NANJING UNIVERSITY

Craig Roberts … http://inp.nju.edu.cn/

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Empirical Determination of the Pion Mass Distribution

Quantum Chromodynamics

$$L = \frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \bar{\psi}\left[\gamma \cdot \partial_x + m + ig\, \frac{\lambda^a}{2} \gamma \cdot A^a(x)\right] \psi(x)$$

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - f^{abc} A^b_\mu(x) A^c_\nu(x)$$

- One-line Lagrangian – expressed in terms of gluon and quark partons
- Which are NOT the degrees-of-freedom measured in detectors

Questions

- What are the asymptotic detectable degrees-of-freedom?
- How are they built from the Lagrangian degrees-of-freedom?
- Is QCD really the theory of strong interactions?
- Is QCD really a theory? ⇒ Implications far beyond Standard Model

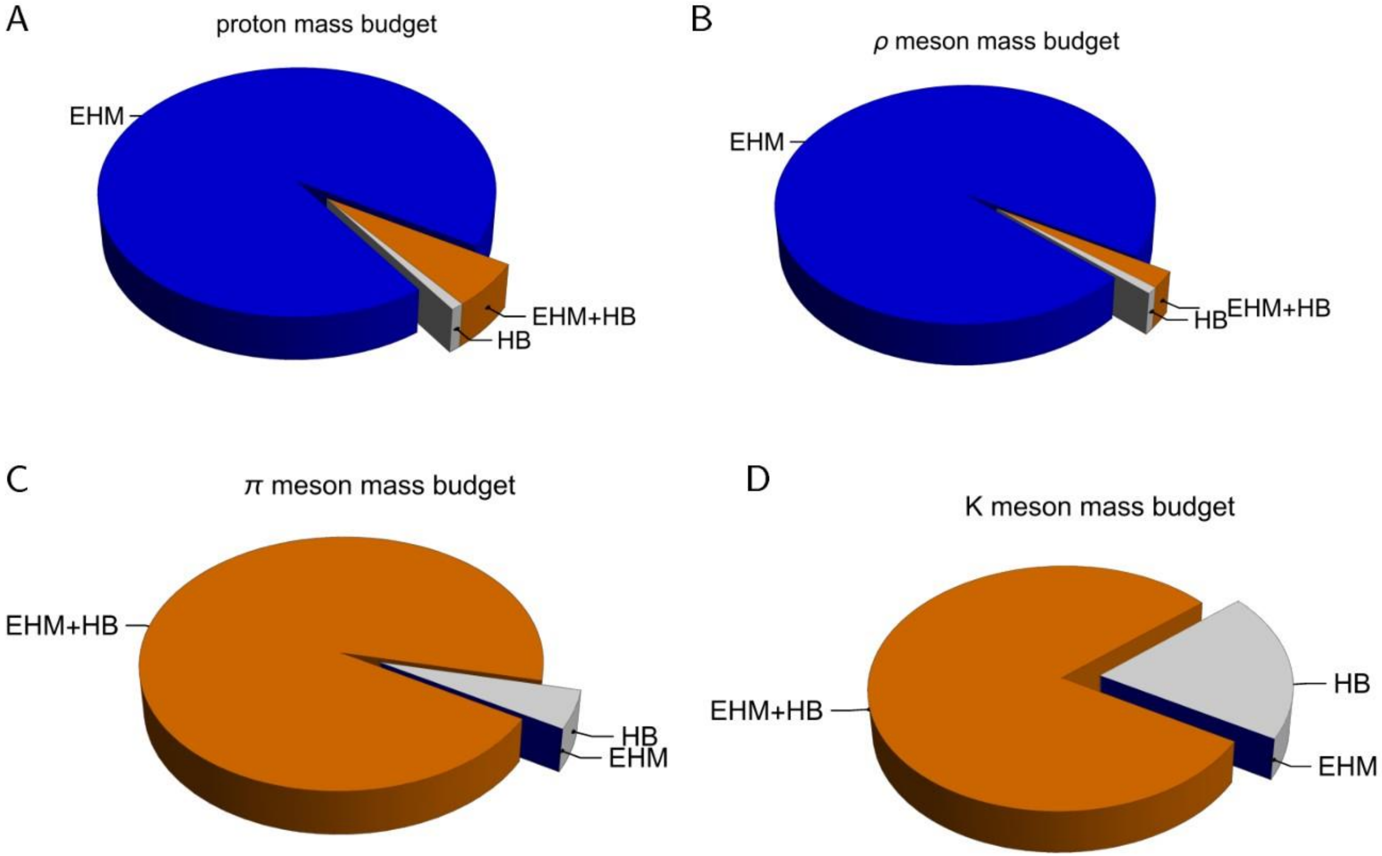
Charting Emergence of Mass

- Proton was discovered 100 years ago … It is stable; hence, an ideal target in experiments
- But, just as studying hydrogen atom ground state didn't give us QED,
  focusing on ground state of only one form of hadron matter will not solve QCD
- New era is dawning … High energy + high luminosity
  ⇒ Science can move beyond the monomaniacal focus on the proton
- Precision studies of the structure of
  - Nature's most fundamental Nambu-Goldstone bosons (π & K) will become possible
  - Baryon excited states
    - ✓ Baryons are the most fundamental three-body systems in Nature
    - ✓ If we don't understand how QCD, a Poincaré-invariant quantum field theory, builds each of the baryons in the complete spectrum, then we don't understand Nature.
- EHM is not immutable
  - its manifestations are manifold
  - experience ⇒ each hadron reveals different facets
  - One piece does not complete a puzzle

Emergence of Hadron Mass - Basic Questions

- What is the origin of EHM?
- Does it lie within QCD?
- What are the connections with …
  - Gluon and quark confinement?
  - Dynamical chiral symmetry breaking (DCSB)?
  - Nambu-Goldstone modes = π & K?
- What is the role of Higgs in modulating observable properties of hadrons?
  - Without Higgs mechanism of mass generation, π and K would be indistinguishable
- What is and wherefrom mass?

Proton and ρ-meson mass budgets are practically identical



π- and K-meson mass budgets are essentially/completely different from those of proton and ρ

G E N E S I S

GLUON

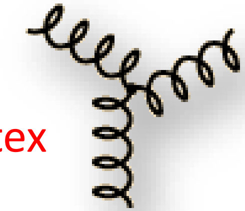
Modern Understanding
Grew Slowly from *Ancient* Origins

- More than 40 years ago

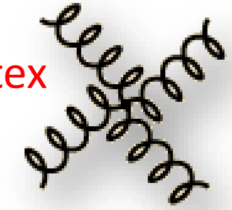
  Dynamical mass generation in continuum quantum chromodynamics,
  J.M. Cornwall, Phys. Rev. D **26** (1981) 1453 … ~ 1070 citations

- Owing to strong self-interactions, gluon partons ⇒ gluon quasiparticles, described by a mass function that is large at infrared momenta

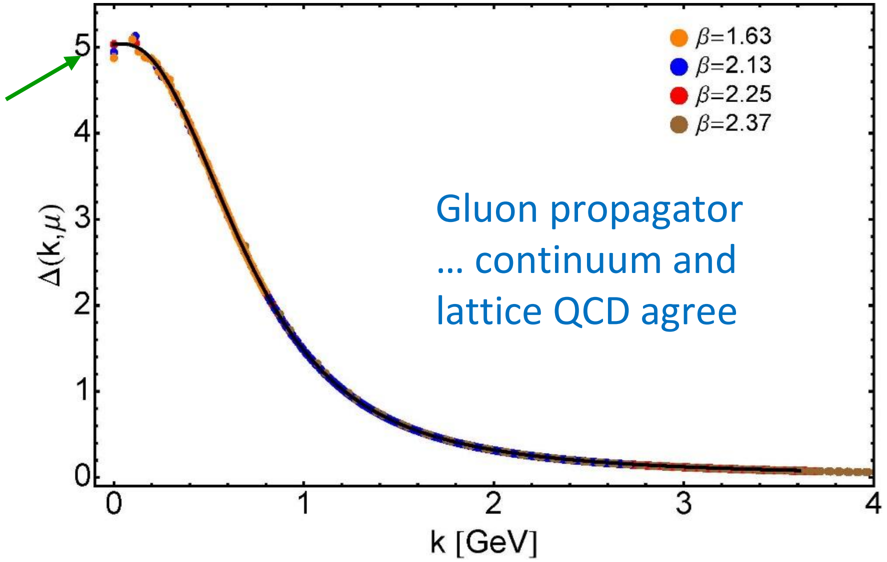
3-gluon vertex



4-gluon vertex



$\frac{1}{m_g^2}$



Truly <u>mass from nothing</u>
An interacting theory, written in terms of massless gluon fields, produces dressed gluon fields that are characterised by a mass function that is large at infrared momenta

- ✓ *QCD fact*
- ✓ *Continuum theory and lattice simulations agree*
- ✓ *Empirical verification?*

Modern Understanding

Grew Slowly from ... Origins

EHM means Gluons are massive

- More than 40 year

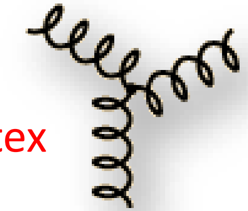
  Dynamical mass ger ... mics,

  J.M. Cornwall, Phys.

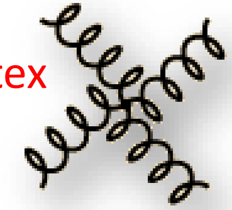
- Owing to strong se ... uasiparticles,

  described by a ... omenta

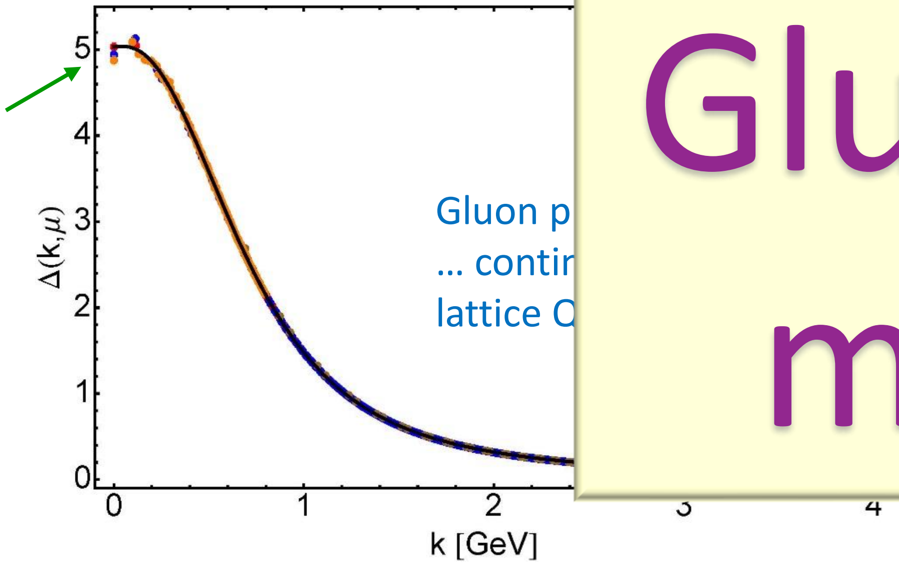
3-gluon vertex



4-gluon vertex



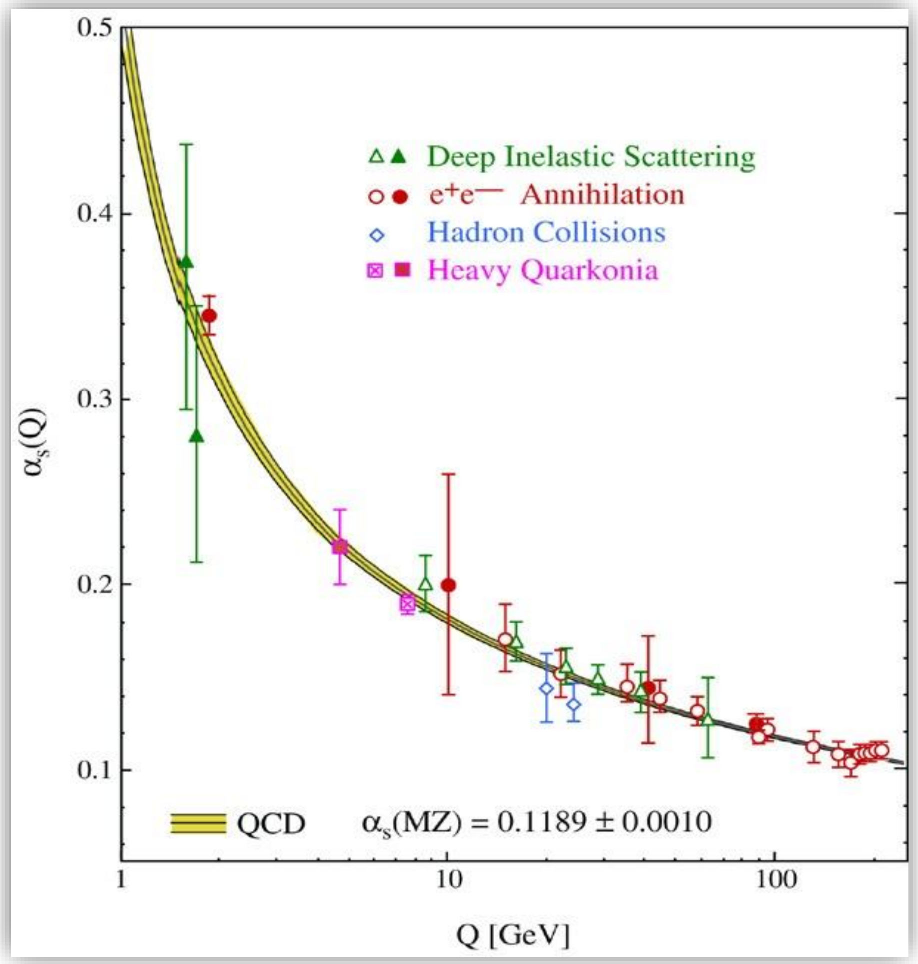
$\frac{1}{m_g^2}$



- ✓ *QCD fact*
- ✓ *Continuum theory and lattice simulations agree*
- ✓ *Empirical verification?*

This is where we live

⇐
What's happening out here?!



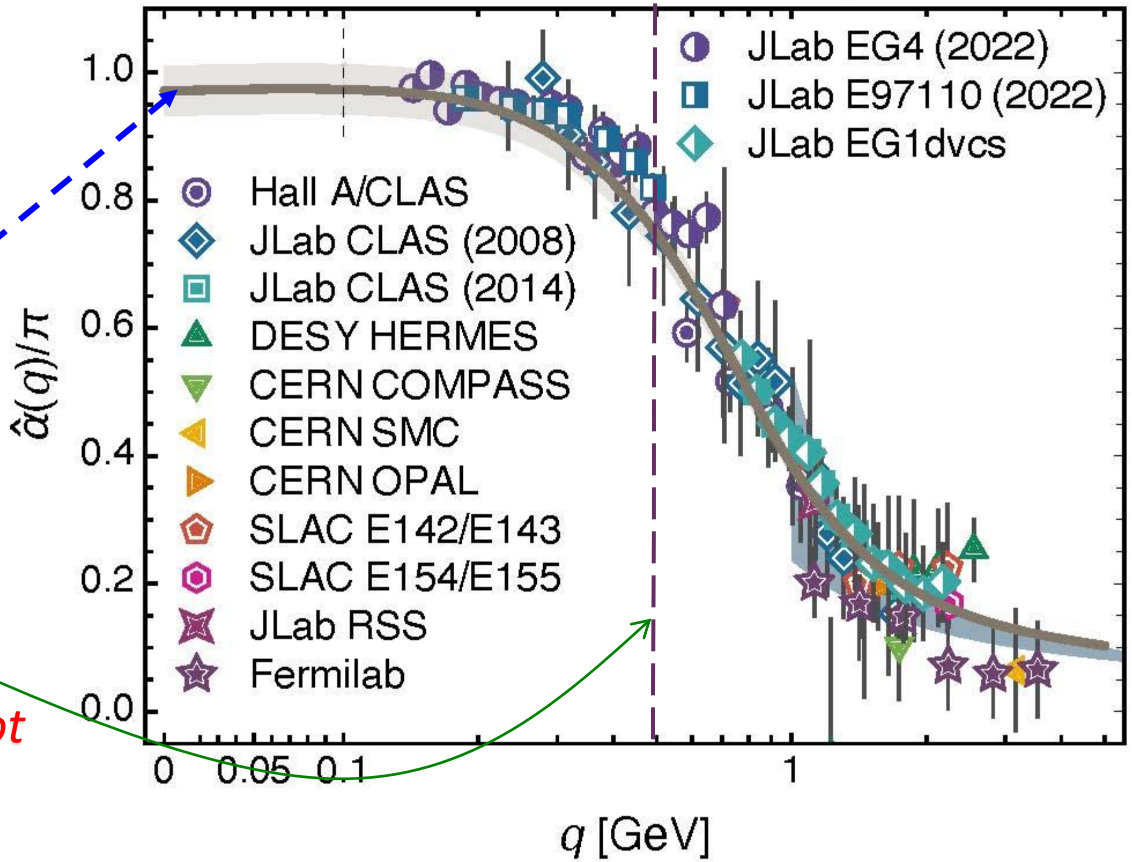
QCD's Running Coupling

Effective charge from lattice QCD, *Zhu-Fang Cui, Jin-Li Zhang et al.*, *NJU-INP 014/19*, *arXiv:1912.08232 [hep-ph]*, *Chin. Phys. C 44 (2020) 083102/1-10*

2204 total downloads

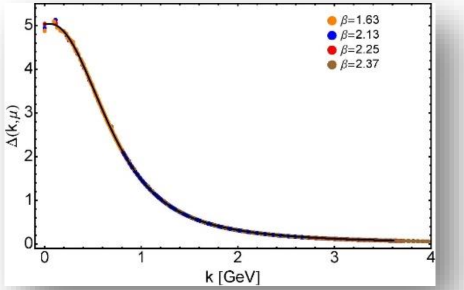
Process independent effective charge = running coupling

- Modern theory enables unique QCD analogue of "Gell-Mann – Low" running charge to be rigorously defined and calculated
- Analysis of QCD's gauge sector yields a *parameter-free prediction*
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$
- No Landau Pole
  - *"Infrared Slavery" picture – linear potential – is not correct explanation of confinement*
- Below $k \sim \widehat{m}_0$, interactions become scale independent, just as they were in the Lagrangian; so, QCD becomes practically conformal again



- *The QCD Running Coupling,* A. Deur, S. J. Brodsky and G. F. de Teramond, Prog. Part. Nucl. Phys. **90** (2016) 1-74
- *Process independent strong running coupling* Daniele Binosi *et al.*, arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7
- *Experimental determination of the QCD effective charge $\alpha_{g1}(Q)$.* A. Deur; V. Burkert; J.-P. Chen; W. Korsch, Particles 5 (2022) 171

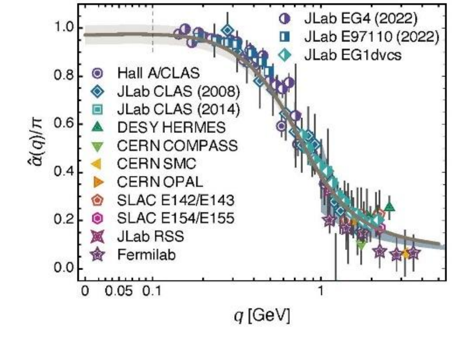
EHM Basics

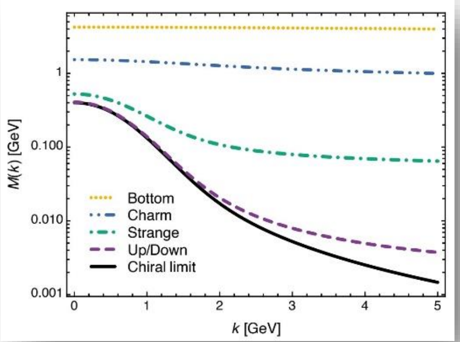


- Absent Higgs boson couplings, the Lagrangian of QCD is scale invariant
- Yet …
  - Massless gluons become massive
  - A momentum-dependent charge is produced
  - Massless quarks become massive
- EHM is expressed in
  EVERY strong interaction observable
- Challenge to Theory =

  Elucidate all observable consequences of these phenomena
  and highlight the paths to measuring them
- Challenge to Experiment =
  Test the theory predictions so that
  the boundaries of the Standard Model
  can finally be drawn

THREE PILLARS OF EHM





JLab Beam Energy (GeV)	Fraction EHM mapped (%)
6	≈ 35
12	≈ 50
22	≈ 90

$$\Psi(x,t)$$

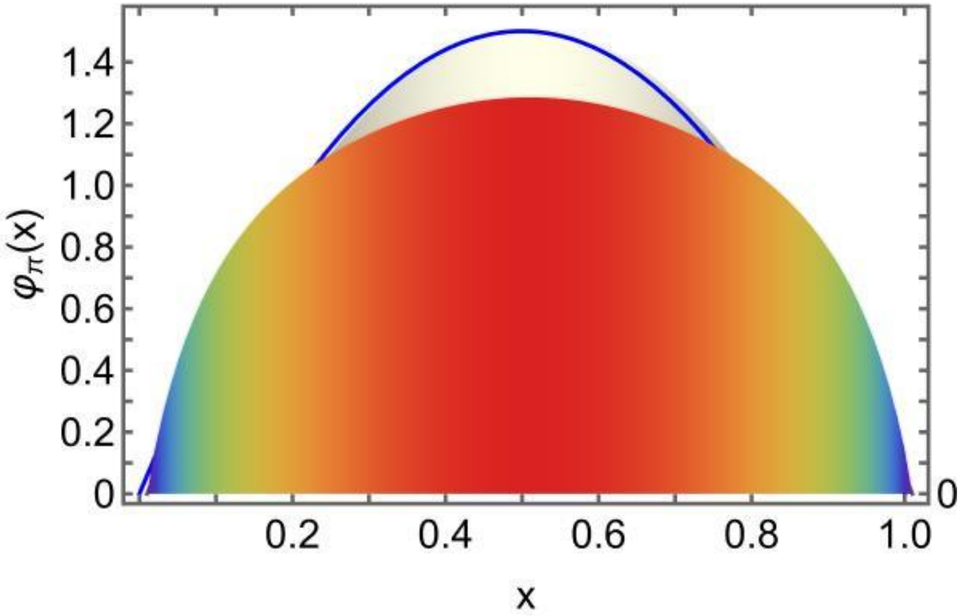
π & K DAs & Form Factors

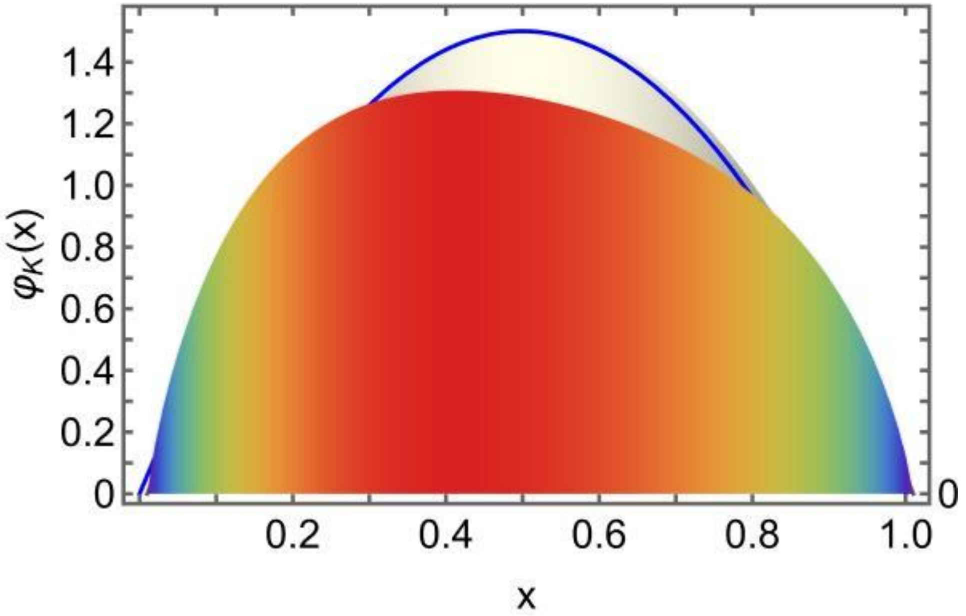
Wave Functions of Nambu Goldstone Bosons

- Physics Goals:
  - Pion and kaon distribution amplitudes (DAs – $\varphi_{\pi,K}$)
  - Nearest thing in quantum field theory to a Schrödinger wave function
  - Consequently, fundamental to understanding π and K structure.
- Scientific Context:
  - For 40 years, the x-dependence of the pion's dominant DA has been controversial.
  - Modern theory ⇒ EHM is expressed in the x-dependence of pion and kaon DAs
  - Pion DA is a direct measure of the dressed-quark running mass in the chiral limit.
  - Kaon DA is asymmetric around the midpoint of its domain of support ($0<x<1$)
    - Degree of asymmetry is signature of constructive interference between EHM and HB mass-generating mechanisms

Insights into the Emergence of Mass from Studies of Pion and Kaon Structure, Craig D. Roberts, David G. Richards, Tanja Horn and Lei Chang, NJU-INP 034/21, arXiv: 2102.01765 [hep-ph], Prog. Part. Nucl. Phys. **120** (2021) 103883/1-65

π & K DAs cf. asymptotic profile





- EHM generates broadening in both π & K
- EHM + Higgs-boson interference is responsible for skewing in kaon
  - HB-only $\Rightarrow$ peak shifted to $\frac{m_u}{m_s} \times \frac{1}{2} \approx 0.02$ … *wrong*
  - Instead, EHM*HB for u and s quarks … $\frac{\text{EHM } m_u \to M_u}{\text{EHM } m_s \to M_s} \times \frac{1}{2} \approx 0.4$

LFWF (ψ_H) Factorisation

See, e.g., Sec. 3A in
Insights into the Emergence of Mass from Studies of Pion and Kaon Structure, C.D. Roberts, D.G. Richards, T. Horn and L. Chang,
NJU-INP 034/21, arXiv: 2102.01765 [hep-ph],
Prog. Part. Nucl. Phys. **120** (2021) 103883/1-65

- Basic correspondence between DAs and DFs

$$\varphi_H(x;\zeta) \propto \int^{\zeta} d^2k_\perp\, \psi_H(x,\vec{k}_\perp;P)\,, \quad \boldsymbol{q}^H(x;\zeta) \propto \int^{\zeta} d^2k_\perp\, |\psi_H(x,\vec{k}_\perp;P)|^2$$

- Hadron scale, $\zeta = \zeta_H$, is – by definition – the scale whereat dressed-parton degrees of freedom carry all of a given hadron's properties
- At ζ_H, factorised LFWF is an excellent approximation:

$$\psi_H(x,k_\perp^2;\zeta_H) \approx \tilde{\varphi}_H(x;\zeta_H)\tilde{\psi}_H(k_\perp^2;\zeta_H)$$

- Hence, $\boldsymbol{q}^H(x;\zeta_H) \approx \tilde{\varphi}_H^2(x;\zeta_H)$;

  so, EHM-induced broadening in DA entails broadening of DF

GPD Overlap Representation

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\psi_{\mathrm{P}}^{u*}\left(x_-,k_{\perp-}^2;\zeta_{\mathcal{H}}\right)\psi_{\mathrm{P}}^{u}\left(x_+,k_{\perp+}^2;\zeta_{\mathcal{H}}\right)$$

- $P = p' - p, \Delta = p' - p, t = -\Delta^2, \xi = -\frac{\mathrm{n}\cdot\Delta}{2n\cdot P}, x_{\mp} = \frac{x\mp\xi}{x\mp\xi}, k_{\perp\mp} = k_\perp \pm \frac{\Delta_\perp(1-x)}{1\mp\xi}, |x| \geq \xi$
- Using Factorised LFWF ⇒

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

$$\Delta_\perp^2 = \Delta^2(1-\xi^2) + 4\xi^2 m_\pi^2\,,$$
$$z^2 = \Delta_\perp^2(1-x)^2/(1-\xi^2)^2\,.$$

- Does not produce factorised GPD. Instead, GPD with simplified dependence on $(x, x\Delta_\perp)$

QCD Effective Charge

P1: There exists at least one effective charge, $\alpha_{1\ell}(k^2)$, such that, when used to integrate the leading-order perturbative DGLAP equations, an all-orders exact evolution scheme for parton DFs is defined

- P1 does not need proof.
  - It is true – by definition
  - As explained in the pioneering work of Grunberg
- Such charges need not be process-independent
- Nevertheless, an efficacious PI charge is not excluded
- PI charge described earlier – page 9 – has proved suitable.

Renormalization Scheme Independent QCD and QED: The Method of Effective Charges

G. Grunberg (Ecole Polytechnique)

Jul, 1982

24 pages

Published in: *Phys.Rev.D* 29 (1984) 2315-2338

DOI: 10.1103/PhysRevD.29.2315

Report number: Print-82-0721 (ECOLE POLY)

View in: OSTI Information Bridge Server, ADS Abstract Service

cite

610 citations

- Connections with experiment & other nonperturbative extensions of QCD's running coupling are given elsewhere, e.g.,
  - A. Deur, S. J. Brodsky, G. F. de Teramond, *The QCD Running Coupling*, Prog. Part. Nucl. Phys. 90 (2016) 1–74.
  - A. Deur, S. J. Brodsky, C. D. Roberts, *Running Couplings and Effective Charges in QCD,* (almost complete)

Preprint no. NJU-INP 070/23

Empirical Determination of the Pion Mass Distribution

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(Dated: 2023 February 14)

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Existing pion+nucleus Drell-Yan and electron+pion scattering data are used to develop ensembles of model-independent representations of the pion generalised parton distribution (GPD). Therewith, one arrives at a data-driven prediction for the pion mass distribution form factor, θ_2. Compared with the pion elastic electromagnetic form factor, θ_2 is harder: the ratio of the radii derived from these two form factors is $r_\pi^{\theta_2}/r_\pi = 0.79(3)$. Our data-driven predictions for the pion GPD, related form factors and distributions should serve as valuable constraints on theories of pion structure.

Data … $u^\pi(x,\zeta)$

- Extant data relevant to extraction of $u^\pi(x,\zeta)$ have been obtained using Drell-Yan process
  - ✓ New data will be obtained using Drell-Yan (AMBER) and Sullivan processes (JLab, EIC, EicC)
- Most malleable set of Drell-Yan data [E615] was collected at large scale, *viz.* $\zeta_5 = 5.2\ \mathrm{GeV}$
  - So, pQCD can be employed in analysis and a reasonable interpretative basis is provided by quark and gluon parton degrees-of-freedom used to define QCD Lagrangian density.
- On the other hand, continuum calculation of $u^\pi(x,\zeta)$ is best begun at $\zeta_H \ll \zeta_5$ whereat dressed-quark and -antiquark quasiparticle degrees-of-freedom can be used to deliver symmetry-preserving, parameter-free <u>predictions</u> for pion properties
- Of particular importance and utility is fact that …
  - ✓ bound-state's quasiparticle degrees-of-freedom carry all measurable properties of hadron at this scale, including the light-front momentum;
  - ✓ hence: $u^\pi(x,\zeta_H) = u^\pi(1-x,\zeta_H)$

P1 … Data … $u^\pi(x,\zeta_H)$

- Given P1, then experiment and theory can be joined at $\zeta = \zeta_H$ because P1 and symmetry of DF entail the following evolution equation (γ_0^n are anomalous dimensions, $\gamma_0^0 = 0$)

$$\langle x^n\rangle_{u_\pi}^{\zeta} = \langle x^n\rangle_{u_\pi}^{\zeta_{\mathcal{H}}} \left(\langle 2x\rangle_{u_\pi}^{\zeta}\right)^{\gamma_0^n/\gamma_0^1}$$

- Using this evolution equation, any set of valence-quark DF Mellin moments at scale ζ can be converted into an equivalent set of ζ_H moments
- Hence, one has direct mapping between u^π at any two scales.
- Crucially, kernel of mapping is the leading moment: $\langle 2x\rangle_{u^\pi}^{\zeta}$;

So, although existence of $\alpha_{1\ell}(k^2)$ is essential, its pointwise form is largely immaterial.

P1 … Data … $u^{\pi}(x,\zeta_H)$

Concerning pion parton distributions, Z.-F. Cui (崔著钫), M. Ding (丁明慧) et al., NJU-INP 053/21, e-Print: 2112.09210 [hep-ph], Eur. Phys. J. A **58** (2022) 10/1-14

- Analysing [E615] data using methods that ensure consistency with QCD endpoint ($x \simeq 0,1$) constraints, then connections with SM properties can be drawn
- Two such studies are available:
  - Aicher *et al*. (2010) – $u_A^{\pi}(x;\zeta_5)$
  - Barry *et al*. (2021), refined in Cui, Ding, *et al*. (2022) – $u_B^{\pi}(x;\zeta_5)$
- We don't bind ourselves to any single fit to E615 data.
- Instead, consider large array of possibilities with the same qualitative character.
- Achieved by supposing that fair approximation to any such DF is provided by

$$u^{\pi}(x;[\alpha_i];\zeta) = n_u^{\zeta} x^{\alpha_1^{\zeta}} (1-x)^{\alpha_2^{\zeta}} (1+\alpha_3^{\zeta} x^2)$$

- Weak assumption [Cui, Ding et al., 2022]: any of the forms commonly used in fitting data would serve equally well.

P1 … Data … $u^{\pi}(x, \zeta_H)$ … S1

- Then proceed thus …

i. Determine central values of $\{\alpha_i^{\zeta} | i = 1,2,3\}$ via least-squares fit to $u^{\pi}_{A(B)}(x; \zeta_5)$-based data

ii. Generate new vector $\{\alpha_i^{\zeta} | i = 1,2,3\}$, each element of which is distributed randomly around its best-fit value

iii. Using the DF obtained therewith, evaluate

$$\chi^2 = \sum_{l=1}^{N} \frac{(u^{\pi}_{\mathrm{A(B)}}(x_l; [\alpha_i]; \zeta_5) - u_j)^2}{\delta_l^2}$$

where $\{(x_l, u_l \pm \delta_l) | l = 1, \ldots, N = 40\}$ are the measured x points in the E615 data set.

This $\{\alpha_i^{\zeta}\}$ configuration is accepted with probability

$$\mathcal{P} = \frac{P(\chi^2; d)}{P(\chi_0^2; d)}\,, \quad P(y; d) = \frac{(1/2)^{d/2}}{\Gamma(d/2)} y^{d/2-1} \mathrm{e}^{-y/2}$$

$d = N - 3$, & $\chi_0^2 \approx d$ locates maximum of the χ^2-probability density

iv. Repeat (ii) & (iii) until one has a $K > 100$-member ensemble of DFs in both cases A, B.

P1 … Data … $u^\pi(x,\zeta_H)$ … S2

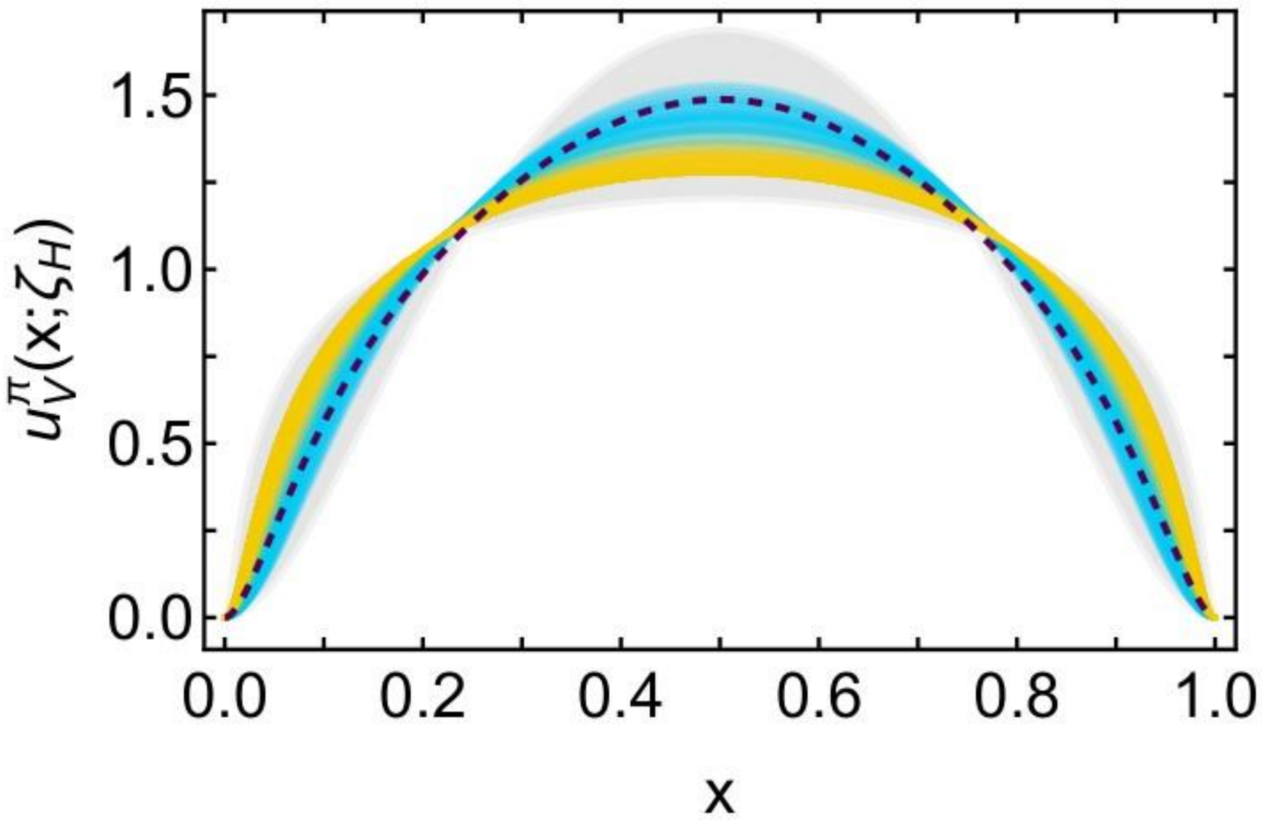
- These ensembles are representative of ζ_5 E615 A(B) data analyses.
- Using P1 evolution equation, each member of an ensemble can be evolved to $\zeta = \zeta_H$:
- Calculate M=large number of $u^\pi_{A(B)}(x;\zeta_5)$ moments
- Evolve each moment to $\zeta = \zeta_H$
- Reconstruct equivalent hadron-scale DF in form

$$u^\pi(x;\zeta_H) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$

  by choosing ρ such that a best least-squares fit is obtained to target set of ζ_H moments.
- This function is efficacious because
  - ✓ it is symmetric, as required by character of hadron scale
  - ✓ consistent with QCD constraints on the endpoint behaviour of valence-quark DFs
  - ✓ flexible enough to express the dilation that EHM is known to produce in pion DFs

P1 … Data … $u^{\pi}(x,\zeta_H)$ … S1 & S2

- Ensembles of $\zeta_5 \rightarrow \zeta_H$ pion valence-quark DF replicas
  - Blue – $u_A^{\pi}(x;\zeta_5)$
  - Orange – $u_B^{\pi}(x;\zeta_5)$
- Comparison curves
  - dashed purple: CSM valence-quark DF
  - grey band: Ensemble of valence-quark DFs developed from results obtained using lattice Schwinger function methods
- Evidently, the data-driven results are a fair match with modern theory predictions



Kaon and pion parton distributions, Zhu-Fang Cui, Minghui Ding et al., NJU-INP 020/20, Eur. Phys. J. C 80 (2020) 1064/1-20

Reconstructing the GPD

- GPD from separable LFWF via overlap

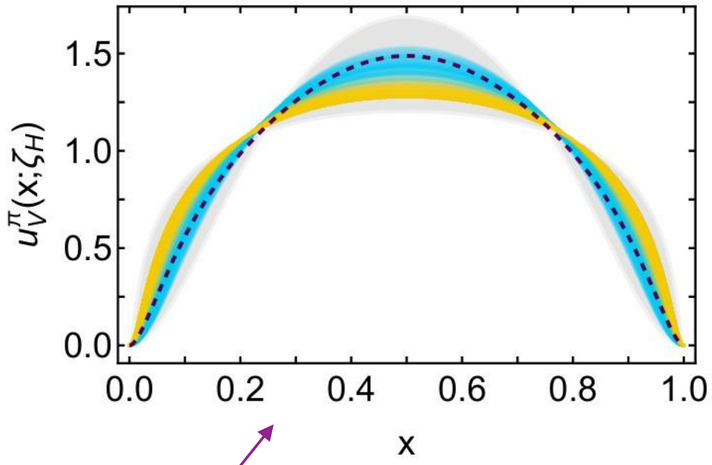
$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

- Pion elastic em form factor

$$F_\pi(\Delta^2) \equiv F_\pi^u(\Delta^2) = \int_{-1}^{1} dx\, H_\pi^u(x,0,-\Delta^2;\zeta_H) = \int_0^1 dx\, u^\pi(x;\zeta_H)\,\Phi^\pi(\Delta^2 x^2;\zeta_H)\,,$$

- With $u^\pi_{A(B)}(x;\zeta_H)$ in hand, Φ^π can be built from data on $F_\pi(\Delta^2)$ …
  - J. Volmer *et al.*, Phys. Rev. Lett. **86** (2001) 1713–1716
  - T. Horn *et al.*, Phys. Rev. Lett. **97** (2006) 192001
  - V. Tadevosyan *et al.*, Phys. Rev. C **75** (2007) 055205
  - H. P. Blok *et al.*, Phys. Rev. C **78** (2008) 045202
  - G. Huber *et al.*, Phys. Rev. C **78** (2008) 045203.

Reconstructing the GPD … S3



- Neglecting scaling violations, $F_\pi(\Delta^2) \propto \frac{1}{\Delta^2}$ on $\Delta^2 \gg m_p^2$;
- So, use a [1,2] Padé approximant to complete GPD ($y = \Delta^2$)

$$\Phi^\pi(y;\zeta_H) = \frac{1+\lambda y}{1+\beta y+\gamma^2 y^2}\,,\ \lambda = \beta - \frac{r_\pi^2}{6\langle x^2\rangle_{u_\pi}^{\zeta_H}}$$

- For every member of ensembles obtained via S1, S2 – drawn in Figure:
  - use $F_\pi(\Delta^2)$ GPD sum rule to obtain best fit to available data,
    in form $\{(\Delta_l^2, \mathrm{F}_l \pm \delta_l)|l = 1, \ldots, L = 9\}$,
    by minimisation of χ^2 in analogy with $u_{A(B)}^\pi(x;\zeta_H)$ procedure
- (β,γ) are fitting parameters & λ is fixed by $r_\pi = 0.64(2)$fm
  Thereby obtain $100_A + 100_B$ pairs (β,γ)

Pion charge radius from pion+electron elastic scattering data,
Z.-F. Cui et al., Phys. Lett. B 822 (2021) 136631.

Reconstructing the GPD … S4

Pion charge radius from pion+electron elastic scattering data, Zhu-Fang Cui (崔著钫) *et al.*, NJU-INP 047/21, arXiv:2108.04948 [hep-ph], Phys. Lett. B **822** (2021) 136631/1-5

- Finally, introduce a spread in r_π values, generated using Gaussian distribution based on $r_\pi = 0.64(2)\text{fm}$
  - Width is twice that calculated in [Cui:2021] ⇒ conservatively constrained r_π distribution
- In concert with this r_π distribution, values of (β, γ) are varied at random about the best-fit values obtained in S3
- Then, select data-driven GPD, H, using GPD $F_\pi(\Delta^2)$ sum rule and Φ^π Padé with probability

$$P_H = P_{\{H|u^\pi\}} P_{u^\pi}$$

where both the conditional probability of the GPD, given a particular DF = $P_{\{H|u^\pi\}}$,
and that of the DF = P_{u^π},
are given by analogues of the earlier expressions,
based on appropriate χ^2 functions, defined from DF and F_π data

- This final step is repeated for every member of each DF ensemble.
- Thereby arrive at data-driven GPDs

Reconstructing the GPD … S3 & S4

- Validation of $F_\pi(\Delta^2)$ part of the procedure

  Compare result of described procedure with data
  - Blue – based on $u_A^\pi(x;\zeta_5)$ ensembles
  - Orange – based on $u_B^\pi(x;\zeta_5)$ ensembles
- Comparison curves
  - dashed purple – CSM prediction for $F_\pi(\Delta^2)$
  - grey band – Ensemble of $F_\pi(\Delta^2)$ results developed from valence-quark DFs based on results obtained using lattice Schwinger function methods
- Once again, data-driven results are in accord with modern theory predictions.

See, e.g., Sec. 4B in
Insights into the Emergence of Mass from Studies of Pion and Kaon Structure, C.D. Roberts, D.G. Richards, T. Horn and L. Chang, NJU-INP 034/21, arXiv: 2102.01765 [hep-ph], Prog. Part. Nucl. Phys. **120** (2021) 103883/1-65

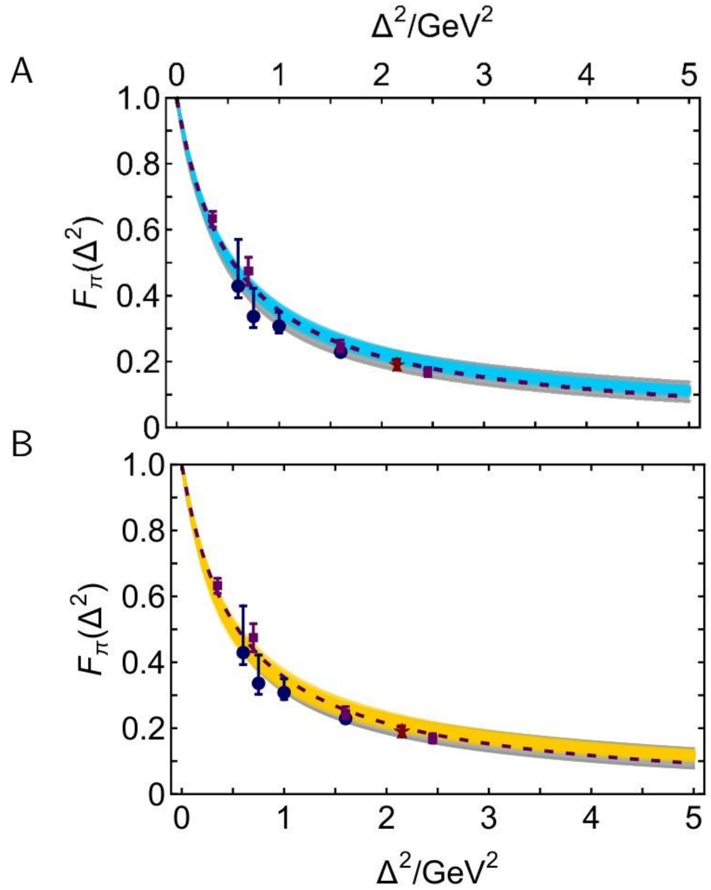


FIG. 2. Pion elastic electromagnetic form factor, $F_\pi(\Delta^2)$, obtained from Eq. (11) using the GPD ensembles generated via S4. *Panel A.* DFs $u_A^\pi(x;\zeta_\mathcal{H})$ [63] (blue band). *Panel B.* DFs $u_B^\pi(x;\zeta_\mathcal{H})$ [54, Sec. 8] (orange band). Comparison curves: dashed purple – $F_\pi(\Delta^2)$ calculated using CSMs [21, Sec. 4B], [45]; grey band – $F_\pi(\Delta^2)$ ensemble obtained with valence-quark DFs developed in Ref. [55] from results obtained using lattice Schwinger function methods [65–67]. The form factor data are from Refs. [36–40].

Pion Generalised Parton Distribution

- Pion GPDs, reconstructed from available analyses of relevant Drell-Yan and electron+pion scattering data
  - Different ensembles only marginally compatible, owing to differences between analyses in [Aicher:2010cb, Barry:2021osv]
  - Yet, both agree with lQCD based ensembles, within mutual uncertainties, because lQCD-constrained ensemble has large uncertainty – improvement of lQCD results needed
  - CSM prediction favours $u_A^\pi(x;\zeta_5)$ ensemble
- In all cases, support of the valence-quark GPD becomes increasingly concentrated in the neighbourhood $x \simeq 1$ with increasing Δ^2
  - Namely, greater probe momentum focuses attention on domain in which one valence-quark carries a large fraction of the pion's light-front momentum

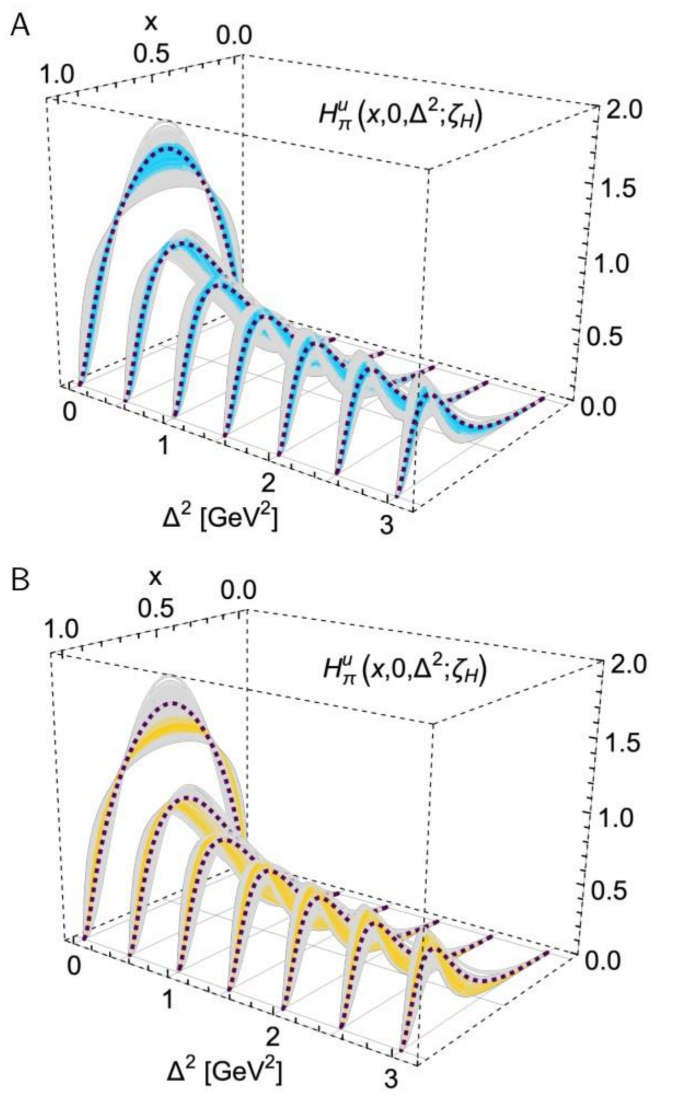


FIG. 3. Pion GPDs. *Panel A.* Working with DFs $u_A^\pi(x;\zeta_H)$ [63] – blue band. *Panel B.* Using DFs $u_B^\pi(x;\zeta_H)$ [54, Sec. 8] – orange band. Comparison curves, both panels: CSM prediction in Refs. [59, 72] – dashed purple curve; GPD ensemble generated from valence-quark DFs developed in Ref. [55], obtained from results computed using lattice Schwinger function methods [65–67] – grey band.

Revealing pion and kaon structure via generalised parton distributions, K. Raya, Z. –F. Cui (崔著钫) et al., NJU-INP 051/21, e-Print: 2109.11686 [hep-ph], Chin. Phys. C **46** (01) (2022) 013107/1-22

π mass distribution

$$\theta_2^\pi(\Delta^2) = \int_{-1}^{1} dx\, 2x\, H_\pi^u(x,0,-\Delta^2;\zeta_{\mathcal{H}})$$

- Expressed as first Mellin-moment of the pion GPD
- Principal, dynamical coefficient in the expectation value of the QCD energy-momentum tensor in the pion

  = pion gravitational current

$$\Lambda^g_{\mu\nu}(K,Q) = 2K_\mu K_\nu \theta_2^\pi(Q^2) + \tfrac{1}{2}[Q^2\delta_{\mu\nu} - Q_\mu Q_\nu]\theta_1^\pi(Q^2) + 2m_\pi^2\delta_{\mu\nu}\bar{c}^\pi(Q^2)$$

- Plainly … $\theta_2(\Delta^2)$ is harder than the $F_\pi(\Delta^2)$

  i.e., the distribution of mass in the pion is more compact than the distribution of electric charge.

- *This is an empirical fact*

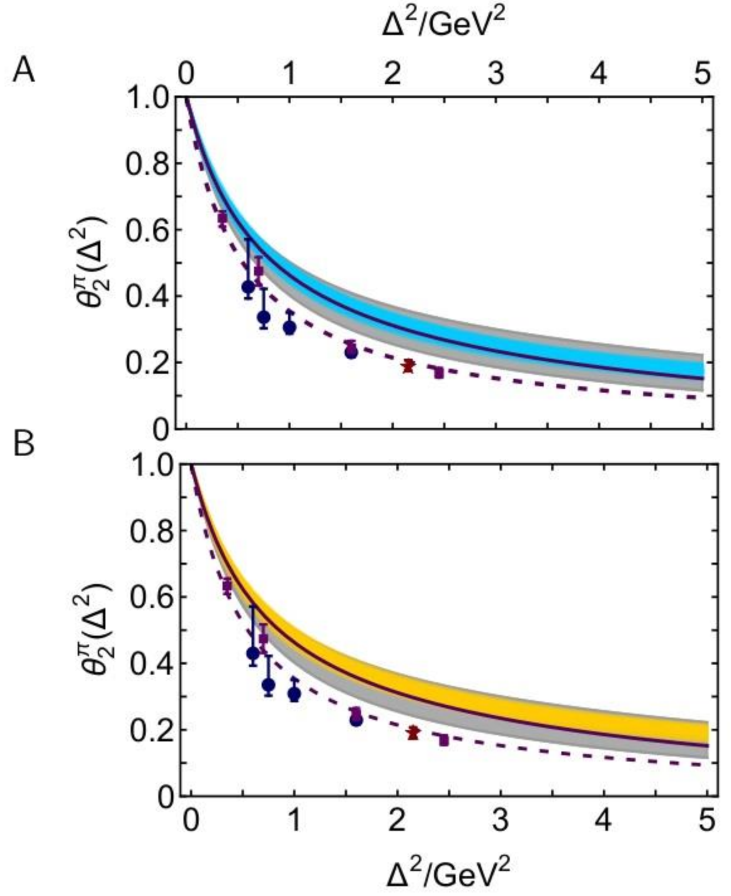


FIG. 4. Pion mass distribution form factor, $\theta_2^\pi(\Delta^2)$. *Panel A*. Developed from the $u_A^\pi(x;\zeta_{\mathcal{H}})$ ensemble [63] – blue band. *Panel B*. Developed from the $u_B^\pi(x;\zeta_{\mathcal{H}})$ ensemble [54, Sec. 8] – orange band. Comparison curves, both panels: CSM prediction for $\theta_2^\pi(\Delta^2)$ in Refs. [59, 72] – solid purple; GPD ensemble generated from valence-quark DFs developed in Ref. [55] using lQCD results [65–67] – grey band. In addition, each panel displays the CSM prediction for $F_\pi(\Delta^2)$ [21, Sec. 4B], [45] – dashed purple curve. The data are those for $F_\pi(\Delta^2)$ from Refs. [36–40], included so as to highlight the precision required to distinguish the mass and electromagnetic form factors.

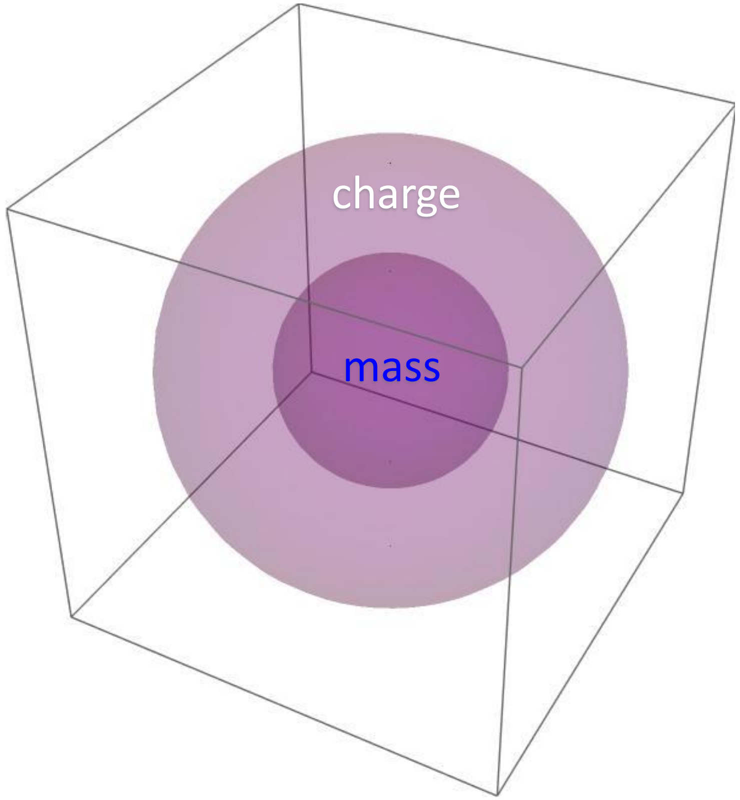
π mass radius

	A	B	lQCD
$r_\pi^{\theta_2}$	0.518(16)	0.498(14)	0.512(21)

- Comparison value for charge radius: $r_\pi = 0.64(2)$
- Data-driven prediction: $\frac{r_\pi^{\theta_2}}{r_\pi} = 0.79(3)$
- Translates into volume ratio = $0.49(6)$

Pion mass distribution is concentrated

within just 50% of the volume of the charge distribution



π mass distribution is harder

Odd function

Even function

- Empirical fact is readily understood mathematically

$$\theta_2(\Delta^2) - F_\pi(\Delta^2) = \int_0^1 dx\, \underbrace{(1-2x)}_{\text{Odd function}}\, u^\pi(x;\zeta_H)\, \underbrace{\Phi^\pi(\Delta^2 x^2;\zeta_H)}_{\text{Even function}}$$

- Baryon number and change conservation $\Rightarrow \theta_2\ (\Delta^2 = 0) - F_\pi(\Delta^2 = 0) = 0$
- In order to produce a realistic pion em form factor, $\Phi^\pi(z^2;\zeta_H)$ must be
  - non-negative
  - monotonically decreasing

  function of its argument
- So, domain of negative support is suppressed more than domain of positive support

  Hence, $\forall\Delta^2 > 0,\ \ \theta_2(\Delta^2) - F_\pi(\Delta^2) > 0$

π mass distribution is harder

- Empirical fact is also readily understood physically
- Pion wave function (hence, pion GPD) is independent of the probe.
  - It's the same whether a photon or graviton is the probing object.
- However, probe itself focuses on different features of the target constituents
  - Target quark carries same charge, irrespective of its momentum.
    - So, pion LFWF alone controls distribution of charge.
  - Gravitational interaction of target quark depends on its momentum. (The current = the energy momentum tensor)
    - Pion effective mass distribution therefore depends on interference between quark momentum growth and LFWF momentum suppression with increasing $\Delta^2 x^2$.
    - This pushes support to a larger momentum domain in the pion = smaller distance domain.

π mass distribution is harder

- The difference between mass and charge radii grows with strength of EHM-induced broadening of pion (DA) DF because …

  Broadening magnifies endpoint differences, accentuating low-x positive support in integrand

$$\theta_2(\Delta^2) - F_\pi(\Delta^2) = \int_0^1 dx\,(1-2x) u^\pi(x;\zeta_H)\Phi^\pi(\Delta^2 x^2;\zeta_H)$$

- This broadening is also manifested in $\Delta^2 x^2$-dependence of Φ^π
  - Broadening of DF is expression of smoothing in the momentum dependence of Φ^π

- "*Is there a specific aspect of the pion GPDs driven by the data which is responsible?*"
- Yes. EHM induced broadening of pion LFWF = GPD.

  Given GPD overlap representation, they're the same thing.

Data on large-momentum tail of $F_\pi(\Delta^2)$ and endpoint behaviour of $u^\pi(x;\zeta_H)$ will enable tighter constraints on mass-charge radii difference

IPS GPD and LF-transverse densities

- Impact Parameter Space GPD:

$$u^\pi(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(|b_\perp|\Delta)\, H_\pi^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

- LF-transverse density, integral over IPS GPD:

$$\rho^\pi_{\{F,\theta_2\}}(|b_\perp|) = \int_{-1}^1 dx\, \{1, 2x\} u^\pi(x, b_\perp^2; \zeta_{\mathcal{H}})$$
$$= \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(|b_\perp|\Delta)\{F_\pi(\Delta^2), \theta_2(\Delta^2)\}$$

- Evidently, consistent with the analysis presented above, the pion's light-front transverse mass distribution is more compact than the analogous charge distribution.
- Moreover, the data-built results are consistent with modern theory predictions.

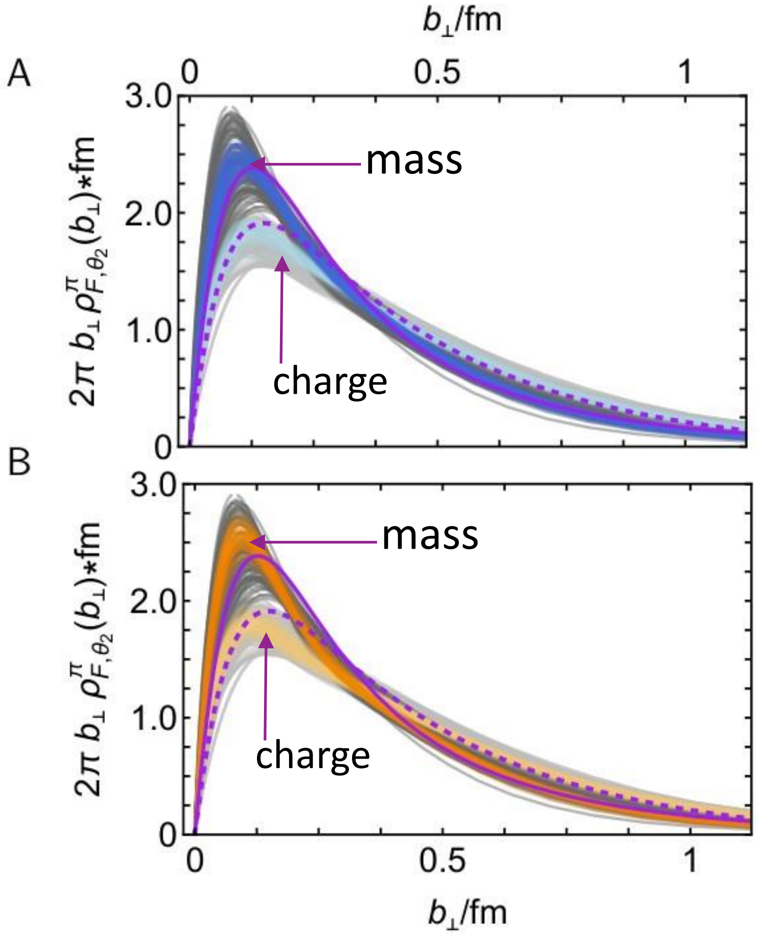


FIG. 6. Light-front transverse density distributions, Eqs. (20), built from: $u^\pi_{\rm A}(x;\zeta_{\mathcal{H}})$ ensemble [63] – *Panel A* (charge = light blue, mass = dark blue) ; and $u^\pi_{\rm B}(x;\zeta_{\mathcal{H}})$ ensemble [54, Sec. 8] – *Panel B* (charge = light orange, mass = dark orange). Comparison curves, both panels: dashed purple curve – charge density calculated using CSM prediction for $F_\pi(\Delta^2)$ [21, Sec. 4B], [45]; solid purple curve – mass density obtained using CSM prediction for $\theta_2^\pi(\Delta^2)$ in Refs. [59, 72]; silver band – lattice-based charge density ensemble; and grey band – lattice-based mass density ensemble.

Predictions for π & K pressure profiles

- Kaon and pion pressure & sheer-force profiles

$$\Lambda^{g}_{\mu\nu}(K,Q) = 2K_\mu K_\nu \theta_2^\pi(Q^2) + \tfrac{1}{2}[Q^2\delta_{\mu\nu} - Q_\mu Q_\nu]\theta_1^\pi(Q^2) + 2m_\pi^2\delta_{\mu\nu}\bar{c}^\pi(Q^2)$$

- Need θ_2 <u>and</u> θ_1: θ_1 requires knowledge of ERBL region
- Pressure is positive and large in neighbourhood $\mathrm{r} \simeq 0$
  - Meson's dressed-valence constituents are pushing away from each other at small separation
- Pressure switches sign as separation becomes large, indicating transition to domain whereupon confinement forces exert their influence on the pair
- Analogous profiles can be drawn for neutron stars.
  - $\mathrm{r} \simeq 0$ pressures therein of ≈ 0.1GeV/fm
- Meson & neutron star core pressures are on same scale.

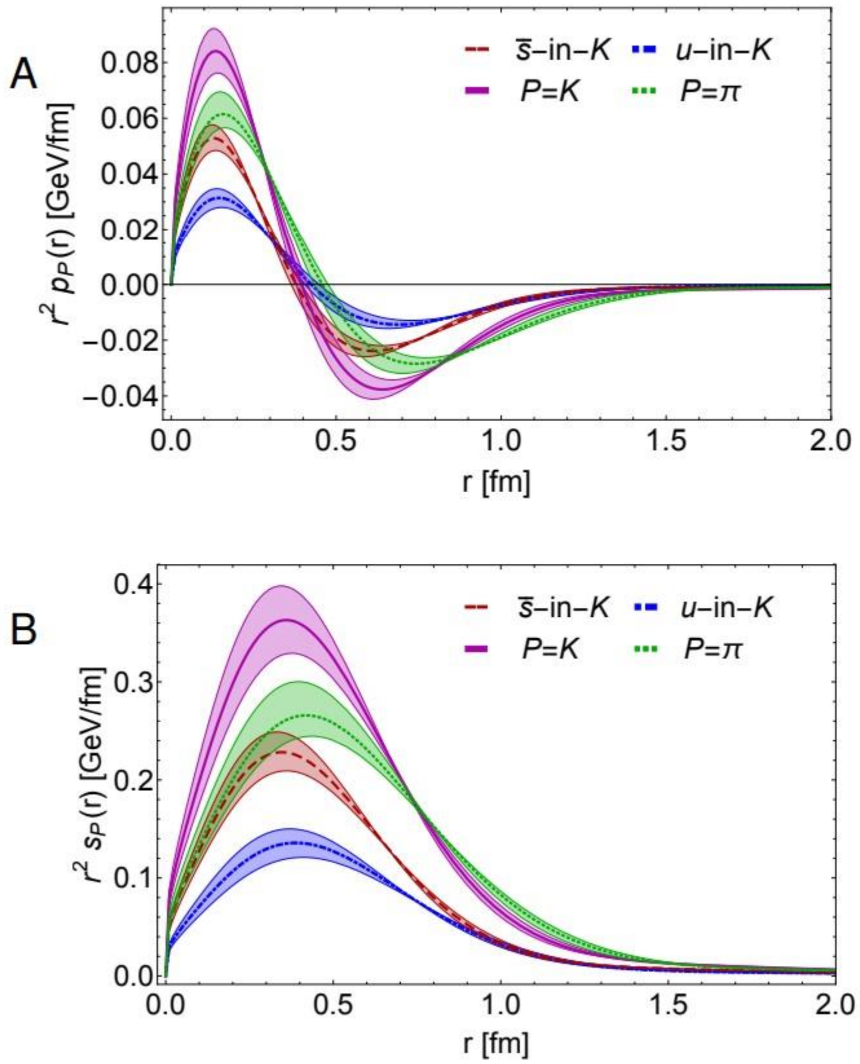


Figure 6: *Upper panel* – A. Pressure profiles in the kaon, Eq. (29a). *Lower panel* – B. Shear pressure profiles, Eq. (29b). Legend. Solid magenta curve and band: total K results obtained herein using Eq. (25) with $M_u = M_u(1 \pm 0.1)$, $M_{\bar{s}} = M_{\bar{s}}(1 \pm 0.1)$. Individual u-in-K (dot-dashed blue) and $\bar{s}$-in-K (dashed red) are also displayed. Green dotted curve and band reproduce Fig. 5 pion profiles.

Revealing pion and kaon structure via generalised parton distributions, <u>K. Raya</u>, Z. –F. Cui (崔著钫) et al., <u>NJU-INP 051/21</u>, <u>e-Print: 2109.11686 [hep-ph]</u>, Chin. Phys. C **46** (01) (2022) 013107/1-22

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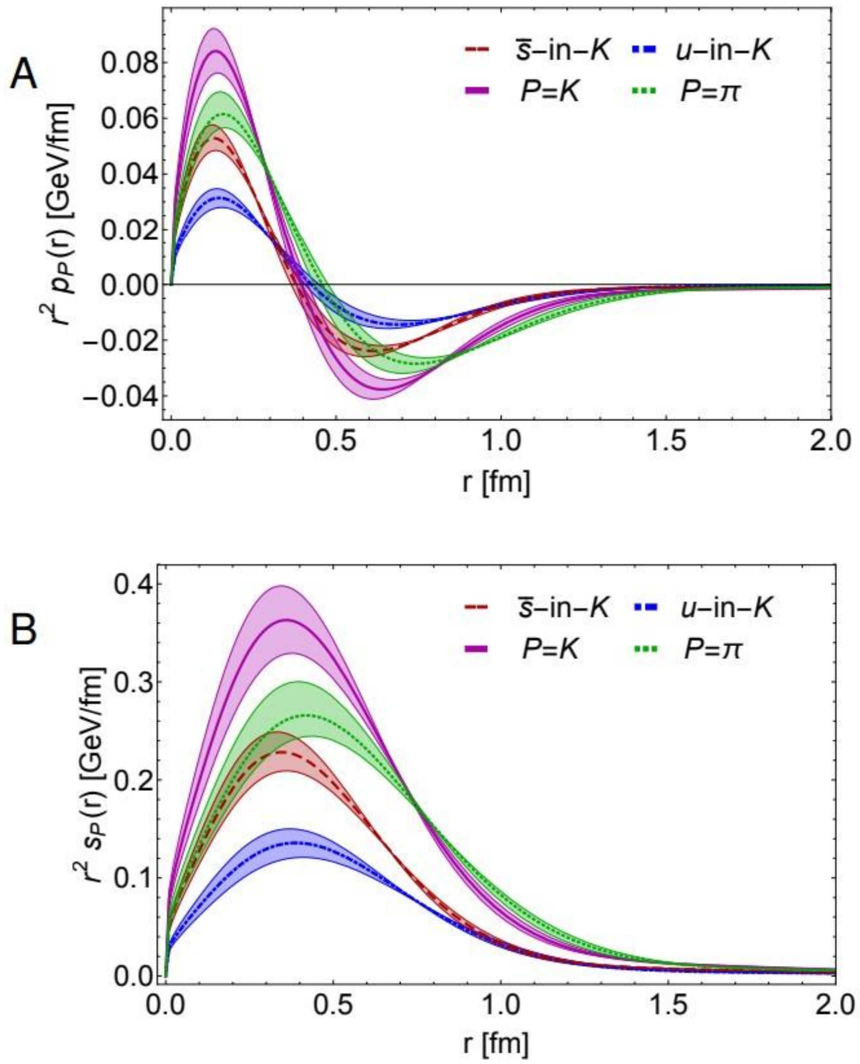
- Need θ_2 and θ_1: θ_1 requires knowledge of ERBL region
- Shear force measures strength of forces within meson which work to deform it.
- These forces are maximal where pressure changes sign
  - *i.e.*, where forces driving quark and antiquark apart are overwhelmed by attractive confinement pressure.
- Pressure-based confinement radius

$$r_\pi^c = 0.45(3)\text{fm},$$
$$i.e.,\ 0.70(4) r_\pi$$



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Predictions for π & K pressure profiles

- Kaon and pion pressure & sheer-force profiles

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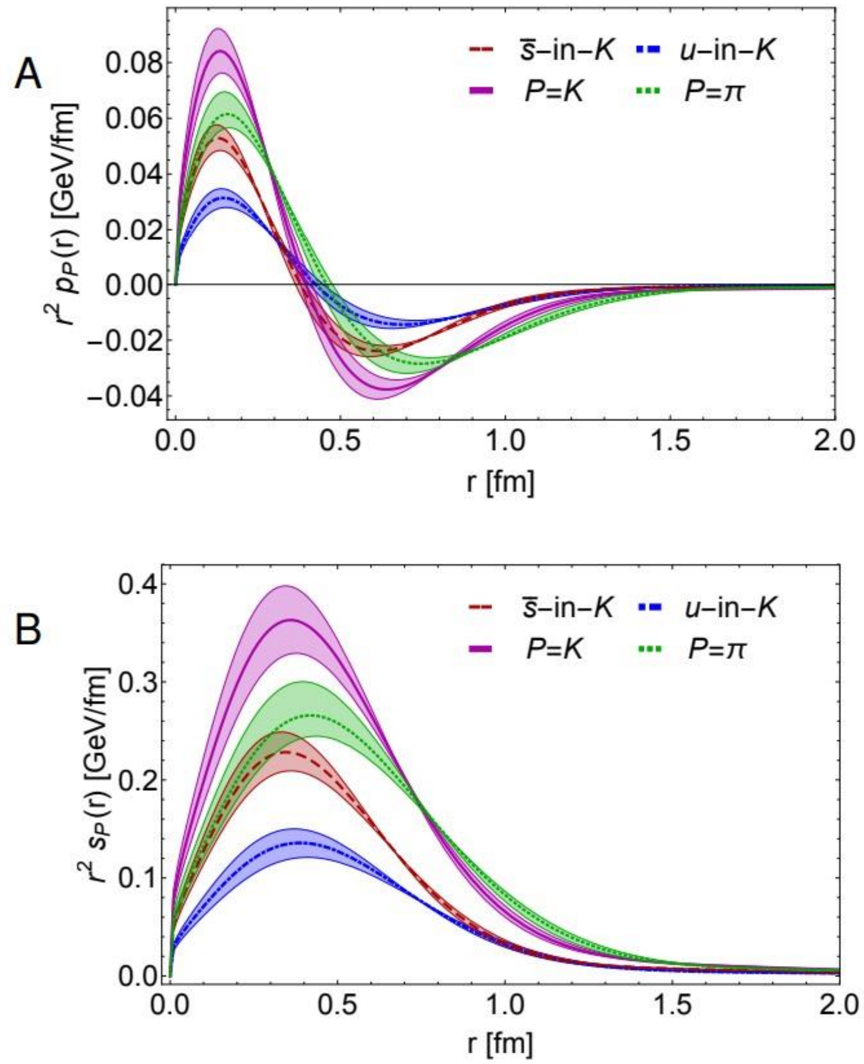
- Need θ_2 and θ_1: θ_1 requires knowledge of ERBL region
- Measured by pressure radius
  - K is $\approx$ 15% smaller than π
  - K core pressure is 20% larger
- Kaon's $\bar{s}$-quark *cf*. partner u-quark
  - contributes a greater fraction of the total K pressure
  - peak/trough intensities are greater
  - associated distributions are localised nearer to $r = 0$.
- Mapped to proton (q+qq) $\Rightarrow$ similar effects in flavour separation of proton profiles, with $u^p \leftrightarrow \bar{s}^K$ and $d^p \leftrightarrow u^K$



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Perspective

- Improvement of these our data-driven predictions must await
  - new data relevant to pion DFs and improvements in associated analysis methods
  - pion form factor data that extends to larger momentum transfers than currently available.
- No similar analysis for the kaon will be possible before analogous empirical information becomes available.
  - kaon charge radius can't be considered known because kaon elastic form factor data are sketchy
  - only eight data relevant to kaon valence-quark DFs are available
- Given importance of contrasting pion and proton mass distributions in search for an understanding of EHM, completing kindred analysis that leads to data-driven ensembles of proton GPDs should be given high priority
  - Precise elastic form factor data are available
  - However, despite wealth of data relevant to proton DFs, improved analyses are required, including, *e.g.*, effects of next-to-leading-logarithm resummation.
  - Meanwhile, one might begin with existing theory predictions that provide a unified description of pion and proton DFs


Thankyou