

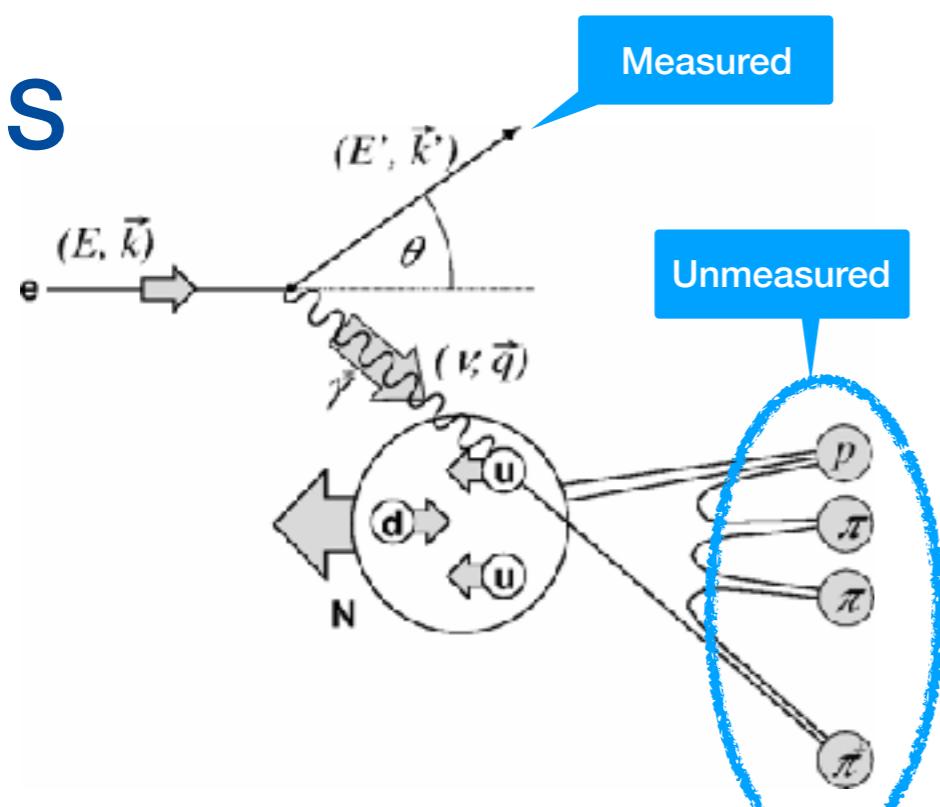
PION'S UNPOLARIZED TMD PDF FROM QCD'S DYSON-SCHWINGER EQUATIONS

Chao Shi

Argonne National Laboratory

Background : DIS & SIDIS

DIS

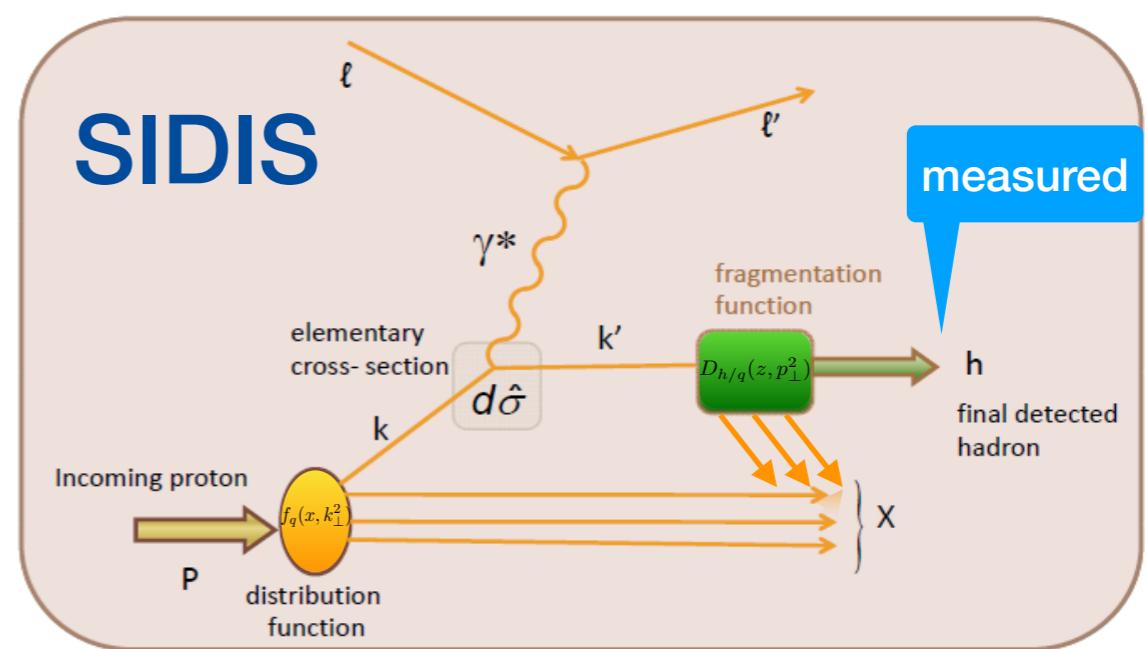


Point-like quark constituents reside in the nucleon (1990 Nobel Prize)

Collinear factorization ($Q^2 \gg \Lambda_{\text{QCD}}^2$):

$$d\sigma^{DIS} \sim \sum_i \int_x^1 \frac{dy}{y} C_i(x/y, Q^2/\mu_F^2; \alpha_s(\mu_R)) f_i(y, \mu_F^2)$$

- Convolution of non-perturbative, long-distance physics with the elementary, short-distance, hard-scattering interaction.
- Some important and interesting information of the structure of the hadron is lost, as all other degrees of freedom are averaged over.

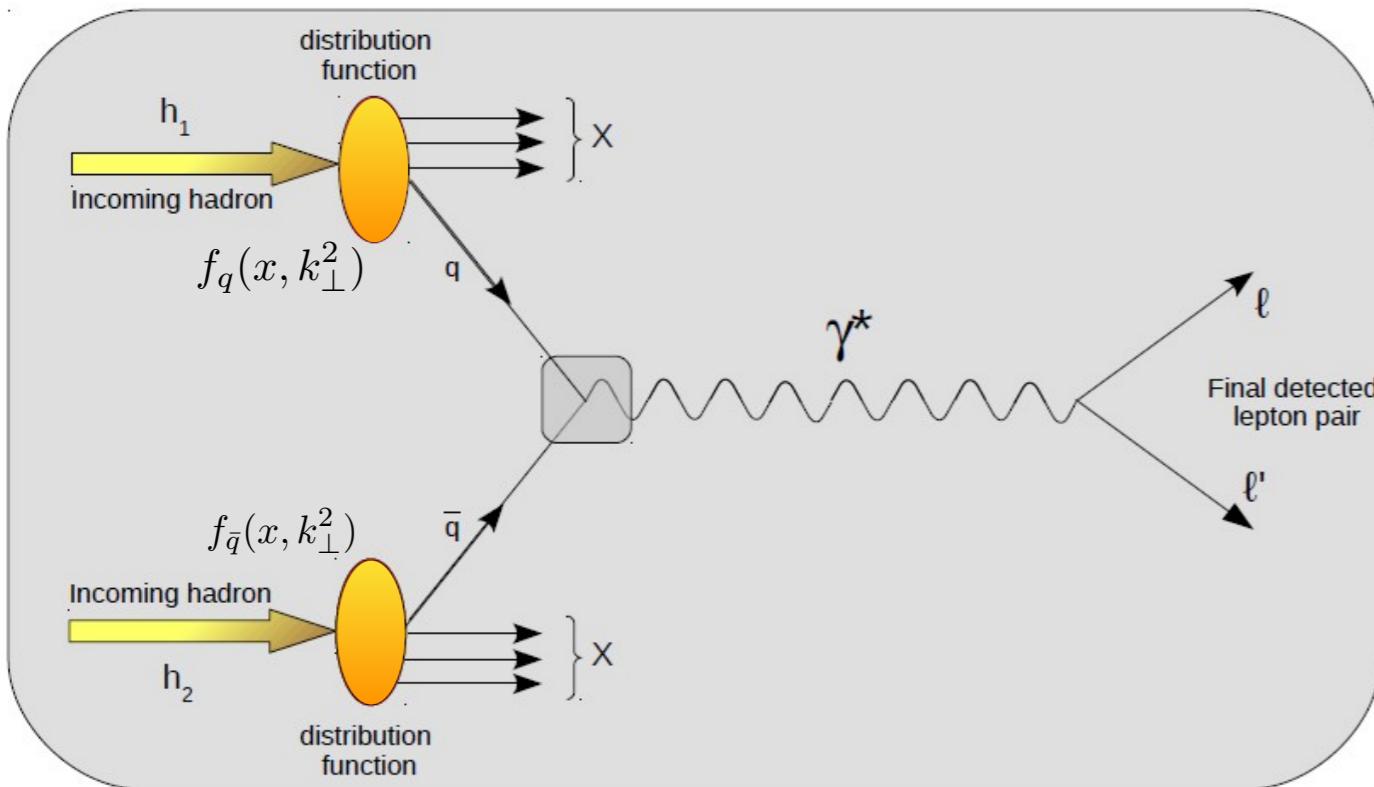


TMD factorization ($Q^2 \gg \Lambda_{\text{QCD}}^2$):

$$d\sigma_{lp \rightarrow lhX} = \sum_q f_{q/P}(x, k_\perp^2; Q^2) \otimes d\hat{\sigma}^{lq \rightarrow lq} \otimes D_{h/q}(z, p_\perp^2; Q^2)$$

- One additional outgoing meson is measured.
- The PDFs depend not only on x , but also on the transverse motion of partons inside the hadron.
- Both TMD PDFs and TMD FFs are accessible in SIDIS.

Background : Drell-Yan process (TMD PDF)



In the DY experiment, the outgoing lepton pair is measured, hence the transverse momentum of the di-lepton .

The annihilated quark-antiquark encodes the partons' intrinsic transverse motion, giving rise to the di-lepton of low transverse momentum.

TMD factorization: $\sigma_{Drell-Yan} \sim f_q(x, k_{\perp}^2) \otimes f_{\bar{q}}(x, k_{\perp}^2) \otimes \hat{\sigma}_{q\bar{q} \rightarrow l^+l^-}$

TMD PDFs

TMD PDFs

In DY, structure functions are expressed as convolutions of TMD PDFs. Since no outgoing hadron is measured, TMD FFs are not needed.

Access to pion's TMD PDFs.

Background : TMD PDFs

• **TMD PDFs (transverse momentum dependent PDFs) describe the transverse motion of partons and spin-orbit correlations inside hadrons.**

Correlation function (SIDIS):

$$\Phi_{ij}(x, \mathbf{k}_\perp, S) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(k^+ \xi_- - \mathbf{k}_\perp \cdot \xi_\perp)} \langle P, S | \bar{\psi}_j(0) \mathcal{U}_{(0,+\infty)}^{n-} \mathcal{U}_{(+\infty,\xi)}^{n-} \psi_i(\xi) | P, S \rangle \Big|_{\xi^+=0},$$

with hadron momentum P and spin $S = (\Lambda \frac{P^+}{M}, -\Lambda \frac{P^+}{M}, \mathbf{S}_\perp)$

The TMDs enter the general decomposition of the quark-quark correlation (leading twist).

$$\begin{aligned} \Phi(x, \mathbf{k}_\perp, S) = & \frac{1}{2} \left\{ f_1 \not{h}_+ - f_{1T}^\perp \frac{\epsilon_T^{ij} \mathbf{k}_\perp^i S_\perp^j}{M} \not{h}_+ + \Lambda g_{1L} \gamma_5 \not{h}_+ + \frac{(\mathbf{k}_\perp \cdot \mathbf{S}_\perp)}{M} g_{1T} \gamma_5 \not{h}_+ + h_{1T} \frac{[\not{S}_\perp, \not{h}_+]}{2} \gamma_5 \right. \\ & \left. + \Lambda h_{1L}^\perp \frac{[\not{k}_\perp, \not{h}_+]}{2M} \gamma_5 + \frac{(\mathbf{k}_\perp \cdot \mathbf{S}_\perp)}{M} h_{1T}^\perp \frac{[\not{k}_\perp, \not{h}_+]}{2M} \gamma_5 + i h_1^\perp \frac{[\not{k}_\perp, \not{h}_+]}{2M} \right\}, \end{aligned}$$

QUARKS	<i>unpolarized</i>	<i>chiral</i>	<i>transverse</i>
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_{1T}, h_{1T}^\perp

- Rows indicate the hadron polarization and columns indicate quark's.
- Circled ones survive when integrated over the transverse momentum and they are linked to the familiar parton density, helicity and transversity distributions.
- For spinless pion, only the unpolarized TMD PDF and the Boer-Mulders function survive.

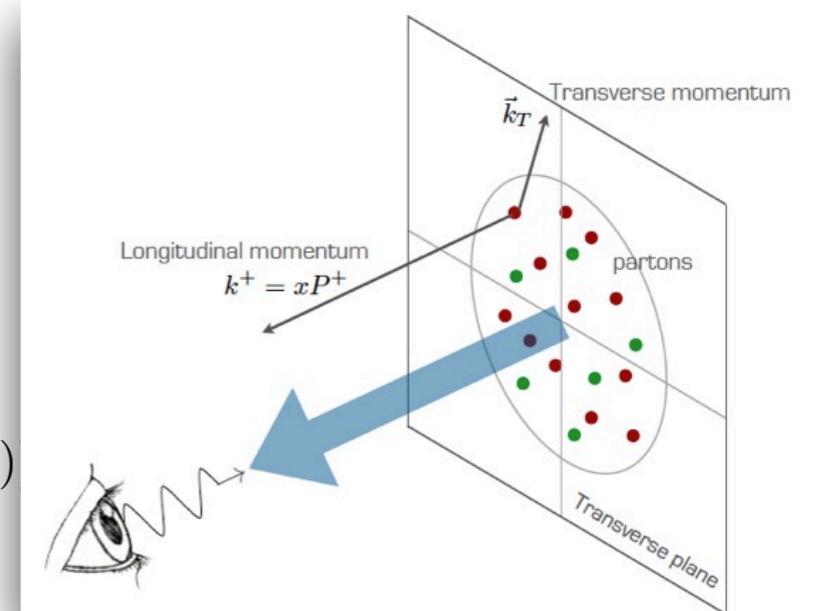
Background : Pion TMD PDFs

For pion, the TMDs can be projected out as:

$$f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

$$\frac{\epsilon^{ij} k_\perp^j}{M_\pi} h_{1,\pi}^\perp(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) i\sigma^{i+} \gamma_5 q(\xi^-, \xi_\perp) | \pi(P) \rangle$$

with process-dependent gauge links not shown explicitly.



- The unpolarized TMD PDF $f_{1,\pi}^\perp(x, k_\perp^2)$ describes the distribution of unpolarized partons carrying the longitudinal momentum fraction x of the pion, and the transverse momentum k_\perp .
- The Boer-Mulders function $h_{1,\pi}^\perp(x, k_\perp^2)$ describes a spin-orbit correlation of transversely polarized partons, which is chirally odd (chirality of partons flipped by operator) and time-reversal odd (flips sign under time reversal transformation). The Wilson line is very important in getting a non-vanishing Boer-Mulders function that changes its sign from SIDIS to DY.

$f_{1,\pi}(x, k_\perp^2)$ Global fits based on TMD evolution with phenomenological assumptions on its form. Formalism under quantitative examination.

$h_{1,\pi}^\perp(x, k_\perp^2)$ Phenomenological analysis and a few first attempts. Model and ansatz.

Calculation? Non-perturbative QCD required!

Nonperturbative QCD

Nonperturbative QCD

Parton distributions amplitudes/functions:

- 1.generalized parton distributions (GPD)**
- 3.transverse momentum distributions (TMD)**
- 2.parton distribution amplitude (PDA)**
- 4.etc...**

Nonperturbative QCD

QCD

$$\mathcal{L}_{\text{QCD}} = \bar{q}_i \gamma^\mu (i\partial_\mu - g_s t^a A_\mu^a - m_i) q_i - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f^{abc} A_\mu^b A_\nu^c$$

$$\alpha_s = \frac{g_s^2}{4\pi}$$

Parton distributions amplitudes/functions:

1. generalized parton distributions (GPD)
3. transverse momentum distributions (TMD)
2. parton distribution amplitude (PDA)
4. etc...

Nonperturbative QCD

QCD

$$\mathcal{L}_{\text{QCD}} = \bar{q}_i \gamma^\mu (i\partial_\mu - g_s t^a A_\mu^a - m_i) q_i - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f^{abc} A_\mu^b A_\nu^c$$

$$\alpha_s = \frac{g_s^2}{4\pi}$$

Nonperturbative QCD methods

1. ADS/QCD (Holographic QCD)
2. **Dyson-Schwinger equations.**
3. Effective theories and models, e.g., ChiPT, NJL model...
4. Light front QCD.
5. Lattice QCD.
6. QCD sum rules.
- etc...

Parton distributions amplitudes/functions:

1. generalized parton distributions (GPD)
3. transverse momentum distributions (TMD)
2. parton distribution amplitude (PDA)
4. etc...

DSEs: QCD

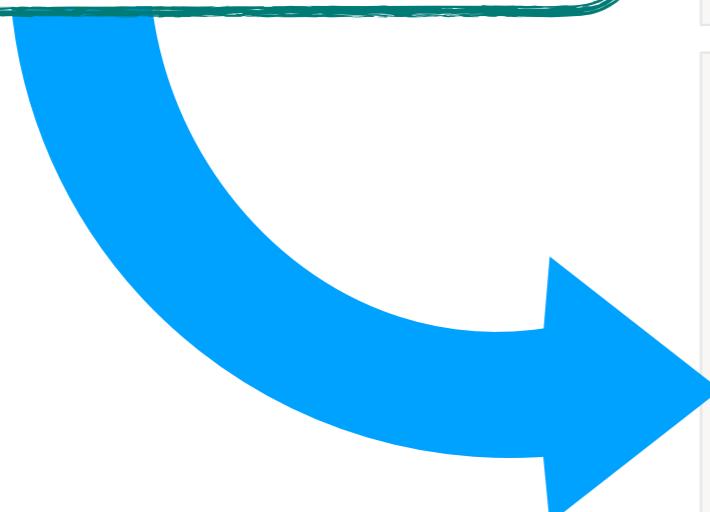
 **Dyson-Schwinger equations:** general relations between Green functions in quantum field theories.

Derivation:

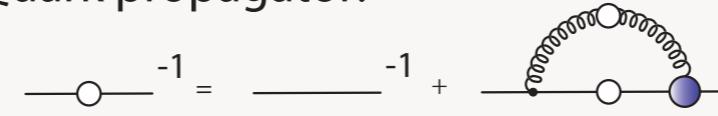
1. Quantum Field Theory
2. Path Integral formulation
3. Invariance of generating functional

$$\int \mathcal{D}\psi \frac{\delta}{\delta \psi_\alpha} e^{i[S + \int d^4y J_\alpha \psi_\alpha]} \Big|_{J_\alpha=0} = 0$$

(Non-perturbative)



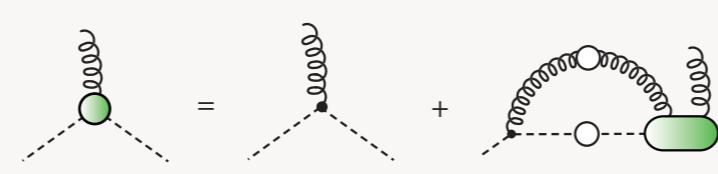
Quark propagator:



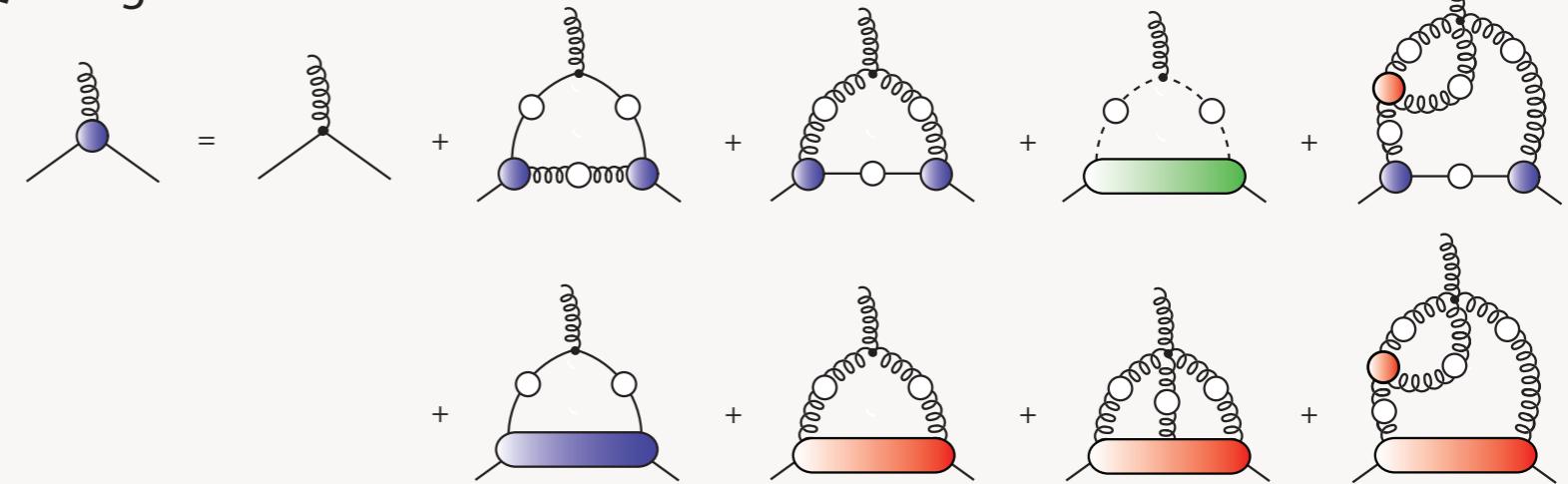
Ghost propagator:



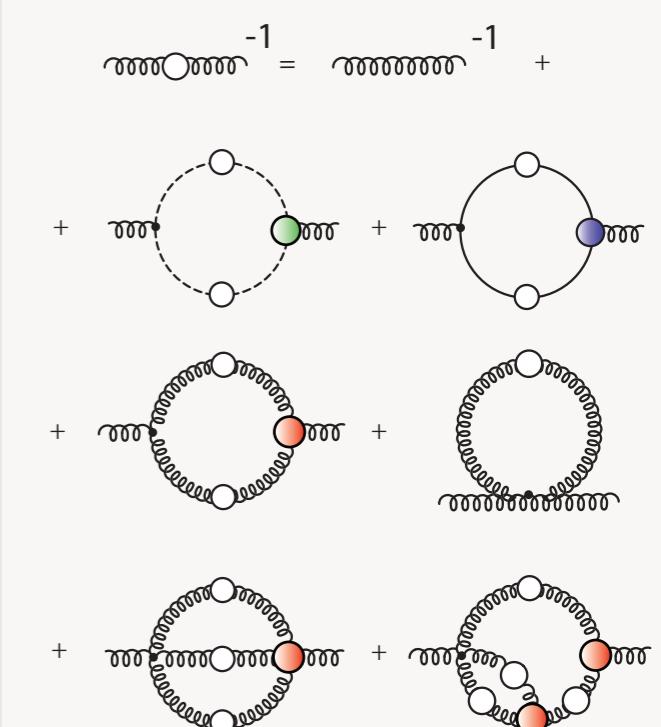
Ghost-gluon vertex:



Quark-gluon vertex:

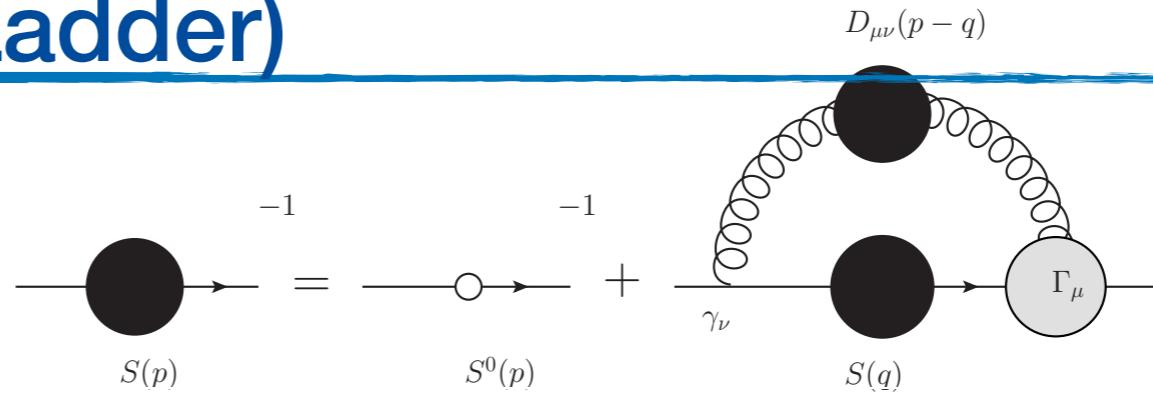


Gluon propagator:

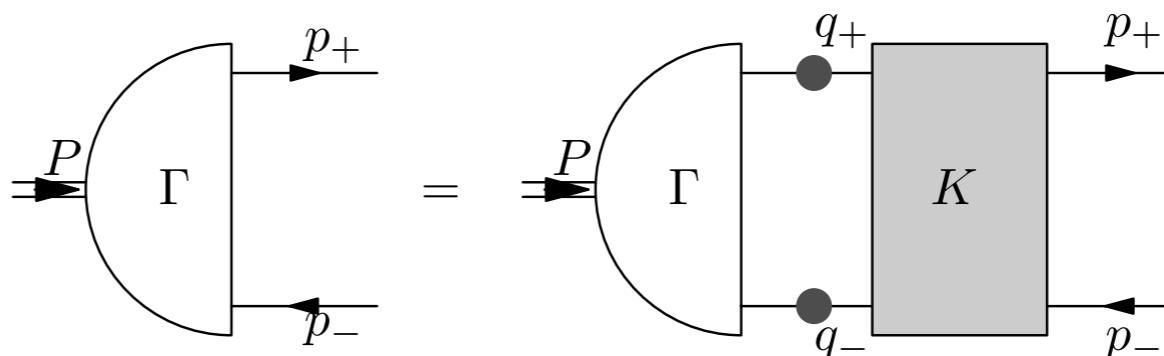


DSEs: Pion (Rainbow-Ladder)

Quark's DSE:

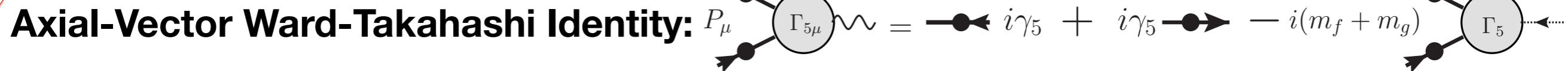
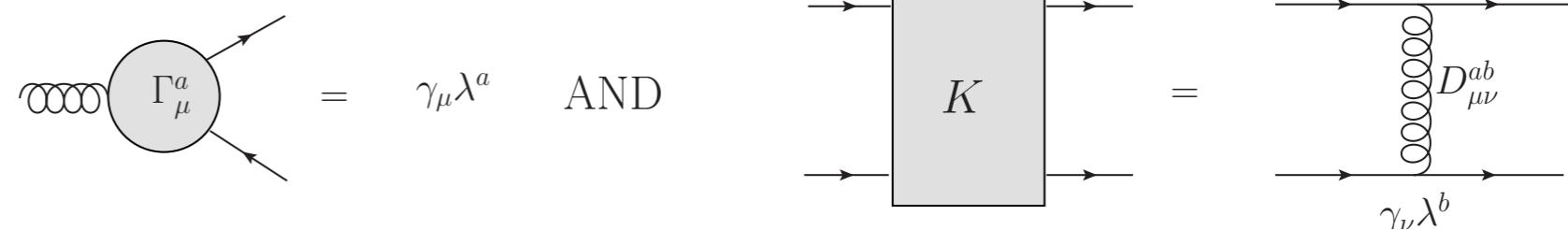


Pion's BSE:



Constrain

Rainbow-Ladder:



$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-)$$

$$- i[m_f + m_g] \Gamma_5^{fg}(k; P),$$

(P. Maris, C.D. Roberts and P. C. Tandy, PLB1998)

DSEs: Pion (Beyond Rainbow-Ladder)

Inhomogeneous BSE:

$$\Gamma_{5\mu}(k; P) = Z_2 \gamma_5 \gamma_\mu$$

$$- Z_2 \int_{dq} \mathcal{G}(k - q) D_{\rho\sigma}^{\text{free}}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S(q_+) \times \Gamma_{5\mu}(q; P) S(q_-) \frac{\lambda^a}{2} \tilde{\Gamma}_\beta(q_-, k_-)$$

$$+ Z_1 \int_{dq} g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \times \frac{\lambda^a}{2} \Lambda_{5\mu\beta}(k, q; P)$$

(Lei Chang and C.D. Roberts PRL2009)

Beyond RL

(Lei Chang, YX Liu and C.D. Roberts PRL2011,

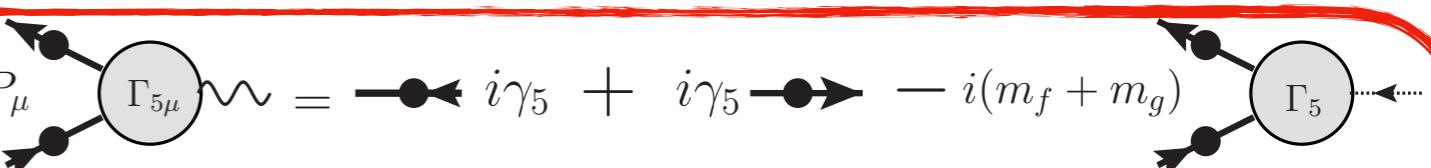
$$\Gamma_\mu(p_1, p_2) = \Gamma_\mu^{\text{BC}}(p_1, p_2) + \Gamma_\mu^{\text{acm}}(p_1, p_2)$$

$$2\Lambda_{5\beta(\mu)} = [\tilde{\Gamma}_\beta(q_+, k_+) + \gamma_5 \tilde{\Gamma}_\beta(q_-, k_-) \gamma_5] \times \frac{1}{S^{-1}(k_+) + S^{-1}(-k_-)} \Gamma_{5(\mu)}(k; P)$$

$$+ \Gamma_{5(\mu)}(q; P) \frac{1}{S^{-1}(-q_+) + S^{-1}(q_-)} \times [\gamma_5 \tilde{\Gamma}_\beta(q_+, k_+) \gamma_5 + \tilde{\Gamma}_\beta(q_-, k_-)]$$

Constrain

Axial-Vector Ward-Takahashi Identity:



$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-)$$

$$- i[m_f + m_g] \Gamma_5^{fg}(k; P),$$

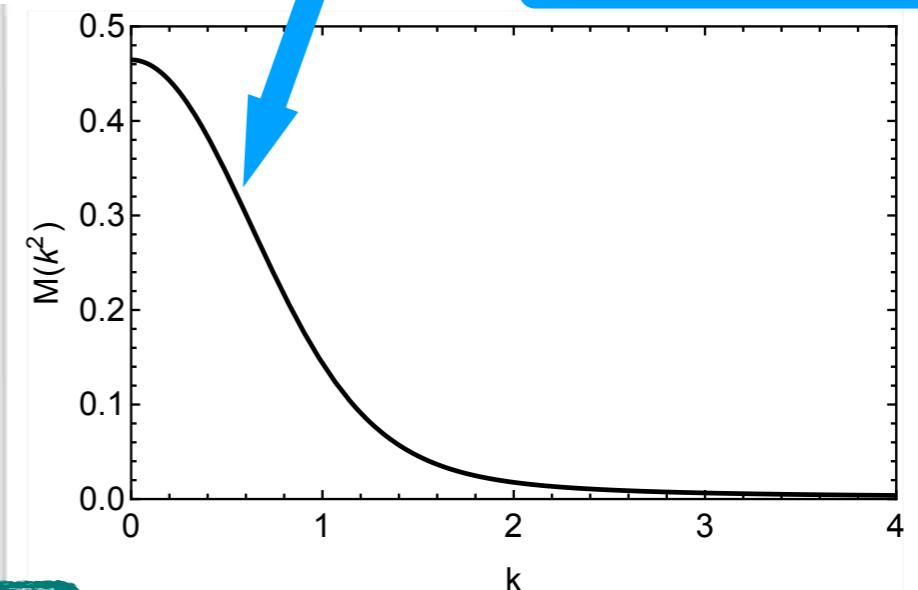
DSEs: $S(q)$ and $\Gamma(q; P)$

dressed quark propagator:

$$S(q) = \sum_{i=1}^N \left[\frac{Z_i}{i\gamma \cdot q + m_i} + \frac{Z_i^*}{i\gamma \cdot q + m_i^*} \right] = \frac{Z(k^2)}{ip + M(k^2)}$$

Significantly enhanced mass function, signals the DCSB.

Complex conjugate poles, **NO** poles on the real axis, consistent with confinement!



pion BS amplitude:

$$\Gamma_P(q; P) = \gamma_5 [iE_P(q; P) + \gamma \cdot P F_P(q; P) + \gamma \cdot q G_P(q; P) + \sigma_{\mu\nu} q_\mu P_\nu H_P(q; P)]$$

Dominant Contributions in BSA

$$\mathcal{F}(k; P) = \mathcal{F}^i(k; P) + \mathcal{F}^u(k; P)$$

correct power law in the UV part of BSA generated by one-gluon exchange

$$\mathcal{F}^i(k; P) = c_F^i \int_{-1}^1 dz \rho_{\sigma_F^i}(z) \left[a_F^i \hat{\Delta}_{\Lambda_F^i}^4(k_z^2) + (1 - a_F^i) \hat{\Delta}_{\Lambda_F^i}^5(k_z^2) \right]$$

strongly enhanced IR part of BSA, generated by DCSB.

$$\mathcal{F}^u(k; P) = c_F^u \int_{-1}^1 dz \rho_{\sigma_F^u}(z) \hat{\Delta}_{\Lambda_F^u}(k_z^2),$$

$$\text{with } \hat{\Delta}_\Lambda(s) = \Lambda^2 / (s + \Lambda^2), \quad k_z^2 = k^2 + z k \cdot P$$

(Lei Chang et al, PRL2013)

TMD PDF: Two Approaches

Fully Dressed Quark propagator & Pion Bethe-Salpeter amplitude



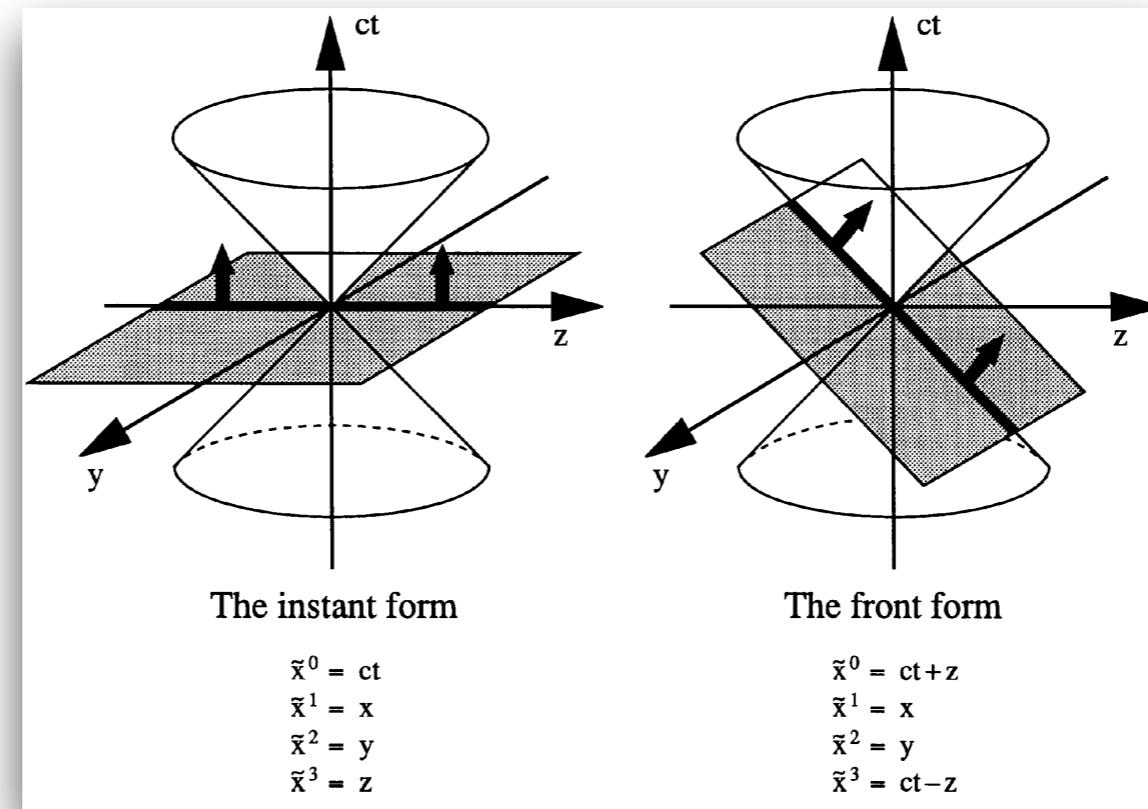
Two approaches:

- **Light-front approach** Extract from pion's Bethe-Salpeter wave functions the LFWFs and calculate TMDs using overlap representation.
- **Covariant approach** Compute the triangle diagrams in terms of fully covariant propagators/vertices with appropriate truncations.



$$\text{TMD: } f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

Light-front QCD



- 💡 A useful alternative to ordinary equal-time quantization: light-front time $x^+ = ct + z$, spatial coordinate $x^- = ct - z$.
- 💡 A natural formalism in describing hard hadron scattering, where the particles move at near speed of light. The PDF, GPDs and TMDs are all defined within the front form.
- 💡 A relativistic description of bound systems in terms of quantum-mechanical-like wave functions, i.e., the light front wave functions (LFWFs). The LFWFs are total momentum-independent (only internal coordinates/momentum) and frame-independent (boost invariant). They keep all the non-perturbative dynamical information of the hadron's internal structure.

Light-front approach

$$f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \boldsymbol{\xi}_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \boldsymbol{\xi}_\perp) | \pi(P) \rangle.$$

Light-front approach

$$f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

↓ Decomposition

$$q_{(+)}(\xi^-, \xi_\perp) + q_{(-)}(\xi^-, \xi_\perp)$$

↓ canonical expansion

$$q_{(+)}(\xi^+ = 0, \xi^-, \xi_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda [b_\lambda(k) u(k\lambda) e^{-i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)} + d_\lambda^+(k) \nu(k\lambda) e^{i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)}]$$

Light-front approach

$$f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

Fock Expansion

↓ Decomposition
 $q_{(+)}(\xi^-, \xi_\perp) + q_{(-)}(\xi^-, \xi_\perp)$
↓ canonical expansion

$q_{(+)}(\xi^+ = 0, \xi^-, \xi_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda [b_\lambda(k) u(k\lambda) e^{-i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)} + d_\lambda^+(k) \nu(k\lambda) e^{i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)}]$

$$|\pi^+(P)\rangle_{L_z=0} = \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x(1-x)}} \psi_{\uparrow\downarrow}(x, k_\perp) [b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\downarrow i}^\dagger(1-x, -k_\perp) - b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\uparrow i}^\dagger(1-x, -k_\perp)] |0\rangle$$

$$|\pi^+(P)\rangle_{|L_z|=1} = \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x(1-x)}} \psi_{\uparrow\uparrow}(x, k_\perp) [(k_1 - ik_2) b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\uparrow i}^\dagger(1-x, -k_\perp) + (k_1 + ik_2) b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\downarrow i}^\dagger(1-x, -k_\perp)] |0\rangle$$

←

Light-front approach

$$f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

Fock Expansion

↓ Decomposition
 $q_{(+)}(\xi^-, \xi_\perp) + q_{(-)}(\xi^-, \xi_\perp)$
↓ canonical expansion

$$q_{(+)}(\xi^+ = 0, \xi^-, \xi_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda [b_\lambda(k) u(k\lambda) e^{-i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)} + d_\lambda^+(k) \nu(k\lambda) e^{i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)}]$$

$$|\pi^+(P)\rangle_{L_z=0} = \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x(1-x)}} \psi_{\uparrow\downarrow}(x, k_\perp) [b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\downarrow i}^\dagger(1-x, -k_\perp) - b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\uparrow i}^\dagger(1-x, -k_\perp)] |0\rangle$$

$$|\pi^+(P)\rangle_{|L_z|=1} = \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x(1-x)}} \psi_{\uparrow\uparrow}(x, k_\perp) [(k_1 - ik_2) b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\uparrow i}^\dagger(1-x, -k_\perp) + (k_1 + ik_2) b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\downarrow i}^\dagger(1-x, -k_\perp)] |0\rangle$$

LFWFs: spin-antiparallel and spin-parallel, later denoted as $\psi_0(x, k_\perp^2)$ & $\psi_1(x, k_\perp^2)$.

Light-front approach

(M. Burkardt et al, PLB 2002)

$$f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

Fock Expansion

↓ Decomposition
 $q_{(+)}(\xi^-, \xi_\perp) + q_{(-)}(\xi^-, \xi_\perp)$
↓ canonical expansion

$q_{(+)}(\xi^+ = 0, \xi^-, \xi_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda [b_\lambda(k) u(k\lambda) e^{-i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)} + d_\lambda^+(k) \nu(k\lambda) e^{i(k^+ \xi^- - \vec{k}_\perp \vec{\xi}_\perp)}]$

$$|\pi^+(P)\rangle_{L_z=0} = \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x(1-x)}} \psi_{\uparrow\downarrow}(x, k_\perp) [b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\downarrow i}^\dagger(1-x, -k_\perp) - b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\uparrow i}^\dagger(1-x, -k_\perp)] |0\rangle$$

$$|\pi^+(P)\rangle_{|L_z|=1} = \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x(1-x)}} \psi_{\uparrow\uparrow}(x, k_\perp) [(k_1 - ik_2) b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\uparrow i}^\dagger(1-x, -k_\perp) + (k_1 + ik_2) b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\downarrow i}^\dagger(1-x, -k_\perp)] |0\rangle$$

LFWFs: spin-antiparallel and spin-parallel, later denoted as $\psi_0(x, k_\perp^2)$ & $\psi_1(x, k_\perp^2)$.

★ $f_{1,\pi}(x, \mathbf{k}_\perp^2) = |\psi_{\uparrow\downarrow}(x, k_\perp^2)|^2 + k_\perp^2 |\psi_{\uparrow\uparrow}(x, k_\perp^2)|^2$

LFWFs

- The LFWFs can be computed by diagonalizing the light-front QCD Hamiltonian operator.
- A great challenge to computation.
- Effective interactions have been used, i.e., holographic QCD.

Extraction from Bethe-Salpeter wave function:

Fourier transformation of LFWF

Correlation function & LFWFs:

$$\langle 0 | \bar{d}_+(0) \gamma^+ \gamma_5 u_+(\xi^-, \xi_\perp) | \pi^+(P) \rangle = i\sqrt{6} P^+ \psi_0(\xi^-, \xi_\perp),$$
$$\langle 0 | \bar{d}_+(0) \sigma^{+i} \gamma_5 u_+(\xi^-, \xi_\perp) | \pi^+(P) \rangle = -i\sqrt{6} P^+ \partial^i \psi_1(\xi^-, \xi_\perp).$$

M. Burkardt et al, PLB 2002

Correlation function & BS wave function:

$$\langle 0 | T\{\psi(\xi)\bar{\psi}(0)\} | \pi(P) \rangle = e^{-iP\xi/2} \int dk^4 e^{-i(k-P/2)\xi} \chi(k, P)$$

$$\chi(k; P) = S(k) \Gamma(k; P) S(k - P)$$

LFWFs & BS wave function:

$$\psi_0(x, k_\perp^2) = \sqrt{3}i \int \frac{d\hat{k}^2}{2\pi} \text{Tr}_D [\not{n} \gamma_5 \chi(k; P)] \delta(xP \cdot n - k \cdot n),$$

$$\psi_1(x, k_\perp^2) = \frac{\sqrt{3}}{\tilde{k}^2} \int \frac{d\hat{k}^2}{2\pi} \text{Tr}_D [\sigma^{\mu\nu} n_\mu \tilde{k}_\nu \gamma_5 \chi(k; P)] \delta(xP \cdot n - k \cdot n),$$

where $\hat{k} = (k^0, \vec{k}^3)$, $\tilde{k} = (0, \vec{k}_\perp, 0)$ and $n^2 = 0$

LFWFs:

Reconstruction from moments

$$\langle x^m \rangle_{k_\perp^2} = \int_0^1 dx x^m \psi_0(x, k_\perp^2) \stackrel{E}{=} \frac{\sqrt{3}}{|P \cdot n|} \int \frac{d\hat{k}^2}{2\pi} \left(\frac{k \cdot n}{P \cdot n} \right)^m \text{Tr}_D [\not{h} \gamma_5 S(k) \Gamma(k; P) S(k - P)]$$

Direct computation

$$\int_0^1 dx x^m \psi_0(x, k_\perp^2) \stackrel{E}{=} \frac{\sqrt{3}}{|P \cdot n|} \int \frac{d\hat{k}^2}{2\pi} \left(\frac{k \cdot n}{P \cdot n} \right)^m \text{Tr}_D [\not{h} \gamma_5 S(k_-) \Gamma(k; P) S(k_+)]$$

$$\sum_{\alpha} \frac{z_{\alpha}}{ip + m_{\alpha}}$$

$$i\gamma_5 \sum_{\beta} \int_{-1}^1 dz \rho_{\beta}(z) \frac{U_{\beta}}{(k^2 + z k \cdot P + \Lambda_{\beta}^2)^{\beta}}$$

$$\sum_{\gamma} \frac{z_{\gamma}}{ip + m_{\gamma}}$$

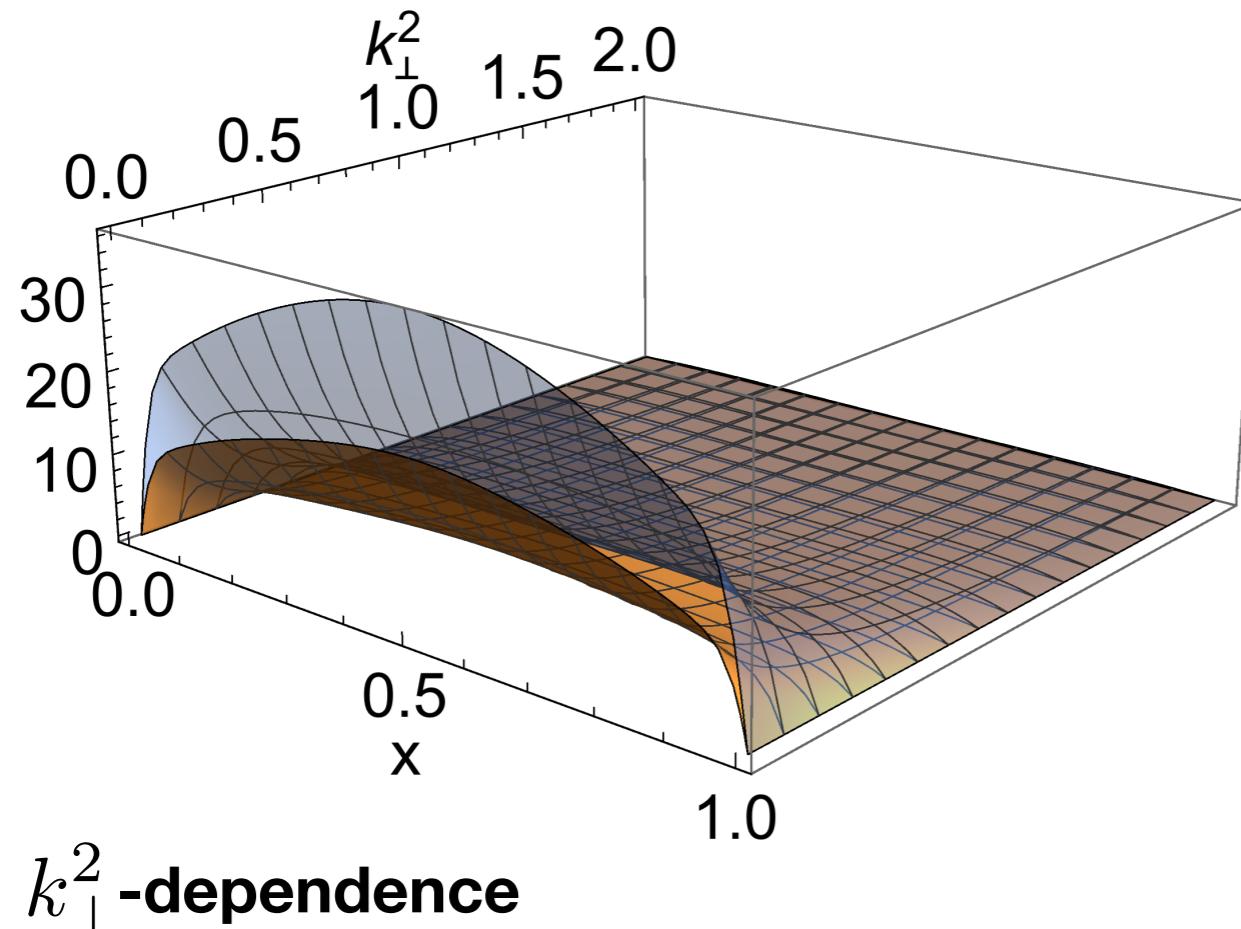
- Feynman parameterization
- change of integral variables (Cedric Mezrag)

$$\int_0^1 dx' x'^m \int dy' dz' \psi'_0(x', y', z', k_\perp^2)$$

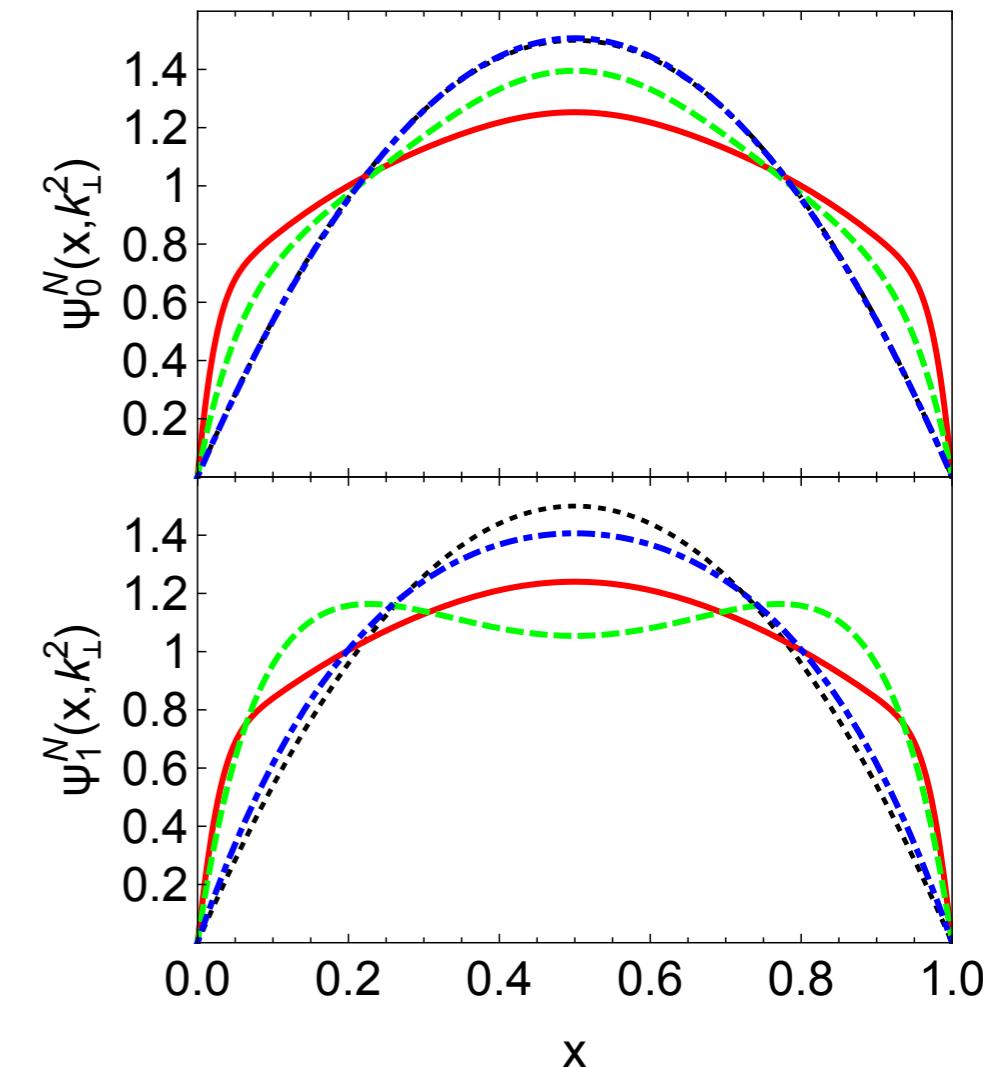
Conclusion:

$$\psi_0(x, k_\perp^2) = \int dy' dz' \psi'_0(x, y', z', k_\perp^2)$$

LFWFs: $\psi_0(x, k_\perp^2)$ & $\psi_1(x, k_\perp^2)$



$$\psi_i^N(x, k_\perp^2) = \frac{\psi_i(x, k_\perp^2)}{\int_0^1 dx \psi_i(x, k_\perp^2)}.$$

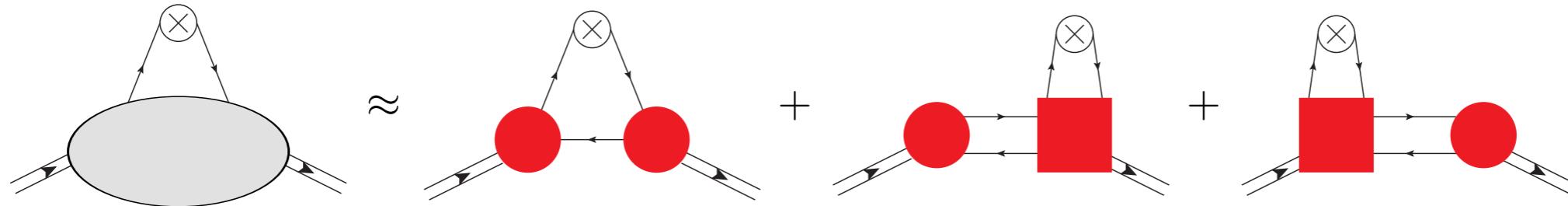


- Strong support at IR, a consequence of the DCSB which generates significant strength in the infrared region of BS wave function.
- In the ultraviolet region, $\psi_0(x, k_\perp^2) \sim 1/k_\perp^2$ and $\psi_1(x, k_\perp^2) \sim 1/k_\perp^4$. Embed in DSEs (one-gluon exchange) and agree with pQCD prediction. (Ji et al, EPJC 33,75)

- Symmetric respect to $x=1/2$, u-d iso-spin symmetry.
- $\psi_0(x, k_\perp^2)$ broad concave curves shrinks to the conformal limit $6x(1-x)$ as k_T increases.

Covariant approach

$$f(x, k_\perp^2) = -\frac{1}{2} \int \frac{d^2 \hat{k}}{(2\pi)^4} \delta(k \cdot n - x P \cdot n) \text{Tr}_{\text{CD}}[i\eta G(k, P)]$$



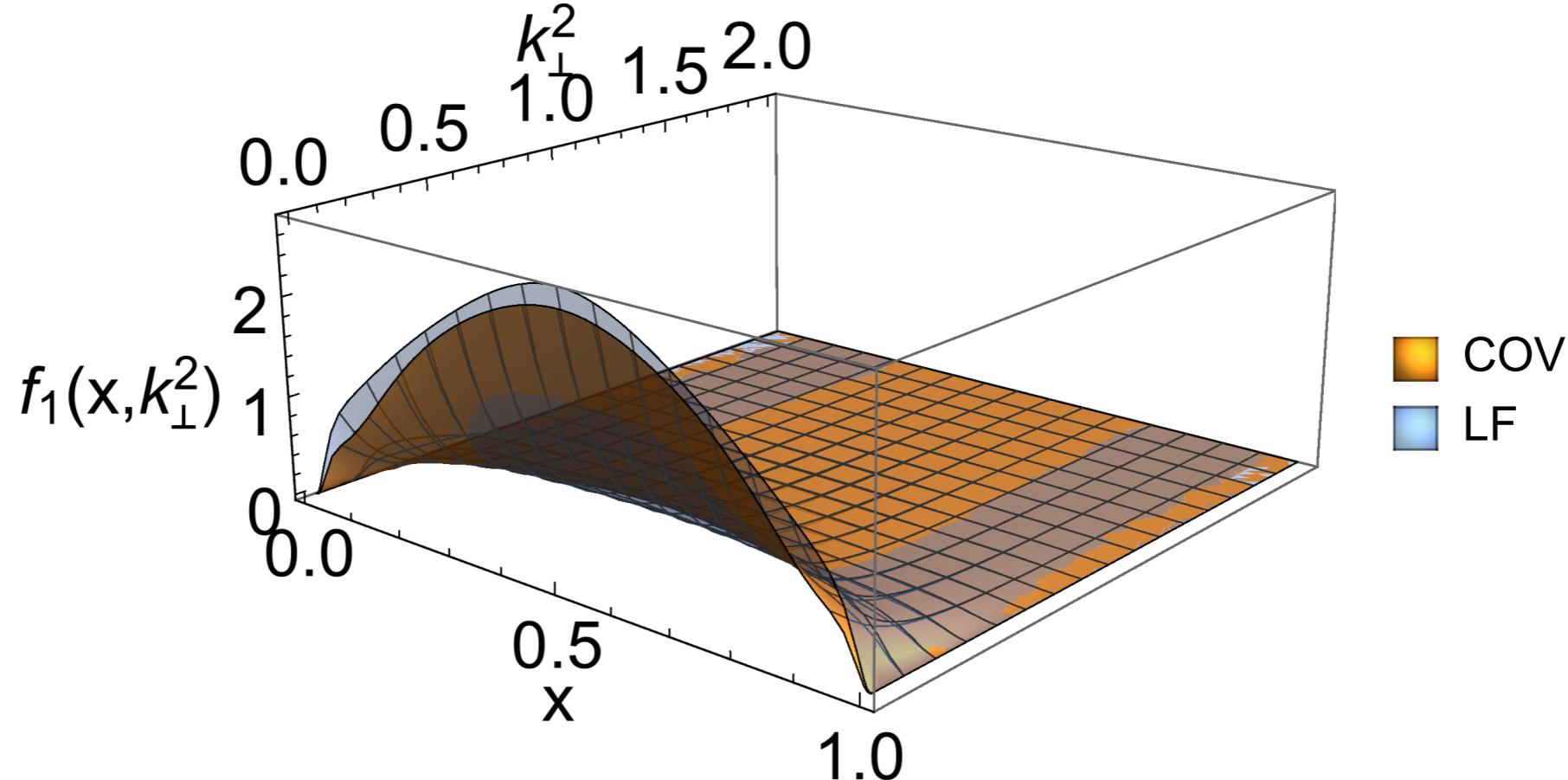
Full contribution in RL truncation. Additional contribution from the twist-two operators acting on the Bethe-Salpeter amplitude. Respects the nonlocal properties of the pion wave function.

Give pion's pure valence picture as two fully dressed quarks, which carry all the pion's light-front momentum at a characteristic hadronic scale. (Lei Chang et al PLB 2014)

$$f(x, k_\perp^2, \mu_0) = -\frac{N_c}{2} \int \frac{d\hat{k}^2}{(2\pi)^4} \delta(n \cdot k - xn \cdot P) \text{Tr}_D \{ n \cdot \partial_k [\bar{\Gamma}_\pi(k - P/2; -P) S(k)] \Gamma_\pi(k - P/2; P) S(k - P) \}.$$

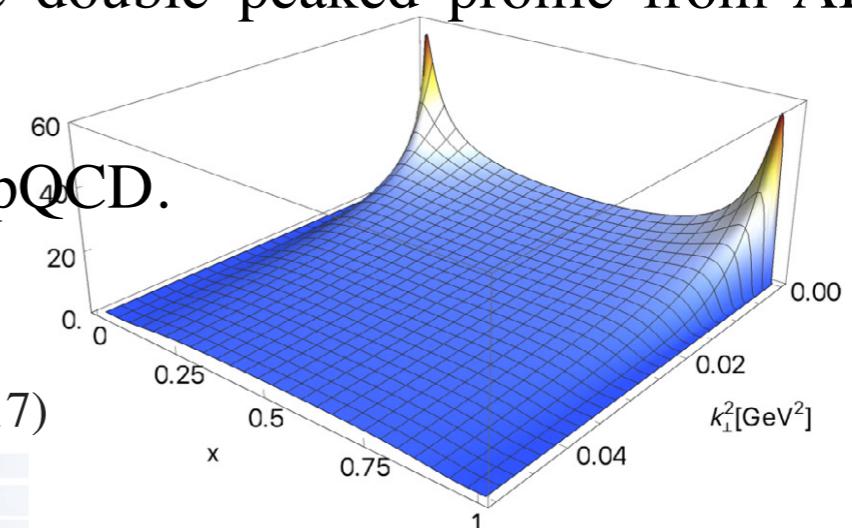
Extracting TMD: Moments -> Feynman parameterization -> change of integral variables.

TMDs from two approaches



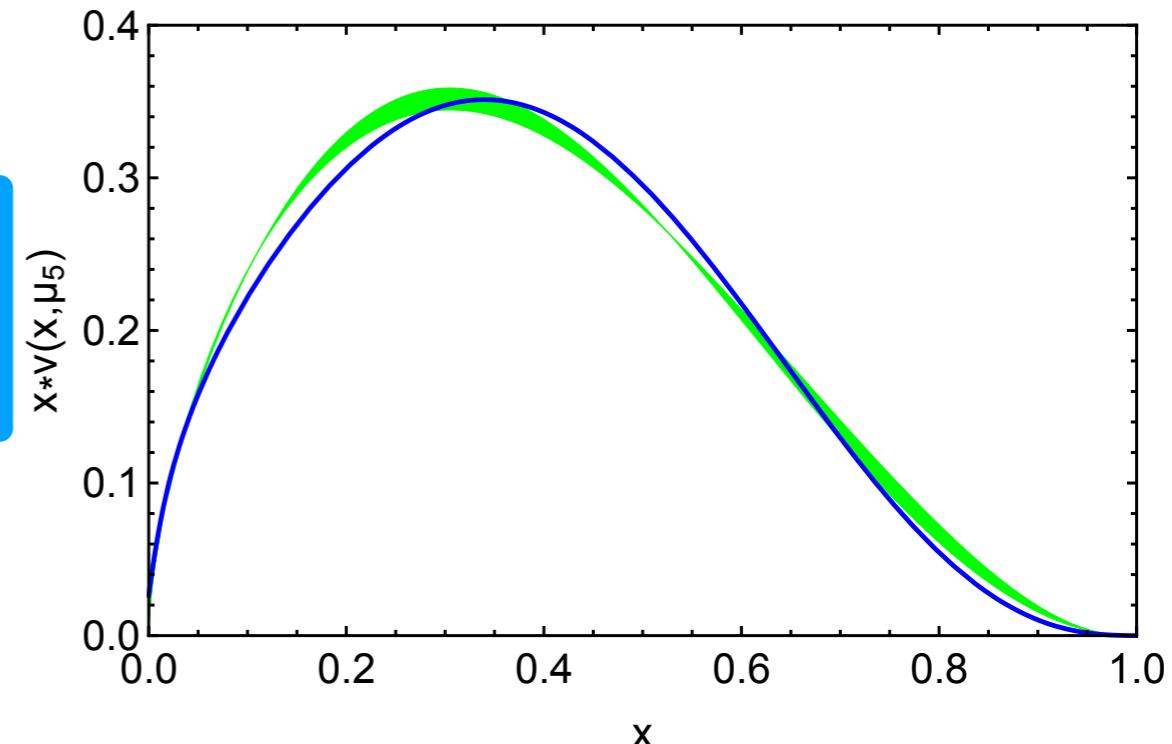
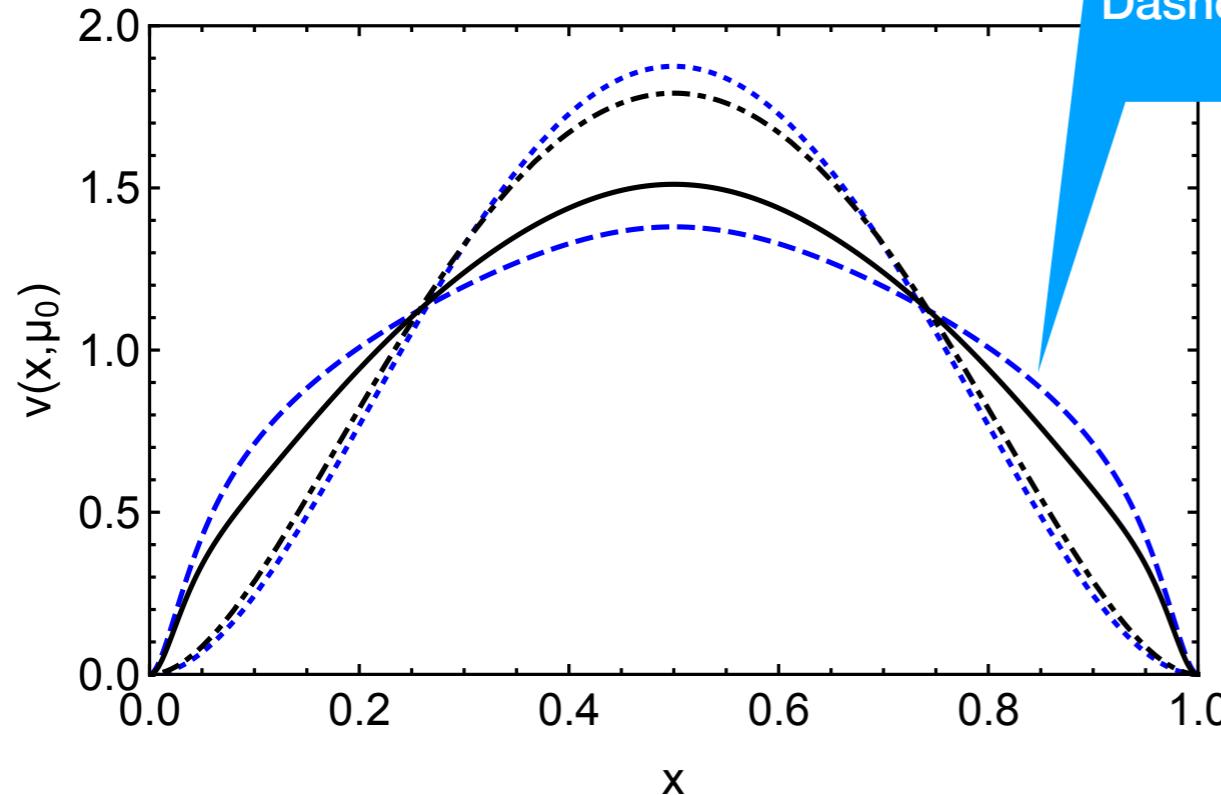
- Unpolarized TMD obtained from the two approaches are close, e.g., their mean square root of deviation is $\sigma = \sqrt{\int_0^1 dx \int_0^1 dk_\perp^2 \left(\frac{f_1^{COV}(x, k_\perp^2) - f_1^{LF}(x, k_\perp^2)}{f_1^{COV}(x, k_\perp^2)} \right)^2} \approx 16\%$
- Both symmetric with respect to $x=1/2$, isospin symmetry manifest.
- Significant strength in the infrared which is relevant for the W-term in TMD factorization. Importantly, they are smooth and unimodal instead of the double peaked profile from ADS QCD.
- They both decrease as $1/k_\perp^4$ in the ultraviolet, agree with pQCD.

(A. Bacchetta, S. Cotogno, B. Pasquini PLB2017)



TMD: Valence PDF

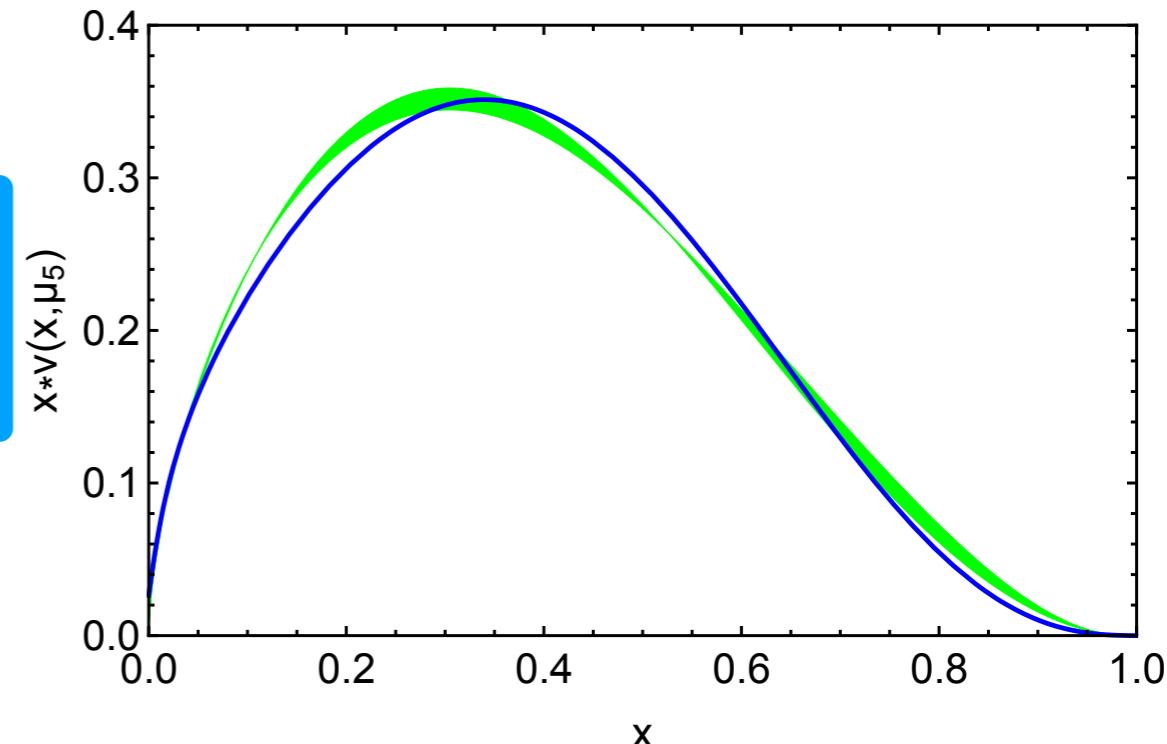
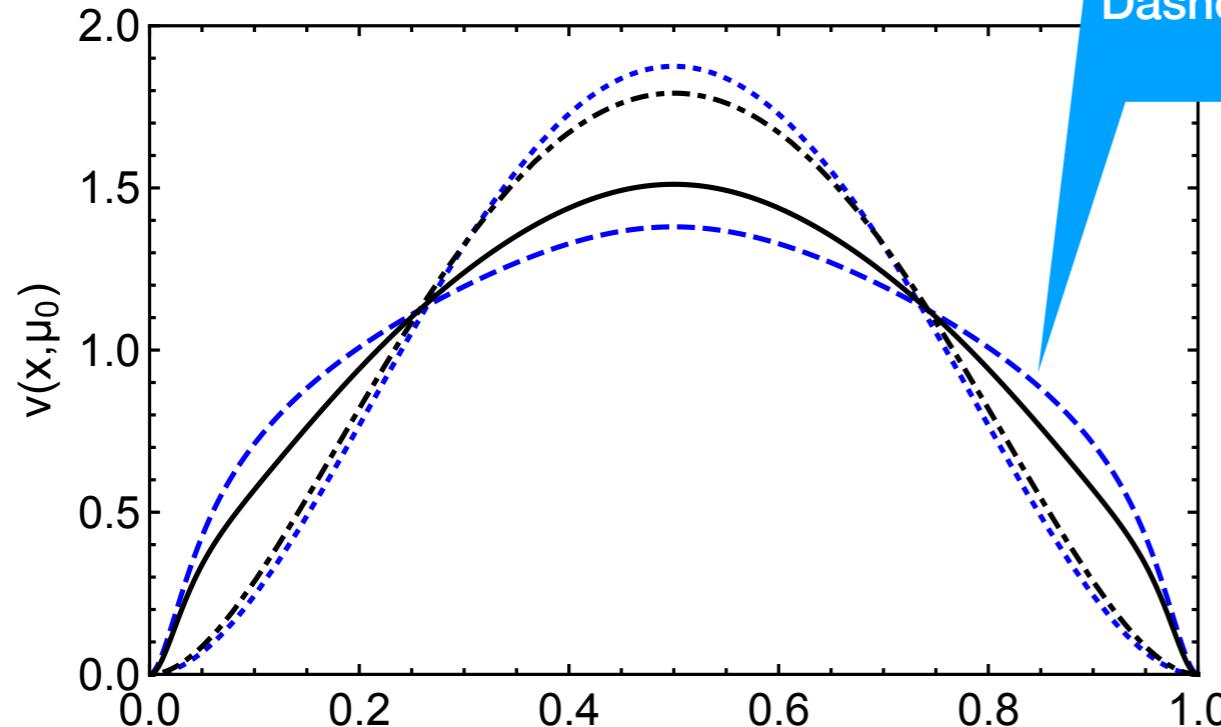
$$v(x; \mu_0) = \int \frac{dk_\perp^2}{(2\pi)^3} f_{1,\pi}(x, k_\perp^2; \mu_0)$$



NLO DGLAP evolution on $v(x, \mu_0)$ performed with the QCDNUM package. $\alpha_s(1\text{GeV}) = 0.491$ and VFNS is taken. The initial scale $\mu_0 = 520$ MeV produces $\langle x \rangle_v^{\mu_2} \sim 0.24$ at $\mu_2 = 2\text{GeV}$, close to the πN Drell-Yan data analysis $2\langle x \rangle_v^{\mu_2} = 0.47(2)$ (Sutton1991, Gluck1999) and lattice simulation $2\langle x \rangle_v^{\mu_2} = 0.48(4)$ (Detmold2003).

TMD: Valence PDF

$$v(x; \mu_0) = \int \frac{dk_\perp^2}{(2\pi)^3} f_{1,\pi}(x, k_\perp^2; \mu_0)$$



NLO DGLAP evolution on $v(x, \mu_0)$ performed with the QCDNUM package. $\alpha_s(1\text{GeV}) = 0.491$ and VFNS is taken. The initial scale $\mu_0 = 520$ MeV produces $\langle x \rangle_v^{\mu_2} \sim 0.24$ at $\mu_2 = 2\text{GeV}$, close to the πN Drell-Yan data analysis $2\langle x \rangle_v^{\mu_2} = 0.47(2)$ (Sutton1991, Gluck1999) and lattice simulation $2\langle x \rangle_v^{\mu_2} = 0.48(4)$ (Detmold2003).

- 📌 The valence PDF obtained from LF & COV approaches generate the green band. The green band is narrow, suggesting a good convergence of two approaches.
- 📌 The green band is close to Aicher et al's (Aicher PRL 2010) NLO analysis on E615 employing the soft-gluon re-summation (blue solid curve).
- 📌 Decrease as $(1-x)^{2+\beta}$ ($\beta>0$) at large x , a consequence of $(1-x)^2$ behavior at hadronic scale and DGLAP evolution. Consistent with pQCD prediction.
- 📌 Note: considering gluon contribution at hadronic scale would increase initial scale.

TMD: Gaussian Ansatz

Gaussian ansatz with non-constant $\langle k_{\perp}^2(x) \rangle_G$

$$f_{1,G}^q(x, k_{\perp}^2) = f_1^q(x, 0) \exp(-k_{\perp}^2 / \langle k_{\perp}^2(x) \rangle_G),$$

$$\langle k_{\perp}^2(x) \rangle_G = f^q(x) / (\pi f_1^q(x, 0)).$$

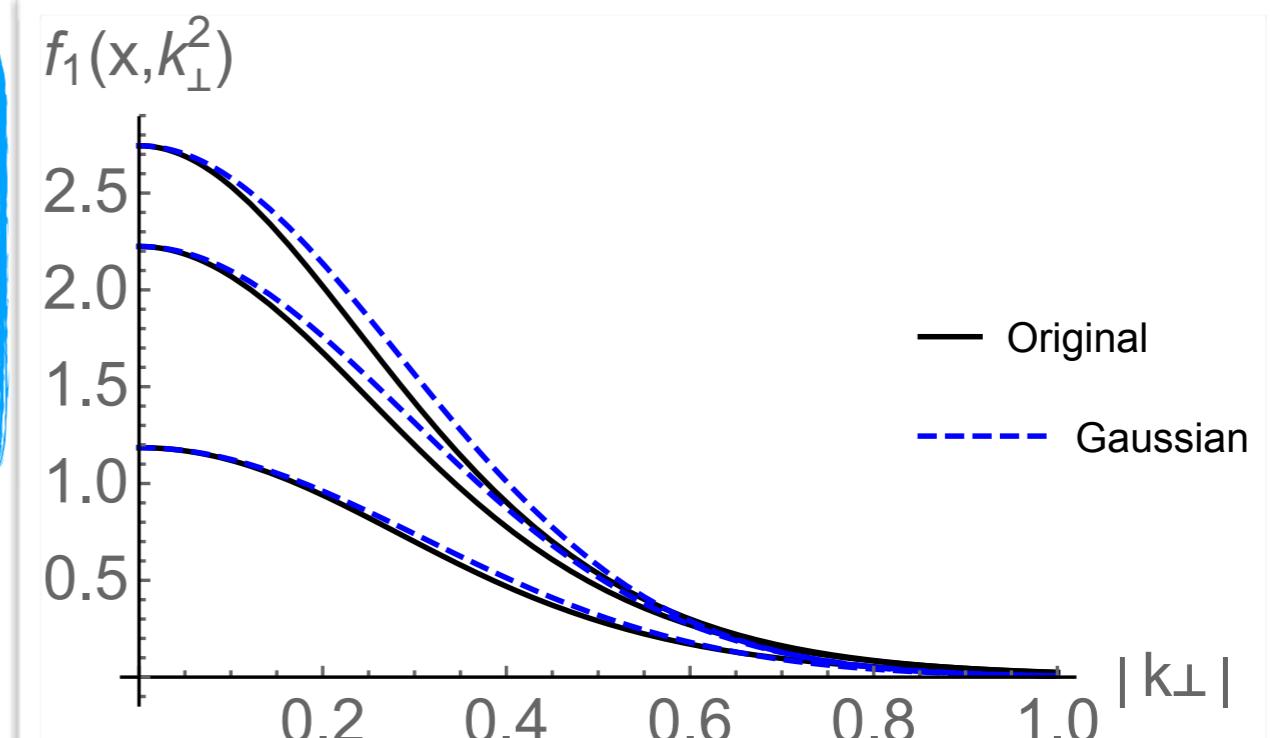


TMD: Gaussian Ansatz

Gaussian ansatz with non-constant $\langle k_\perp^2(x) \rangle_G$

$$f_{1,G}^q(x, k_\perp^2) = f_1^q(x, 0) \exp(-k_\perp^2 / \langle k_\perp^2(x) \rangle_G),$$

$$\langle k_\perp^2(x) \rangle_G = f^q(x) / (\pi f_1^q(x, 0)).$$



From up to bottom, $x=0.5, 0.3, 0.1$ respectively

- The Gaussian Ansatz approximates the profile our TMDs when k_\perp is not large.
- $\langle k_\perp^2(x) \rangle_G$ varies about 15% when x in [0.1,0.9]. Separable k_\perp and x dependence works as an approximation. (Invalid at higher k_\perp)
- $\langle k_\perp^2 \rangle_G \approx 0.19(1)$ GeV 2 within the general estimated region $[(0.3\text{GeV})^2, (0.5\text{GeV})^2]$. (Stanley Brodsky PRD2011).
- TMD evolution?

TMD evolution:

The renormalization group (RG) equation TMD PDF:

$$\mu^2 \frac{d}{d\mu^2} F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta) = \frac{1}{2} \gamma_F^f(\mu, \zeta) F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta) = -\mathcal{D}^f(\mu, \vec{b}) F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta).$$

The scale μ is the standard RG scale, with the additional rapidity factorization scale ζ to regularize the light-cone divergence arising from Wilson lines. They were usually chosen to be the same order of scattering scale.

$$F_{f \leftarrow h}(x, \vec{b}; \mu_f, \zeta_f) = \exp \left[\int_P \left(\gamma_F^f(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}^f(\mu, \vec{b}) \frac{d\zeta}{\zeta} \right) \right] F_{f \leftarrow h}(x, \vec{b}; \mu_i, \zeta_i)$$

TMD PDF at initial scale,
non-perturbative input.

Contains a non-perturbative term of the anomalous dimension D at large b , i.e., $-g_2 b^2$

TMD evolution:

b*-prescription:

$$b^* = b_{\max} \left(\frac{1 - e^{-b_{\perp}^4/b_{\max}^4}}{1 - e^{-b_{\perp}^4/b_{\min}^4}} \right)^{1/4}$$

It saturates at the cut off b_{\max} where non-perturbative (small k_T) TMD PDF dominates. Matching the large and small b_T behavior.

Extraction of partonic transverse momentum distributions from semi-inclusive deep-inelastic scattering, Drell-Yan and Z-boson production (Alessandro Bacchetta et al, JHEP06(2017)081)

- HERMES and COMPASS, SIDIS, $10 \text{ GeV}^2 > Q^2 > 1.4 \text{ GeV}^2$
- E288 and E605, DY, $100 \text{ GeV}^2 > Q^2 > 16 \text{ GeV}^2$, $0.36 > x_F > 0$

ζ -prescription: $(\mu_i, \zeta_i) \rightarrow (\mu_i, \zeta_\mu)$ instead of $(\mu_i, \zeta_i) \rightarrow (\mu_f, \zeta_f)$ where $\mu^2 \frac{dF(x, \mathbf{b}; \mu, \zeta_\mu)}{d\mu^2} = 0$

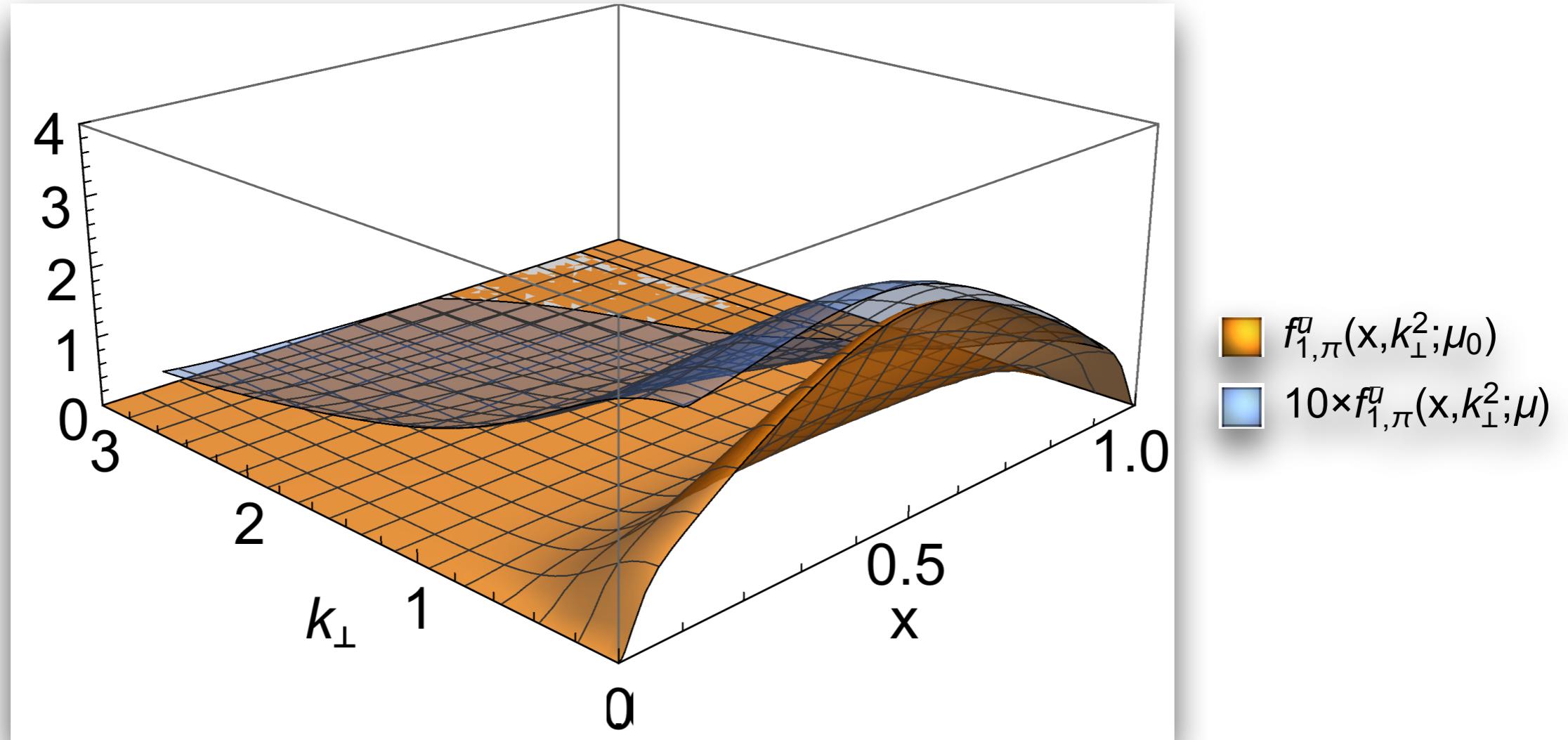
Allows to minimize the impact of perturbative logarithms in a large range of scales and does not generate undesired power corrections.

Analysis of vector boson production within TMD factorization (Alexey Vladimirov et al, Eur. Phys. J. C (2018) 78:89)

- E288: Drell-Yan process, at $4 < Q < 14 \text{ GeV}$.
- CDF/D0: Z-boson production at $\sqrt{s} = 1.8, 1.96 \text{ TeV}$.
- ATLAS/CMS/LHCb: Z-boson production at $s = 7, 8, 13 \text{ TeV}$.
- ATLAS: Vector boson production outside the Z-peak ($46 < Q < 66$ and $116 < Q < 150 \text{ GeV}$) at $\sqrt{s} = 8 \text{ TeV}$.

arTeMiDe code

TMD evolution:



- Evolution has a significant effect, leading to approximately an order of magnitude of suppression at small k_T , and a broad tail at larger k_T .
- The evolved TMD PDF at smaller x is broader than that at large x (No longer symmetric in x , No factorized k_T dependence).

Experiment?



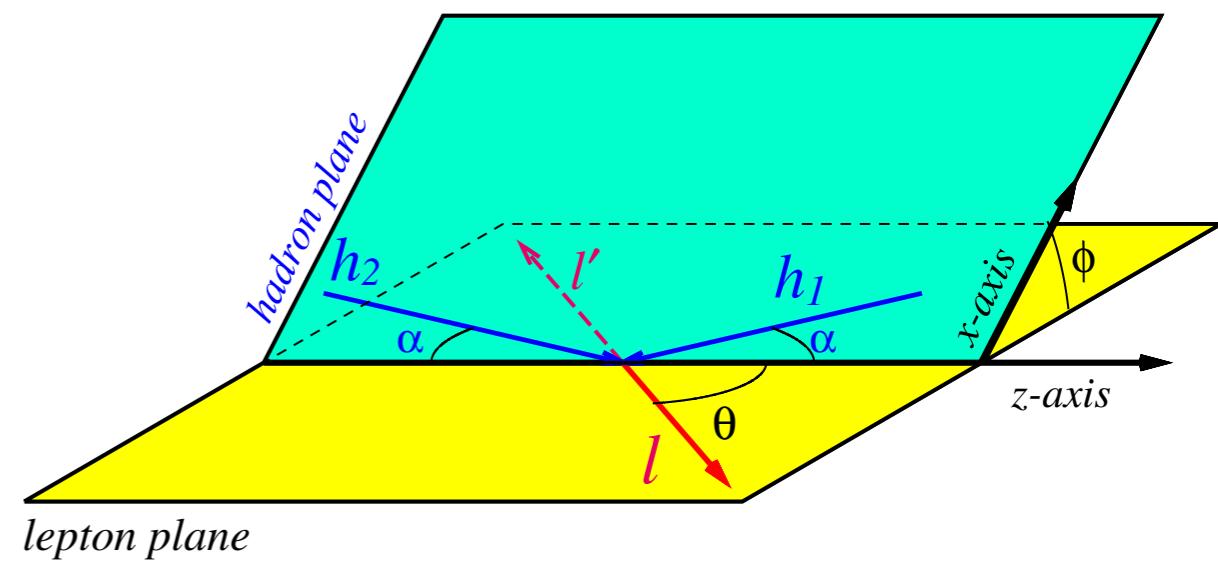
Drell-Yan Process

Lorentz invariant
structure functions

General form of the DY cross section:

$$\frac{d\sigma}{d^4 q d\Omega} = \frac{\alpha_{\text{em}}^2}{F q^2} \left\{ \begin{array}{l} ((1 + \cos^2\theta) F_{UU}^1 + (1 - \cos^2\theta) F_{UU}^2 + \sin 2\theta \cos \phi F_{UU}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{UU}^{\cos 2\phi}) \\ + S_{aL} (\sin 2\theta \sin \phi F_{LU}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{LU}^{\sin 2\phi}) + S_{bL} (\sin 2\theta \sin \phi F_{UL}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{UL}^{\sin 2\phi}) \\ + |\vec{S}_{aT}| [\sin \phi_a ((1 + \cos^2\theta) F_{TU}^1 + (1 - \cos^2\theta) F_{TU}^2 + \sin 2\theta \cos \phi F_{TU}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{TU}^{\cos 2\phi}) \\ + \cos \phi_a (\sin 2\theta \sin \phi F_{TU}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{TU}^{\sin 2\phi})] + |\vec{S}_{bT}| [\sin \phi_b ((1 + \cos^2\theta) F_{UT}^1 + (1 - \cos^2\theta) F_{UT}^2 \\ + \sin 2\theta \cos \phi F_{UT}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{UT}^{\cos 2\phi}) + \cos \phi_b (\sin 2\theta \sin \phi F_{UT}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{UT}^{\sin 2\phi})] \\ + S_{aL} S_{bL} ((1 + \cos^2\theta) F_{LL}^1 + (1 - \cos^2\theta) F_{LL}^2 + \sin 2\theta \cos \phi F_{LL}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{LL}^{\cos 2\phi}) \\ + S_{aL} |\vec{S}_{bT}| [\cos \phi_b ((1 + \cos^2\theta) F_{LT}^1 + (1 - \cos^2\theta) F_{LT}^2 + \sin 2\theta \cos \phi F_{LT}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{LT}^{\cos 2\phi}) \\ + \sin \phi_b (\sin 2\theta \sin \phi F_{LT}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{LT}^{\sin 2\phi})] + |\vec{S}_{aT}| S_{bL} [\cos \phi_a ((1 + \cos^2\theta) F_{TL}^1 + (1 - \cos^2\theta) F_{TL}^2 \\ + \sin 2\theta \cos \phi F_{TL}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{TL}^{\cos 2\phi}) + \sin \phi_a (\sin 2\theta \sin \phi F_{TL}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{TL}^{\sin 2\phi})] \\ + |\vec{S}_{aT}| |\vec{S}_{bT}| [\cos(\phi_a + \phi_b) ((1 + \cos^2\theta) F_{TT}^1 + (1 - \cos^2\theta) F_{TT}^2 + \sin 2\theta \cos \phi F_{TT}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{TT}^{\cos 2\phi}) \\ + \cos(\phi_a - \phi_b) ((1 + \cos^2\theta) \bar{F}_{TT}^1 + (1 - \cos^2\theta) \bar{F}_{TT}^2 + \sin 2\theta \cos \phi \bar{F}_{TT}^{\cos\phi} + \sin^2\theta \cos 2\phi \bar{F}_{TT}^{\cos 2\phi}) \\ + \sin(\phi_a + \phi_b) (\sin 2\theta \sin \phi F_{TT}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{TT}^{\sin 2\phi}) \\ + \sin(\phi_a - \phi_b) (\sin 2\theta \sin \phi \bar{F}_{TT}^{\sin\phi} + \sin^2\theta \sin 2\phi \bar{F}_{TT}^{\sin 2\phi})] \end{array} \right\}$$

q is the di-lepton momentum, θ and ϕ are the angles in the Collins-Soper frame.



Collins-Soper frame

Drell-Yan Process

Experiment (E615)

Transverse momentum dependence parameterized by function $P(q_T; x_F, m_{\mu\mu})$ (**DATA!**)

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} = \frac{d^2\sigma}{dx_\pi dx_N} P(q_T; x_F, m_{\mu\mu}).$$

$$q^0 = \frac{\sqrt{s}}{2}(x_\pi + x_N)$$

$$q^3 = \frac{\sqrt{3}}{2}(x_\pi - x_N)$$

"Experimental study of muon pairs produced by 252-GeV pions on tungsten", Conway, J.S. et al.
Phys.Rev. D39 (1989) 92-122.

Theory

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} \propto |q_T| F_{uu}^1(x_\pi, x_N, q_T) \quad (\text{leading twist})$$

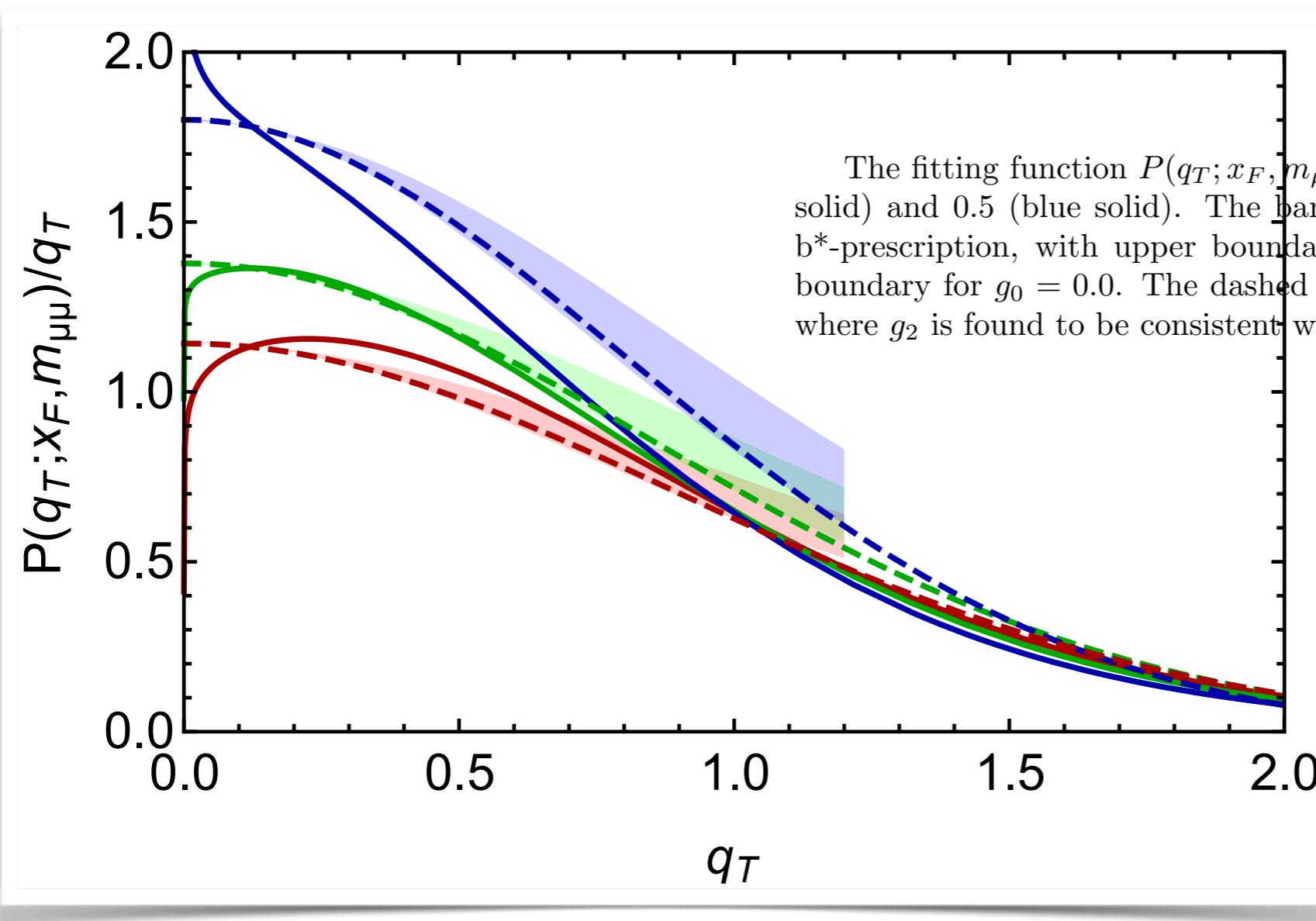
TMD formalism: $F_{UU}^1(x_1, x_2, q_T) = \frac{1}{N_c} \sum_a e_a^2 \int d^2 k_{1\perp} d^2 k_{2\perp} \delta^{(2)}(\mathbf{q}_T - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) f_{1,\pi}^{\bar{a}}(x_1, \mathbf{k}_{1\perp}^2) f_{1,N}^a(x_2, \mathbf{k}_{2\perp}^2)$

offer by DSEs&evolution

borrow from global analysis

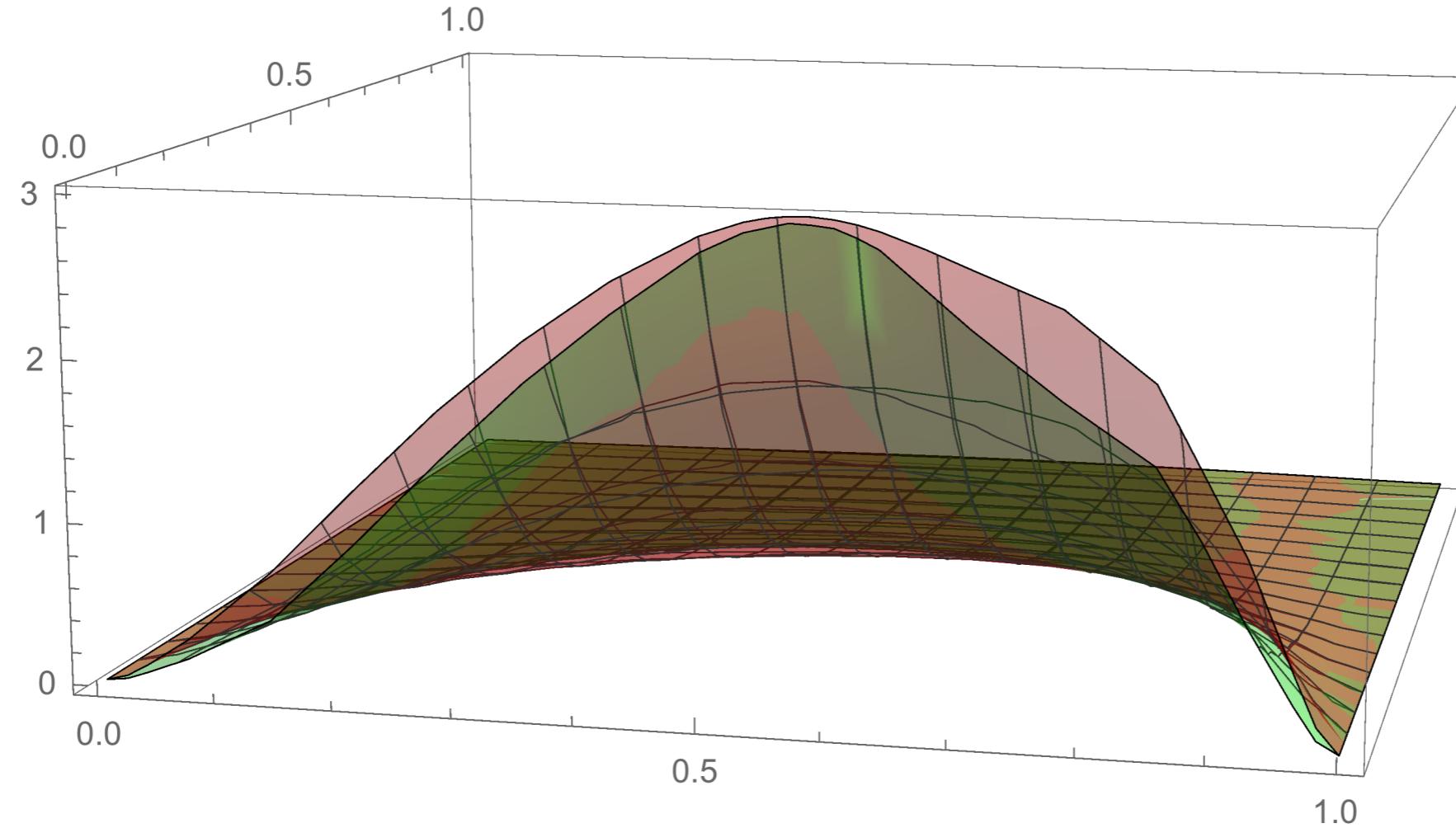
Examine: $P(q_T; x_F, m_{\mu\mu}) \propto |q_T| F_{UU}^1(q_T; x_F, \tau)$

Drell-Yan Process



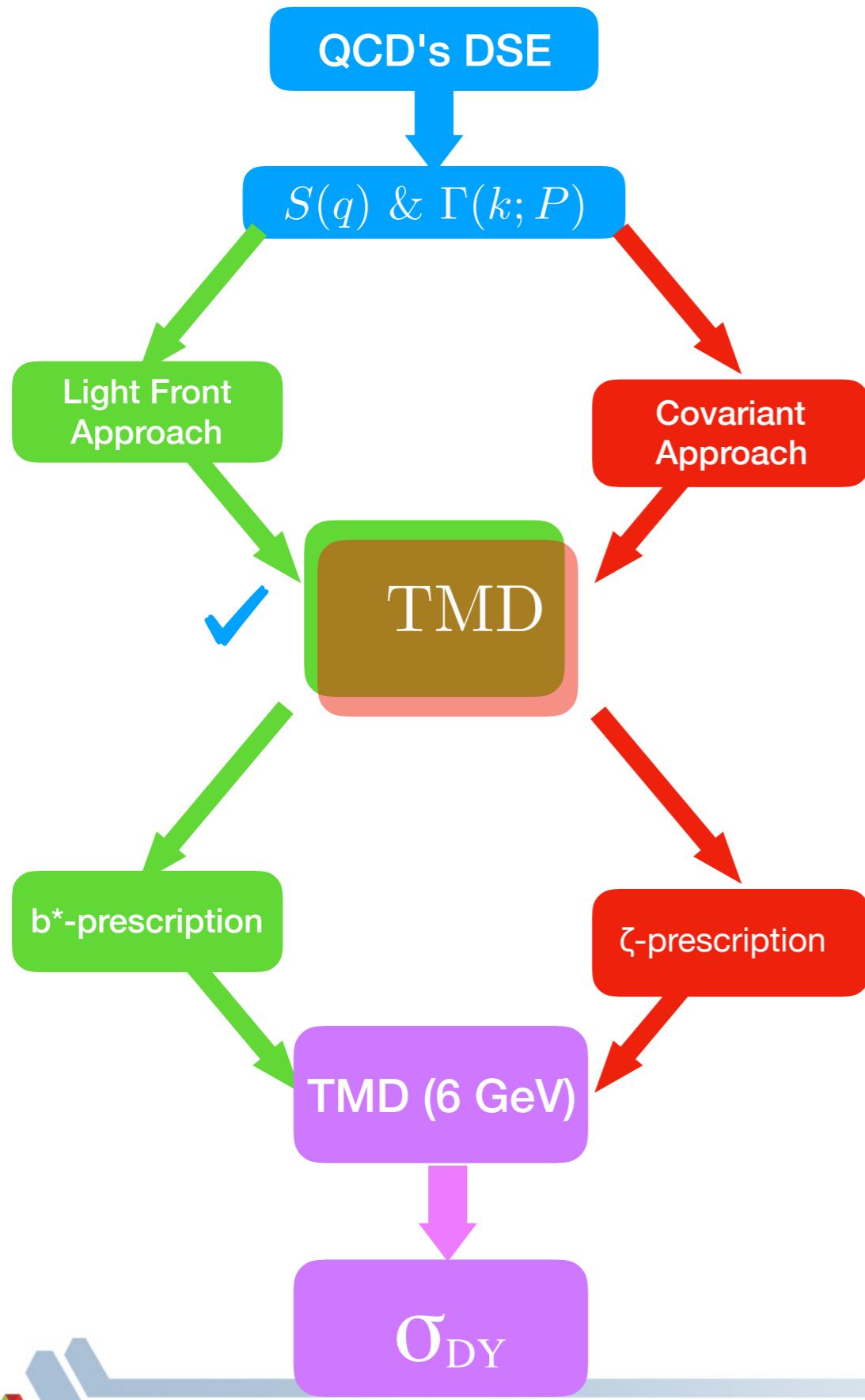
- Our results using two evolution schemes generally agree with E615 measurement. In particular, when g_2 goes to zero as suggested by ζ -prescription at higher order. The deviation is less than 10% for $x_F = 0$ and $x_F = 0.25$, and increases to 30% at most for $x_F = 0.5$.
- Our calculation also shows the TMD formalism becomes less valid as x_F goes larger, where TMD formalism is believed to be less valid. Berger- Brodsky effect with pion distribution amplitude.

Flavor Dependence of TMD PDF in Kaon



- Non-symmetric in $x=0.5$, skewed with s quark carrying more longitudinal momentum fraction.
- $f^u(x, k_\perp^2) = f^s(1 - x, k_\perp^2)$, not the simple flavor independence in k_\perp .
- The width of transverse momentum increases by about 10%, m_s/m_u masked by DCSB effect.

Summary



Findings & Conclusion

1. Based on two approaches, the TMDs at hadronic scale have the smooth and unimodal, different from holographic QCD prediction.
2. Gaussian ansatz serves as an approximation to TMD in the IR at hadronic scale.
3. Evolved TMD gets suppressed in magnitude with a broader tail.
4. Good agreement with E615 data on the transverse momentum dependence in the TMD region.
5. Flavor dependence in TMD PDF from kaon.

Outlook

1. Boer-Mulders function: careful treatment on the Wilson line; positivity relation to be satisfied.
2. Nucleon: DSEs ready; LFWF projections ready; formulation and computation!

Thank
you