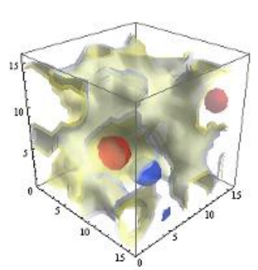
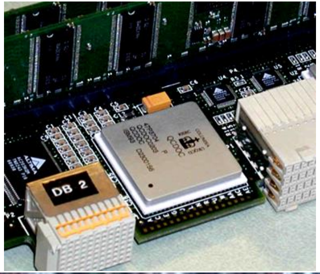
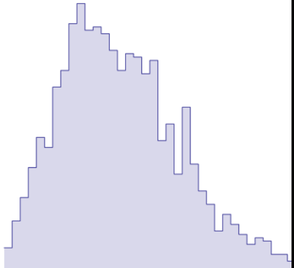
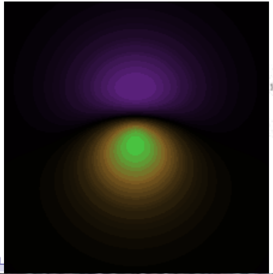
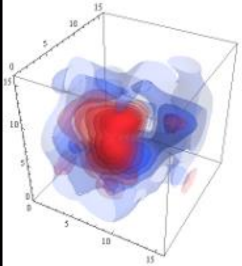
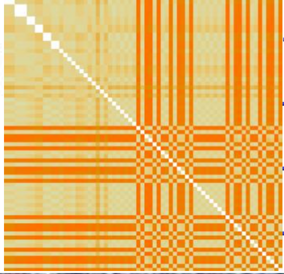
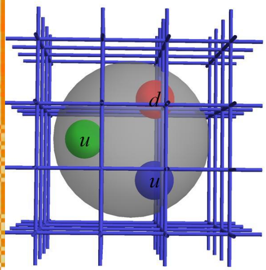








Lattice Bjorken-x Dependence of Pion & Kaon Structure

Huey-Wen Lin

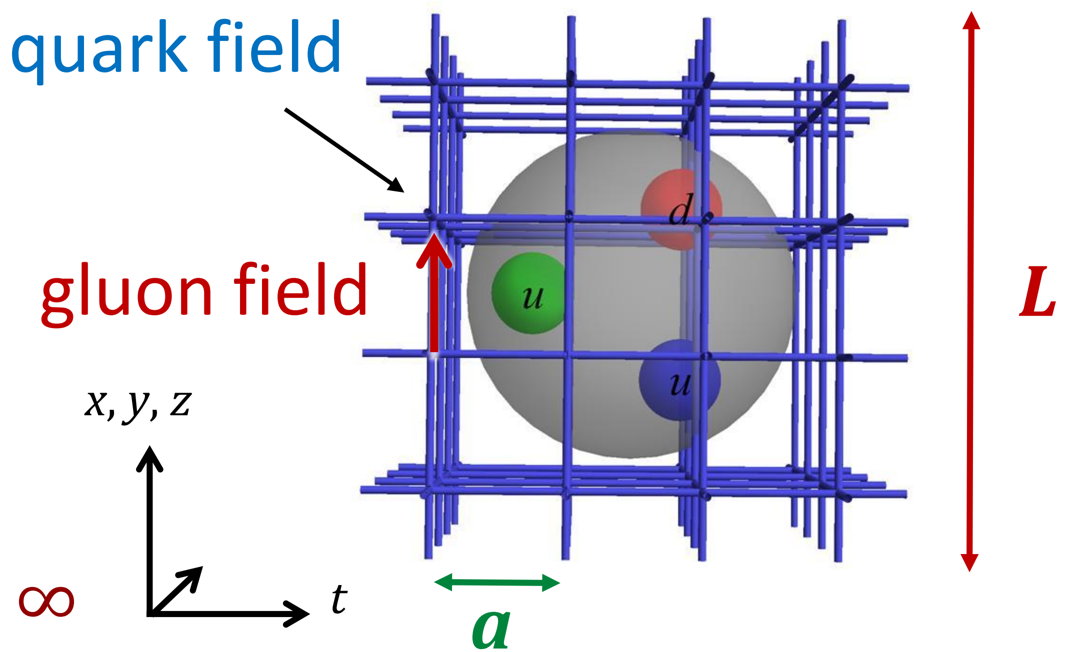
Lattice QCD 101

- § Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- § Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)} O\left(\bar{\psi},\psi,A\right)$$

in **Euclidian** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



- § Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

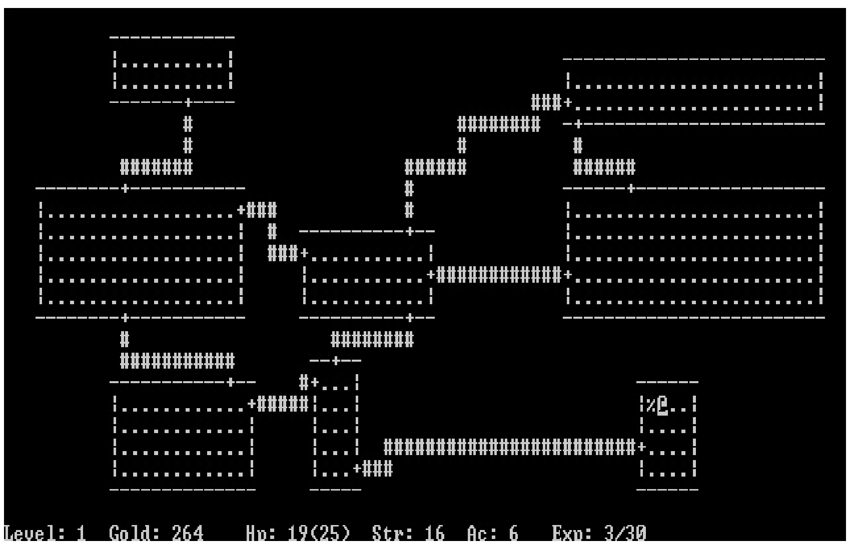
Are We There Yet?

- § Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven’t we solved QCD yet?
- § Progress is limited by computational resources

Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven’t we solved QCD yet?
- Progress is limited by computational resources

1980s



Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

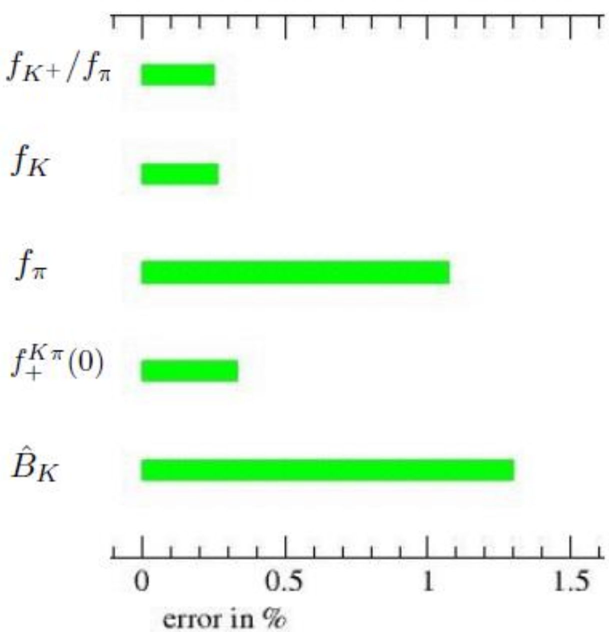
Successful Examples

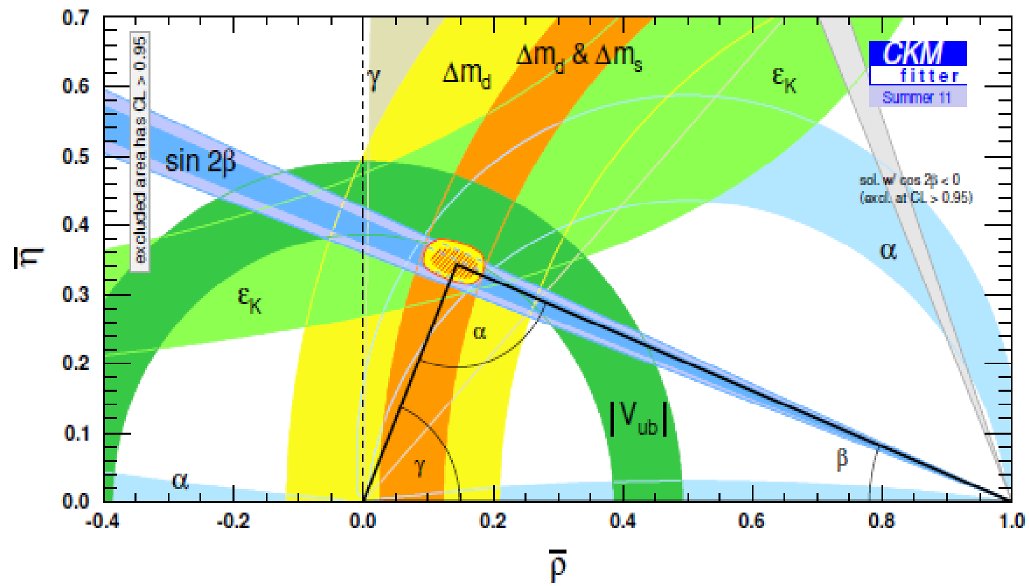
§ Lattice flavor physics provides precise inputs from the SM

A. El-Khadra, Sep. 2015, INT workshop "QCD for New Physics at the Precision Frontier"

- Very precise results in many meson systems

errors (in %) (preliminary) FLAG-3 averages





§ Precision calculations of hadron structure have begun

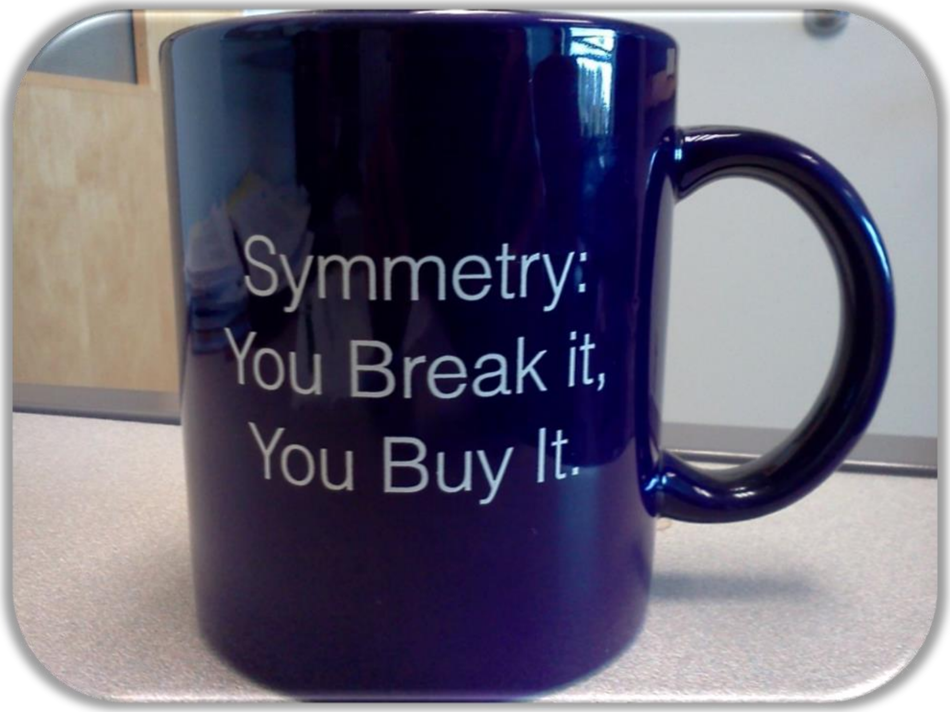
DAs on the Lattice

- Lightcone definition
  - $\langle 0 | \bar{d}(-z)\gamma_\mu\gamma_5[-z,z]u(z) | \pi^+ \rangle = i f_\pi p_\mu \int_0^1 d\xi\, e^{-i\xi p\cdot z}\phi_\pi(\xi,\mu)$
    - Nonlocal matrix elements with Wilson line $[-z,z]$ connecting u and $\bar{d}$
    - $\xi = x-(1-x) = 2x-1$
- Lattice calculations rely on operator product expansion, only provide moments
  - $\langle \xi^n \rangle = \int d\xi\, \xi^n \phi(\xi,\mu)$
  - $\langle \xi^0 \rangle = 1, \quad \langle \xi^2 \rangle \to \left\langle 0 \left| \bar{d}\overleftrightarrow{D}_{(\mu}\overleftrightarrow{D}_\nu\gamma_{\rho)}\gamma_5 u \right| \pi(p) \right\rangle \ldots$
- In principle, use inverse transformation to retrieve true DA

$$\phi(\xi,\mu) = 6x(1-x)\left(1 + \sum_{n=2,4,\ldots} a_n^\pi(\mu) C_n^{3/2}(2x-1)\right)$$

Problem with Moments

- For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- Relative error grows in higher moments
  - Calculation would be costly and difficult

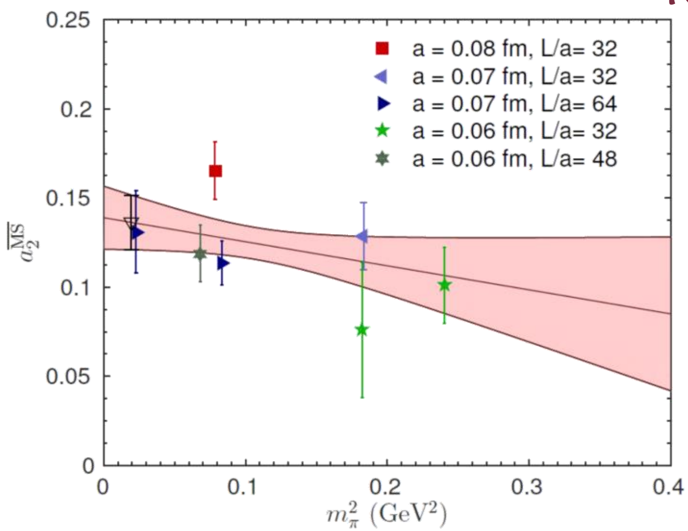


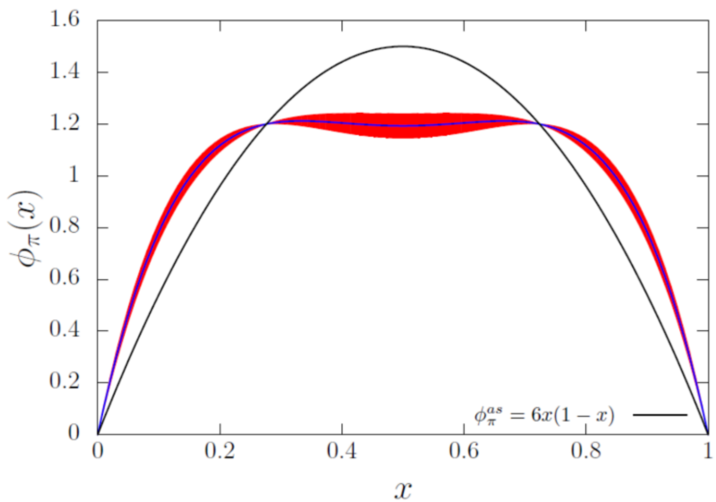
State of the Art Lattice Moments

§ Pion DAs from Lattice Moments

- $a_2^\pi, \langle \xi^2 \rangle$ from $\left\langle 0 \left| \bar{d} \overleftrightarrow{D}_{(\mu} \overleftrightarrow{D}_\nu \gamma_{\rho)} \gamma_5 u \right| \pi(p) \right\rangle$
- RQCD: $N_f = 2, m_\pi \in [150, 490]$ MeV, $m_\pi L \in [3.4, 6.7]$

RQCD, 1503.03656





- RQCD: $a_2^{\overline{\rm MS}}(2\ \text{GeV}) = 0.136(15)(15)$ B1503.03656 (RQCD)
- RBC/UKQCD: $a_2^{\overline{\rm MS}}(2\ \text{GeV}) = 0.233(30)(60)$ Arthur 1011.5906
- QCDSF/UKQCD: $a_2^{\overline{\rm MS}}(2\ \text{GeV}) = 0.211(114)$ Braun hep-lat/0606012

Problem with Reconstruction

§ Possible reconstructions

- Using Gegenbauer polynomial expansion of the pion DA

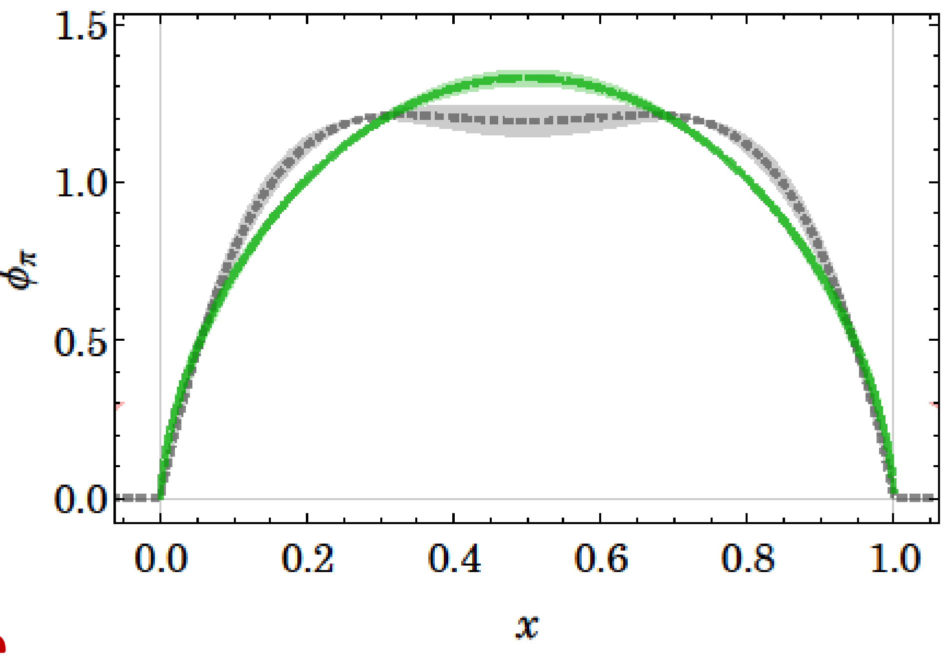
$$6x(1-x)\left(1+a_2C_2^{3/2}(2x-1)\right)$$

Using $a_2 = 0.136(15)(15)$

- Another through

$$A\left(x(1-x)\right)^B$$

Using $\langle\xi^2\rangle = 0.2361(41)(39)$



LP3 Collaboration, 1702.00008

§ A new approach is needed to extract DAs on the lattice

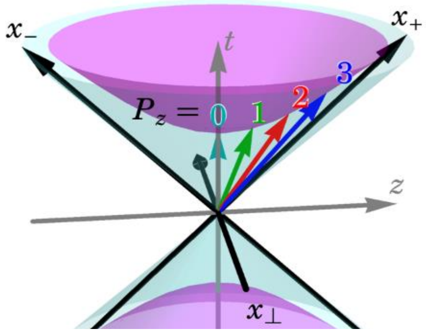
A Promising "New" Direction
with Examples of PDFs

A New Direction

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

§ Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)



§ Finite-momentum quark distribution (quasi-distribution)

- Suggested operator:

$$\tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$x = k_z/P_z$

Lattice z coordinate

hadron momentum $P_\mu = \{P_0, 0, 0, P_z\}$

Product of lattice gauge links

§ Take the infinite-P_z limit to recover lightcone functions

- Just another limit to take, like taking $a \to 0$ or $V \to \infty$

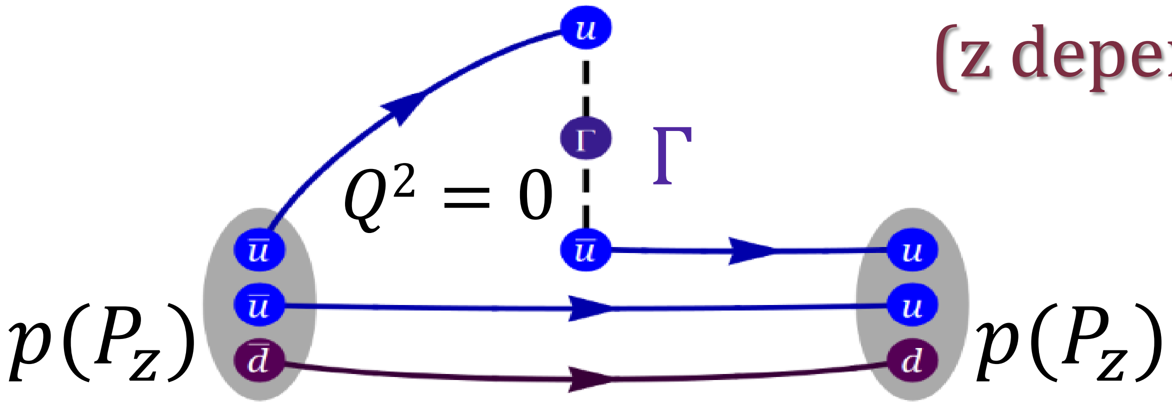
A New Direction

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

(z dependence)



2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \Gamma \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

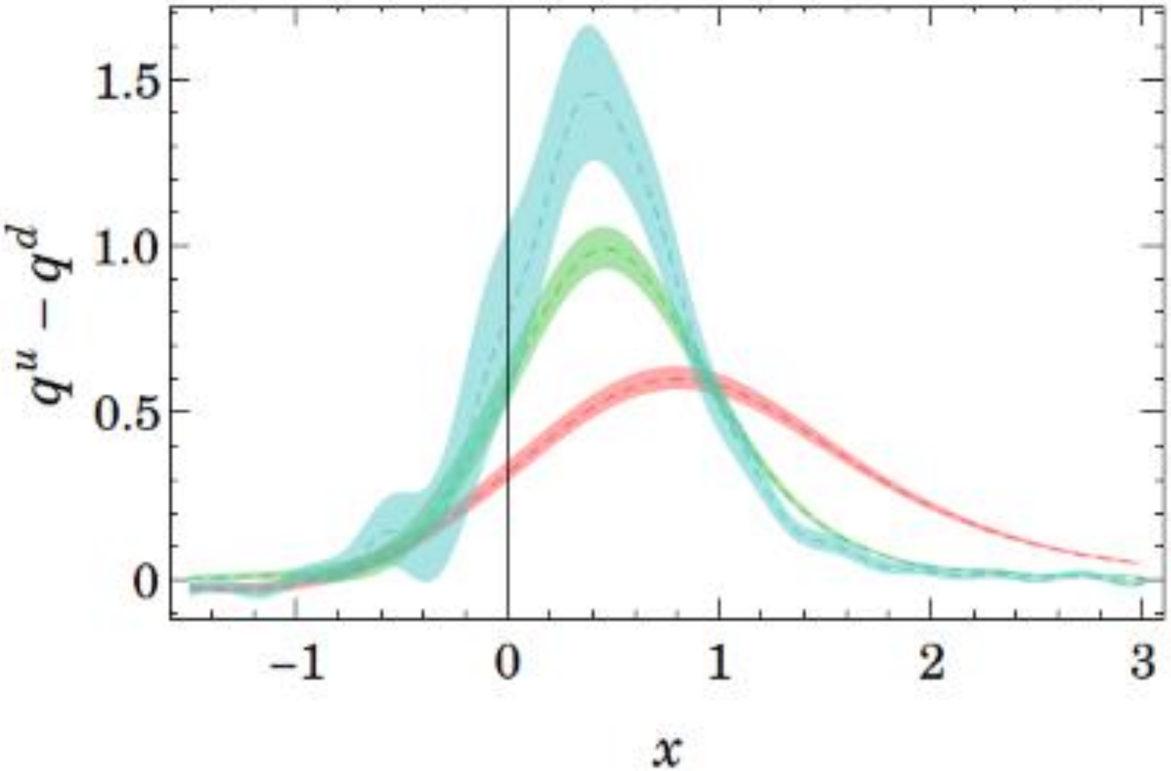
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Quasi-Distribution

§ Exploratory study $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_L$$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

Improvement?

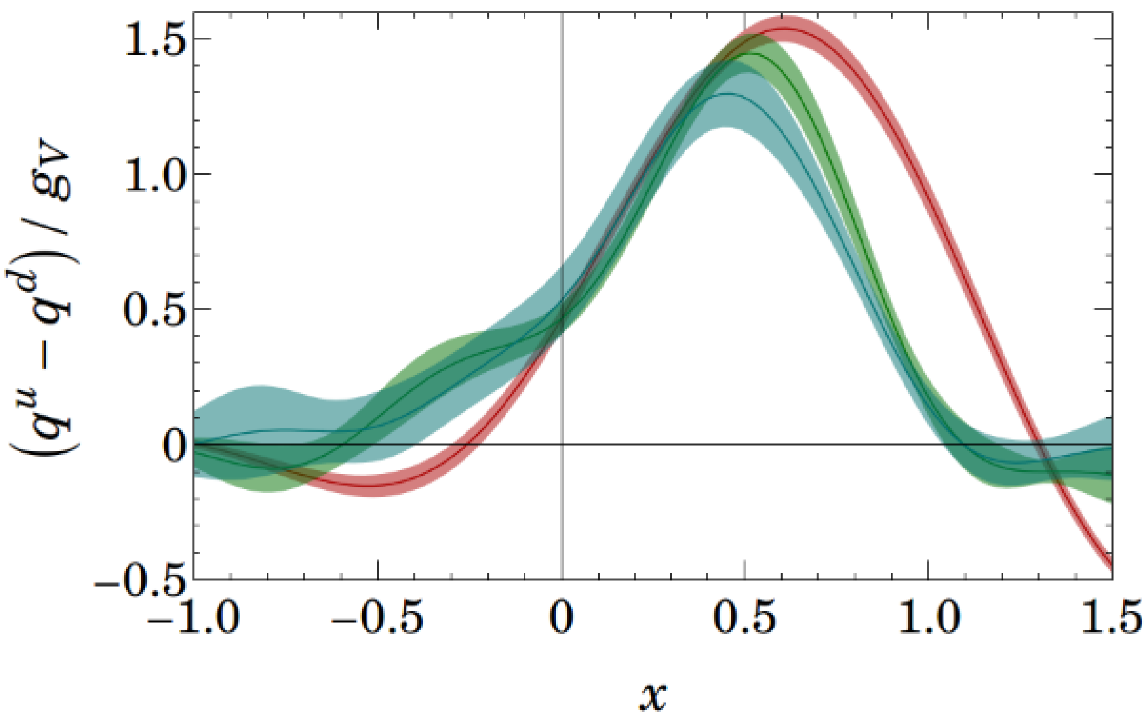
Work out leading-P_z corrections

Distribution after Matching

§ Exploratory study $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)

- Take ratios (partially cancel statistical and systematic errors)

$$q_{\text{norm}}(x,\mu,P_z) = \frac{q(x,\mu,P_z)}{\int dx\, q(x,\mu,P_z)} \times g_V^{\overline{\text{MS}}}(2\text{ GeV})$$



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

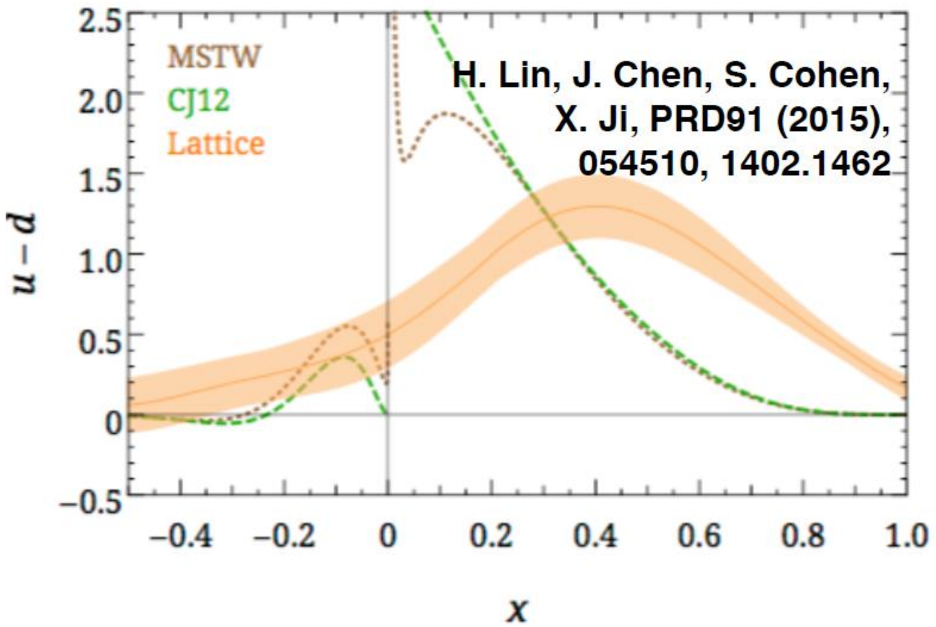
No significant finite-momentum effect seen for P_z>1

A New Direction

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

§ From 2014 to 2018



§ First result in 2014

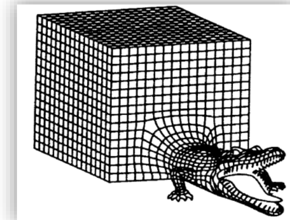
- $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)
- Largest $P_z \approx 1.3$ GeV
- 1-loop $\overline{\text{MS}}$ matching + target-mass correction

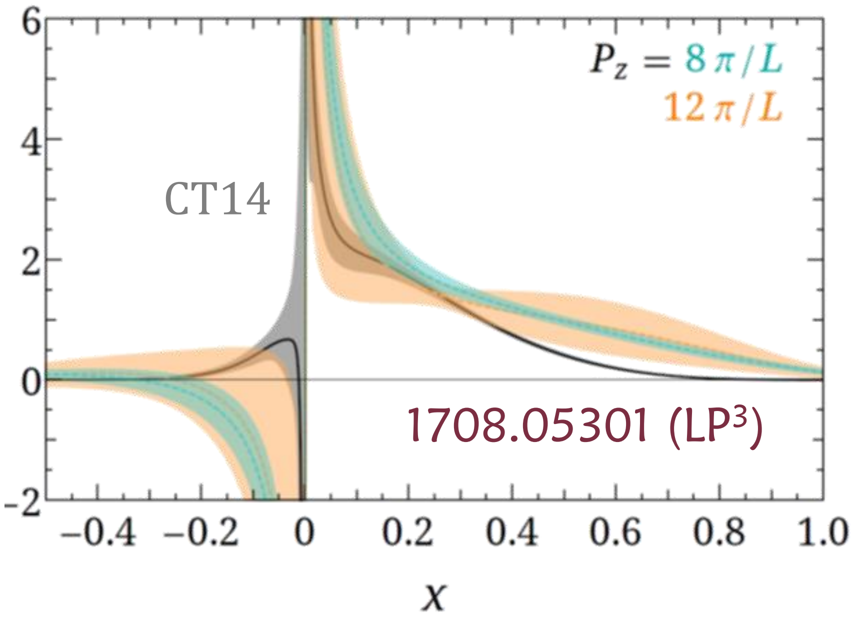
§ Updated results in 2017/18

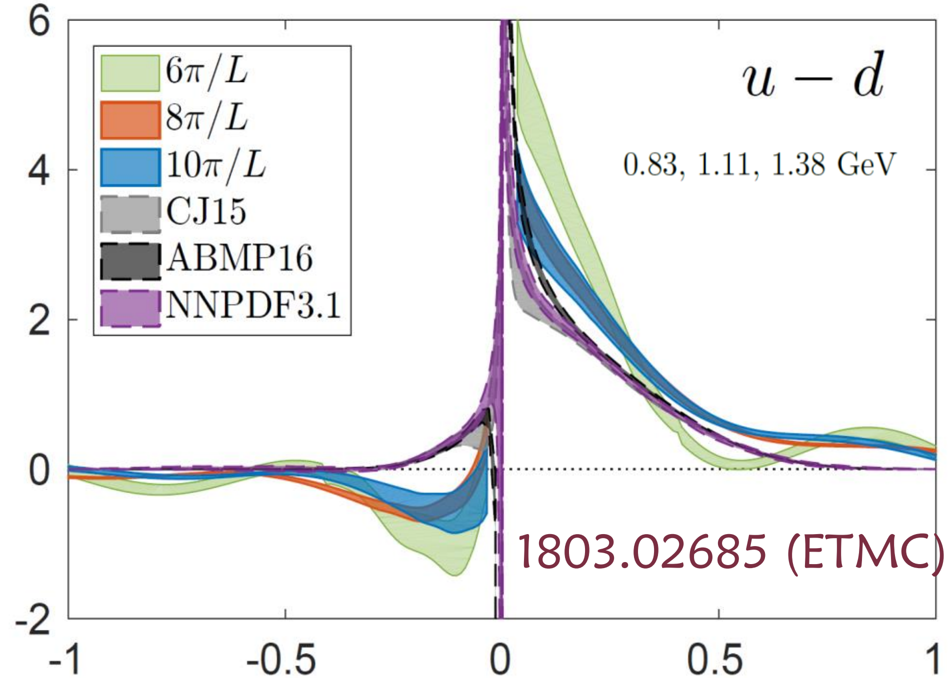
- Improved quasi-distribution definition
- RI/MOM nonperturbative renormalization and corresponding matching to lightcone distribution

Physical Pion Mass Results

- Exciting! Two collaborations' results at physical pion mass
  - Boost $P_z \approx 1.4$ GeV
  - Study of systematics still needed



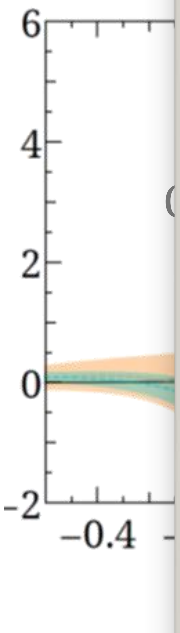


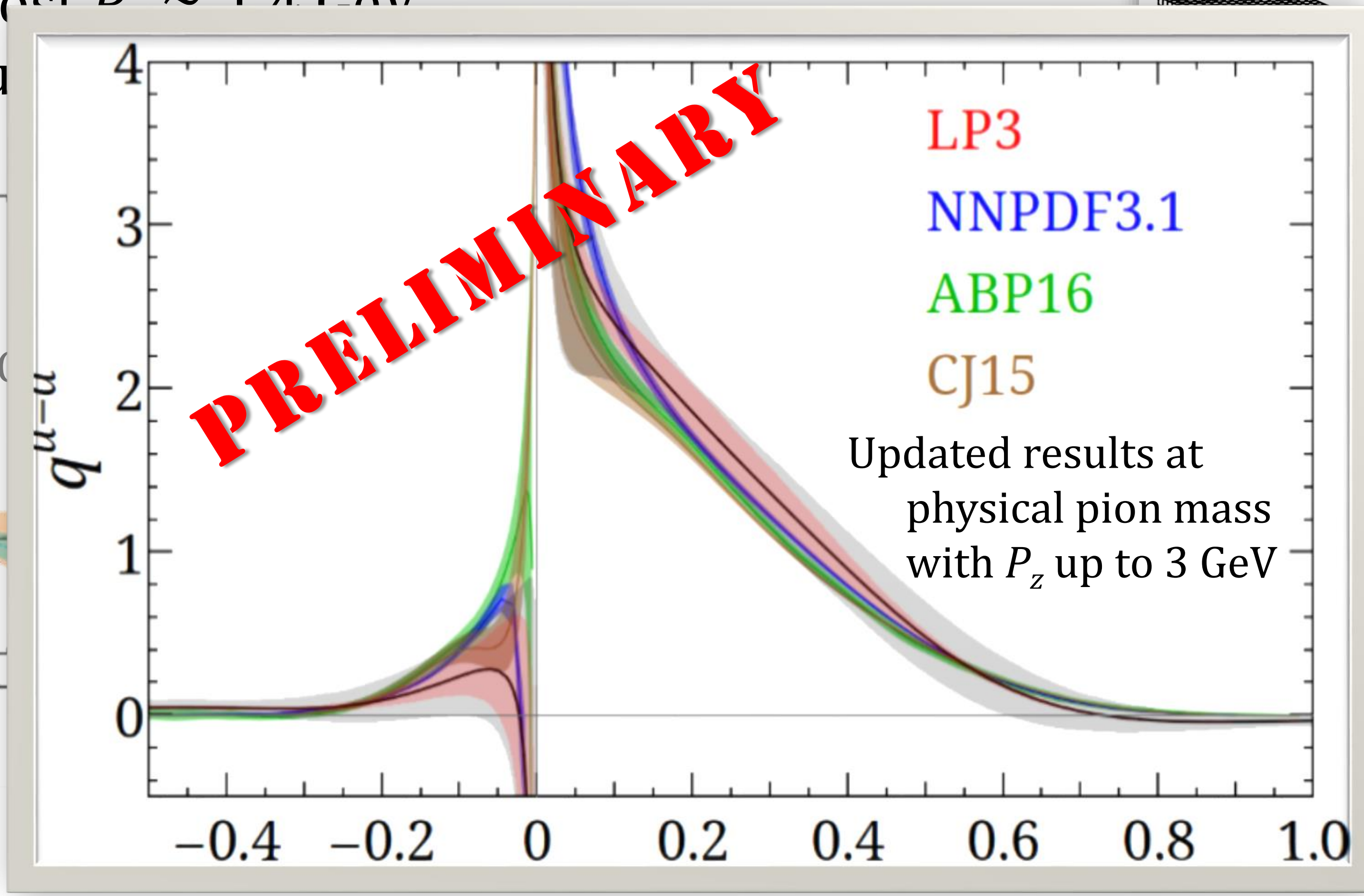


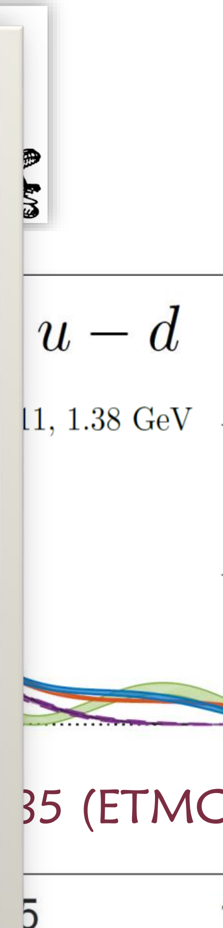
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \sim 1.4$ GeV
- Stu







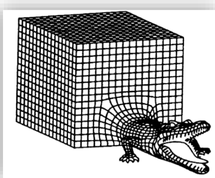
Back to Meson PDA

Meson Distribution Amplitude

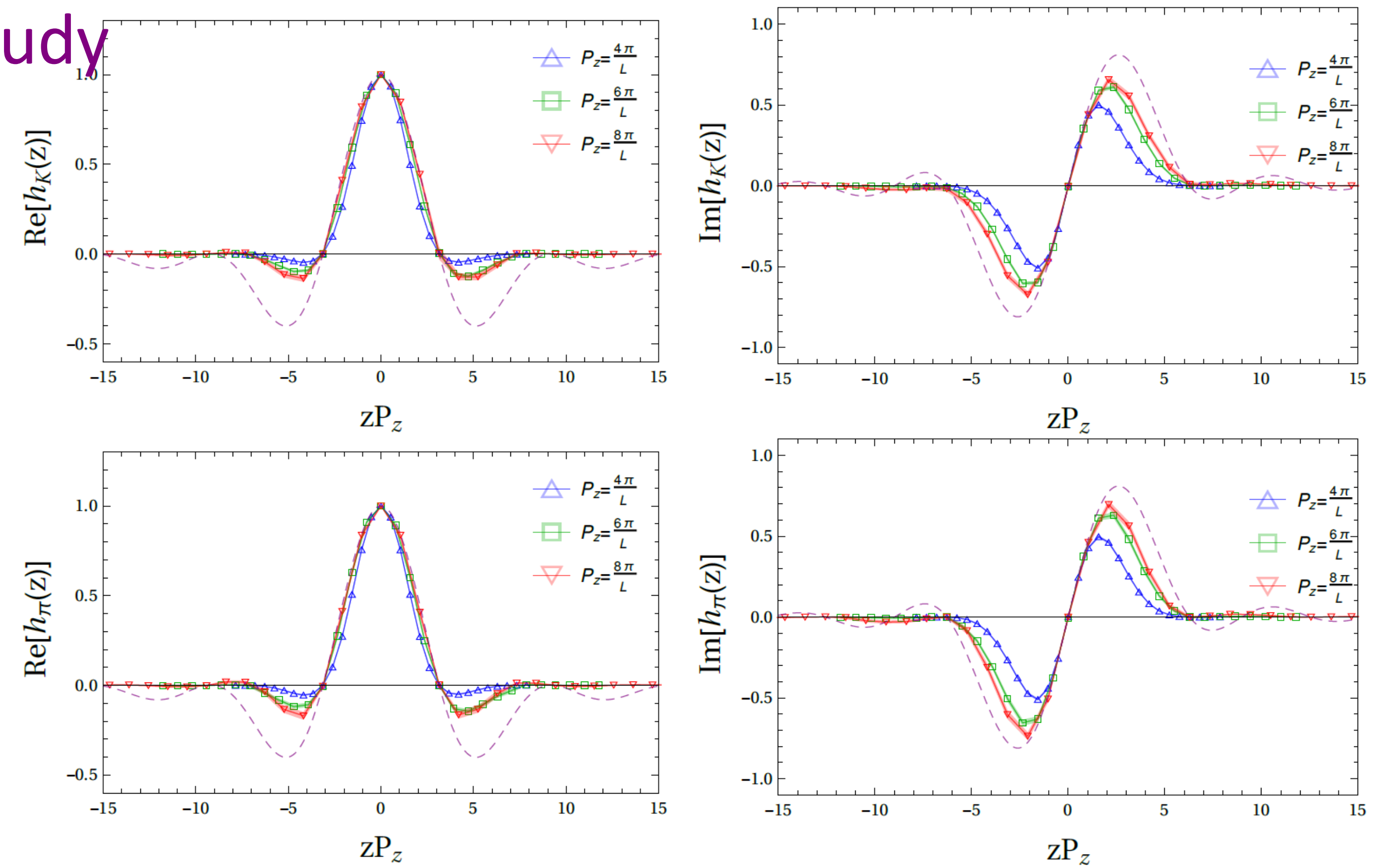
- Supposed to be the simplest structure to calculate for any beyond-moment approach
  - In reality, not so trivial

$P_z \in \{1.72, 2.15, 2.58\}$ GeV

- Exploratory study
  - $M_\pi \approx 310$ MeV
    $a \approx 0.12$ fm
    $(M_\pi L \approx 4.5)$

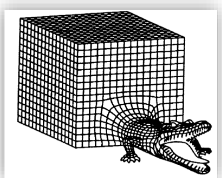


1712.10025 (LP³)



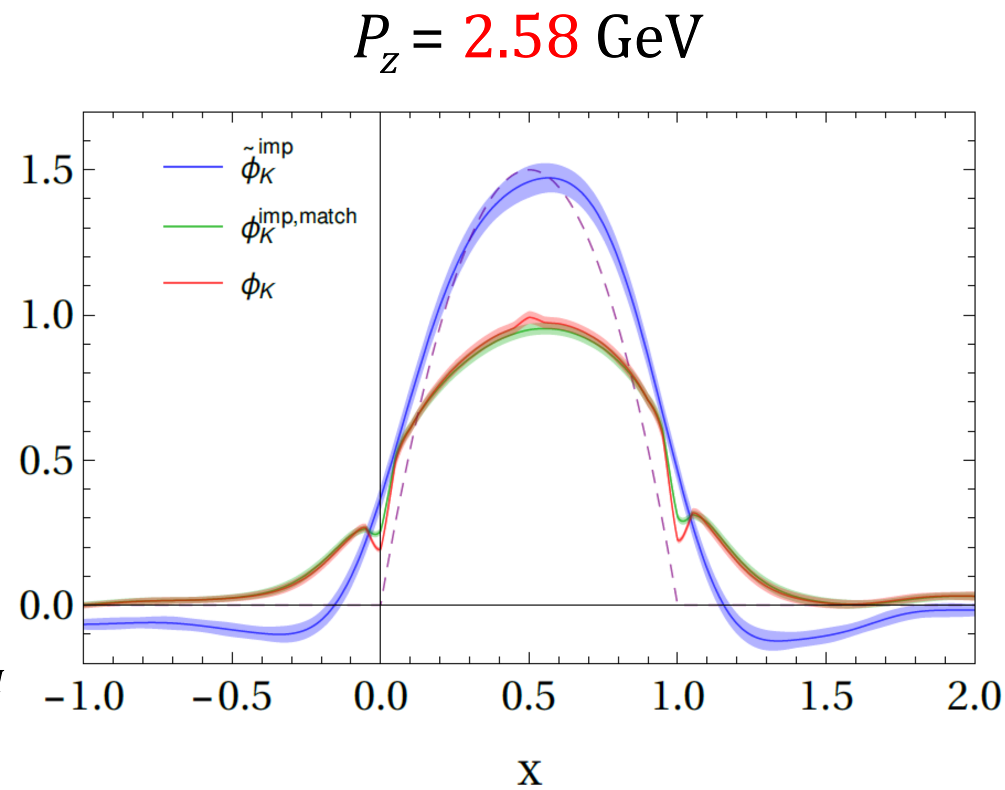
Kaon Distribution Amplitude

- § Supposed to be the simplest structure to calculate for any beyond-moment approach
  - In reality, not so trivial
- § Exploratory study: kaon
  - $M_\pi \approx 310$ MeV
    $a \approx 0.12$ fm
    $(M_\pi L \approx 4.5)$



1712.10025 (LP³)

$$\tilde{\phi}_{\text{imp}}(x, P_z) = \frac{i}{f_\pi}\int \frac{dz}{2\pi} e^{-i(x-1)P_z z - \delta m|z|} h_M$$



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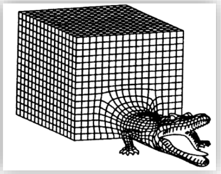

1712.10025 (LP[3])

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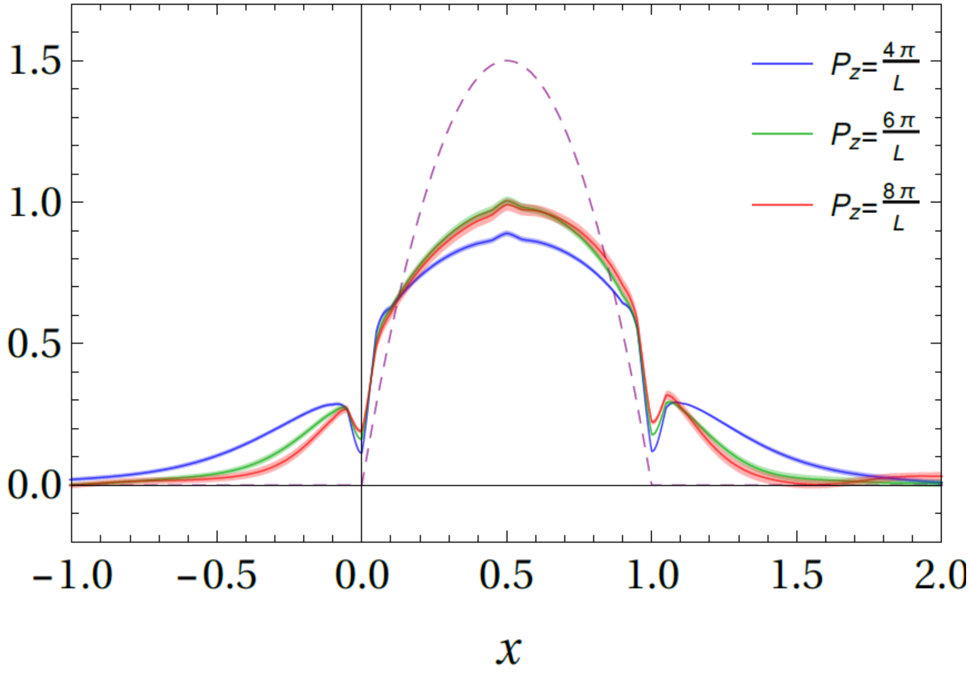
$P_z \in \{1.72, 2.15, 2.58\}$ GeV



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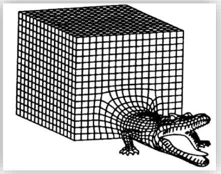
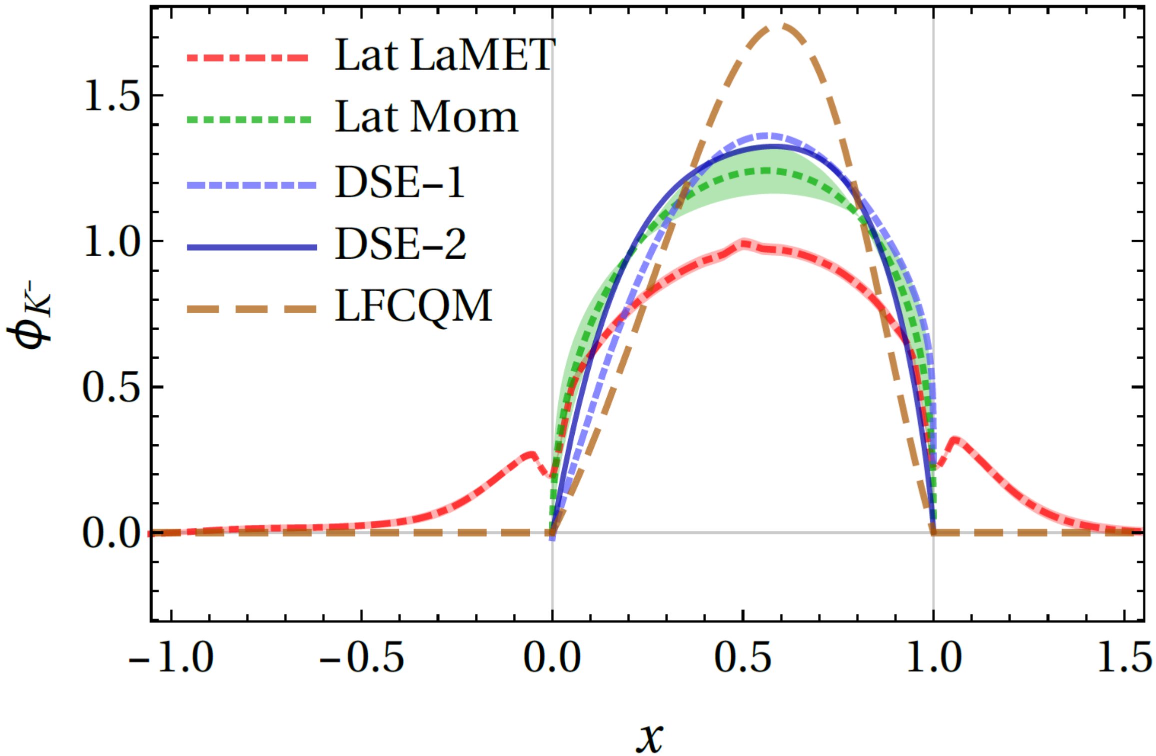


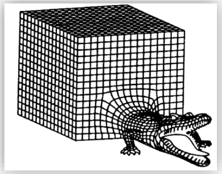


1712.10025 (LP[3])

1503.03656(RQCD)
1301.0324 (DSE)
1512.07260 (LFCQM)

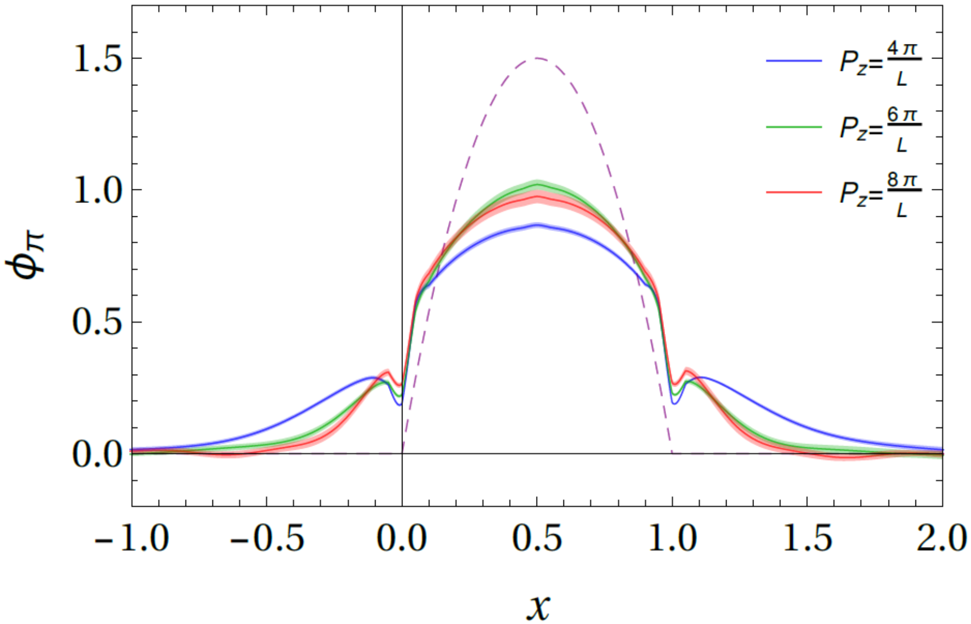
Pion Distribution Amplitude

- Supposed to be the simplest structure to calculate for any beyond-moment approach
  - In reality, not so trivial
- Updated study: pion
  - $M_\pi \approx 310$ MeV
    $a \approx 0.12$ fm
    $(M_\pi L \approx 4.5)$



1712.10025 (LP[3]);
Larger P_z than our first PDA
study in 1702.00008

$P_z \in \{1.72, 2.15, 2.58\}$ GeV



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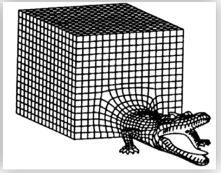
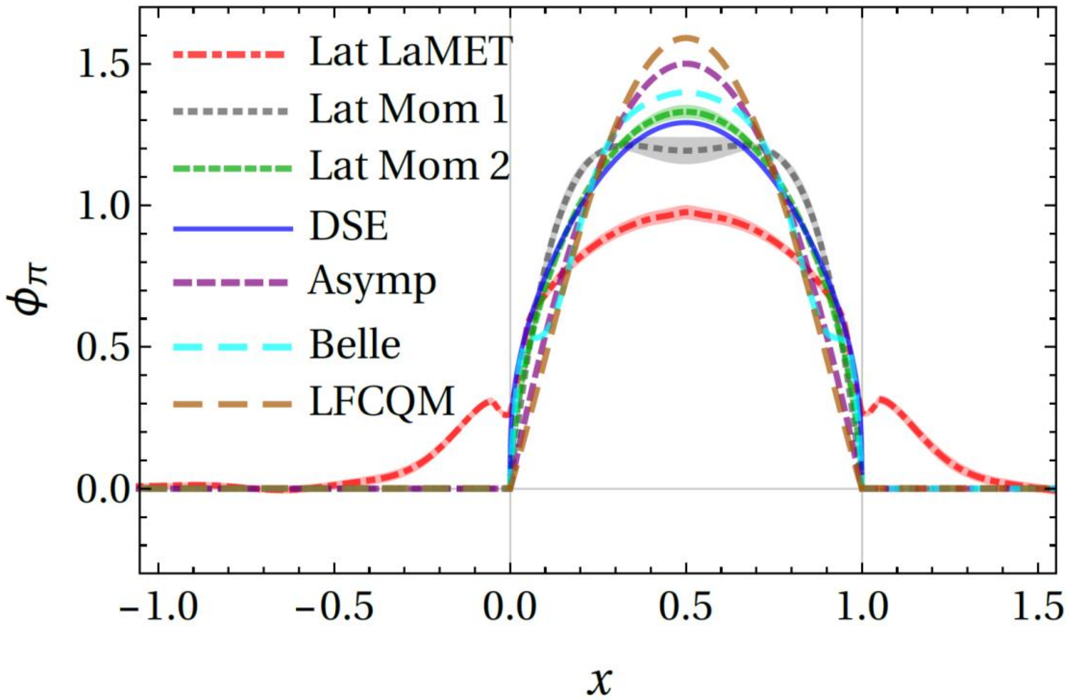


1712.10025 (LP[3])

1503.03656(RQCD)
1301.0324 (DSE)
1206.2968 (Belle)
1512.07260 (LFCQM)

$P_z \in \{1.72, 2.15, 2.58\}$ GeV



Pion Distribution Amplitude

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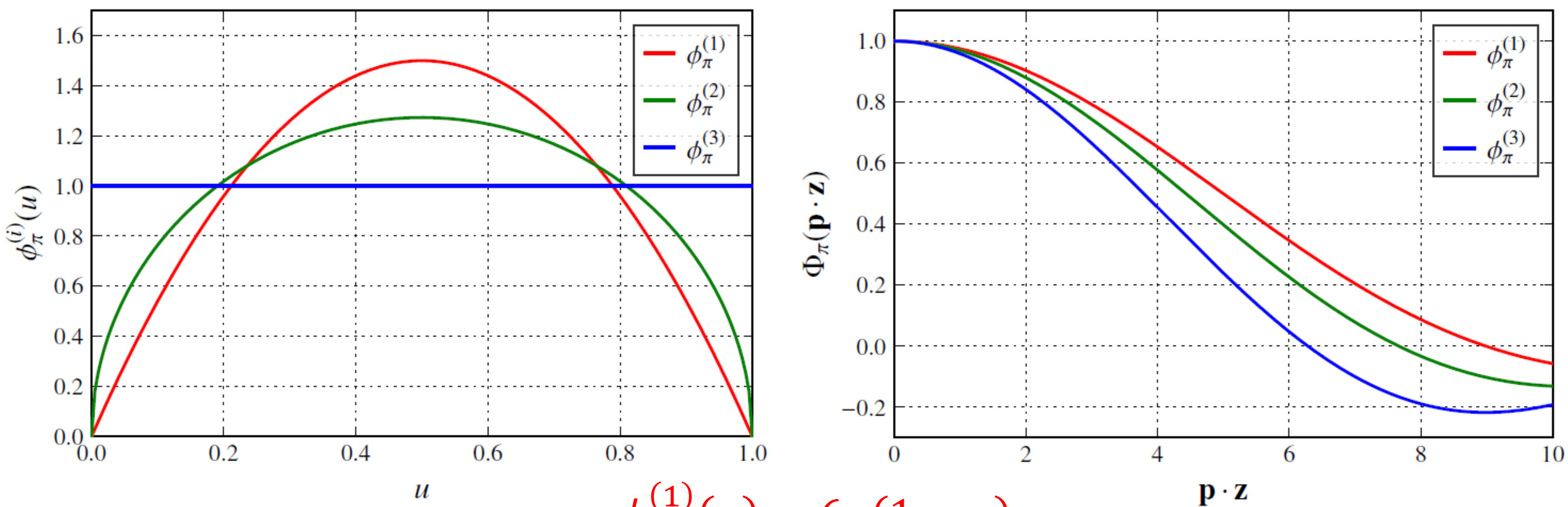
- Position space 1709.04325(RQCD)



$$\phi_\pi^{(1)}(u) = 6u(1-u)$$

$$\phi_\pi^{(2)}(u) = \frac{8}{\pi}\sqrt{u(1-u)}$$

$$\phi_\pi^{(3)} = 1$$

Pion Distribution Amplitude

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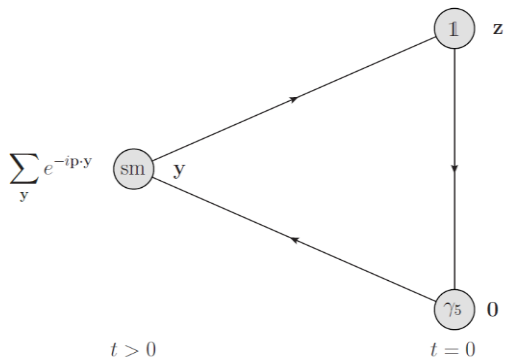
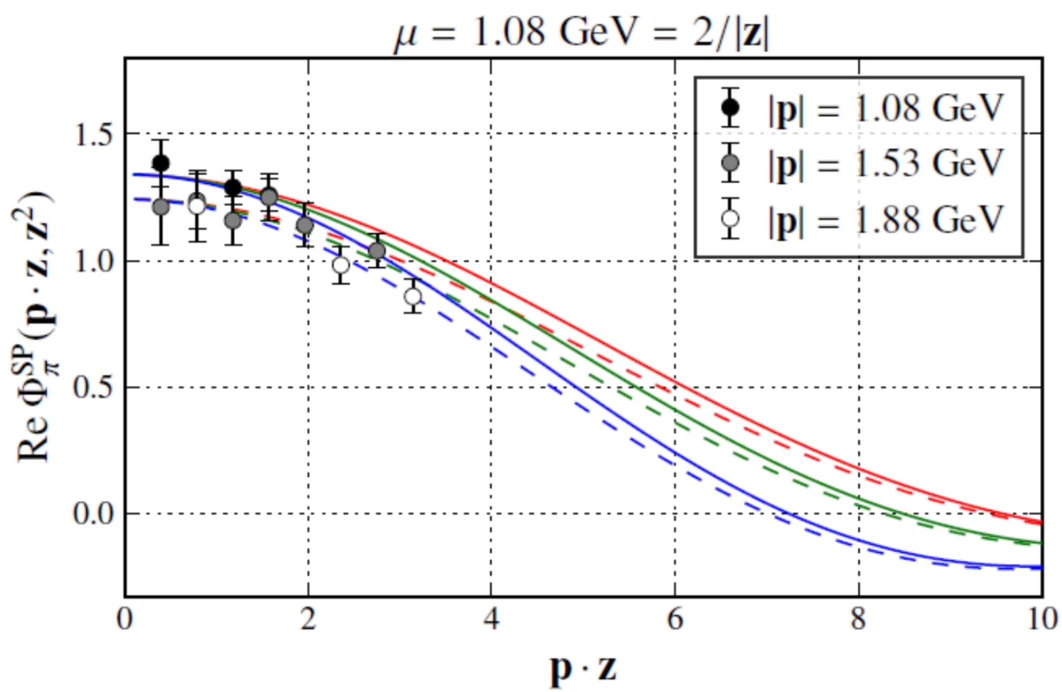
$P_z \in \{1.72, 2.15, 2.58\}$ GeV

- § Exploratory study: Comparison with other approaches

$M_\pi \approx$ **295 MeV**, $a \approx 0.07$ fm
1709.04325(RQCD)





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1712.10025 (LP[3])

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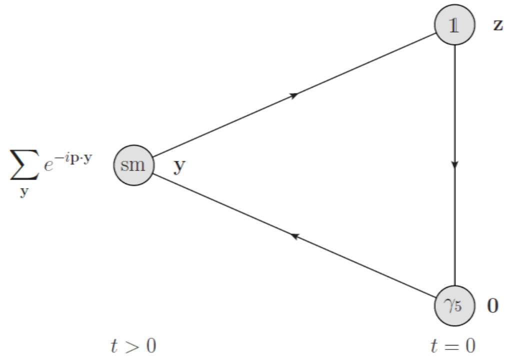
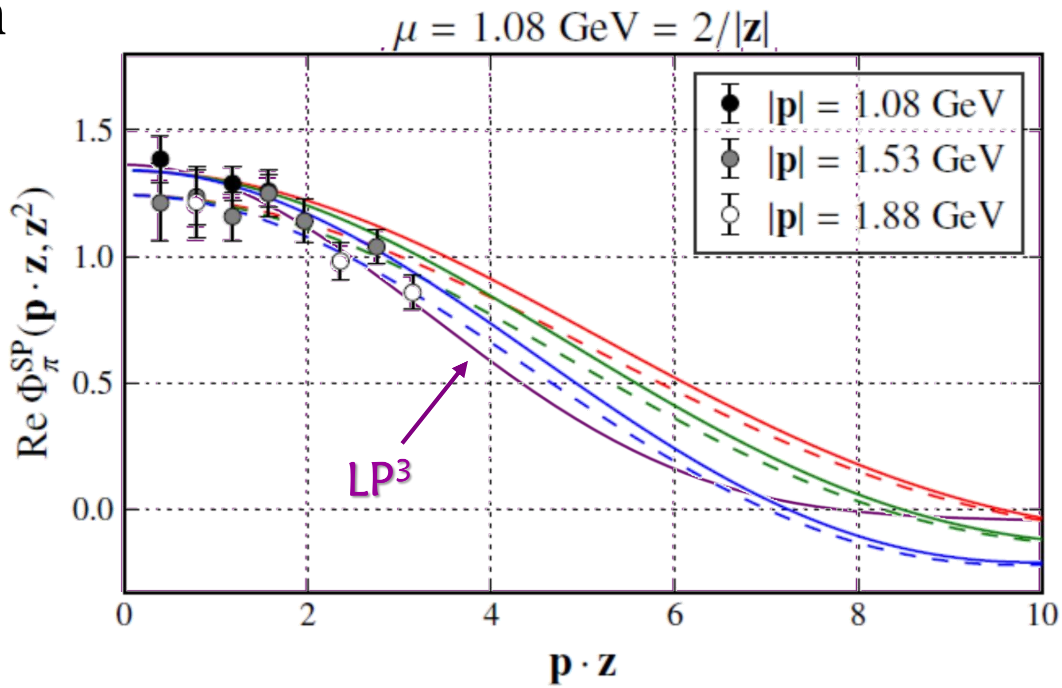
1709.04325(RQCD)



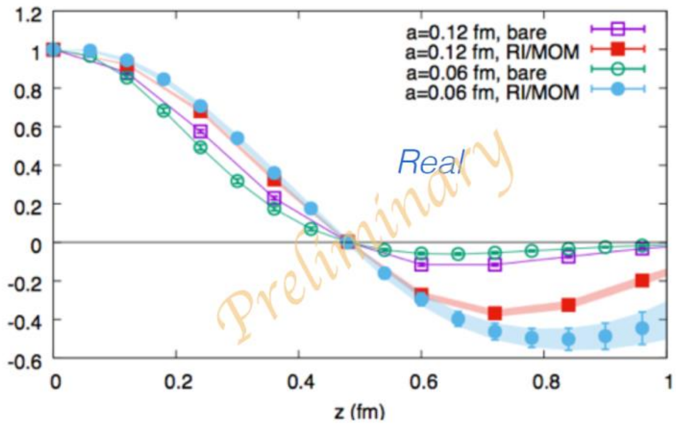


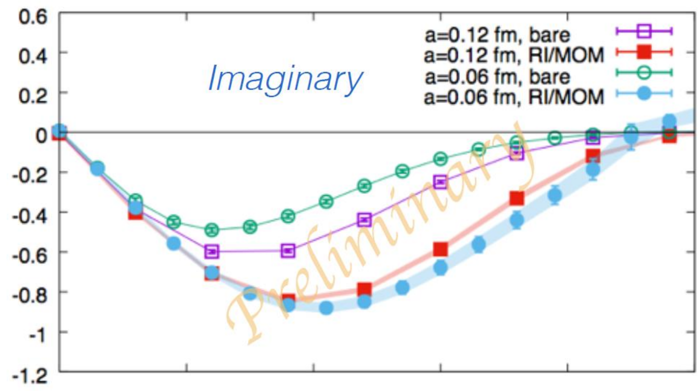
Pion Distribution Amplitude

§ No power divergence observed

$\langle \eta_s(P_z = 1.3\text{GeV}) | \bar{\psi}(z)\gamma_z\gamma_5 U_z(z,0)\psi(0)|0\rangle$

$M_\pi \approx$ **310 MeV**, $a \approx 0.06$ and 0.12 fm





§ Ongoing investigation

- Finite-volume effects
- Truncation systematic in Fourier transformation is more significant in the meson DA case than nucleon PDF
  - Not so simple after all
- Future plan: Larger zP_z to reduce truncation effects
- Smaller lattice spacing needed to avoid lattice artifacts
  - $(aP_z)^n$ errors, $a \approx 0.06$ fm

SU(3) Symmetry Breaking

- Exp: large difference in direct CPV of

$$B^{\pm} \to \pi^0 K^{\pm} \text{ and } B^0 \to \pi^{\mp} K^{\pm}$$

$A_{CP}(\pi^0 K^{\pm}) = 0.050 \pm 0.025 \quad A_{CP}(\pi^{\mp} K^{\pm}) = -0.098^{+0.012}_{-0.011}$ (HFAG)

$$D^0 \to K^+ K^- \text{ and } D^0 \to \pi^+ \pi^-$$

$A_{CP}(D^0 \to \pi^+\pi^-) = (+0.22 \pm 0.24 \pm 0.11)\%$ and $A_{CP}(D^0 \to K^+K^-) = (-0.24 \pm 0.22 \pm 0.09)\%$ (CDF)

1712.10025 (LP[3])

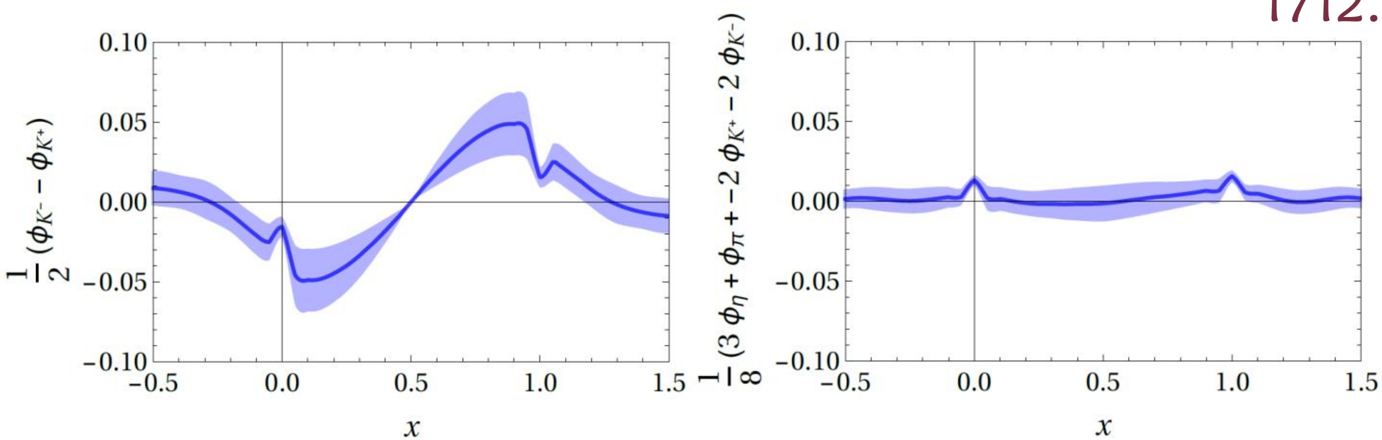


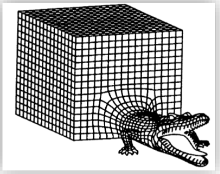
Figure 7. Results for flavor SU(3) symmetry breaking: $\delta_{\mathrm{SU}(3),1} = (\phi_{K^-} - \phi_{K^+})/2$ (left) and $\delta_{\mathrm{SU}(3),2} = (\phi_\pi + 3\phi_\eta - 2\phi_{K^+} - 2\phi_{K^-})/8$ (right) using the corrected distribution of $P_z = 8\pi/L$. Our results support the ChPT [4] prediction $\delta_{\mathrm{SU}(3),1} > \delta_{\mathrm{SU}(3),2}$.

Pion PDF

Pion PDF

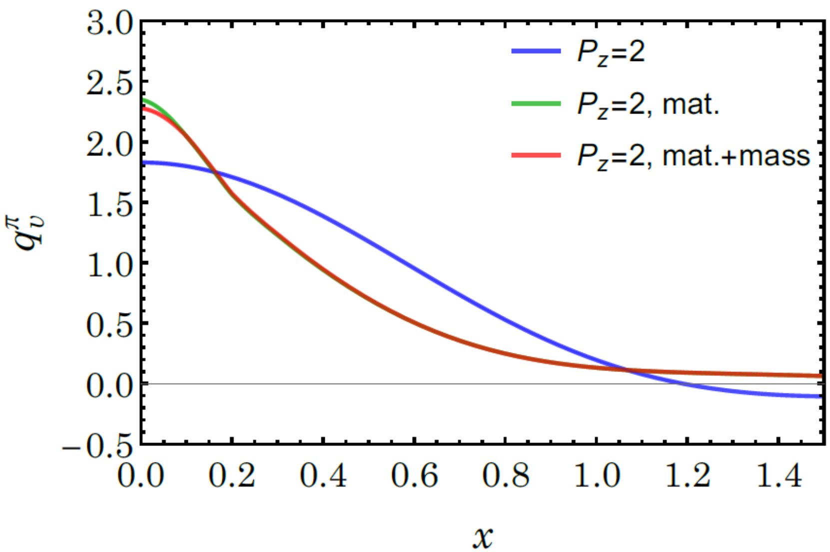
- Not trivial to calculate in reality either
- The first lattice exploratory study

1804.01483 (LP[3])

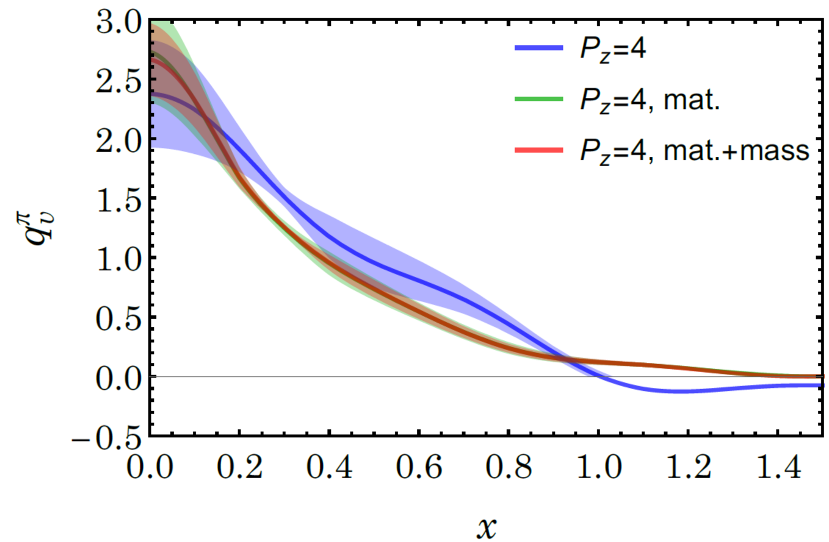


- $M_\pi \approx 310$ MeV, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)

$P_z = 0.86$ GeV



$P_z = 1.72$ GeV



Pion PDF

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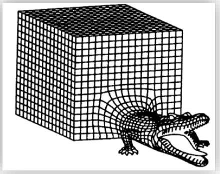
- The first lattice exploratory study

1804.01483 (LP[3])



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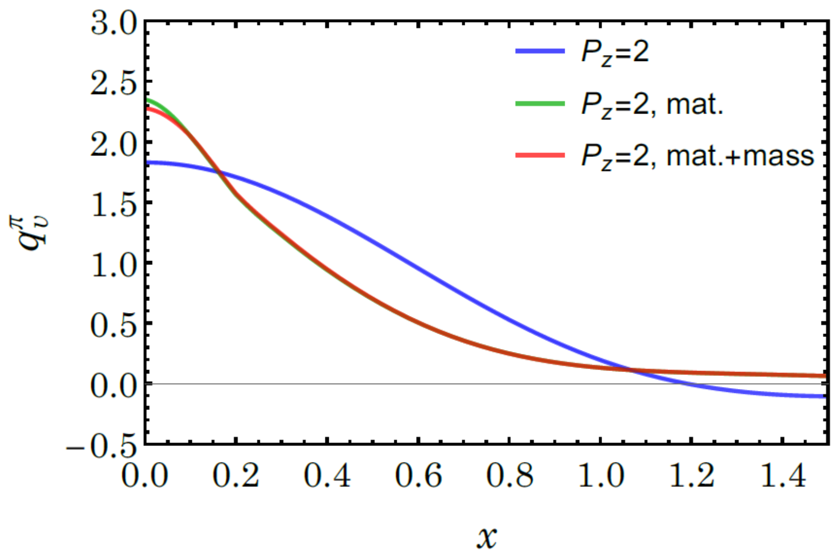
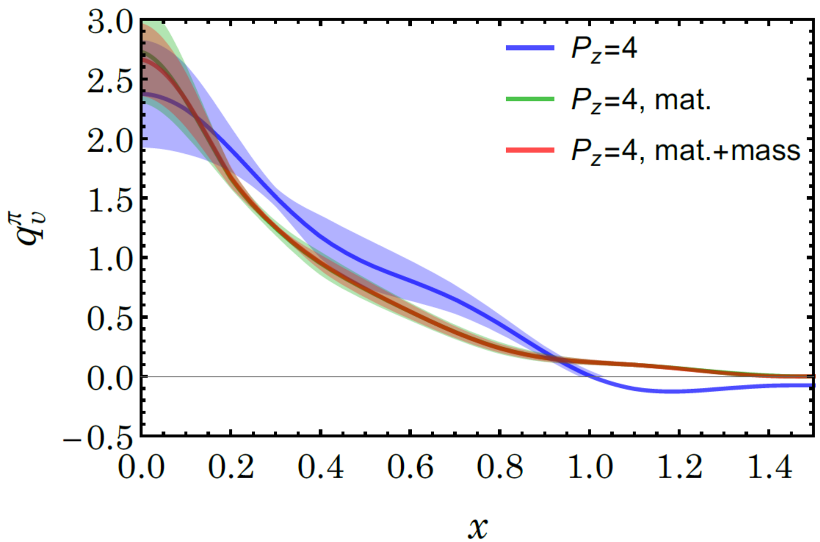


$P_z = 1.72$ GeV



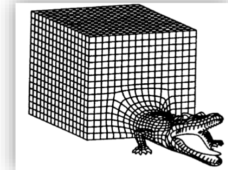
- Study of systematics needed
  - Lattice-spacing, finite-volume, larger P_z, …

Pion PDF

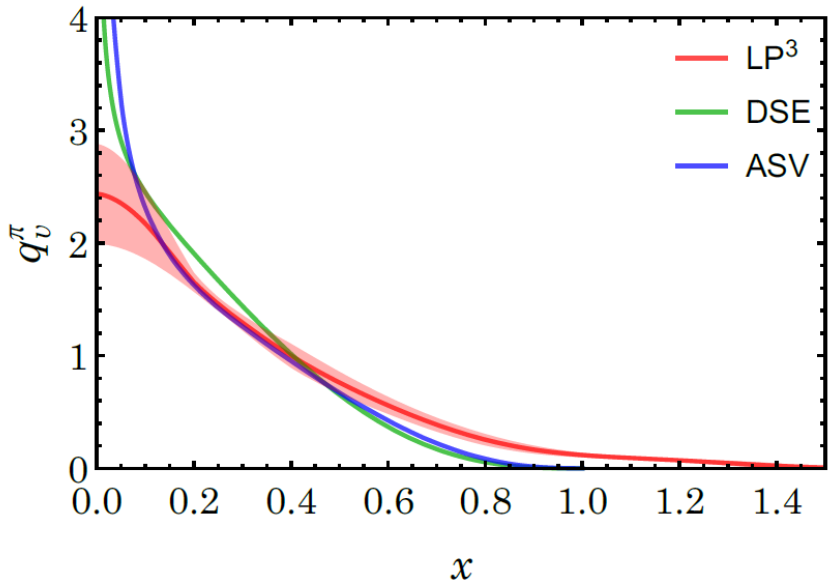
§ Not trivial to calculate in reality either

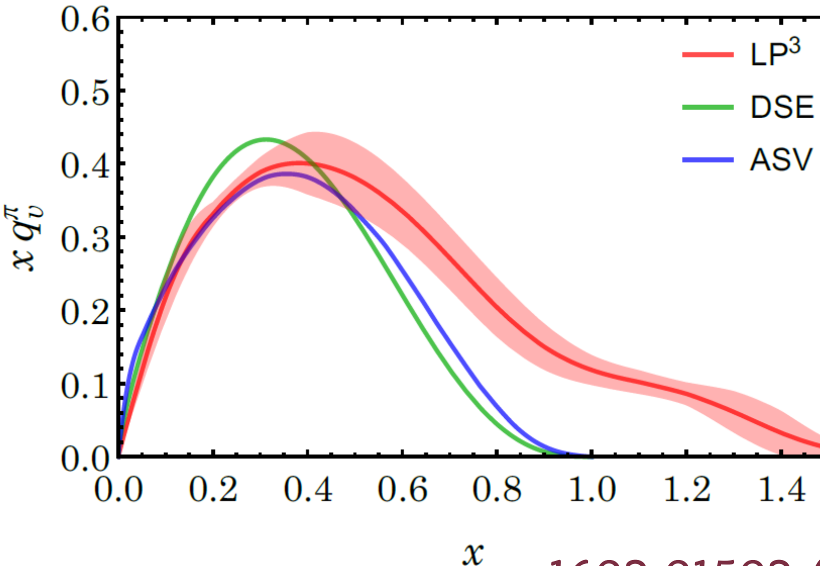
§ The first lattice exploratory study

1804.01483 (LP[3])



- $M_\pi \approx 310$ MeV, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)





1602.01502 (DSE)
1009.2481(ASV)

§ Study of systematics needed

- Lattice-spacing, finite-volume, larger P_z, …

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

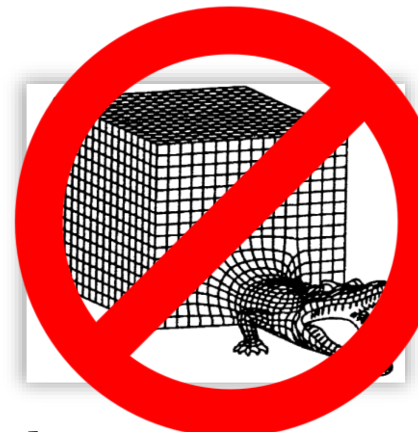
They carry secret plans to build the most powerful

Summary & Outlook

Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full x-distribution

- Most importantly, this can be done with today's computer
- Nucleon PDF seems reasonable
- First look into the meson PDA and pion PDF



§ More study of systematic uncertainty is needed for meson structures

- Finite-volume, larger momentum boost, finer lattice-spacing ensembles, higher-order matching…

Challenge = Opportunity