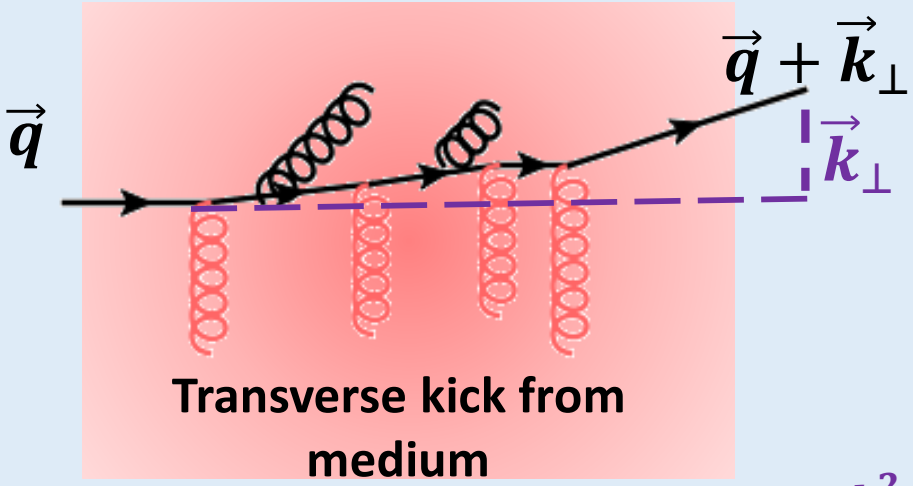


Computing $\hat{q}$ for cold nuclear matter or hot QGP from first principle is quite challenging

QGP is locally thermalized and highly non-perturbative



Transport parameter $\hat{q}(\vec{r}, t) = \frac{<k_\perp^2>}{L}$

First principles calculation: Lattice QCD to compute $\hat{q}$

Lattice formulation of $\hat{q}$

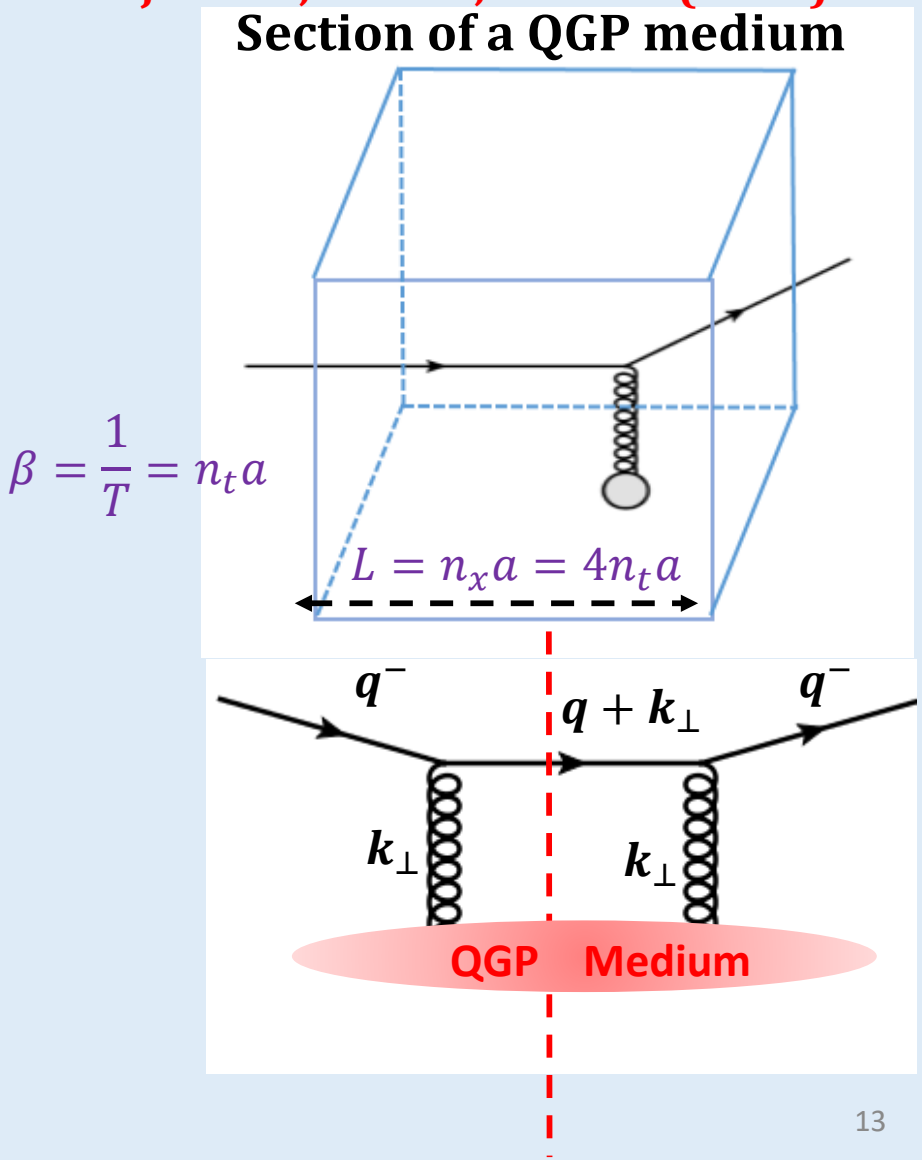
A. Majumder, PRC 87, 034905 (2013)

- Simplest process: A leading quark propagating through **hot plasma (gluons only)** at temperature T

$$\boldsymbol{q = \left(\mu^2/2q^-, q^-, 0\right) = \left(\lambda^2, 1, 0\right)Q; \quad Hard\ scale = Q; \lambda \ll 1}$$

$$\boldsymbol{k = \left(k^+, k^-, k_\perp 0\right) = \left(\lambda^2, \lambda^2, \lambda\right)Q; Glauber\ gluon}$$

- Life time of quark, $\boldsymbol{\tau \geq 4n_t a = \frac{4}{T}}$



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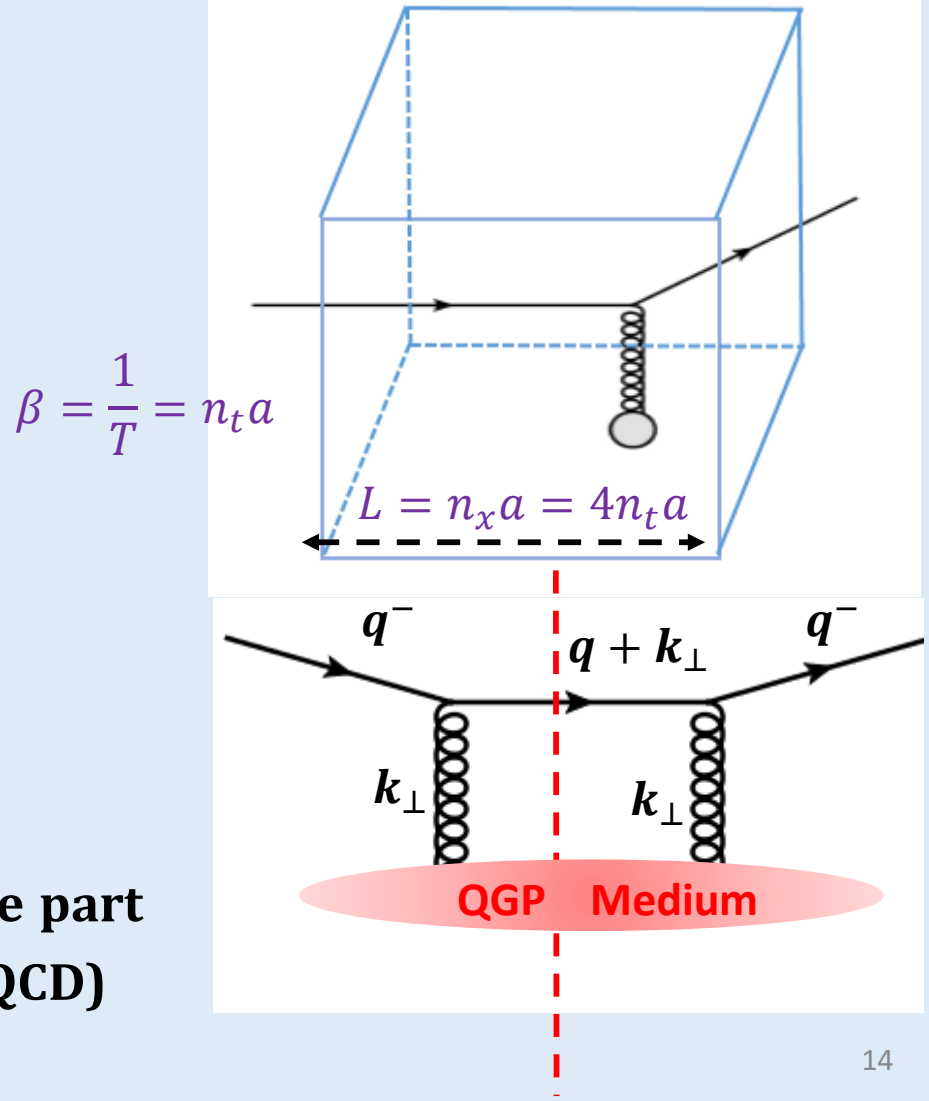
- Life time of quark, $\tau \geq 4n_t a = \frac{4}{T}$

$$\boxed{\hat{q}(\vec{r}, t) = \sum_k k_\perp^2 \frac{Disc[W(k)]}{L}}$$

$$\boxed{\hat{q} = \frac{4\pi^2 \alpha_s}{N_c} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} d^2 k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp . \vec{y}_\perp} \times \langle M | F_\perp^{+\mu}(y^-, y_\perp) F_{\perp\mu}^{+}(0) | M \rangle}$$

Non-perturbative part (Lattice QCD)

Section of a QGP medium



Constructing a more general expression as $\hat{Q}$

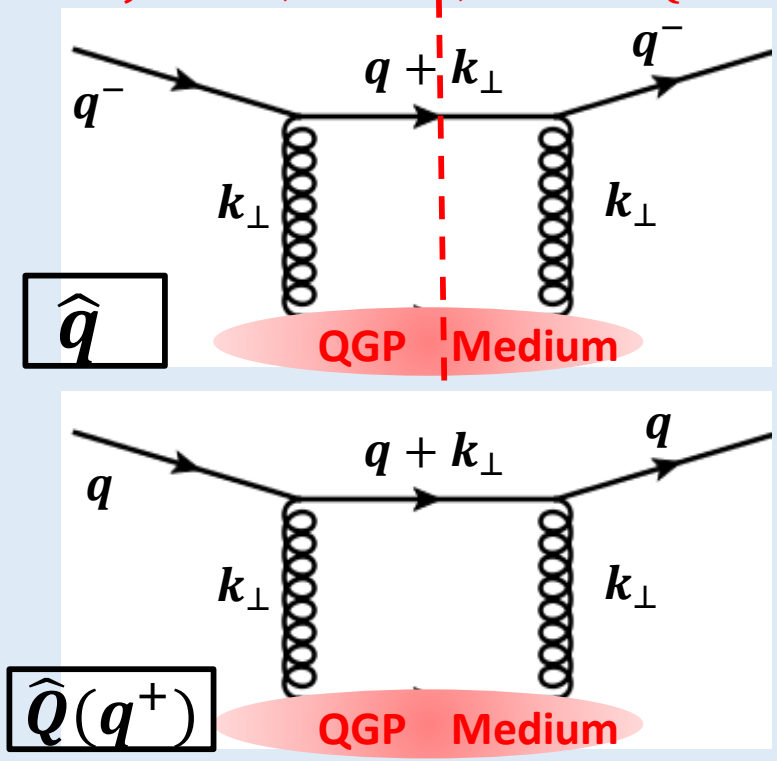
A. Majumder, PRC 87, 034905 (2013)

❖ **Physical form of $\hat{q}$**

$$\hat{q} = \frac{4\pi^2\alpha_s}{N_c}\int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp.\vec{y}_\perp} \langle M|F_\perp^{+\mu}(y^-, y_\perp)F_{\perp\mu}^{+}(0)|M\rangle$$

❖ **General form of $\hat{q}$:** with q^- *is Fixed*; $q_\perp = 0$; q^+ *is variable*

$$\hat{Q}(q^+) = \frac{4\pi^2\alpha_s}{N_c}\int \frac{d^4y d^4k}{2\pi^4} e^{iky}\, 2q^- \frac{\langle M|F_\perp^{+\mu}(0)\, F_{\perp\mu}^{+}(y)|M\rangle}{(q+k)^2 + i\epsilon}$$



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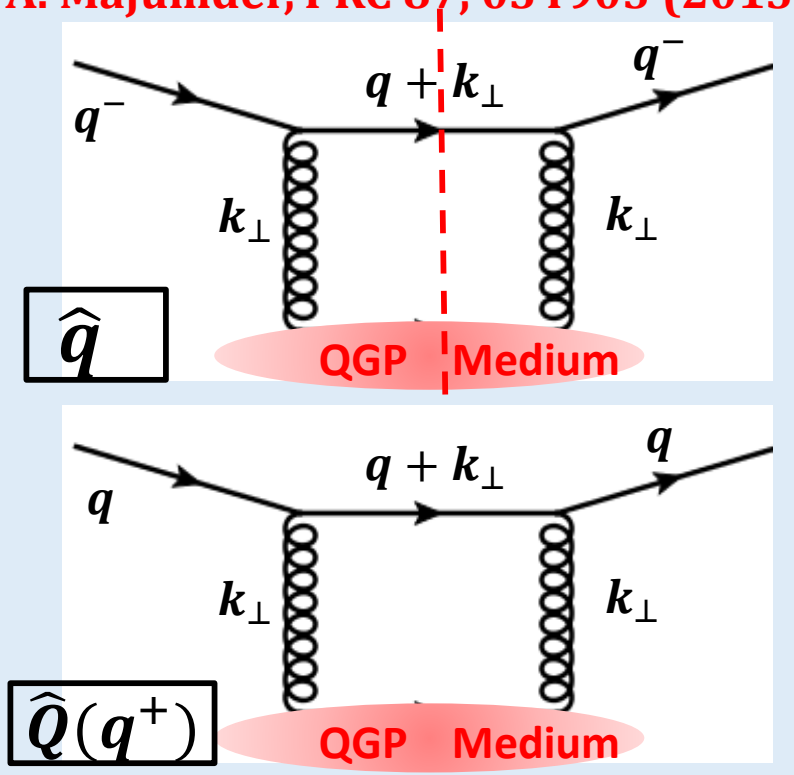
1) When $q^+ \sim T$

$$\mathrm{Disc}\left[\hat{Q}(q^+)\right]_{at\ q^+\sim T} = \hat{q}$$



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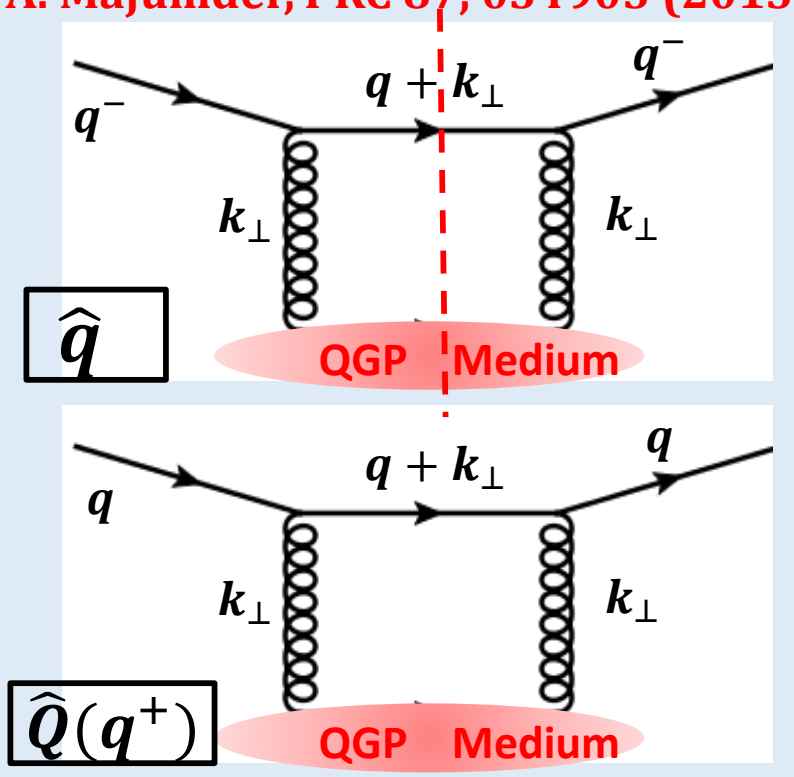

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2) When $q^+ = -q^-$

$$\frac{1}{(q+k)^2} \simeq \frac{1}{-2q^-q^- + 2q^-(k^+ - k^-)} = -\frac{1}{2(q^-)^2}\left[1 - \left(\frac{k^+ - k^-}{q^-}\right)\right]^{-1} = -\frac{1}{2(q^-)^2}\left[1 - \left(\frac{\sqrt{2}k_z}{q^-}\right)\right]^{-1} = -\frac{1}{2(q^-)^2}\left[\sum_{n=0}^{\infty}\left(\frac{\sqrt{2}k_z}{q^-}\right)^n\right]$$

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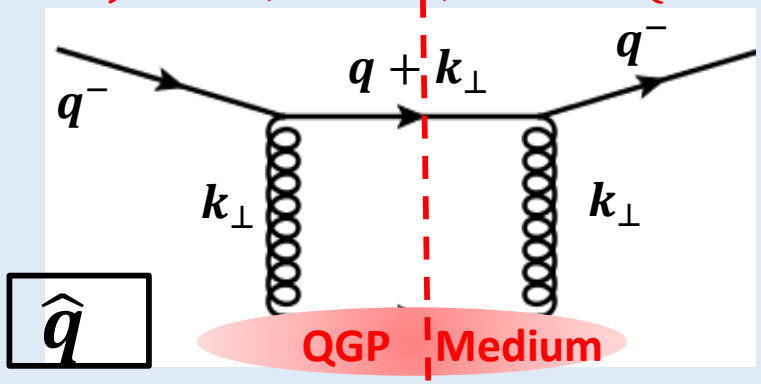

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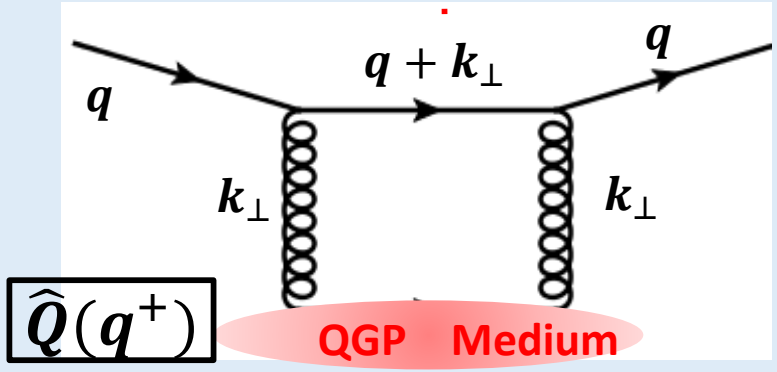

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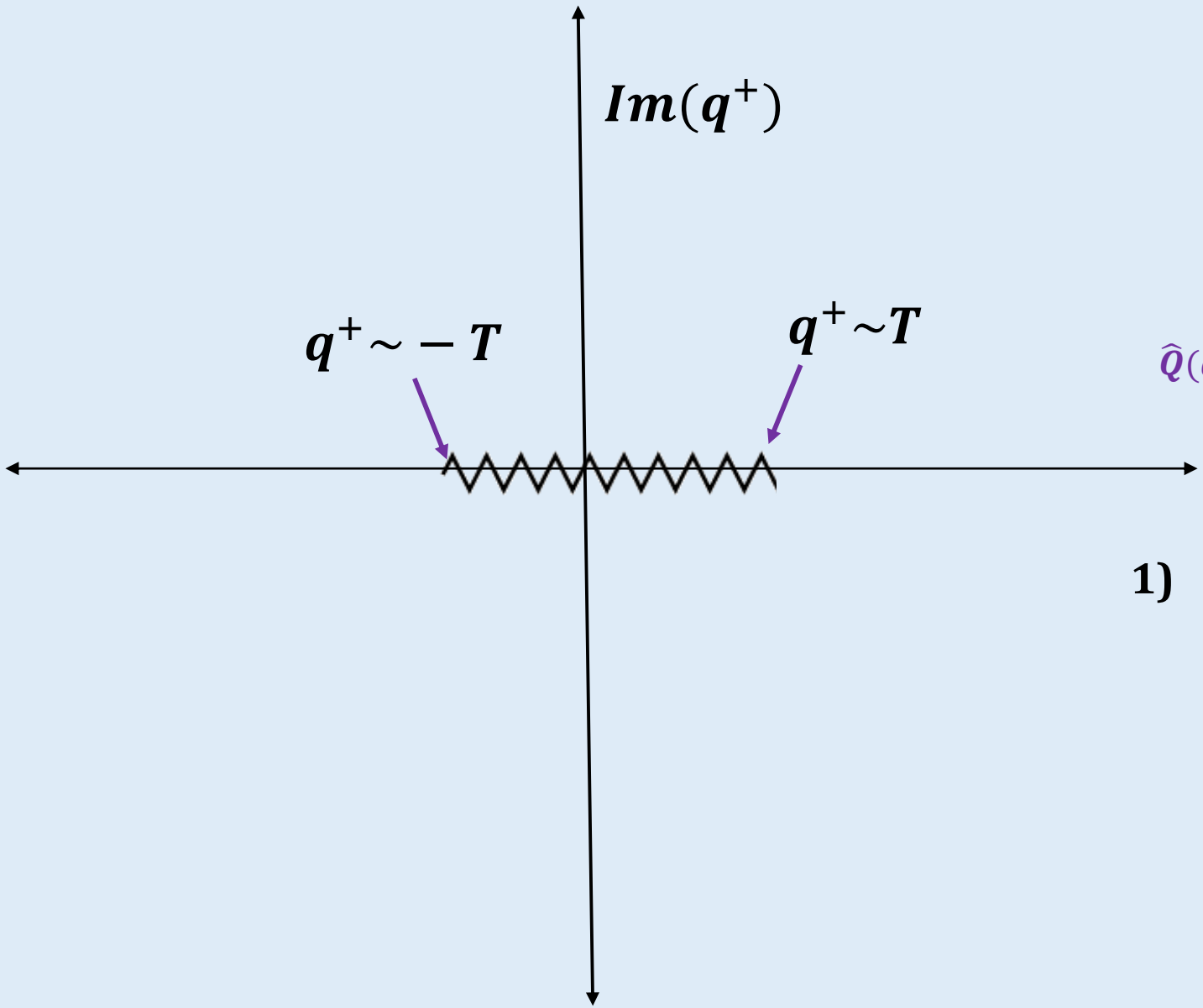
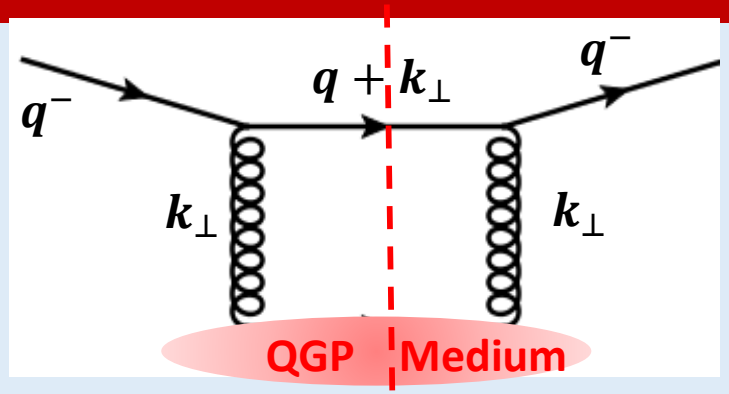
$$\hat{Q}(q^+ = -q^-) = \frac{4\pi^2\alpha_s}{N_c\, q^-}\langle M|F_\perp^{+\mu}(0)\sum_{n=0}^{\infty}\left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F_{\perp\mu}^{+}(0)\,|M\rangle$$

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$





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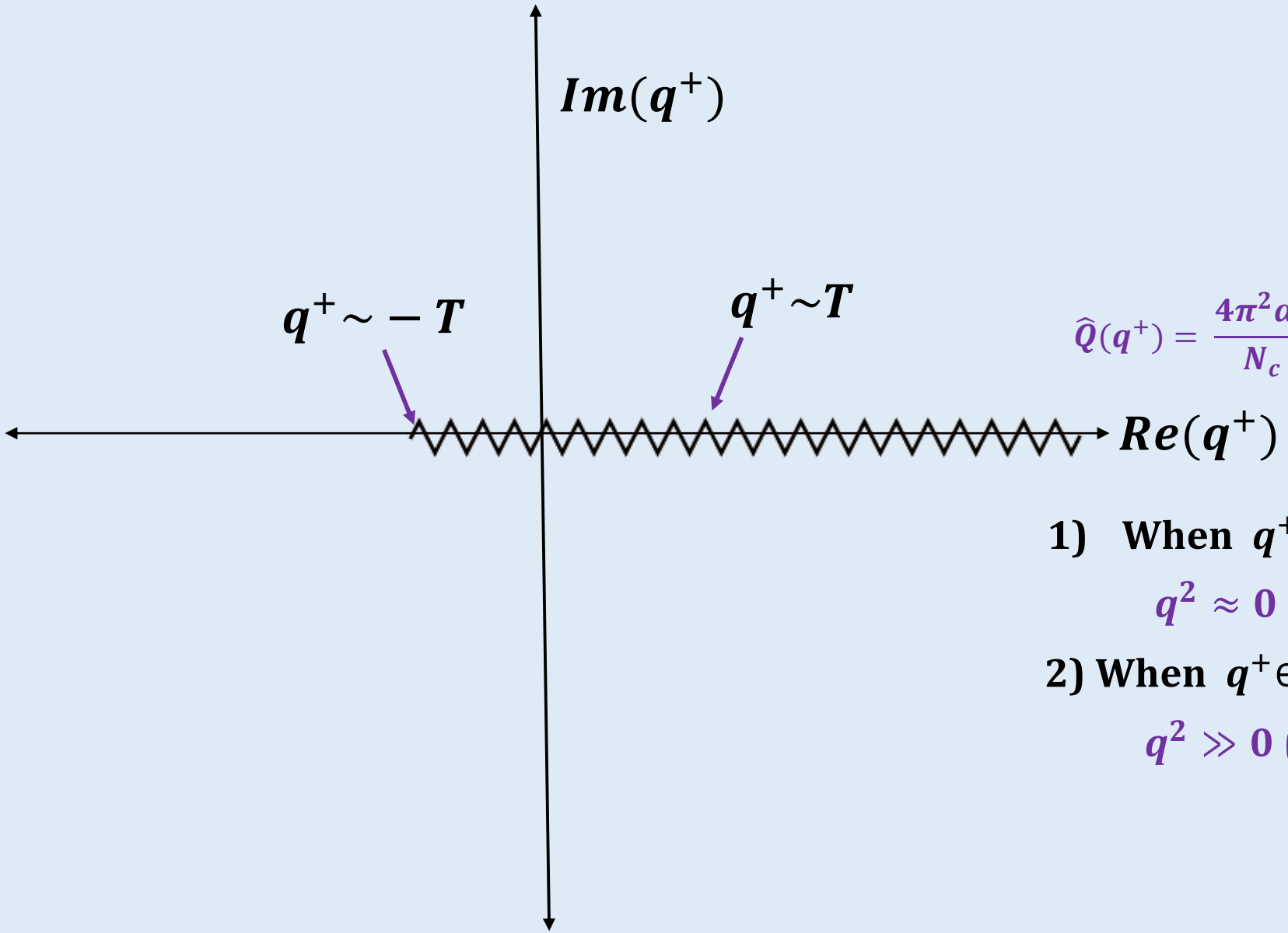
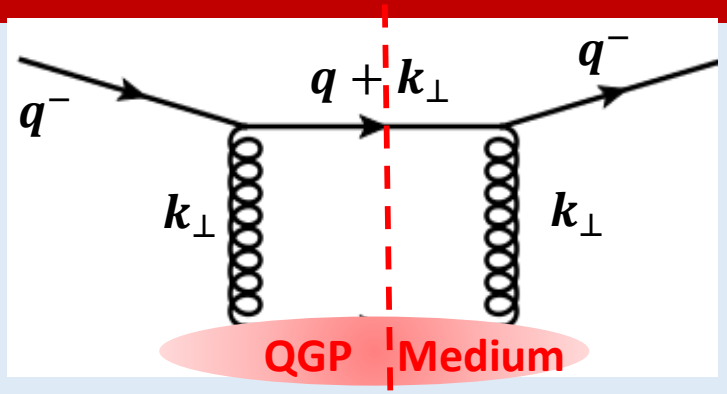
1) When $q^+ \in \#[-T, T]$

$q^2 \approx 0$ (in-medium scattering)

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$





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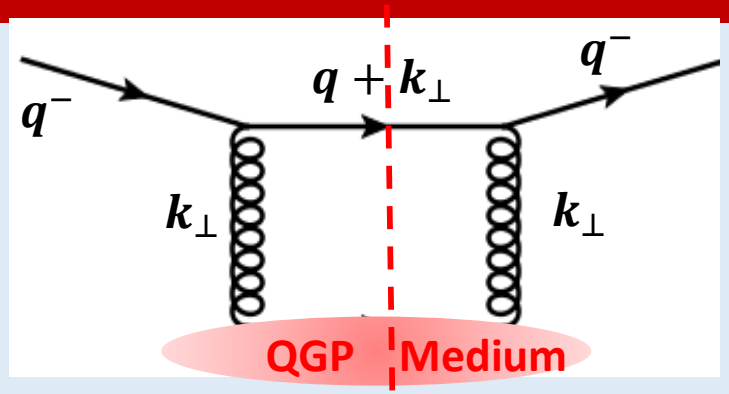
1) When $q^+ \in [-\#T, \#T]$

$q^2 \approx 0$ (in-medium scattering)

2) When $q^+ \in [\#T, +\infty)$

$q^2 \gg 0$ (Bremsstrahlung radiation)

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$



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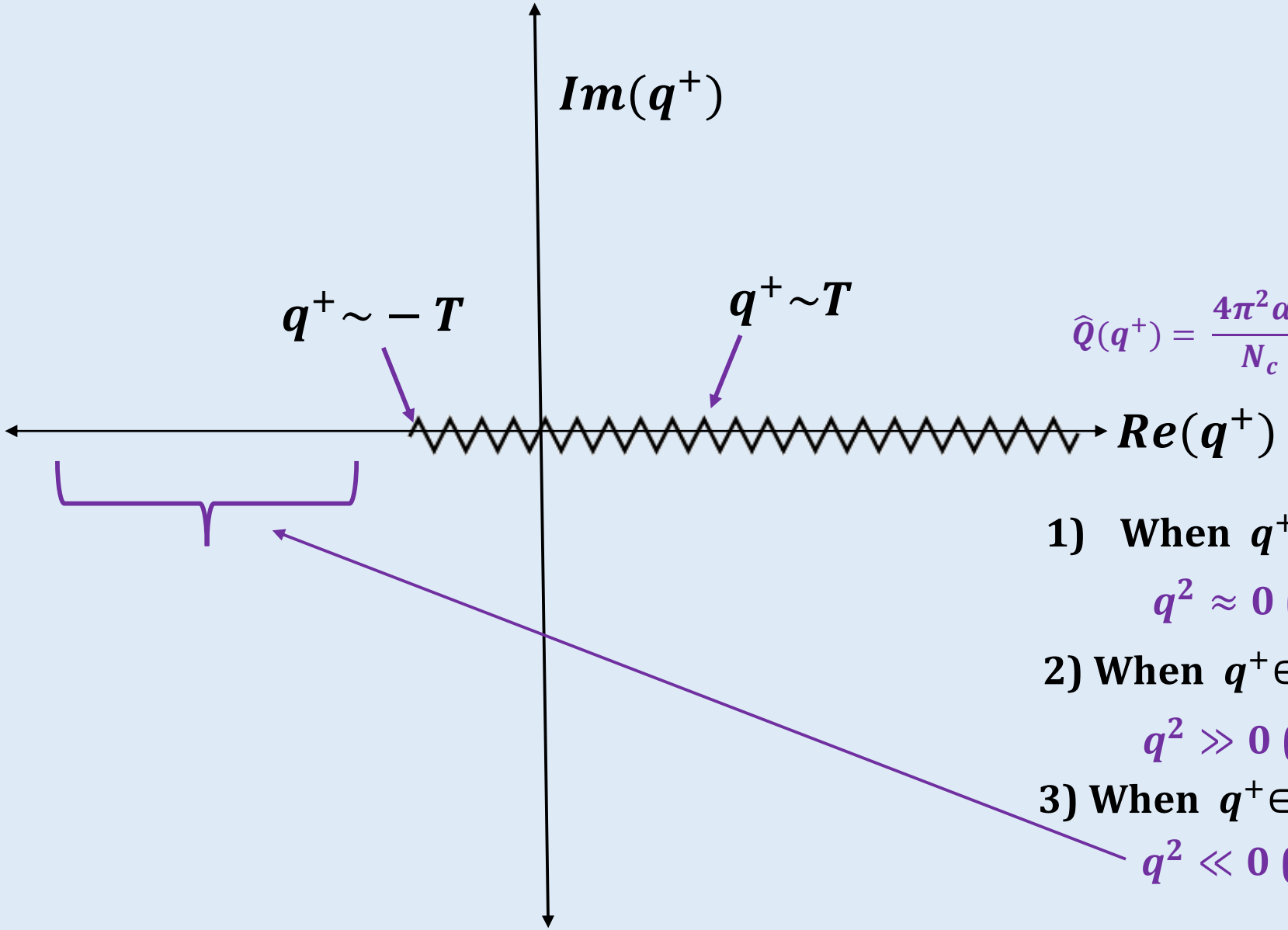

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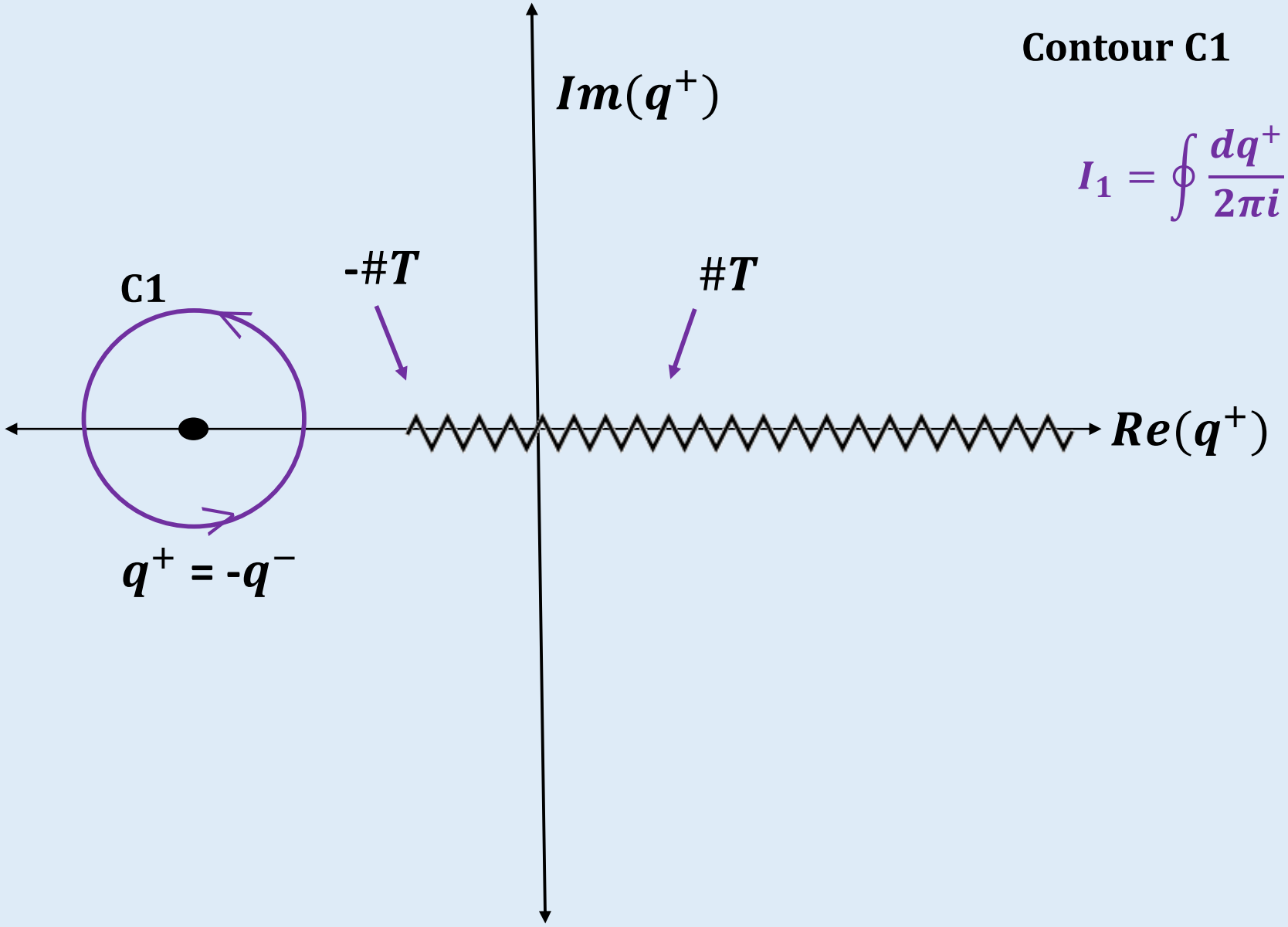
3) When $q^+ \in (-\infty, -\#T]$

$q^2 \ll 0$ (Space-like) ; $\lim_{q^+ \to -\infty} \mathrm{Disc}[\hat{Q}(q^+)] = 0$

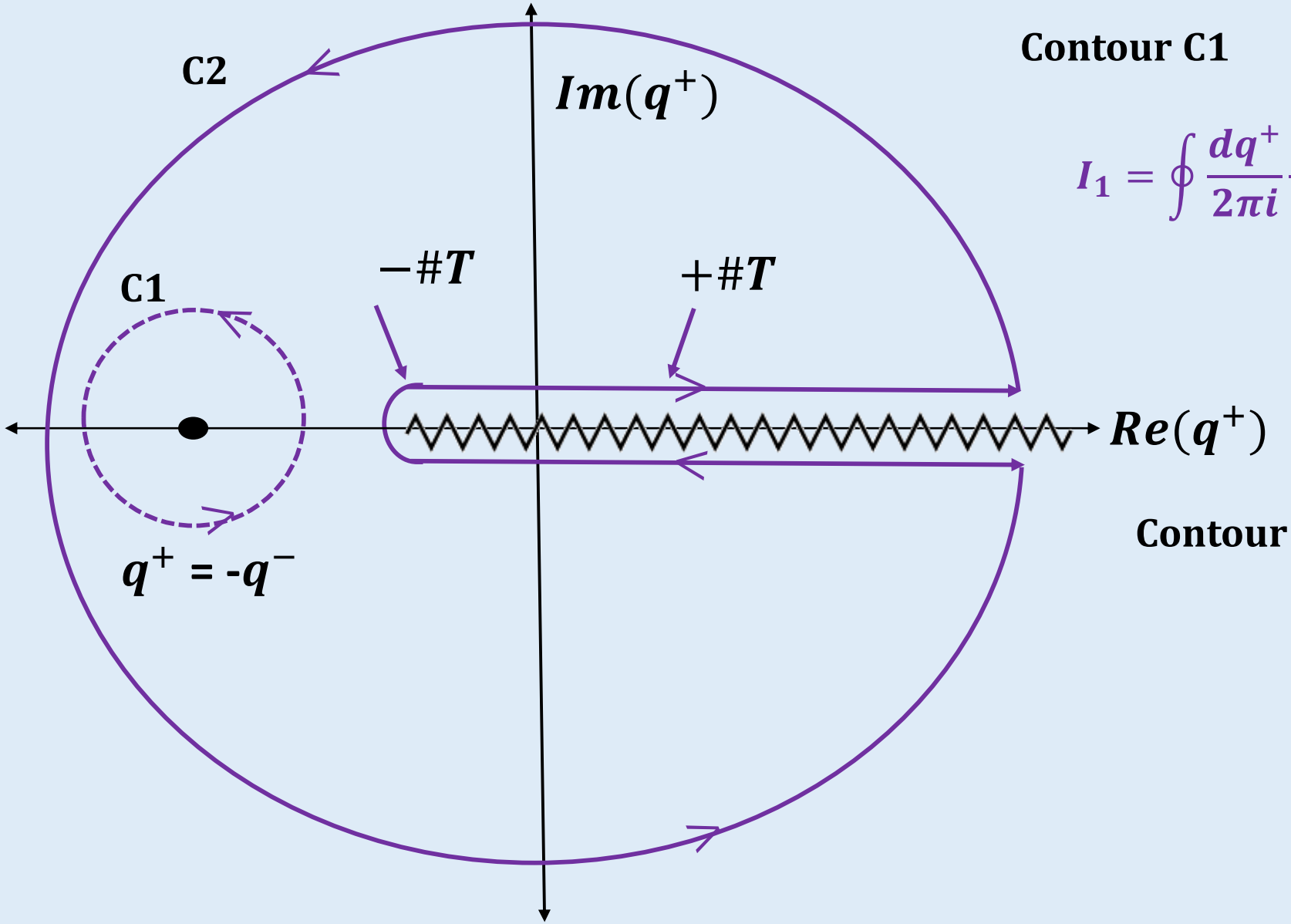
Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$



Contour C1

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$



Contour C1

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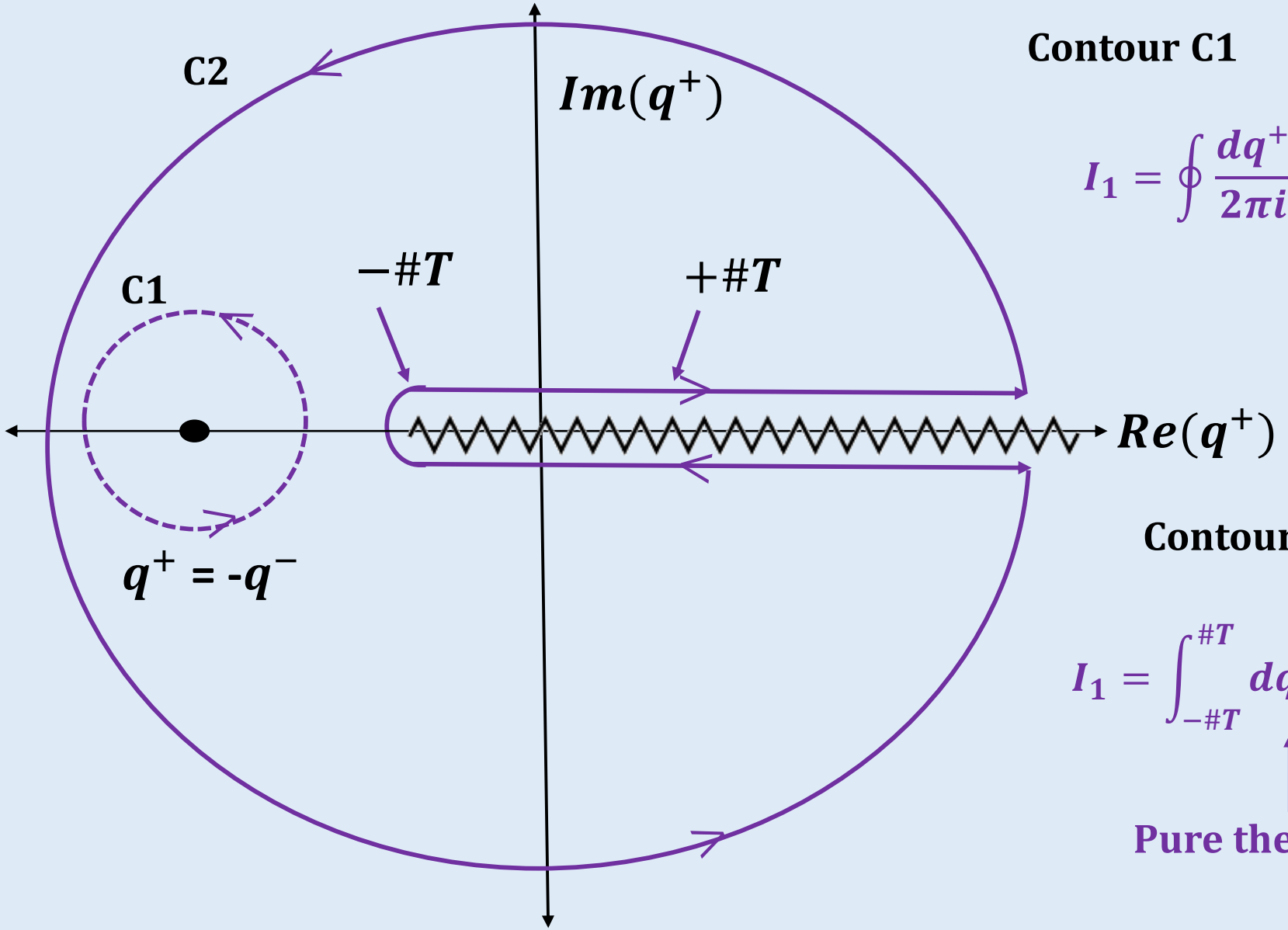
Contour C2: On stretching it to infinity

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$



Contour C1

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Contour C2: On stretching it to infinity

$$I_1 = \int_{-\#T}^{\#T} dq^+ \frac{\hat{q}(q^+)}{q^+ + q^-} + \int_0^\infty dq^+ \frac{Disc[\hat{Q}(q^+)]}{q^+ + q^-}$$

Pure thermal part **Pure Vacuum part**

$\hat{q}$ as a series of local operators

❖ **Physical form of $\hat{q}$ at LO:**

$$\hat{q} = \frac{4\sqrt{2}\pi^2\alpha_s}{N_c T}\langle M|F_{\perp}^{+\mu}(0)\sum_{n=0}^{\infty}\left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F_{\perp\mu}^{+}(0)\,|M\rangle_{(Thermal-Vacuum)}$$

$\hat{q}$ as a series of local operators

Xiangdong Ji, PRL 110, 262002 (2013)
Parton PDF operator product expansion with D_z derivatives

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Rotating to Euclidean space:

$$x^0 \to -ix^4; \quad A^0 \to iA^4$$

$$\Longrightarrow \quad F^{0i} \to iF^{4i}$$

LO operators:

$$\sum_{i=1}^{2} Trace\left[F^{3i}F^{3i} - F^{4i}F^{4i}\right] + 2i\sum_{i=1}^{2} Trace\left[F^{3i}F^{4i}\right]$$

Uncrossed operator **Crossed operator**

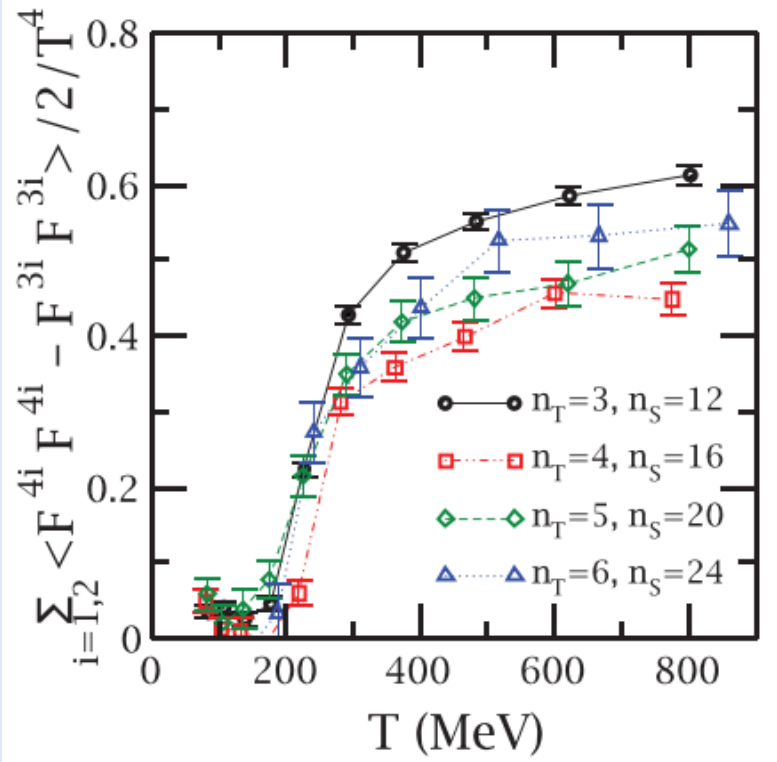
LO operators with D_z derivative:

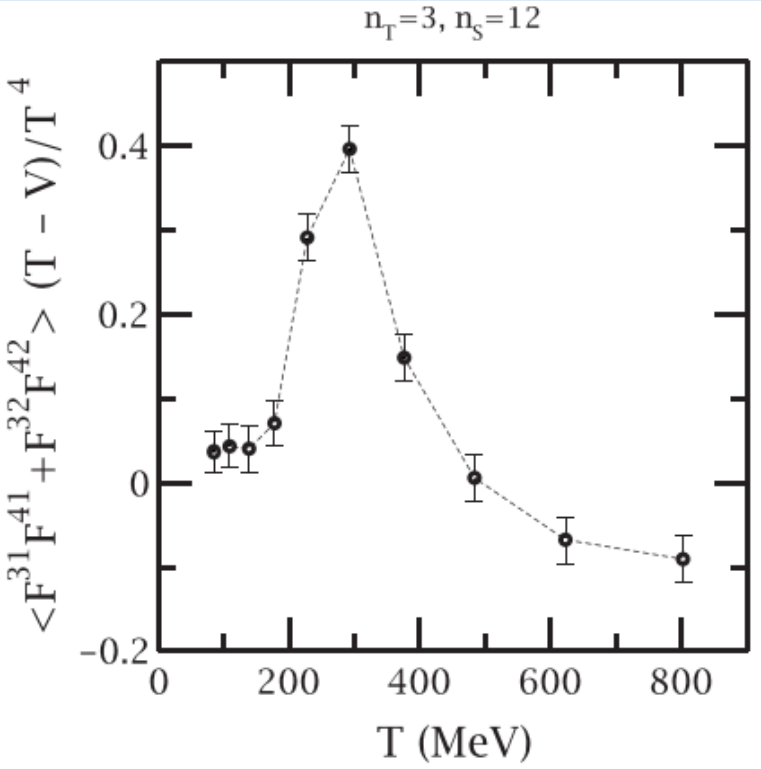
$$\sum_{i=1}^{2} Trace\left[F^{3i}D_zF^{3i} - F^{4i}D_zF^{4i}\right] + i\sum_{i=1}^{2} Trace\left[F^{3i}D_zF^{4i} + F^{4i}D_zF^{3i}\right]$$

Operators in quenched SU(2) plasma

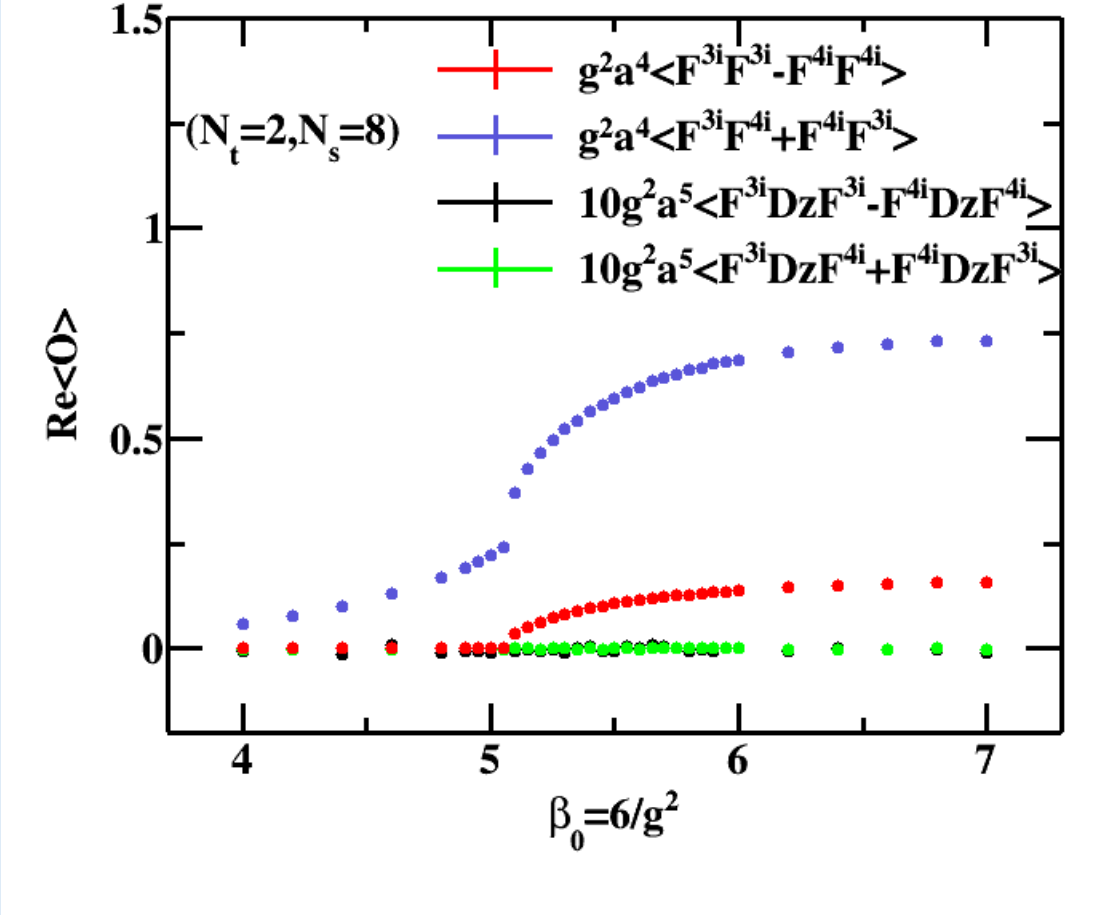
- **Average over 5000 configuration**
- **Transition temperature $T_c \in [170, 350]$ MeV**
- **Crossed correlator is small for T ~ 400 MeV**

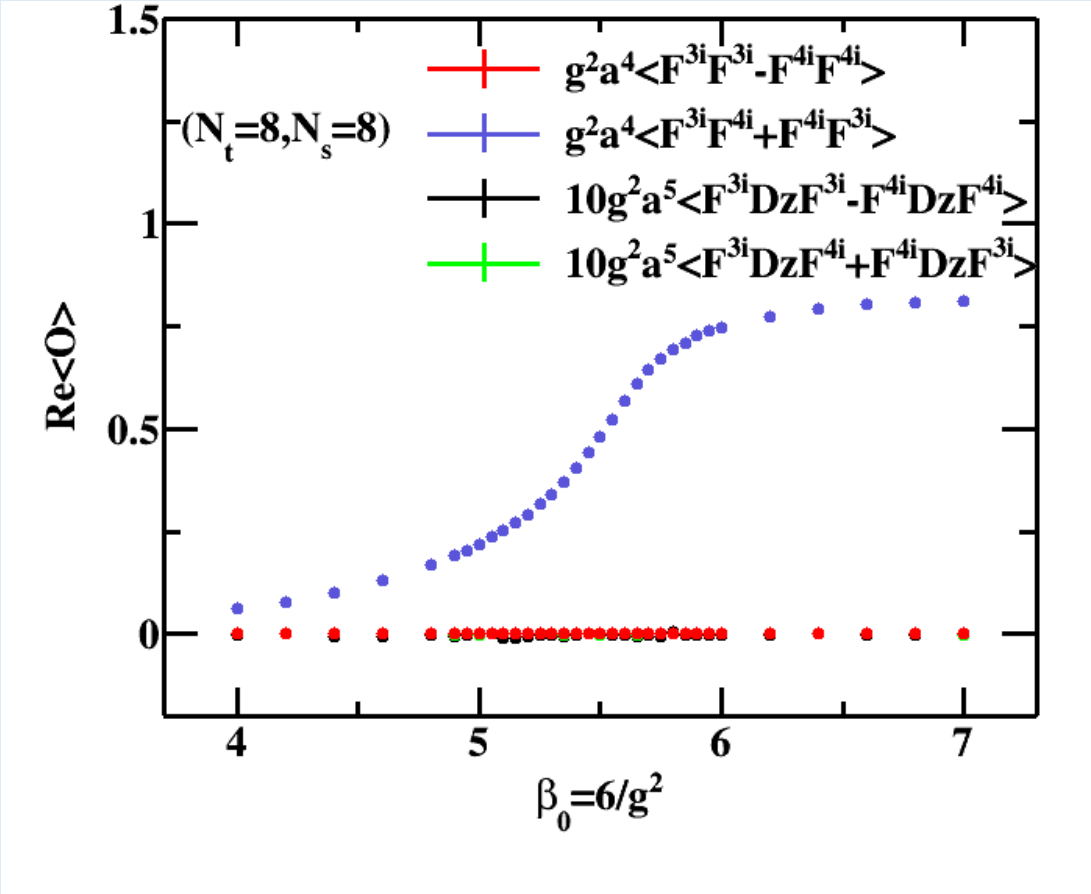
A. Majumder, PRC 87, 034905 (2013)





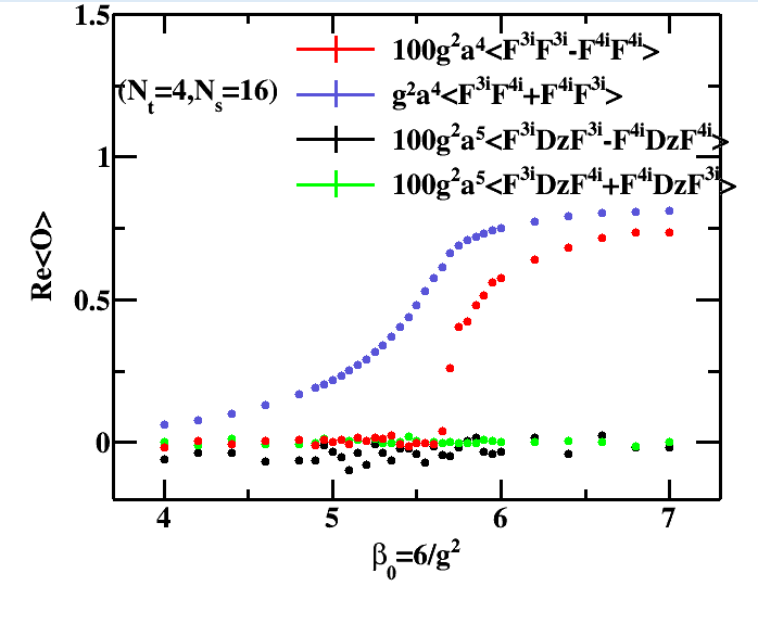
Operators in quenched SU(3) plasma

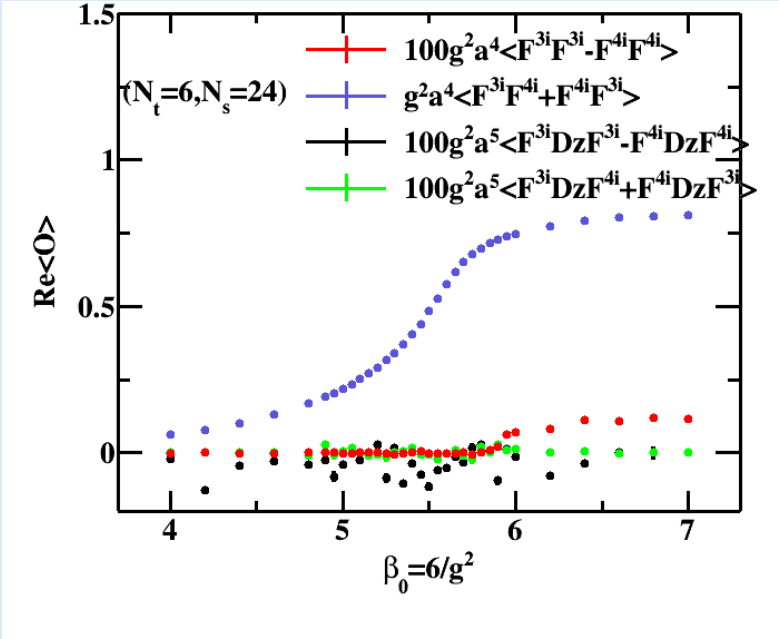


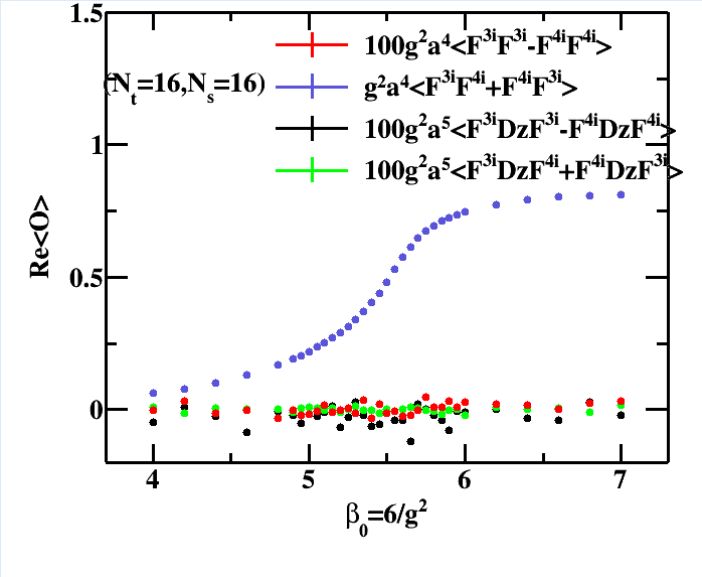


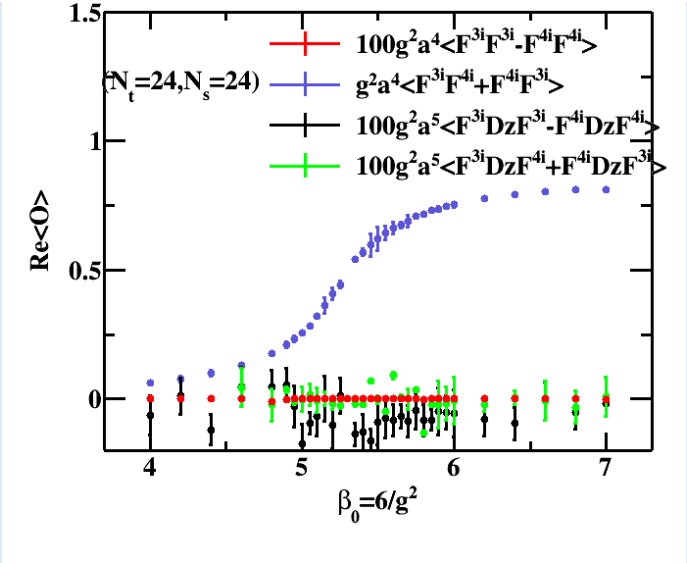
Preliminary
(in collaboration with Chiho Nonaka)

Operators in quenched SU(3) plasma



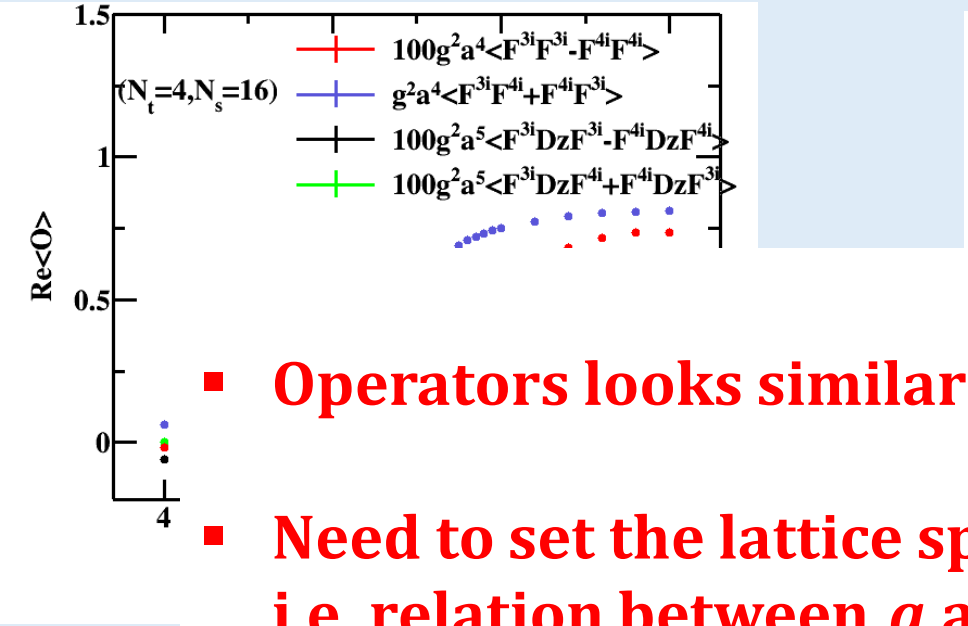


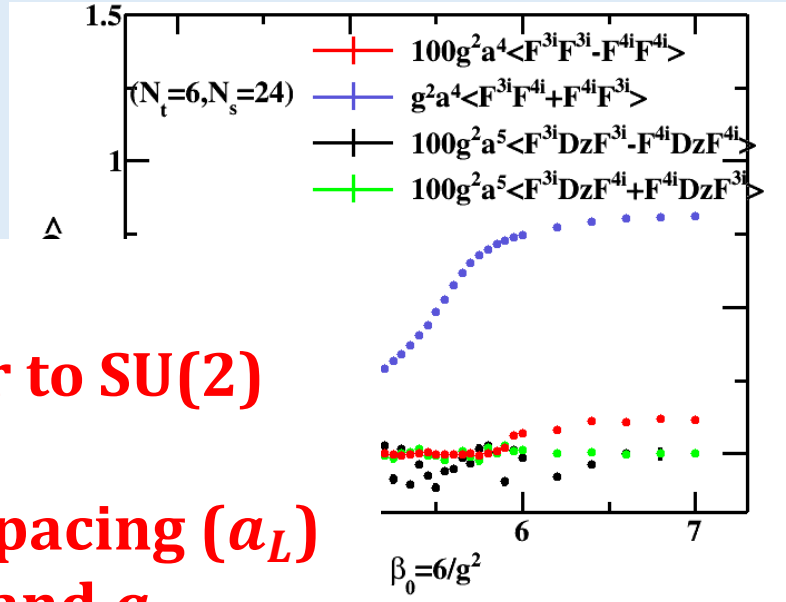




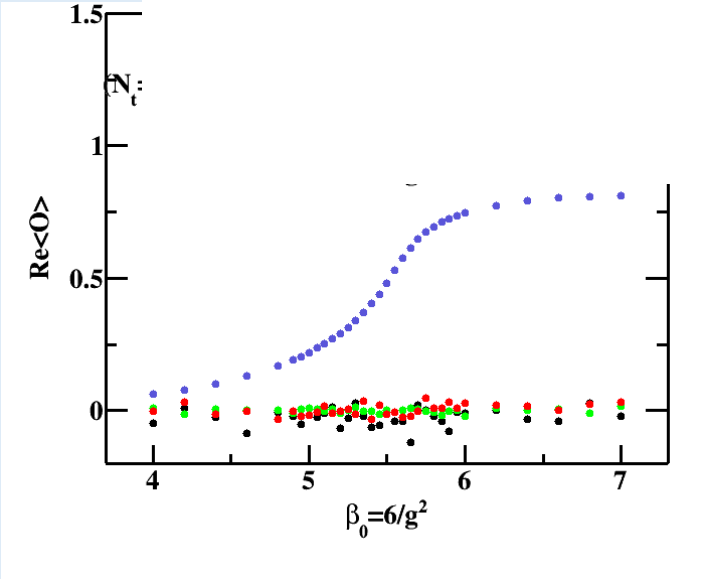
Preliminary (in collaboration with Chiho Nonaka)

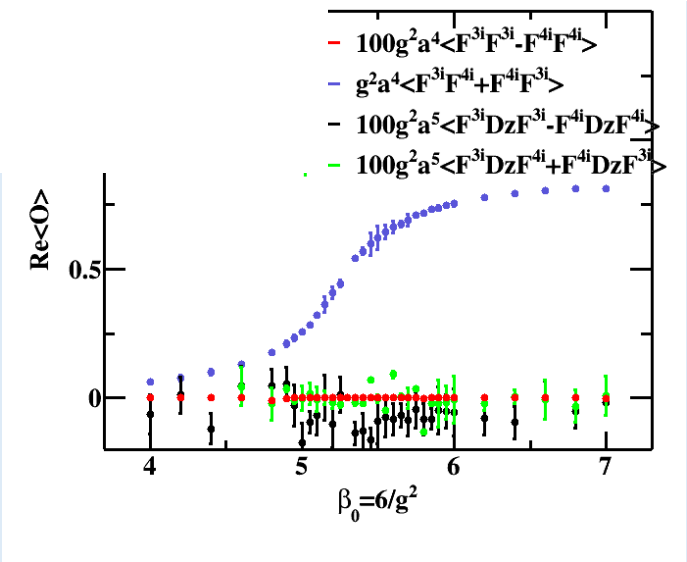
Operators in quenched SU(3) plasma





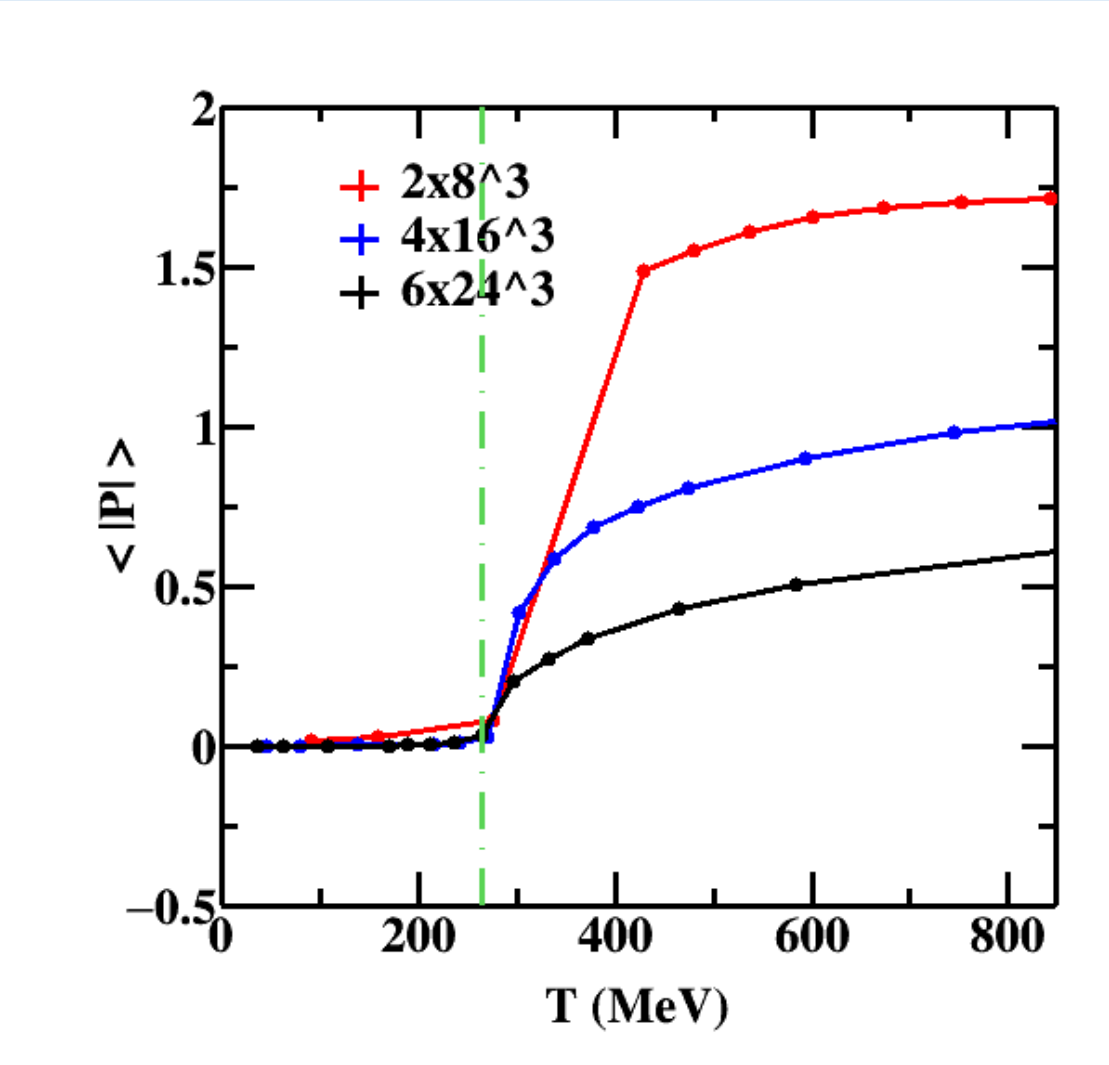
- **Operators looks similar to SU(2)**
- **Need to set the lattice spacing (a_L) i.e. relation between g and a_L**





Preliminary (in collaboration with Chiho Nonaka)

Scale setting on the lattice using Polyakov loop



- **Expectation value of Polyakov loop:**

$$P = \frac{1}{n_x n_y n_z} tr\left[\sum_{\vec{r}} \prod_{n=0}^{n_t-1} U_4(na,\vec{r})\right]$$

- **Two loop beta function**

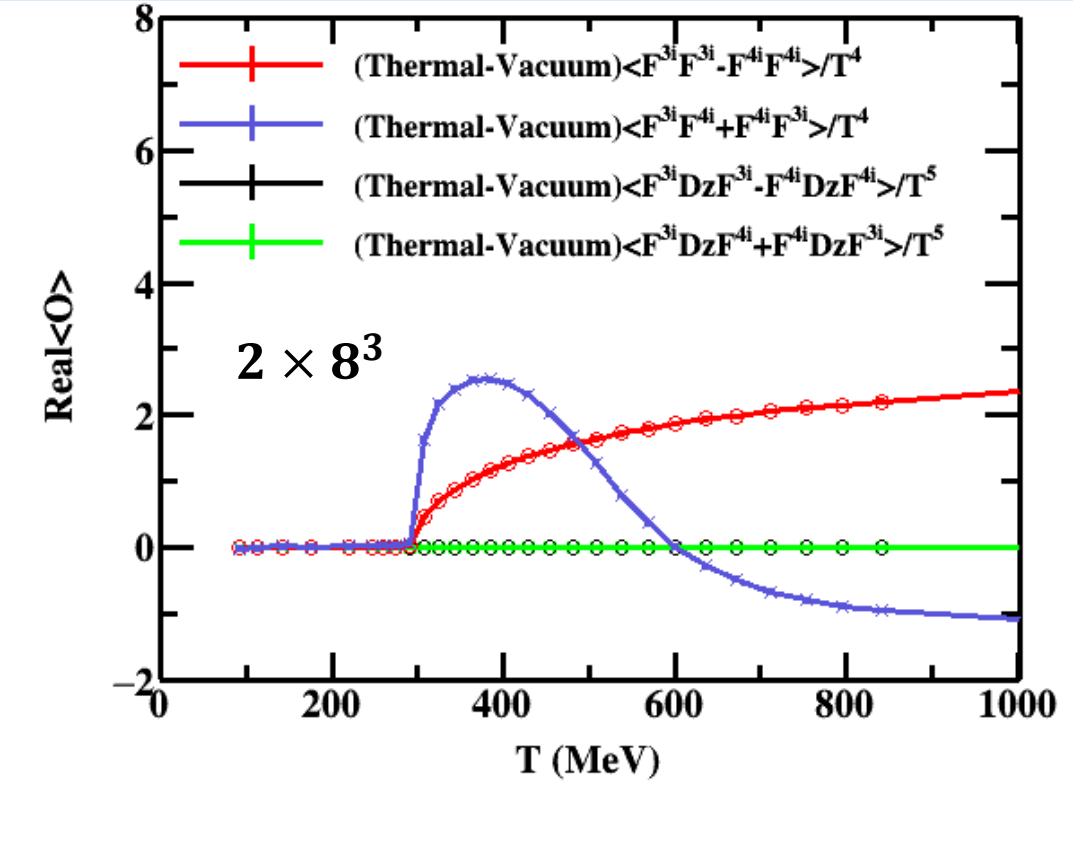
$$a_L = \frac{1}{\Lambda_L}\left(\frac{11}{16\pi^2 g^2}\right)^{-\frac{51}{121}} exp\left(-\frac{8\pi^2}{11g^2}\right)$$

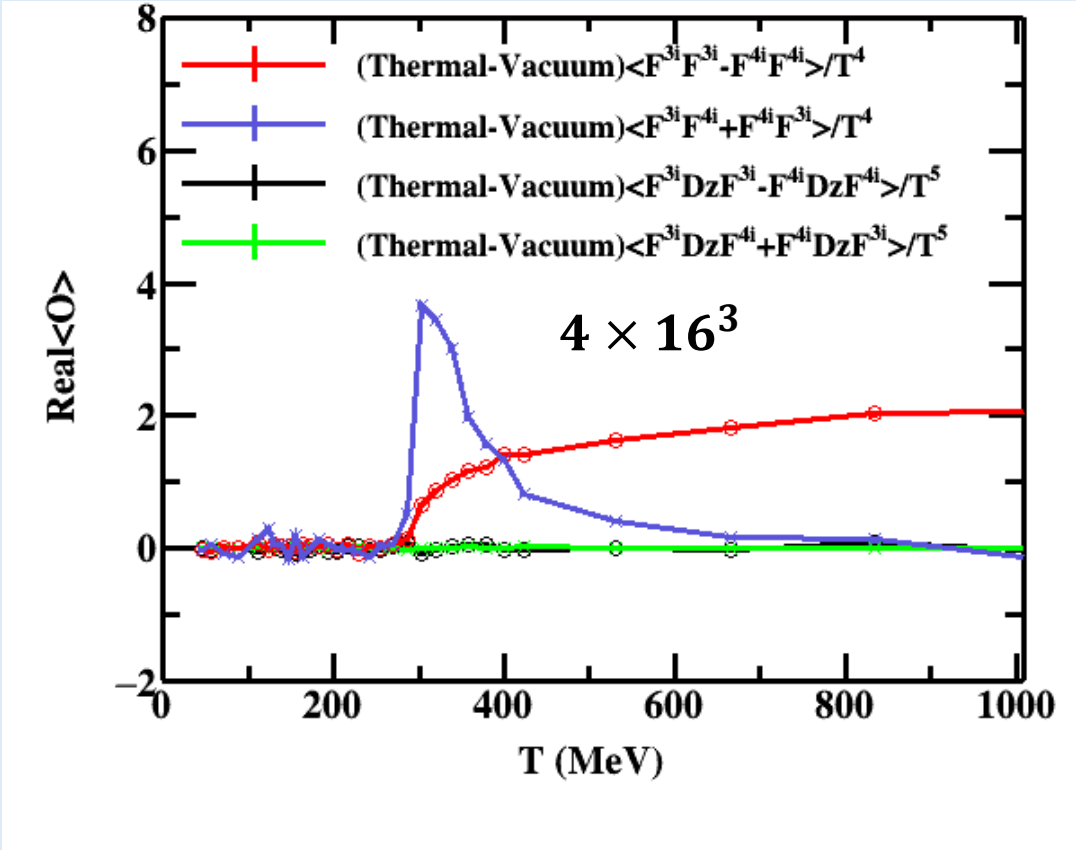
$$Temperature, T = \frac{1}{n_t a_L}$$ (Pure SU(3))

- **Nonperturbative correction**

$$Tune\ \frac{T_c}{\Lambda_L}\ is\ independent\ of\ g$$

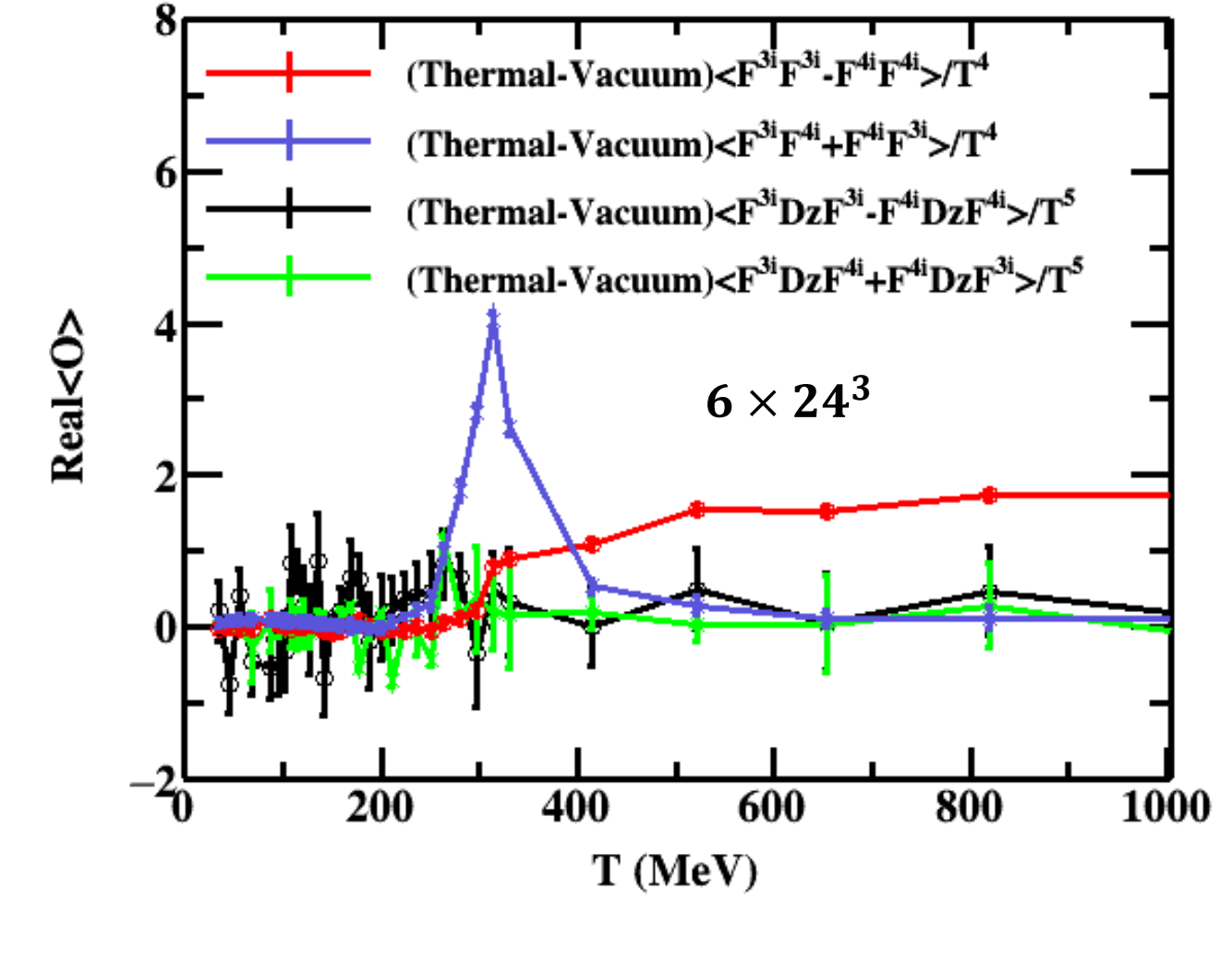
Real part of FF correlator in quenched SU(3)





- **Uncrossed correlator is dominant at high temperature**
- **Crossed correlator goes to zero at high temperature**
- **Correlator with Dz derivative are suppressed**

Real part of FF correlator in quenched SU(3)



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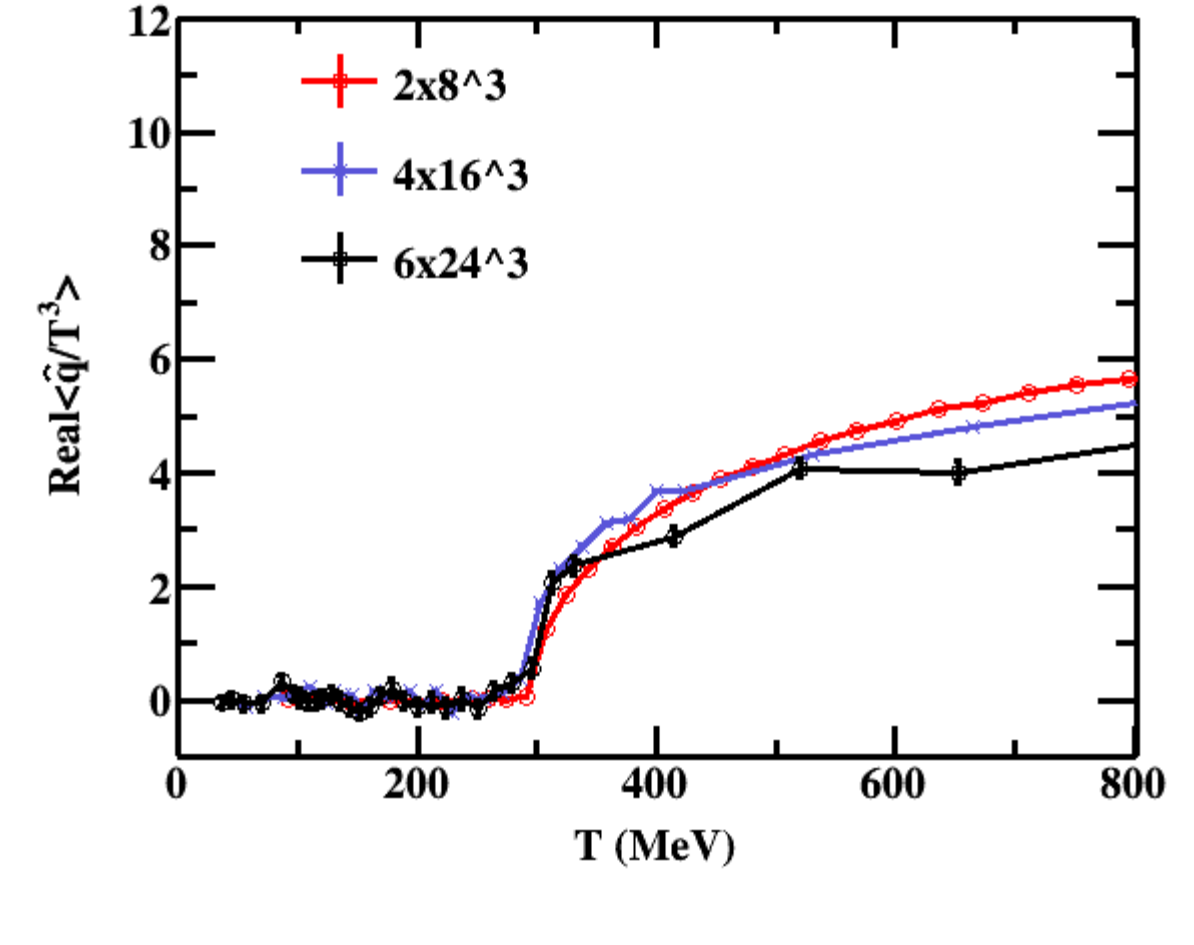
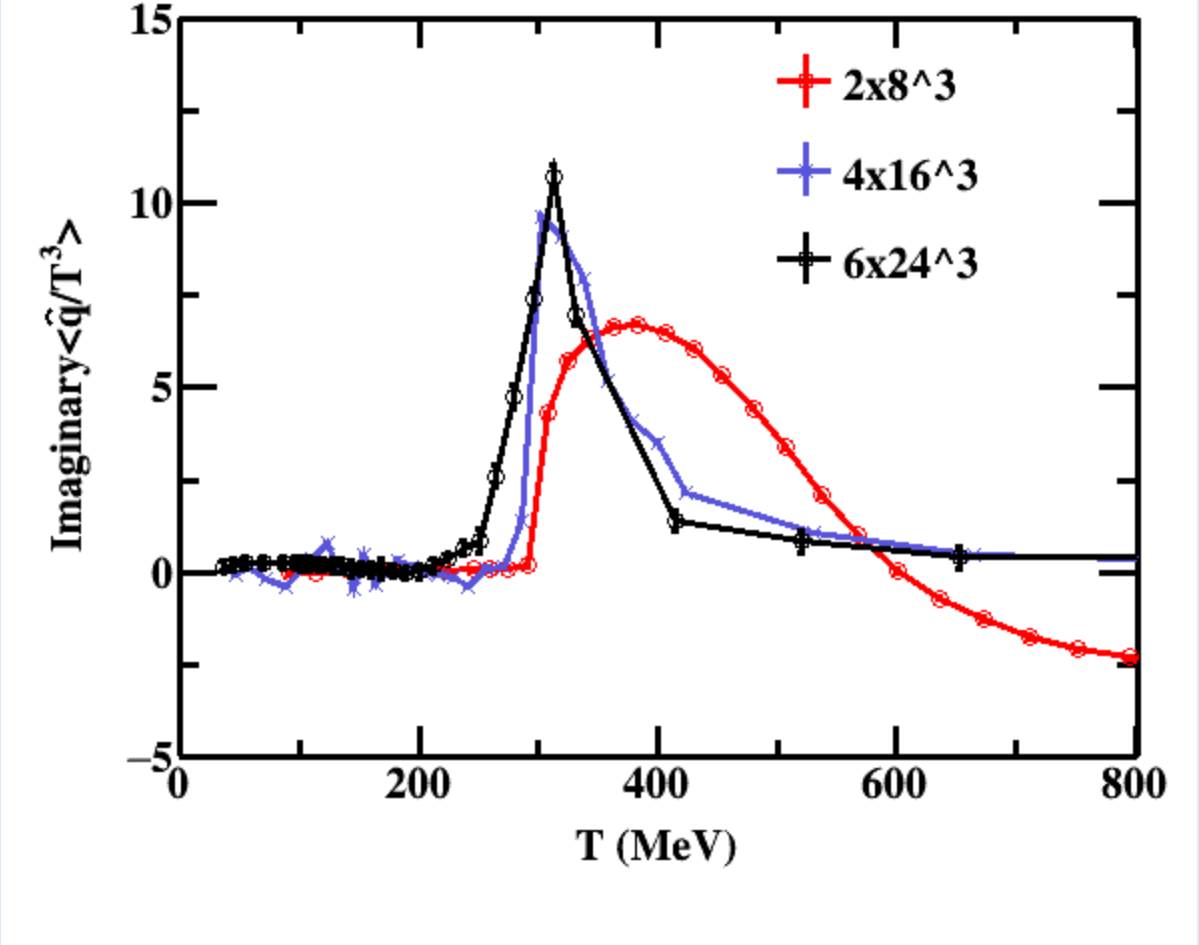
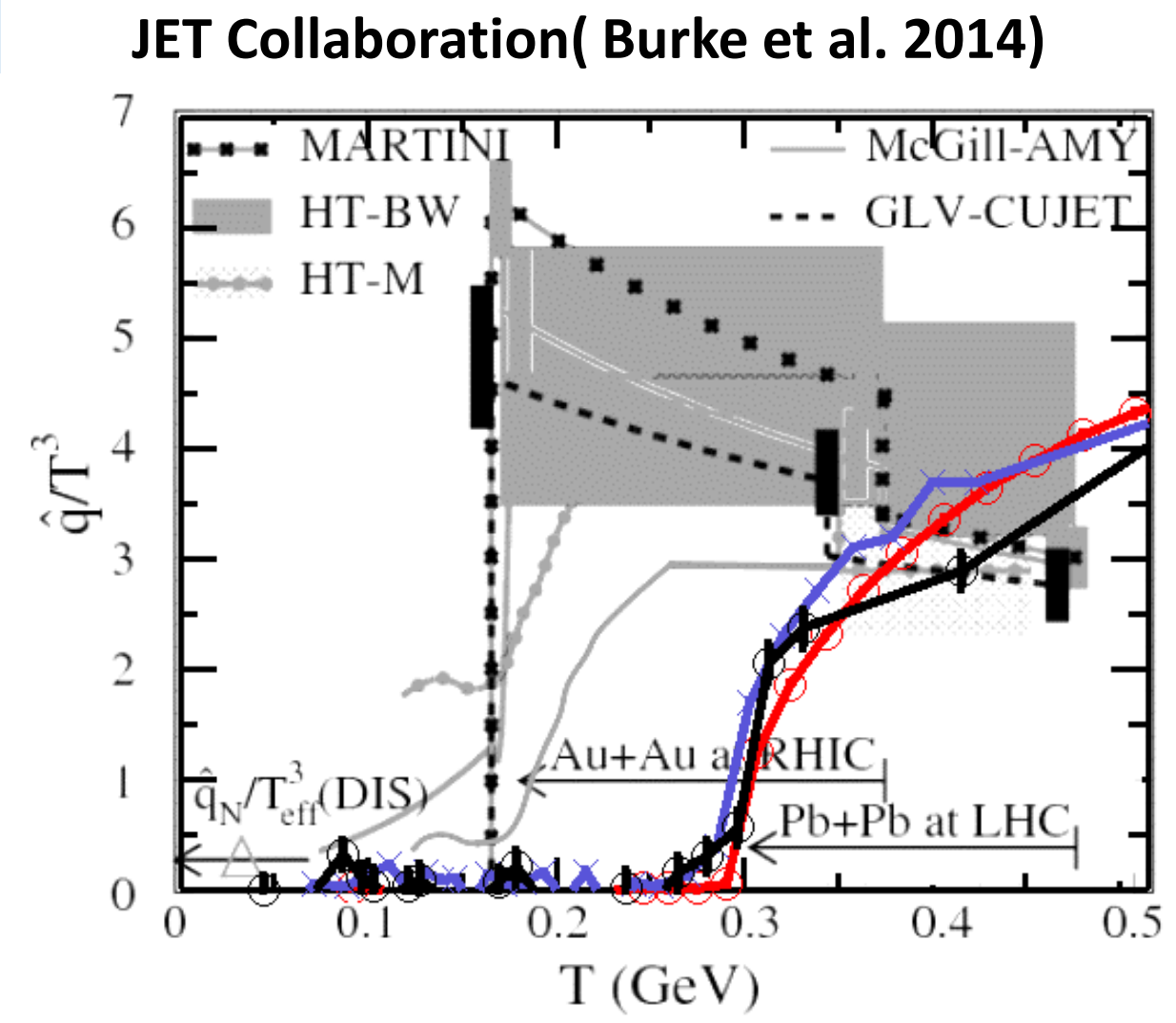
$\hat{q}$ in quenched SU(3) plasma





$\hat{q}$ in quenched SU(3) plasma



Quenched SU(3) Lattice

- 2x8^3
- 4x16^3
- 6x24^3

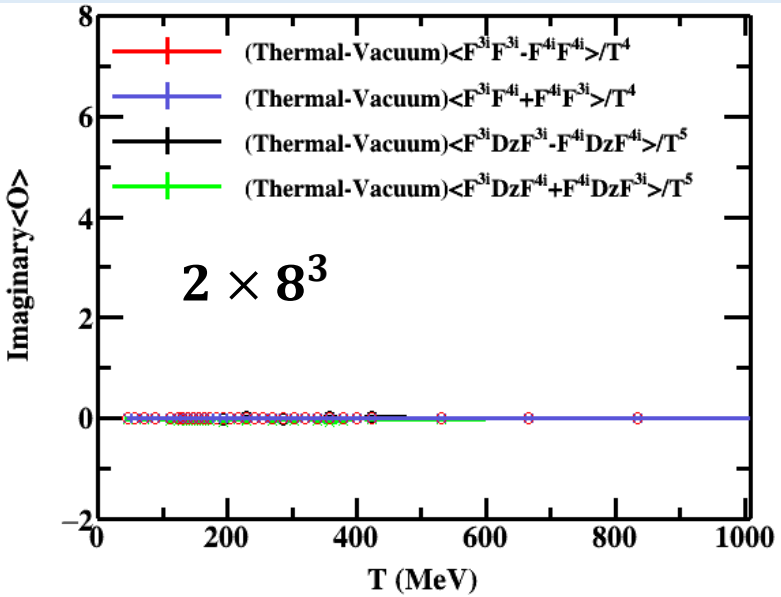
Summary and Future work

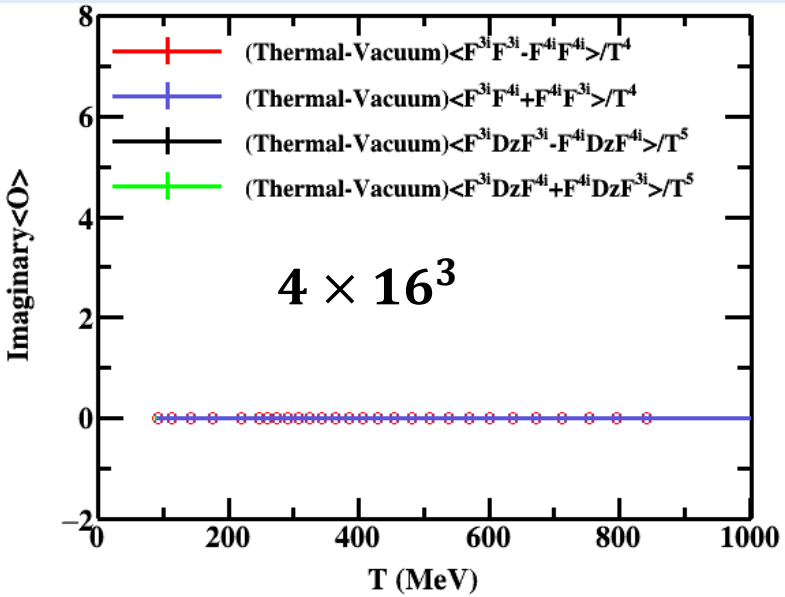
- First calculation of $\hat{q}$ on SU(3) quenched plasma
  - Analytic continuation to deep Euclidean space and expressed as local operators
  - Scale setting using perturbative loop beta function with non-perturbative correction using Polyakov loop.
  - Real part of $\hat{q}$ goes as T^3 for $T > 400$ MeV
  - Real part of $\hat{q}$ shows scaling behavior (Nt=2,4 and 6)
  - Imaginary part goes to 0 for $T > 400$ MeV
- $\hat{q}$ is the one transport coefficient which can be identically defined in cold nuclear matter and hot QGP
  - It gives a window into studying the change of the gluon distribution in cold nuclear matter vs QGP.
- Future work
  - Extend calculation using Improved Action and bigger lattice size
  - Include radiation diagram contributions
  - Extend to unquenched plasma (QGP)

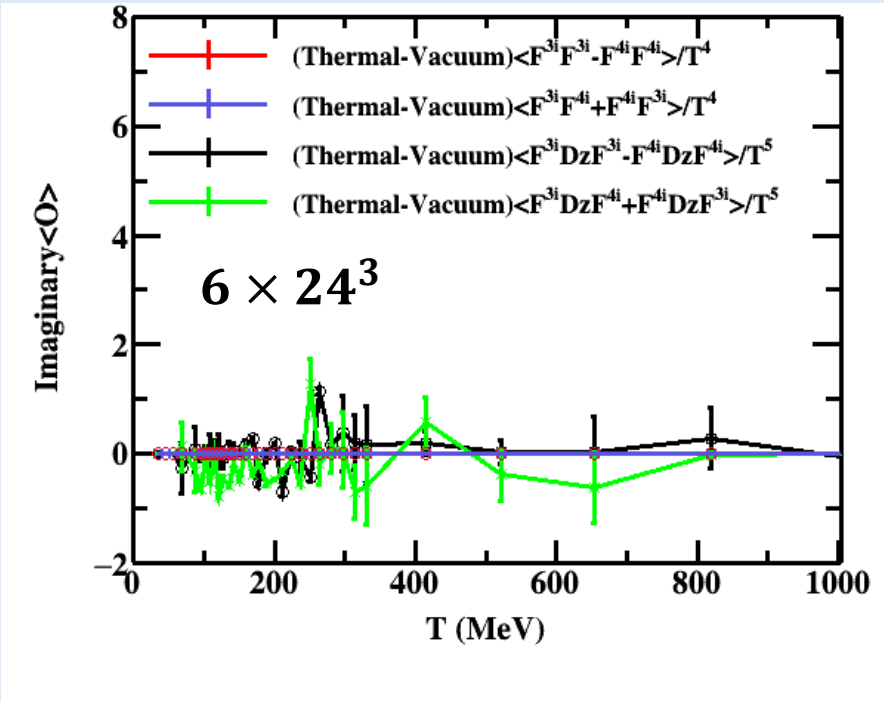
Thanks to group members and colleagues

- ❖Abhijit Majumder
- ❖Chiho Nonaka (Nagoya University)
- ❖ShanShan Cao, Yasuki Tachibana and Chathuranga Sirimana

Imaginary part of FF correlator in quenched SU(3)







- **Imaginary part of FF correlator does not contribute**