Mass and Structure of the Pion and Kaon

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High-priority science questions

The NAS Assessment of a U.S. based Electron Ion Collider identified three high-priority science questions

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

What do we know about hadron masses?

- The three current quarks needed to define the nucleon quantum numbers contribute only \( \sim 1\% \) to its mass
- In chiral limit nucleon mass \( \sim 900 \text{ MeV} \); *Higgs mechanism is largely irrelevant for visible mass*
- The chiral symmetry of \( \mathcal{L}_{QCD} \) is dynamically broken \( \Rightarrow \sim 500 \text{ MeV} \) mass splittings in hadron spectrum & massless Goldstone bosons in chiral limit (\( \pi, K, \eta \))

Therefore understanding the nucleon mass is not sufficient

- must also understand the mass of the pion (\( u\bar{d}, \ldots \)) and kaon (\( u\bar{s}, \ldots \))
**High-priority science questions**

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- **What do we know about hadron masses?**
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  - The chiral symmetry of $\mathcal{L}_{QCD}$ is dynamically broken $\Rightarrow \sim 500$ MeV mass splittings in hadron spectrum & massless Goldstone bosons in chiral limit ($\pi, K, \eta$).
  - Therefore understanding the nucleon mass is not sufficient, must also understand the mass of the pion ($u\bar{d}, \ldots$) and kaon ($u\bar{s}, \ldots$).
**Story of the Pion and Kaon**

- In 1935 Yukawa postulated a strongly interacting particle [“(π-) meson”] as a mediator for the strong nuclear force.

- In 1947 both the π and shortly afterwards the K were discovered from cosmic ray tracks in a photographic emulsion.

- Today, pion still regarded as the mediator of the strong force in *ab initio* approaches to nuclear structure; the kaon has played an important role in establishing quark model, and understanding flavor breaking & CP violation.

- Formally the pion and kaon are now understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD.

- This dichotomous nature has numerous ramifications near chiral limit *e.g.*:

  \[
  f_\pi^2 m_\pi^2 \approx \frac{1}{2} (m_u + m_d) \langle \psi \bar{\psi} \rangle, \quad H_{\pi}^{u-d}(x, \xi \to 1, 0) = \phi_\pi \left( \frac{1 + x}{2} \right), \quad D_\pi(0) = -1
  \]

- Perturbative QCD can also make predictions for pion and kaon structure – therefore π and K provide an ideal laboratory to test and understand QCD.
What we know about the Pion and Kaon

- Pion and kaon structure is slowly being revealed using: $\pi^-/K^-$ beams at CERN; Sullivan type experiments at Jefferson Lab; $\pi^-$ beams at Fermilab; and $e^+e^- \rightarrow \pi^+\pi^-, K^+K^-$ in the time-like region.

- 40 years of experiments has revealed e.g.:
  - $r_\pi = 0.672 \pm 0.008$, $r_{K^+} = 0.560 \pm 0.031$, $r_{K^0} = -0.277 \pm 0.018$

- Still a lot more to learn about pion and kaon structure:
  - quark and gluon PDFs; TMDs including Boer-Mulders function; $q, g \rightarrow \pi/K$ fragmentation functions, quark and gluon GPDs; gravitational form factors.
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Hadron Masses in QCD

Quark/gluon contributions to masses (& angular momentum) are accessed via matrix elements of QCD’s (symmetric) energy-momentum tensor

\[ T^{\mu\nu} = T^{\nu\mu}, \quad \partial_\mu T^{\mu\nu} = \partial_\mu T_q^{\mu\nu} + \partial_\mu T_g^{\mu\nu} = 0, \quad T^{\mu\nu} = \overline{T}^{\mu\nu}_{[\text{traceless}]} + \hat{T}^{\mu\nu}_{[\text{trace}]} \]

Renormalized (perturbatively) trace piece of \( T^{\mu\nu} \) takes the form

\[ T^{\mu}_{\mu} = \sum_{q=u,d,s} m_q (1 + \gamma_m) \overline{\psi}_q \psi_q + \frac{\beta(g)}{2g} F^{\mu\nu,a} F^a_{\mu\nu} \]

quark mass term

trace anomaly

At zero momentum transfer

\[ \langle p \left| T^{\mu\nu} \right| p \rangle = 2 p^\mu p^\nu \implies \langle p \left| T^{\mu}_{\mu} \right| p \rangle = 2 m^2 \]

in chiral limit – at a large renormalization scale – entire hadron mass from gluons!

Dmitri Kharzeev – Proton Mass workshops at Temple University and ECT*

Understanding difference in pion and proton is key to hadron masses:

\[ \langle \pi \left| T^{\mu}_{\mu} \right| \pi \rangle = 2 m^2_\pi \xrightarrow{\text{chiral limit}} 0, \quad \langle N \left| T^{\mu}_{\mu} \right| N \rangle = 2 m^2_N \]
Xiangdong Ji proposed hadron mass decomposition \cite{PRL74,1071,1995;PRD52,271,1995}

\[
mp = \left. \frac{\langle p \mid \int d^3x \, T^{00}(0, \vec{x}) \mid p \rangle}{\langle p \mid p \rangle} \right|_{\text{at rest}} = M_q + M_g + M_m + M_a
\]

\[
M_q = \frac{3}{4} (a - b) \, m_p, \quad M_g = \frac{3}{4} (1 - a) \, m_p, \quad M_m = b \, m_p, \quad M_a = \frac{1}{4} (1 - b) \, m_p,
\]

- \(a\) = quark momentum fraction, \(b\) related to sigma-term or anomaly contribution
- \cite{CedrickLorce,EPJC78,2018} for decomposition with pressure effects

**Ji’s proton mass decomposition**
- quark energy (29%)
- gluon energy (34%)
- quark mass (17%)
- trace anomaly (20%)

**Ji’s pion mass decomposition**
- quark energy (0%)
- gluon energy (38%)
- quark mass (50%)
- trace anomaly (20%)

In chiral limit \((m_q \to 0)\) pion has no rest frame \((m_\pi = 0)\) – how to interpret Ji’s pion mass decomposition? Perhaps in limit as \(m_\pi \to 0\).
**Hadron Mass Decomposition – Experiment**

- Gravitational form factors of the pion:
  \[
  \langle p' \mid T_{q,g}^{\mu\nu} \mid p \rangle = A_q^{q,g}(t) P^\mu P^\nu + D_q^{q,g}(t) \left( q^\mu q^\nu - q^2 g^{\mu\nu} \right) + \bar{c}_q^{q,g}(t) g^{\mu\nu}
  \]

- A graviton probe can only measure \( T_{q,q}^{\mu\nu} = T_{q}^{\mu\nu} + T_{g}^{\mu\nu} \), where
  \[
  A^q(0) + A^g(0) = 1, \quad D^q(0) + D^g(0) \rightarrow 0 - 1, \quad \partial_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \bar{c}_q(t) = -\bar{c}_g(t)
  \]

- However, GPDs can access both \( T_{q,q}^{\mu\nu} \) and \( T_{g}^{\mu\nu} \)
  \[
  \int dx \, x H_{q,g}^{q,g}(x, \xi, t) = A_q^{q,g}(t) + \xi^2 D_q^{q,g}(t)
  \]

- measuring pion and kaon GPDs would shed-light on mass and confinement

- Trace anomaly contribution can be accessed through \( J/\psi, \Upsilon \) production at threshold

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**EIC UGM 2018**
At an EIC – and Jefferson Lab – pion and kaon structure can be accessed via the so-called Sullivan processes

- initial pion/kaon is off mass-shell – need extrapolation to pole
- proven results for form factors – what about quark and gluon PDFs, TMDs, GPDs, etc, at an EIC?

Explored this ideal at a series of workshops on “Pion and Kaon Structure at an Electron–Ion Collider” (PIEIC)

QCD’s Dyson-Schwinger Equations

- The equations of motion of QCD \(\iff\) QCD’s Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability \(\implies\) must implement a symmetry preserving truncation

- The most important DSE is QCD’s gap equation \(\iff\) quark propagator

- ingredients – dressed gluon propagator & dressed quark-gluon vertex

\[
S(p) = \frac{Z(p^2)}{i\gamma + M(p^2)}
\]

- mass function, \(M(p^2)\), exhibits dynamical mass generation, even in chiral limit

- Hadron masses are generated by dynamical chiral symmetry breaking – caused by a cloud of gluons around a quark


Rapid acquisition of mass is effect of gluon cloud

**Hadron masses are generated by dynamical chiral symmetry breaking – caused by a cloud of gluons around a quark**
Pion & Kaon Wave Functions
Calculating and Predicting Pion Structure

In QFT a two-body bound state (e.g. a pion, kaon, etc) is described by the Bethe-Salpeter equation (BSE):

\[
\Gamma = \Gamma K = \Gamma K + \ldots
\]

- the kernel must yield a solution that encapsulates the consequences of DCSB, e.g., in chiral limit
  \( m_\pi = 0 \) & \( m_\pi^2 \propto m_u + m_d \)

- Pion BSE wave function has the general form
  \[
  \chi_\pi(p, k) = S(k) \left[ E_\pi(p, k) + \not{p} F_\pi(p, k) + \not{k} k \cdot p G(p, k) + i\sigma^{\mu\nu} k_\mu p_\nu H(p, k) \right] \gamma_5 S(k - p)
  \]

- BSE wave function \( \rightarrow \) light-front wave functions (LFWFs)
  \( \rightarrow \) parton distribution amplitudes (PDAs)

\[
\psi(x, k_T) = \int dk^- \chi_{\text{BSE}}(p, k), \quad \varphi(x) = \int d^2k_T \psi(x, k_T)
\]
Pion’s Parton Distribution Amplitude

- pion’s PDA – \( \varphi_\pi(x) \): is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state’s valence Fock state
- it’s a function of the light-cone momentum fraction \( x = \frac{k^+}{p^+} \) and the scale \( Q^2 \)
- asymptotic result is: \( \varphi_\pi^{\text{asy}}(x) = 6x(1-x) \)

\[ Q^2 F_\pi(Q^2) \rightarrow 16\pi f_\pi^2 \alpha_s(Q^2) \]

\[ Q^2 F_{\gamma^*\gamma\pi}(Q^2) \rightarrow 2f_\pi \]

PDAs enter numerous hard exclusive scattering processes

[Farrar, Jackson; Lepage, Brodsky; Radyushkin, Efremov]
Both DSE results – each using a different Bethe-Salpeter kernel – exhibit a pronounced broadening compared with the asymptotic pion PDA. The scale of calculation is given by the renormalization point $\xi = 2 \text{ GeV}$. A realization of DCSB on the light-front shows that the pion’s PDA remains broad & concave for all accessible scales in current and conceivable experiments. Broading of PDA influences the $Q^2$ evolution of the pion’s EM form factor.
Pion PDA from Lattice QCD

Currently, lattice QCD can determine only one non-trivial moment \( e.g. \)
\[
\int dx \ (2x - 1)^2 \varphi_\pi(x) = 0.2361 \ (41) \ (39)
\]


- scale is \( Q^2 = 4 \text{ GeV}^2 \)

Standard practice to fit first coefficient of “asymptotic expansion” to moment

\[
\varphi_\pi(x, Q^2) = 6x (1-x) \left[ 1 + \sum_{n=2,4,...} a^{3/2}_n (Q^2) C^{3/2}_n (2x - 1) \right]
\]

- however this expansion is guaranteed to converge rapidly only when \( Q^2 \rightarrow \infty \)
- method results in a double-humped pion PDA – not supported by BSE WFs

Advocate using a generalized expansion

\[
\varphi_\pi(x, Q^2) = N_\alpha x^{\alpha} (1-x)^{\alpha} \left[ 1 + \sum_{n=2,4,...} a^{\alpha+1/2}_n (Q^2) C^{\alpha+1/2}_n (2x - 1) \right]
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Find good agreement with DSE result
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EIC UGM 2018
Pion & Kaon Form Factors
Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_\pi(Q^2)$ at $Q^2 \approx 6 \text{ GeV}^2$

- Magnitude of this product is determined by strength of DCSB at all accessible scales

- The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \quad Q^2 \gg \Lambda^2_{\text{QCD}} \quad 16 \pi f_\pi^2 \alpha_s(Q^2) \omega_\pi^2; \quad \omega_\pi = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi_\pi(x)$$

- Find consistency between the direct pion form factor calculation and the QCD hard-scattering formula – if DSE pion PDA is used

- 15% disagreement may be explained by higher order/higher-twist corrections

- At an EIC preliminary studies [Garth Huber – PIEIC 2018] suggest pion form factor can be measured to $Q^2 \gtrsim 30 \text{ GeV}^2$
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Form factors must be a sensitive measure of confinement in QCD

- but what are they telling us?
- consider quark-sector kaon form factors: $K^+ = u \bar{s}$

Find remarkable flavor dependence of $K$ form factors

- $s$-quark much harder than the $u$-quark
- confinement? If probe strikes a light $u$-quark it is much harder for the hadron to remain intact – compared to when an $s$ quark is struck
Pion PDFs
**Pion PDFs**

Longstanding pQCD prediction [Farrar & Jackson (1975); Lepage & Brodsky (1980)] that pion PDF near $x = 1$ behaves as: $q(x) \simeq (1 - x)^2$

Pion-induced Drell-Yan data (Conway) and a resent analysis (Sato), also including leading-neutron data, find $q(x) \sim (1 - x)^1$ near $x = 1$

soft-gluon resummation effects (Aicher) may explain this discrepancy

DSEs predict $q(x) \simeq (1 - x)^2$ near $x = 1$, which is related to the $1/k^2$ dependence of the BSE interaction kernel at large relative momentum

However, both the pQCD and DSE predictions need only set in very near $x \simeq 1$, the observed $q(x) \simeq (1 - x)$ behavior could be real where data exists
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Pion & Kaon Tomography
Pion and Kaon LFWFs


Pion has two leading Fock-state LFWFs: $\psi_{\uparrow\downarrow}(x, k_T^2) \& \psi_{\uparrow\uparrow}(x, k_T^2)$

- many remarkable properties: frame-independent; have a probability interpretation, boosts are kinematical
- LFWFs give access to a tomography of hadrons – TMDs and GPDs
- DSE result finds broad (almost) concave functions at hadronic scales, with features at small $k_T^2$ driven by DCSB
- at large $k_T^2$ find same power-law behavior as predicted by perturbative QCD
- in this domain: $\psi_0(x, k_T^2) \propto x(1 - x)/k_T^2 \& \psi_1(x, k_T^2) \propto x(1 - x)/k_T^4$
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See talk by Chao Shi – Wed in 3D Imaging
Using pion’s LFWFs straightforward to make predictions for pion TMDs

\[ f(x, k_T^2) \propto |\psi_{\uparrow\downarrow}(x, k_T^2)|^2 + k_T^2 |\psi_{\uparrow\uparrow}(x, k_T^2)|^2 \]

- numerous features inherited from LFWFs: TMDs are broad function as a result of DCSB and peak at zero relative momentum \((x = 1/2)\)
- evolution from model scale \((\mu = 0.52 \text{ GeV})\) to \(\mu = 6 \text{ GeV}\) results in significant broadening in \(\langle k_T^2 \rangle\), from 0.16 GeV\(^2\) to 0.69 GeV\(^2\)
- Need careful treatment of gauge link to study pion Boer-Mulders function
Measuring the pion TMDs will be a challenge, however progress can be made now by studying the $q \rightarrow \pi$ TMD fragmentation functions.

Fragmentation functions are particularly important and interesting:

- potentially fragmentation functions can shed the most light on confinement and DCSB – because they describe how a fast moving (massless) quark or gluon becomes a tower of hadrons.

Also interesting tool with which to probe color entanglement at an EIC.

- over what length scales can colored correlations be observed?
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Straightforward to make predictions for pion and kaon GPDs from overlaps of LFWFs – only one type of GPD at leading twist

Impact parameter dependent parton distributions are given by

\[ q(x, b_T^2) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i \Delta_T \cdot b_T} H(x, 0, -\Delta_T^2) \]

- IP–PDFs have a probability interpretation, and as \( x \to 1 \) must have \( b_T^2 \to 0 \)
- \( q(x, b_T^2) \) peaks near \( x \simeq 1, b_T^2 \simeq 0 \) because phase space is reduced here, however this region contributes very little to the PDF
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See talk by Chao Shi – Wed in 3D Imaging
Conclusion

- QCD is exhibits numerous phenomena not apparent in the Lagrangian
  - confinement, DCSB, hadron masses, etc
- Building an EIC to understand these phenomena in the nucleon is crucial
  - however our understanding of QCD will only begin to be comprehensive when we also understand the pion and kaon at a similar level
- Using the DSEs find that DCSB drives numerous effects in QCD e.g.
  - hadron masses & confinement
  - broad pion and kaon PDAs, PDFs, TMDs, and GPDs; \( Q^2 F_\pi(Q^2) \)
- Much work remains in experiment and theory to understand the pion and kaon
  - need effort from lattice, pQCD, and models

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