Complementarity of experimental and lattice QCD data on pion parton distributions

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EIC Meson SF Meeting 2/25/2022

Jefferson Lab Angular Momentum Collaboration

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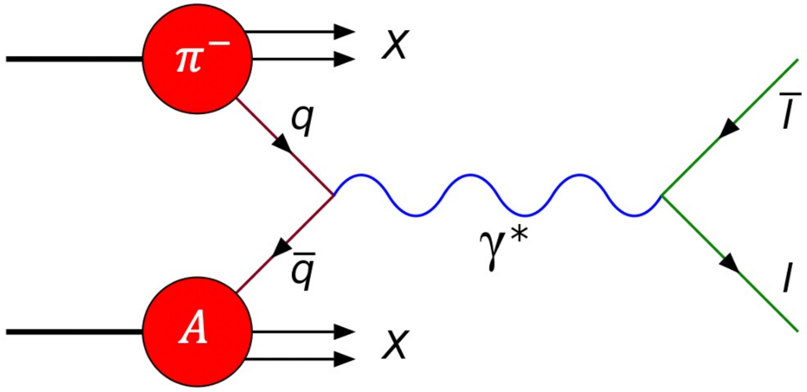
Exploring the Nature of Matter

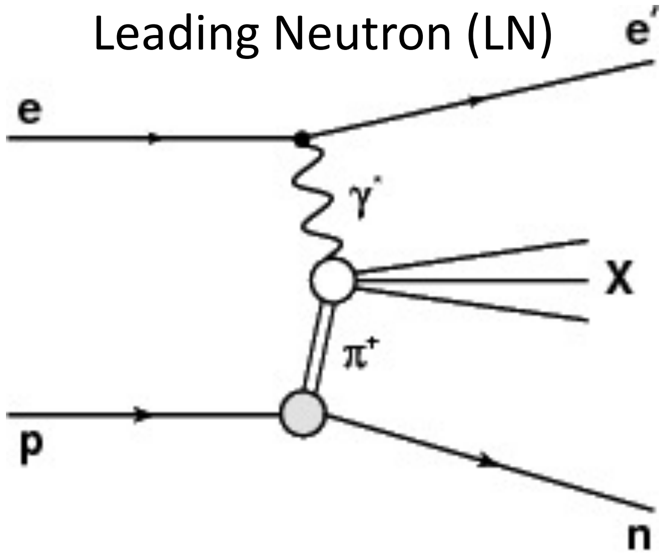
Background and Motivation

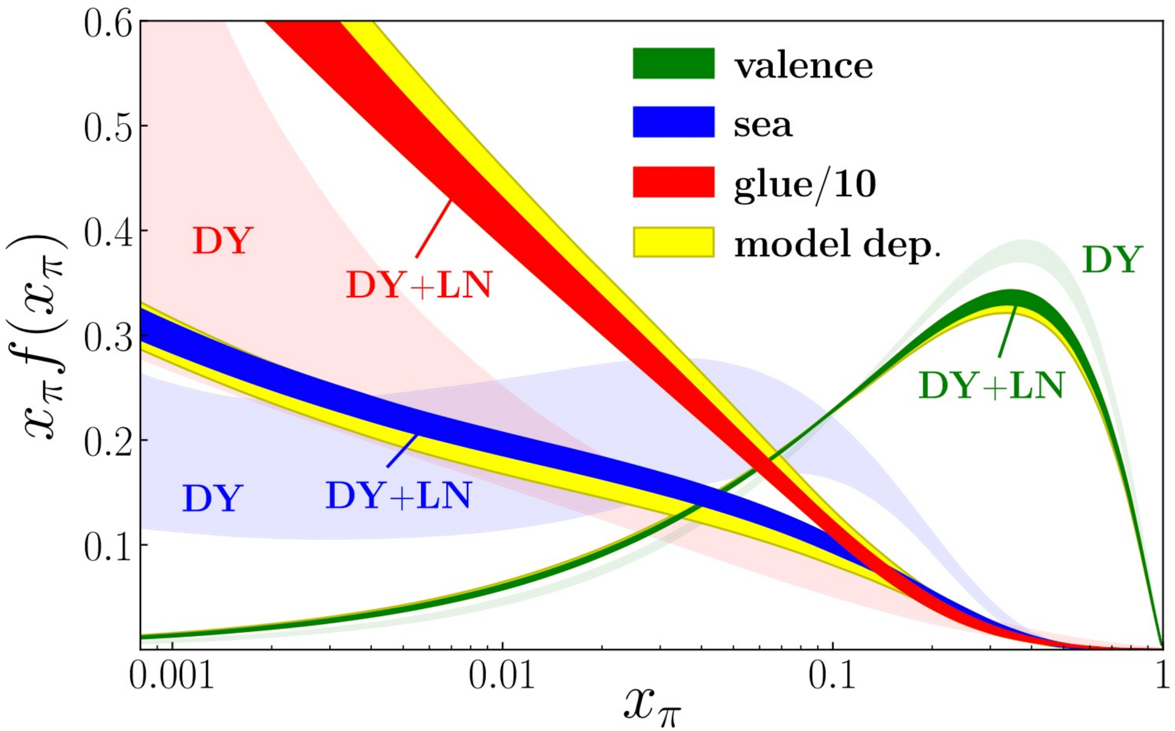
Experiments to probe pion structure

Drell-Yan (DY)



Leading Neutron (LN)





PHYSICAL REVIEW LETTERS **121**, 152001 (2018)

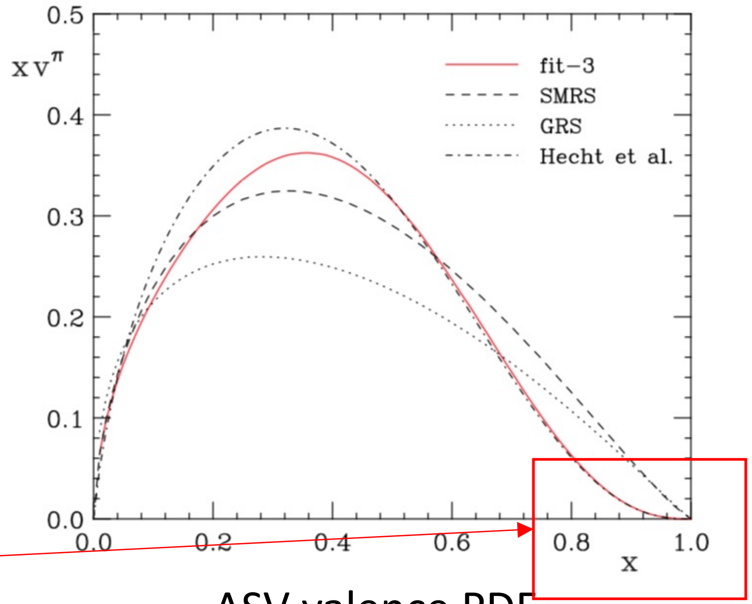
Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

Large-x_π behavior

- Generally, the parametrization lends a behavior as $x_\pi \rightarrow 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a fixed order analysis, we find $\beta \approx 1$
- Debate whether $\beta = 1$ or $\beta = 2$
- Connection with pQCD expects $\beta = 2$
- Aicher, Schaefer, Vogelsang (ASV) found $\boxed{\beta = 2}$ with threshold resummation



ASV valence PDF

Phys. Rev. Lett. **105**, 114023 (2011).

Include Threshold Resummation in DY

- ASV analysis got $(1-x)^2$ behavior using threshold resummation, while all NLO analyses follow $(1-x)$

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

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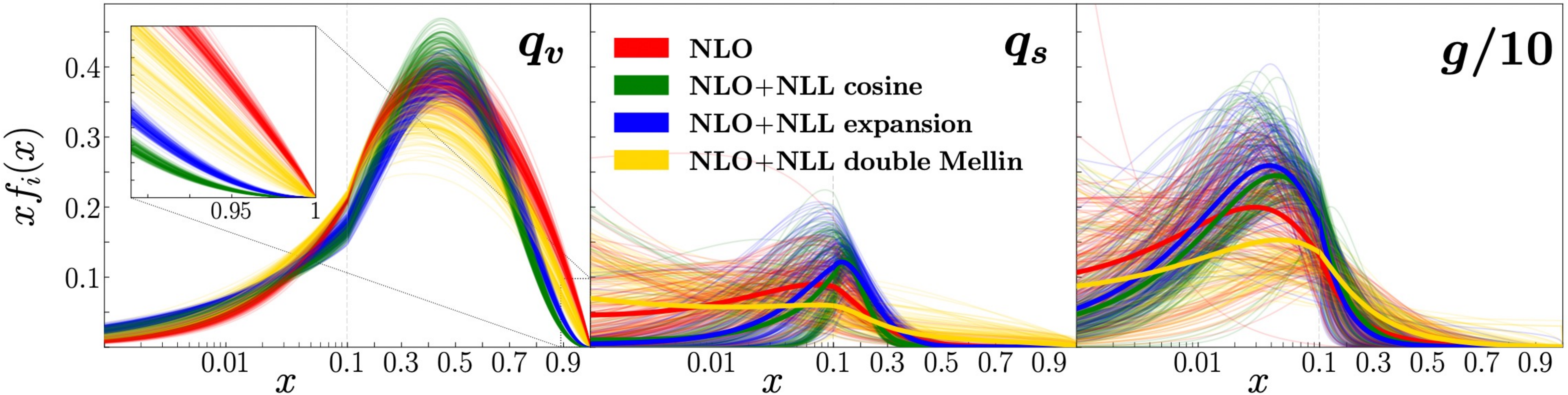
(Received 18 August 2021; revised 22 October 2021; accepted 26 October 2021; published 29 November 2021)

Resulting PDFs



- Large x behavior of q_v **highly sensitive** to method of resummation

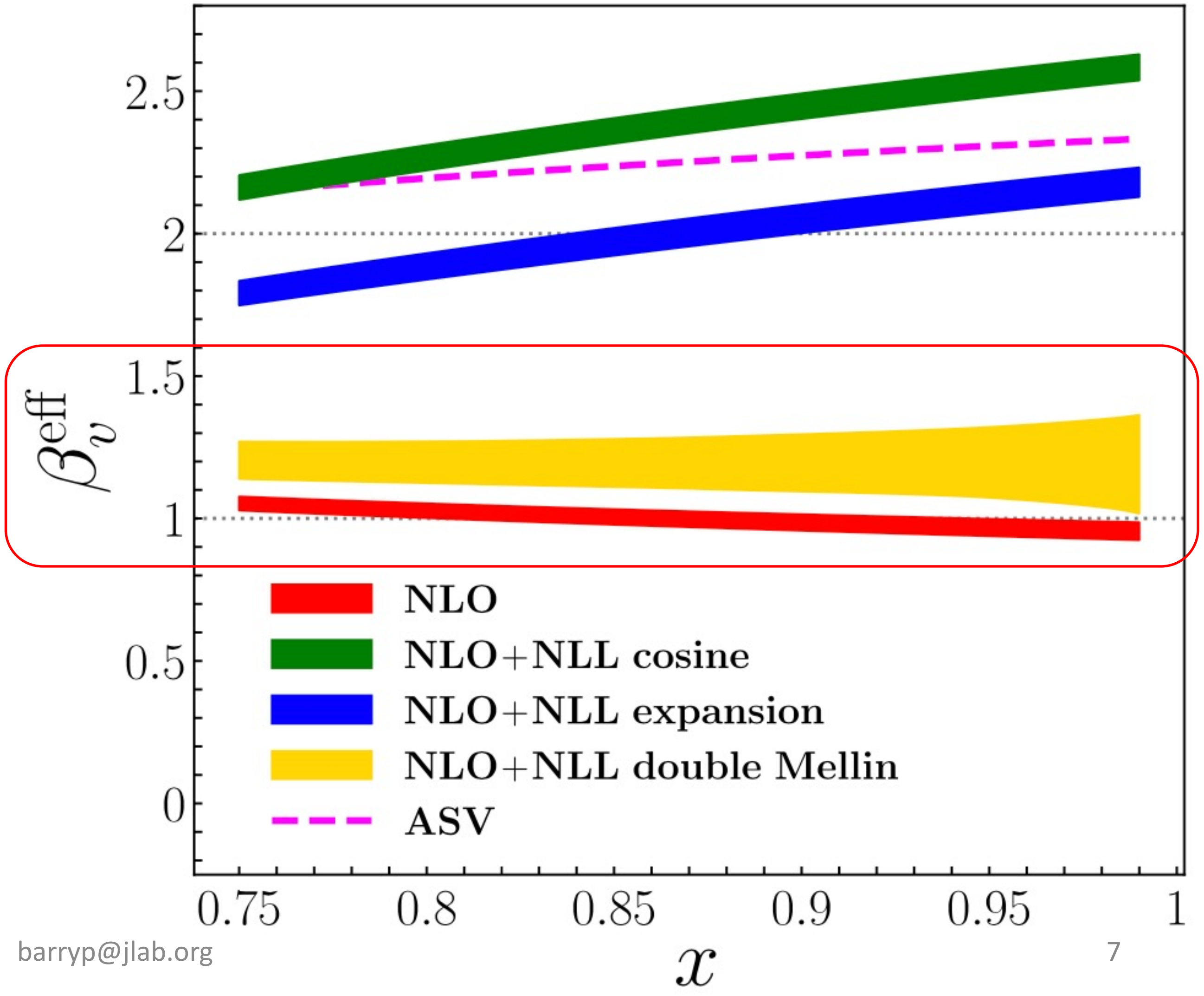
Current data do not distinguish between NLO and NLO+NLL

Method	χ^2/npts
NLO	0.85
NLO+NLL cosine	1.29
NLO+NLL expansion	0.95
NLO+NLL double Mellin	0.80

Slightly disfavored

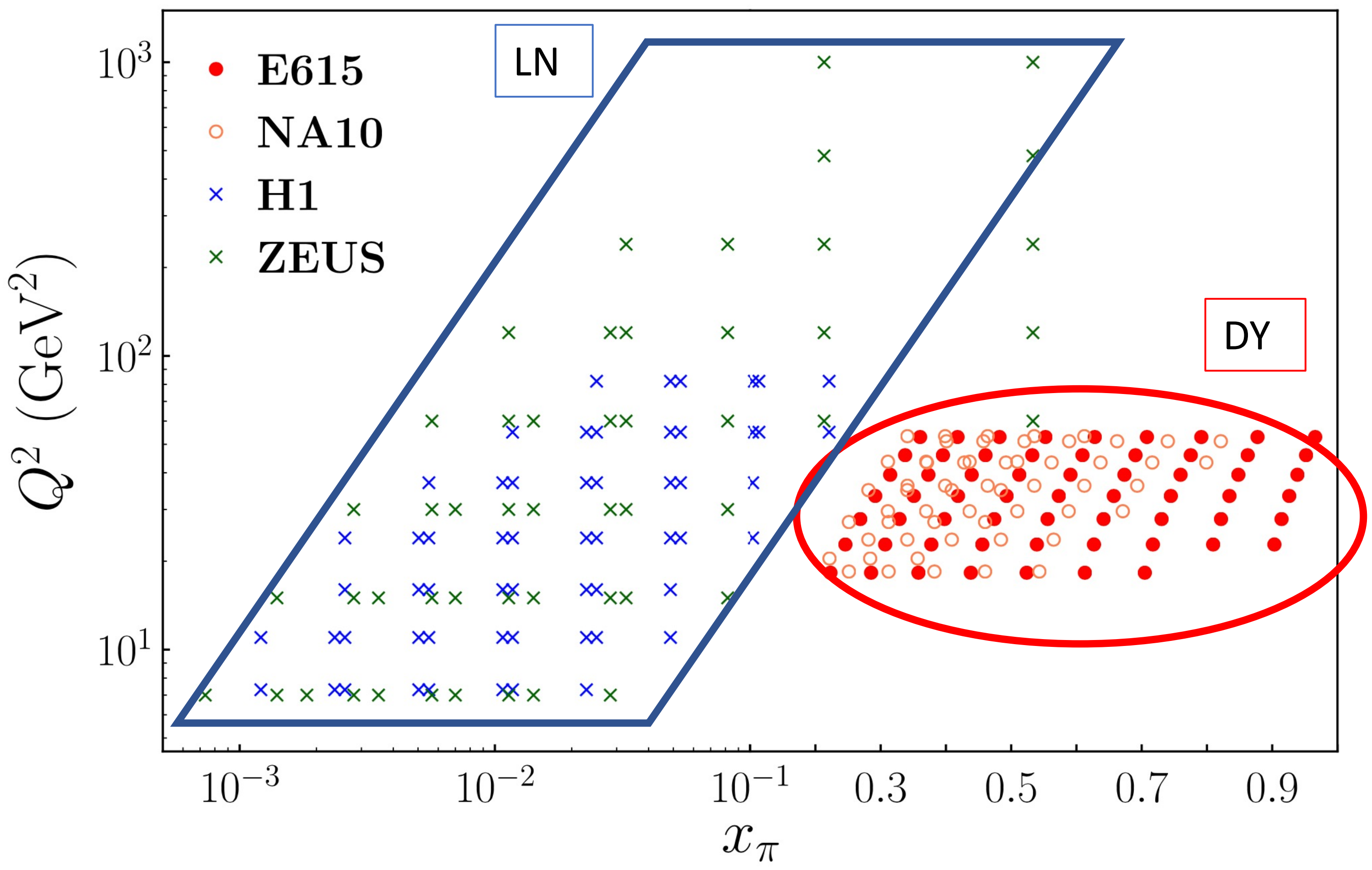
What we believe to be theoretically better

- Take more seriously the red and yellow
- $\beta_v^{\text{eff}} \sim 1 - 1.2$, much closer to 1 than 2



Datasets -- Kinematics

- Not much kinematic overlap
- Need more observables!



Lattice QCD observables

How to relate PDFs with lattice observables?

- Make use of good lattice cross sections and appropriate matching coefficients

$$\boxed{\begin{aligned}\sigma_{n/h}(\omega,\xi^2) &\equiv \langle h(p)|T\{\mathcal{O}_n(\xi)\}|h(p)\rangle \\ &= \sum_i f_{i/h}(x,\mu^2)\otimes K_{n/i}(x\omega,\xi^2,\mu^2) \\ &\qquad + O(\xi^2\Lambda^2_{\mathrm{QCD}}),\end{aligned}}$$

- Structure just like experimental cross sections – good for global analysis

Reduced Ioffe time pseudo-distribution (Rp-ITD)

- Lorentz-invariant Ioffe time pseudo-distribution:

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p | \boxed{\bar{\psi}(0)} \gamma^0 \boxed{\mathcal{W}(z;0)} \boxed{\psi(z)} | p \rangle$$

Bare quark and antiquark fields

Gauge link

"Ioffe time"

$$\boxed{\nu = p \cdot z}$$

$$z = (0,0,0, z_3)$$

Observable is the *reduced* Ioffe time pseudo-distribution (Rp-ITD)

$$\boxed{\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}}$$

Ratio cancels UV divergences

Fitting the Data and Systematic Corrections

Valence quark distribution in pion

Wilson coefficients for matching

$$\mathrm{Re}\,\mathfrak{M}(\nu, z^2) = \int_0^1 \mathrm{d}x\, \boxed{q_v(x, \mu_{\mathrm{lat}})}\, \boxed{\mathcal{C}^{\mathrm{Rp-ITD}}\left(x\nu, z^2, \mu_{\mathrm{lat}}\right)} + \boxed{z^2 B_1(\nu)} + \boxed{\frac{a}{|z|} P_1(\nu)} + \boxed{e^{-m_\pi (L-z)} F_1(\nu)} + \boxed{\ldots}$$

Integration lower bound is 0

Other potential systematic corrections the data is not sensitive to

<u>Systematic corrections to parametrize</u>

- $z^2 B_1(\nu)$: power corrections
- $\frac{a}{|z|} P_1(\nu)$: lattice spacing errors
- $e^{-m_\pi (L-z)} F_1(\nu)$: finite volume corrections

Integration limits

- Notice the integral over x goes $0 \to 1$
- However, the integral for experimental values goes from $x_{\min} \to 1$
- Sensitivity to threshold corrections comes at large x where the PDF is sharply falling, entire integration range of x is not sensitive to threshold regions
- Do not perform threshold resummation for lattice observables

Current-current (CC) correlators

- Pair of vector and axial-vector currents

$$\Sigma^{\alpha\beta}_{VA}(z,p) = z^4 Z_V Z_A \langle p| \, [\bar{\psi}\gamma^\alpha\psi](z) \, [\bar{\psi}\gamma^\beta\gamma^5\psi](0)|p\rangle + (V \leftrightarrow A)$$

- $Z_{V,A}$ are renormalization constants
- Calculate 4-point function as opposed to Rp-ITD (3-point function)

Current-current correlator matching

Valence quark distribution in pion

Wilson coefficients for matching

$$T_1(\nu, z^2) = \int_0^1 \mathrm{d}x \, \boxed{q_v(x, \mu_{\mathrm{lat}})} \, \boxed{\mathcal{C}^{\mathrm{CC}}(x\nu, z^2, \mu_{\mathrm{lat}})} + \boxed{z^2 B_1(\nu)} + \boxed{aR_1(\nu)} + \ldots,$$

Integration lower bound is 0

Very noisy data, fit a subset of the systematics to ensure PDF stability

- $z^2 B_1(\nu)$: power corrections
- $aR_1(\nu)$: discretization corrections

Rp-ITD Analysis Results

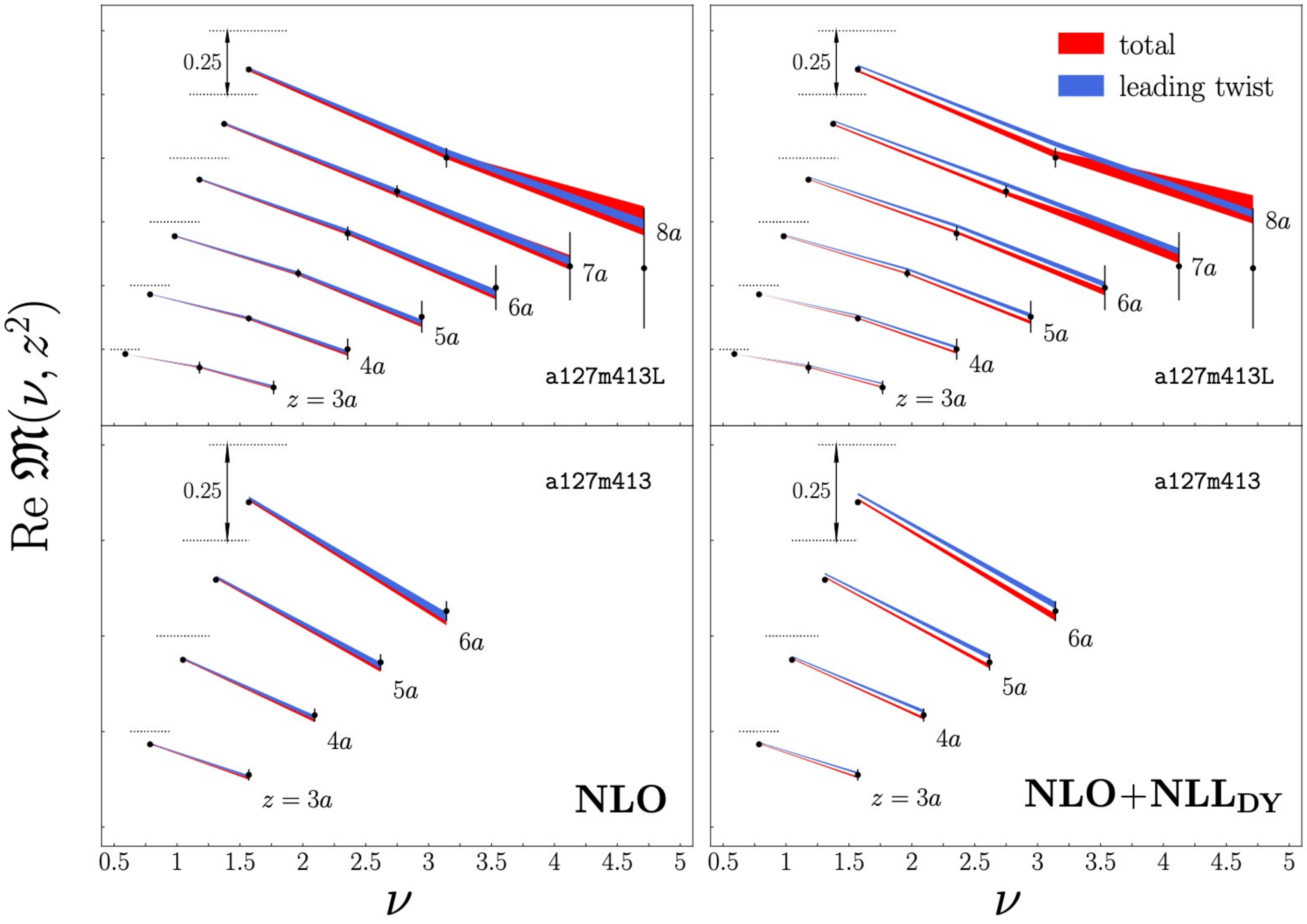
Goodness of fit

- Scenario A: experimental data alone
- Scenario B: experimental + lattice, no systematics
- Scenario C: experimental + lattice, with systematics

			Scenario A		Scenario B		Scenario C	
			NLO	$+\mathrm{NLL}_{\mathrm{DY}}$	NLO	$+\mathrm{NLL}_{\mathrm{DY}}$	NLO	$+\mathrm{NLL}_{\mathrm{DY}}$
Process	Experiment	N_{dat}	$\overline{\chi}^2$		$\overline{\chi}^2$		$\overline{\chi}^2$	
DY	E615	61	0.84	0.82	0.84	0.82	0.83	0.82
	NA10 (194 GeV)	36	0.53	0.53	0.52	0.54	0.53	0.55
	NA10 (286 GeV)	20	0.80	0.81	0.78	0.79	0.79	0.87
LN	H1	58	0.37	0.35	0.38	0.39	0.37	0.37
	ZEUS	50	1.49	1.48	1.60	1.69	1.59	1.60
Rp-ITD	a127m413L	18	–	–	1.05	1.06	1.05	1.06
	a127m413	8	–	–	1.97	2.63	1.15	1.42
Total		**251**	**0.81**	**0.80**	**0.89**	**0.92**	**0.86**	**0.87**

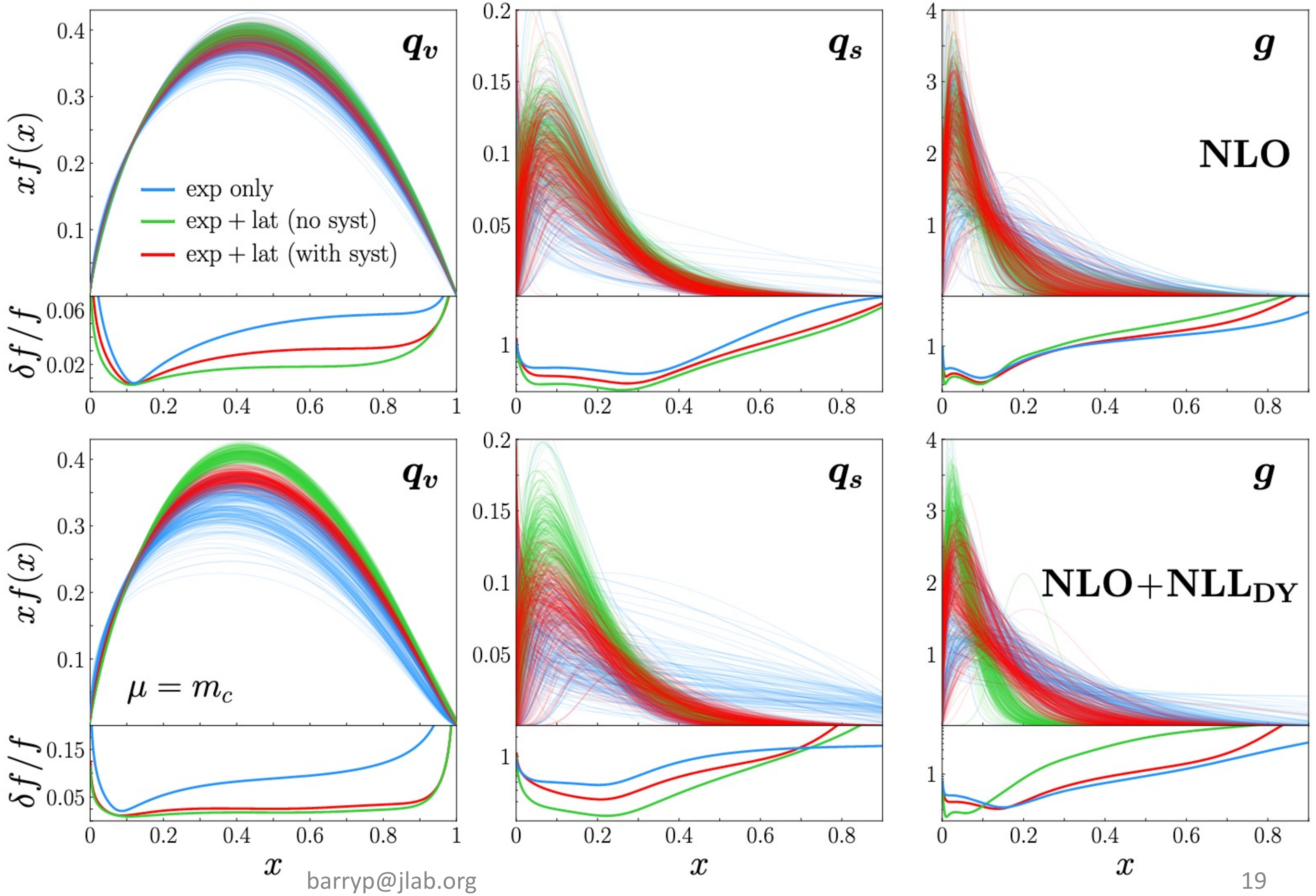
Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



Resulting PDFs

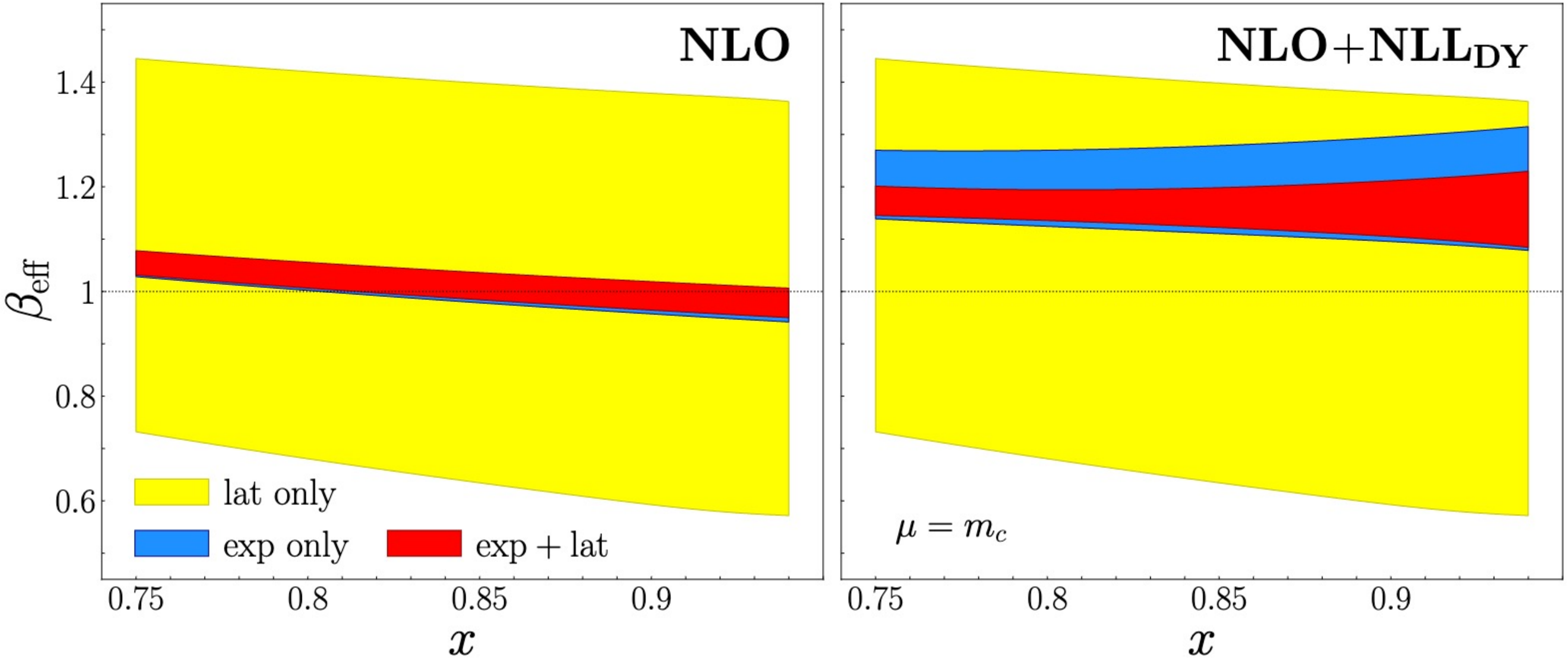
- PDFs and relative uncertainties
- Including lattice reduces uncertainties
- NLO+NLL$_{DY}$ changes a lot – unstable under new data



Effective β from $(1-x)^{\beta_{\text{eff}}}$

$$\beta_{\text{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}$$

Calculations from QCD do not predict $\beta_{\text{eff}} = 2$

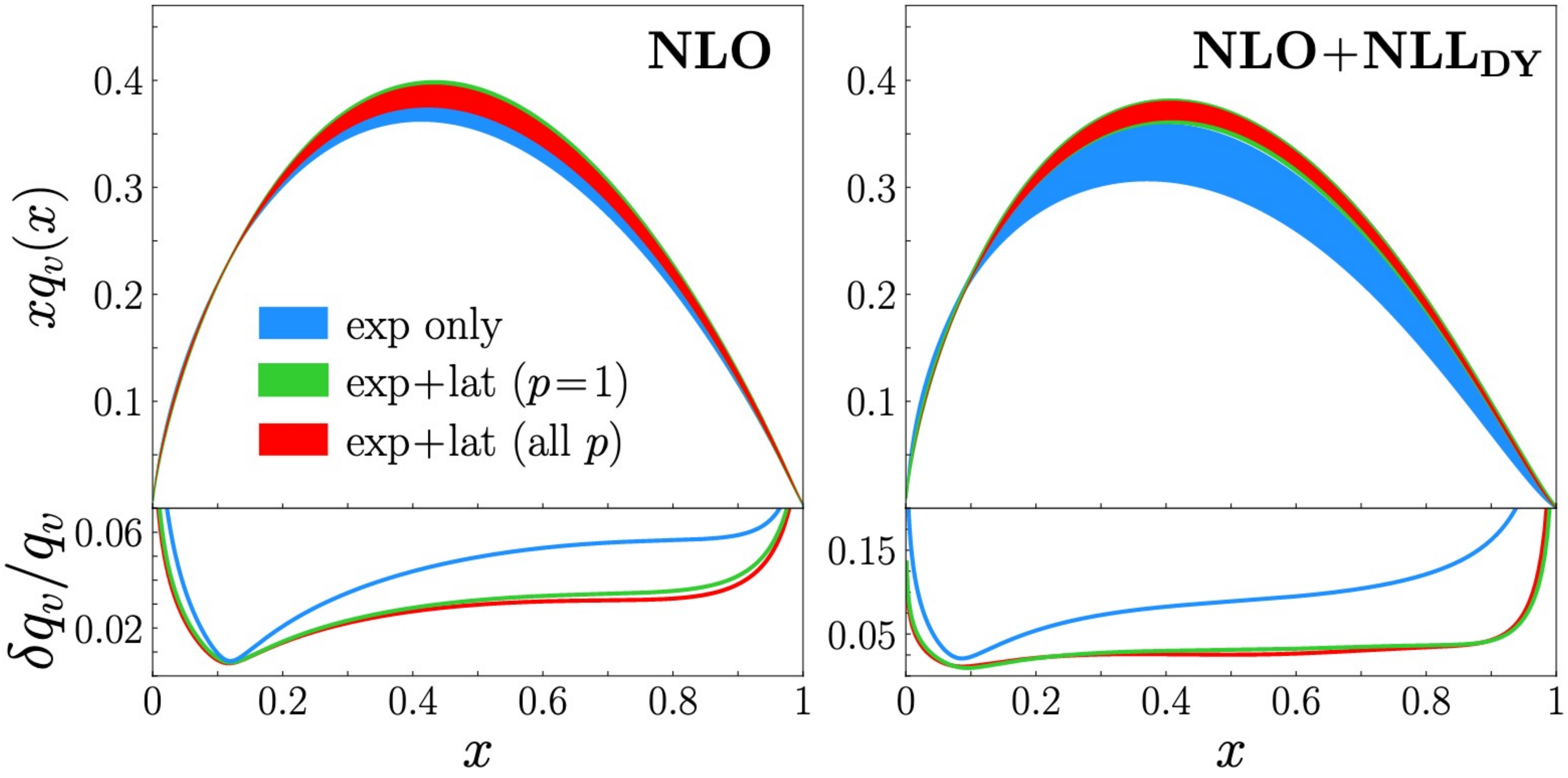


Fitting only the $p = 1$ points

- Most precise points, but not large range in Ioffe time
- Through analysis containing *only* lattice data, would not be sufficient to get a large x description of PDF

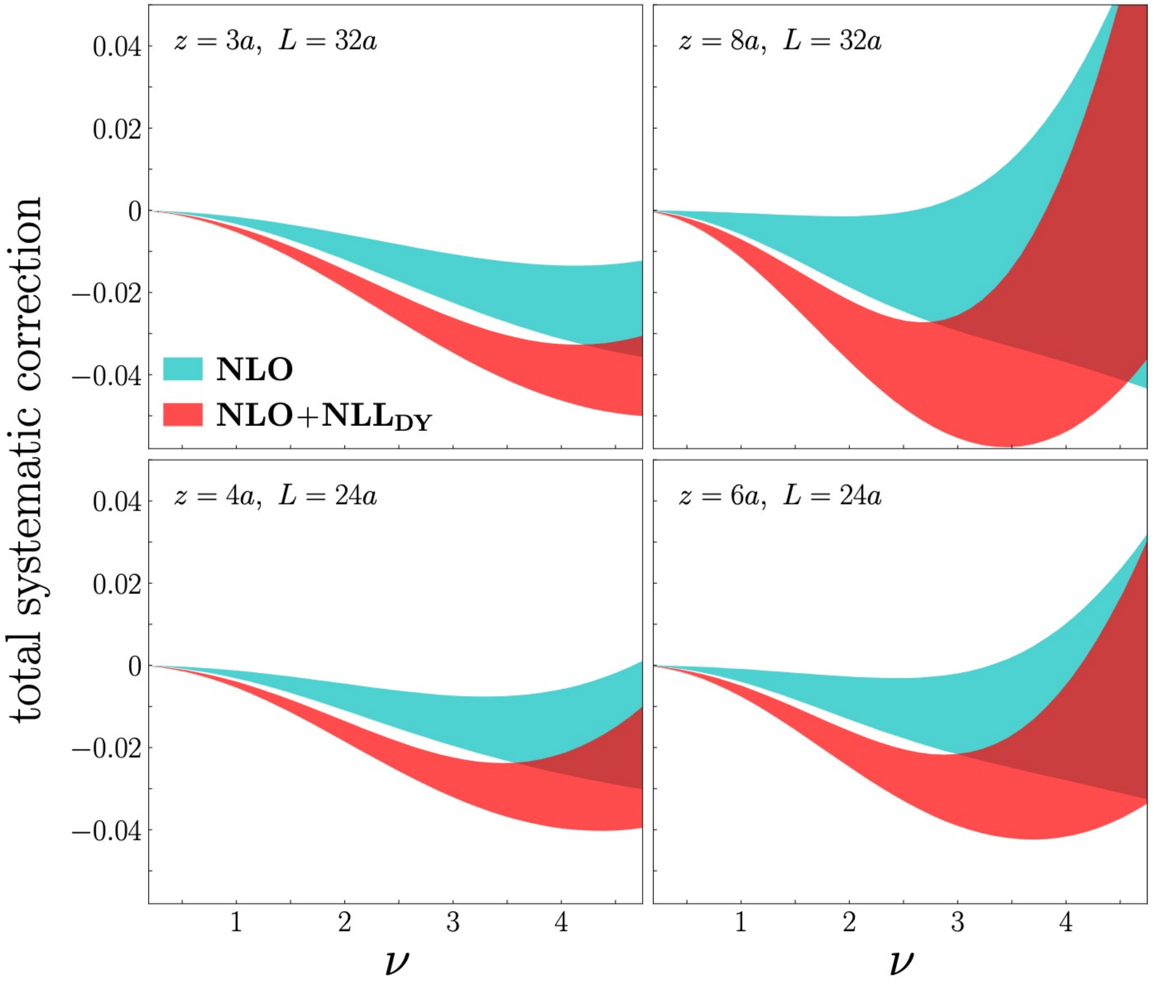
Resulting low-momentum PDFs

- These momentum points do entire job!



Quantifying Systematic Corrections

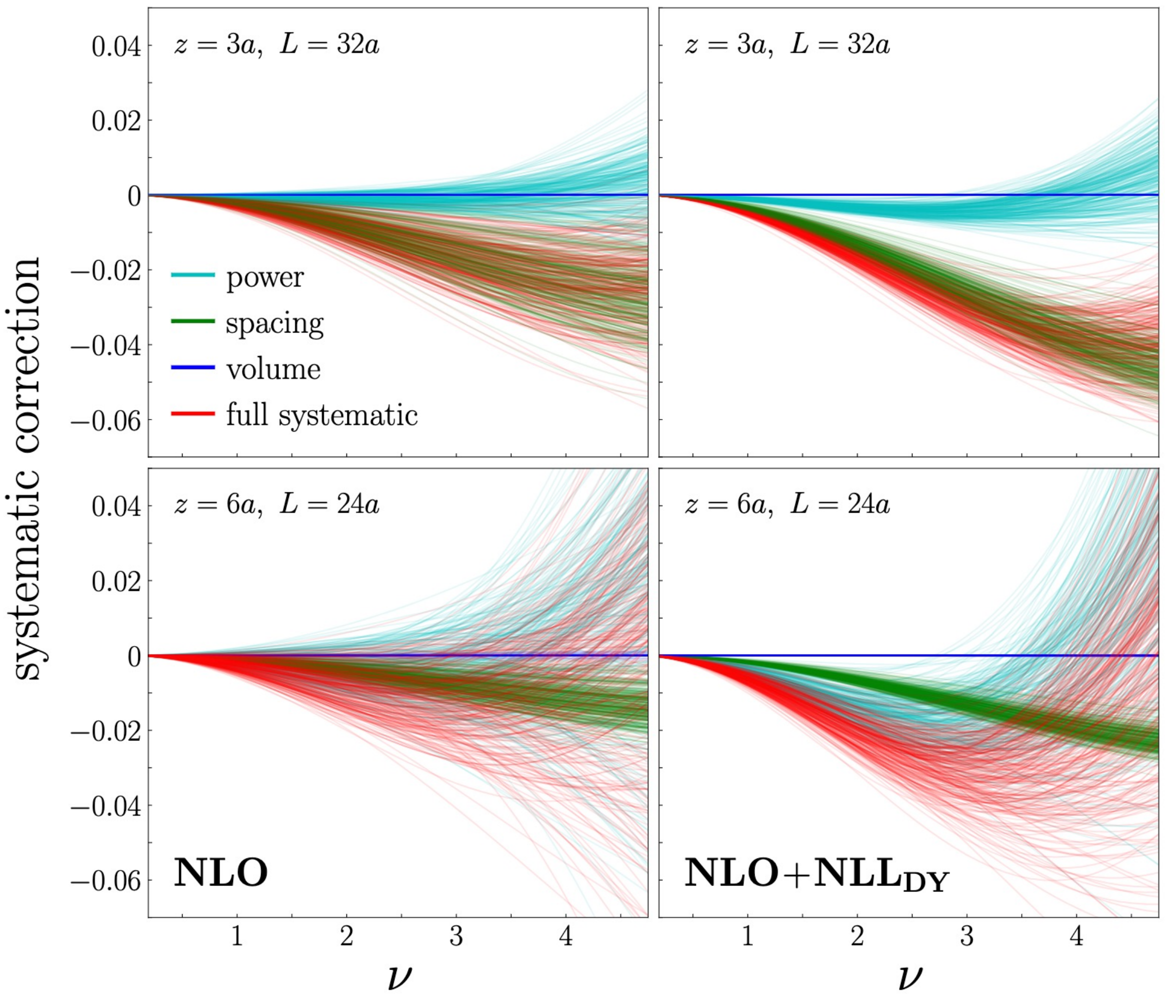
- Do systematic corrections agree within the DY theories?
  - No!
- Have a min/max estimation for the systematic corrections



Quantifying individual systematics

- Breaking down by the 3 systematics

$$z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu)$$



- Dominance of power or spacing corrections depends on z
- Finite volume corrections don't matter

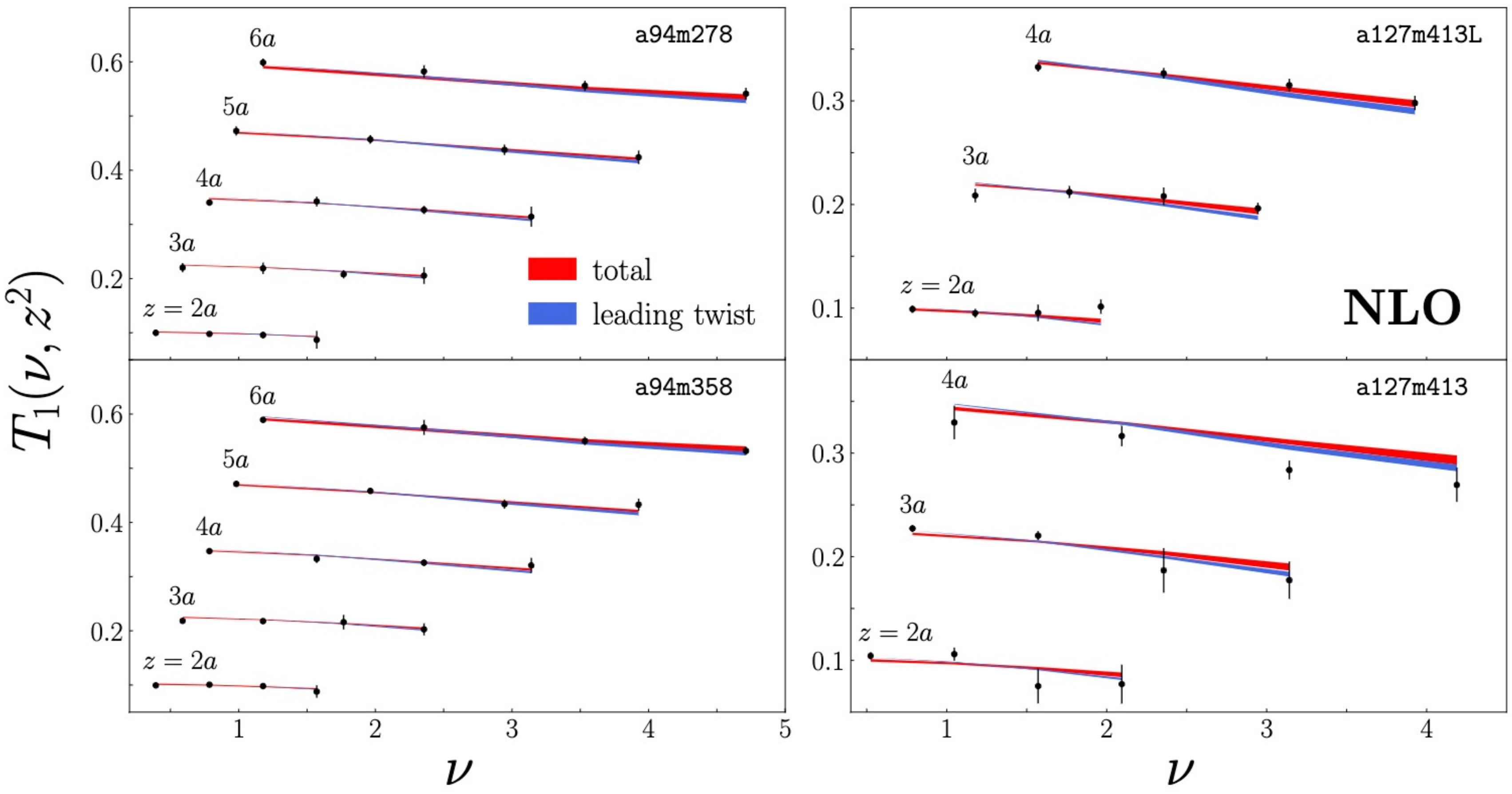
CC correlator analysis results

Resulting χ^2

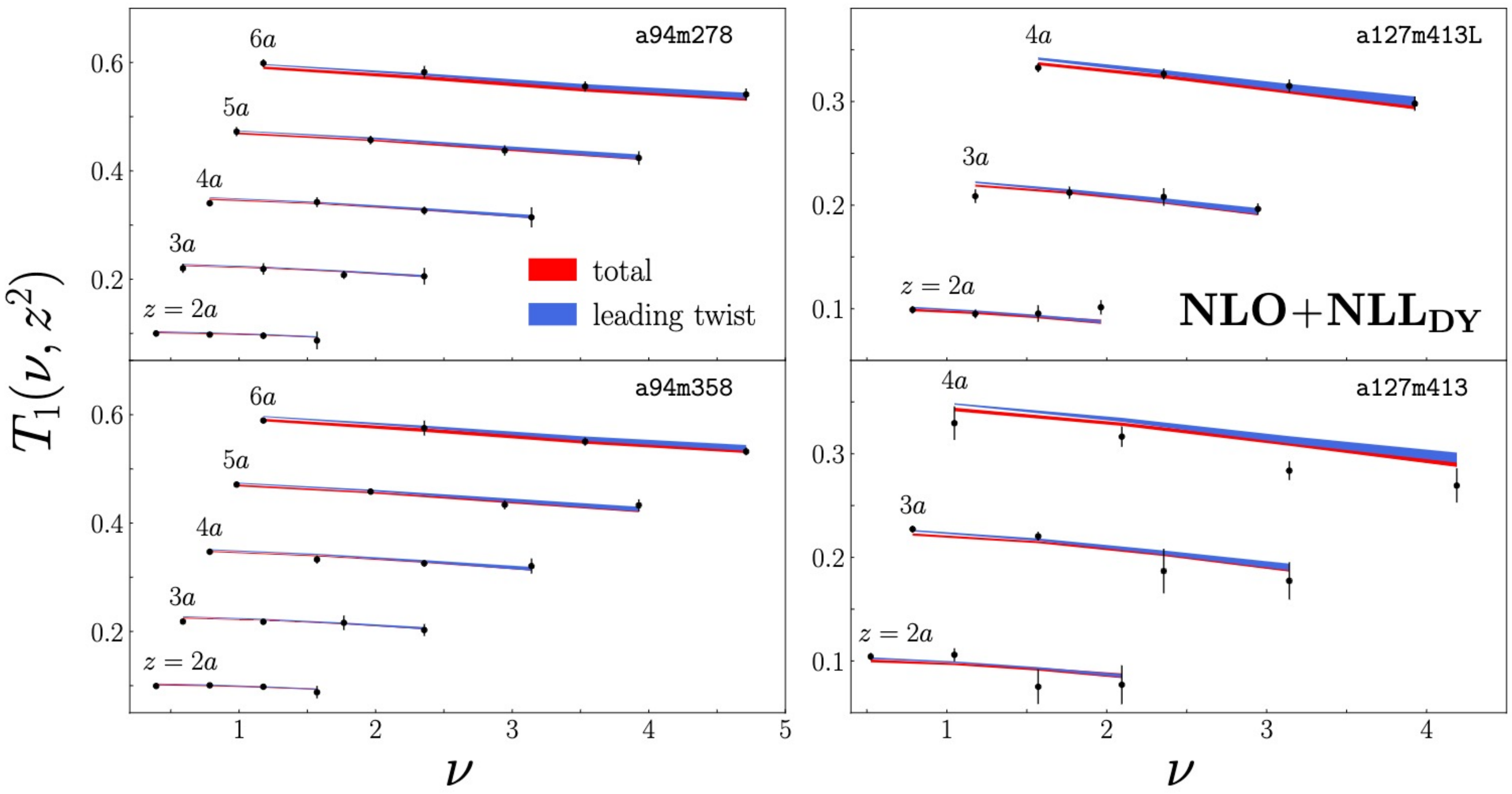
- Good overall agreement
- Exception being `a127m413`: the smaller lattice volume ensemble

Process	Experiment	N_{dat}	NLO $\overline{\chi}^2$	NLO+NLL$_{\mathrm{DY}}$ $\overline{\chi}^2$
DY	E615 (x_F, Q)	61	0.83	0.81
	NA10 (194 GeV) (x_F, Q)	36	0.55	0.54
	NA10 (286 GeV) (x_F, Q)	20	0.85	0.86
LN	H1	58	0.37	0.35
	ZEUS	50	1.56	1.55
CC	a94m278	20	0.33	0.33
	a94m358	20	0.45	0.45
	a127m413L	12	0.72	0.77
	a127m413	12	1.98	1.90
Total		**289**	**0.81**	**0.80**

Agreement with data

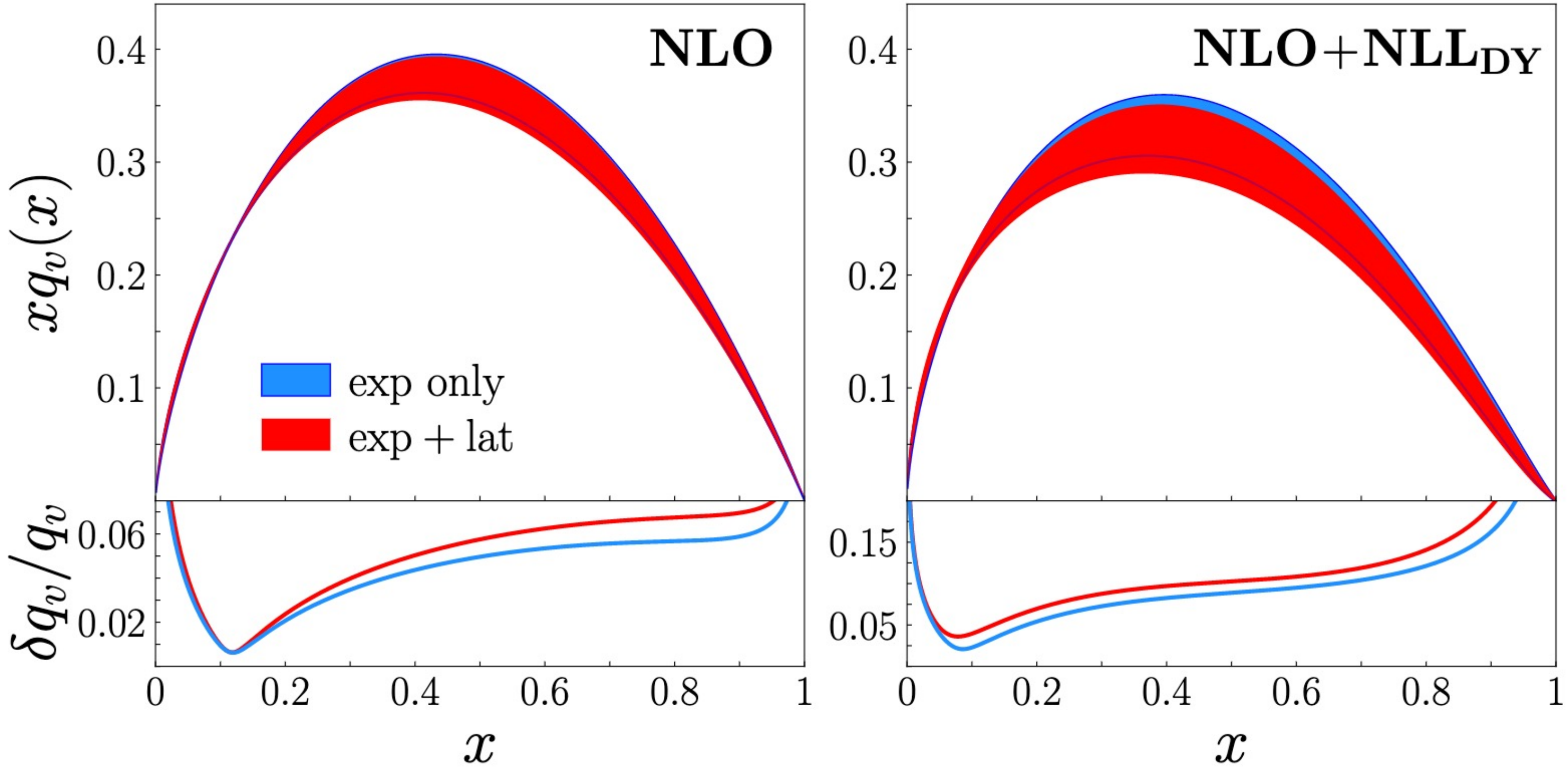


Agreement with data



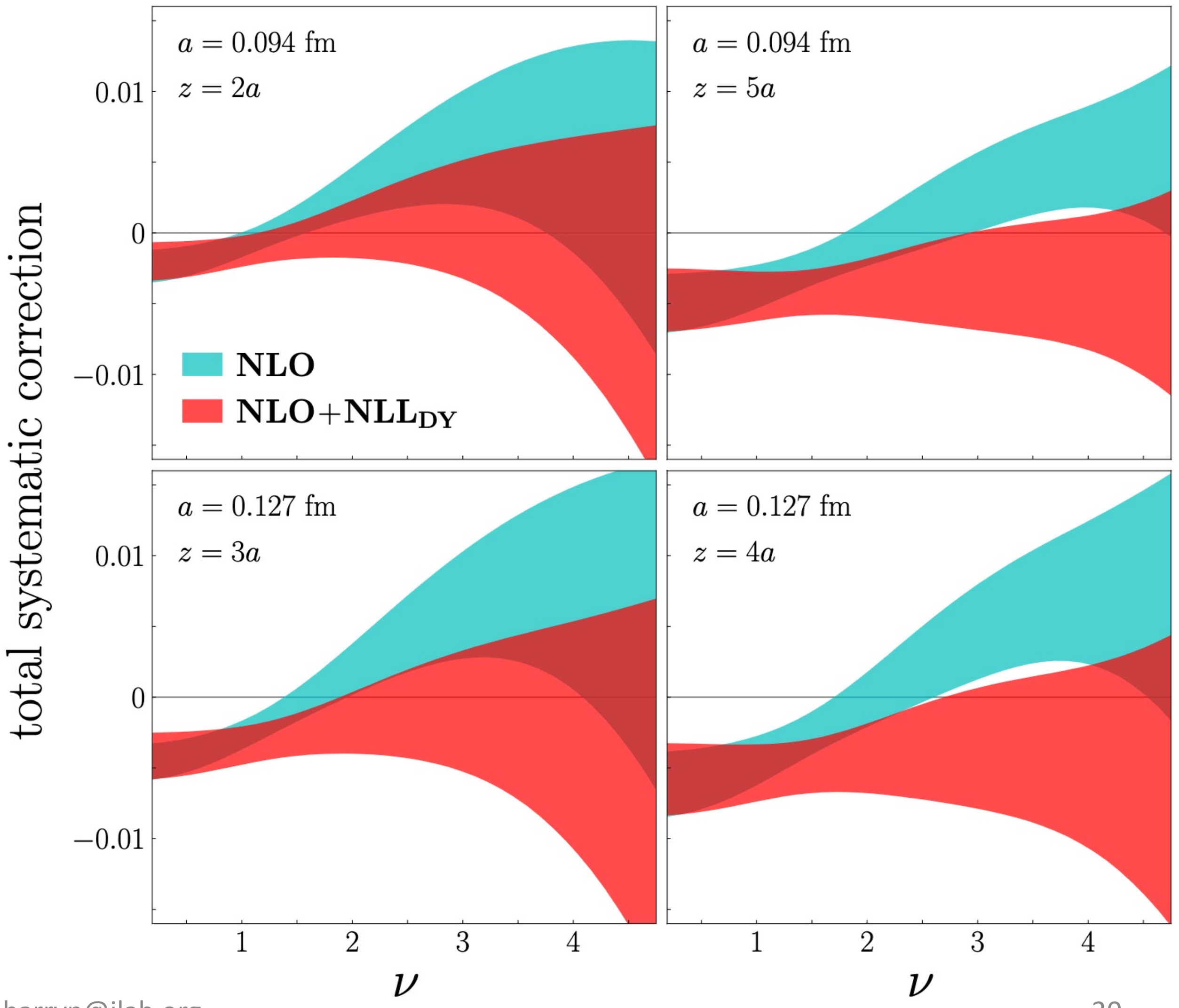
PDFs

- PDFs are almost identical before and after inclusion of lattice data
- CC correlators have no pull!



Quantifying systematics – total

- Different DY methods give different signs, but similar trends
- Large uncertainties at small z

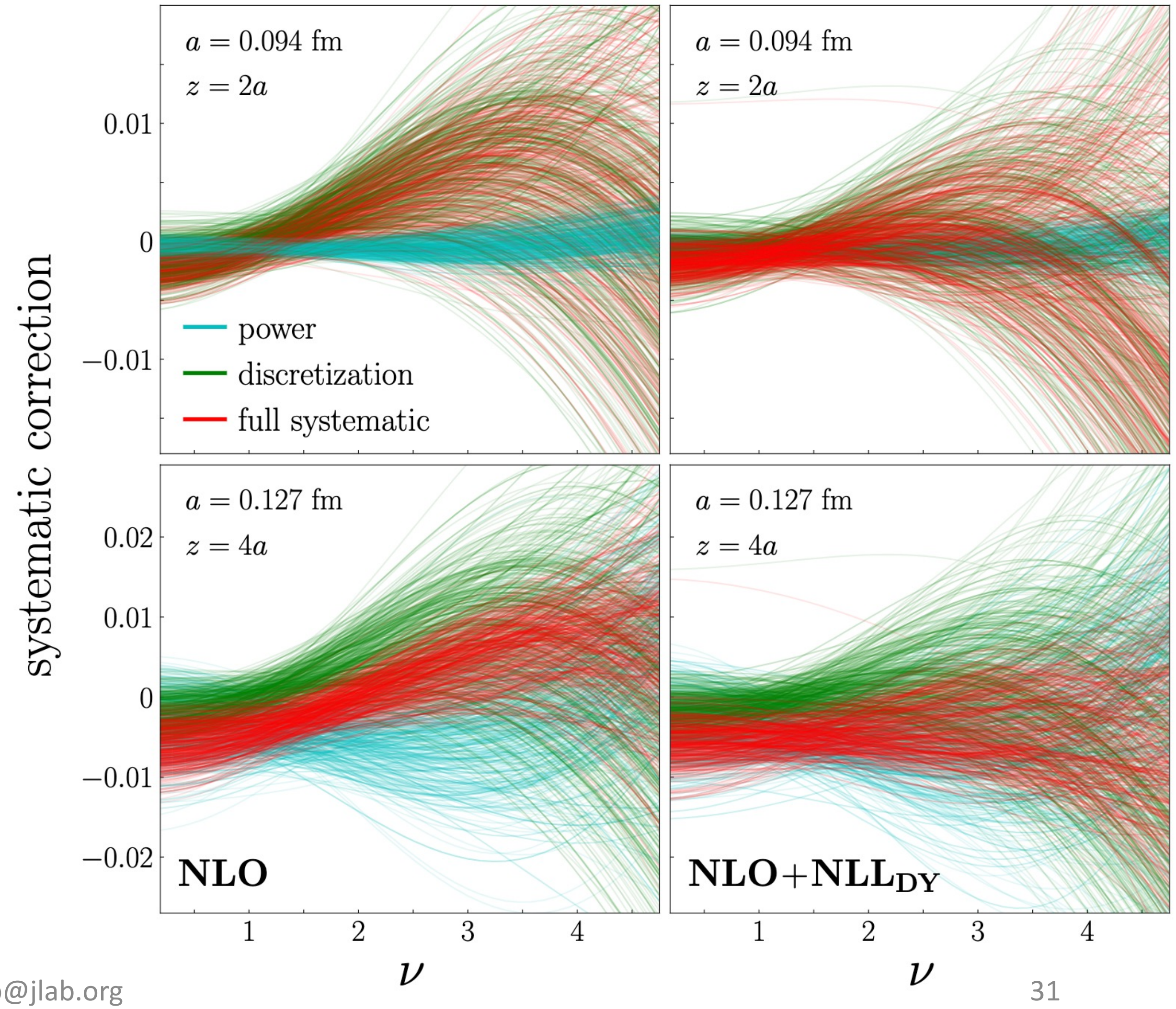


Quantifying systematics

- Each of two systematics

$$z^2 B_1(\nu) + a R_1(\nu)$$

- Some tension between the two types, effectively canceling



Conclusions/Outlook

Conclusions and Outlook

- Further control of the lattice systematic corrections needed to further impact – calculate at small lattice spacing a
- Low-momentum lattice data can be used when combined with experimental data
- Extend methodology to observables that are not well constrained by experimental data – helicity PDFs, transversity PDFs, etc.
- Future experiments such as TDIS at JLab and EIC and DY measurements can provide checks of universality in kinematic regions similar to the lattice data

Backup

Deriving resummation expressions – MF

Claim: yellow terms give rise to the resummation expressions

$$
\begin{aligned}
\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right] \\
+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ +1-z\right]\right. \\
\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ +\left[\frac{1}{1-y}\right]_+\right)-2(1-z)\right]\right\}
\end{aligned}
$$

Claim: Red terms are power suppressed in $(1-z)$ and wouldn't contribute to the same order as the yellow terms

Generalized Threshold resummation

- Write the (z, y) coefficients in terms of (z_a, z_b), and for the red terms, you get:

$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right)=\mathrm{d}z_a\mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\left[1+\mathcal{O}(1-z_a,1-z_b)\right].$$

- This is *not* power suppressed in $(1-z_a)$ or $(1-z_b)$ but instead the same order as the leading power in the soft limit
- Generalized threshold resummation in the soft limit does not agree with the MF methods

Parametrizing the systematic effects

- Use a basis of Jacobi polynomials and Taylor expand

$$\mathrm{Re}B_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, b_n,$$

$$\mathrm{Re}P_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, p_n,$$

$$\mathrm{Re}F_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, f_n,$$

$$\sigma_{0,n}(\nu) = \int_0^1 dx\ \cos(\nu x)\, x^a (1-x)^b\, J_n(x),$$

- Expanded b_n, p_n, f_n, which are free parameters in the fit

Begin at $n = 1$ to ensure at $\nu = 0$ the observable $== 1$

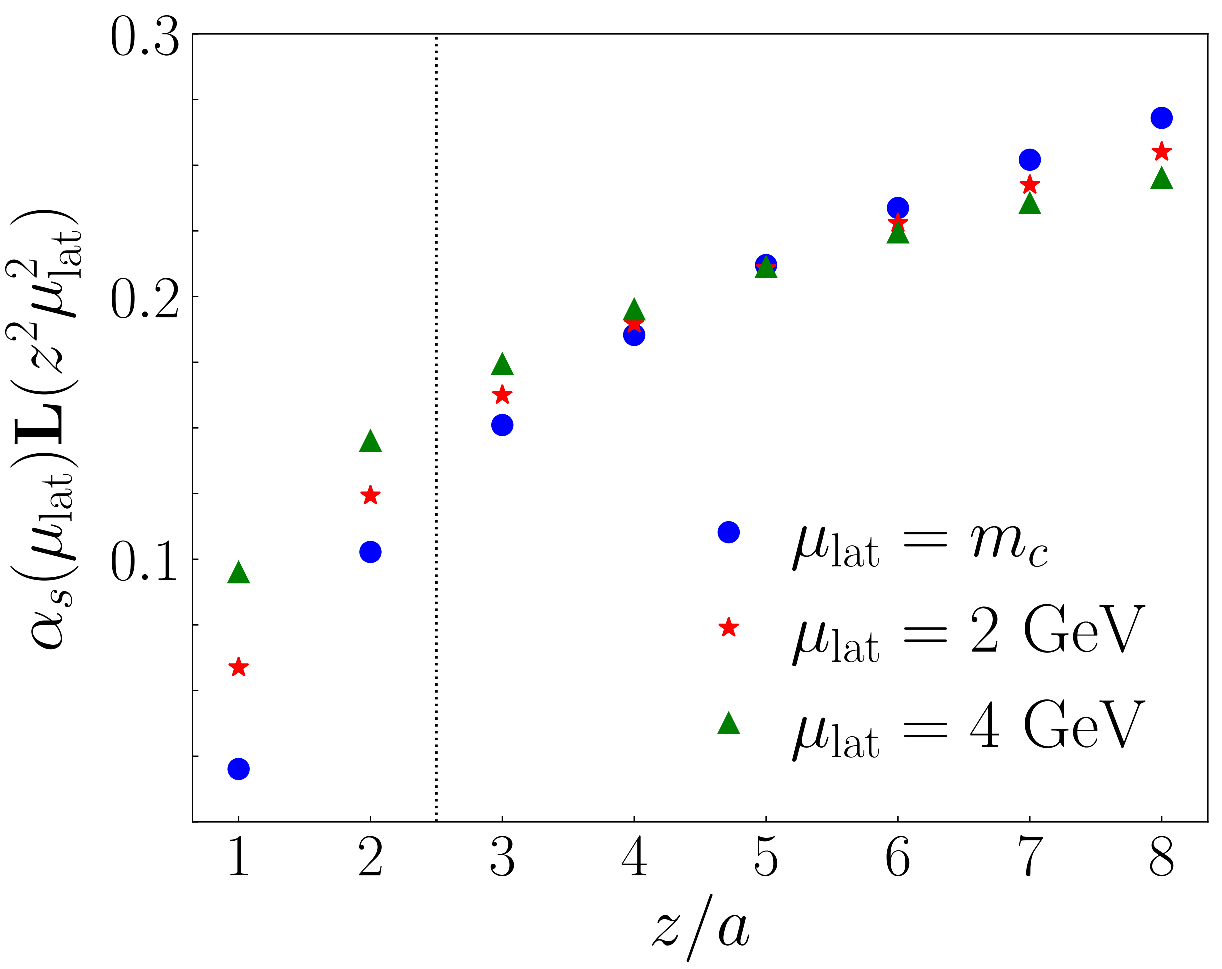
Multiple scale problem

$$\sigma_{n/h}(\omega, \xi^2) \equiv \langle h(p)|T\{\mathcal{O}_n(\xi)\}|h(p)\rangle$$
$$= \sum_i f_{i/h}(x, \mu^2) \otimes K_{n/i}(x\omega, \xi^2, \mu^2)$$

- LHS (1st equation): Lattice QCD data are calculated using QCD and must be renormalized to the continuum limit and have renormalization constants – unlike experimental cross sections!!
  - Related with lattice spacing.
- RHS: two scales – renormalization scale to specify PDF, factorization scale to get hard coefficients

Perturbation expansion is OK

- At the expense of a small α_s, the product with the logarithm is under control
- Choose $\mu_{\rm lat} = 2$ GeV unless otherwise specified



Methodology

Parametrization of PDFs

$$f(x,\mu_0^2) = \frac{N_f\, x^{\alpha_f}(1-x)^{\beta_f}(1+\gamma_f x^2)}{B(\alpha_f+2,\beta_f+1)+\gamma_f B(\alpha_f+4,\beta_f+1)},$$

Experimental data

$$\chi_{\mathrm{e}}^2(\boldsymbol{a},\mathrm{data}) = \sum_i \left[\frac{d_i^e - \sum_k r_k^e\, \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e}\right]^2 + \left(\frac{1-n_e}{\delta n_e}\right)^2 + \sum_k \left(r_k^e\right)^2,$$

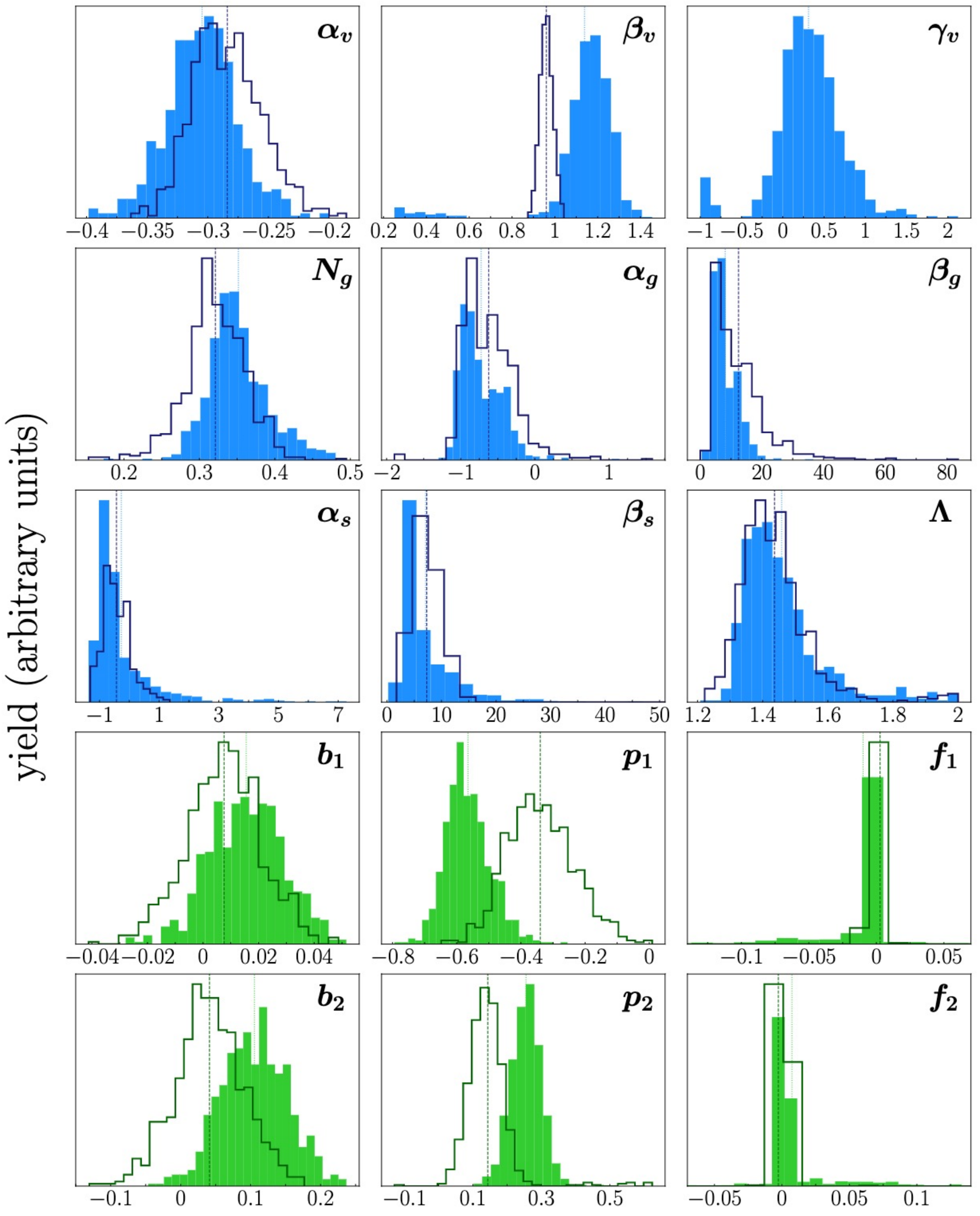
Lattice data

$$\chi_\lambda^2(\boldsymbol{a},\mathrm{data}) = \left(\boldsymbol{D}^\lambda - \boldsymbol{T}^\lambda(\boldsymbol{a})\right)^T V_\lambda^{-1}\left(\boldsymbol{D}^\lambda - \boldsymbol{T}^\lambda(\boldsymbol{a})\right).$$

Covariance matrix

Histograms of parameters

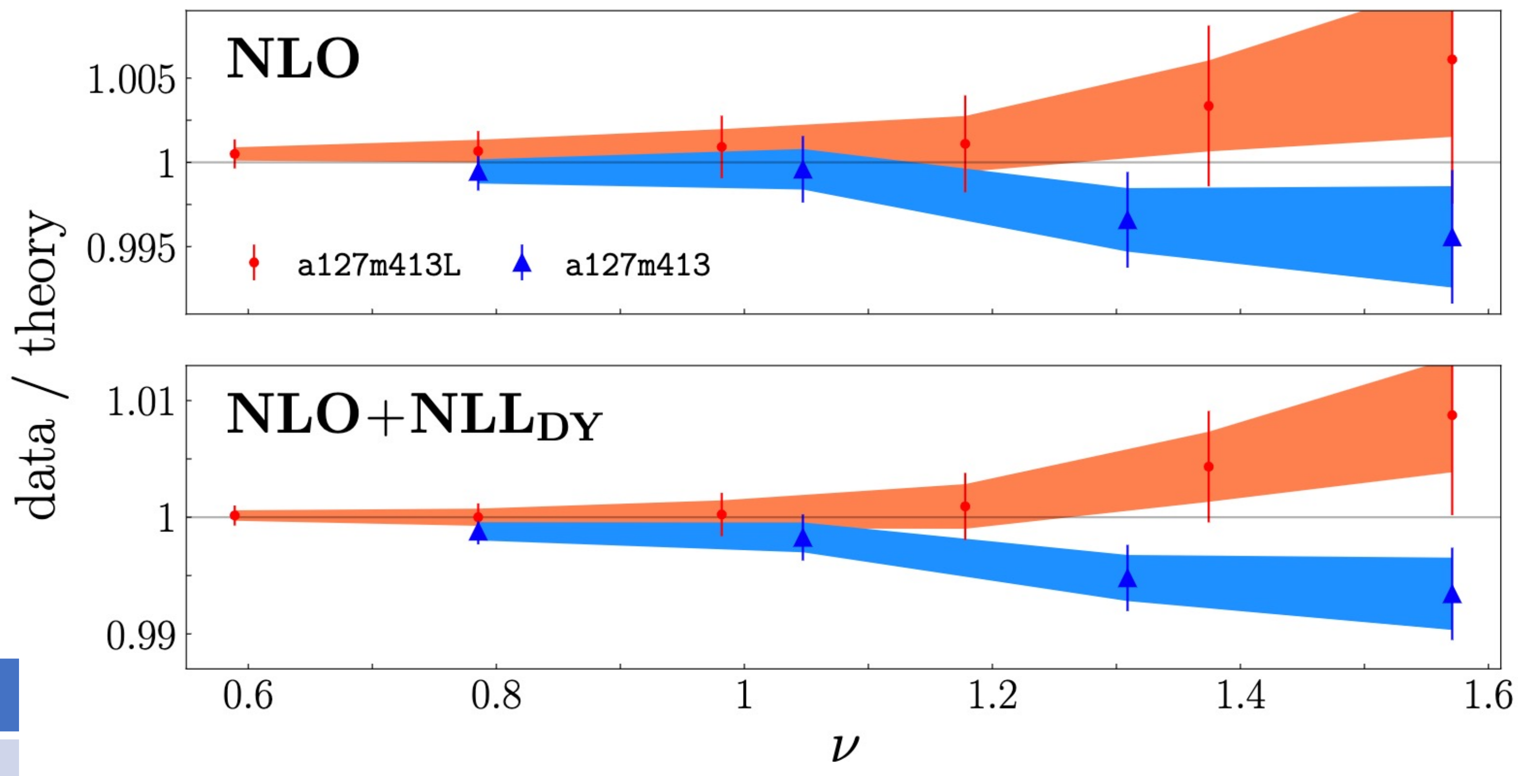
- Outlined – NLO
- Filled – NLO+NLL_{DY}
- All distributions well peaked



Data and theory comparison

- Each bin of z contains 3 momentum points, but only fitting to 1 momentum point
- Overall χ^2 are similar, but the fits to these are

Dataset	NLO ($\bar{\chi}^2$)	NLO+NLL$_{DY}$ ($\bar{\chi}^2$)
a127m413L	0.76	0.81
a127m413	1.28	1.45



Scale Variation

- Do we capture systematic uncertainty from choosing $\mu_{\text{lat}} = 2$ GeV?
- Central values within uncertainty band – not a big issue