# Wide-angle Compton scattering

P. Kroll

Universitaet Wuppertal

workshop on 'New opportunities with high intensity photon sources at JLab'

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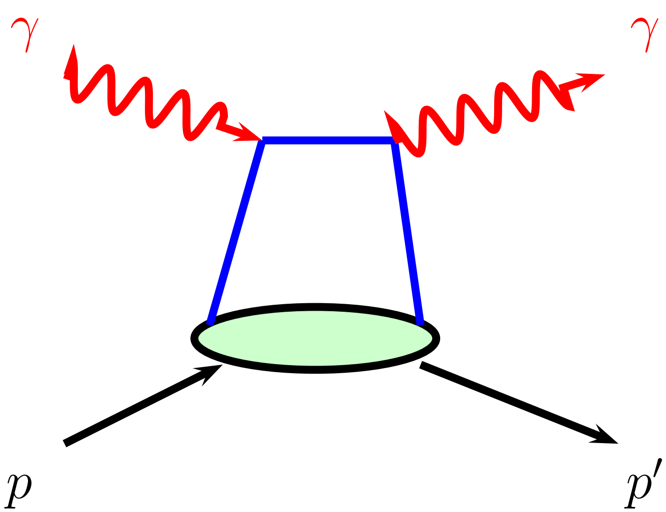
**Outline:**

- **Handbag factorization in excl. reactions**
- **WACS**
- **The Compton form factors**
- **Spin effects**
- **Photoproduction of mesons**
- **Summary**

# Handbag factorization in excl. reactions

(for Compton scattering and corresponding processes)




only one active parton
(others are spectators)

hard process: $\gamma^{(*)} q \to \gamma q$

soft physics: GPDs

two classes of hard exclusive reactions:

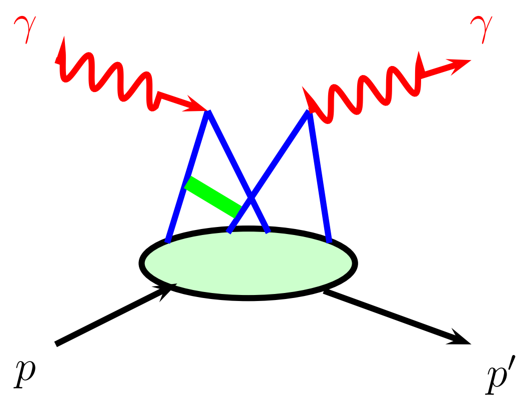
DEEP VIRTUAL $\quad Q^2$ large $\quad -t/Q^2 \ll 1$

complementary

WIDE-ANGLE $\quad -t(-u)$ large $\quad Q^2/(-t) < 1$

# Other topologies

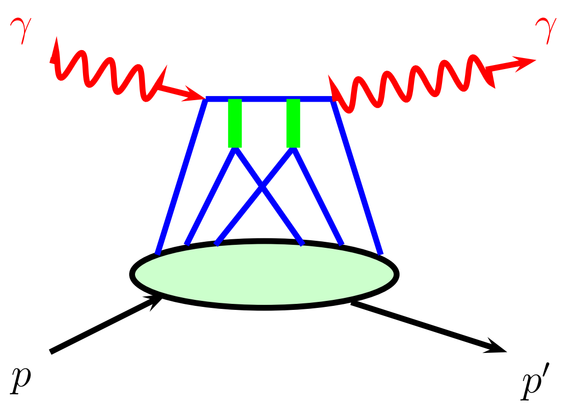
two active partons




cat's ears

large virtualities or large intrinsic
transverse momenta occur in diagram
expected to be suppressed since
at least one hard gluon is to be added

three active partons



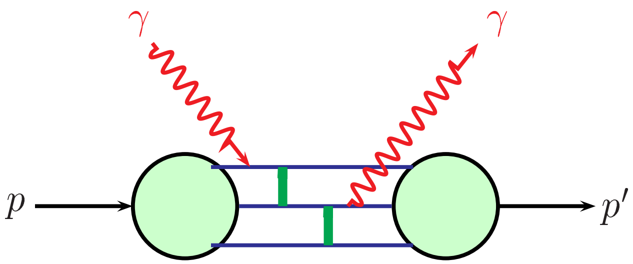

at least two hard gluons required

valence quark approximation $\Longrightarrow$

# Leading-twist result

Brodsky-Lepage (80)

collinear factorization




all valence quarks participate in hard scattering

$\Longrightarrow$ blob decays into two

quarks are moving collinear with their parent hadron

different factorization scheme

hard process: $\gamma qqq \to \gamma qqq$ soft physics: DA $\Phi_p = \Phi_p(x_1, x_2, x_3)$

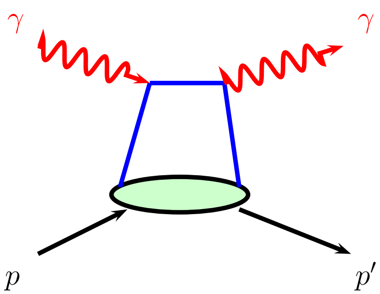
$$\mathcal{M} \sim \Phi_p \otimes \mathcal{H} \otimes \Phi_p$$

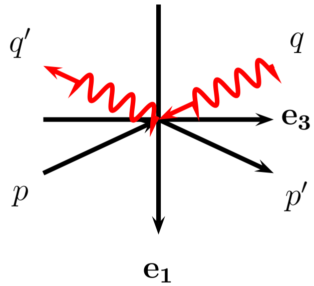
asymptotically dominant (handbag formally a 'power' correction)

for momentum transfer of order of $10\mathrm{GeV}^2$ strongly suppressed $(10^{-2} - 10^{-3})$

onset of leading-twist region probably above $-t(-u) > 100\,\mathrm{GeV}^2$

# The handbag contribution to WACS




$s, -t, -u \gg \Lambda^2$

$\Lambda \sim \mathcal{O}(1\mathrm{GeV})$

typical hadronic scale

- work in a symmetric frame: (otherwise additional contr.)

$$p^{(\prime)} = [p^+, \frac{m^2 - t/4}{2p^+}, \pm\mathbf{\Delta}_\perp] \qquad \xi = \frac{(p-p')^+}{(p+p')^+} = 0 \qquad t = -\Delta_\perp^2$$

- assumption:
  parton virtualities $k_i^2 < \Lambda^2$ , intrinsic transverse momenta $k_{\perp i}^2/x_i < \Lambda^2$

- consequences — propagators poles avoided
  $\hat{s} = (k_j + q)^2 \simeq (p+q)^2 = s$ — active partons approximately on-shell
  $\hat{u} = (k_j - q')^2 \simeq (p - q')^2 = u$ — collinear with parent hadrons
  and $x_j, x_j' \simeq 1$

- physical situation: hard photon-parton scattering and
  soft emission and reabsorption of partons by hadrons

# The Compton amplitudes

Radyushkin hep-ph/9803316; DFJK hep-ph/9811253; Huang-K.-Morii hep-ph/0110208

$s,\ -t,\ -u \gg \Lambda^2$

$$\mathcal{M}_{\mu'+,\mu+} = 2\pi\alpha_{\rm em}\Big\{\mathcal{H}_{\mu'+,\mu+}\big[R_V + R_A\big] + \mathcal{H}_{\mu'-,\mu-}\big[R_V - R_A\big]\Big\}$$

$$\mathcal{M}_{\mu'-,\mu+} = \pi\alpha_{\rm em}\frac{\sqrt{-t}}{m}\Big\{\mathcal{H}_{\mu'+,\mu+} + \mathcal{H}_{\mu'-,\mu-}\Big\}R_T$$

$$R_V(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x} H_v^q(x,\xi=0,t)$$

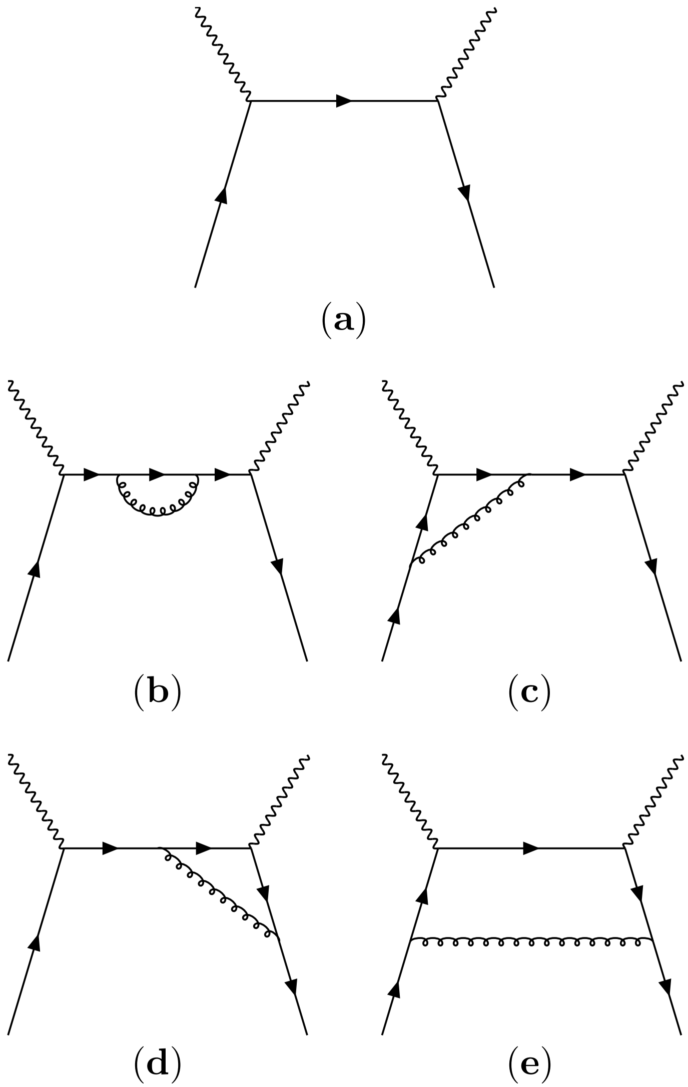
$$R_A(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x} \widetilde{H}_v^q(x,\xi=0,t)$$

$$R_T(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x} E_v^q(x,\xi=0,t)$$

$\widetilde{E}$ decouples at $\xi=0$; $\quad H_v^q = H^q - H^{\bar{q}}$ (sea quarks neglected)

$R_i$ to be evaluated from $\xi=0$ GPDs Diehl-K 1302.4604

# Subprocess amplitudes to NLO




$$\mathcal{H}^{LO}_{++++} = 2\sqrt{-s/u}$$

$$\mathcal{H}^{LO}_{-+-+} = 2\sqrt{-u/s}$$

NLO provide

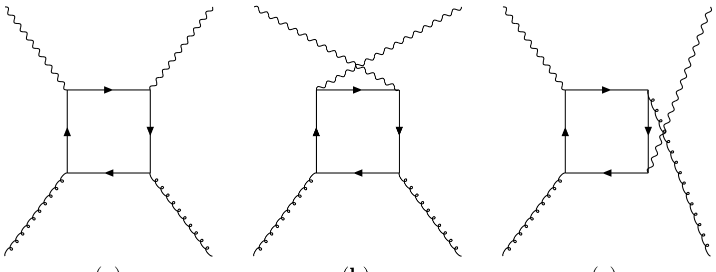
photon helicity flip

$$\mathcal{H}^{NLO}_{-+,++} = -\frac{\alpha_s}{2\pi} C_F \left[ \sqrt{\frac{s}{-u}} + \sqrt{\frac{-u}{s}} \right]$$

and imaginary parts and log. corrections to $\mathcal{H}^{LO}_{++++}$ and $\mathcal{H}^{LO}_{-+-+}$

Huang-K-Morii (01)

# $\gamma g \to \gamma g$




requires gluonic GPDs
and new form factors

$$R_V^g(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x^2} H^g(x,0;t)$$

$$xg(x) = H^g(x,0,0)$$

$R_A^g$, $R_T^g$ analogously, gluon helicity flips occur (even more form factors)
$R_V^g$ expected to be the most important one, others are neglected

$$\mathcal{M}^g_{\mu'+,\mu+} = 2\pi\alpha_{\rm em}\left[\mathcal{H}^g_{\mu'+,\mu+} + \mathcal{H}^g_{\mu'-,\mu-}\right] R_V^g$$

to be added to quark amplitudes

# Analysis of nucleon form factors

need for Compton ffs, i.e. need for GPDs at large $-t$

deeply virtual processes provide GPDs only at small $-t$

but large $-t$ GPDs from nucleon ffs through sum rules:

$$F_i^{p(n)} = e_u F_i^{u(d)} + e_d F_i^{d(u)}\,, \qquad F_i^a = \int_0^1 dx K_v^a(x,\xi=0,t)$$

Dirac (Pauli) ff: $K = H(E)$ (normalization from $\kappa_q = \int_0^1 dx E_v^q(x,\xi=t=0)$)

axial form factor: $\widetilde{H}$

ansatz $K_i^a(x,\xi=0,t) = k_i^a(x) e^{t f_i^a(x)}$

profile fct: $f_i^a = (B_i^a + \alpha_i'^a \ln 1/x)(1-x)^3 + A_i^a x(1-x)^2$

forward limits $H: \quad q(x)$ $\widetilde{H}: \quad \Delta q(x)$ (see below)

$E$: $e_i = N_i x^{\alpha_i}(1-x)^{\beta_i}$ additional parameters

DFJK hep-ph/0408173; update: Diehl-K, 1302.4604 fit to all

$G_M^i, G_E^i/G_M^i$ data ($i=p,n$) and use of ABM11, DSSV09 parton densities

# Estimate of proton radius

Approx: distance between active parton and cluster of spectators

work in hadron's center of momentum frame

$\sum x_i \mathbf{b_i} = 0$

Fourier transform of $H$

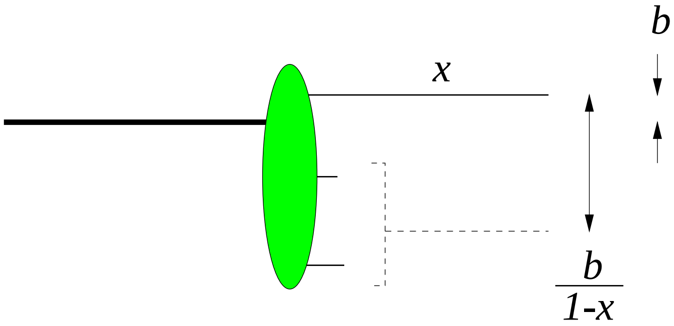
$$q(x, \mathbf{b}) = \frac{1}{4\pi} \frac{q(x)}{f_q(x)} \exp\left[-b^2/(4f_q(x))\right]$$

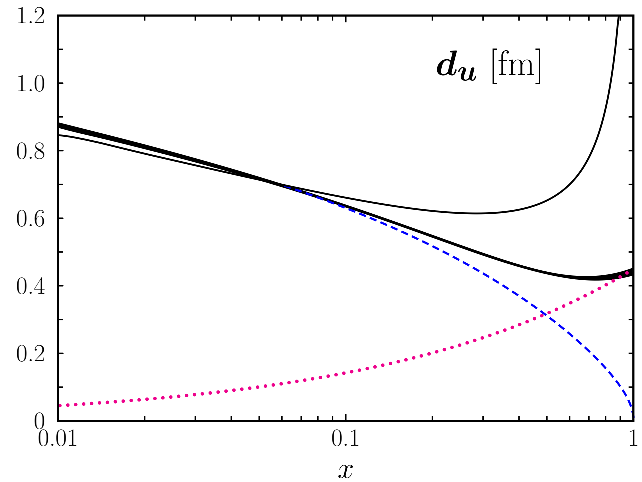
$$d_q(x) = \frac{\sqrt{\langle b^2 \rangle_x^q}}{1-x} = \frac{2\sqrt{f_q(x)}}{1-x} \to 2\sqrt{A_q}$$

for $x \to 1$

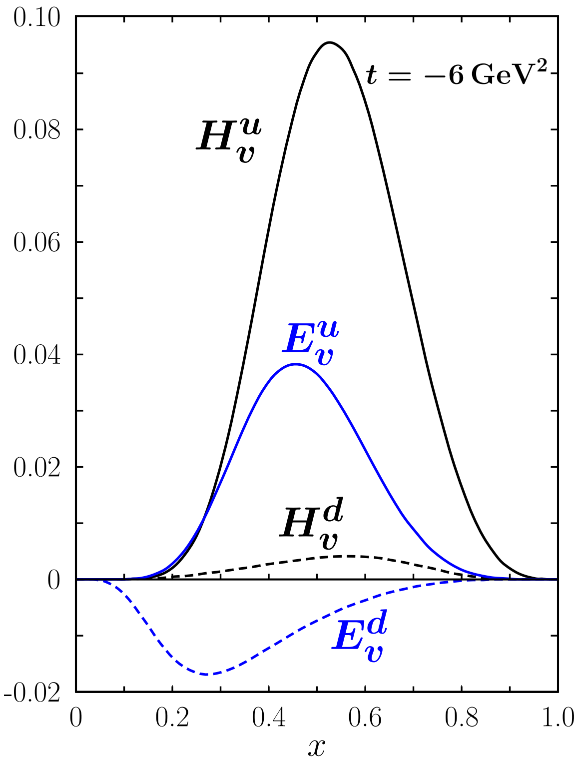
Regge-type term, A term, full profile fct

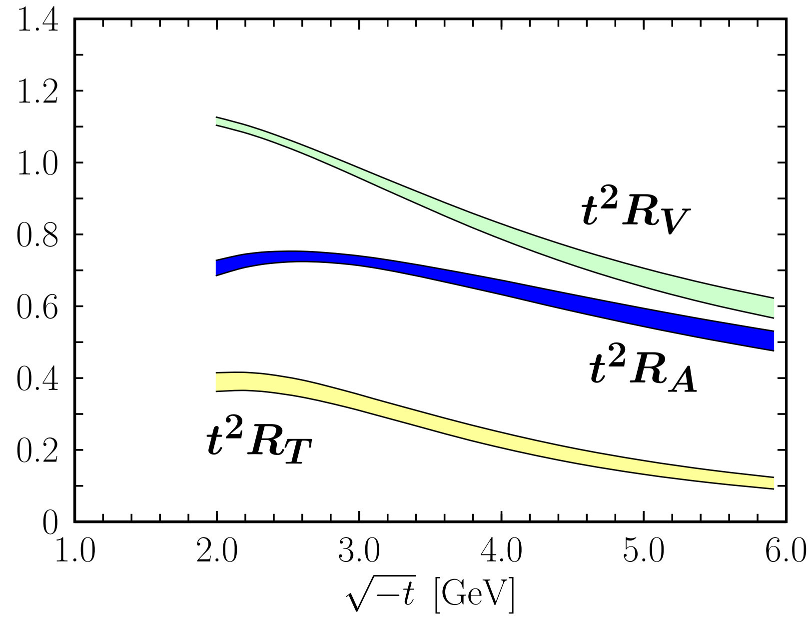
Regge-like profile fct can (only) be used at small $x$ (small $-t$)



# GPDs and Compton ffs




$K_v^q \sim x^{-\alpha_q(0)-t\alpha'_q}$ at small $x$

$K_v^q \sim (1-x)^{\beta_q}$ at large $x$

pronounced peak (scale 2 GeV)

strong $x \leftrightarrow t$ correlation

position moves to larger $x$ and

becomes narrower with increasing $-t$

under control of large $x$

$-t \to \infty :$ $x_{\mathrm{av}} \to 1$

$1/x$ becomes negligible

Compton ffs $\Longrightarrow$

flavor ffs $\times$ charges squared

# Large$-t$ behavior of flavor form factors

at large $t$: dominance of narrow region of large $x$:

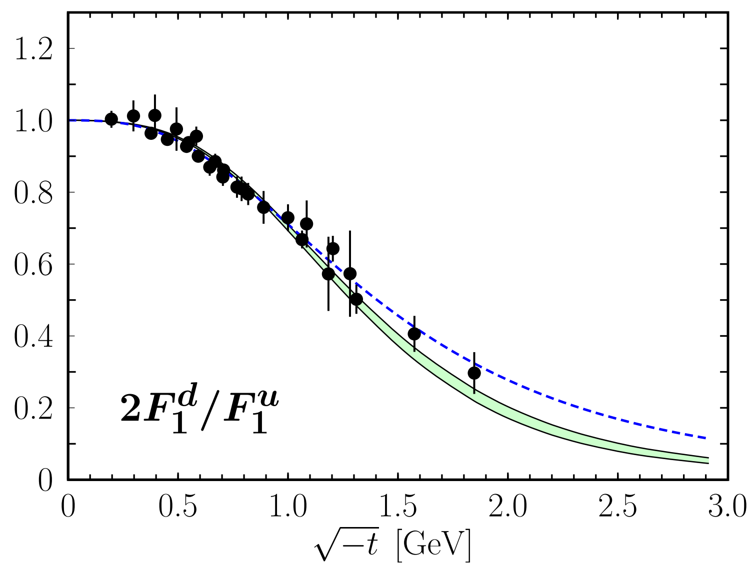
$q_v \sim (1-x)^{\beta_q}$, $f_q \sim A_q(1-x)^2$ (analogously for $F_2^q$)

Saddle point method provides $1 - x_s = \left(\frac{2}{\beta_q} A_q |t|\right)^{-1/2}$ $F_1^q \sim |t|^{-(1+\beta_q)/2}$

active parton carries most of proton momentum while the spectators are soft

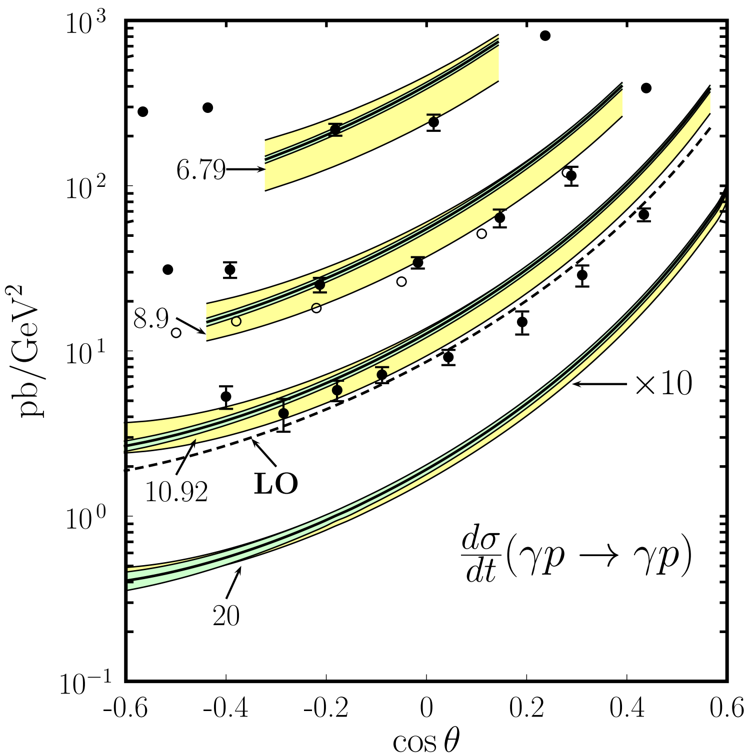
region of Feynman mechanism (similar to Drell-Yan)

power laws from wave fct overlaps: Dagaonkar-Jain-Ralston (14)




ABM PDFs: $\beta_u \simeq 3.4$, $\beta_d \simeq 5$,

# The Compton cross section




$$\frac{d\sigma}{dt} = \frac{d\hat\sigma}{dt}\left\{\frac{1}{2}\,\frac{(s-u)^2}{s^2+u^2}\left[R_V^2(t) + \frac{-t}{4m^2}\,R_T^2(t)\right] + \frac{1}{2}\,\frac{t^2}{s^2+u^2}R_A^2(t)\right\} + \mathcal{O}(\alpha_s)$$

$$\frac{d\hat\sigma}{dt} = 2\pi\frac{\alpha_{\rm em}^2}{s^2}\left[-\frac{u}{s} - \frac{s}{u}\right]$$

Klein-Nishina cross section

$-t, -u > 2.5$ GeV$^2$

data: JLab E99-114

form factors from $\xi = 0$ anlaysis

neutron target: at large $-t$ $\quad H^d << H^u$ $\quad$ Diehl-K (13)

$$\Longrightarrow F_1^n(t)/F_1^p(t) \simeq e_d/e_u \qquad R_V^n/R_V^p \simeq (e_d/e_u)^2$$

# Light-cone helicities versus c.m.s helicities

$\mathcal{M}$ in light-cone helicity basis

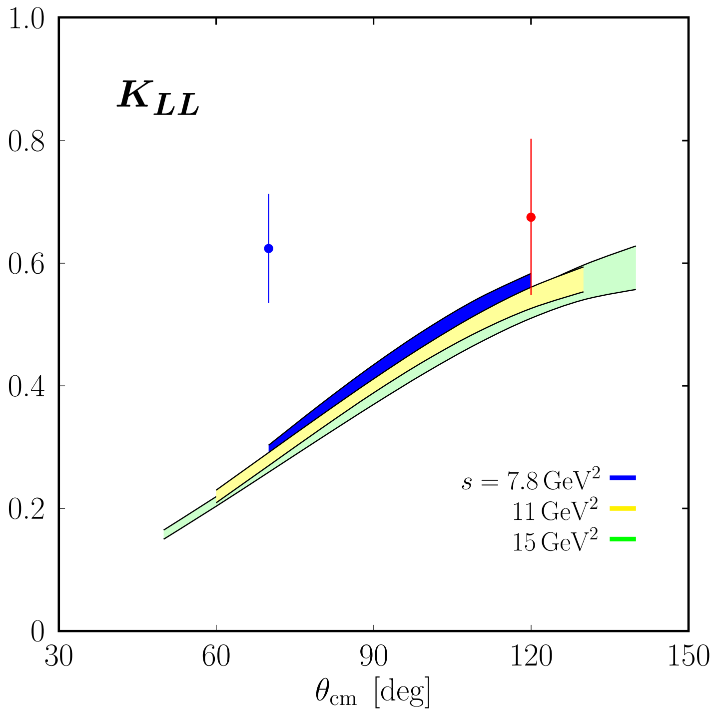
c.m.s. helicity amplitudes more convenient for comparison with experiment

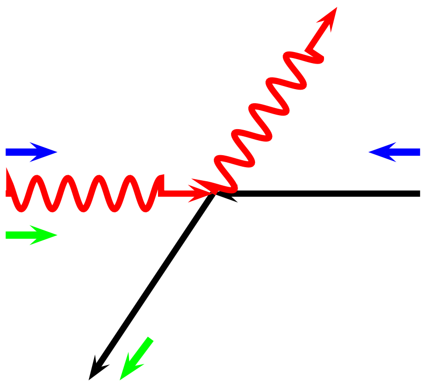
Transformation: Diehl hep-ph/0101335

$$\Phi_{\mu'\nu',\mu\nu} = \mathcal{M}_{\mu'\nu',\mu\nu} + \frac{\beta}{2}\Big[(-1)^{1/2-\nu'}\mathcal{M}_{\mu'-\nu',\mu\nu} + (-1)^{1/2+\nu'}\mathcal{M}_{\mu'\nu',\mu-\nu}\Big] + \mathcal{O}(\Lambda^2/t)$$

$$\beta = \frac{2m}{\sqrt{s}}\frac{\sqrt{-t}}{\sqrt{s}+\sqrt{-u}}$$

# Helicity correlation $A_{LL}$ and $K_{LL}$



Klein-Nishina result

$\hat{A}_{LL} = \hat{K}_{LL} = \frac{s^2-u^2}{s^2+u^2}$

$A_{LL} = K_{LL} \simeq \hat{A}_{LL}\,\frac{R_A}{R_V}$

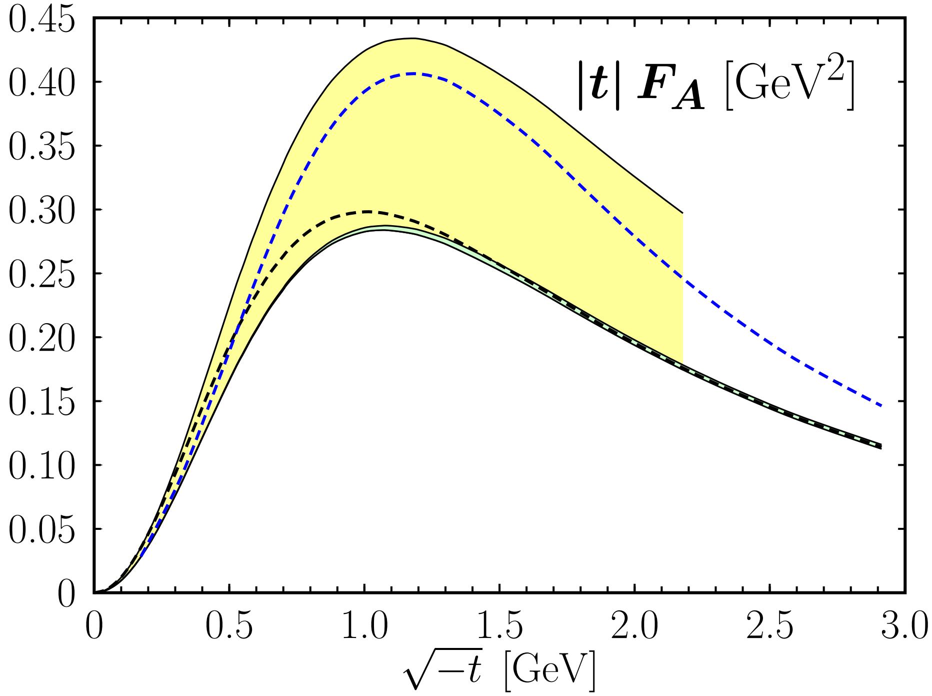
JLab E99-114 ($s = 6.9\mathrm{GeV}^2 \quad u = -1.04\mathrm{GeV}^2$)

JLab E07-002 ($s = 7.8\mathrm{GeV}^2 \quad t = -2.1\mathrm{GeV}^2$)

application of handbag mechanism is at the limits

$R_A$ badly known ( $\Phi_{--,++} = -\Phi_{-+,-+}$, of $\mathcal{O}(\alpha_s)$)

# The axial form factor




sum rule:

$$F_A(t) \;=\; \int_0^1 dx \Big[\widetilde{H}_v^u - \widetilde{H}_v^d\Big] \;+\; 2\int_0^1 dx \Big[\widetilde{H}^{\bar{u}} - \widetilde{H}^{\bar{d}}\Big]$$

no new data; old data (covering a fairly large range of $t$): Kitagaki et al (83)
in form of dipole parametrization (yellow band)

no attempt to analyse it like $H, E$ $\Delta q$ from DSSV(09)

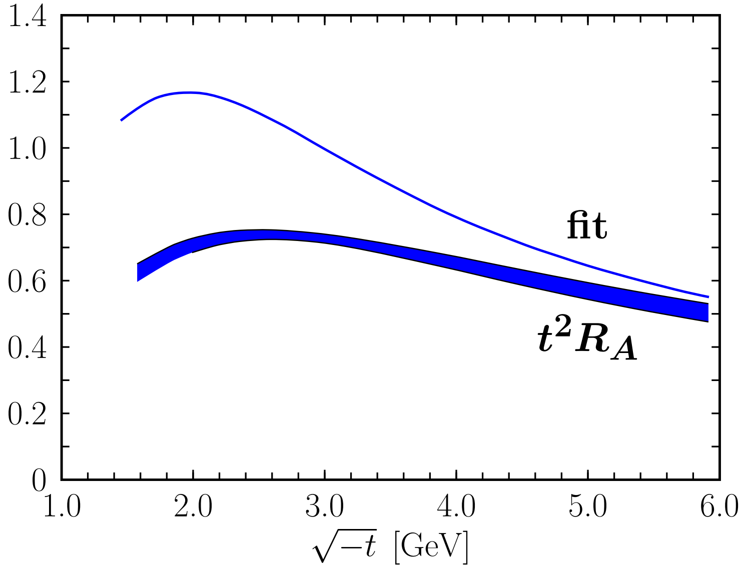
use $\widetilde{H}^q = \Delta q(x) \exp\left[t \tilde{f}_q(x,t)\right]$ assume $\tilde{f}_q = f_q$ (green band)

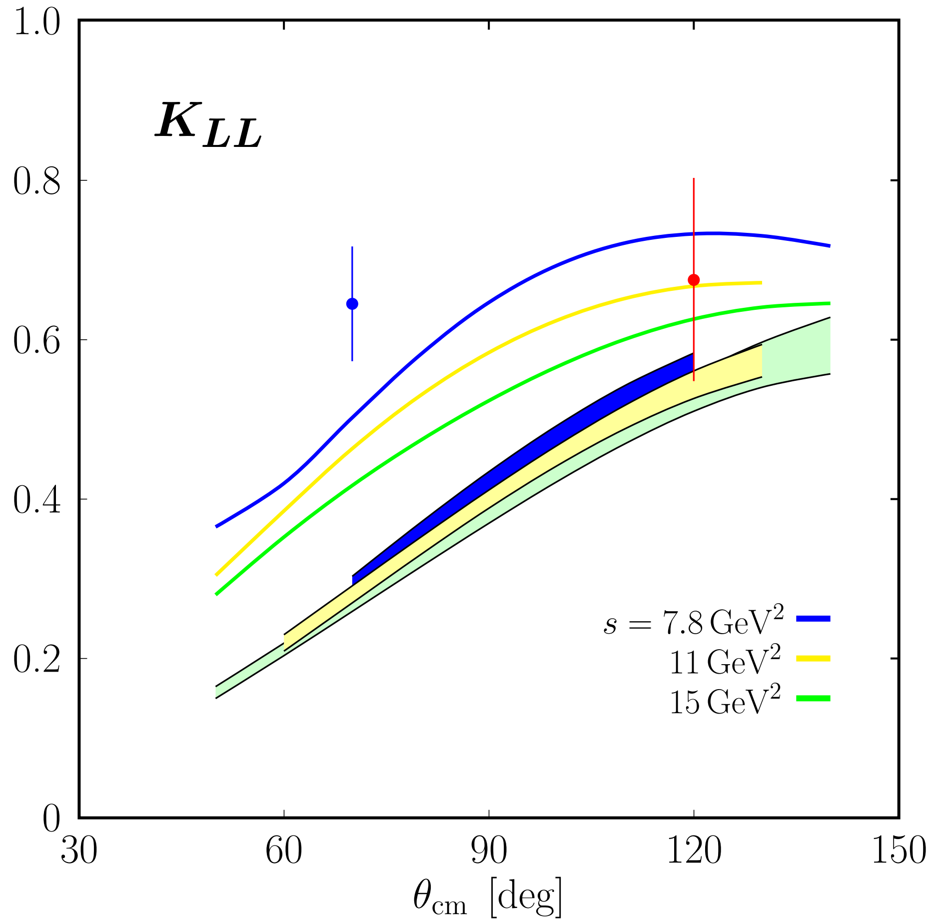
density interpretation of $q_v(x,b^2) \pm \Delta q_v(x,b^2)$ implies $\tilde{f}_q \leq f_q$

$\tilde{f}_q < f_q$ increases $F_A$: 50% increase of $F_A$ possible (blue dashed line)

black dashed line: with an estimate of sea contribution

# New result for $K_{LL}$



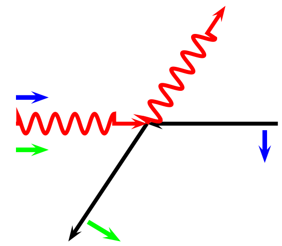




colored lines with new profile function for $\widetilde{H}$

# More on spin effects

$\hat{A}_{LS} = 0 \qquad \beta = \frac{2m}{\sqrt{s}} \frac{\sqrt{-t}}{\sqrt{s}+\sqrt{-u}}$

helicity correlation $A_{LS}, K_{LS}$

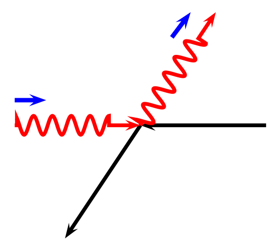


$A_{LS} = -K_{LS} \simeq \frac{R_A}{R_V} \hat{A}_{LL} \left( \frac{\sqrt{-t}}{2m} \frac{R_T}{R_V} - \beta \right)$

$K_{LS} = \quad 0.114 \pm 0.083 \pm 0.04$ (E99-114)

$= -0.0890 \pm 0.059 \pm 0.04$ (E07-002)

$= -0.02 \pm 0.02$ (theory)



$D_{LL} = 1 + \mathcal{O}(\alpha_s^2)$

requires photon helicity flip $\sim \mathcal{H}^{\rm NLO}_{-+,++}$

$\Sigma \frac{d\sigma}{dt} = \frac{1}{2} \left[ \frac{d\sigma_\perp}{dt} - \frac{d\sigma_\parallel}{dt} \right]$

linear photon polarization

$\Sigma \simeq -4\alpha_s/(3\pi)$ mild dependence on ff

notoriously difficult, large uncertainty

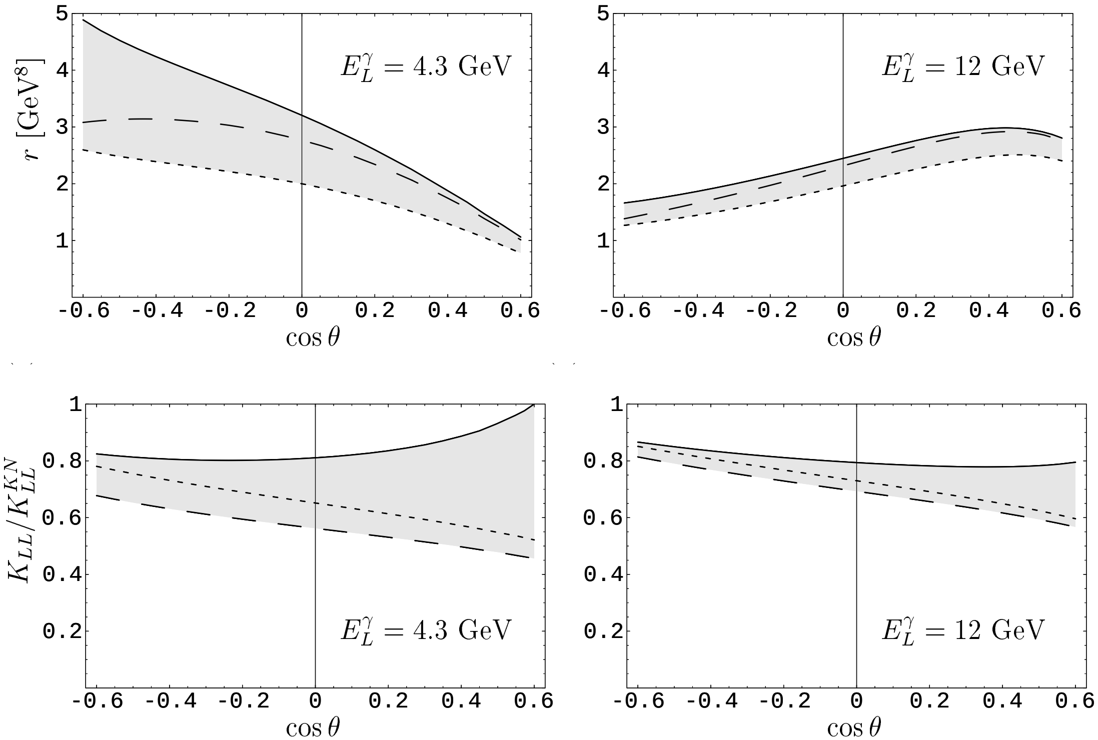
transv. proton polarization

proton helicity flip $\simeq R_T$ and

phase difference from NLO (from gluon contr.)

small $< 0.1$

# Target mass corrections




$d\sigma/d\hat{\sigma}$

$K_{LL}/\hat{K}_{LL}$

| | | | | |
|---|---|---|---|---|
| Scenario 1: | $\hat{s} = s$ | $\hat{t} = t$ | $\hat{u} = u$ | full |
| Scenario 2: | $\hat{s} = s - m^2$ | $\hat{t} = t$ | $\hat{u} = u - m^2$ | long-dashed |
| Scenario 3: | $\hat{s} = s - m^2$ | $\hat{t} = -\frac{\hat{s}}{2}(1 - \cos\theta)$ | $\hat{u} = -\hat{s} - \hat{t}$ | dashed |

Diehl-Feldmann-Huang-K, hep-ph/0212138

# Other approaches

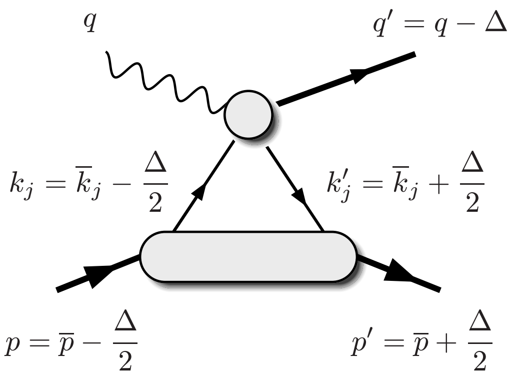
all base on the handbag but different corrections

Miller(04): Constituent quark model:
massive quarks; differences for $\theta \to 180^\circ$
elm and Compton form factors from wave fct overlaps
$A_{LL} \neq K_{LL}$; $D_{LL}$ becomes small, even changes sign;
$K_{LS} = A_{LS} = \Sigma = 0$

Kivel-Vanderhaeghen (15): Soft collinear effective theory
leading term equal to handbag for $R_V = R_A$
$R_V$ fixed from RCS cross section $\qquad A_{LL} \simeq K_{LL}$ and $K_{LS} \simeq 0.2$;

# Photoproduction of pions

handbag contribution $(s,\, -t,\, -u \gg \Lambda^2)$



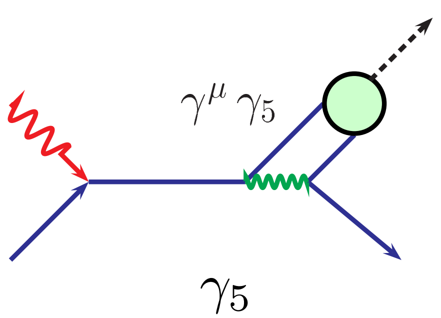

$$\mathcal{M}^{\pi}_{0+\mu+} = \frac{e}{2}\Big\{\mathcal{H}^{\pi}_{0+\mu+}\,[R^{\pi}_{V} + R^{\pi}_{A}] + \mathcal{H}^{\pi}_{0-\mu-}\,[R^{\pi}_{V} - R^{\pi}_{A}]\Big\}$$

$$\mathcal{M}^{\pi}_{0+\mu-} = \frac{e}{2}\Big\{\mathcal{H}^{\pi}_{0+\mu+} + \mathcal{H}^{\pi}_{0-\mu-}\Big\}R^{\pi}_{T}$$

For each flavor: $R^{\pi q}_i \simeq R^q_i$ known, universality (sea quarks neglected)

$$R^{\pi^0}_i = 1/\sqrt{2}\,\left[e_u R^u_i - e_d R^d_i\right] \qquad R^{\pi^+}_i = R^{\pi^-}_i = R^u_i - R^d_i$$

leading twist formation of meson (expected to hold for $s, -t, -u \gg \Lambda$)




cross section much too small but many good features (pol., charge ratios,...)

Huang *et al* hep-ph/0005318, hep-ph/0309071

transversity GPDs and twist-3 $\pi$ w.f.?

# Summary

- The handbag mechanism in wide-angle Compton scattering
  requires $s, -t, -u \gg 1\text{GeV}^2$
  can be extended to VCS provided $Q^2 < -t$ DFJK(99)
  and to photoproduction of mesons as well as $\gamma\gamma \to h\bar{h}$
- The Compton amplitude factorizes in a hard photon-parton scattering -calculated to NLO - and in soft form factors - representing $1/x$-moments of GPDs at $\xi = 0$
- With the Compton form factors evaluated from GPDs obtained in an analysis of nucleon elm FFs the handbag provides predictions for polarized and unpolarized Compton scattering in fair agreement with JLab experiments ($d\sigma/dt$, $K_{LL}$, $K_{LS}$) despite the rather low energies at which the data are available
- The handbag contribution is a realization of the Feynman mechanism
  a correction to the leading-twist one
  seems to dominate for momentum transfers of the order of 10 $\text{GeV}^2$

# Polarization states of the proton

defined as spin eigestates of $\vec{A} \cdot \vec{\sigma}$

$\sigma_i$ Pauli matrices, $\vec{A}$ any of the unit vectors

$$\vec{L}^{(\prime)} = \frac{\vec{p}^{(\prime)}}{|\vec{p}^{(\prime)}|} \quad \vec{N} = \frac{\vec{L} \times \vec{L}^{(\prime)}}{|\vec{L} \times \vec{L}^{(\prime)}|} \quad \vec{S}^{(\prime)} = \frac{\vec{N} \times \vec{L}^{(\prime)}}{|\vec{N} \times \vec{L}^{(\prime)}|}$$

$\vec{p}, \vec{p}^{(\prime)}$ denote 3-momenta of incoming and outgoing protons,resp.

e.g. scattering plane 1-3: $\vec{N} = \vec{e}_2 \quad \vec{L} = -\vec{e}_3 \quad \vec{S} = -\vec{e}_1$

sideway polarization $| \Rightarrow \rangle = \frac{1}{\sqrt{2}} \left[ |+\rangle - |-\rangle \right]$