Pion and proton TMD phenomenology

Patrick Barry, Leonard Gamberg, Wally Melnitchouk, Eric Moffat, Daniel Pitonyak, Alexei Prokudin, Nobuo Sato

Meson SF, Jan 19th, 2023

Jefferson Lab
Exploring the Nature of Matter

Light-front correlator

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$$
$$= \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\big[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\big],$$

$$b \equiv (b^-, 0^+, \boldsymbol{b}_T)$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- We can learn about the coordinate space correlations of quark fields in hadrons

Factorization for low-q_T Drell-Yan

- Like collinear observable, a hard part with two functions that describe structure of beam and target
- So called "W"-term, valid only at low-q_T

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau \mathrm{d}Y \mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2,\mu) \int \mathrm{d}^2 b_T \, e^{i b_T \cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2),$$

Small b_T operator product expansion

- At small b_T, the TMDPDF can be described in terms of its OPE:

$$\tilde{f}_{f/h}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{f/j}(x/\xi, b_T; \zeta_F, \mu) f_{j/h}(\xi; \mu) + \mathcal{O}((\Lambda_{\mathrm{QCD}} b_T)^a)$$

- where $\tilde{C}$ are the Wilson coefficients, and $f_{j/h}$ is the collinear PDF
- Breaks down when b_T gets large

b_* prescription

- A common approach to regulating large b_T behavior

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics

- At small b_T, $b_*(b_T) = b_T$
- At large b_T, $b_*(b_T) = b_{\max}$

Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large b_T behavior

Completely general – independent of quark, hadron, PDF or FF

$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{j/H}(x, \boldsymbol{b}_{\mathrm{T}}; b_{\max})} = \frac{\tilde{f}_{j/H}(x, \boldsymbol{b}_{\mathrm{T}}; \zeta, \mu)}{\tilde{f}_{j/H}(x, \boldsymbol{b}_*; \zeta, \mu)} e^{g_K(b_{\mathrm{T}}; b_{\max}) \ln(\sqrt{\zeta}/Q_0)}.$$

Full description of the TMD

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)$$
$$\times \exp\left\{ -g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q) \right\}$$

- Have individual pieces that are sensitive to low-b_T spectrum (perturbative) and the high-b_T (non-perturbative)

TMD factorization in Drell-Yan

- In small-q_{T} region, use the Collins-Soper-Sterman (CSS) formalism and b_* prescription

Can these data constrain the pion collinear PDF?

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q))\int\frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2}e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}$$

$$\times\, e^{-g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})}\int_{x_A}^1\frac{\mathrm{d}\xi_A}{\xi_A}\boxed{f_{j_A/A}(\xi_A;\mu_{b_*})}\,\tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})}\int_{x_B}^1\frac{\mathrm{d}\xi_B}{\xi_B}f_{j_B/B}(\xi_B;\mu_{b_*})\,\tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\exp\left\{-\boxed{g_K}(b_{\mathrm{T}};b_{\max})\ln\frac{Q^2}{Q_0^2}+\tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2}+\int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu'))-\ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Non-perturbative pieces

Perturbative pieces

Non-perturbative piece of the CS kernel

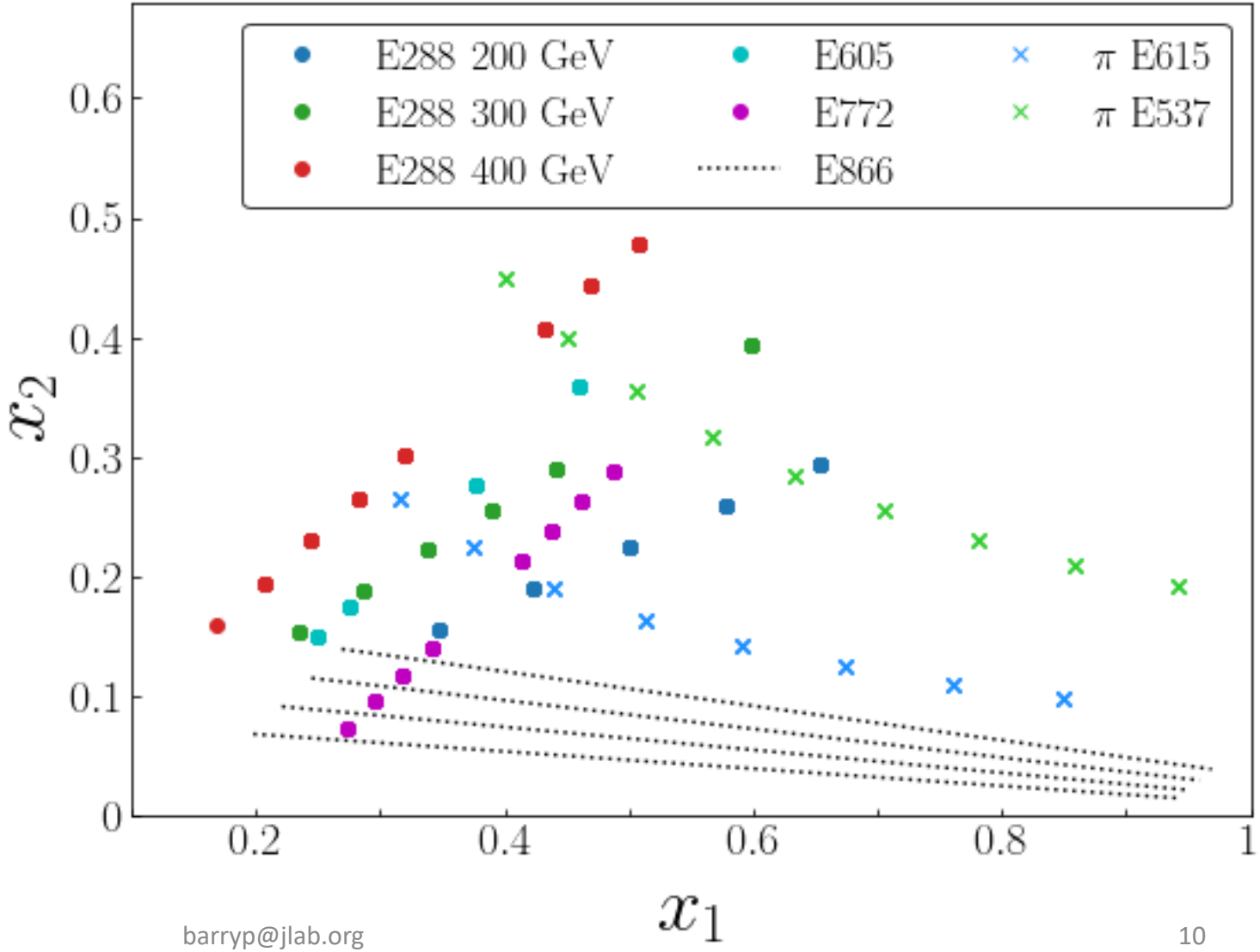
Datasets in the analysis

Expt.	$\sqrt{s}$ (GeV)	Reaction	Observable	Q (GeV)	x_F or y	$N_{\text{pts.}}$
E288 [39]	19.4	$p + Pt \to \ell^+\ell^- X$	$E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$	4 – 9	$y = 0.4$	38
E288 [39]	23.8	$p + Pt \to \ell^+\ell^- X$	$E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$	4 – 12	$y = 0.21$	48
E288 [39]	24.7	$p + Pt \to \ell^+\ell^- X$	$E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$	4 – 14	$y = 0.03$	74
E605 [40]	38.8	$p + Cu \to \ell^+\ell^- X$	$E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$	7 – 18	$x_F = 0.1$	49
E772 [41]	38.8	$p + D \to \ell^+\ell^- X$	$E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$	5 – 15	$0.1 \le x_F \le 0.3$	61
E866 [50]	38.8	$p + Fe \to \ell^+\ell^- X$	R_{FeBe}	4 – 8	$0.13 \le x_F \le 0.93$	10
E866 [50]	38.8	$p + W \to \ell^+\ell^- X$	R_{WBe}	4 – 8	$0.13 \le x_F \le 0.93$	10
E537 [38]	15.3	$\pi^- + W \to \ell^+\ell^- X$	$\mathrm{d}^2\sigma/\mathrm{d}x_F\mathrm{d}q_T$	4 – 9	$0 < x_F < 0.8$	48
E615 [4]	21.8	$\pi^- + W \to \ell^+\ell^- X$	$\mathrm{d}^2\sigma/\mathrm{d}x_F\mathrm{d}q_T^2$	4.05 – 8.55	$0 < x_F < 0.8$	45

- Total of 383 number of points
- All fixed target, low-energy data

Kinematics in x_1, x_2

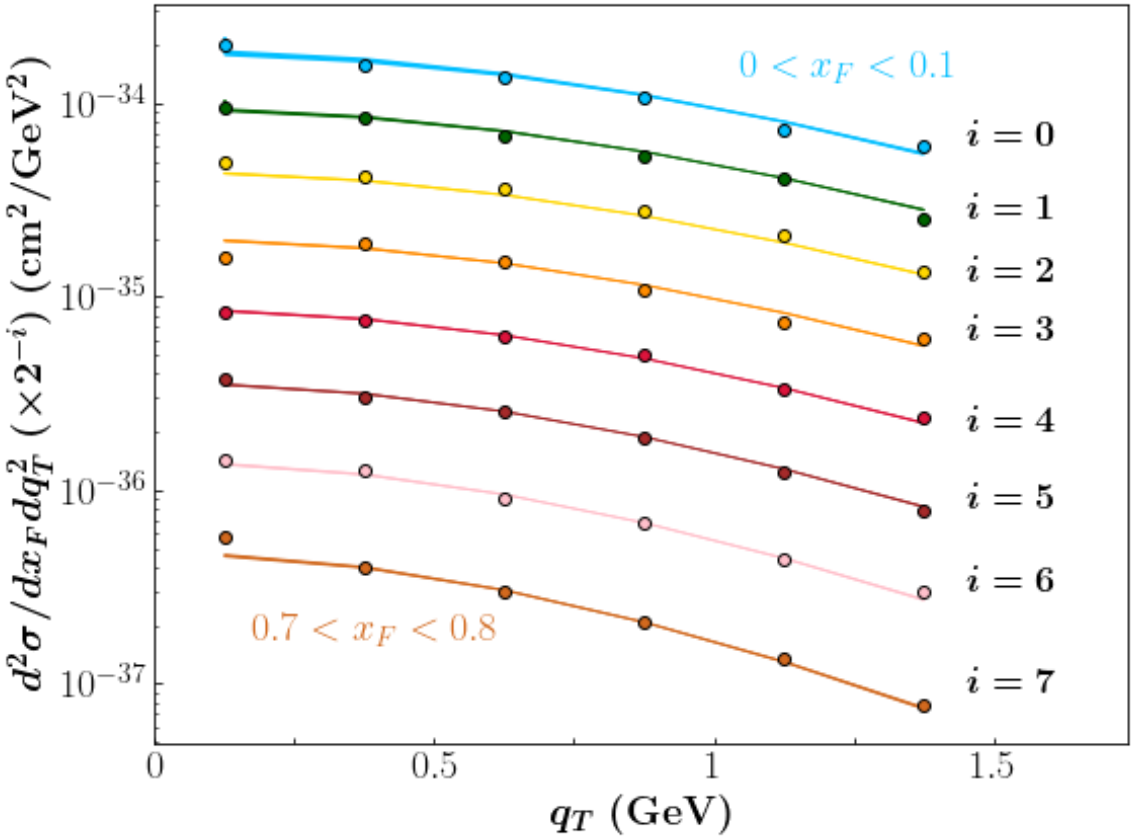
- Using the kinematic mid-point from each of the bins, we show the range in x_1 and x_2



Data and theory agreement

- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	$\sqrt{s}$ GeV	χ^2/np	Z-score
q_T-*integr. DY*	E615 [37]	21.8	0.86	0.76
$\pi W \to \mu^+\mu^- X$	NA10 [38]	19.1	0.54	2.27
	NA10 [38]	23.2	0.91	0.18
Leading neutron	H1 [73]	318.7	0.36	4.61
$ep \to e'nX$	ZEUS [74]	300.3	1.48	2.16
q_T-*dep. pA DY*	E288 [67]	19.4	0.93	0.25
$pA \to \mu^+\mu^- X$	E288 [67]	23.8	1.33	1.54
	E288 [67]	24.7	0.95	0.23
	E605 [68]	38.8	1.07	0.39
	E772 [69]	38.8	2.41	5.74
	E866 (Fe/Be) [70]	38.8	1.07	0.29
	E866 (W/Be) [70]	38.8	0.89	0.11
q_T-*dep.* πA *DY*	E615 [37]	21.8	1.61	2.58
$\pi W \to \mu^+\mu^- X$	E537 [71]	15.3	1.11	0.57
Total			1.15	2.55



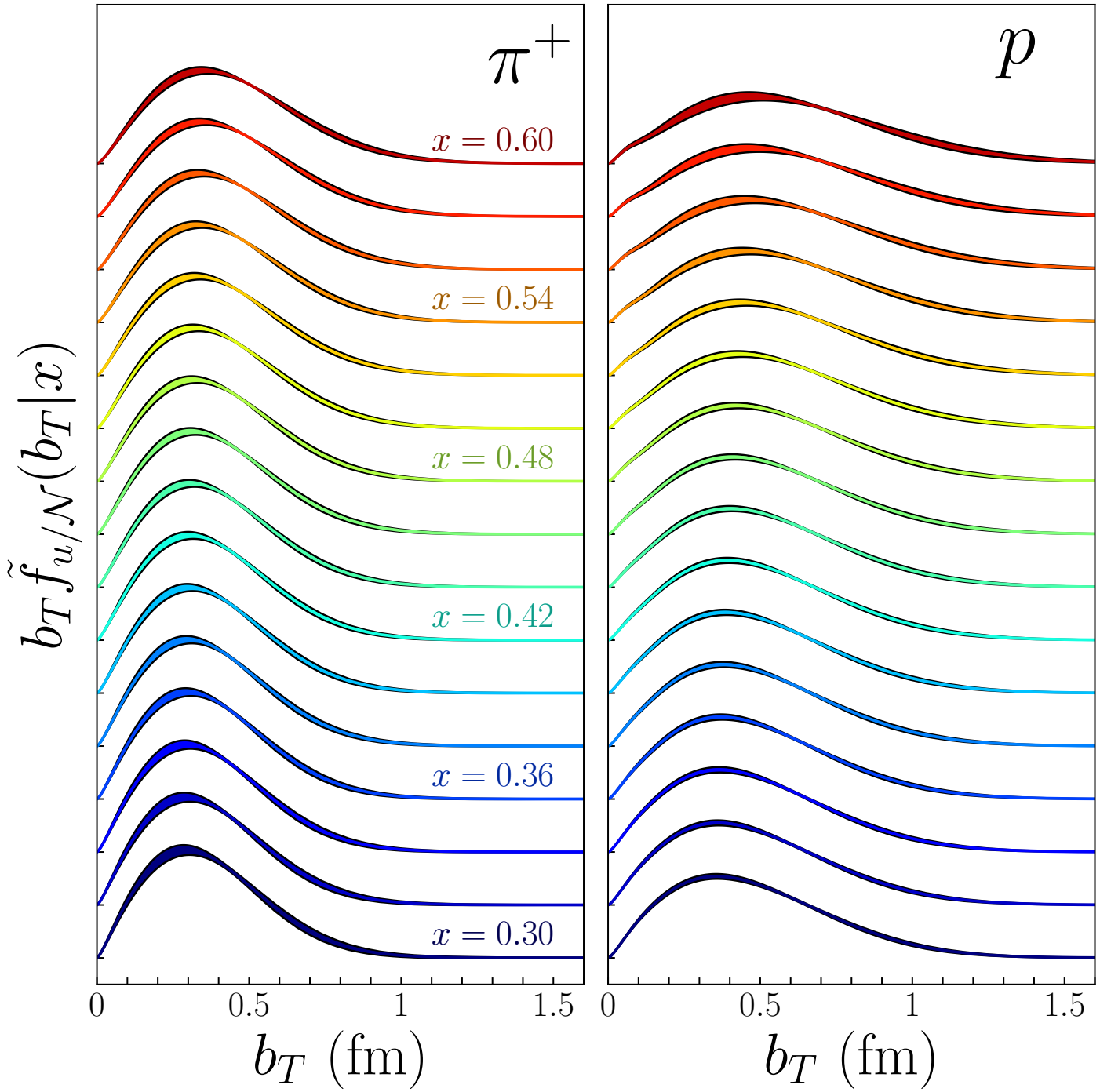
Conditional density

- We define a quantity in which describes the ratio of the 2-dimensional density to the integrated, b_T-independent number density

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)} \,.$$

Resulting TMD PDFs of proton and pion

- Shown in the range where pion and proton are both constrained
- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



Average b_T

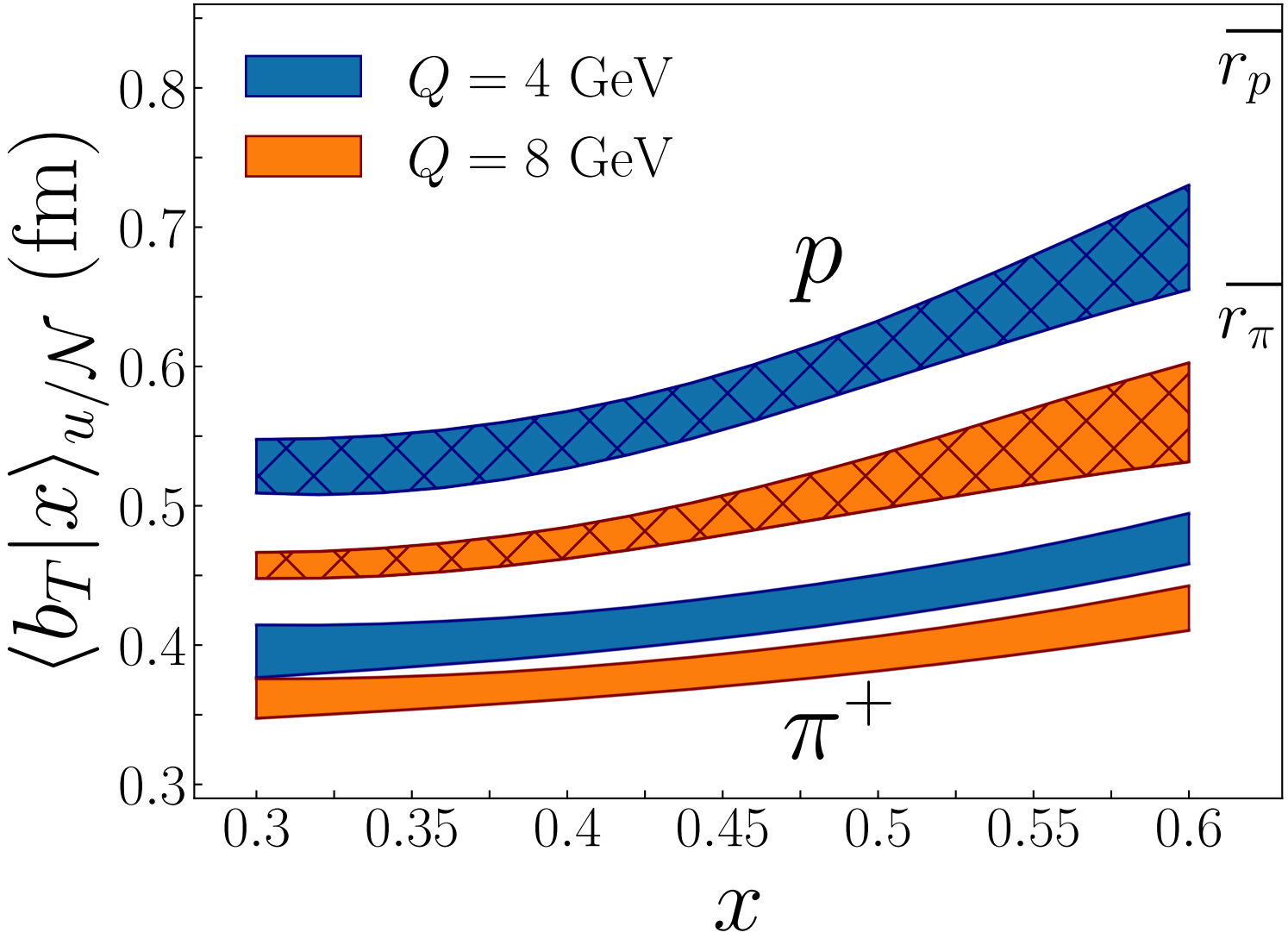
- The conditional expectation value of b_T for a given x

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int \mathrm{d}^2 \boldsymbol{b}_T \, b_T \, \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$

- Shows a measure of the transverse correlation in coordinate space of the quark in a hadron for a given x

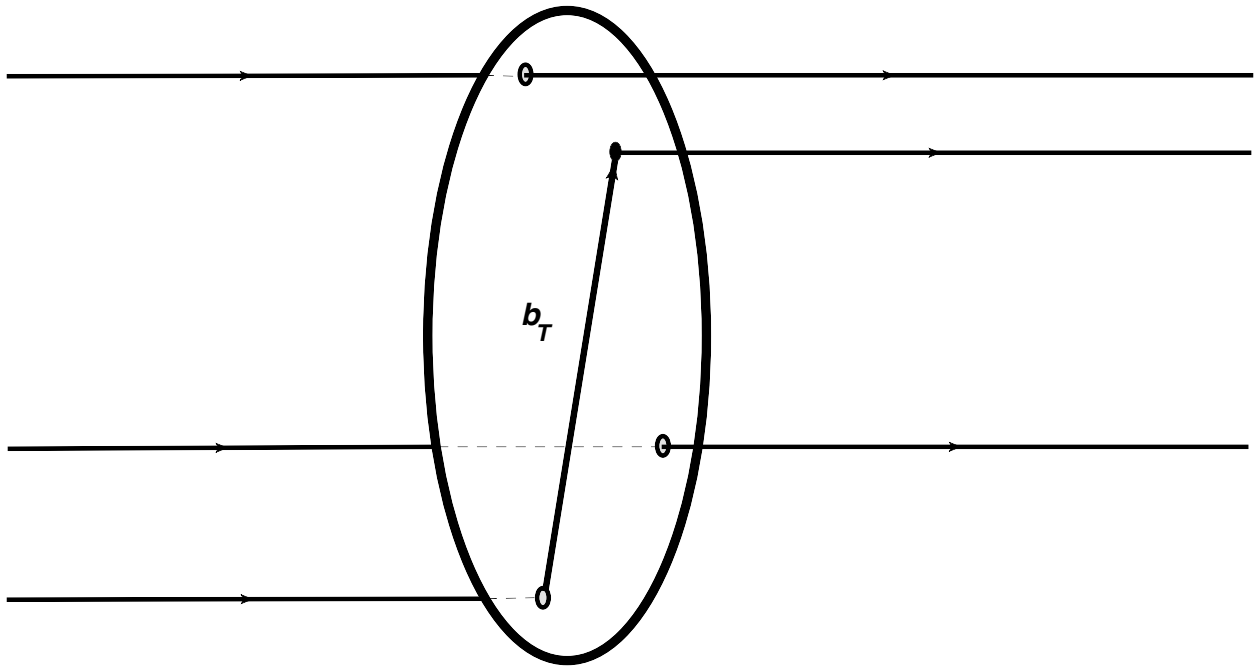
Resulting average b_T

- Up quark in proton is ~ 1.2 times bigger than that of pion
- Pion's $\langle b_T | x \rangle$ is $5.3 - 7.5\sigma$ smaller than proton in this range
- Decreases as x decreases



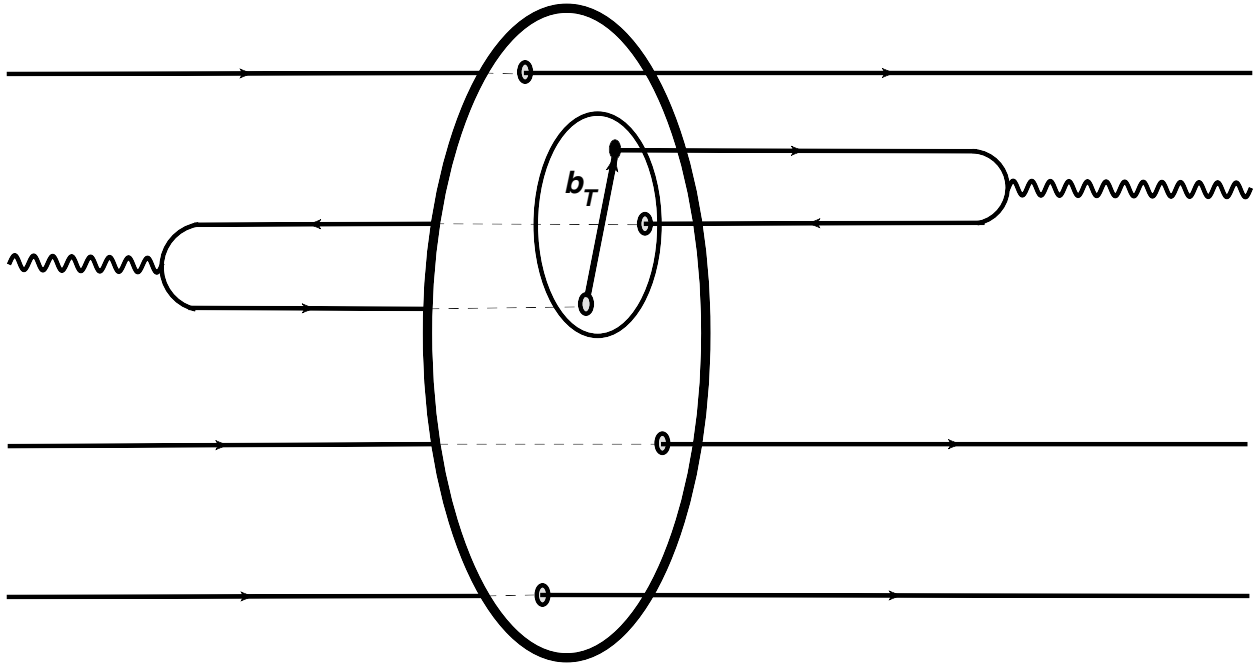
Possible explanation

- At large x, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron



Possible explanation

- At small x, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system



Future work – pion SIDIS

$$eN \rightarrow e'N'\pi X$$

- Measure an outgoing pion in the TDIS experiment
- Gives us another observable sensitive to pion TMDs
  - Needed for tests of universality