# Mass and Structure of the Pion and Kaon

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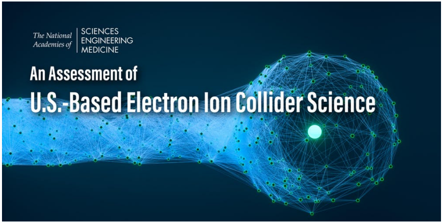
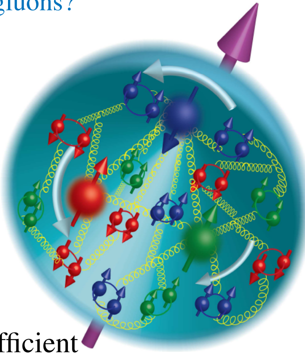
# *High-priority science questions*

- The NAS *Assessment of a U.S. based Electron Ion Collider* identified three high-priority science questions
  - How does the mass of the nucleon arise?
  - How does the spin of the nucleon arise?
  - What are the emergent properties of dense systems of gluons?

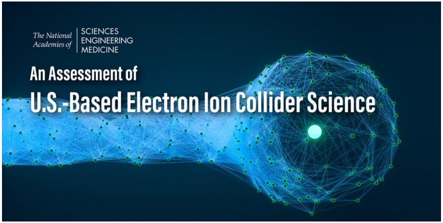



- What do we know about hadron masses?
  - The three current quarks needed to define the nucleon quantum numbers contribute only $\sim 1\%$ to its mass
  - In chiral limit nucleon mass $\sim 900\,\text{MeV}$; *Higgs mechanism is largely irrelevant for visible mass*
  - The chiral symmetry of $\mathcal{L}_{\text{QCD}}$ is dynamically broken $\Longrightarrow \sim 500\,\text{MeV}$ mass splittings in hadron spectrum & massless Goldstone bosons in chiral limit $(\pi,\ K,\ \eta)$



- Therefore understanding the nucleon mass is not sufficient
  - must also understand the mass of the pion $(u\bar{d},\ldots)$ and kaon $(u\bar{s},\ldots)$

# High-priority science questions

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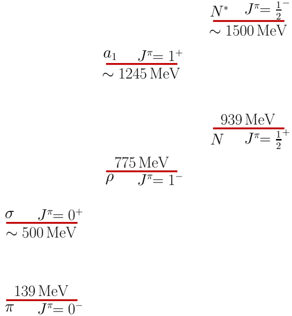


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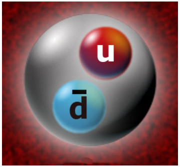
# *Story of the Pion and Kaon*

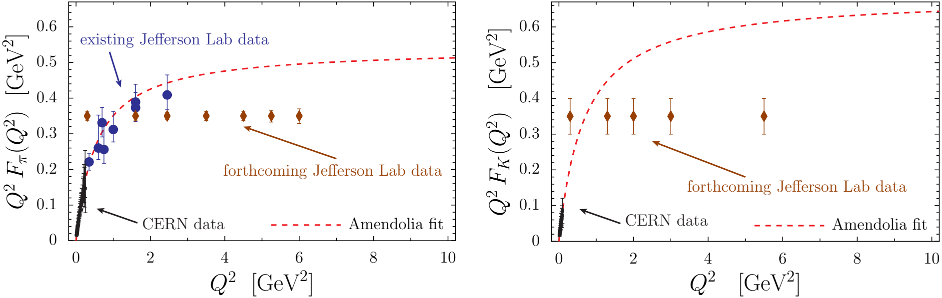



- In 1935 Yukawa postulated a strongly interacting particle [“($\pi$-) meson”] as a mediator for the strong nuclear force
  - in 1947 both the $\pi$ and shortly afterwards the $K$ were discovered from cosmic ray tracks in a photographic emulsion
- Today, pion still regarded as the mediator of the strong force in *ab inito* approaches to nuclear structure; the kaon has played an important role in establishing quark model, and understanding flavor breaking & CP violation
- Formally the pion and kaon are now understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD
- This dichotomous nature has numerous ramifications near chiral limit *e.g.*:

$$f_\pi^2\, m_\pi^2 \simeq \frac{1}{2}\,(m_u + m_d)\,\langle\psi\bar{\psi}\rangle\,, \;\; H_\pi^{u-d}(x, \xi \to 1, 0) = \phi_\pi\left(\frac{1+x}{2}\right)\,, \;\; D_\pi(0) = -1$$

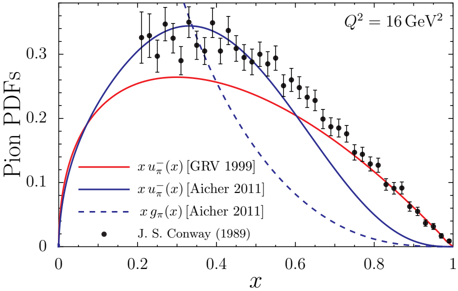
- Perturbative QCD can also make predictions for pion and kaon structure – therefore $\pi$ and $K$ provide an ideal laboratory to test and understand QCD

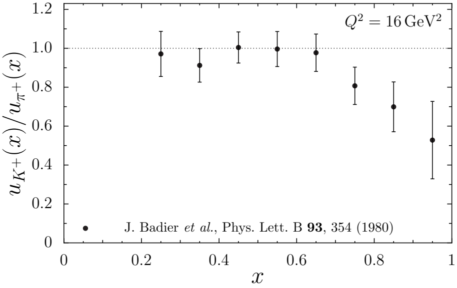
# What we know about the Pion and Kaon




- Pion and kaon structure is slowly being revealed using: $\pi^-/K^-$ beams at CERN; Sullivan type experiments at Jefferson Lab; $\pi^-$ beams at Fermilab; and $e^+e^- \to \pi^+\pi^-,\ K^+K^-$ in the time-like region
- 40 years of experiments has revealed *e.g.*:
  - $r_\pi = 0.672 \pm 0.008,\ r_{K^+} = 0.560 \pm 0.031,\ r_{K^0} = -0.277 \pm 0.018$
- Still a lot more to learn about pion and kaon structure:
  - quark and gluon PDFs; TMDs including Boer-Mulders function; $q, g \to \pi/K$ fragmentation functions, quark and gluon GPDs; gravitational form factors

# What we know about the Pion and Kaon

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# Hadron Masses in QCD

- Quark/gluon contributions to masses (& angular momentum) are accessed via matrix elements of QCD's (symmetric) energy-momentum tensor

$$T^{\mu\nu} = T^{\nu\mu}, \qquad \partial_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu}_q + \partial_\mu T^{\mu\nu}_g = 0, \qquad T^{\mu\nu} = \underset{\text{[traceless]}}{\overline{T}^{\mu\nu}} + \underset{\text{[trace]}}{\widehat{T}^{\mu\nu}}$$

- Renormalized (perturbatively) trace piece of $T^{\mu\nu}$ takes the form

$$T^\mu_\mu = \sum_{q=u,d,s} \underbrace{m_q\left(1+\gamma_m\right)\overline{\psi}_q\psi_q}_{\text{quark mass term}} + \underbrace{\frac{\beta(g)}{2\,g}\,F^{\mu\nu,a}F^a_{\mu\nu}}_{\text{trace anomaly}}$$

- At zero momentum transfer

$$\langle p\,|T^{\mu\nu}|\,p\rangle = 2\,p^\mu p^\nu \quad \Longrightarrow \quad \left\langle p\,\middle|T^\mu_\mu\middle|\,p\right\rangle = 2\,m^2$$

  - in chiral limit – at a large renormalization scale – entire hadron mass from gluons!
  - Dmitri Kharzeev – Proton Mass workshops at Temple University and ECT*

- Understanding difference in pion and proton is key to hadron masses:

$$\left\langle \pi\,\middle|T^\mu_\mu\middle|\,\pi\right\rangle = 2\,m_\pi^2 \overset{\text{chiral limit}}{\to} 0, \qquad \left\langle N\,\middle|T^\mu_\mu\middle|\,N\right\rangle = 2\,m_N^2$$

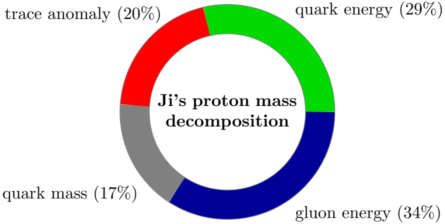
# Rest Frame Hadron Mass Decompositions

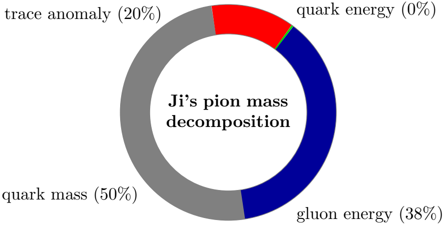
- Xiangdong Ji proposed hadron mass decomposition [PRL **74**, 1071 (1995); PRD **52**, 271 (1995)]

$$m_p = \frac{\langle p \left| \int d^3x \, T^{00}(0,\vec{x}) \right| p \rangle}{\langle p|p \rangle}\Bigg|_{\text{at rest}} = \underbrace{M_q \quad + \quad M_g}_{\text{quark and gluon energies}} + \underbrace{M_m}_{\text{quark mass}} + \underbrace{M_a}_{\text{trace anomaly}}$$

$$M_q = \tfrac{3}{4}\,(a-b)\,m_p, \quad M_g = \tfrac{3}{4}\,(1-a)\,m_p, \quad M_m = b\,m_p, \quad M_a = \tfrac{1}{4}\,(1-b)\,m_p,$$

  - $a$ = quark momentum fraction, $b$ related to sigma-term or anomaly contribution
  - [See Cédric Lorcé, EPJC **78**, (2018) for decomposition with pressure effects]




- In chiral limit ($m_q \to 0$) pion has no rest frame ($m_\pi = 0$) – how to interpret Ji's pion mass decomposition? Perhaps in limit as $m_\pi \to 0$.

# Hadron Mass Decomposition – Experiment

- Gravitational form factors of the pion:

$$\langle p' \,| T^{\mu\nu}_{q,g} |\, p\rangle = A^{q,g}_{\pi}(t)\, P^{\mu}\, P^{\nu} + D^{q,g}_{\pi}(t)\left(q^{\mu}q^{\nu} - q^2 g^{\mu\nu}\right) + \bar{c}^{q,g}_{\pi}(t)\, g^{\mu\nu}$$

- A graviton probe can only measure $T^{\mu\nu} = T^{\mu\nu}_q + T^{\mu\nu}_g$, where

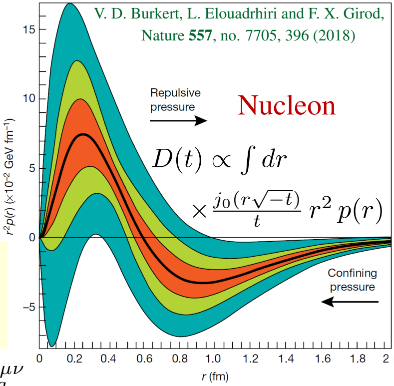
$$A^q(0) + A^g(0) = 1, \qquad D^q_\pi(0) + D^g_\pi(0) \overset{m_q \to 0}{=} -1$$
$$\partial_\mu T^{\mu\nu} = 0 \implies \bar{c}^q_\pi(t) = -\bar{c}^g_\pi(t)$$

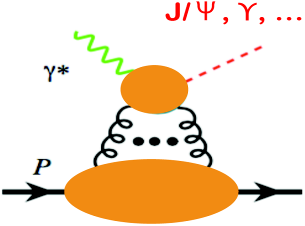
- However, GPDs can access both $T^{\mu\nu}_q$ and $T^{\mu\nu}_g$

$$\int dx\, x\, H^{q,g}_{\pi}(x,\xi,t) = A^{q,g}_{\pi}(t) + \xi^2\, D^{q,g}_{\pi}(t)$$

  - measuring pion and kaon GPDs would shed-light on mass and confinement

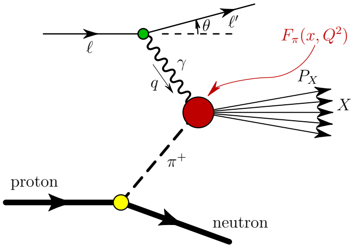
- Trace anomaly contribution can be accessed through $J/\psi, \Upsilon$ production at threshold

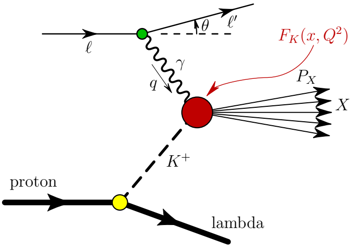



See talk by Sylvester Joosten

# Pion & Kaon Structure at an EIC

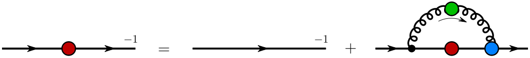



- At an EIC – and Jefferson Lab – pion and kaon structure can be accessed via the so-called *Sullivan* processes
  - initial pion/kaon is off mass-shell – need extrapolation to pole
  - proven results for form factors – what about quark and gluon PDFs, TMDs, GPDs, *etc*, at an EIC?
- Explored this ideal at a series of workshops on "Pion and Kaon Structure at an Electron–Ion Collider" (PIEIC)
  - 1–2 June 2017, Argonne National Laboratory www.phy.anl.gov/theory/pieic2017/
  - 24–25 May 2018, The Catholic University of America www.jlab.org/conferences/pieic18/

# QCD's Dyson-Schwinger Equations

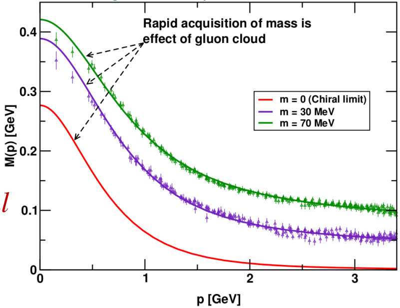
- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability $\Longrightarrow$ must implement a symmetry preserving truncation
- The most important DSE is QCD's gap equation $\Longrightarrow$ quark propagator




  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

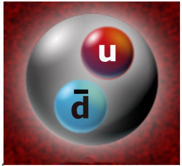
[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



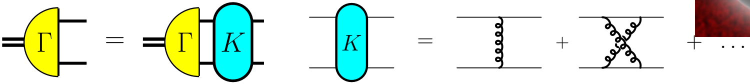

- mass function, $M(p^2)$, exhibits dynamical mass generation, even in chiral limit
- *Hadron masses are generated by dynamical chiral symmetry breaking – caused by a cloud of gluons around a quark*

# Pion & Kaon Wave Functions

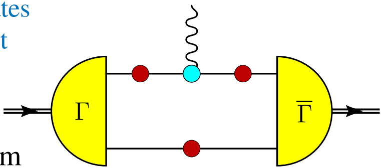
# *Calculating and Predicting Pion Structure*




- In QFT a two-body bound state (*e.g.* a pion, kaon, *etc*) is described by the Bethe-Salpeter equation (BSE):




  - the kernel must yield a solution that encapsulates the consequences of DCSB, *e.g.*, in chiral limit $m_\pi = 0$ & $m_\pi^2 \propto m_u + m_d$




- Pion BSE wave function has the general form

$$\chi_\pi(p,k) = S(k)\Big[E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, \mathcal{G}(p,k) + i\sigma^{\mu\nu}k_\mu p_\nu\, \mathcal{H}(p,k)\Big]\gamma_5\, S(k-p)$$

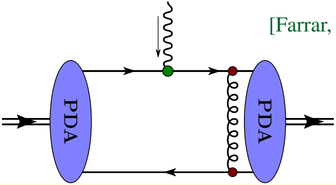
- BSE wave function $\Longrightarrow$ light-front wave functions (LFWFs)
  $\Longrightarrow$ parton distribution amplitudes (PDAs)

$$\psi(x,\boldsymbol{k}_T) = \int dk^-\ \chi_{\text{BSE}}(p,k), \qquad \varphi(x) = \int d^2\boldsymbol{k}_T\ \psi(x,\boldsymbol{k}_T)$$

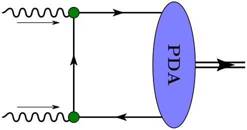
# *Pion's Parton Distribution Amplitude*

- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale $Q^2$
  - asymptotic result is: $\varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

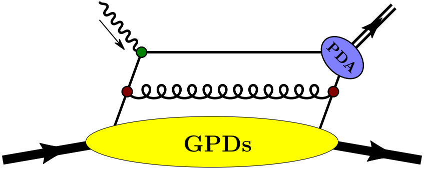
[Farrar, Jackson; Lepage, Brodsky; Radyushkin, Efremov]

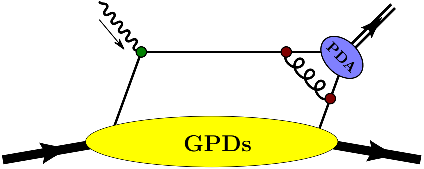



$$Q^2\,F_\pi(Q^2) \to 16\pi\,f_\pi^2\,\alpha_s(Q^2)$$




$$Q^2\,F_{\gamma^*\gamma\pi}(Q^2) \to 2\,f_\pi$$

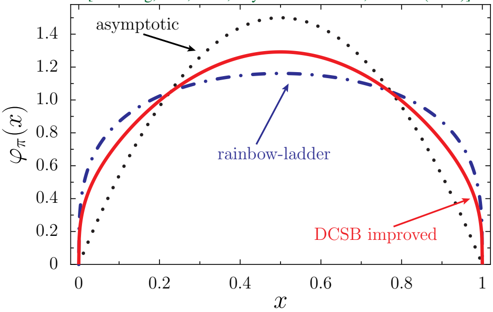


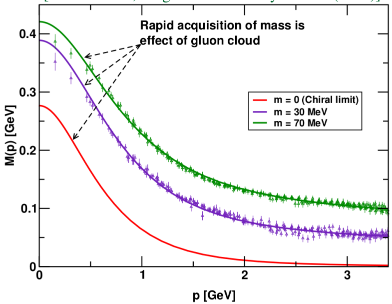

- PDAs enter numerous hard exclusive scattering processes

# Pion PDA from the DSEs

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]




[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]




- Both DSE results – each using a different Bethe-Salpeter kernel – exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\xi = 2\,\text{GeV}$
- A realization of DCSB on the light-front
- *ERBL evolution demonstrates that the pion's PDA remains broad & concave for all accessible scales in current and conceivable experiments*
- Broading of PDA influences the $Q^2$ evolution of the pion's EM form factor

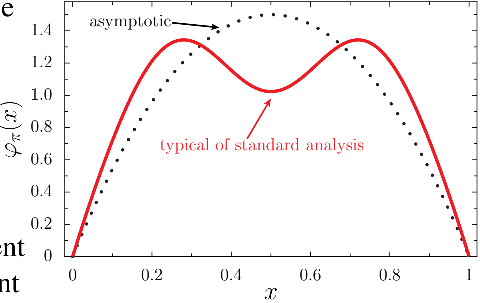
# Pion PDA from Lattice QCD

- Currently, lattice QCD can determine only one non-trivial moment *e.g.*

$$\int dx\,(2\,x-1)^2\varphi_\pi(x) = 0.2361\,(41)\,(39)$$

[V. M. Braun, *et al.*, Phys. Rev. D **92**, no. 1, 014504 (2015)]

  - scale is $Q^2 = 4\,\mathrm{GeV}^2$




- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1+\sum\nolimits_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - method results in a *double-humped* pion PDA – not supported by BSE WFs

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\,x^\alpha(1-x)^\alpha\left[1+\sum\nolimits_{n=2,\,4,\ldots} a_n^{\alpha+1/2}(Q^2)\,C_n^{\alpha+1/2}(2x-1)\right]$$

- Find good agreement with DSE result

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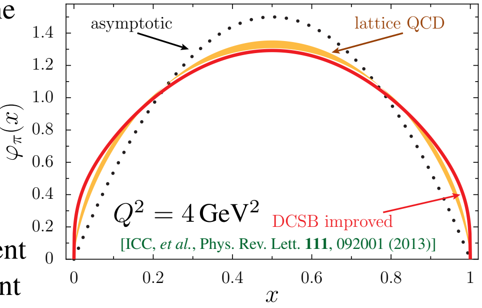


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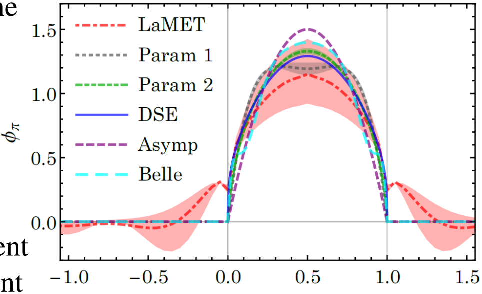
[J. H. Zhang, J. W. Chen, *et al.*, Phys. Rev. D **95**, no. 9, 094514 (2017)]

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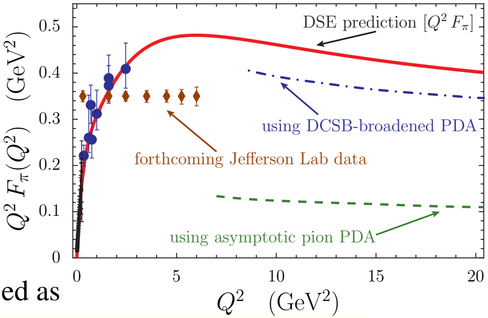
# Pion & Kaon Form Factors

# *Pion Elastic Form Factor*

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2\,F_\pi(Q^2)$ at $Q^2 \approx 6\,\mathrm{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]




- The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\mathrm{QCD}}}{\sim} 16\,\pi\,f_\pi^2\,\alpha_s(Q^2)\,w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\,\frac{1}{x}\,\varphi_\pi(x)$$

- Find consistency between the *direct pion form factor calculation* and the QCD hard-scattering formula – if DSE pion PDA is used
  - 15% disagreement may be explained by higher order/higher-twist corrections
- At an EIC preliminary studies [Garth Huber – PIEIC 2018] suggest pion form factor can be measured to $Q^2 \gtrsim 30\,\mathrm{GeV}^2$

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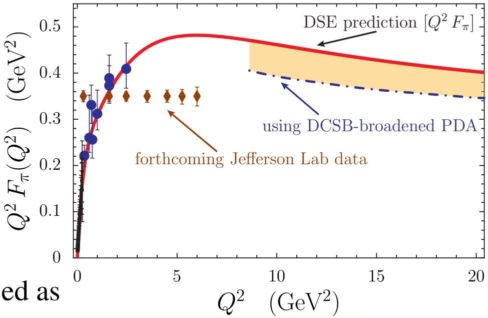


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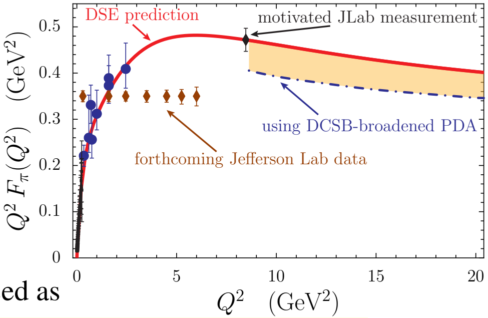
$$Q^2 F_\pi(Q^2) \overset{Q^2\gg\Lambda^2_{\mathrm{QCD}}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

- Find consistency between the *direct pion form factor calculation* and the QCD hard-scattering formula – if DSE pion PDA is used
  - 15% disagreement may be explained by higher order/higher-twist corrections
- At an EIC preliminary studies [Garth Huber – PIEIC 2018] suggest pion form factor can be measured to $Q^2 \gtrsim 30\,\mathrm{GeV}^2$

# *Pion Elastic Form Factor*

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2\,F_\pi(Q^2)$ at $Q^2 \approx 6\,\mathrm{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]

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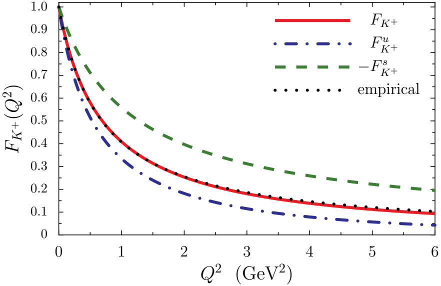
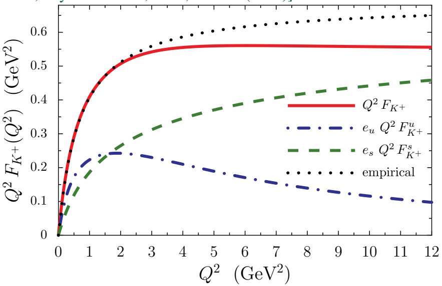
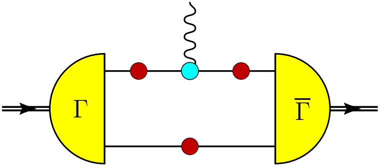
# *Form Factors and Confinement*

[P. T. P. Hutauruk, ICC and A. W. Thomas, Phys. Rev. C **94**, no. 3, 035201 (2016)]




- Form factors must be a sensitive measure of confinement in QCD
  - but what are they telling us?
  - consider quark-sector kaon form factors: $K^+ = u\bar{s}$

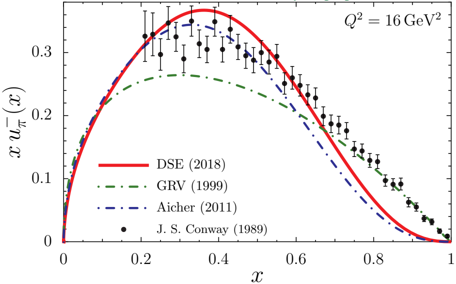



- Find remarkable flavor dependence of $K$ form factors
  - $s$-quark much harder than the $u$-quark
  - confinement? If probe strikes a light $u$-quark it is much harder for the hadron to remain intact – compared to when an $s$ quark is struck

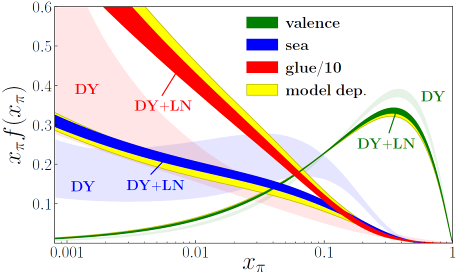
# Pion PDFs

# Pion PDFs

[Bednar, Cloët, and Tandy, in preparation]



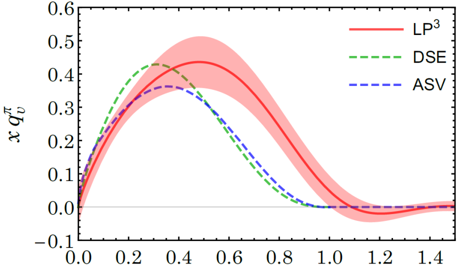

[Barry, Sato, Melnitchouk, and Ji, arXiv:1804.01965 [hep-ph]]




- Longstanding pQCD prediction [Farrar & Jackson (1975); Lepage & Brodsky (1980)] that pion PDF near $x = 1$ behaves as: $q(x) \simeq (1-x)^2$
- Pion-induced Drell-Yan data (Conway) and a resent analysis (Sato), also including leading-neutron data, find $q(x) \sim (1-x)^1$ near $x = 1$
  - soft-gluon resummation effects (Aicher) may explain this discrepancy
- DSEs predict $q(x) \simeq (1-x)^2$ near $x = 1$, which is related to the $1/k^2$ dependence of the BSE interaction kernel at large relative momentum
- However, both the pQCD and DSE predictions need only set in very near $x \simeq 1$, the observed $q(x) \simeq (1-x)$ behavior could be real where data exists

# *Pion PDFs*

[J. W. Chen, L. Jin, H. W. Lin, *et al.*, arXiv:1804.01483 [hep-lat]]

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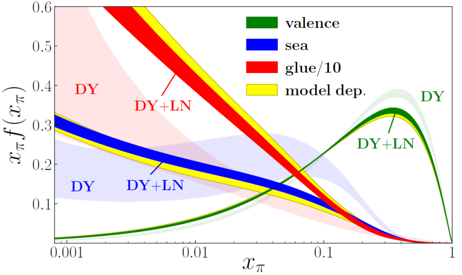


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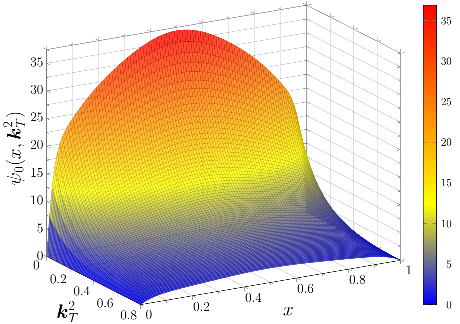
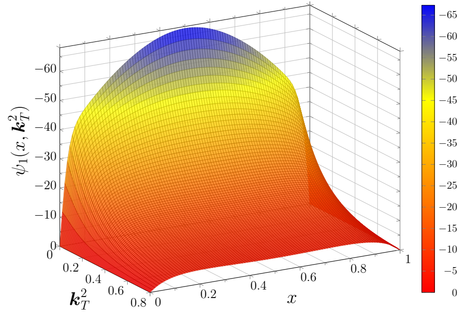
# Pion & Kaon Tomography

# Pion and Kaon LFWFs

[C. Shi and I. C. Cloët, arXiv:1806.04799 [nucl-th]]

See talk by Chao Shi – Wed in 3D Imaging




- Pion has two leading Fock-state LFWFs: $\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)$ & $\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)$
  - many remarkable properties: frame-independent; have a probability interpretation, boosts are kinematical
  - LFWFs give access to a tomography of hadrons – TMDs and GPDs
- DSE result finds broad (almost) concave functions at hadronic scales, with features at small $\boldsymbol{k}_T^2$ driven by DCSB
  - at large $\boldsymbol{k}_T^2$ find same power-law behavior as predicted by perturbative QCD
  - in this domain: $\psi_0(x, \boldsymbol{k}_T^2) \propto x(1-x)/\boldsymbol{k}_T^2$ & $\psi_1(x, \boldsymbol{k}_T^2) \propto x(1-x)/\boldsymbol{k}_T^4$

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[C. Shi and I. C. Cloët, arXiv:1806.04799 [nucl-th]]

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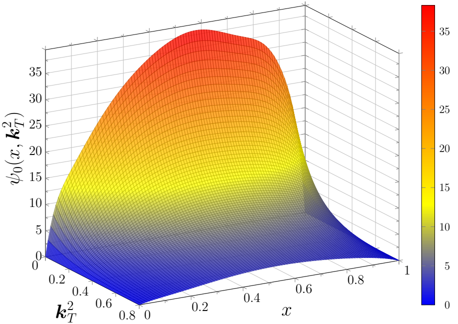


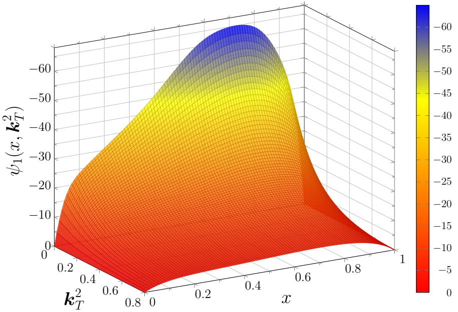


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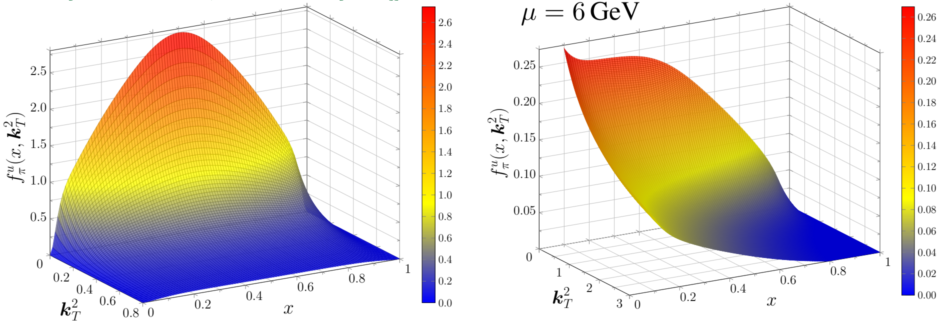
# Pion $T$-even TMD

[C. Shi and I. C. Cloët, arXiv:1806.04799 [nucl-th]]



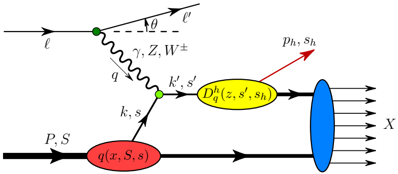

- Using pion's LFWFs straightforward to make predictions for pion TMDs

$$f(x, \boldsymbol{k}_T^2) \propto \left|\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)\right|^2 + \boldsymbol{k}_T^2 \left|\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)\right|^2$$

  - numerous features inherited from LFWFs: TMDs are broad function as a result of DCSB and peak at zero relative momentum ($x = 1/2$)
  - evolution from model scale ($\mu = 0.52\,\text{GeV}$) to $\mu = 6\,\text{GeV}$ results in significant broadening in $\langle \boldsymbol{k}_T^2 \rangle$, from $0.16\,\text{GeV}^2$ to $0.69\,\text{GeV}^2$
- Need careful treatment of gauge link to study pion Boer-Mulders function

# Probing Transverse Momentum

| leading twist | | quark polarization | | |
|---|---|---|---|---|
| | | unpolarized [U] | longitudinal [L] | transverse [T] |
| nucleon polarization | U | $f_1 =$ unpolarized | | $h_1^\perp = \; - $ Boer-Mulders |
| | L | | $g_1 = \; - $ helicity | $h_{1L}^\perp = \; - $ worm gear 1 |
| | T | $f_{1T}^\perp = \; - $ Sivers | $g_{1T}^\perp = \; - $ worm gear 2 | $h_1 = \; - $ transversity<br>$h_{1T}^\perp = \; - $ pretzelosity |




- Measuring the pion TMDs will be a challenge, however progress can be made now by studing the $q \to \pi$ TMD fragmentation functions
- Fragmentation functions are particularly important and interesting
  - *potentially fragmentation functions can shed the most light on confinement and DCSB – because they describe how a fast moving (massless) quark or gluon becomes a tower of hadrons*
- Also interesting tool with which to probe color entanglement at an EIC
  - over what length scales can colored correlations be observed?

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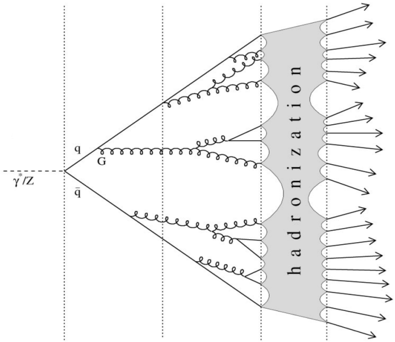


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# Pion and Kaon GPDs

[C. Shi and I. C. Cloët, arXiv:1806.04799 [nucl-th]]

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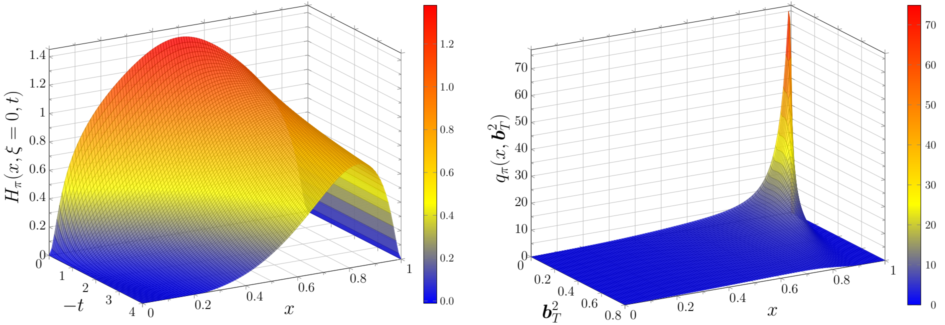


- Straightforward to make predictions for pion and kaon GPDs from overlaps of LFWFs – only one type of GPD at leading twist
- Impact parameter dependent parton distributions are given by

$$q(x, \boldsymbol{b}_T) = \int \frac{d^2\,\boldsymbol{\Delta}_T}{(2\pi)^2}\, e^{-i\,\boldsymbol{\Delta}_T\cdot \boldsymbol{b}_T}\, H(x, 0, -\boldsymbol{\Delta}_T^2)$$

  - IP–PDFs have a probability interpretation, and as $x \to 1$ must have $\boldsymbol{b}_T^2 \to 0$
  - $q(x, \boldsymbol{b}_T^2)$ peaks near $x \simeq 1$, $\boldsymbol{b}_T^2 \simeq 0$ because phase space is reduced here, however this region contributes very little to the PDF

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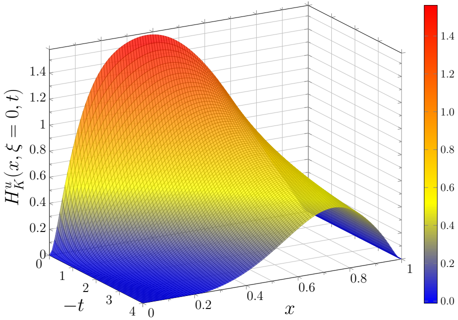


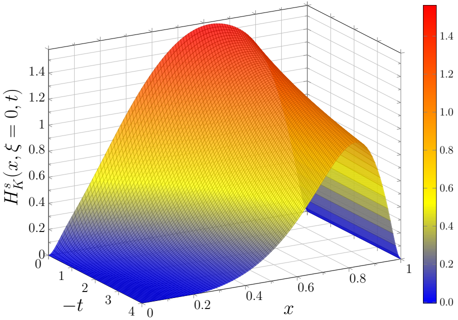


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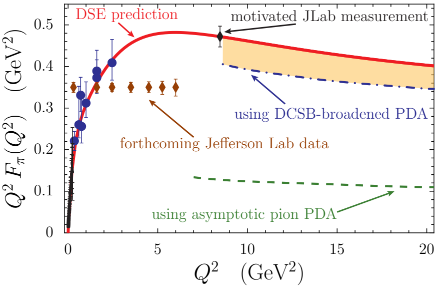
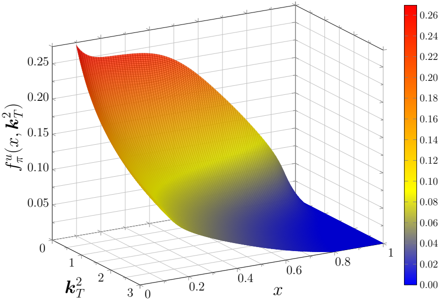
# Conclusion

- QCD is exhibits numerous phenomena not apparent in the Lagrangian
  - confinement, DCSB, hadron masses, etc
- Building an EIC to understand these phenomena in the nucleon is crucial
  - however our understanding of QCD will only begin to be comprehensive when we also understand the pion and kaon at a similar level
- Using the DSEs find that DCSB drives numerous effects in QCD *e.g.*
  - hadron masses & confinement
  - broad pion and kaon PDAs, PDFs, TMDs, and GPDs; $Q^2\, F_\pi(Q^2)$
- Much work remains in experiment and theory to understand the pion and kaon
  - need effort from lattice, pQCD, and models

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]




[C. Shi and I. C. Cloët, arXiv:1806.04799 [nucl-th]]