# An brief overview of <br> Generalised Partons Distributions 

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Introduction : probing the internal structure of matter

## Scattering experiments

A key tool to understand the structure of matter

- Fraunhofer diffraction


Simulation of Fraunhofer diffraction due to a rectangle slit.
source : Wikimedia Commons

- Far field diffraction
- Diffraction
$\rightarrow$ Fourier transform of transmission coefficient


## Scattering experiments

A key tool to understand the structure of matter

- Fraunhofer diffraction
- X-ray scattering


Silicium crystal diffractive pattern
source: UK's national synchrotron

- X-ray wavelength $\rightarrow \lambda \simeq$ typical size
- Bragg Law
- Diffraction pattern
$\rightarrow$ Fourier transform of electronic density
- Provide information on the cristal structure


## Scattering experiments

A key tool to understand the structure of matter

- Fraunhofer diffraction
- X-ray scattering
- Rutherford experiment

- $\alpha$ particles scattering on a gold foil
- Some of which are scattered at large angles
- Invalidate the Thomson Model (Plum Pudding)
- Allows to develop the Rutherford planetary model
source : Wikimedia Commons


## A pattern a study matter

- Scattering without breaking
- Fourier transform relation between matter structure and diffraction figure
- Repeat itself for different orders of magnitude
- Can we extend that to hadron structure?


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## Large virtuality and factorisation

- When the photon is strongly virtual : $Q^{2}=-q^{2} \gg M^{2}, t$

- Decomposition of DVCS between perturbative (green) and non-perturbative (blue) subparts.
- Perturbative part $\rightarrow$ description of the interaction between the probe and a parton inside hadron
- Non-perturbative part : description of a parton hadron amplitude called Generalised Partons Distributions (GPDs)
- GPDs is where the information on the hadrons structure lies.


# Generalised Parton Distributions 

## References

- General review on GPDs:
M. Diehl, Phys.Rept., 2003, 388, 41-277
A. Belitsky and A. Radyushkin, Phys.Rept., 2005, 418, 1-387
- Modern phenomenological applications K. Kumericki et al., Eur. Phys. J., 2016, A52, 157
- Future experimental opportunities EIC Yellow Report, arXiv:2103.05419


## Definitions and some properties

## Formal Definition for the pion

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\begin{aligned}
& H_{\pi}^{q}(x, \xi, t)=\left.\frac{1}{2} \int \frac{e^{i x P^{+} z^{-}}}{2 \pi}\left\langle P+\frac{\Delta}{2}\right| \bar{\psi}^{q}\left(-\frac{z}{2}\right) \gamma^{+} \psi^{q}\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle \mathrm{d} z^{-}\right|_{z^{+}=0, z=0} \\
& H_{\pi}^{g}(x, \xi, t)=\left.\frac{1}{2} \int \frac{e^{i x P^{+} z^{-}}}{2 \pi}\left\langle P+\frac{\Delta}{2}\right| G^{+\mu}\left(-\frac{z}{2}\right) G_{\mu}^{+}\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle \mathrm{d} z^{-}\right|_{z^{+}=0, z=0}
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- $t=\Delta^{2}$ : the Mandelstam variable
- Caveat! In gauges other than the lightcone one, a Wilson line is necessary to make the GPDs gauge invariant


## Kinematical Range

Different values of $(x, \xi)$ yields different lightfront interpretations:


- Modifies our understanding of what is probed
- Different type of contributions
- It determines two big regions
- Relevant for evolution equations
- $|\xi|>1$ region of Generalised Distribution Amplitudes (GDA)


## Connection with the PDF

Coming back to the definition:

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$$
\begin{aligned}
& H_{\pi}^{q}(x, 0,0)=q(x) \Theta(x)-\bar{q}(-x) \Theta(-x) \\
& H_{\pi}^{g}(x, 0,0)=x g(x) \Theta(x)-x g(-x) \Theta(-x)
\end{aligned}
$$

In the limit $(\xi, t) \rightarrow(0,0)$, one recover the PDFs.

## Connection with the form factor

Looking at the quark definition:

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\begin{aligned}
\int \mathrm{d} x H_{\pi}^{q}(x, \xi, t) & =\left.\frac{1}{2} \int \delta\left(P^{+} z^{-}\right)\left\langle P+\frac{\Delta}{2}\right| \bar{\psi}^{q}\left(-\frac{z}{2}\right) \gamma^{+} \psi^{q}\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle \mathrm{d} z^{-}\right|_{z^{+}=0, z=0} \\
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We recover the pion electromagnetique Form Factor

## GPD and the hadron $2+1$ Structure

Prerequisite

- Hadron description in coordinate space: position of its center of mass in the transverse plane


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- Necessary to define a "center of mass" of the hadron!
- Turn to Galileen subgroup acting in the 2D transverse plane
- It yields a centre of mass w.r.t. the $p_{i}^{+}$

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b_{\perp}=\frac{\sum_{i} p_{i}^{+} b_{\perp}^{i}}{\sum_{i} p_{i}^{+}}
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## Immediate consequences for GPDs

GPDs encode a kick in the momentum fraction along the lightfront of $2 \xi$ $\rightarrow$ unless $\xi=0$ the "centre of mass" is modified between the initial and final Proton

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A probabilistic interpretation can be obtained only for $\xi=0$

## GPD and the hadron $2+1$ Structure

Examples of $2+1 \mathrm{D}$ pictures

$$
\rho\left(x, \tilde{b}_{\perp}\right)=\int \frac{\mathrm{d}^{2} \Delta_{\perp}}{(2 \pi)^{2}} e^{i \Delta_{\perp} \tilde{b}_{\perp}} H\left(x, 0,-\Delta_{\perp}^{2}\right)
$$

M. Burkardt, PRD 62 (2000) 071503, PRD 66 (2002) 119903 (erratum)

Computations

fig. from C. Mezrag et al., PLB 741 (2015) 190-196

## Place of GPDs in the Hadron physics context


figure from A. Accardi et al., Eur.Phys.J.A 52 (2016) 9, 268

## Interpretation of GPDs II

Connection to the Energy-Momentum Tensor


How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence
C. Lorcé et al., PLB 776 (2018) 38-47, M. Polyakov and P. Schweitzer, IJMPA 33 (2018) 26, 1830025
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\left\langle p^{\prime}\right| T_{q, g}^{\mu \nu}|p\rangle=2 P^{\mu} P^{\nu} A_{q, g}(t ; \mu)+\frac{1}{2}\left(\Delta^{\mu} \Delta^{\nu}-g^{\mu \nu} \Delta^{2}\right) C_{q, g}(t ; \mu)+2 M^{2} g^{\mu \nu} \bar{C}_{q, g}(t ; \mu)
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$$
\int_{-1}^{1} \mathrm{~d} \times \times H_{q}(x, \xi, t ; \mu)=A_{q}(t ; \mu)+\xi^{2} C_{q}(t ; \mu)
$$

- Ji sum rule (nucleon)
- Fluid mechanics analogy
M.V. Polyakov PLB 555, 57-62 (2003)


## Connection with experimental data

- GPDs are not directly connected to experimental cross sections
- They enter the description of experimental amplitude through Compton Form Factor:

$$
\underbrace{\mathcal{H}\left(\xi, t, Q^{2}\right)}_{\text {Exp. Amplitude }}=\int_{-1}^{1} \frac{\mathrm{~d} x}{\xi} \underbrace{C\left(\frac{x}{\xi}, \frac{Q^{2}}{\mu^{2}} ; \alpha_{s}\right)}_{p Q C D} H\left(x, \xi, t, \mu^{2}\right)
$$

- The coefficient function $C$ is computed using pertubative QCD up to a given order in $\alpha_{S}$.
- This yield a deconvolution problem that we will discuss later


## Questions ?

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- We looked for a way to performed internal tomography of hadrons (similarly to X-ray cristallography for instance)
- We introduced Deeply Virtual Compton Scattering (DVCS) as an exclusive process
- We introduced GPDs as a way to parametrise DVCS
- We realised that GPDs contained the 3D information we are after and are connected to the energy momentum tensor
- We saw that GPDs are connected to data through a convolution


## Evolution properties of GPDs

## UV singularities of operators

- Coming back to the operator definition of GPDs:

$$
\left\langle\pi, P+\frac{\Delta}{2}\right| \bar{\psi}\left(-\frac{z}{2}\right) \gamma^{+} \psi\left(\frac{z}{2}\right)\left|\pi, P-\frac{\Delta}{2}\right\rangle
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Two approaches (among others)

- Renormalisation of local operators
- Renormalisation using "in partons" matrix elements


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- However, it is necessary to choose a scheme which is independent of the external states

For that purpose, $\overline{\mathrm{MS}}$ is well suited GPDs (3D structure, pressure) become scheme dependent!

## First order computation

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## Final result

$$
H^{i}(x, \xi, t, \mu)=\int_{-1}^{1} \frac{\mathrm{~d} y}{|y|} Z_{i, j}\left(\frac{x}{y}, \frac{\xi}{x}, \alpha_{s}(\mu), \epsilon\right) H_{r e g}^{j}(y, \xi, t, \epsilon)
$$

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## Renormalisation Group

- Knowing the GPD at a scale $\mu$ we want to know how it behaves at $\mu+\mathrm{d} \mu$
- we describe perturbatively the impact of this $\mathrm{d} \mu$ leap

$$
H(x, \xi, t, \mu+\mathrm{d} \mu)-H(x, \xi, t, \mu)
$$

- we obtain like this a first-order integro-differential equation
- $\alpha_{S}$ becomes "exponentiated"


## Evolution equations for GPDs

## Non-Singlet Case

$$
\frac{\mathrm{d} H_{N S}^{q}(x, \xi, t, \mu)}{\mathrm{d} \ln (\mu)}=\frac{\alpha_{s}(\mu)}{4 \pi} \int_{0}^{1} \frac{\mathrm{~d} y}{y} \mathcal{P}_{q \leftarrow q}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right) H_{N S}^{q}(y, \xi, t, \mu)
$$

## Singlet Case

$$
\left.\binom{\frac{\mathrm{d} H_{S}^{q}(x, \xi, t, \mu)}{\mathrm{d} \ln (\mu)}}{\frac{\mathrm{d} H^{\xi}(x, \xi, t, \mu)}{\mathrm{d} \ln (\mu)}}=\frac{\alpha_{s}(\mu)}{4 \pi} \int_{0}^{1} \frac{\mathrm{~d} y}{y}\left(\begin{array}{ll}
\mathcal{P}_{q \leftarrow q}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right) & \mathcal{P}_{q \leftarrow g}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right. \\
\mathcal{P}_{g \leftarrow q}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right) & \mathcal{P}_{g \leftarrow g}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right.
\end{array}\right) . \begin{array}{l}
H_{S}^{q}(y, \xi, t, \mu) \\
H^{g}(y, \xi, t, \mu)
\end{array}\right)
$$

## Evolution equations for GPDs

## Non-Singlet Case

$$
\frac{\mathrm{d} H_{N S}^{q}(x, \xi, t, \mu)}{\mathrm{d} \ln (\mu)}=\frac{\alpha_{s}(\mu)}{4 \pi} \int_{0}^{1} \frac{\mathrm{~d} y}{y} \mathcal{P}_{q \leftarrow q}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right) H_{N S}^{q}(y, \xi, t, \mu)
$$

## Singlet Case

$$
\binom{\frac{\mathrm{d} H_{S}^{q}(x, \xi, t, \mu)}{\mathrm{d}(\mathrm{In}(\mu)}}{\frac{\mathrm{d} H^{\xi}(, \xi, t, \mu)}{\mathrm{d} \ln (\mu)}}=\frac{\alpha_{s}(\mu)}{4 \pi} \int_{0}^{1} \frac{\mathrm{~d} y}{y}\left(\begin{array}{ll}
\mathcal{P}_{q \leftarrow q}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right) & \mathcal{P}_{q \leftarrow g}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right) \\
\mathcal{P}_{g \leftarrow q}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right) & \mathcal{P}_{g \leftarrow g}^{0}\left(\frac{x}{y}, \frac{\xi}{x}\right)
\end{array}\right)\binom{H_{S}^{q}(y, \xi, t, \mu)}{H^{g}(y, \xi, t, \mu)}
$$

The $\mathcal{P}$ distributions can in principle be computed in pQCD

## DGLAP connection

- Splitting function have been computed at:
- LO $\left(\alpha_{s}\right)$
D. Mueller et al., Fortsch.Phys. 42 101-141, 1994
X. Ji PRD55, 7114-7125, 1997
A. Radyushkin, PRD56, 5524-5557, 1997
- NLO $\left(\alpha_{S}^{2}\right)$
- $\mathrm{N} 2 \mathrm{LO}\left(\alpha_{s}^{3}\right)$
A. Belitsky et al., Nucl.Phys. B574, 347-406, 2000 V.M. Braun et al., JHEP, vol. 02, p. 191, 2019
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$$
\lim _{\xi \rightarrow 0} \mathcal{P}\left(\frac{x}{y}, \frac{\xi}{x}\right)=P_{D G L A P}\left(\frac{x}{y}\right)
$$

## Other properties

- Charge conservation: the electromagnetic for factor is independent of $\mu$ (observable)
- Energy-Momentum Conservation: $\int \mathrm{d} x x(q(x)+g(x))$ is independent of $\mu$
- Continuity at the crossover lines $|x|=|\xi|$


## Questions ?

## Questions ?

- We needed to take care of singularities, typical of QFT
- We introduced renormalisation constants, a renormalisation scheme and a scale
- Quantities related to GPDs become scale and scheme dependent
- We introduced an integro-differential equation to describe the scale dependence
- Experimental data do not depend on the scale and scheme (in principle)


# The Nucleon 

## Nucleon vs. Pion

Main difference: spin-1/2 $\rightarrow$ more tensorial structures!

$$
\begin{aligned}
& \left.\frac{1}{2} \int \frac{e^{i x P^{+} z^{-}}}{2 \pi}\left\langle P+\frac{\Delta}{2}\right| \bar{\psi}^{q}\left(-\frac{z}{2}\right) \gamma^{+} \psi^{q}\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle \mathrm{d} z^{-}\right|_{z^{+}=0, z=0} \\
& =\frac{1}{2 P^{+}}\left[H^{q}(x, \xi, t) \bar{u} \gamma^{+} u+E^{q}(x, \xi, t) \bar{u} \frac{i \sigma^{+\alpha} \Delta_{\alpha}}{2 M} u\right] . \\
& \left.\frac{1}{2} \int \frac{e^{i x P^{+} z^{-}}}{2 \pi}\left\langle P+\frac{\Delta}{2}\right| \bar{\psi}^{q}\left(-\frac{z}{2}\right) \gamma^{+} \gamma_{5} \psi^{q}\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle \mathrm{d} z^{-}\right|_{z^{+}=0, z=0} \\
& =\frac{1}{2 P^{+}}\left[\tilde{H}^{q}(x, \xi, t) \bar{u} \gamma^{+} \gamma_{5} u+\tilde{E}^{q}(x, \xi, t) \bar{u} \frac{\gamma_{5} \Delta^{+}}{2 M} u\right] .
\end{aligned}
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\end{aligned}
$$

The nucleon has 4 chiral-even and 4 chiral-odd quark GPDs. All previous properties apply

## Probing GPDs through exclusive processes

## Experimental connection to GPDs

## Observables (cross sections, asymmetries ...)

## Experimental connection to GPDs



## Experimental connection to GPDs



## Experimental connection to GPDs



- CFFs play today a central role in our understanding of GPDs
- Extraction generally focused on CFFs


## Deep Virtual Compton Scattering



- Best studied experimental process connected to GPDs
$\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12


## Deep Virtual Compton Scattering



- Best studied experimental process connected to GPDs
$\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler ( BH ) process
- Blessing: Interference term boosted w.r.t. pure DVCS one
- Curse: access to the angular modulation of the pure DVCS part difficult
M. Defurne et al., Nature Commun. 8 (2017) 1, 1408


## Theory of DVCS


where
The differential $e p \rightarrow e p \gamma$ cross section is given by

$$
\frac{d^{5} \sigma}{d x_{B} d Q^{2} d|t| d \phi d \phi_{S}}=\frac{\alpha^{3} x_{B}}{16 \pi^{24} \sqrt{1+\epsilon^{2}}}|\mathcal{T}|^{2}
$$

e.g. K. Kumericki et al., EPJ A 52 (2016) 6, 157

$$
|\mathcal{T}|^{2}=\left|\mathcal{T}_{\mathrm{BH}}+\mathcal{T}_{\mathrm{DVCS}}\right|^{2}=\left|\mathcal{T}_{\mathrm{BH}}\right|^{2}+\left|\mathcal{T}_{\mathrm{DVCS}}\right|^{2}+\mathcal{J}
$$

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$$

The different contributions are then analysed in terms of harmonics of $\phi$ :

$$
\begin{aligned}
& \mathcal{J} \propto c_{0}^{\mathcal{J}}+\sum_{n=1}^{3}\left[c_{n}^{\mathcal{J}} \cos (n \phi)+s_{n}^{\mathcal{J}} \sin (n \phi)\right] \\
& \left|\mathcal{T}_{\mathrm{DVCS}}\right|^{2} \propto c_{0}+\sum_{n=1}^{2}\left[c_{n} \cos (n \phi)+s_{n} \sin (n \phi)\right]
\end{aligned}
$$

## Strategy

- The coefficients are extracted from data
- GPDs enter the description of the coefficients through Compton Form Factors


## Recent CFF extractions


M. Cuic̀ et al., PRL 125, (2020), 232005

H. Moutarde et al., EPJC 79, (2019), 614

- Recent effort on bias reduction in CFF extraction (ANN)
additional ongoing studies, J. Grigsby et al., PRD 104 (2021) 016001
- Studies of ANN architecture to fulfil GPDs properties (dispersion relation,polynomiality,...)
- Recent efforts on propagation of uncertainties (allowing impact studies for JLAB12, EIC and EicC)


## QCD corrections to DVCS

- At LO, the DVCS coefficient function is a QED one


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- At NLO, gluon GPDs play a significant role in DVCS



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H. Moutarde et al., PRD 87 (2013) 5, 054029
- Recent N2LO studies, impact needs to be assessed
V. Braun et al., JHEP 09 (2020) 117


## Finite $t$ corrections

Kinematical corrections in $t / Q^{2}$ and $M^{2} / Q^{2}$
V. Braun et al., PRL 109 (2012), 242001


- Sizeable even for $t / Q^{2} \sim 0.1$
- Not currently included in global fits.
- Difficulty for probabilistic interpretation (Hankle transform)


## Dispersion relation and the D-term

- At all orders in $\alpha_{S}$, dispersion relations relate the real and imaginary parts of the CFF.
I. Anikin and O. Teryaev, PRD 76056007
M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932


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\operatorname{Re}(\mathcal{H}(\xi, t))=\frac{1}{\pi} \int_{-1}^{1} \mathrm{~d} x \operatorname{Im}(\mathcal{H}(x, t))\left[\frac{1}{\xi-x}-\frac{1}{\xi+x}\right]+\underbrace{2 \int_{-1}^{1} \mathrm{~d} \alpha \frac{D(\alpha, t)}{1-\alpha}}_{\text {Independent of } \xi}
$$

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$\underbrace{\operatorname{Re}(\mathcal{H}(\xi, t))}_{\text {Extracted from data }}=\frac{1}{\pi} \int_{-1}^{1} \mathrm{~d} x \underbrace{\operatorname{Im}(\mathcal{H}(x, t))}_{\text {Extracted from data }}\left[\frac{1}{\xi-x}-\frac{1}{\xi+x}\right]+2 \int_{-1}^{1} \mathrm{~d} \alpha \frac{D(\alpha, t)}{1-\alpha}$
- $D(\alpha, t)$ is related to the EMT (pressure and shear forces)
M.V. Polyakov PLB 555, 57-62 (2003)


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Extracted from data
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figure from H. Dutrieux et al., Eur.Phys.J.C 81 (2021) 4

The DVCS deconvolution problem I
From CFF to GPDs


## The DVCS deconvolution problem I

 From CFF to GPDs

- It has been known for a long time that this is not the case at LO Due to dispersion relations, any GPD vanishing on $x= \pm \xi$ would not contribute to DVCS at LO (neglecting D-term contributions).


## The DVCS deconvolution problem I

 From CFF to GPDs

- It has been known for a long time that this is not the case at LO Due to dispersion relations, any GPD vanishing on $x= \pm \xi$ would not contribute to DVCS at LO (neglecting D-term contributions).
- Are QCD corrections improving the situation?


## Shadow GPDs

## CFF Definition

$$
\underbrace{\mathcal{H}\left(\xi, t, Q^{2}\right)}_{\text {Observable }}=\int_{-1}^{1} \frac{\mathrm{~d} x}{\xi} \underbrace{T\left(\frac{x}{\xi}, \frac{Q^{2}}{\mu^{2}}, \alpha_{s}\left(\mu^{2}\right)\right)}_{\text {Perturbative DVCS kernel }} H\left(x, \xi, t, \mu^{2}\right)
$$

## Shadow GPDs

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$$

## Shadow GPD definition

We define shadow GPD $H^{(n)}$ of order $n$ such that when $C$ is expanded in powers of $\alpha_{s}$ up to $n$ one has:

$$
\begin{aligned}
& 0=\int_{-1}^{1} \frac{\mathrm{~d} x}{\xi} C^{(n)}\left(\frac{x}{\xi}, \frac{Q^{2}}{\mu_{0}^{2}}, \alpha_{s}\left(\mu_{0}^{2}\right)\right) H^{(n)}\left(x, \xi, t, \mu_{0}^{2}\right) \quad \text { invisible in DVCS } \\
& 0=H^{(n)}(x, 0,0) \quad \text { invisible in DIS }
\end{aligned}
$$

A part of the GPD functional space is invisible to DVCS and DIS combined

## The DVCS deconvolution problem II




## The DVCS deconvolution problem II




GPDs

## The DVCS deconvolution problem II




## Sullivan processes



- Tested at JLab 6 Huber et al.,PRC78, 045203
- Planned for JLab 12

Aguilar et al., EPJA 55 10, 190

- Envisioned at EIC and EicC see EIC Yellow Report and EicC white paper

- Not done at JLab 6
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## DVCS on virtual Pion Target



- Question already raised in 2008 for JLab 12. Amrath et al., EPJC 58, 179-192
- Would such processes be measurable at the future EIC and EicC? Answering the question of measurability of DVCS requires:
- A pion GPD model
- An evolution code
- A phenomenological code able to compute amplitudes from GPDs
- An event generator simulating how many events could be detected


## Sullivan DVCS at the EIC



- Sullivan DVCS seems measurable at the EIC
- Our model predicts a sign flip of the Beam Spin Asymmetry due to gluons


## Timelike Compton Scattering



- Amplitude related to the DVCS one ( $Q^{2} \rightarrow-Q^{2}, \ldots$ )
$\rightarrow$ theoretical development for DVCS can be extended to TCS
E. Berger et al., EPJC 23 (2002) 675
- Excellent test of GPD universality but not the best option to solve the deconvolution problem


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- Excellent test of GPD universality but not the best option to solve the deconvolution problem
- Interferes with the Bethe-Heitler (BH) process
- Same type of final states as exclusive quarkonium production


## TCS: Recent results




O. Grocholski et al., EPJC 80, (2020) 61

- DVCS Data-driven prediction for TCS at LO and NLO
- First experimental measurement at JLab through forward-backward asymmetry (interference term)
P. Chatagnon et al.,arXiv:2108.11746
- Measurable at the LHC in UPC ?


## Deep Virtual Meson Production



- Factorization proven for $\gamma_{L}^{*}$
J. Collins et al., PRD 56 (1997) 2982-3006
- Same GPDs than previously
- Depends on the meson DA
- Formalism available at NLO
D. Müller et al., Nucl.Phys.B 884 (2014) 438-546


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- Leading-order access to gluon GPDs
- Factorisation proven $\neq$ factorisation visible at achievable $Q^{2}$
- Leading-twist dominance at a given $Q^{2}$ is process-dependent $\rightarrow$ for DVMP it can change between mesons.
- At JLab kinematics, higher-twist contributions are very strong $\rightarrow$ hide factorisation of $\sigma_{L}$


## Status of DVMP

- $\pi^{0}$ electroproduction
- $\sigma_{T}>\sigma_{L}$ at JLab 6 and likely at JLab 12 kinematics $\left(Q^{2}=8.3 \mathrm{GeV}^{2}\right)$
M. Dlamini et al., Phys.Rev.Lett. 127 (2021) 15, 152301
- No extraction of $\sigma_{L}$ at JLab 12 yet
- Model-dependent treatment of $\sigma_{T}$ using higher-twist contributions
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$$
\text { see e.g. L. Favart, EPJA } 52 \text { (2016) 6, } 158
$$

- $\sigma_{T} \neq 0$ though $\rho_{0 ; T}$ production vanishes at leading twist $\rightarrow$ No LT access to chiral-odd GPDs.
M. Diehl et al., PRD 59 (1999) 034023
- Sizeable higher-twist effects need to be understood
I. Anikin et al., PRD 84 (2011) 054004


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DVMP is as interesting as challenging Additional data would be more than welcome

## PARTONS and Gepard

## PARTONS

partons.cea.fr


Gepard
calculon.phy.hr/gpd/server/index.html

K. Kumericki, EPJ Web Conf. 112 (2016) 01012


- Differences : models, evolution, ...


## Physics impact

These integrated softwares are the mandatory path toward reliable multichannel analyses.

## First NLO DVCS-DVMP multichannel analysis



M. Cuic et al., JHEP 12 (2023) 192

First NLO-multichannel analysis regarding in the GPDs community

## Conclusion

## Summary

- Introduction to GPDs and their place in hadron structure studies
- Evolution of GPD
- Connection to experimental processes


## Conclusion

- GPD field is as complicated as interesting
- Many theoretical and phenomenological works remain required
- Forthcoming facilities will likely shed new light on them
- Progresses in ab-initio computations (continuum and lattice) expected to be significant in the forthcoming years


## Thank you for your attention! Some final questions?

