Pion and Kaon Structure on the Lattice

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Outline

• Pion and Kaon EM Form Factors
  – Experimental Motivation
  – Direct Calculation in Lattice QCD
  – Approach to Partonic Degrees of Freedom

• Pion and Kaon PDFs
  • Why the pion?
  • Good Lattice Cross sections

• Future Opportunities
Pion and Kaon Form Factors
Pion EM form factor

Paradigm for LQCD Calculations of matrix elements

\[ \langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2) \]

where

\[ V_\mu = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d \]

\[ -Q^2 = [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}_i)^2 \]
Pion Experimental Summary

Charge Radius

Partonic DOF

\[ Q^2 \rightarrow 30 \text{ GeV}^2 \text{ at future EIC} \]

Thanks to Bipasha Chakraborty

G. Huber, D. Gaskell, T. Horn
PR12-16-003
Anatomy of Pion Form Factor Calculation

\[ \Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x}, \vec{y}} \langle 0 | \phi(\vec{x}, t_f) V_{\mu}(\vec{y}, t) \phi^\dagger(\vec{0}, 0) | 0 \rangle e^{-i\vec{p} \cdot \vec{x}} e^{-i\vec{q} \cdot \vec{y}} \]

Resolution of unity – insert states

\[ \langle 0 | \phi(0) | \pi, \vec{p} + \vec{q} \rangle \langle \pi, \vec{p} + \vec{q} | V_{\mu}(0) | \pi, \vec{p} \rangle \langle \pi, \vec{p} | \phi^\dagger | 0 \rangle e^{-E(\vec{p}(t-t_i))} e^{-E(\vec{p}+\vec{q})(t_f-t)} \]

\[ \Gamma_{\pi^+\pi^+}(t, 0; \vec{p}) = \sum_{\vec{x}} \langle 0 | \phi(\vec{x}, t_f) \phi^\dagger(0) | 0 \rangle e^{-i\vec{p} \cdot \vec{x}} \]

\[ \propto |\langle 0 | \phi(0) | \pi, \vec{p} \rangle|^2 e^{-E(\vec{p})t} \]
Pion Form Factor - I

\[ \langle r^2 \rangle = 6 \frac{dF(q^2)}{dq^2} \bigg|_{q^2=0} \]

Briceno, Chakraborty, Edwards, Kusno, Orginos, DGR, Winter

HadSpec, PRD 91 (2015)
Pion Form Factor - II

- Challenge to reach high momenta
  - discretization errors \( p \leq 1/a \)
- Signal-to-noise ratio

\[
C(t, \vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t, \vec{x}) \mathcal{O}^\dagger(0, 0) \mid 0 \rangle e^{-i\vec{p} \cdot \vec{x}} \rightarrow e^{-E(\vec{p})t}
\]

\[
C_{\sqrt{\sigma^2}}(t, \vec{p}) \rightarrow e^{-m_\pi t}
\]

- Boosted interpolating operators
  - \textit{Bali et al.}, Phys. Rev. D 93, 094515 (2016)

- \textit{Variational Method}

- \textit{Feynman-Hellmann method}

- \textit{Probe correlators at small t}
Variational Method

- Solve generalized eigenvalue equation

\[ C(t) v^{(N)}(t, t_0) = \lambda_N(t, t_0) C(t_0) v^{(N)}(t, t_0). \]
\[ \lambda_N(t, t_0) \longrightarrow e^{-E_N(t-t_0)}, \]

- Find optimal interpolating operator, coupling to lowest state

\[ \mathcal{O}_{N, \text{proj}} = v_i^{(N)} \mathcal{O}_i \]

- Implement using distillation M.Pearson et al., arXiv:0905.2160

\[ C_{3pt} \to \langle 0 \mid \mathcal{O}_{\text{proj}}(\vec{p}_f, t_f) V_\mu(\tau) \mathcal{O}_{\text{proj}}(\vec{p}_i, t_i) \mid 0 \rangle; \bar{q} = \vec{p}_f - \vec{p}_i \]

Feynman-Hellmann method

\[ H = H_0 + \lambda H_\lambda \]
\[ \frac{\partial E_n}{\partial \lambda} = \langle n \mid H_\lambda \mid n \rangle \]

Reduces to calculation of energy-shift of two-point functions but repeat the calculation for each operator
Form Factor at high $Q^2$

Form factor at high momenta achievable

Feynman-Hellmann method

Preliminary
Kaon Form Factor

\[ m_\pi \approx 310 \text{ MeV} \]

Kaponen arXiv:1710.07754
Pion and Kaon PDFs

PDFs

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use Operator-Product-Expansion to formulate in terms of Mellin Moments
    \[
    q(x, \mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^-} P^+ \langle P | \bar{\psi}(\xi^-)\gamma^+ e^{-ig \int_{\eta^-}^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) | P \rangle 
    \]
    \[
    \langle P | \bar{\psi}\gamma_{\mu_1}(\gamma_5) D_{\mu_2} \ldots D_{\mu_n} \psi | P \rangle \rightarrow P_{\mu_1} \ldots P_{\mu_n} a^{(n)}
    \]

- Moment Methods

  KF Liu, SJ Dong, PRL72, 1790 (1994)

- Hadronic Tensor (HT)
  \[
  W_{\mu\nu} = \frac{1}{4\pi} \int d^4z e^{i q \cdot z} \langle p | J_{\mu}(z)^\dagger J_{\nu}(0) | p \rangle
  \]
  \[
  C_4(p, q, \tau) = \sum_{\bar{x}_f} e^{-i p \cdot \bar{x}_f} \sum_{\bar{x}_2, \bar{x}_1} e^{-i q \cdot (\bar{x}_2 - \bar{x}_1)} \langle N(\bar{x}_f, t_f) J_{\mu}(\bar{x}_2, t_2) J_{\nu}(\bar{x}_1, t_1) \bar{N}(\bar{0}, t_0) \rangle
  \]
  This is a \textbf{four-point} function.
• Quasi-PDF (qPDF) interpreted in LaMET (Large Momentum Effective Theory) was proposed by X. Ji

\[ q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P | \bar{\psi}(z) \gamma^z e^{-ig \int_0^z dz' A^z(z')} \psi(0) | P \rangle + O((\Lambda^2/(P^z)^2), M^2/(P^z)^2) \]

Quasi distributions approach light-cone distributions in limit of large \( P^z \)

\[ q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z \left( \frac{x}{y}, \frac{\mu}{P^z} \right) q(y, \mu^2) + O(\Lambda^2/(P^z)^2, M^2/(P^z)^2) \]

• Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of loffe Time. \( \nu = p \cdot z \)

A. Radyushkin, PLB767 (2017)
Recent qPDF and pPDF Results

Orginos, Radyushkin, Karpie and Zafeiropoulos, arXiv:1706.05373

- Pseudo-PDF
- Quenched
- pion mass 600 MeV
Challenges/Questions

Relation between qPDF and pPDF approaches
– Both integrals of Ioffe-Time Distribution Function
– Should yield same PDF after matching and systematic controls

\[ P(x, z_0^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} M(\nu, z_0^2) \]

\[ Q(x, p_z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} M(\nu, \frac{\nu^2}{p_z^2}) \]
Lattice Cross Sections

- Good “Lattice Cross Sections” (LCS) (Ma and Qiu, Phys. Rev. Lett. 120 022003)

\[
\sigma_n(\omega, \xi^2, P^2) = \langle P \mid T\{O_n(\xi)\} \mid P \rangle
\]

where

\[
\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + O(\xi^2 \Lambda_{QCD}^2)
\]

Calculated in perturbation theory

Structure function

Expressed in coordinate space

Short distance scale

Calculated in LQCD

Factorize in \( \omega = P \cdot \xi, \xi^2 P^2 \)

providing \( \xi << \frac{1}{\Lambda_{QCD}} \)

Momentum space

\[
\tilde{\sigma}(\tilde{\omega}, q^2 P^2) \equiv \int \frac{d^4\xi}{\xi^4} \sigma(P \cdot \xi, \xi^2, P^2)
\]

\( \tilde{\omega} = 1/x_B \)
Lattice Cross Sections - II

- Quasi- and Pseudo-distributions particular case

\[ \mathcal{O}(\xi) = \bar{\psi}(0) \Gamma W(0, 0 + \xi) \psi(\xi) \]

- Current-current correlators, e.g.

\[ \mathcal{O}_S(\xi) = \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi) [\bar{\psi}_q \psi](0) \]
\[ \mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_q'](\xi) [\bar{\psi}_q' \xi \cdot \gamma \psi](0) \]

- Wilson Line

- Gauge-invariant

- Renormalization straightforward
Pion PDFs

- u distribution of FNAL E615 to leading order
- C12-15-006 at Hall A will look at structure of pion
- C12-15-006A at Hall A will look at structure of Kaon
- No free pion target

Discrepancy in large-x behavior of pion distribution

de Teramond, Liu, Sufian, Dosch, Brodsky, Deur, PRL (2018)
Pion PDFs - II

• Pion less computationally demanding than nucleon
  – Larger signal-to-noise ratio

\[ C(t, \vec{p}) \equiv \sum \langle 0 | \mathcal{O}(t, \vec{x}) \mathcal{O}^\dagger(0, 0) | 0 \rangle e^{-i\vec{p} \cdot \vec{x}} \rightarrow e^{-E(\vec{p})t} \]

\[ C_{\sqrt{\sigma^2}}(t, \vec{p}) \rightarrow \begin{cases} 
  e^{-m_\pi t} & \text{Mesons} \\
  e^{-(3m_\pi/2)t} & \text{Baryons}
\end{cases} \]

• Important constraint on systematic uncertainty is understanding operator renormalization
  – Operator renormalization “independent” of external states

• Admits simple computational methodology
  – Coordinate-space currents computationally demanding in lattice QCD
Quasi-Distribution of Pion

\[ m_\pi \simeq 300 \text{ MeV} \]

LP3, arXiv:1804.01483
Momentum conservation

\[
\langle \Pi(-p')|\mathcal{O}_J(x_0)\mathcal{O}_J(\xi)|\Pi(-p')\rangle = \\
= \sum_{y,z} e^{i(p'.z - p.y)} \langle \bar{q}_\Pi q(z,T) \bar{Q} J_2 Q(x_0 + \xi,t) \bar{q} J_1 q(x_0,t) \bar{q} \Gamma_\Pi q(y,0) \rangle \\
= \sum_{y,z} e^{i(p'.z - p.y)} \text{Tr}[J_2 G_Q(x_0 + \xi,t;x_0,t) J_1 G(x_0,t;y,0) \Gamma_\Pi G(y,0;z,T) \Gamma_\Pi G(z,T;x_0 + \xi,t)]
\]

Straightforward computational setup using sequential-source method:
Computational Setup

Momentum conservation

\[ \langle \Pi(-p')|O_{J_1}(x_0)O_{J_2}(\xi)|\Pi(-p') \rangle = \sum_{y,z} e^{i(p',z-p,y)} \langle \bar{q} \Gamma_{\Pi} q(z,T) \bar{Q} \ J_2 \ Q(x_0 + \xi, t) \bar{q} \ J_1 \ q(x_0, t) \bar{q} \ \Gamma_{\Pi} \ q(y, 0) \rangle \]

\[ = \sum_{y,z} e^{i(p',z-p,y)} \text{Tr}[J_2 \ G_Q(x_0 + \xi, t; x_0, t) \ J_1 \ G(x_0, t; y, 0) \ \Gamma_{\Pi} \ G(y, 0; z, T) \ \Gamma_{\Pi} \ G(z, T; x_0 + \xi, t)] \]

Straightforward computational setup using sequential-source method:

\[ D(Z, T; w)H(w; x_0, t) = \sum_{y} e^{-i p \cdot y} \Gamma_{\Pi} G(y, 0; x_0, t) \]
Computational Setup

Momentum projection

\[ m_{u/d} \leq m_Q \leq m_s \]

\[ \langle \Pi(-p')| O_{J_1}(x_0) O_{J_2}(\xi)|\Pi(-p') \rangle = \]
\[ = \sum_{y,z} e^{i(p' \cdot z - p \cdot y)} \langle \bar{q} \Gamma_{\Pi} q(z, T) \bar{Q} J_2 Q(x_0 + \xi, t) \bar{q} J_1 q(x_0, t) \bar{q} \Gamma_{\Pi} q(y, 0) \rangle \]
\[ = \sum_{y,z} e^{i(p' \cdot z - p \cdot y)} \text{Tr}[J_2 G_Q(x_0 + \xi, t; x_0, t) J_1 G(x_0, t; y, 0) \Gamma_{\Pi} G(y, 0; z, T) \Gamma_{\Pi} G(z, T; x_0 + \xi, t)] \]

Momentum conservation

Straightforward computational setup using sequential-source method:

\[ D(Z, T; w) H(w; x_0, t) = \sum_y e^{-i p \cdot y} \Gamma_{\Pi} G(y, 0; x_0, t) \]
\[ D(s; w) \tilde{H}(w; x_0, t) = \sum_z e^{i p \cdot z} \Gamma_{\Pi} H(z, T; x_0, t) \]
Preliminary Results

- 2+1 Flavor clover-fermion action
  \[ a \approx 0.12, 0.09 \text{ fm} \]
  \[ m_\pi \approx 400, 440 \text{ MeV} \]

\[ J_1 = V, \ J_2 = A \]

- 110 configurations
- Single source point for current \( J_1 \)

Clear isolation of pion matrix element
Preliminary Results - II

Clear signal in real part to $p$ around 1 GeV

Imaginary part only to small values of $p$
Challenges/Questions

High spatial momentum and lattice systematics

Boosted interpolating operators


Inverse Problem - common to all approaches

\[
\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + O(\xi^2 \Lambda_{QCD}^2)
\]

Calculate on Lattice

Extract PDF?

Calculate in PQCD
Finite Volume Effects

Briceno, Guerrero, Hansen and Monahan, arXiv:1805.01304

Typically $m_\pi L \simeq 4$

Future? \{ $\xi$ short-distance $m_\pi \rightarrow m_\pi^{\text{phys}}$ \}
Pion Quark Distribution Amplitude

“Pseudo” quark distribution amplitudes

\[ T(\omega, z^2) = \langle \pi(p) \mid [\bar{u}Q](z/2)[\bar{Q}u](-z/2) \mid 0 \rangle \]

Bali et al., arXiv:1709.04325

Quark Distribution Pictures → Ioffe pseudo-time
Pion QDA - II

Fixed $z^2$
Summary

• Pion Form Factors at high momenta with reach comparable to 12 GeV at JLab

• Calculation of current-current correlators for pion and kaon in progress for variety of local operators
  – Important to understand finite-volume effects
  – Extending calculation to close-to-physical
    \[ m_\pi \simeq 170 \text{ MeV} \]
    \[ 64^3 \times 128 \text{ Lattices} \]

• Variety of lattice cross sections - including pseudo PDFs - on same ensemble of lattices.
Outlook

Momentum projection

\[ p' \]

\[ J_1 \]

\[ (x_0, t) \]

\[ (x_0 + \xi, t) \]

\[ J_2 \]

Momentum conservation

\[ (z, T) \]

\[ (y, 0) \]

Momentum projection

TMDs \[ \leftrightarrow \]

\[ \text{Compute for any } \xi \text{ and } p, p' \leftrightarrow \]

GPDs

\[ Q \]

\[ s \]

\[ x \]

\[ t \]

\[ J \]

\[ M \]

\[ W \]

\[ \eta \]

\[ \eta' \]

\[ \chi \]

\[ Q \]

\[ p \]

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