

HIGH-DENSITY PARTON PHYSICS

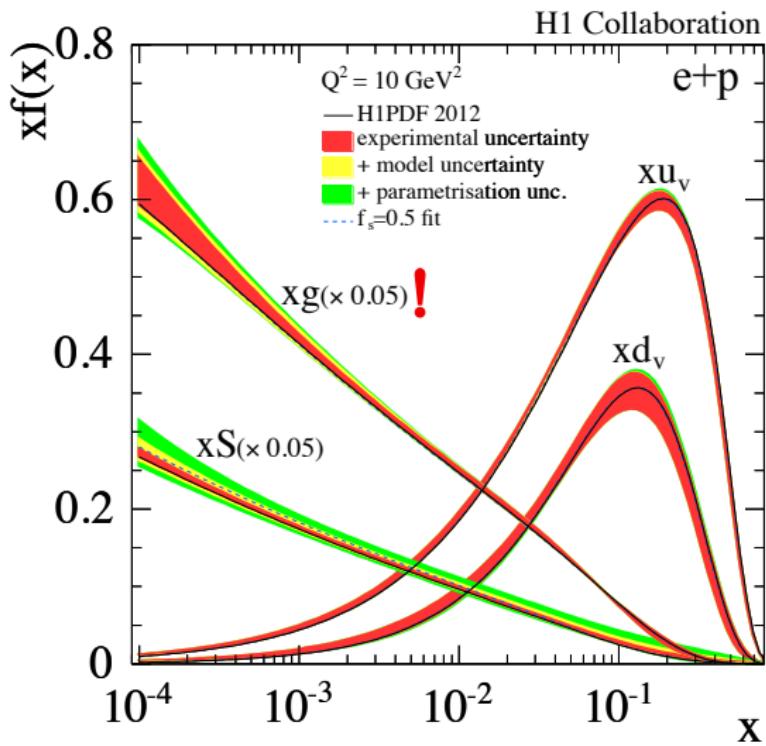
Vladimir Skokov



OUTLINE

- Introduction: small x evolution, saturation, ...
- What to look for?
- From another angle:
 - color memory
 - entanglement entropy
- Conclusions

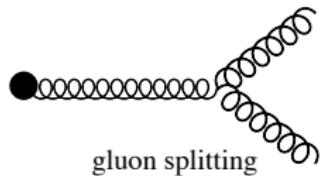
GLUON DENSITY AT SMALL x



EVOLUTION EQUATIONS: RADIATION DOMINATED REGIME

- BFKL equation resums powers of $\alpha_s \ln \frac{1}{x}$; no multiple rescattering

$$\frac{\partial}{\partial \ln \frac{1}{x}} N(x, k^2) = \alpha_s N_c K_{\text{BFKL}} \otimes N(x, k^2)$$



I. Balitsky, V. Fadin, E. Kuraev, & L. Lipatov, '78

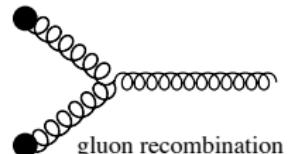
- Emission of new gluons as x decreases; the emission is proportional to N
- Gluon density increases without bound
violating quantum mechanical black disk limit for total cross section

EVOLUTION EQUATIONS: NONLINEAR REGIME

- At small x , parton recombination modifies evolution (large N_c)

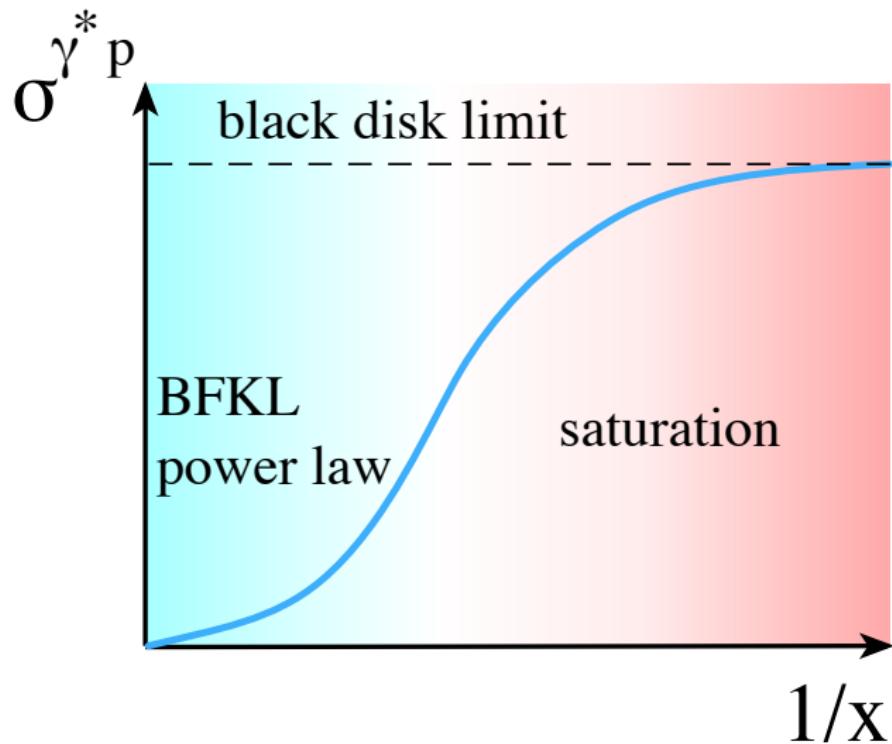
$$\frac{\partial}{\partial \ln \frac{1}{x}} N(x, k^2) = \alpha_s N_c K_{\text{BFKL}} \otimes N(x, k^2) - \alpha_s N_c N^2(x, k^2)$$

I. Balitsky '96; Yu. Kovchegov '99



- Emission of new gluons as x decreases; emission is proportional to N
- Recombination \leadsto reduction of gluon number in wave function

BK AND BLACK DISK LIMIT



- Nonlinear BK evolution respects black disk limit

EVOLUTION EQUATIONS: BEYOND LARGE N_c LIMIT

- BK was formally derived in large N_c limit
- JIMWLK evolution equation overcomes this deficiency;
it provides straightforward recipe of computing any operator constructed from arbitrary number of Wilson lines

$$\frac{\partial W[\alpha]}{\partial \ln \frac{1}{x}} = \frac{1}{2} \int dz_1 dz_2 \frac{\delta}{\delta \alpha_\eta^a(z_1)} \chi^{ab}(z_1, z_2)[\alpha] \frac{\delta}{\delta \alpha_\eta^b(z_2)} W_\eta[\alpha], \quad U = P \exp \left\{ ig \int_{-\infty}^{\infty} dx^- t^a a^a(x^-, \mathbf{x}_\perp) \right\}$$

Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner, '97-'02

- Extension of JIMWLK beyond leading order
 - running coupling corrections
 - NLO JIMWLK

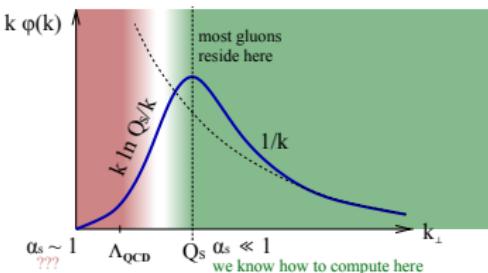
T. Lappi, H. Mäntysaari '13

I. Balitsky & G. A. Chirilli; A. Grabovsky; S. Caron-Huot
A. Kovner, M. Lublinsky, & Y. Mulian '14

EVOLUTION EQUATIONS: SUMMARY

- Gluon saturation leads to formation of transverse scale, Q_s , defined by average density
- At small distances $\Delta x < Q_s^{-1}$; system is dilute
- At large distances $\Delta x > Q_s^{-1}$; system is saturated;
 - there are much less gluons than perturbatively expected
 - intrinsically non-linear regime of QCD
 - gluon field $A_\mu \sim 1/g$ is strong: semi-classical picture
⇒ Color Glass Condensate as an EFT

- When $Q_s \gg \Lambda_{\text{QCD}}$, theory is calculable
nonlinear dynamics ⇒ experimental observables



SYSTEMATICS OF SATURATION MOMENTUM

- The higher Q_s the better!
- Two knobs: energy ($1/x$) and atomic number
- BK evolution equation dictates

$$Q_s^2(x) \propto \left(\frac{1}{x}\right)^\lambda$$

for simplicity $\lambda = 1/3$

- For nuclear target: small x gluons have coherence length $l_{\text{coh}} \sim \frac{1}{x m_N} \gg R_A$

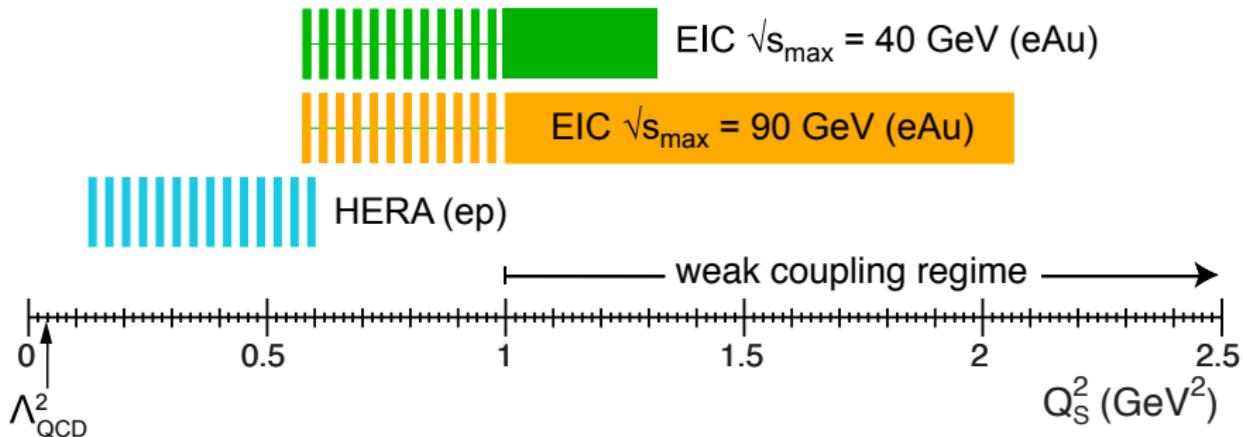
$$Q_s^2(x, A) \propto A^{1/3} \left(\frac{1}{x}\right)^\lambda \approx \left(\frac{A}{x}\right)^{1/3}$$

$$Q_s^2(x, Au) \approx 6 Q_s^2(x, p)$$

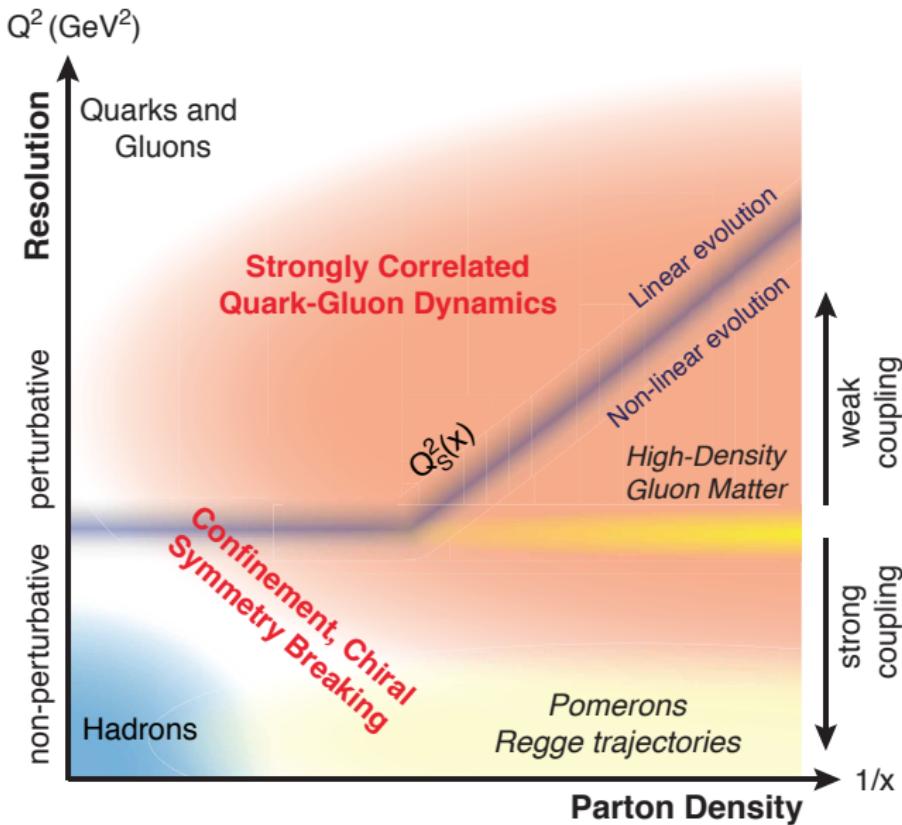
- To study saturation probe heavy nucleus at high energy
- To minimize strong final state interactions: e as a probe!

ENERGY DEPENDENCE OF Q_s

$x \leq 0.01$



HIGH ENERGY “PHASE DIAGRAM” OF QCD

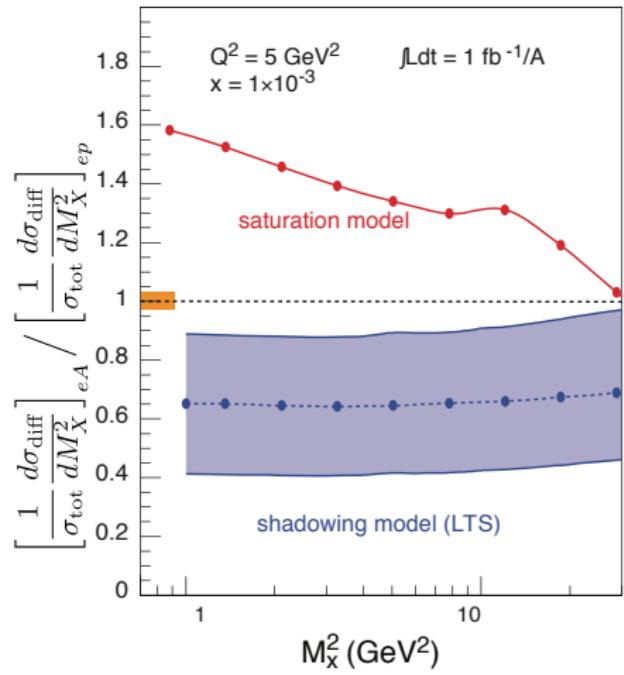
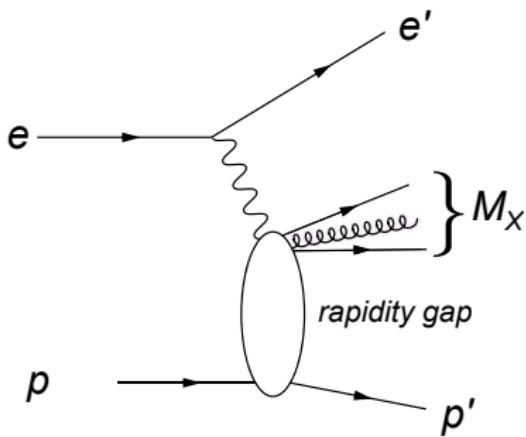


WHAT TO LOOK FOR?

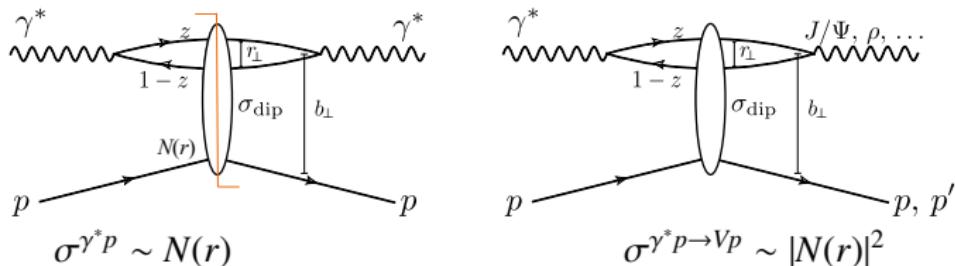
- Intrinsic transverse momentum: Q_s is a momentum scale
- Vary x or/and A
 - intrinsic QCD scale, Λ_{QCD} is constant, while Q_s increases with energy $1/x$ and A
- Increase Q^2 to “turn off” saturation

DIFFRACTION

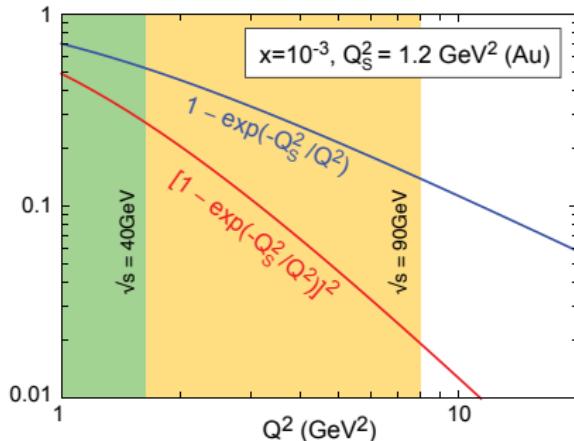
- General idea: $\sigma_{\text{diff}}^{\gamma^* p} / \sigma_{\text{tot}}^{\gamma^* p}$ is maximal at black disk limit
Saturation \Leftrightarrow Large number of diffractive events



DIFFRACTIVE VECTOR MESON PRODUCTION



- Dipole amplitude includes saturation dynamics
- Illustrative case study: $Q^2 \sim 1/r^2$:



E. Aschenauer et al, arXiv:1708.01527

SYSTEMATICS OF DIFFRACTIVE VECTOR MESON PRODUCTION

- Saturation effects and Q^2 and A scaling properties:

	saturated \parallel , low Q^2	dilute \parallel , high Q^2	saturated \perp , low Q^2	dilute \perp , high Q^2
$\frac{d\sigma^{\gamma^*+A \rightarrow V+A}}{dt} \Big _{t=0}$	$Q^2 A^{4/3}$	$Q^{-6} A^2$	$Q^0 A^{4/3}$	$Q^{-8} A^2$
$\sigma^{\gamma^*+A \rightarrow V+A}$	$Q^2 A^{2/3}$	$Q^{-6} A^{4/3}$	$Q^0 A^{2/3}$	$Q^{-8} A^{4/3}$

H. Mantysaari & R. Venugopalan, '17

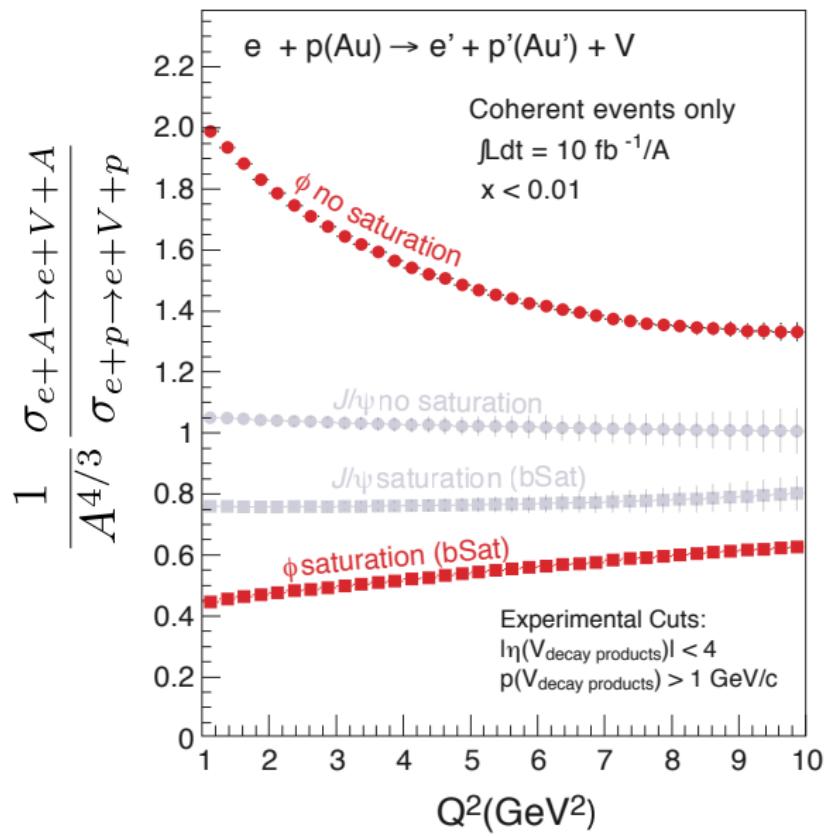
SYSTEMATICS OF DIFFRACTIVE VECTOR MESON PRODUCTION

- Saturation effects and Q^2 and A scaling properties:

	saturated , low Q^2	dilute , high Q^2
$\sigma^{\gamma^*+A \rightarrow V+A}$	$Q^2 A^{2/3}$	$Q^{-6} A^{4/3}$

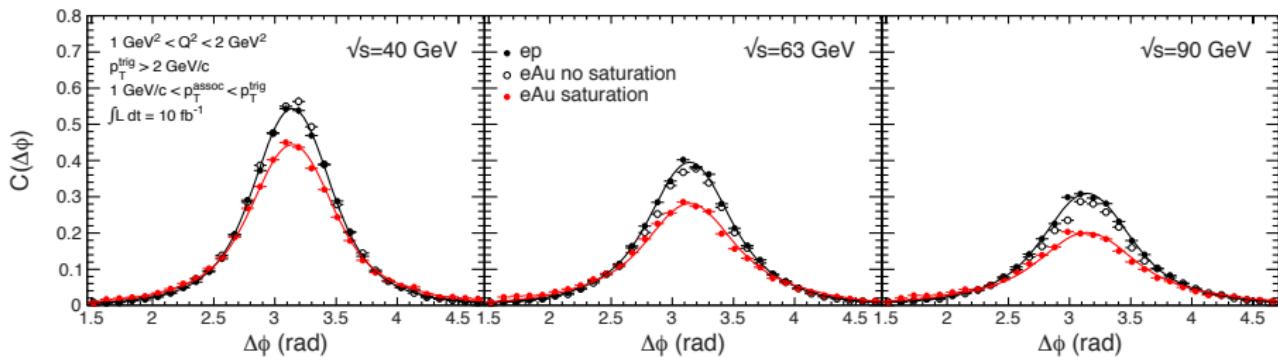
H. Mantysaari & R. Venugopalan, '17

DIFFRACTIVE VECTOR MESON PRODUCTION



DIHADRON CORRELATIONS

- Access to details of p/A structure through correlations between partons
- Two key measurements:
 - back-to-back correlation in $e+A \rightarrow e' + h_1 + h_2 + X$
~ Weizsäcker-Williams gluon distribution $xG^{(1)}$
 - structure of back-to-back correlation in $e+A \rightarrow e' + j_1 + j_2 + X$
~ linearly polarized WW gluon distribution $xh^{(1)}$



- Saturation \Rightarrow suppression of away-side peak

E. Aschenauer et al, arXiv:1708.01527
L. Zheng, E. Aschenauer, J. H. Lee, & B. W. Xia, '14

STRUCTURE OF BACK-TO-BACK CORRELATION: DIJET MEASUREMENT

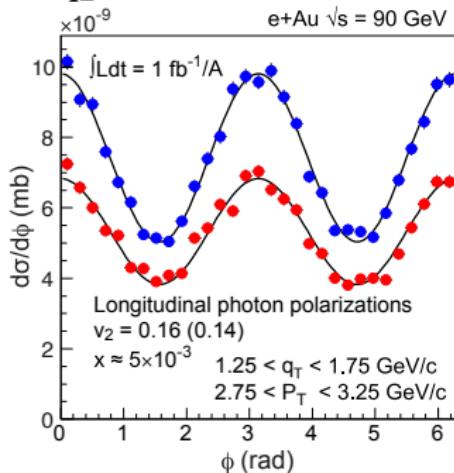
- Dijet production at small x (Long. γ^* is shown only)

$$E_1 E_2 \frac{d\sigma^{\gamma_L^* A \rightarrow q\bar{q}X}}{d^3 k_1 d^3 k_2 d^2 b} = \alpha_{em} e_q^2 \alpha_s z^2 (1-z)^2 \frac{8\epsilon_f^2 P_\perp^2}{(P_\perp^2 + \epsilon_f^2)^4} \left[x \mathbf{G}^{(1)}(x, q_\perp) + \underbrace{\cos(2\phi)}_{\text{Longitudinal photon polarization}} x \mathbf{h}_\perp^{(1)}(x, q_\perp) \right]$$

F. Dominguez, C. Marquet, B.-W. Xiao, & F. Yuan, '11

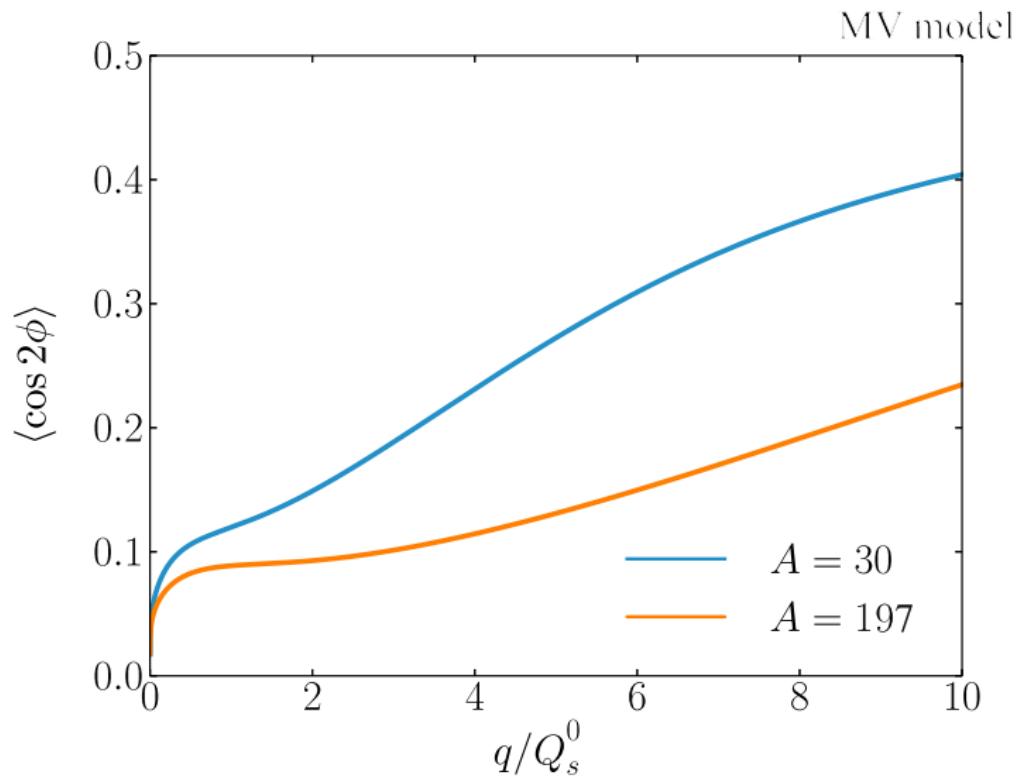
A. Metz and J. Zhou, '11

Dijet transverse momentum $\mathbf{P}_\perp = (\mathbf{k}_{1\perp} - \mathbf{k}_{2\perp})/2$; momentum imbalance $\mathbf{q}_\perp = \mathbf{k}_{1\perp} + \mathbf{k}_{2\perp}$
 ϕ is the angle between \mathbf{P}_\perp and \mathbf{q}_\perp



E. Aschenauer et al, arXiv:1708.01527
A. Dumitru, T. Ullrich, & V.S., in progress

STRUCTURE OF BACK-TO-BACK CORRELATION: A DEPENDENCE



- McLerran-Venugopalan model predicts suppression with increasing A

CONNECTION TO CHIRAL MAGNETIC EFFECT

- Hint of Chiral Magnetic Effect, $\mathbf{j} = \sigma_\chi \mathbf{B}$, in A+A
- Key ingredient is $\sigma_\chi \Leftrightarrow$ fluctuations of divergence of Chern-Simons currents.
At early times

$$\propto g^4 N_c^4 \left[\left(G_{(A_1)}^{(1)}(x_\perp, y_\perp) \right)^2 \left(G_{(A_2)}^{(1)}(x_\perp, y_\perp) \right)^2 - \left(h_{\perp(A_1)}^{(1)}(x_\perp, y_\perp) \right)^2 \left(h_{\perp(A_2)}^{(1)}(x_\perp, y_\perp) \right)^2 \right]$$

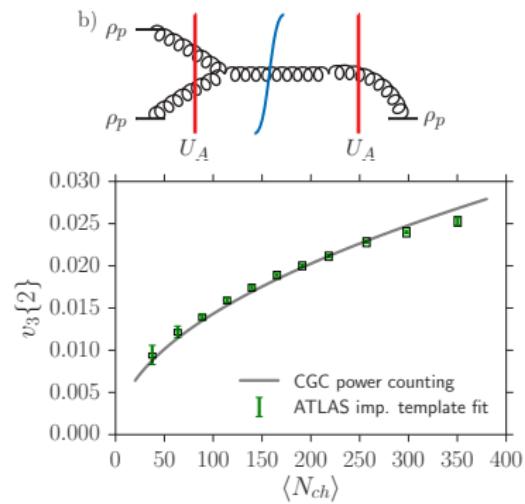
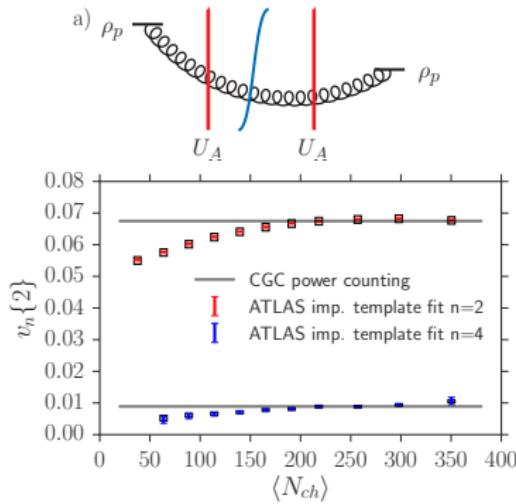
T. Lappi & S. Schlichting, '17

- $G^{(1)}$ and $h_{\perp}^{(1)}$ are Weizsäcker-Williams gluons distributions
- Nontrivial link to dijet production in e-p/A collisions

Saturation/CGC framework & Initial state of A+A: P. Tribedy, this afternoon

P-A: OPEN FOR INTERPRETATIONS

- p-A collisions at LHC might have been perfect to study saturation effects
- Potential dominance of final state interactions opens a Pandora box of various interpretations



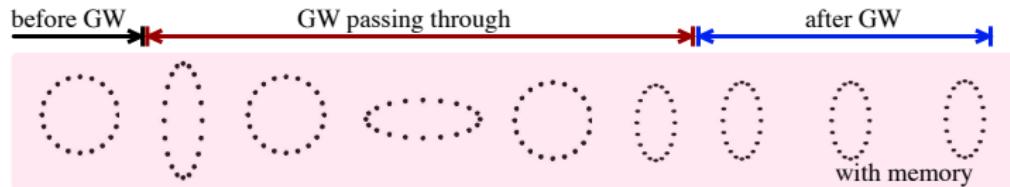
- Odd azimuthal harmonics is a sign of emerging coherence in proton wave function:
the first saturation correction in proton!
Non-zero long-range odd harmonics in high energy p-A as an evidence of saturation!?

FROM ANOTHER ANGLE

GRAVITATIONAL MEMORY EFFECT AND ITS YANG-MILLS ANALOG

GRAVITATIONAL WAVE MEMORY

- Passage of GW \sim distance between a pair of inertial bodies to oscillate
- After passage, oscillations cease
- Final separation between bodies differs from initial one



COLOR MEMORY

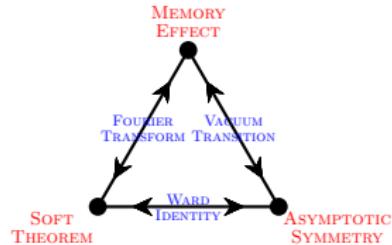
- In *classical* Yang-Mills, pulse of color radiation incident on a color singlet pair
 \leadsto oscillations in relative color
- After pulse passes, permanent color rotation remains

GRAVITATIONAL MEMORY EFFECT AND ITS YANG-MILLS ANALOG

Both memory effects are related to subtle features in infrared

- Soft theorems
- Infinite dimensional asymptotic BMS symmetries

Together with memory effect, they formed the infrared triangle.



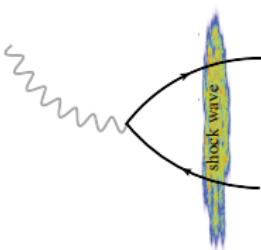
- Color memory is encoded by S-matrix operator

$$\textcolor{orange}{S}(\underline{x}, \underline{y}) = \frac{1}{N} \text{tr } V(\underline{x}) V^+(\underline{y})$$

\equiv to flat connection quantifying color memory

- Dipole scattering amplitude $\sigma_{\gamma^* H}(x, Q) = \int dz \int d^2 r |\Psi_{\gamma^* \rightarrow q\bar{q}}(z, \underline{r}, Q^2)|^2 \times 2 \int d^2 b (1 - \text{Re } \textcolor{orange}{S}(\underline{x}, \underline{y}))$

is related to color memory



M. Pate, A. Raclariu & A. Strominger, '17,
A. Ball, M. Pate, A. M. Raclariu, A. Strominger & R. Venugopalan, '18

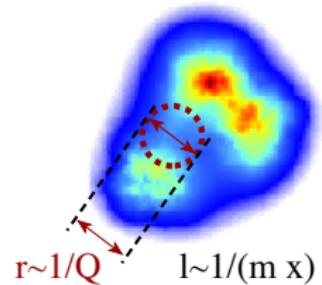
COLOR MEMORY

- Color memory is captured also by higher order correlators of Wilson lines
 - e.g. quadrupole \Leftrightarrow dijet production in DIS at small x
- Electron Ion Collider is L_{aser} I_{nterferometer} S_{pace} A_{ntenna} for color memory observation!
- Besides this intriguing connection,
we also might learn something new from other vertices of the infrared triangle

ENTANGLEMENT ENTROPY AT SMALL x

- DIS at given (x, Q) probes only part **(A)** of p wave function **(B)**

- **(A)** is spatial region of $r \sim 1/Q$ and long. length $l \sim 1/(m_N x)$



- Entanglement entropy: $S_{(A)} = -\text{tr} \hat{\rho}_{(A)} \ln \hat{\rho}_{(A)}$,

where reduced density matrix $\hat{\rho}_{(A)} = \text{tr}_{(B)} \hat{\rho}$

D. Kharzeev, E. Levin '17

- Momentum space entanglement: entanglement entropy between large x **(B)** and small x **(A)** components of hadron wave function

A. Kovner, M. Lublinsky '15, '18

SATURATION FROM PERSPECTIVE OF ENTANGLEMENT ENTROPY

- Solving small x evolution \Rightarrow Maximally entangled state
 - Universal saturated regime \Leftrightarrow Statistical system in thermal equilibrium
 - Loss of “confinement” memory due to evolution
 - \Leftrightarrow Loss of memory due to thermalization ?!
- Why is interesting?
 - Small x evolution as evolution of entanglement entropy?!
 - Entanglement entropy can be defined for arbitrary coupling
 - Entanglement entropy is non local; can be a probe of QCD topology
 - Connection to final state entropy? Thermalization?

D. Kharzeev, E. Levin '17,
A. Kovner, M. Lublinsky '15, '18

CONCLUSIONS

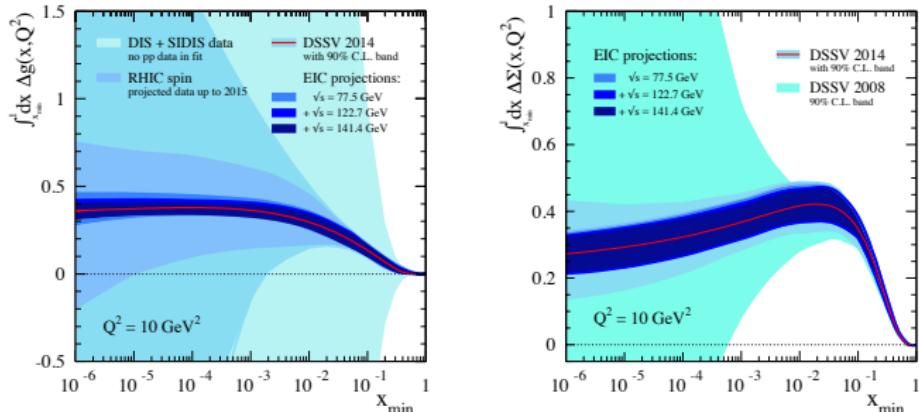
- High gluons density at small $x \Rightarrow$ saturation
- Intrinsically nonlinear regime of QCD: small coupling, large fields $A_\mu \sim 1/g$
Semi-classical description is appropriate
- To probe this new regime: electron + heavy-ion collisions at high energy
- Almost certainly EIC is going to probe deep into saturation regime
Diffraction as a “smoking gun”?
 - ...
- There are novel ways to approach high parton density physics:
 - Color memory
 - Entanglement entropy
 - ...

*More this afternoon: A. Dumitru, Small- x gluon distribution and multiplicity bias
V. S., Saturation & long-range rapidity correlations
P. Tribedy, Saturation & initial state in A+A*



PROTON SPIN: CONSTRAINING SMALL x CONTRIBUTION

- Small x contribution to proton spin may be potentially important.



- Need for theoretical input on small x asymptotics of quark and gluon helicity distributions.
- Resummation parameter

$$\alpha_s \ln^2(1/x)$$

is different from standard small x BFKL/BK/JIMWLK

$$\alpha_s \ln(1/x)$$

$$\Delta G \propto \left(\frac{1}{x}\right)^{\alpha_h^G}, \quad \alpha_g^G \approx 1.88 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$\Delta q \propto \left(\frac{1}{x}\right)^{\alpha_h^q}, \quad \alpha_g^q \approx 2.31 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

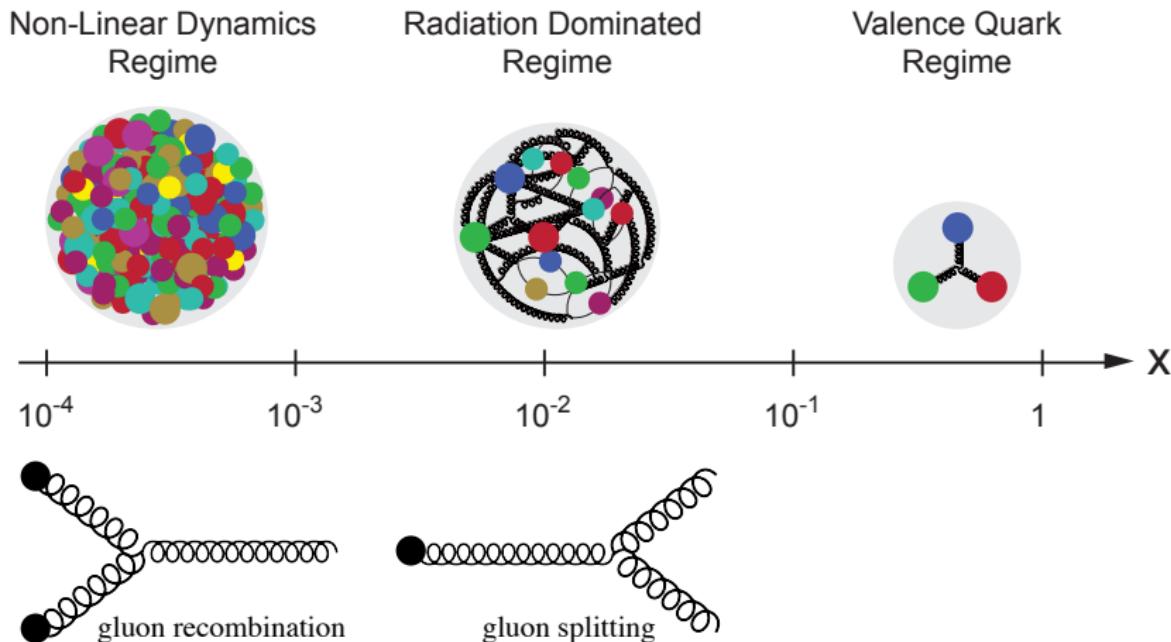
Yu. Kovchegov, D. Pitonyak, M. Sievert

PROTON SPIN: ROLE OF SATURATION

- Current results obtained in *Double Logarithmic Approximation*
are not suited to fully account for saturation dynamics
(Single Logarithmic correction is beyond DLA accuracy)
- Derived saturation corrections:
saturation suppresses evolution of helicity PDFs $\sim \alpha_h^{G,q} \rightarrow 0$.
- Saturation guarantees that at a given Q^2 ,
there is such x below which contribution to proton spin is negligible!

Yu. Kovchegov, D. Pitonyak, M. Sievert

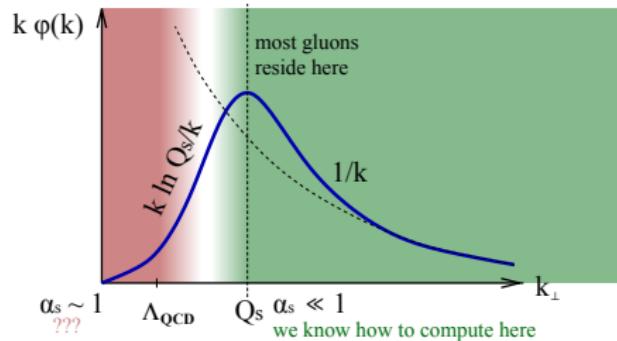
HIGH ENERGY LIMIT OF QCD



Emerging dynamical scale: saturation momentum, Q_s .
Classical Yang-Mills fields at scale λ : $R_{\text{proton}} > \lambda > 1/Q_s$.

SATURATION REGIME/CGC

- High energy \sim high gluon density
 \sim formation of perturbative scale, Q_s
- Particle production is dominated by
 $k_\perp \sim Q_s$
- Weak coupling methods can be applied
 $\alpha_s(Q_s) \ll 1$
- Still non-perturbative, as fields are strong, $A \sim \frac{1}{g} \rightsquigarrow$ non-linearity is important



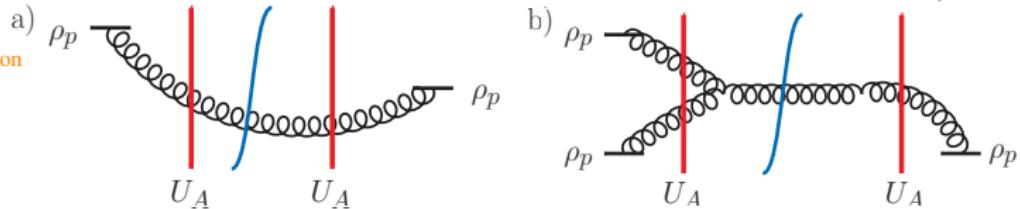
CGC PERSPECTIVE ON v_3

- Leading order and the first saturation correction

$$a) \frac{dN^{\text{even}}(\underline{k})}{d^2k dy} [\rho_p, \rho_t] = \frac{2}{(2\pi)^3} \frac{\delta_{ij}\delta_{lm} + \epsilon_{ij}\epsilon_{lm}}{k^2} \Omega_{ij}^a(\underline{k}) [\Omega_{lm}^a(\underline{k})]^*$$

$$b) \frac{dN^{\text{odd}}(\underline{k})}{d^2k dy} [\rho_p, \rho_T] = \frac{2}{(2\pi)^3} \text{Im} \left\{ \frac{g}{k^2} \int \frac{d^2l}{(2\pi)^2} \frac{\text{Sign}(\underline{k} \times \underline{l})}{l^2 |\underline{k} - \underline{l}|^2} f^{abc} \Omega_{ij}^a(\underline{l}) \Omega_{mn}^b(\underline{k} - \underline{l}) [\Omega_{rp}^c(\underline{k})]^* \right. \\ \left. \left[(\underline{k}^2 \epsilon^{ij} \epsilon^{mn} - \underline{l} \cdot (\underline{k} - \underline{l}) (\epsilon^{ij} \epsilon^{mn} + \delta^{ij} \delta^{mn})) \epsilon^{rp} + 2\underline{k} \cdot (\underline{k} - \underline{l}) \epsilon^{ij} \delta^{mn} \delta^{rp} \right] \right\}$$

Recall that $\Omega \propto \rho_{\text{proton}}$



- Odd azimuthal harmonics is a sign of emerging coherence in proton wave function:
the first saturation correction!

Non-zero long-range odd harmonics in high energy p-A is evidence of saturation!

MULTIPLICITY DEPENDENCE: SCALING ARGUMENT

- Physical two-particle anisotropy coefficients can be simply expressed as

$$v_n^2\{2\}(N_{\text{ch}}) = \int \mathcal{D}\rho_p \mathcal{D}\rho_t W[\rho_p] W[\rho_t] |\mathcal{Q}_n[\rho_p, \rho_t]|^2 \delta\left(\frac{dN}{dy}[\rho_p, \rho_t] - N_{\text{ch}}\right)$$

with

$$\mathcal{Q}_{2n}[\rho_p, \rho_t] = \frac{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i2n\phi} \frac{dN^{\text{even}}(k)}{d^2kdy} [\rho_p, \rho_t]}{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\text{even}}(k)}{d^2kdy} [\rho_p, \rho_t]}, \quad \mathcal{Q}_{2n+1}[\rho_p, \rho_t] = \frac{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i(2n+1)\phi} \frac{dN^{\text{odd}}(k)}{d^2kdy} [\rho_p, \rho_t]}{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\text{odd}}(k)}{d^2kdy} [\rho_p, \rho_t]}$$

- High multiplicity is driven by fluctuations in ρ_p
- To study multiplicity dependence, rescale $\rho_p \rightarrow c \rho_p$
- Under this rescaling:

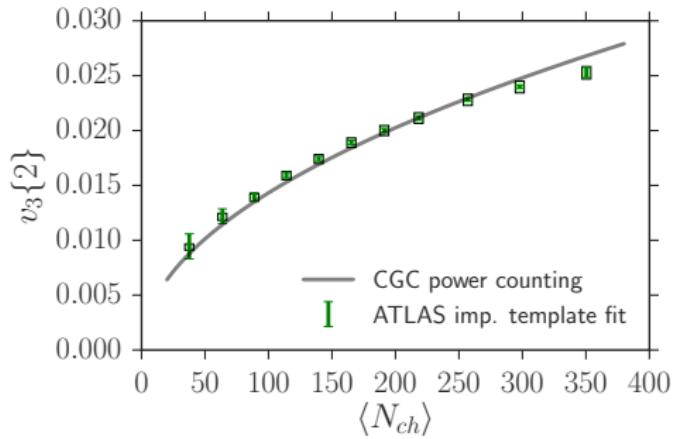
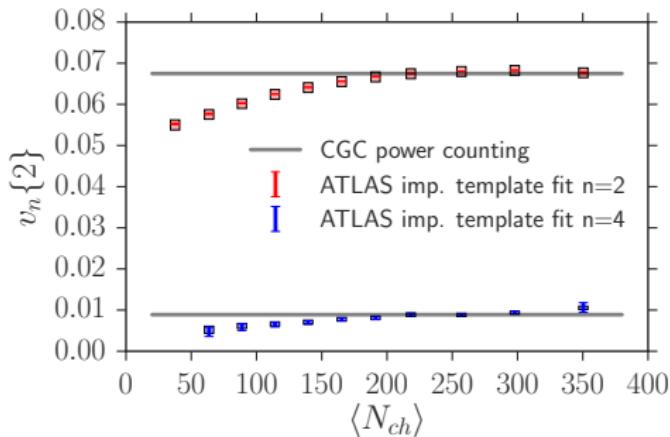
$$\frac{dN}{dy} \rightarrow c \frac{dN}{dy}; \quad v_{2n}^2\{2\} \rightarrow v_{2n}^2\{2\}; \quad v_{2n+1}^2\{2\} \rightarrow c v_{2n+1}^2\{2\}$$

- Therefore in the first approximation: $v_{2n}\{2\}$ is independent of multiplicity

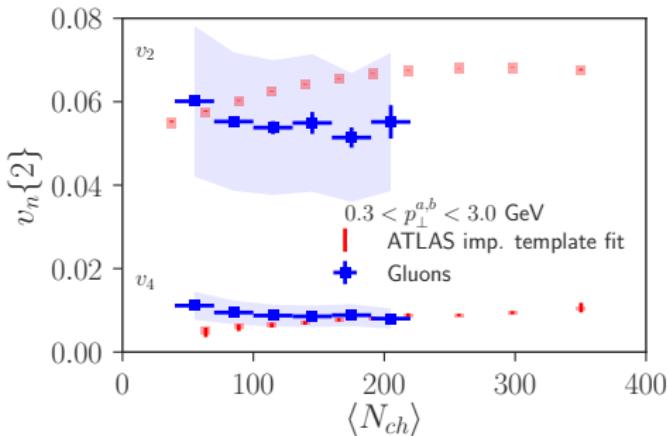
$$v_{2n+1}\{2\} \propto \sqrt{\frac{dN}{dy}}$$

MULTIPLICITY DEPENDENCE: SCALING ARGUMENT

M. Mace , V. S., P. Tribedy, & R. Venugopalan, arXiv:1807.00825



MULTIPLICITY DEPENDENCE: NUMERICAL RESULT



M. Mace , V. S., P. Tribedy, & R. Venugopalan, arXiv:1807.00825



- Multiplicity dependence of integrated v_3
is beyond our computational resources

GLUON DISTRIBUTIONS

Daniel Boer ArXiv 1611.06089

At small x , there are two different unintegrated gluon distributions (UGD):

- **Dipole** gluon distribution ($G^{(2)}$) + linearly polarized partner ($h^{(2)}$).
Appears in many processes. Small x evolution is well understood.
Maximal polarization $xh^{(2)} = xG^{(2)}$
- **Weizsäcker-Williams (WW)** gluon distribution ($G^{(1)}$) + linearly polarized partner ($h^{(1)}$).
Degree of polarization is x - and transverse momentum dependent

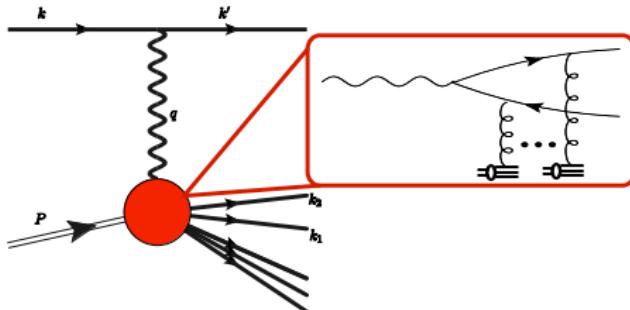
	DIS	DY	SIDIS	$pA \rightarrow \gamma \text{jet} X$	$\begin{array}{l} ep \rightarrow e' Q \bar{Q} X \\ ep \rightarrow e' j_1 j_2 X \end{array}$	$\begin{array}{l} pp \rightarrow \eta_{c,b} X \\ pp \rightarrow H X \end{array}$	$\begin{array}{l} pp \rightarrow J/\psi \gamma X \\ pp \rightarrow T \gamma X \end{array}$	$pA \rightarrow j_1 j_2 X$
$G^{(1)} \text{ (WW)}$	✗	✗	✗	✗	✓	✓	✓	✓
$G^{(2)} \text{ (DP)}$	✓	✓	✓	✓	✗	✗	✗	✓

	$pp \rightarrow \gamma \gamma X$	$pA \rightarrow \gamma^* \text{jet} X$	$\begin{array}{l} ep \rightarrow e' Q \bar{Q} X \\ ep \rightarrow e' j_1 j_2 X \end{array}$	$\begin{array}{l} pp \rightarrow \eta_{c,b} X \\ pp \rightarrow H X \end{array}$	$\begin{array}{l} pp \rightarrow J/\psi \gamma X \\ pp \rightarrow T \gamma X \end{array}$
$h^{(1)} \text{ (WW)}$	✓	✗	✓	✓	✓
$h^{(2)} \text{ (DP)}$	✗	✓	✗	✗	✗

Dijets in DIS: saturation \leadsto decrease of back-to-back dihadron correlation as a probe of $G^{(1)}$

L. Zheng, E. C. Aschenauer, J. H. Lee and B. W. Xia Phys. Rev. D **89**, 7, 074037 (2014)

DIJET PRODUCTION IN DIS AT SMALL X



- DIS dijet production: $\gamma^* A \rightarrow q \bar{q} X$
- Multiple scatterings of (anti) quark are accounted for by resummation:

$$U(\mathbf{x}) = \mathbb{P} \exp \left\{ ig \int dx^- A^+(x^-, \mathbf{x}_\perp) \right\}$$

- In color dipole model this process corresponds to

$$\frac{d\sigma^{\gamma^* A \rightarrow q\bar{q}X}}{d^3 k_1 d^3 k_2} = \text{---} \quad \text{---} \quad \text{---}$$

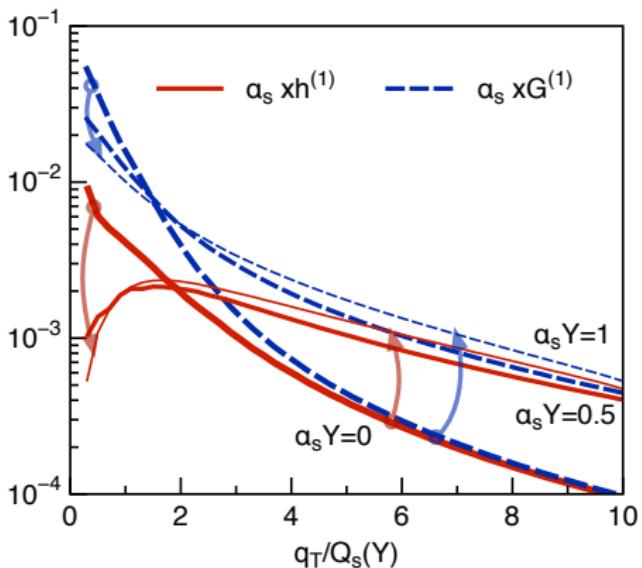
$$N_c \alpha_{em} e_q^2 \delta(p^+ - k_1^+ - k_2^+) \int \frac{d^2 x_1}{(2\pi)^2} \frac{d^2 x_2}{(2\pi)^2} \frac{d^2 y_1}{(2\pi)^2} \frac{d^2 y_2}{(2\pi)^2} \exp(-i\mathbf{k}_1(\mathbf{x}_1 - \mathbf{y}_1) - i\mathbf{k}_2(\mathbf{x}_2 - \mathbf{y}_2))$$

$$\sum_{\gamma\alpha\beta} \psi_{\alpha\beta}^{T,L\gamma}(\mathbf{x}_1 - \mathbf{x}_2) \psi_{\alpha\beta}^{T,L\gamma*}(\mathbf{y}_1 - \mathbf{y}_2) \left[1 + \frac{1}{N_c} \left(\langle \text{Tr } U(\mathbf{x}_1) U^\dagger(\mathbf{y}_1) U(\mathbf{y}_2) U^\dagger(\mathbf{x}_2) \rangle \right. \right.$$

$$\left. \left. - \langle \text{Tr } U(\mathbf{x}_1) U^\dagger(\mathbf{x}_2) \rangle - \langle \text{Tr } U(\mathbf{y}_1) U^\dagger(\mathbf{y}_2) \rangle \right) \right] \quad \uparrow \text{Quadrupole contribution}$$

- Splitting wave function of γ^* with longitudinal momentum p^+ and virtuality Q^2
- This expression can be computed without any further simplifications with **quadrupole**, but no direct relation to WW distribution function

SMALL x EVOLUTION



Reminder of McLerran-Venugopalan model results

$$xh_{\perp}^{(1)} = \frac{S_{\perp}}{2\pi^3 \alpha_s} \frac{N_c^2 - 1}{N_c} \int_0^\infty dr r \frac{J_2(q_{\perp} r)}{r^2 \ln \frac{1}{r^2 \Lambda^2}} \left(1 - \exp \left(-\frac{1}{4} r^2 Q_s^2 \right) \right)$$

$$xG^{(1)} = \frac{S_{\perp}}{2\pi^3 \alpha_s} \frac{N_c^2 - 1}{N_c} \int_0^\infty dr r \frac{J_2(q_{\perp} r)}{r^2} \left(1 - \exp \left(-\frac{1}{4} r^2 Q_s^2 \right) \right)$$

$$\text{Small } q_{\perp} \ll Q_s: xh_{\perp}^{(1)} \propto q_{\perp}^0 \quad xG^{(1)} \propto \ln \frac{Q_s^2}{q_{\perp}^2}$$

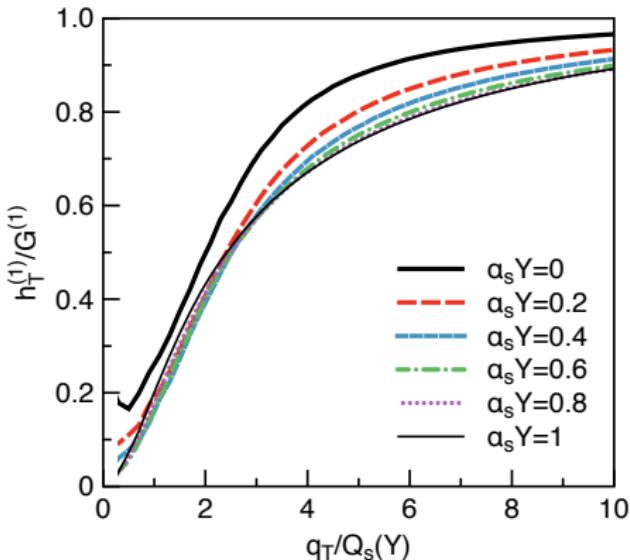
$$\text{Large } q_{\perp} \gg Q_s: xh_{\perp}^{(1)} = xG^{(1)} \propto 1/q_{\perp}^2$$

$$xh^{(1)} \sim \frac{1}{\alpha_s} \left(\frac{g^4 \mu^2}{q^2} - \# \frac{g^8 \mu^4}{q^4} \right), \quad xG^{(1)} \sim \frac{1}{\alpha_s} \left(\frac{g^4 \mu^2}{q^2} + \# \frac{g^8 \mu^4}{q^4} \right)$$

Definition of $Q_s(Y)$: $\langle \text{tr} V^\dagger(0) V(r = \sqrt{2}/Q_s) \rangle = N_c e^{-1/2}$

- at large q_{\perp} , saturation of positivity bound $h_{\perp}^{(1)} \rightarrow G^{(1)}$, as also was found in pert. twist 2 calculations of small x field of fast quark
- at small q_{\perp} , $h_{\perp}^{(1)}/G^{(1)} \rightarrow 0$

SMALL x EVOLUTION II



- Fast departure from MV ($\alpha_s Y = 0$)
- Slow evolution towards smaller x
- Emission of small x gluons reduces degree of polarization.
 q_\perp is scaled by exponentially growing $Q_s(Y)$: ratio at fixed q_\perp decreases with rapidity.
- Approximate scaling at small x .

A. Dumitru, T. Lappi, & V. S. Phys.Rev.Lett. 115 (2015) 25, 252301

