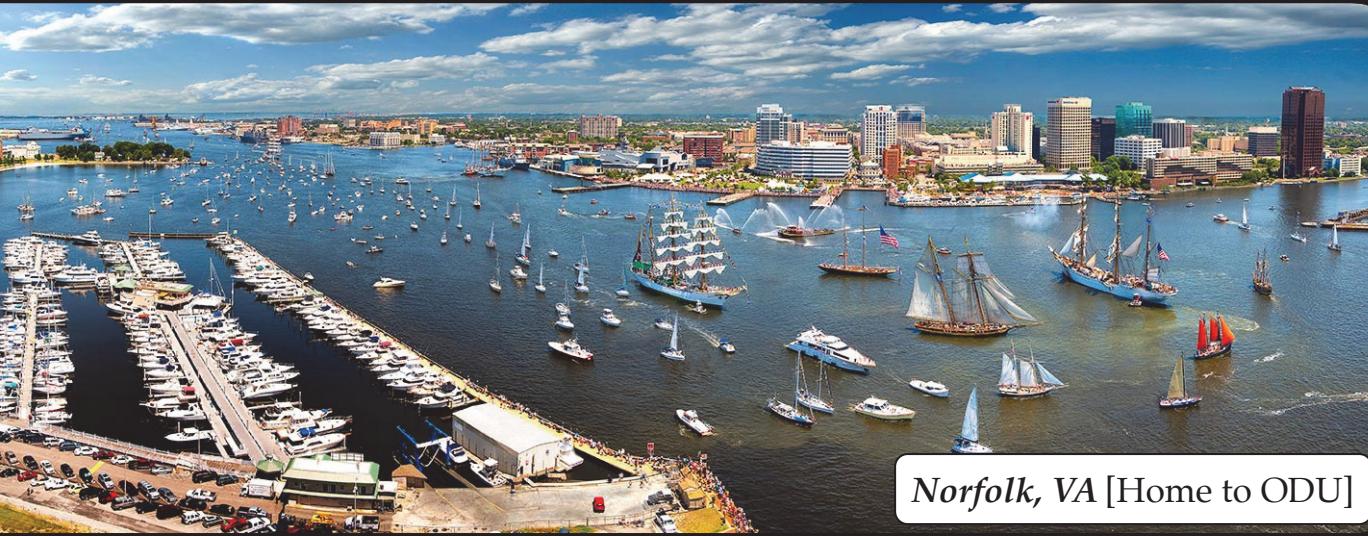


Status of theoretical understanding of $K\pi$ Interactions

Raúl Briceño

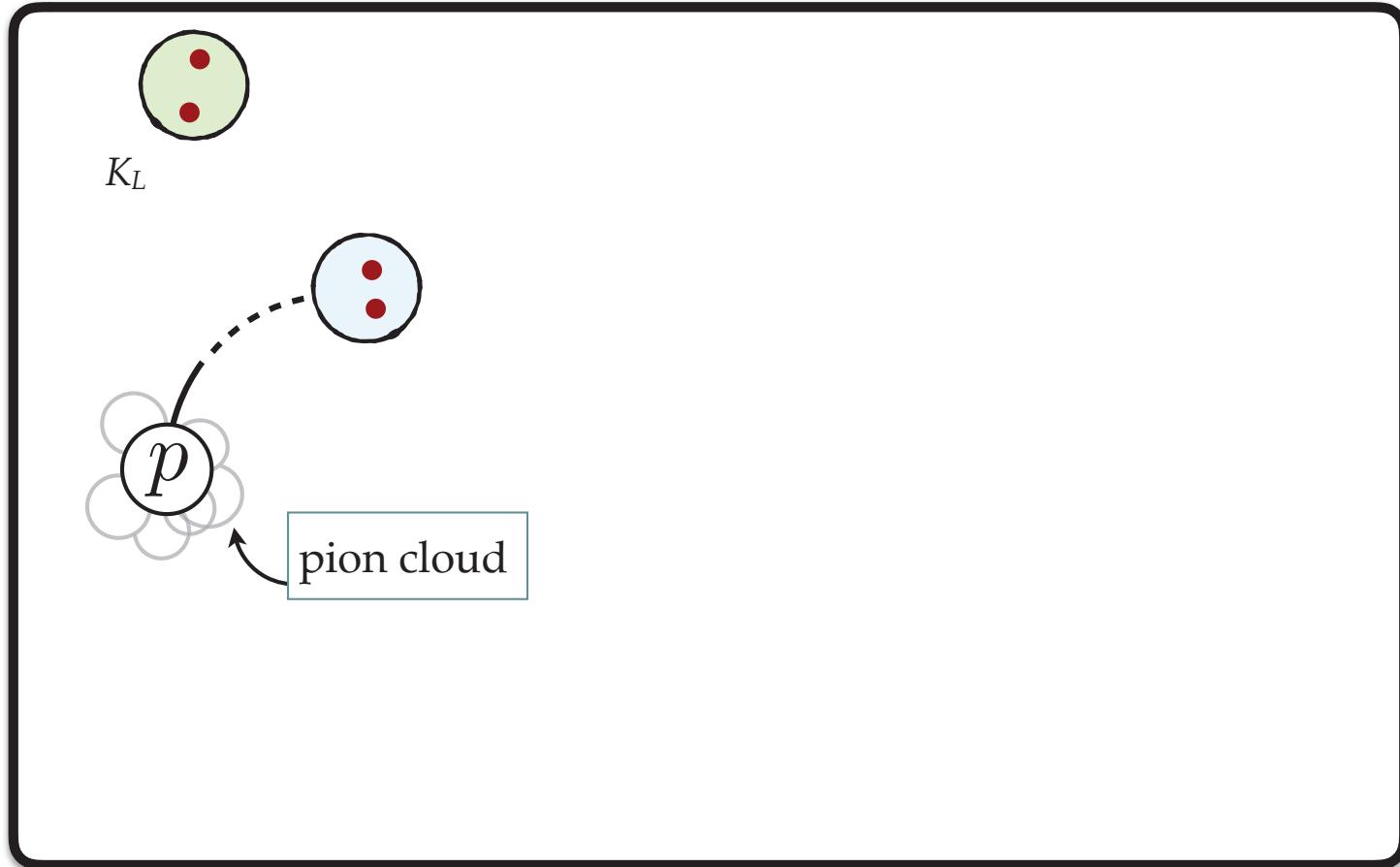


Norfolk, VA [Home to ODU]

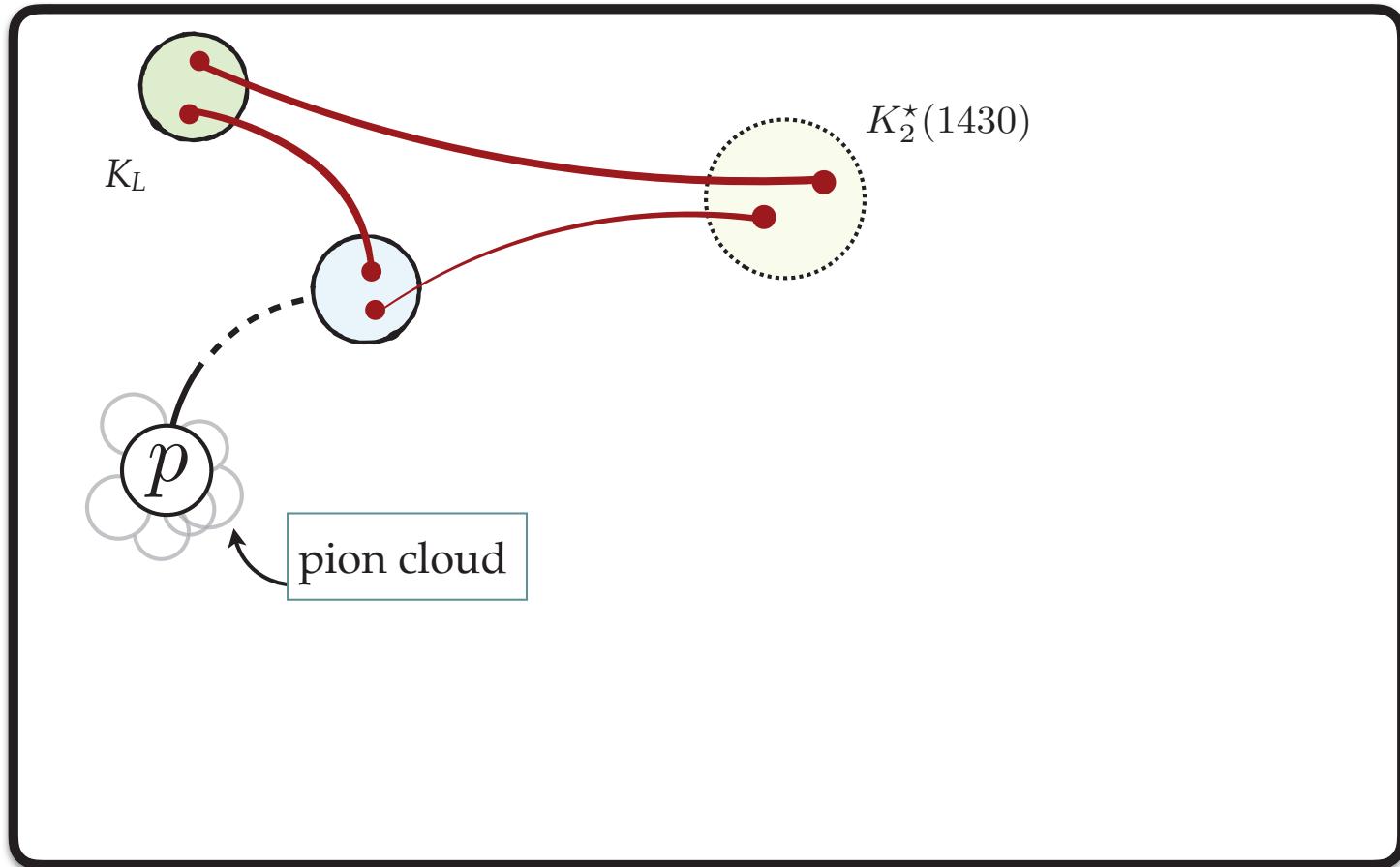


JLab [Newport News, VA]

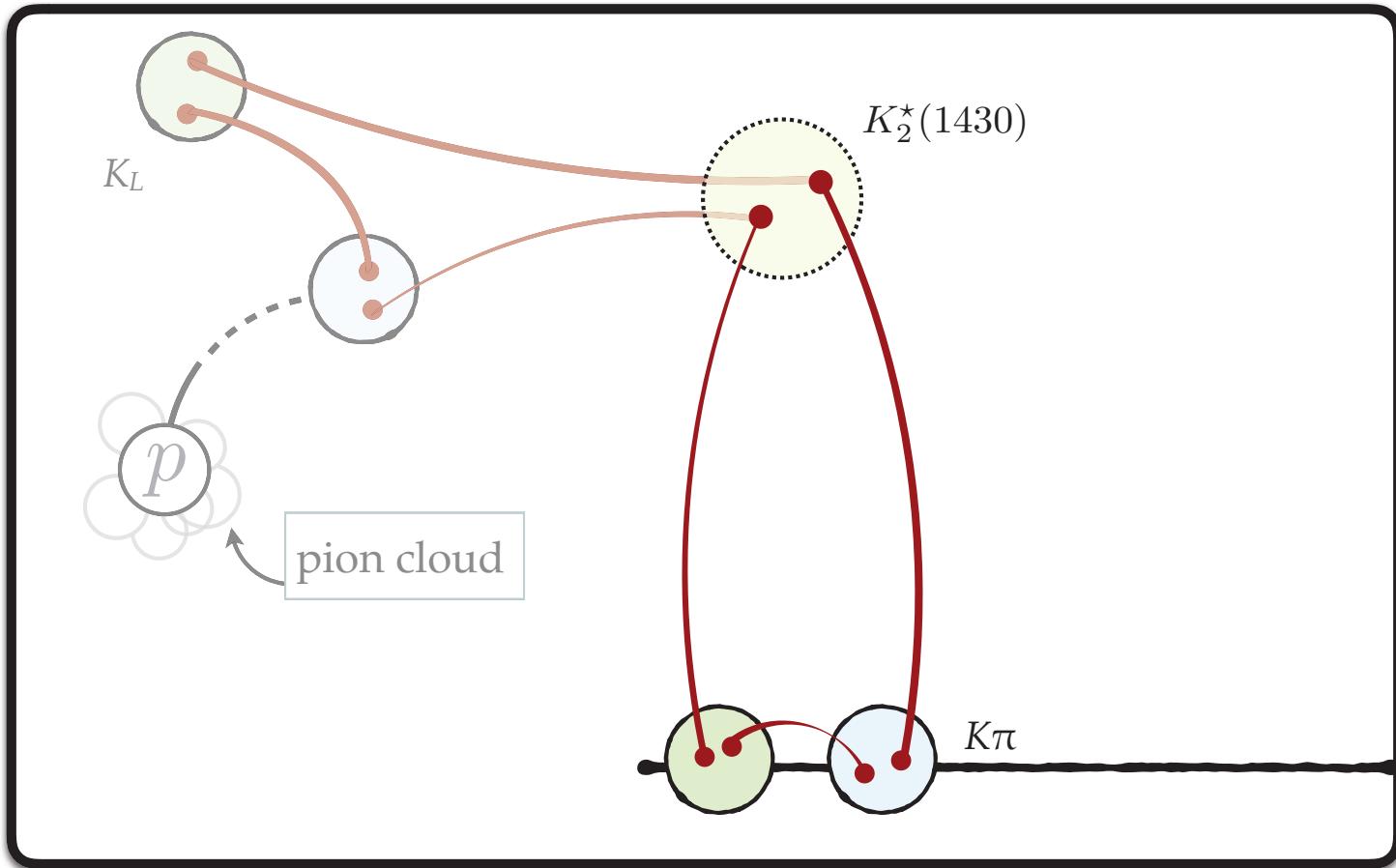
$K\pi$ Interactions



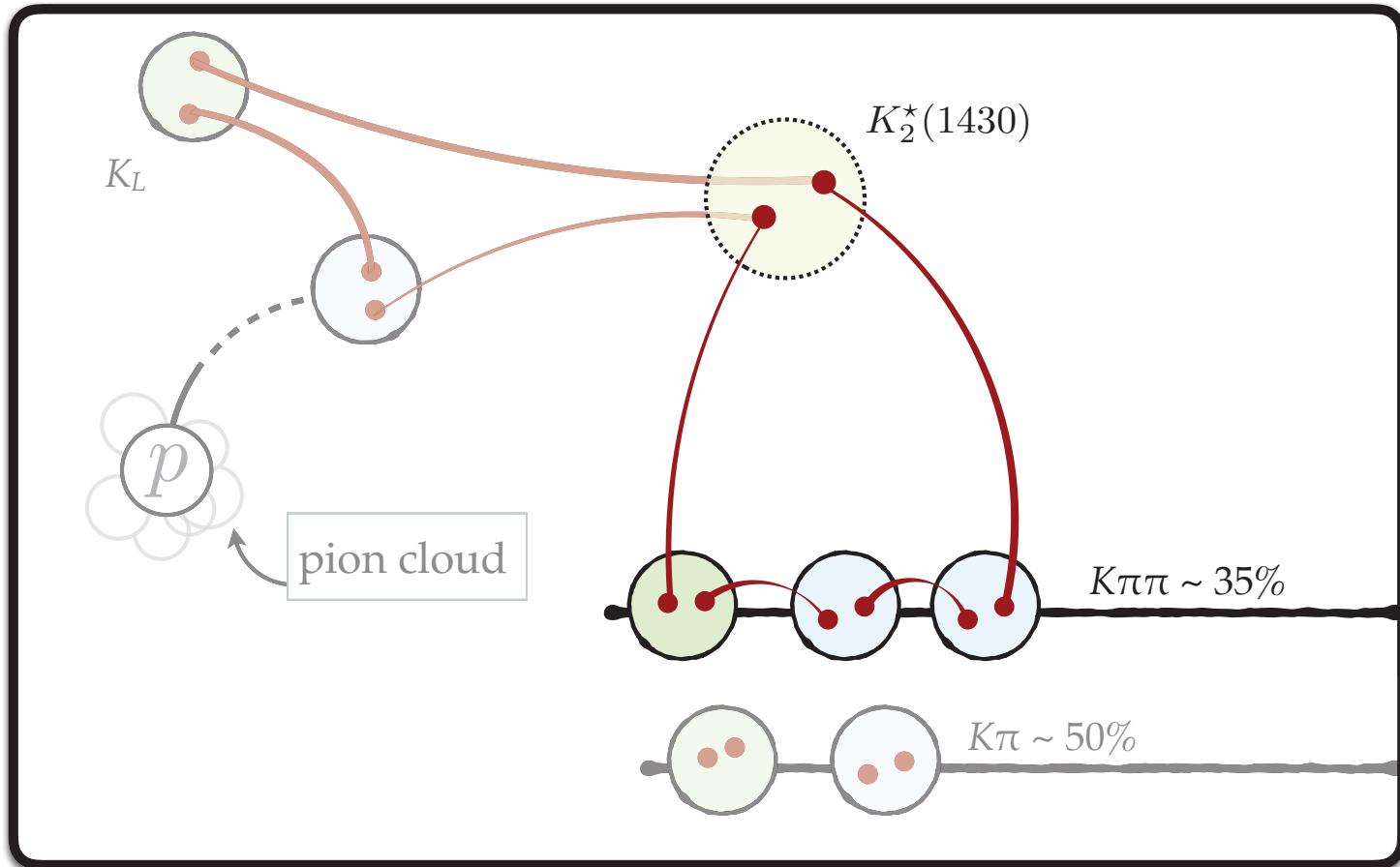
$K\pi$ Interactions



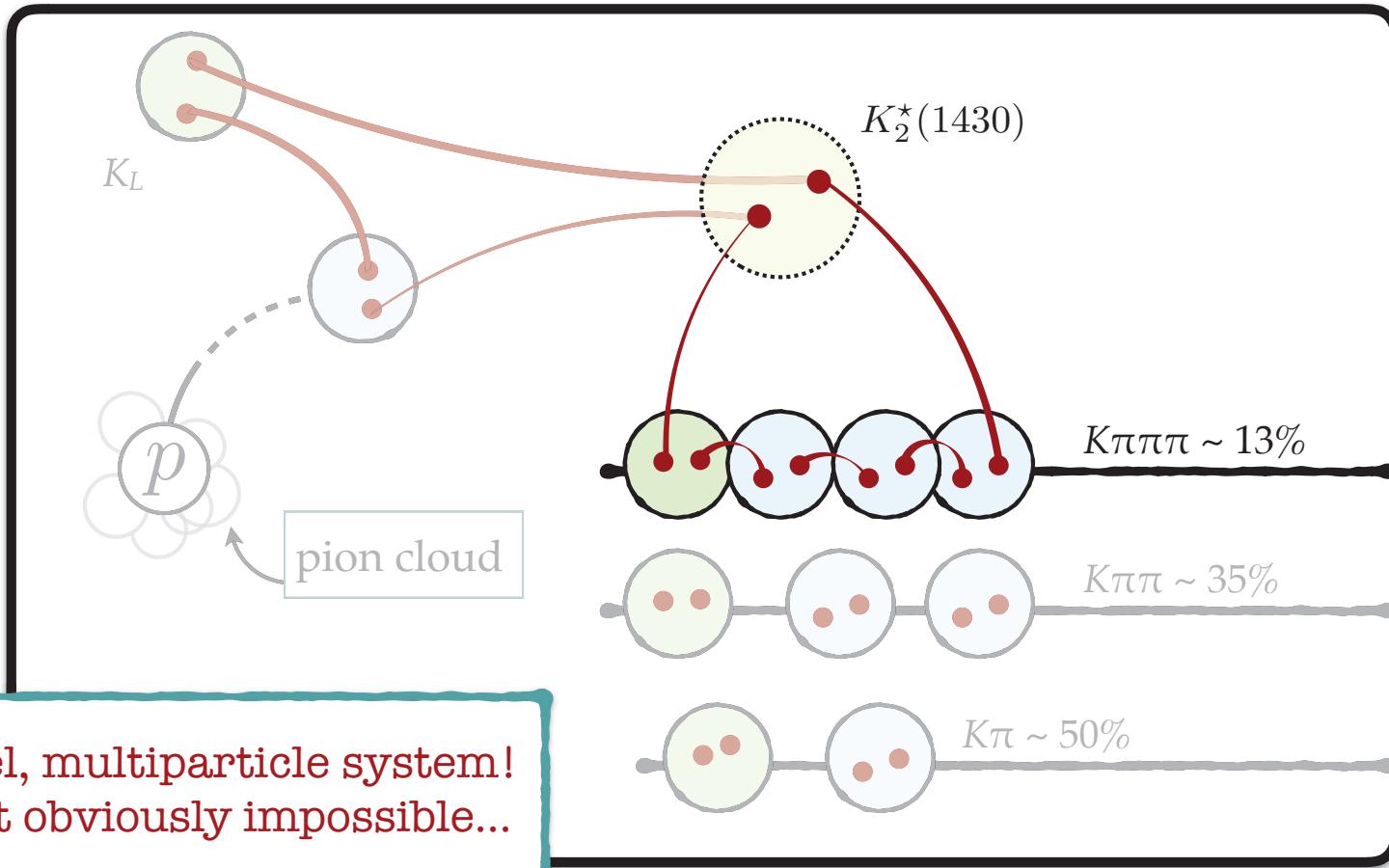
$K\pi$ Interactions



$K\pi$ Interactions



$K\pi$ Interactions



$$|n\rangle_{\text{QCD}} = c_0 \text{ (meson)} + c_1 \text{ (meson + pion cloud)} + c_2 \text{ (meson + pion + pion)} + \dots$$

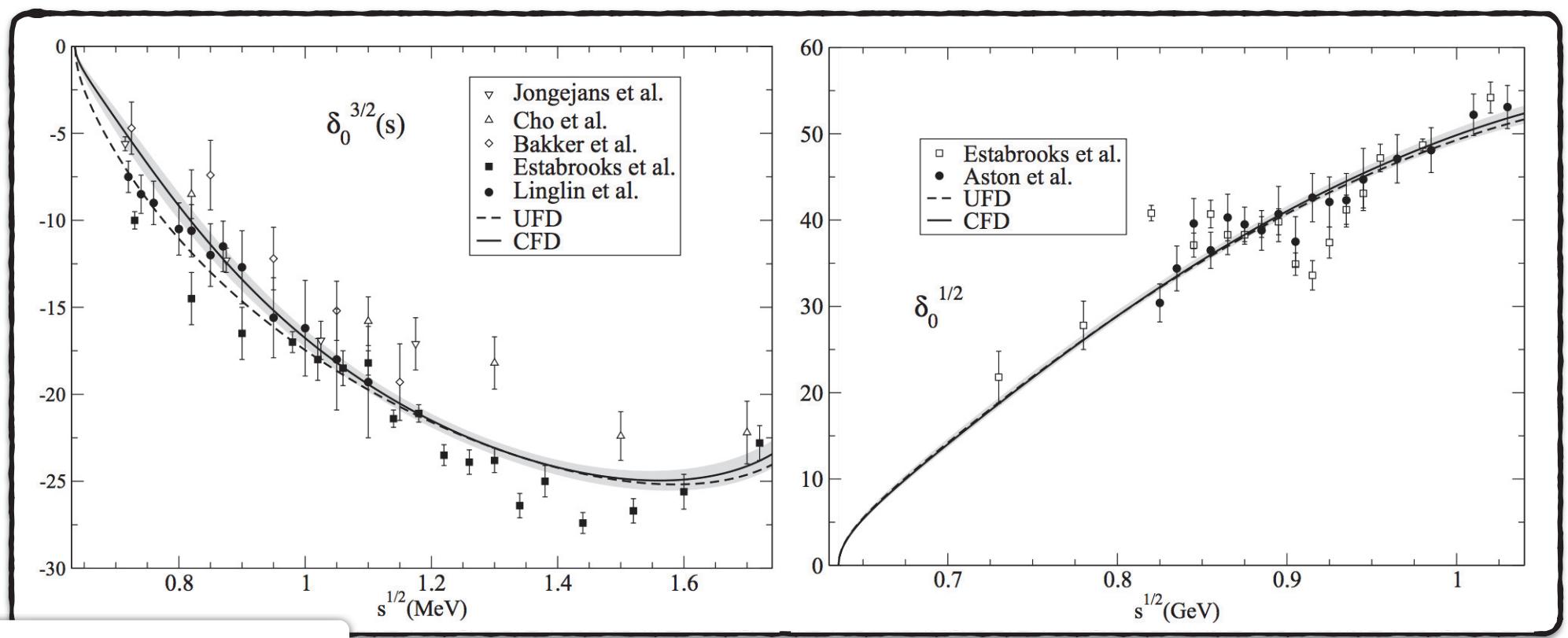
theoretical needs

structural understanding

Theoretical tools

Phenomenological analysis of data

- dispersive techniques,...
- essential for analysis
- zero to little predictive power



Theoretical tools

- Phenomenological analysis of data

- Models

- great for building intuition
- nearly impossible to assess systematics

Theoretical tools

- Phenomenological analysis of data
- Models
- EFT-inspired results

- χ PT and $U\chi$ PT
- systematic expansion about the chiral limit & threshold
- restricted predictive power
- strange quarks are fairly heavy
- systematic errors of $SU(3)$ χ PT: hard to asses

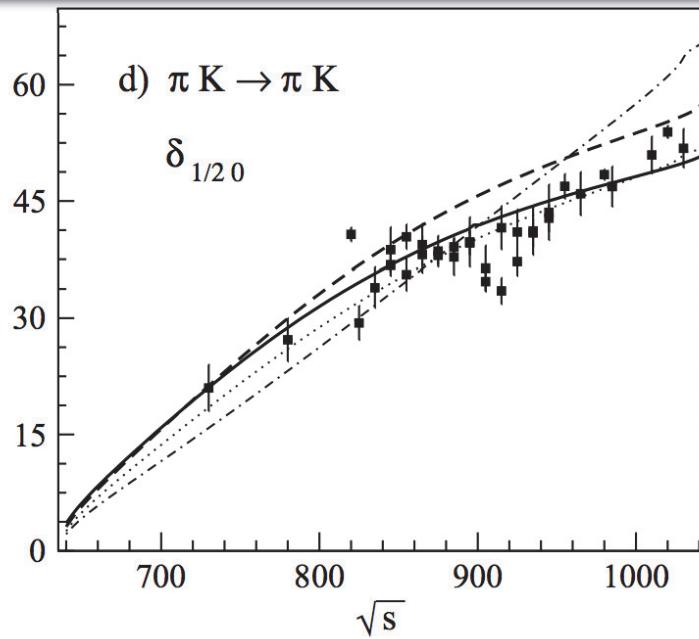
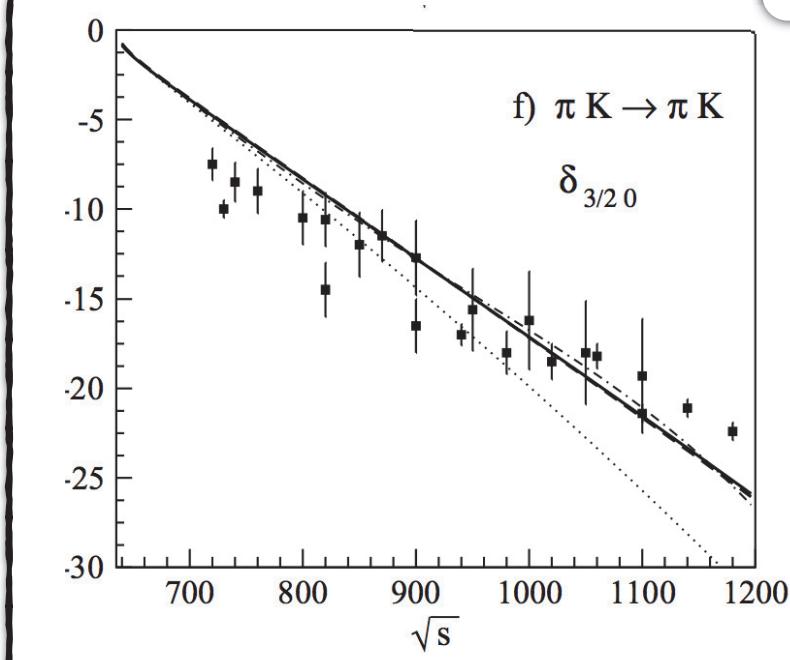
Theoretical tools

- Phenomenological analysis of data

- Models

- EFT-inspired results

"nevertheless, there's been impressive progress. 8 parameters describe all of the low lying partial waves in all of the isospin channels...this shouldn't work this well!"



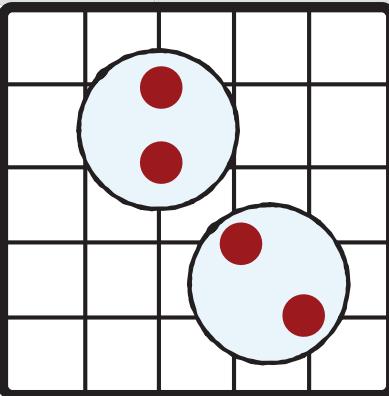
Theoretical tools

- Phenomenological analysis of data
- Models
- EFT-inspired results
- Lattice QCD

- **only systematic tool for studying QCD**
- remarkably challenging
 - one of the more computationally costly theoretical efforts
 - requires a substantial formal effort
 - scattering / reactions are among the most challenging obs.

Lattice QCD in a nutshell

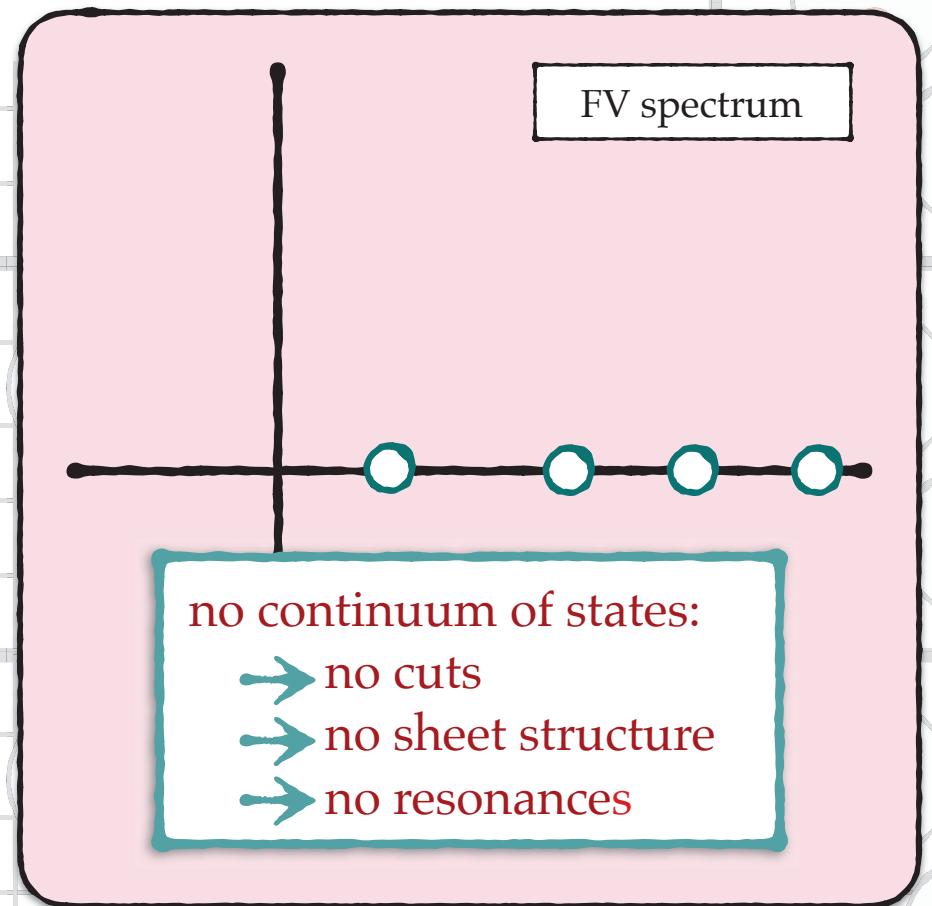
- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses: $m_q \rightarrow m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15 \text{ fm}$
- finite volume



Never free!
No asymptotic states!
No scattering!

Lattice QCD in a nutshell

- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses: $m_q \rightarrow m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15 \text{ fm}$
- finite volume



Status of the field

• Simple properties of QCD stable states [non-composite states]

• physical or lighter quark masses [down to $m_\pi \sim 120$ MeV] 

• non-degenerate light-quark masses: $N_f = 1+1+1+1$ 

• dynamical QED 

• One of the frontiers of lattice QCD: multi-particle physics

• scattering/reactions

• composite states

• bound states

• hadronic resonances

• electrocouplings

Formal development:

• under way

• more needed

Benchmark calculations:

• unphysical quark masses

• exploratory

• proof of principle

• ...

Meet the team - LQCD spectroscopy efforts at JLab and abroad

more formal



Dudek



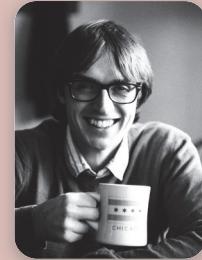
Edwards



Winter



Joó



Wilson



Peardon



Ryan



Thomas



Hansen



Sharpe

more numerical - JLab

more numerical - Europe

Students:
*Johnson, Radhakrishnan,
Cheung, Moss, O Hara, Tims*

had spec

Jefferson Lab

WILLIAM & MARY

OLD DOMINION
UNIVERSITY

UNIVERSITY OF
CAMBRIDGE

W
UNIVERSITY of
WASHINGTON



TRINITY
COLLEGE
DUBLIN

CERN

tiffr

Challenges

Numerical

- operators basis



- correlation functions

Analysis

- spectra



- amplitudes from spectra

- elastic



- inelastic



- three-body



- amplitude analysis



- K matrix



- dispersive tech.

Formalism

- amplitudes from spectra



- elastic



- inelastic

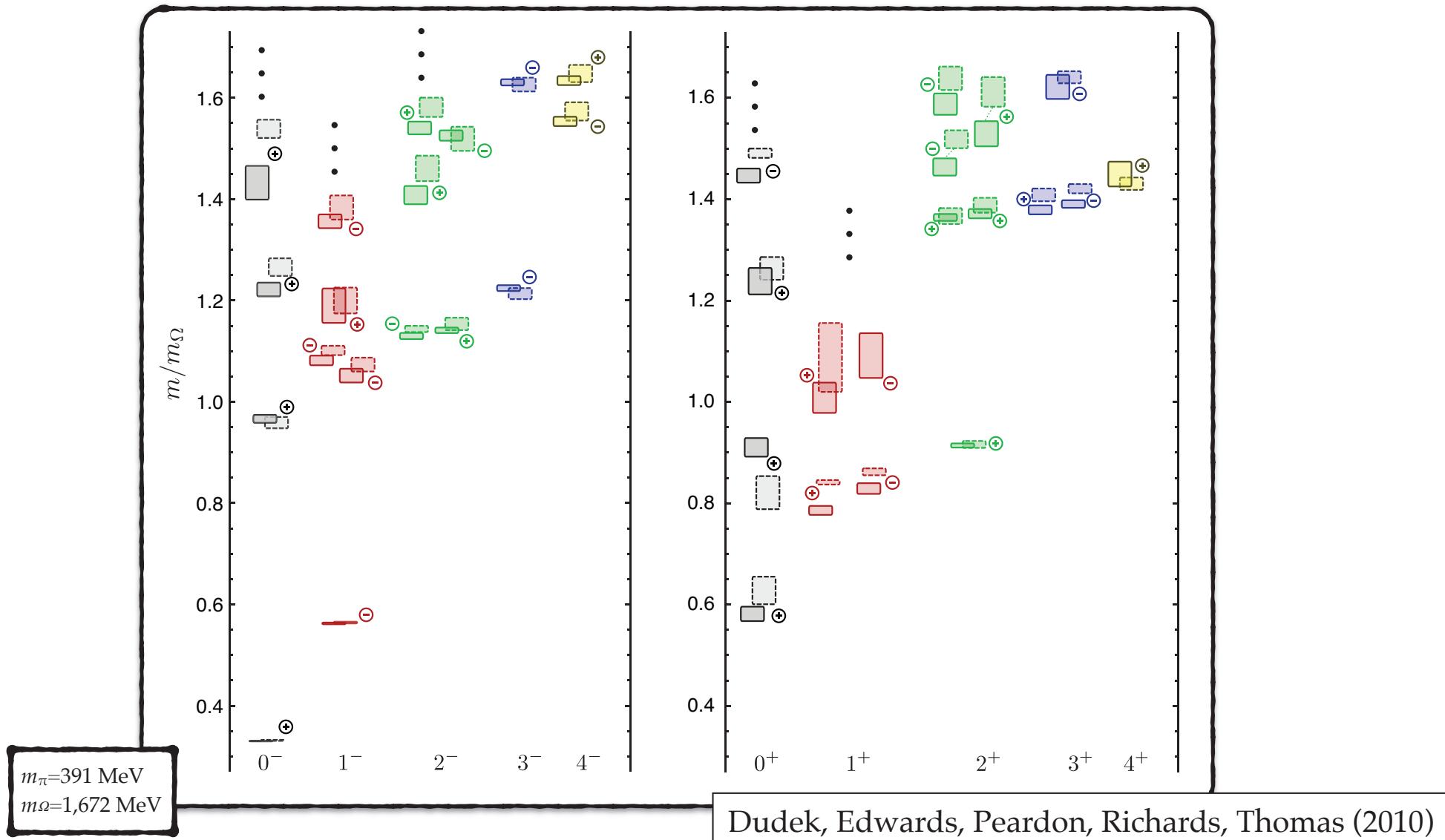


- three-body

New “old-school spectroscopy”

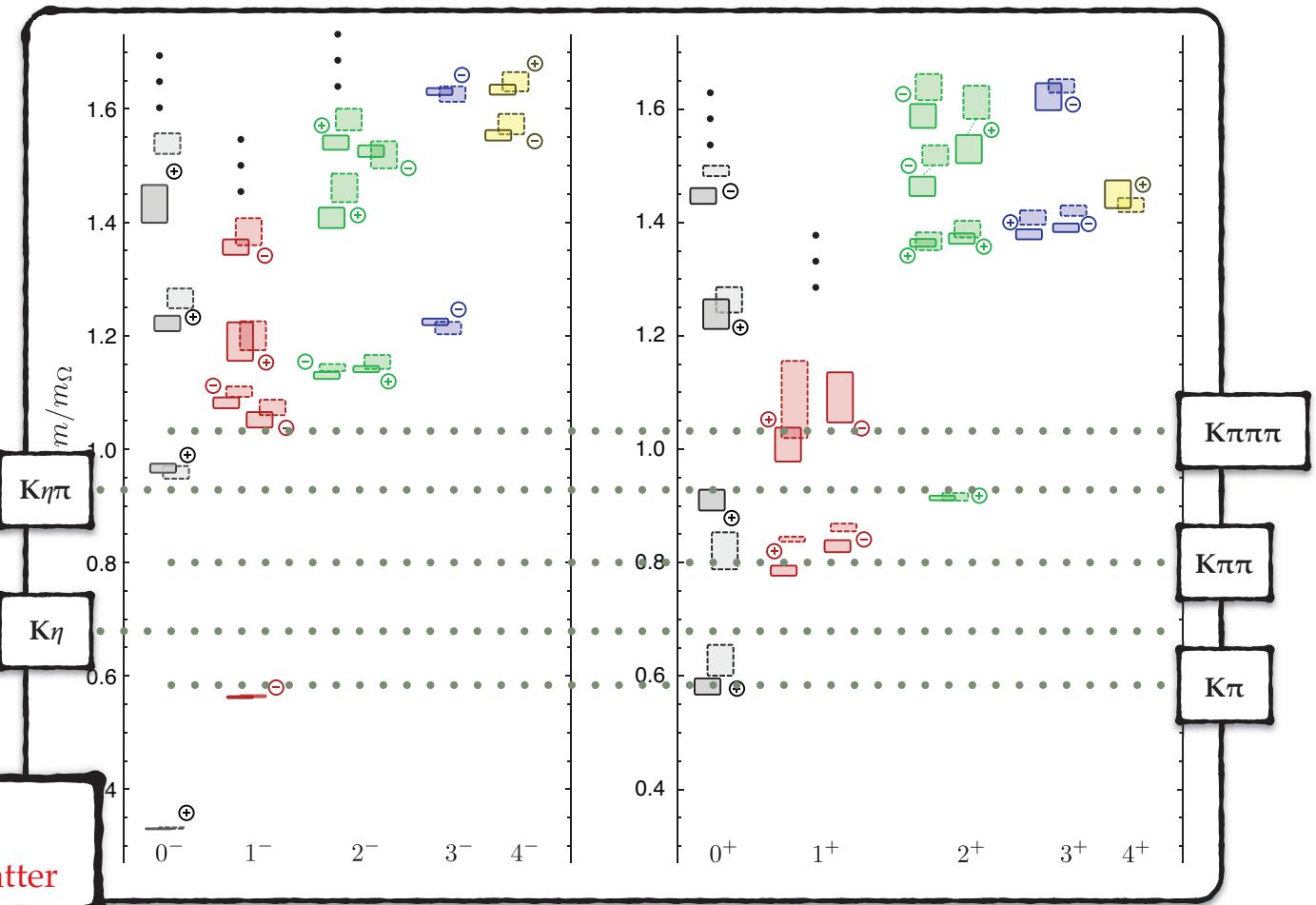
Evaluate: $C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t}$

...a large number [10-30] of local ops, $\mathcal{O}_b \sim \bar{q} \Gamma_b q$



Narrow width approximation

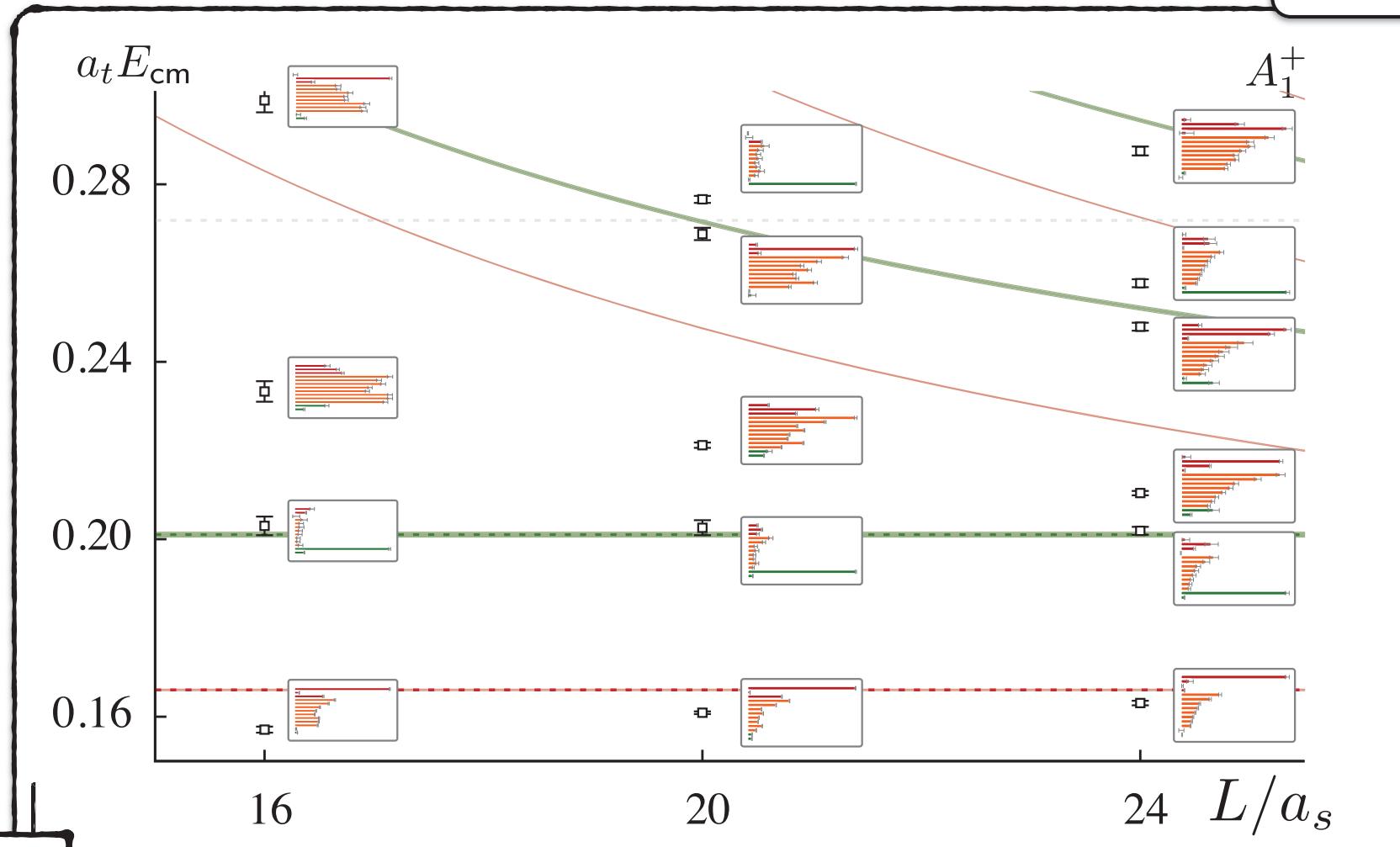
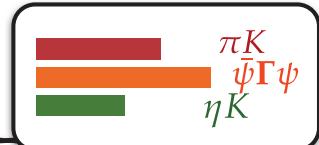
- Op. basis did not include multi-hadron ops: $K\pi, K\eta, K\pi\pi, K\pi\eta, K\eta\eta, \dots$
- Unstable nature of the states ignored
- Finite-volume states are *not* resonances
- Must use Lüscher and its extensions
- Spectrum does suggest where *some* resonance might lie



Spectrum: S-wave dominant

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

Use local and multi-hadron ops: $\bar{\psi}\Gamma\psi, K\pi, K\eta, K\pi\pi, K\pi\eta, K\eta\eta, \dots$



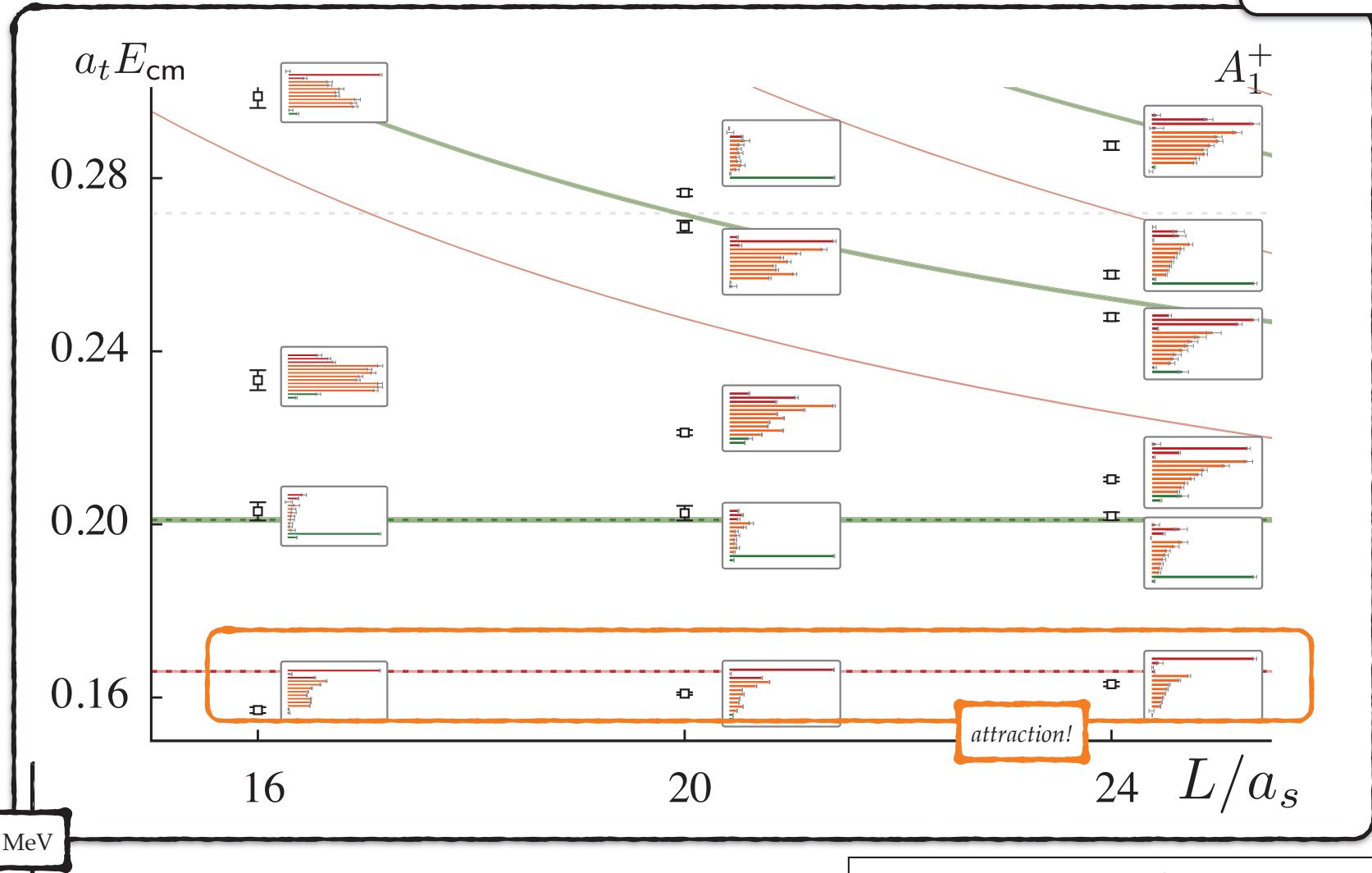
$m_\pi = 391$ MeV

Dudek, Edwards, Thomas, Wilson (2014)

Spectrum: S-wave dominant

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

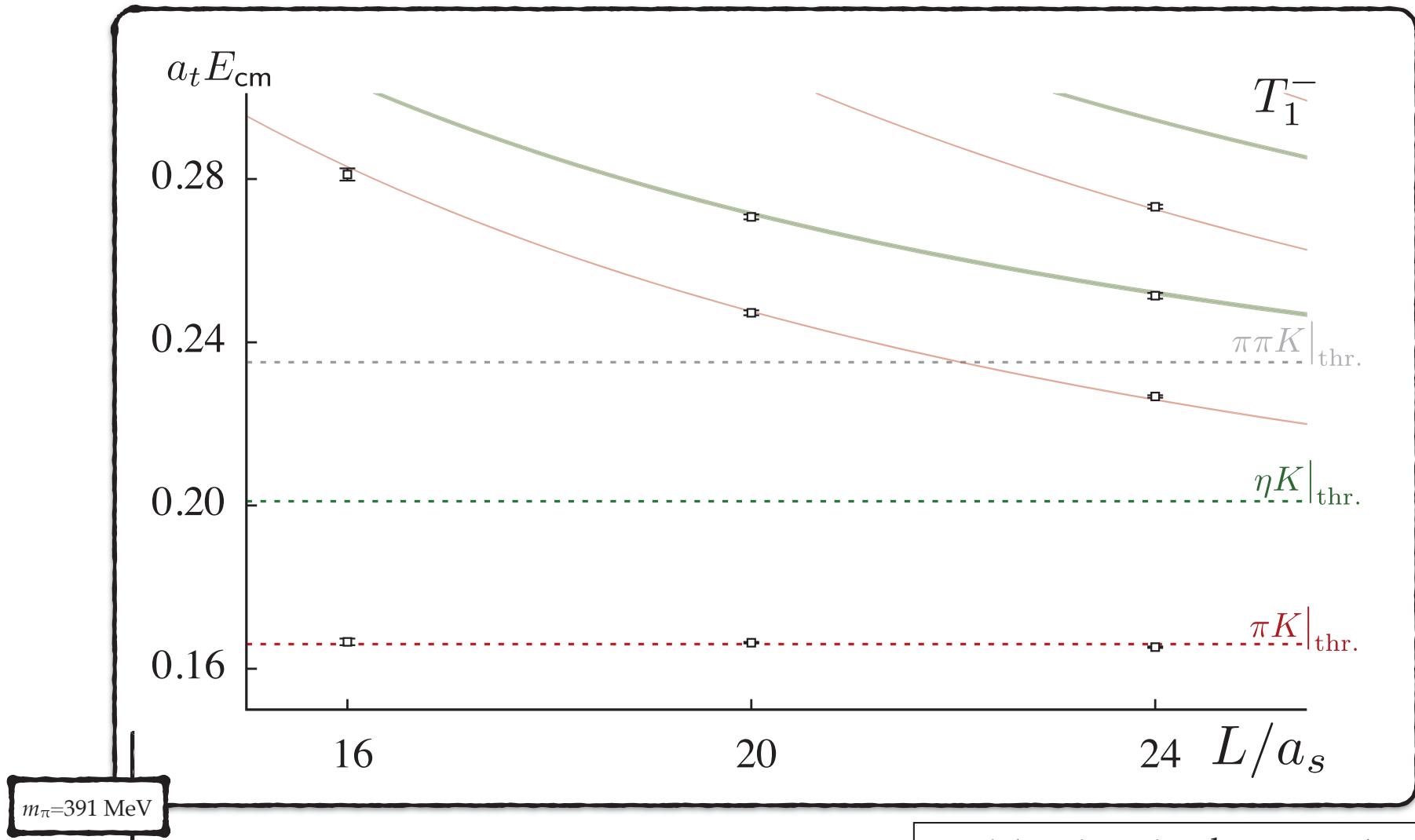
Use local and multi-hadron ops: $\bar{\psi}\Gamma\psi, K\pi, K\eta, K\pi\pi, K\pi\eta, K\eta\eta, \dots$



Spectrum: P-wave dominant

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

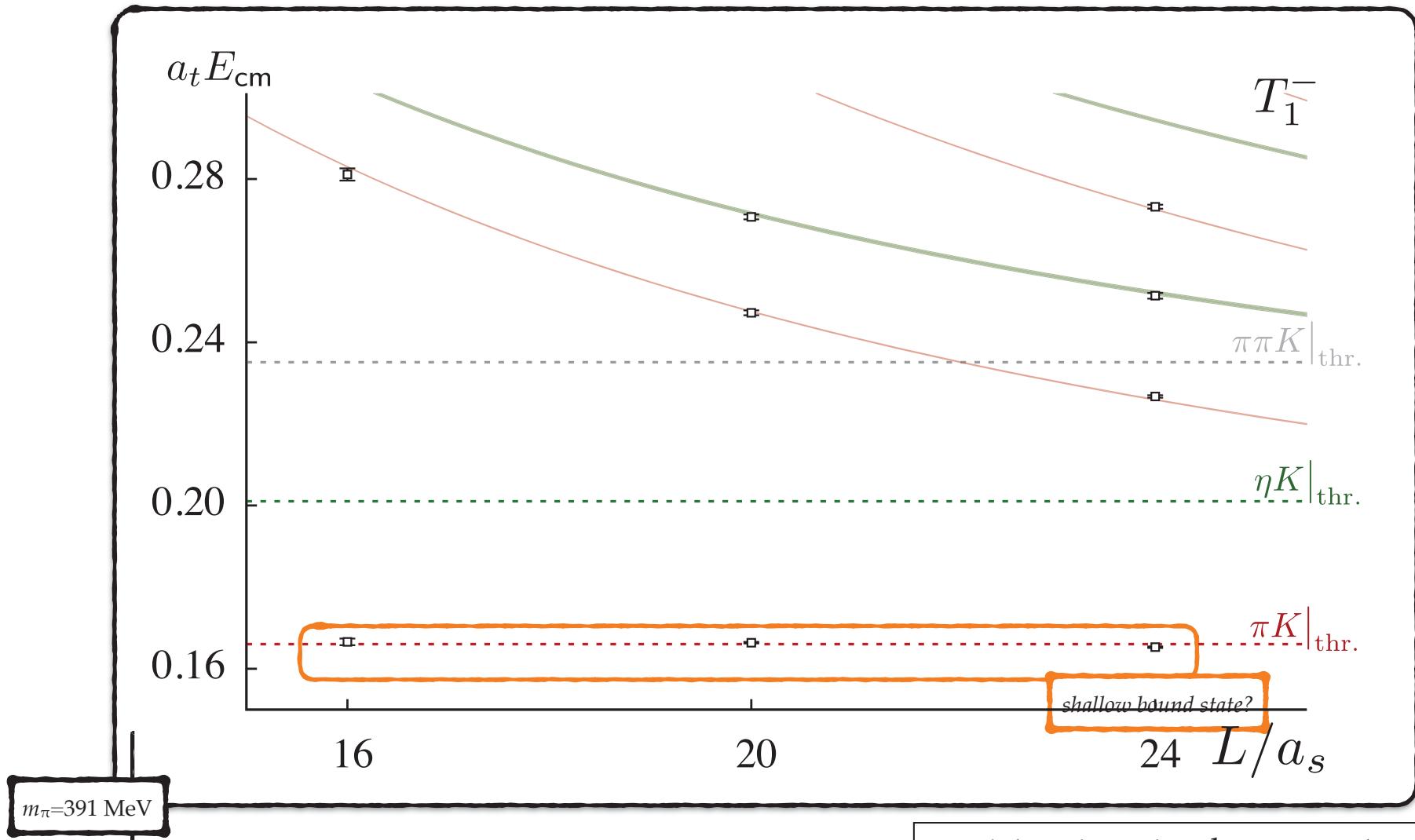
📌 Use local and multi-hadron ops: $\bar{\psi}\Gamma\psi, K\pi, K\eta, \cancel{K\pi\pi}, \cancel{K\pi\eta}, \cancel{K\eta\eta}, \dots$.



Spectrum: P-wave dominant

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

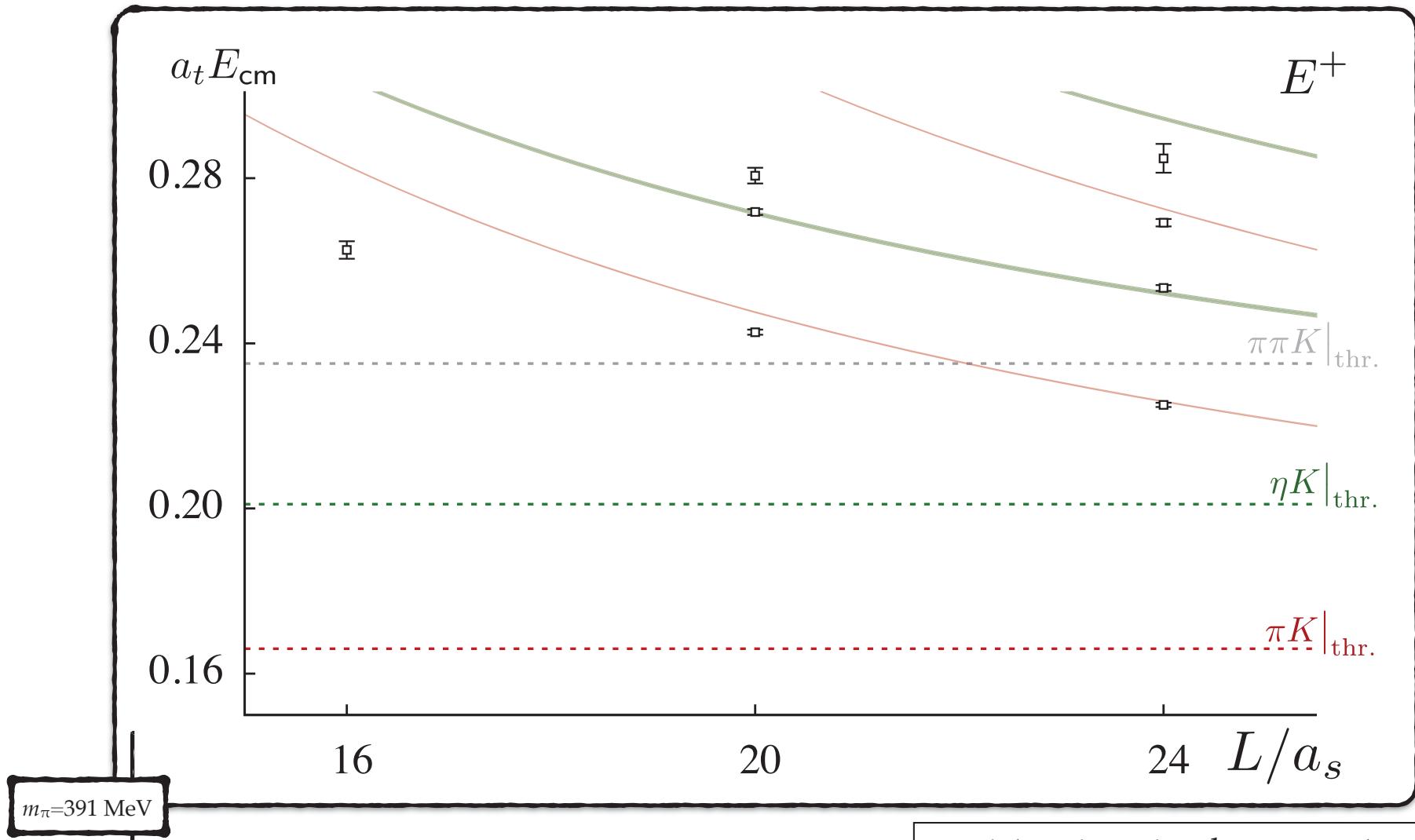
📌 Use local and multi-hadron ops: $\bar{\psi}\Gamma\psi, K\pi, K\eta, \cancel{K\pi\pi}, \cancel{K\pi\eta}, \cancel{K\eta\eta}, \dots$.



Spectrum: D-wave dominant

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

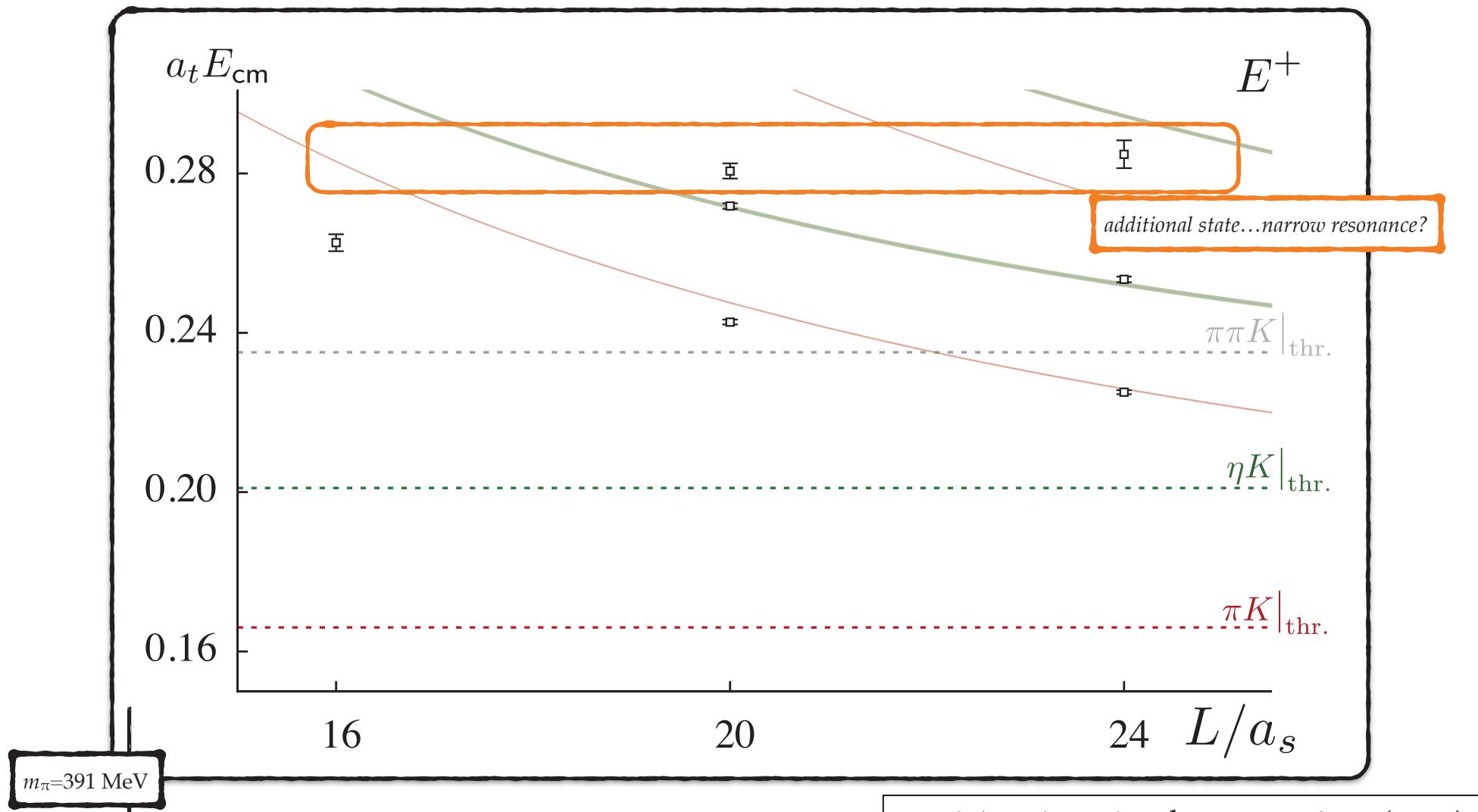
Use local and multi-hadron ops: $\bar{\psi}\Gamma\psi, K\pi, K\eta, \cancel{K\pi\pi}, \cancel{K\pi\eta}, \cancel{K\eta\eta}, \dots$.



Spectrum: D-wave dominant

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

📌 Use local and multi-hadron ops: $\bar{\psi}\Gamma\psi, K\pi, K\eta, \cancel{K\pi\pi}, \cancel{K\pi\eta}, \cancel{K\eta\eta}, \dots$.



Challenges

Numerical

- operators basis



- correlation functions

Analysis

- spectra



- amplitudes from spectra



- elastic



- inelastic



- three-body

- amplitude analysis



- K matrix



- dispersive tech.

Formalism

- amplitudes from spectra



- elastic

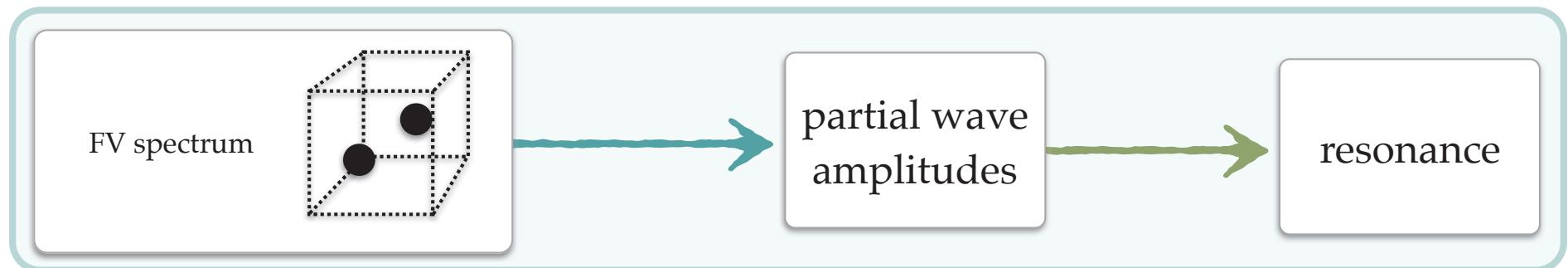


- inelastic



- three-body

Scattering amplitudes

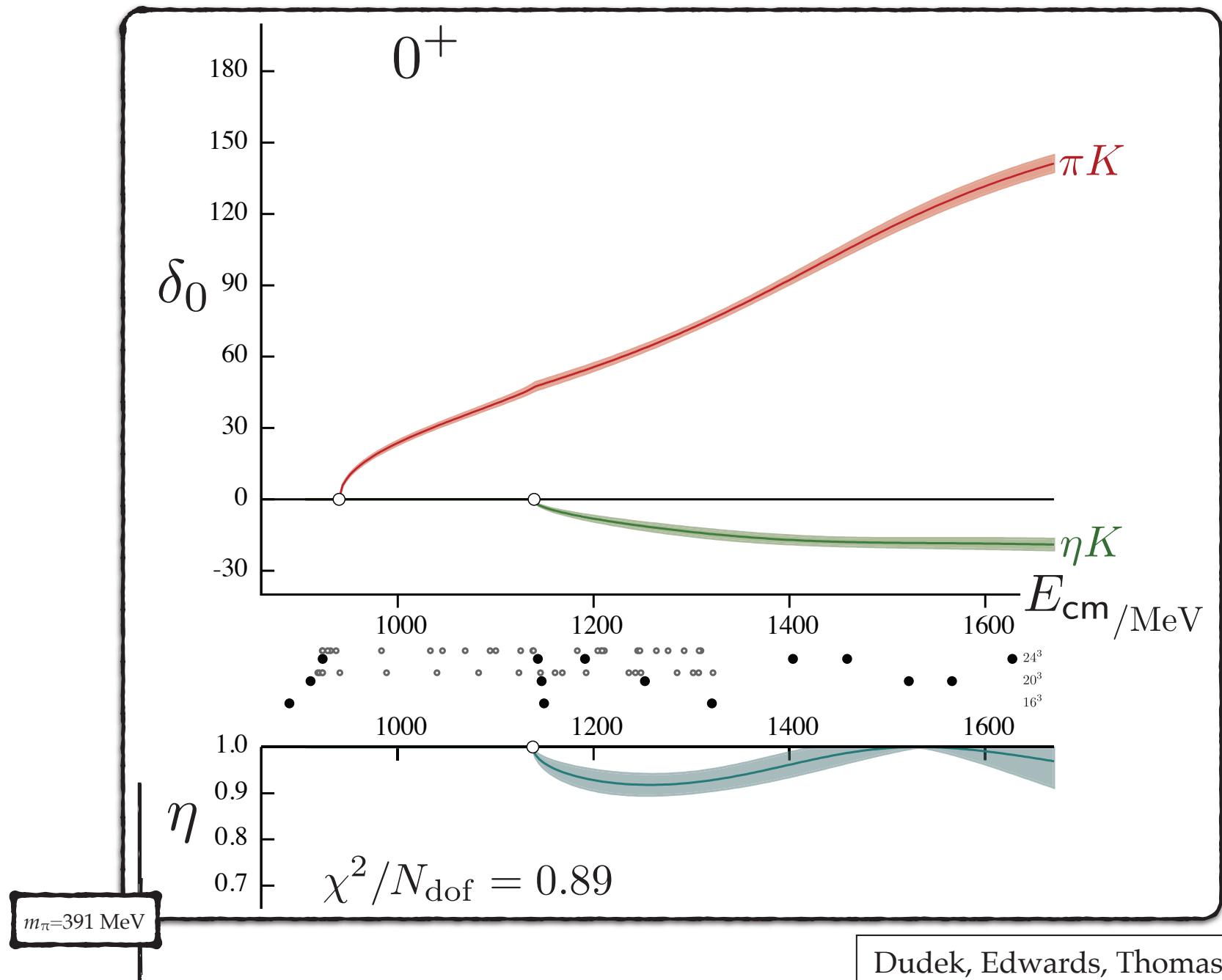


$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

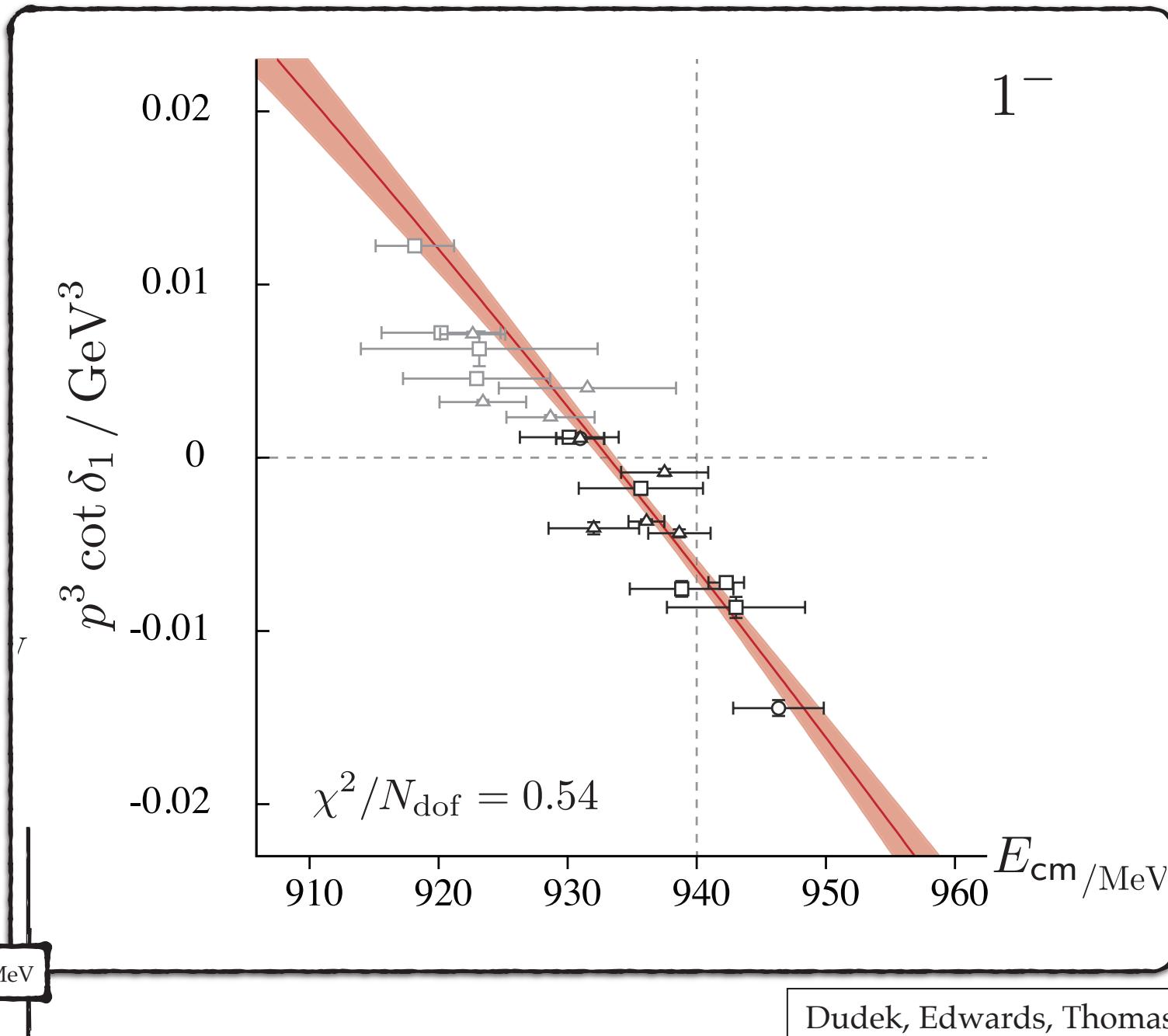
E_L = finite volume spec.
 L = finite volume
 F = known function
 \mathcal{M} = scattering amp.

- ➊ Lüscher (1986, 1991) [elastic scalar bosons]
- ➋ Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- ➋ Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [QFT derivation]
- ➋ Feng, Li, & Liu (2004) [inelastic scalar bosons]
- ➋ Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- ➋ RB (2014) [general 2-body result]

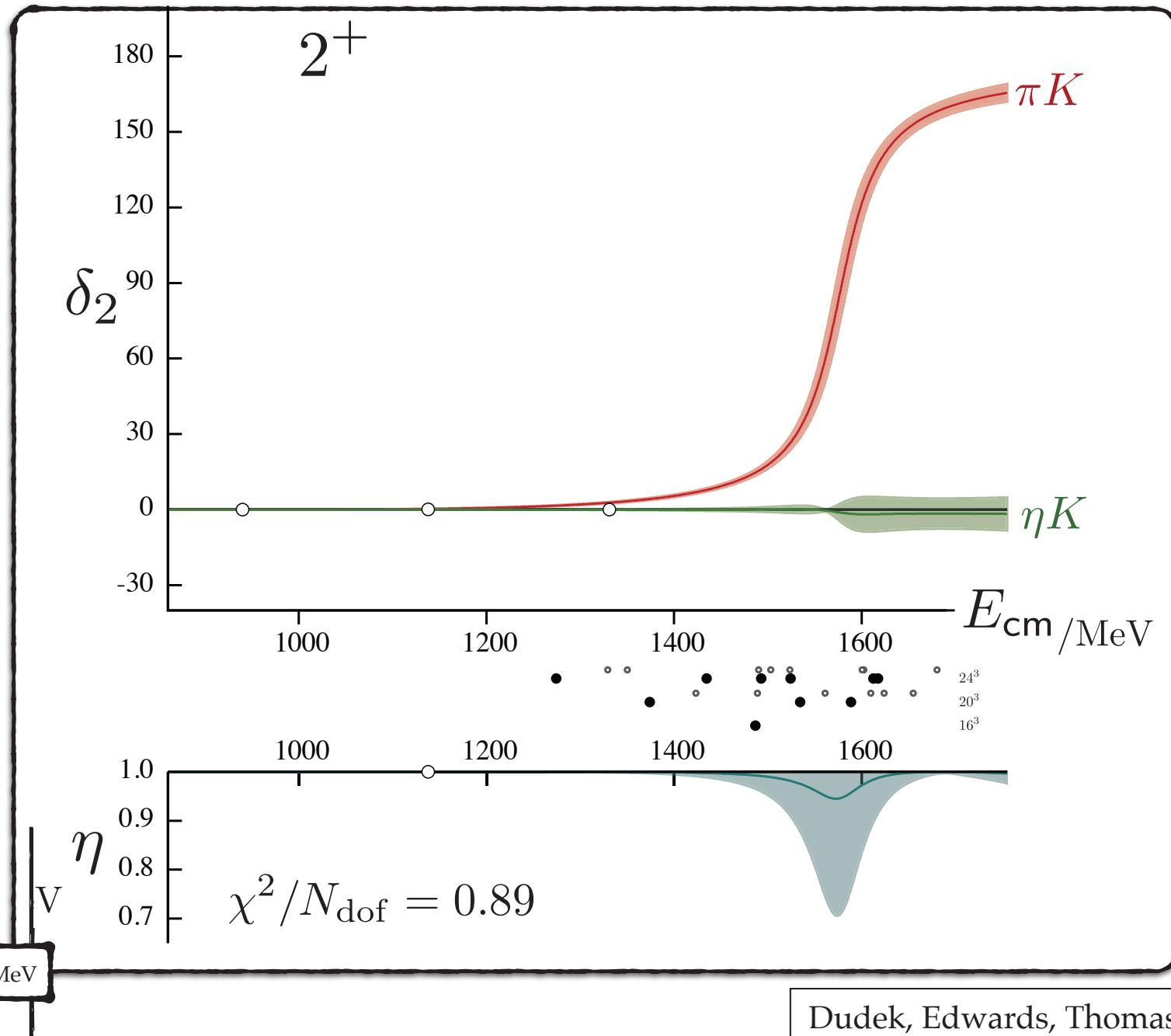
S-wave



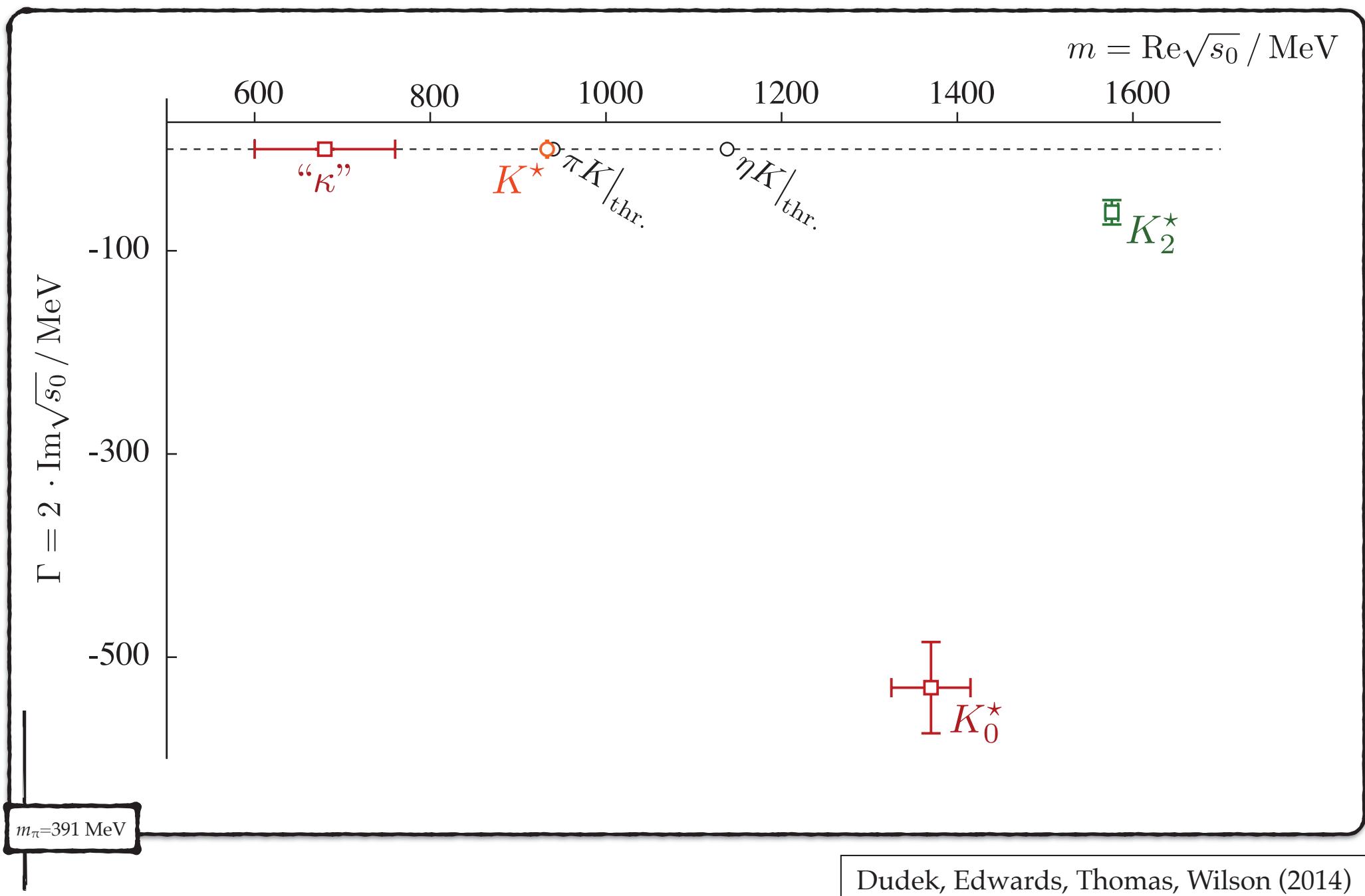
P-wave



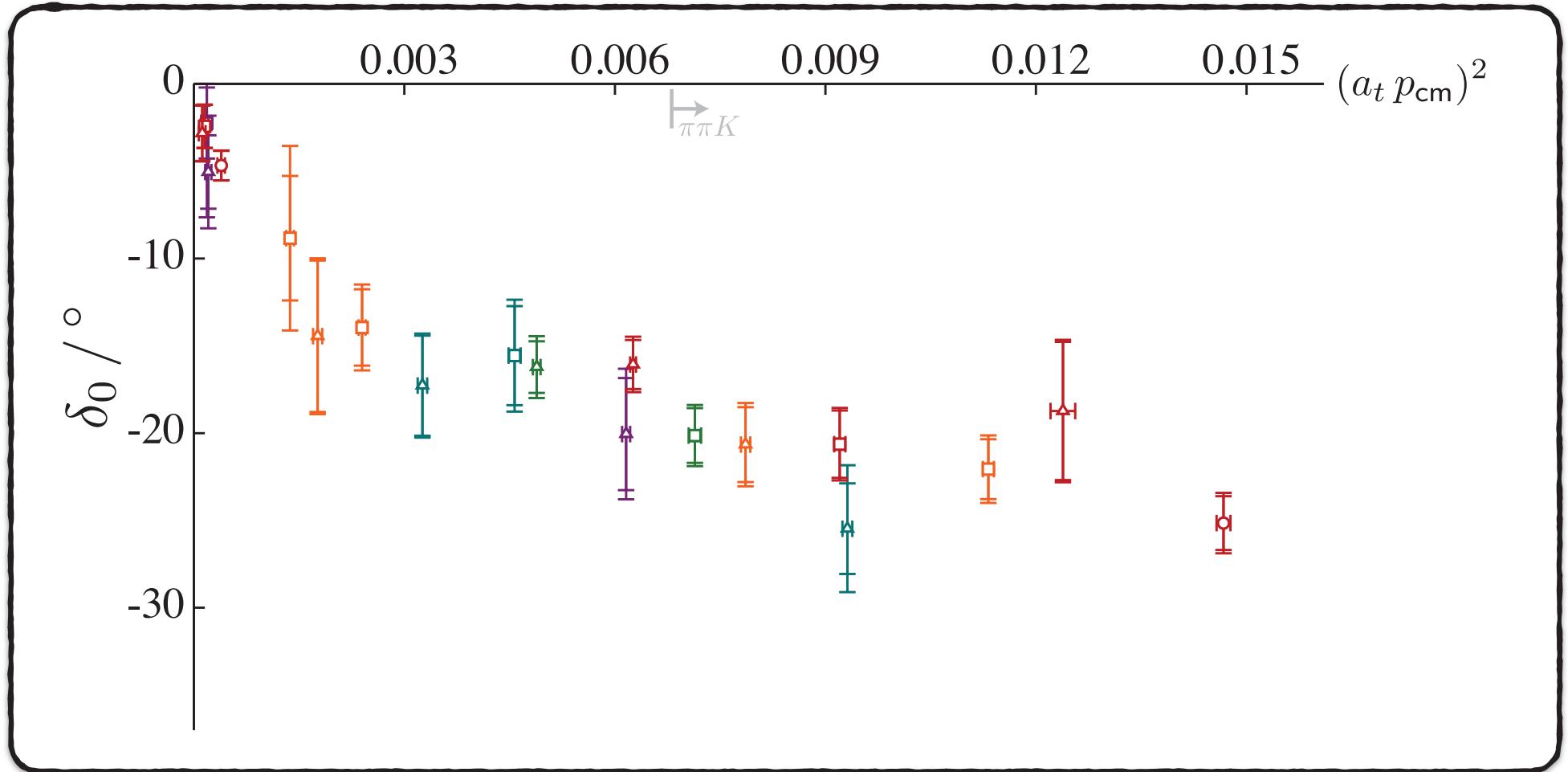
D-wave



Resonances



Isospin=3/2



Challenges

Numerical

- operators basis

- correlation functions



Analysis

- spectra
- amplitudes from spectra
- elastic
- inelastic
- three-body
- amplitude analysis
- K matrix
- dispersive tech.



Formalism

- amplitudes from spectra
- elastic
- inelastic
- three-body



Challenges

Numerical

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- correlation functions



Analysis

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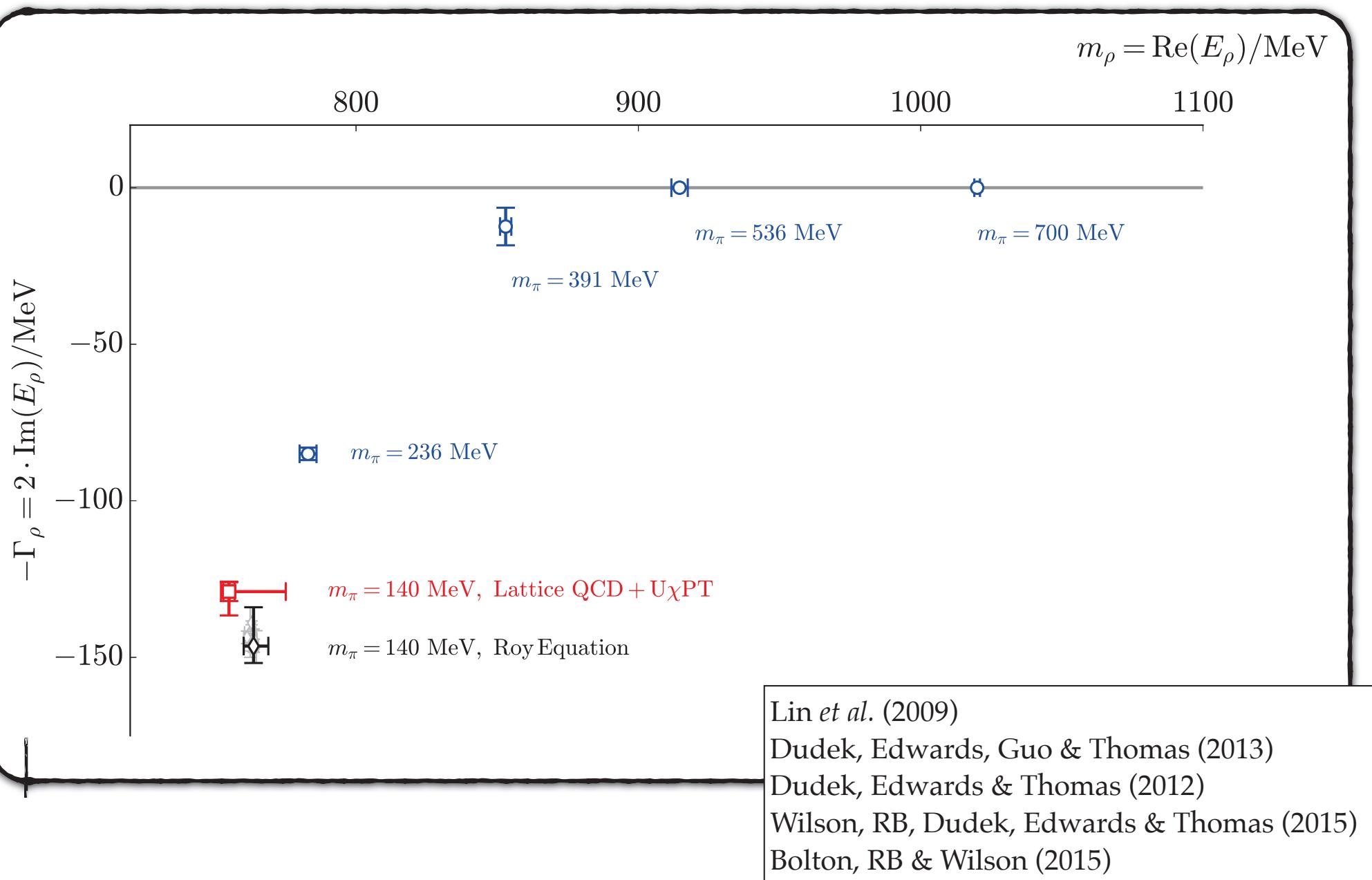
Formalism

- amplitudes from spectra
- elastic
- inelastic
- three-body

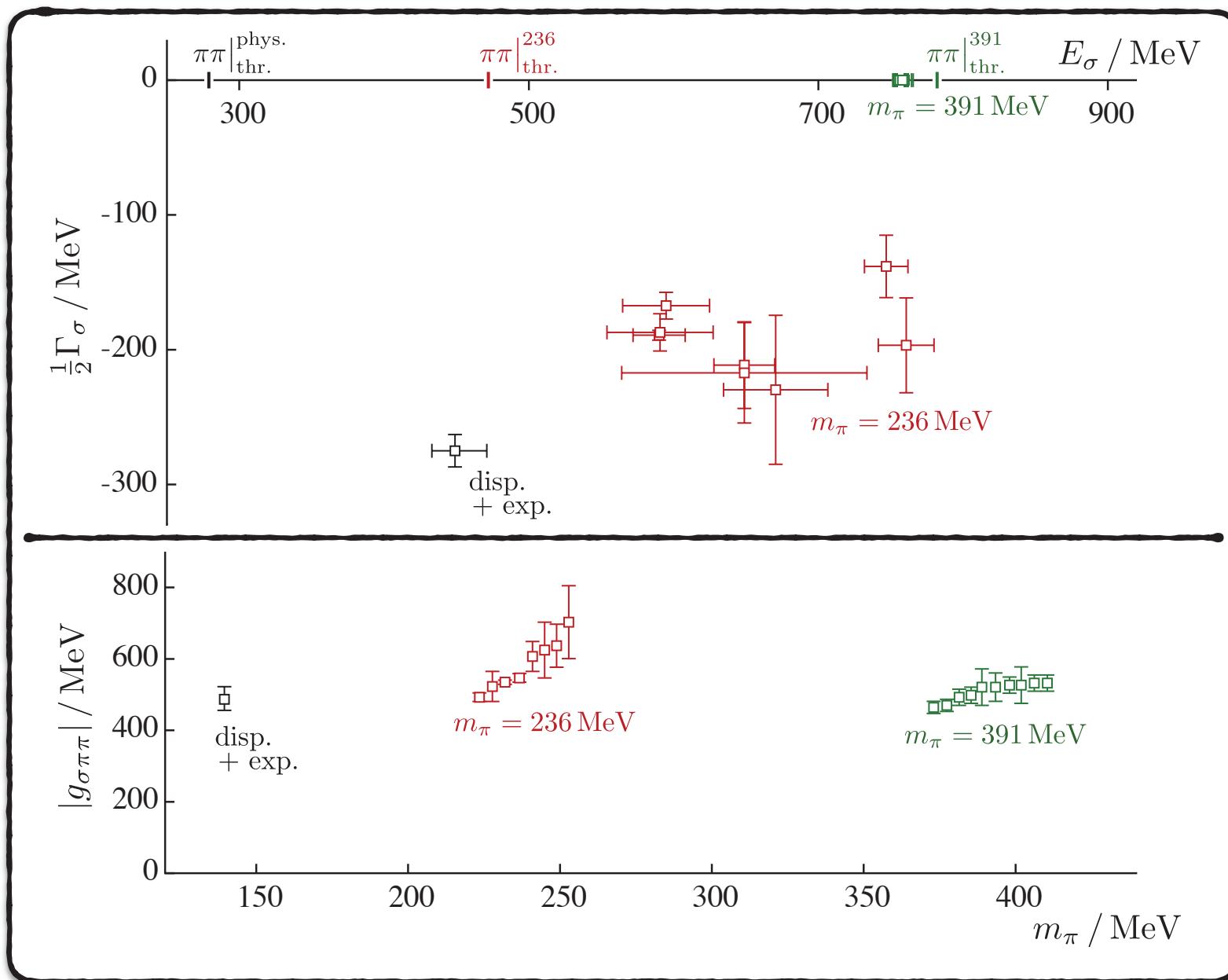


the remaining real challenges for studying resonant systems using physical quark masses

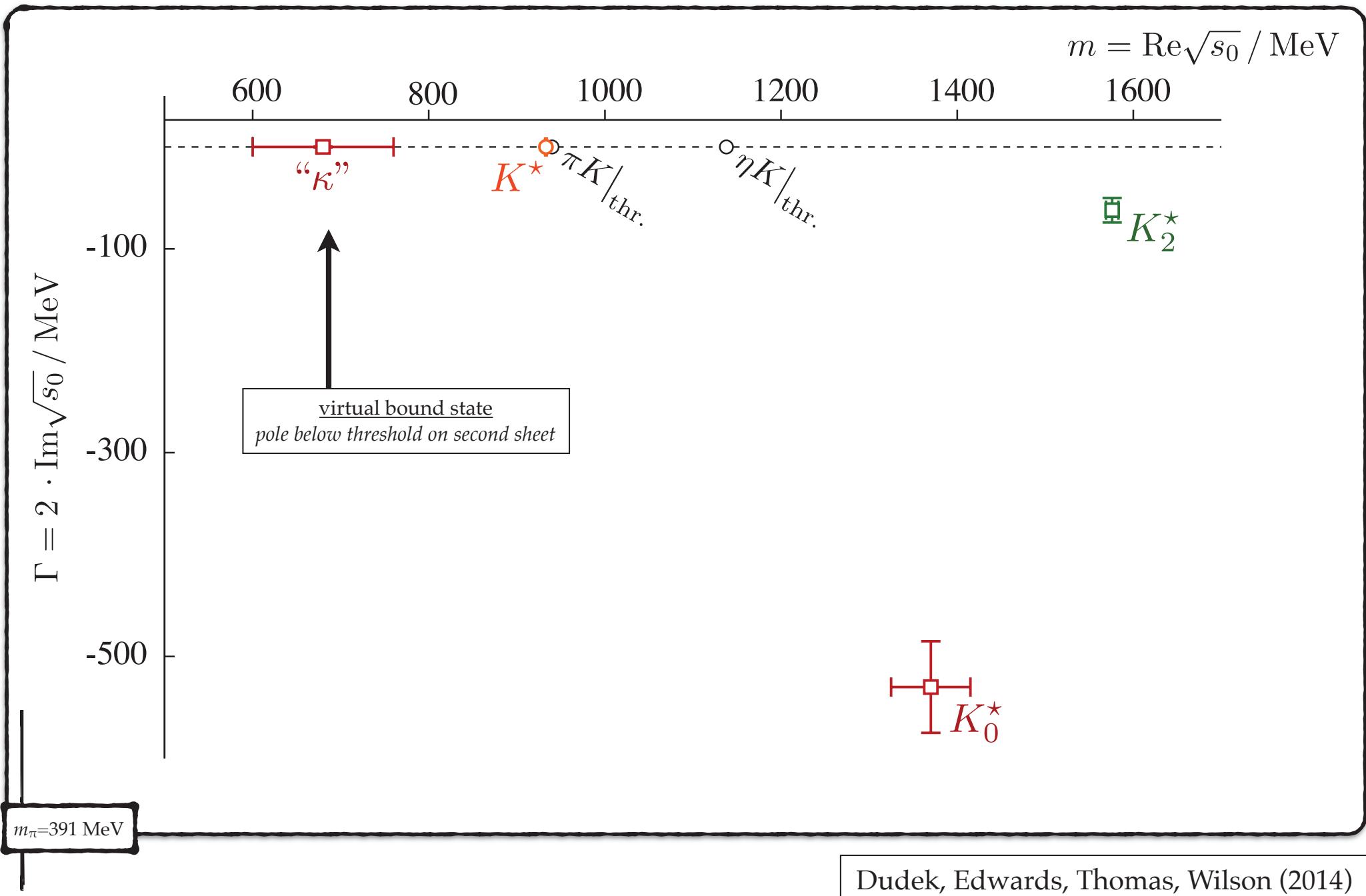
Need for dispersive analysis - the “ ρ ”



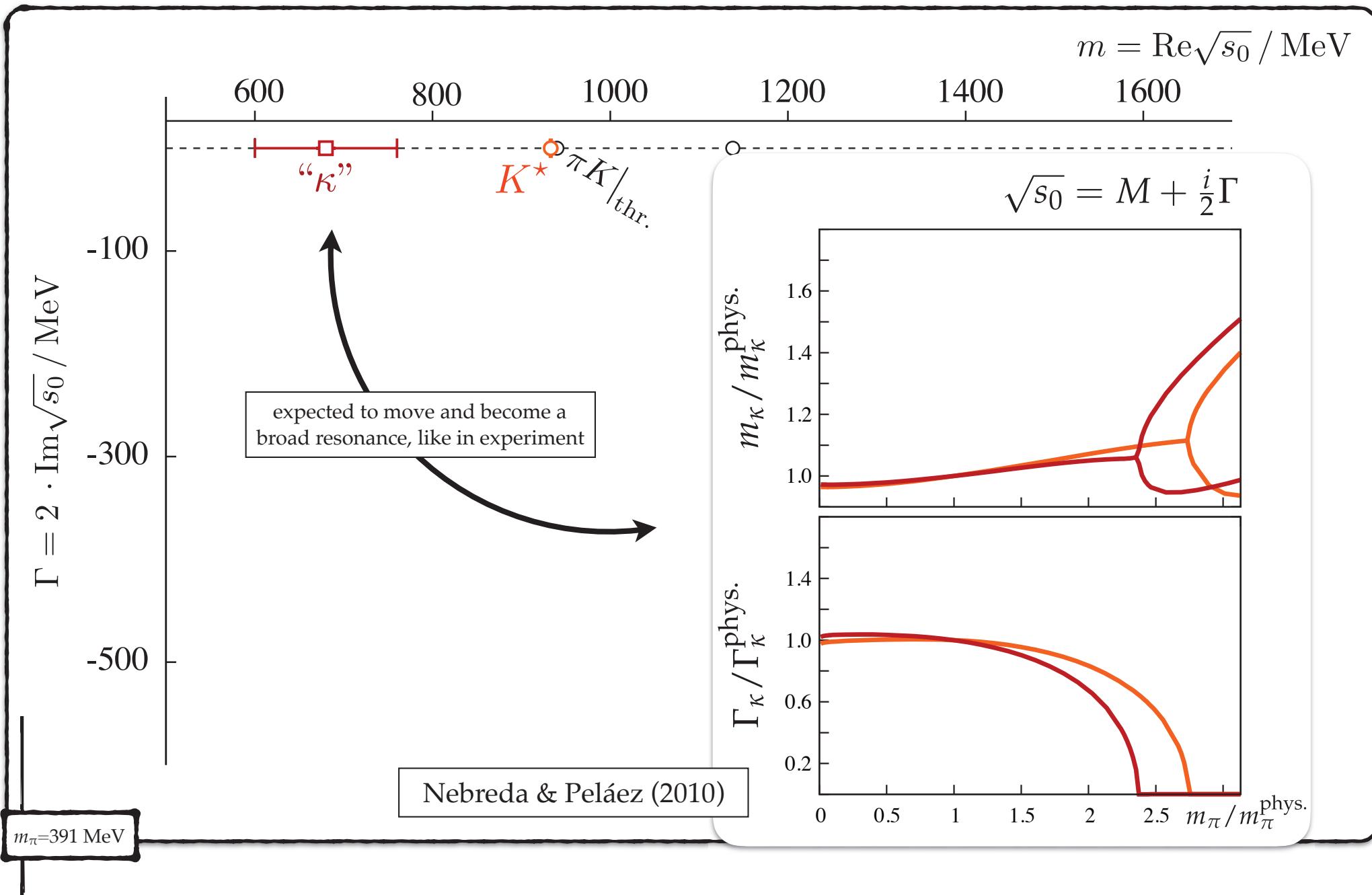
Need for dispersive analysis - the “ σ ”



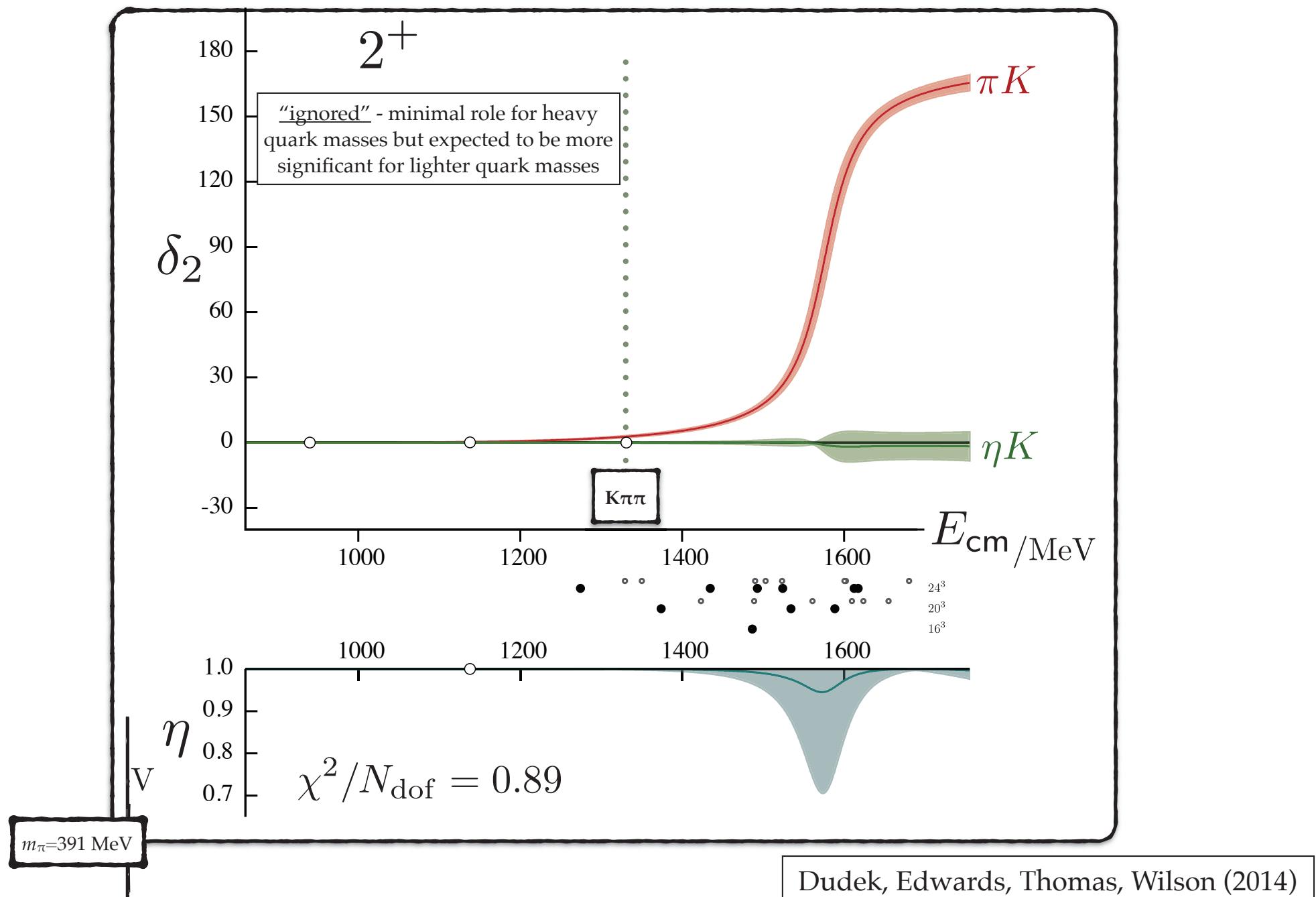
Need for dispersive analysis - the “ κ ” and K^*



Need for dispersive analysis - the “ κ ” and K^*



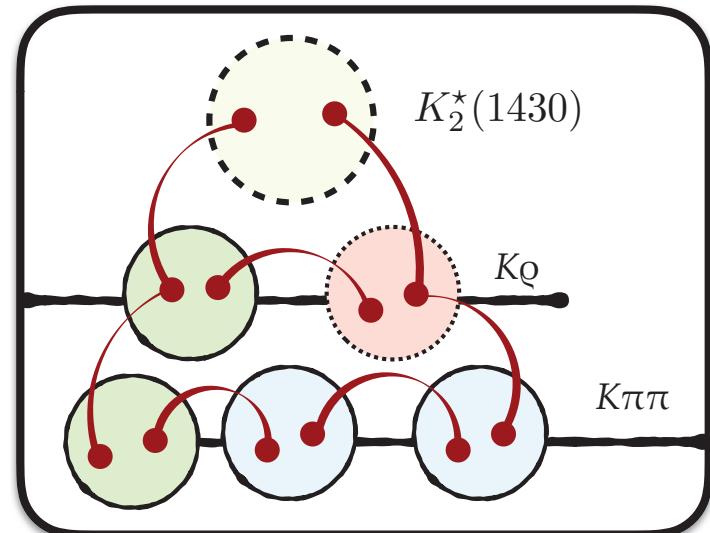
Need for three-body formalism



Need for three-body formalism

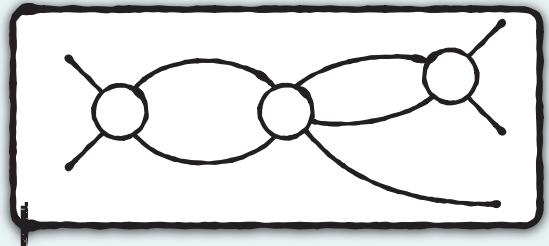
Needed for:

- resonances
- Kaons
- baryons [e.g., the Roper]
- ...
- 3N-force



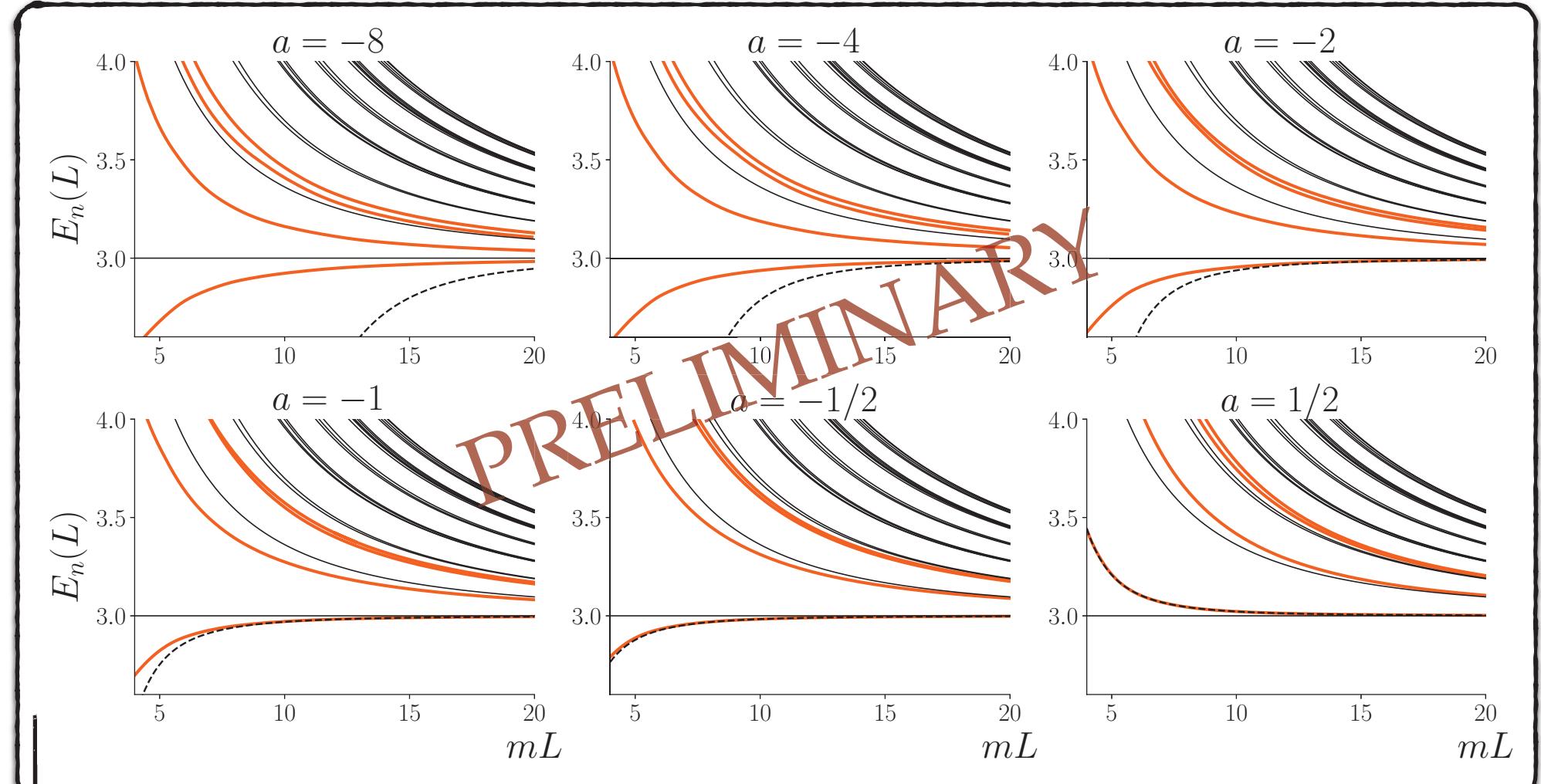
$$\det \left[1 + \begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix} \begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\text{df},3} \end{pmatrix} \right] = 0$$

RB, Hansen & Sharpe (2016)



- Polejaeva & Rusetsky (2012) [spectrum depends on S-matrix]
- RB & Davoudi (2013) [1+shallow bound states]
- Hansen & Sharpe (2014-15) [relativistic $\pi\pi\pi$]
- RB, Hansen & Sharpe (2016) [relativistic coupled, 2-, and 3-mesons]

Need for three-body formalism



analysis code under construction!

RB, Hansen, Sharpe (to appear)

Challenges

Numerical

- operators basis

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- elastic
- inelastic
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- amplitude analysis
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- dispersive tech.



Formalism

- amplitudes from spectra
- elastic
- inelastic
- three-body



underway!

The team and some references

more numerical - JLab



Dudek



Edwards

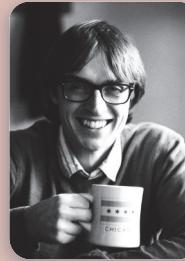


Winter



Joó

more numerical - Europe



Wilson



Peardon



Ryan



Thomas

more formal



Hansen



Sharpe

Meson Spectrum

- JHEP05 021 (2013)
PRD88 094505 (2013)
JHEP07 126 (2011)
PRD83 111502 (2011)
PRD82 034508 (2010)
PRL103 262001 (2009)

Baryon Spectrum

- PRD91 094502 (2015)
PRD90 074504 (2014)
PRD87 054506 (2013)
PRD85 054016 (2012)
PRD84 074508 (2011)

Scattering

- arXiv:1708.06667
PRL118 022002 (2017)
JHEP011 1610 (2016)
PRD93 094506 (2016)
PRD92 094502 (2015)
PRD91 054008 (2015)
PRL113 182001 (2014)
PRD87 034505 (2013)
PRD86 034031 (2012)
PRD83 071504 (2011)

Electroweak

- PRD93 114508 (2016)
PRL115 242001 (2015)
PRD91 114501 (2015)
PRD90 014511 (2014)

Techniques

- arXiv:1709.01417
PRD85 014507 (2012)
PRD80 054506 (2009)
PRD79 034502 (2009)

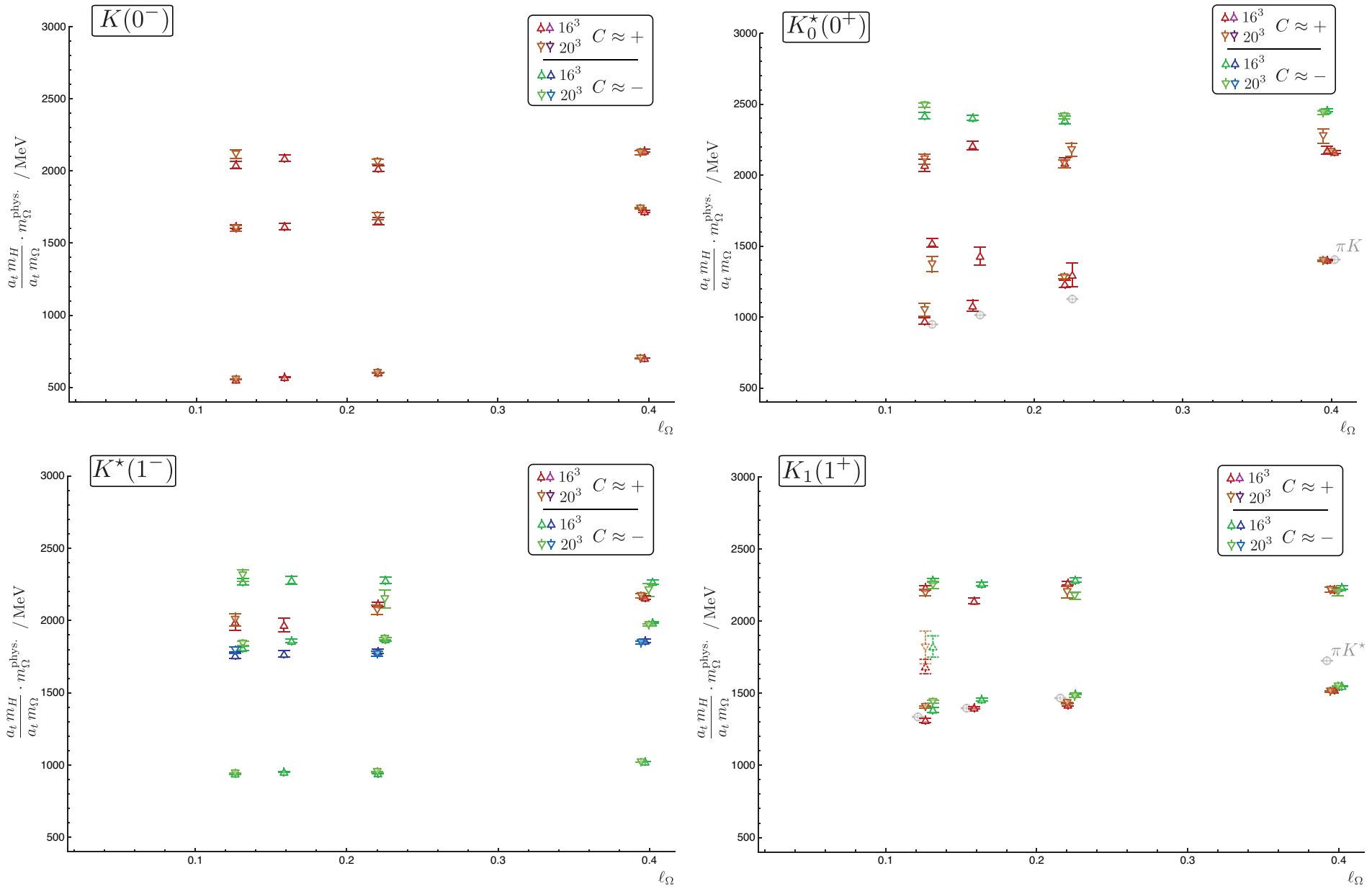
Students:

Johnson, Radhakrishnan,
Cheung, Moss, O Hara, Tims

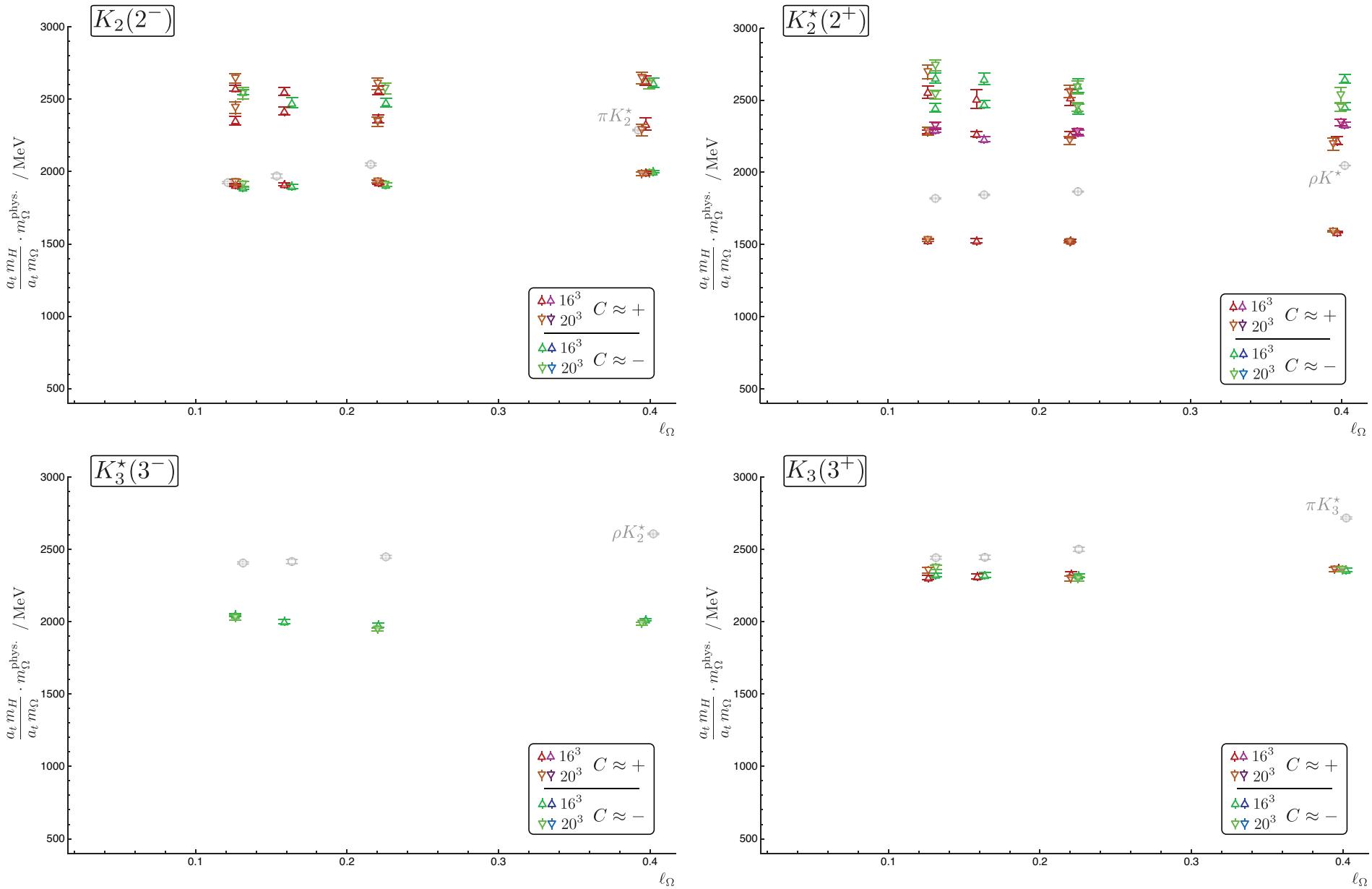
Formalism

- PRD95 074510 (2017)
PRD94 013008 (2016)
PRD92 074509 (2015)
PRD91 034501 (2015)
PRD89 074507 (2014)

Narrow width approximation vs. m_π



Narrow width approximation vs. m_π



Status of the field

- Simple properties of QCD stable states [non-composite states]

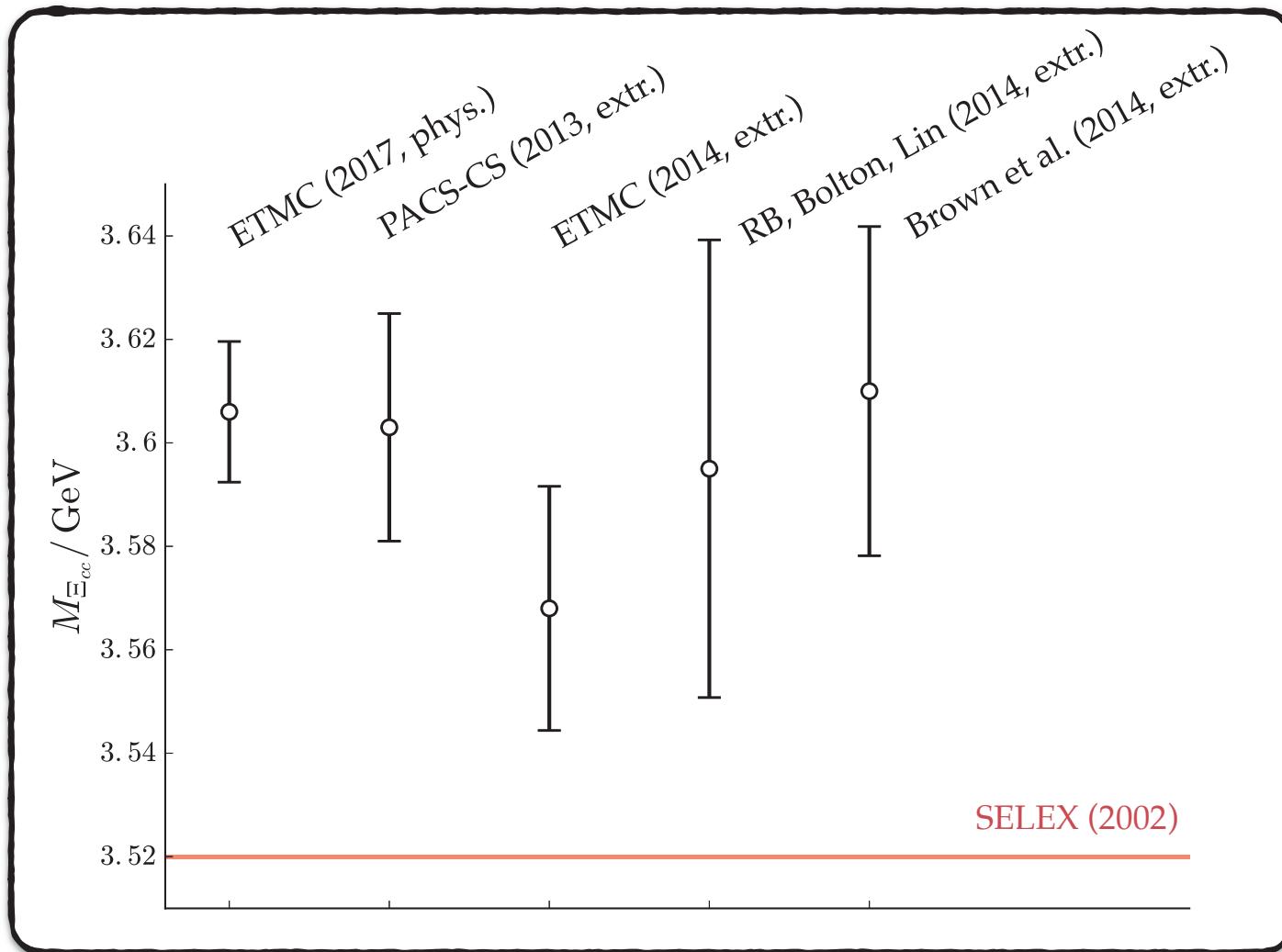
- physical or lighter quark masses [down to $m_\pi \sim 120$ MeV]



- non-degenerate light-quark masses: $N_f = 1+1+1+1$



- dynamical QED



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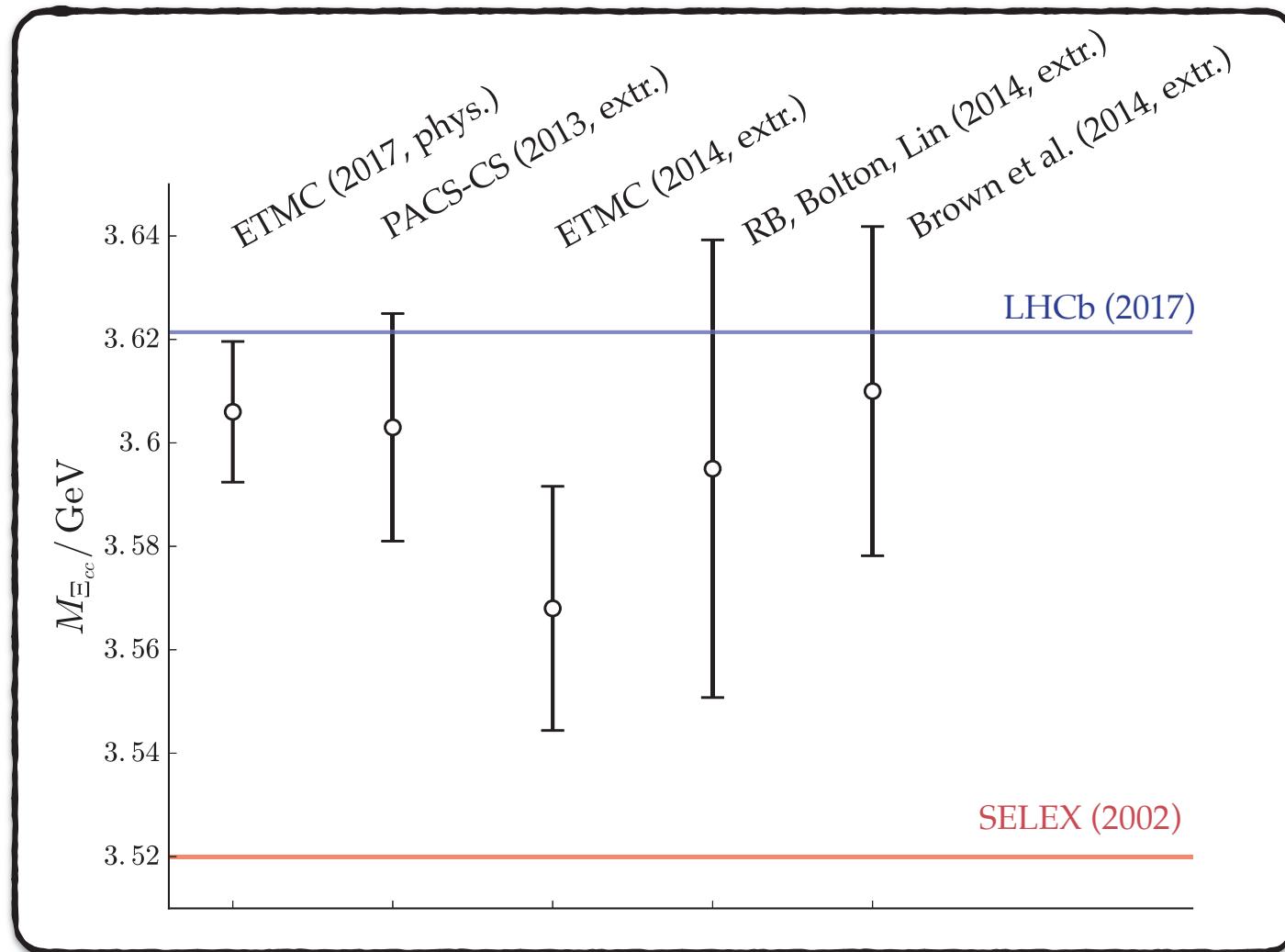
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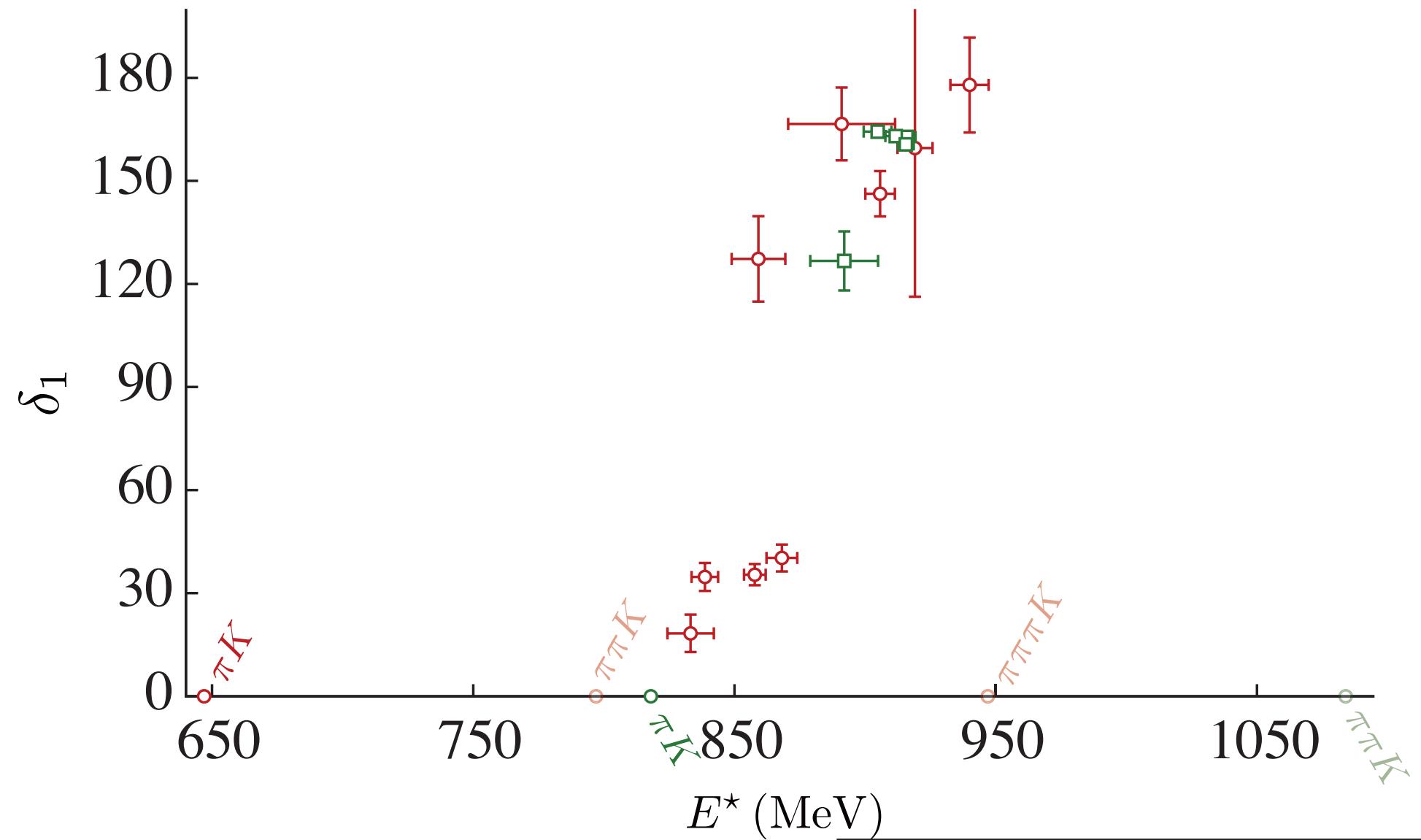
- non-degenerate light-quark masses: $N_f = 1+1+1+1$



- dynamical QED



Other efforts - (I=1/2, P-wave)



$m_\pi \sim 270$ MeV, $N_f=2$, Prelovsek et al. (2013)
 $m_\pi \sim 150$ MeV, $N_f=2$, Bali et al. (2016)

Other efforts - (I=3/2, S-wave)

