# An brief overview of Generalised Partons Distributions

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Nuclear Physics Seminar Series at CUA

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GPDs

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Introduction : probing the internal structure of matter

# Scattering experiments

A key tool to understand the structure of matter



#### Fraunhofer diffraction



Simulation of Fraunhofer diffraction due to a rectangle slit.

source : Wikimedia Commons

- Far field diffraction
- Diffraction  $\rightarrow$  Fourier transform of transmission coefficient

Image: A matrix and a matrix

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# Scattering experiments

A key tool to understand the structure of matter

- Fraunhofer diffraction
- X-ray scattering



Silicium crystal diffractive pattern

source : UK's national synchrotron

- X-ray wavelength  $\rightarrow \lambda \simeq$  typical size
- Bragg Law
- ▶ Diffraction pattern → Fourier transform of electronic density
- Provide information on the cristal structure



# Scattering experiments

A key tool to understand the structure of matter

- Fraunhofer diffraction
- X-ray scattering
- Rutherford experiment



source : Wikimedia Commons

- α particles scattering on a gold foil
- Some of which are scattered at large angles
- Invalidate the Thomson Model (Plum Pudding)
- Allows to develop the Rutherford planetary model



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# A pattern a study matter

- Scattering without breaking
- Fourier transform relation between matter structure and diffraction figure
- Repeat itself for different orders of magnitude
- Can we extend that to hadron structure?



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# Large virtuality and factorisation



• When the photon is strongly virtual :  $Q^2 = -q^2 >> M^2, t$ 



- Decomposition of DVCS between perturbative (green) and non-perturbative (blue) subparts.
- $\bullet$  Perturbative part  $\rightarrow$  description of the interaction between the probe and a parton inside hadron
- Non-perturbative part : description of a parton hadron amplitude called Generalised Partons Distributions (GPDs)
- GPDs is where the information on the hadrons structure lies.

### Generalised Parton Distributions



- General review on GPDs: M. Diehl, Phys.Rept., 2003, 388, 41-277
   A. Belitsky and A. Radyushkin, Phys.Rept., 2005, 418, 1-387
- Modern phenomenological applications
   K. Kumericki *et al.*, Eur. Phys. J., 2016, A52, 157
- Future experimental opportunities EIC Yellow Report, arXiv:2103.05419

### Definitions and some properties



$$\begin{split} H_{\pi}^{q}(x,\xi,t) &= \frac{1}{2} \int \frac{e^{ixP^{+}z^{-}}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^{q}(-\frac{z}{2}) \gamma^{+} \psi^{q}(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle \mathrm{d}z^{-} |_{z^{+}=0,z=0} \\ H_{\pi}^{g}(x,\xi,t) &= \frac{1}{2} \int \frac{e^{ixP^{+}z^{-}}}{2\pi} \langle P + \frac{\Delta}{2} | G^{+\mu}(-\frac{z}{2}) G^{+}_{\mu}(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle \mathrm{d}z^{-} |_{z^{+}=0,z=0} \end{split}$$

D. Müller et al., Fortsch. Phy. 42 101 (1994)

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- $\xi = -2\Delta \cdot n/P \cdot n$  is the skewness parameter  $\xi \simeq \frac{x_B}{2-x_B}$
- $t = \Delta^2$ : the Mandelstam variable



$$\begin{aligned} H_{\pi}^{q}(x,\xi,t) &= \frac{1}{2} \int \frac{e^{ixP^{+}z^{-}}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^{q}(-\frac{z}{2}) \gamma^{+} \psi^{q}(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle \mathrm{d}z^{-} |_{z^{+}=0,z=0} \\ H_{\pi}^{g}(x,\xi,t) &= \frac{1}{2} \int \frac{e^{ixP^{+}z^{-}}}{2\pi} \langle P + \frac{\Delta}{2} | G^{+\mu}(-\frac{z}{2}) G_{\mu}^{+}(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle \mathrm{d}z^{-} |_{z^{+}=0,z=0} \end{aligned}$$

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- Caveat ! In gauges other than the lightcone one, a Wilson line is necessary to make the GPDs gauge invariant

# Kinematical Range

Different values of  $(x, \xi)$  yields different lightfront interpretations:





- Modifies our understanding of what is probed
- Different type of contributions
- It determines two big regions
- Relevant for evolution equations
- $|\xi| > 1$  region of Generalised Distribution Amplitudes (GDA)

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# Connection with the PDF



Coming back to the definition:

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$$\begin{aligned} H^q_\pi(x,0,0) &= q(x)\Theta(x) - \bar{q}(-x)\Theta(-x) \\ H^g_\pi(x,0,0) &= xg(x)\Theta(x) - xg(-x)\Theta(-x) \end{aligned}$$

In the limit  $(\xi, t) \rightarrow (0, 0)$ , one recover the PDFs.

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# Connection with the form factor



Looking at the quark definition:

$$H_{\pi}^{q}(x,\xi,t) = \frac{1}{2} \int \frac{e^{ixP^{+}z^{-}}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^{q}(-\frac{z}{2})\gamma^{+}\psi^{q}(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle \mathrm{d}z^{-} |_{z^{+}=0,z=0}$$

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$$\begin{split} \int \mathrm{d} x \, H^q_\pi(x,\xi,t) &= \frac{1}{2} \int \delta(P^+z^-) \langle P + \frac{\Delta}{2} | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \psi^q(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle \mathrm{d} z^- |_{z^+=0,z=0} \\ &= \frac{1}{2P^+} \langle P + \frac{\Delta}{2} | \bar{\psi}^q(0) \gamma^+ \psi^q(0) | P - \frac{\Delta}{2} \rangle \end{split}$$

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We recover the pion electromagnetique Form Factor

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Prerequisite



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- Necessary to define a "center of mass" of the hadron !
  - Turn to Galileen subgroup acting in the 2D transverse plane
  - It yields a centre of mass w.r.t. the  $p_i^+$

$$b_{\perp} = \frac{\sum_{i} p_{i}^{+} b_{\perp}^{i}}{\sum_{i} p_{i}^{+}}$$

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#### Immediate consequences for GPDs

GPDs encode a kick in the momentum fraction along the lightfront of  $2\xi \rightarrow$  unless  $\xi = 0$  the "centre of mass" is modified between the initial and final Proton

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A probabilistic interpretation can be obtained only for  $\xi = 0$ 

Examples of 2+1D pictures



M. Burkardt, PRD 62 (2000) 071503, PRD 66 (2002) 119903 (erratum)



fig. from C. Mezrag et al., PLB 741 (2015) 190-196



fig. from H. Moutarde et al., EPJ C 78 (2018) 11, 890



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# Place of GPDs in the Hadron physics context



figure from A. Accardi et al., Eur.Phys.J.A 52 (2016) 9, 268

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# Interpretation of GPDs II

Connection to the Energy-Momentum Tensor



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# How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé et al., PLB 776 (2018) 38-47, M. Polyakov and P. Schweitzer, IJMPA 33 (2018) 26, 1830025 C. Lorcé et al., Eur.Phys.J.C 79 (2019) 1, 89

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$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = 2P^{\mu}P^{\nu}A_{q,g}(t;\mu) + \frac{1}{2} \left( \Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^{2} \right) C_{q,g}(t;\mu) + 2M^{2}g^{\mu\nu}\bar{C}_{q,g}(t;\mu)$$

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$$\int_{-1}^{1} \mathrm{d} x \, x \, H_q(x,\xi,t;\mu) = A_q(t;\mu) + \xi^2 C_q(t;\mu)$$

Ji sum rule (nucleon)

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 Fluid mechanics analogy X. Ji, PRL 78, 610-613 (1997)
 M.V. Polyakov PLB 555, 57-62 (2003)



• They enter the description of experimental amplitude through Compton Form Factor:

$$\underbrace{\mathcal{H}(\xi, t, Q^2)}_{\text{Exp. Amplitude}} = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \underbrace{C\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}; \alpha_s\right)}_{pQCD} H(x, \xi, t, \mu^2)$$

- The coefficient function C is computed using pertubative QCD up to a given order in  $\alpha_S$ .
- This yield a deconvolution problem that we will discuss later

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### Questions ?

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# Questions ?

- We looked for a way to performed internal tomography of hadrons (similarly to X-ray cristallography for instance)
- We introduced Deeply Virtual Compton Scattering (DVCS) as an exclusive process
- We introduced GPDs as a way to parametrise DVCS
- We realised that GPDs contained the 3D information we are after and are connected to the energy momentum tensor
- We saw that GPDs are connected to data through a convolution

# Evolution properties of GPDs

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• Coming back to the operator definition of GPDs:

$$\langle \pi, P + \frac{\Delta}{2} | \bar{\psi} \left( -\frac{z}{2} \right) \gamma^+ \psi \left( \frac{z}{2} \right) | \pi, P - \frac{\Delta}{2} \rangle$$

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singular when  $z^2 \rightarrow 0$ 

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Need to renormalise our non-local operator

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• When  $z \to 0$  working with renormalised quark fields  $\psi_R = (Z_2)^{-1} \psi$  is not enough to treat the UV singularity

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#### Two approaches (among others)

- Renormalisation of local operators
- Renormalisation using "in partons" matrix elements

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- Possible to look because the singularity is a property of the operator, *not* of the external states.
- However, it is necessary to *choose* a scheme which is independent of the external states

For that purpose,  $\overline{\rm MS}$  is well suited GPDs (3D structure, pressure) become scheme dependent !



• On top of scheme, one should also choose a gauge, we picked the lightcone one, where  $A^+ = 0$ .

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- We are left in the quark sector with:



• Applying dimensional regularisation, and  $\overline{MS}$  renormalisation.

Final result

$$H^{i}(x,\xi,t,\mu) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} Z_{i,j}\left(\frac{x}{y},\frac{\xi}{x},\alpha_{s}(\mu),\epsilon\right) H^{j}_{reg}(y,\xi,t,\epsilon)$$





 $\bullet\,$  The previous equation is nice, but interesting on a limited range in  $\mu^2$ 

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- $\bullet\,$  The previous equation is nice, but interesting on a limited range in  $\mu^2$
- $\bullet$  On a wide range of  $\mu$  we would expect deviations from  $\alpha_{\mathcal{S}}$  behaviour



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- The previous equation is nice, but interesting on a limited range in  $\mu^2$
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#### Renormalisation Group

- Knowing the GPD at a scale  $\mu$  we want to know how it behaves at  $\mu + \mathrm{d}\mu$
- ullet we describe perturbatively the impact of this  $\mathrm{d}\mu$  leap

$$H(x,\xi,t,\mu+\mathrm{d}\mu)-H(x,\xi,t,\mu)$$

- we obtain like this a first-order integro-differential equation
- α<sub>S</sub> becomes "exponentiated"

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### Non-Singlet Case

$$\frac{\mathrm{d}H^{q}_{NS}(x,\xi,t,\mu)}{\mathrm{d}\ln(\mu)} = \frac{\alpha_{s}(\mu)}{4\pi} \int_{0}^{1} \frac{\mathrm{d}y}{y} \mathcal{P}^{0}_{q \leftarrow q}\left(\frac{x}{y},\frac{\xi}{x}\right) H^{q}_{NS}(y,\xi,t,\mu)$$

### Singlet Case

$$\begin{pmatrix} \frac{\mathrm{d}H^q_{\mathsf{s}}(\mathsf{x},\mathsf{\xi},t,\mu)}{\mathrm{d}\ln(\mu)} \\ \frac{\mathrm{d}H^g(\mathsf{x},\mathsf{\xi},t,\mu)}{\mathrm{d}\ln(\mu)} \end{pmatrix} = \frac{\alpha_{\mathsf{s}}(\mu)}{4\pi} \int_0^1 \frac{\mathrm{d}y}{y} \begin{pmatrix} \mathcal{P}^0_{q\leftarrow q}\left(\frac{\mathsf{x}}{y},\frac{\mathsf{\xi}}{\mathsf{x}}\right) & \mathcal{P}^0_{q\leftarrow g}\left(\frac{\mathsf{x}}{y},\frac{\mathsf{\xi}}{\mathsf{x}}\right) \\ \mathcal{P}^0_{g\leftarrow q}\left(\frac{\mathsf{x}}{y},\frac{\mathsf{\xi}}{\mathsf{x}}\right) & \mathcal{P}^0_{g\leftarrow g}\left(\frac{\mathsf{x}}{y},\frac{\mathsf{\xi}}{\mathsf{x}}\right) \end{pmatrix} \begin{pmatrix} H^q_{\mathsf{s}}(y,\mathsf{\xi},t,\mu) \\ H^g(y,\mathsf{\xi},t,\mu) \end{pmatrix}$$



### Non-Singlet Case

$$\frac{\mathrm{d}H^{q}_{NS}(x,\xi,t,\mu)}{\mathrm{d}\ln(\mu)} = \frac{\alpha_{s}(\mu)}{4\pi} \int_{0}^{1} \frac{\mathrm{d}y}{y} \mathcal{P}^{0}_{q \leftarrow q}\left(\frac{x}{y},\frac{\xi}{x}\right) H^{q}_{NS}(y,\xi,t,\mu)$$

### Singlet Case

$$\begin{pmatrix} \frac{\mathrm{d}H_{\mathcal{S}}^{q}(\mathbf{x},\xi,t,\mu)}{\mathrm{d}\ln(\mu)} \\ \frac{\mathrm{d}H^{\mathcal{S}}(\mathbf{x},\xi,t,\mu)}{\mathrm{d}\ln(\mu)} \end{pmatrix} = \frac{\alpha_{\mathcal{S}}(\mu)}{4\pi} \int_{0}^{1} \frac{\mathrm{d}y}{y} \begin{pmatrix} \mathcal{P}_{q\leftarrow q}^{0}\left(\frac{\mathbf{x}}{y},\frac{\xi}{\mathbf{x}}\right) & \mathcal{P}_{q\leftarrow g}^{0}\left(\frac{\mathbf{x}}{y},\frac{\xi}{\mathbf{x}}\right) \\ \mathcal{P}_{g\leftarrow q}^{0}\left(\frac{\mathbf{x}}{y},\frac{\xi}{\mathbf{x}}\right) & \mathcal{P}_{q\leftarrow g}^{0}\left(\frac{\mathbf{x}}{y},\frac{\xi}{\mathbf{x}}\right) \end{pmatrix} \begin{pmatrix} \mathcal{H}_{\mathcal{S}}^{q}(y,\xi,t,\mu) \\ \mathcal{H}^{\mathcal{B}}(y,\xi,t,\mu) \end{pmatrix}$$

The  $\ensuremath{\mathcal{P}}$  distributions can in principle be computed in pQCD

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# DGLAP connection

- Splitting function have been computed at:
  - ► LO (α<sub>s</sub>)



- D. Mueller et al., Fortsch.Phys. 42 101–141, 1994 X. Ji PRD55, 7114–7125, 1997 A. Radyushkin, PRD56, 5524–5557, 1997
- A. Belitsky et al., Nucl.Phys. B574, 347-406, 2000
   V.M. Braun et al., JHEP, vol. 02, p. 191, 2019

V.M. Braun et al., JHEP 06, 037, 2017.

- ► NLO (α<sup>2</sup><sub>S</sub>)
- ▶ N2LO ( $\alpha_s^3$ )

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# DGLAP connection



LO (α<sub>s</sub>)



NLO (α<sup>2</sup><sub>5</sub>)

A. Belitsky et al., Nucl.Phys. B574, 347-406, 2000
 V.M. Braun et al., JHEP, vol. 02, p. 191, 2019

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N2LO  $(\alpha_s^3)$ 

V.M. Braun et al., JHEP 06, 037, 2017.

In the limit Δ → 0, the H<sup>q</sup>(x, 0, 0, μ) = q(x, μ)
 → immediate consequence: one should recover the DGLAP evolution equations

# DGLAP connection



LO (α<sub>s</sub>)



NLO  $(\alpha_s^2)$ 

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 V.M. Braun et al., JHEP, vol. 02, p. 191, 2019

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In the limit Δ → 0, the H<sup>q</sup>(x, 0, 0, μ) = q(x, μ)
 → immediate consequence: one should recover the DGLAP evolution equations

$$\lim_{\xi \to 0} \mathcal{P}\left(\frac{x}{y}, \frac{\xi}{x}\right) = P_{DGLAP}\left(\frac{x}{y}\right)$$





- Charge conservation: the electromagnetic for factor is independent of  $\mu$  (observable)
- Energy-Momentum Conservation:  $\int \mathrm{d}x x(q(x) + g(x))$  is independent of  $\mu$
- Continuity at the crossover lines  $|x| = |\xi|$

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# Questions ?

# Questions ?

- We needed to take care of singularities, typical of QFT
- We introduced renormalisation constants, a renormalisation scheme and a scale
- Quantities related to GPDs become scale and scheme dependent
- We introduced an integro-differential equation to describe the scale dependence
- Experimental data do not depend on the scale and scheme (in principle)

### The Nucleon

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### Nucleon vs. Pion



Main difference: spin-1/2  $\rightarrow$  more tensorial structures!

$$\begin{split} &\frac{1}{2}\int \frac{e^{i\chi P^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} |\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P - \frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0} \\ &= \frac{1}{2P^+} \bigg[ H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_{\alpha}}{2M}u \bigg]. \end{split}$$

$$\begin{split} &\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} |\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P - \frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0} \\ &= \frac{1}{2P^+} \bigg[ \tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u \bigg]. \end{split}$$

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### Nucleon vs. Pion



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February 1st, 2024

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The nucleon has 4 chiral-even and 4 chiral-odd quark GPDs. All previous properties apply

Cédric Mezrag	(Irfu-DPhN)	
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## Probing GPDs through exclusive processes



Observables (cross sections, asymmetries ...)

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### Experimental connection to GPDs



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### Experimental connection to GPDs







- CFFs play today a central role in our understanding of GPDs
- Extraction generally focused on CFFs



# Deep Virtual Compton Scattering





- Best studied experimental process connected to GPDs
  - $\rightarrow$  Data taken at Hermes, Compass, JLab 6, JLab 12

# Deep Virtual Compton Scattering





- Best studied experimental process connected to GPDs
  - $\rightarrow$  Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

M. Defurne et al., Nature Commun. 8 (2017) 1, 1408
#### Theory of DVCS



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e.g. K. Kumericki et al., EPJ A 52 (2016) 6, 157

where

$$\mathfrak{T}|^2 = |\mathfrak{T}_{\rm BH} + \mathfrak{T}_{\rm DVCS}|^2 = |\mathfrak{T}_{\rm BH}|^2 + |\mathfrak{T}_{\rm DVCS}|^2 + \mathfrak{I} \,.$$

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## Theory of DVCS



The differential  $ep \rightarrow ep\gamma$  cross section is given by

$$\frac{d^5\sigma}{dx_B dQ^2 d|t| d\phi d\phi_S} = \frac{\alpha^3 x_B}{16\pi^{24}\sqrt{1+\epsilon^2}} |\mathfrak{T}|^2 ,$$

e.g. K. Kumericki et al., EPJ A 52 (2016) 6, 157

where

$$|\mathfrak{T}|^2 = |\mathfrak{T}_{\rm BH} + \mathfrak{T}_{\rm DVCS}|^2 = |\mathfrak{T}_{\rm BH}|^2 + |\mathfrak{T}_{\rm DVCS}|^2 + \mathfrak{I} \, .$$

The different contributions are then analysed in terms of harmonics of  $\phi$ :

$$\begin{aligned} \mathbb{J} \propto c_0^{\mathbb{J}} + \sum_{n=1}^3 \left[ c_n^{\mathbb{J}} \cos(n\phi) + s_n^{\mathbb{J}} \sin(n\phi) \right] \\ |\mathbb{T}_{\text{DVCS}}|^2 \propto c_0 + \sum_{n=1}^2 \left[ c_n \cos(n\phi) + s_n \sin(n\phi) \right] \\ \end{aligned}$$

#### Strategy

- The coefficients are extracted from data
- GPDs enter the description of the coefficients through **Compton Form Factors**





#### Recent CFF extractions





• Recent effort on bias reduction in CFF extraction (ANN)

additional ongoing studies, J. Grigsby et al., PRD 104 (2021) 016001

- Studies of ANN architecture to fulfil GPDs properties (dispersion relation, polynomiality, . . . )
- Recent efforts on propagation of uncertainties (allowing impact studies for JLAB12, EIC and EicC)

see e.g. H. Dutrieux et al., EPJA 57 8 250 (2021)



#### • At LO, the DVCS coefficient function is a QED one

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QCD corrections to DVCS



• At LO, the DVCS coefficient function is a QED one

H. Moutarde et al., PRD 87 (2013) 5, 054029

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QCD corrections to DVCS



H. Moutarde et al., PRD 87 (2013) 5, 054029

Recent N2LO studies, impact needs to be assessed

At LO, the DVCS coefficient function is a QED one

V. Braun et al., JHEP 09 (2020) 117

#### Finite t corrections



#### Kinematical corrections in $t/Q^2$ and $M^2/Q^2$

V. Braun et al., PRL 109 (2012), 242001



• Sizeable even for  $t/Q^2 \sim 0.1$ 

M. Defurne et al. PRC 92 (2015) 55202

- Not currently included in global fits.
- Difficulty for probabilistic interpretation (Hankle transform)

February 1<sup>st</sup>, 2024

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• At all orders in  $\alpha_S$ , dispersion relations relate the real and imaginary parts of the CFF.

M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932

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   I. Anikin and O. Teryaev, PRD 76 056007 M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932
- For instance at LO:

$$Re(\mathcal{H}(\xi,t)) = \frac{1}{\pi} \int_{-1}^{1} \mathrm{d}x \ Im(\mathcal{H}(x,t)) \left[\frac{1}{\xi-x} - \frac{1}{\xi+x}\right] + \underbrace{2 \int_{-1}^{1} \mathrm{d}\alpha \frac{D(\alpha,t)}{1-\alpha}}_{\text{Independent of }\xi}$$

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- At all orders in  $\alpha_5$ , dispersion relations relate the real and imaginary parts of the CFF. I. Anikin and O. Tervaev, PRD 76 056007 M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932
- For instance at I O:

$$\underbrace{\operatorname{Re}(\mathcal{H}(\xi,t))}_{\operatorname{racted from data}} = \frac{1}{\pi} \int_{-1}^{1} \mathrm{d}x \underbrace{\operatorname{Im}(\mathcal{H}(x,t))}_{\operatorname{Extracted from data}} \left[ \frac{1}{\xi - x} - \frac{1}{\xi + x} \right] + 2 \int_{-1}^{1} \mathrm{d}\alpha \frac{D(\alpha,t)}{1 - \alpha}$$

Extracted from data

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•  $D(\alpha, t)$  is related to the EMT (pressure and shear forces)

M.V. Polyakov PLB 555, 57-62 (2003)

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Extracted from data

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•  $D(\alpha, t)$  is related to the EMT (pressure and shear forces)



figure from H. Dutrieux et al., Eur.Phys.J.C 81 (2021) 4 M.V. Polyakov PLB 555, 57-62 (2003)

First attempt from JLab 6 GeV data

Burkert et al., Nature 557 (2018) 7705, 396-399

- Tensions with other studies
  - $\rightarrow$  uncontroled model-dependence

K. Kumericki, Nature 570 (2019) 7759, E1-E2
 H. Moutarde *et al.*, Eur.Phys.J.C 79 (2019) 7, 614
 H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

Scheme/scale dependence

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## The DVCS deconvolution problem I $_{\rm From\ CFF\ to\ GPDs}$



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## The DVCS deconvolution problem I $_{\rm From\ CFF\ to\ GPDs}$



 It has been known for a long time that this is not the case at LO Due to dispersion relations, any GPD vanishing on x = ±ξ would not contribute to DVCS at LO (neglecting D-term contributions).

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- It has been known for a long time that this is not the case at LO Due to dispersion relations, any GPD vanishing on x = ±ξ would not contribute to DVCS at LO (neglecting D-term contributions).
- Are QCD corrections improving the situation?

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#### Shadow GPDs



**CFF** Definition

$$\underbrace{\mathcal{H}(\xi, t, Q^2)}_{\text{Observable}} = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \underbrace{\mathcal{T}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_{\mathfrak{s}}(\mu^2)\right)}_{\text{Perturbative DVCS kernel}} H(x, \xi, t, \mu^2)$$

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#### Shadow GPD definition

We define shadow GPD  $H^{(n)}$  of order *n* such that when *C* is expanded in powers of  $\alpha_s$  up to *n* one has:

$$\begin{split} 0 &= \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} C^{(n)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu_0^2}, \alpha_s(\mu_0^2)\right) H^{(n)}(x, \xi, t, \mu_0^2) \quad \text{invisible in DVCS} \\ 0 &= H^{(n)}(x, 0, 0) \quad \text{invisible in DIS} \end{split}$$

A part of the GPD functional space is invisible to DVCS and DIS combined

## The DVCS deconvolution problem II





- NLO analysis of shadow GPDs:
  - Cancelling the line x = ξ is necessary but **no longer** sufficient
  - Additional conditions brought by NLO corrections reduce the size of the "shadow space"...
  - ... but do not reduce it to 0
    - $\rightarrow$  NLO shadow GPDs
      - H. Dutrieux et al., PRD 103 114019 (2021)

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- Evolution
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Multichannel Analysis required to fully determine GPDs

Cédric Mezrag (Irfu-DPhN)

GPDs

February 1<sup>st</sup>, 2024

#### Sullivan processes





- Tested at JLab 6 Huber *et al.*, PRC78, 045203
- Planned for JLab 12
  Aguilar et al., EPJA 55 10, 190
- Envisioned at EIC and EicC see EIC Yellow Report and EicC white paper



- Not done at JLab 6
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GPDs

#### DVCS on virtual Pion Target







• Question already raised in 2008 for JLab 12. Amrath *et al.*, EPJC 58, 179-192

- Would such processes be measurable at the future EIC and EicC? Answering the question of measurability of DVCS requires:
  - A pion GPD model
  - An evolution code
  - A phenomenological code able to compute amplitudes from GPDs
  - An event generator simulating how many events could be detected

#### Sullivan DVCS at the EIC





- Sullivan DVCS seems measurable at the EIC
- Our model predicts a sign flip of the Beam Spin Asymmetry due to gluons

Cédric Mezrag (Irfu-DPhN)

## Timelike Compton Scattering





• Amplitude related to the DVCS one  $(Q^2 \rightarrow -Q^2,...)$  $\rightarrow$  theoretical development for DVCS can be extended to TCS

E. Berger et al., EPJC 23 (2002) 675

• Excellent test of GPD universality but not the best option to solve the deconvolution problem

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- Interferes with the Bethe-Heitler (BH) process
- Same type of final states as exclusive quarkonium production

#### TCS: Recent results







O. Grocholski et al., EPJC 80, (2020) 61

- DVCS Data-driven prediction for TCS at LO and NLO
- First experimental measurement at JLab through forward-backward asymmetry (interference term)

P. Chatagnon et al., arXiv:2108.11746

February 1st, 2024

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• Measurable at the LHC in UPC ?

#### Deep Virtual Meson Production



- Factorization proven for  $\gamma_L^*$ 
  - J. Collins et al., PRD 56 (1997) 2982-3006

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- Same GPDs than previously
- Depends on the meson DA
- Formalism available at NLO
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- Factorisation proven  $\neq$  factorisation visible at achievable  $Q^2$ 
  - Leading-twist dominance at a given  $Q^2$  is process-dependent  $\rightarrow$  for DVMP it can change between mesons.
  - ► At JLab kinematics, higher-twist contributions are very strong
    - $\rightarrow$  hide factorisation of  $\sigma_L$

Cédric Mezrag (Irfu-DPhN)

#### Status of DVMP



- $\bullet \ \pi^0$  electroproduction
  - $\sigma_T > \sigma_L$  at JLab 6 and likely at JLab 12 kinematics ( $Q^2 = 8.3 GeV^2$ )

M. Dlamini et al., Phys.Rev.Lett. 127 (2021) 15, 152301

- No extraction of  $\sigma_L$  at JLab 12 yet
- Model-dependent treatment of  $\sigma_T$  using higher-twist contributions

S. V. Goloskokov and P. Kroll, EPJC 65, 137 (2010)
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see e.g. L. Favart, EPJA 52 (2016) 6, 158

•  $\sigma_T \neq 0$  though  $\rho_{0;T}$  production vanishes at leading twist  $\rightarrow$  No LT access to chiral-odd GPDs.

M. Diehl et al., PRD 59 (1999) 034023

Sizeable higher-twist effects need to be understood

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#### DVMP is as interesting as challenging Additional data would be more than welcome

Cédric Mezrag (Irfu-DPhN)



PARTONS partons.cea.fr



Gepard calculon.phy.hr/gpd/server/index.html



B. Berthou et al., EPJC 78 (2018) 478 Similarities : NLO computations, BM formalism, ANN, . . .

Differences : models, evolution, ...

#### Physics impact

These integrated softwares are the mandatory path toward reliable multichannel analyses.

→

#### First NLO DVCS-DVMP multichannel analysis



M. Cuic et al., JHEP 12 (2023) 192

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First NLO-multichannel analysis regarding in the GPDs community

Cédric	: Mezrag (	Ir	fu-DP	hN)	)

February 1<sup>st</sup>, 2024

## Conclusion



#### Summary

- Introduction to GPDs and their place in hadron structure studies
- Evolution of GPD
- Connection to experimental processes

#### Conclusion

- GPD field is as complicated as interesting
- Many theoretical and phenomenological works remain required
- Forthcoming facilities will likely shed new light on them
- Progresses in ab-initio computations (continuum and lattice) expected to be significant in the forthcoming years

## Thank you for your attention ! Some final questions ?

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