Pion and Kaon Structure on the Lattice

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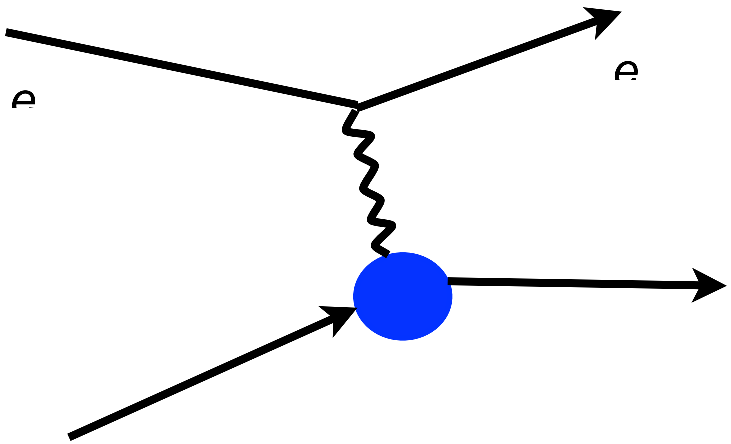
Outline

- Pion and Kaon EM Form Factors
  – Experimental Motivation
  – Direct Calculation in Lattice QCD
  – Approach to Partonic Degrees of Freedom
- Pion and Kaon PDFs
  - Why the pion?
  - Good Lattice Cross sections
- Future Opportunities

Pion and Kaon Form Factors

Pion EM form factor

Paradigm for LQCD Calculations of matrix elements



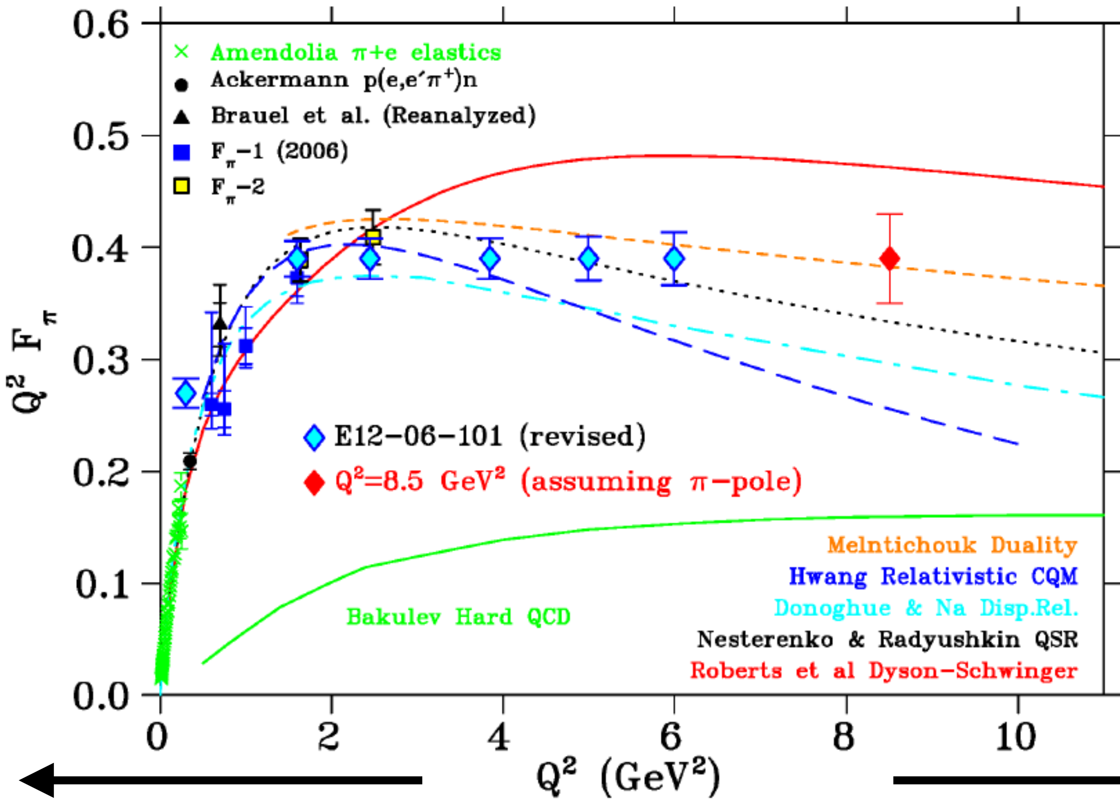
$$\langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2)$$

where

$$\begin{aligned} V_\mu &= \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d \\ -Q^2 &= [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}_i)^2 \end{aligned}$$

Spacelike…

Pion Experimental Summary



Thanks to
Bipasha Chakraborty

G. Huber, D.Gaskell, T. Horn
PR12-16-003

Charge Radius ⟵ ⟶ *Partonic DOF*

$Q^2 \longrightarrow 30\,\mathrm{GeV}^2$ at future EIC

Anatomy of Pion Form Factor Calculation

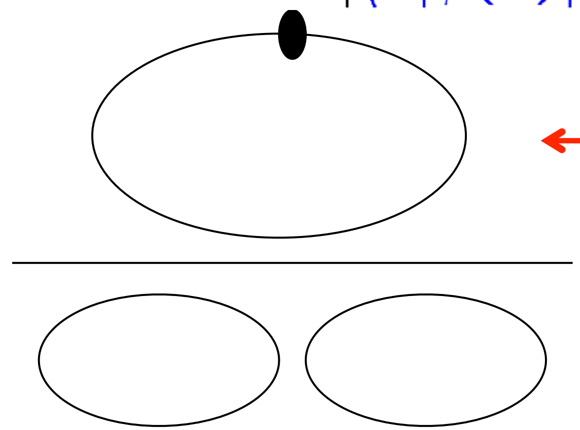
$$\Gamma_{\pi^+\mu\pi^+}(t_f,t;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}}\langle 0|\phi(\vec{x},t_f)V_\mu(\vec{y},t)\phi^\dagger(\vec{0},0)|0\rangle e^{-i\vec{p}\cdot\vec{x}}e^{-i\vec{q}\cdot\vec{y}},$$

Resolution of unity – insert states

$$\langle 0 \mid \phi(0) \mid \pi,\vec{p}+\vec{q}\rangle\langle\pi,\vec{p}+\vec{q} \mid V_\mu(0) \mid \pi,\vec{p}\rangle\langle\pi,\vec{p} \mid \phi^\dagger \mid 0\rangle e^{-E(\vec{p}(t-t_i)}e^{-E(\vec{p}+\vec{q})(t_f-t)}$$

$$\Gamma_{\pi^+\pi^+}(t,0;\vec{p}) = \sum_{\vec{x}}\langle 0 \mid \phi(\vec{x},t_f)\phi^\dagger(0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}}$$

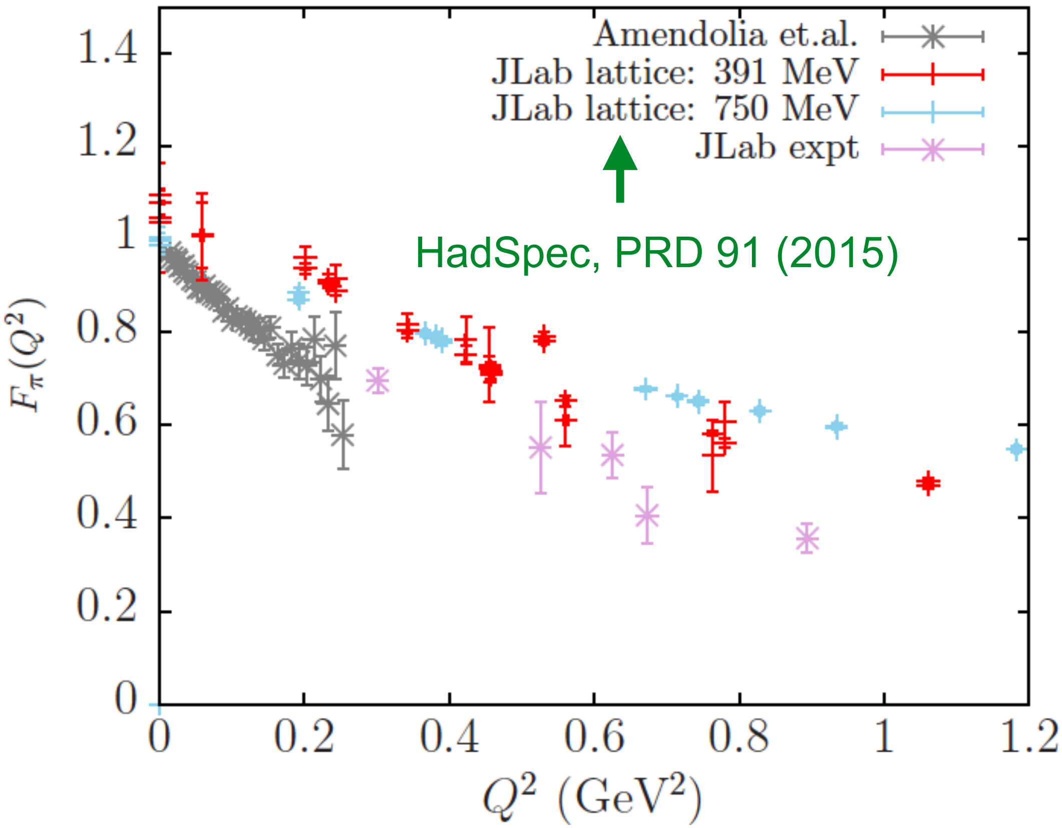
$$\propto |\langle 0|\phi(0)|\pi,\vec{p}\rangle|^2 e^{-E(\vec{p})t}$$



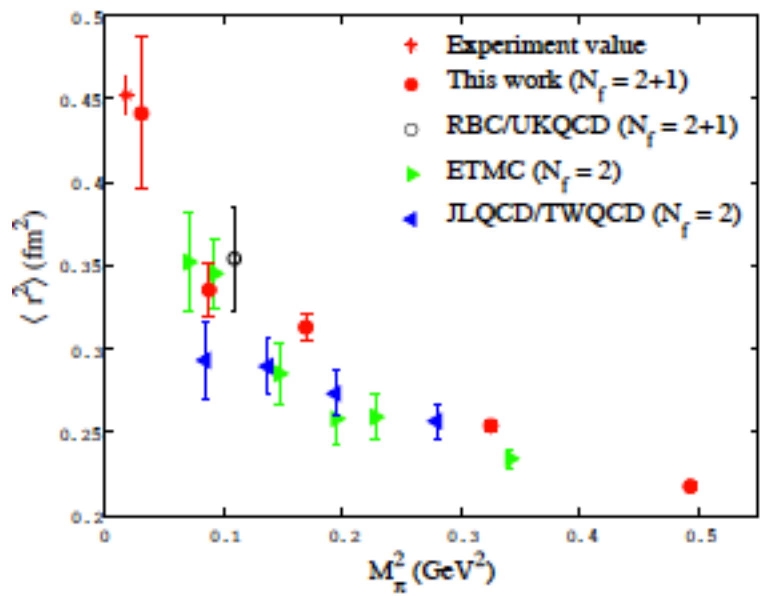
3pt

2pt

Pion Form Factor - I



Briceno, Chakraborty, Edwards, Kusno, Orginos, DGR, Winter



$$\langle r^2 \rangle = 6 \left. \frac{dF(q^2)}{dq^2} \right|_{q^2=0}$$

Pion Form Factor - II

- Challenge to reach high momenta
  - discretization errors $p \leq 1/a$
- Signal-to-noise ratio

$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^\dagger(0,0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}} \to e^{-E(\vec{p})t}$$

$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \longrightarrow e^{-m_\pi t}$$

Probe correlators at small t

Boosted interpolating operators

Bali *et al*., Phys. Rev. D 93, 094515 (2016)

Feynman-Hellmann method

$\boxed{\text{Variational Method}}$

Variational Method

- Solve generalized eigenvalue equation

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$
$$\lambda_N(t,t_0) \longrightarrow e^{-E_N(t-t_0)},$$

- Find *optimal interpolating operator*, coupling to lowest state

$$\mathcal{O}_{N,\mathrm{proj}} = v_i^{(N)}\mathcal{O}_i$$

- Implement using *distillation* M.Peardon et al., arXiv:0905.2160

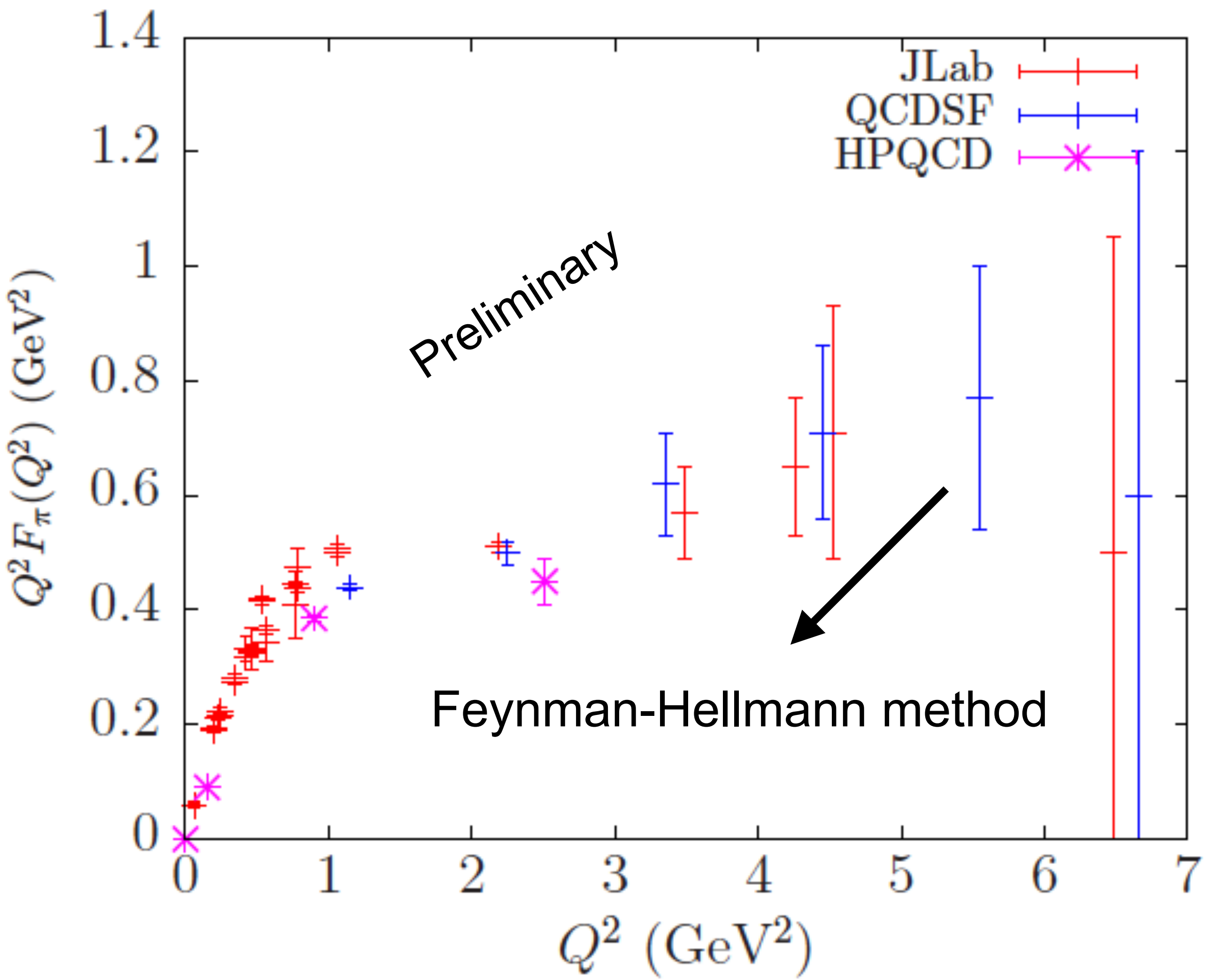
$$C_{3\mathrm{pt}} \to \langle 0 \mid \mathcal{O}_{\mathrm{proj}}(\vec{p}_f, t_f)V_\mu(\tau)\mathcal{O}_{\mathrm{proj}}(\vec{p}_i, t_i) \mid 0\rangle;\ \vec{q} = \vec{p}_f - \vec{p}_i$$

Feynman-Hellmann method

$$\begin{aligned} H &= H_0 + \lambda H_\lambda \\ \frac{\partial E_n}{\partial \lambda} &= \langle n \mid H_\lambda \mid n\rangle \end{aligned}$$

Reduces to calculation of energy-shift of two-point functions ***but*** repeat the calculation for each operator

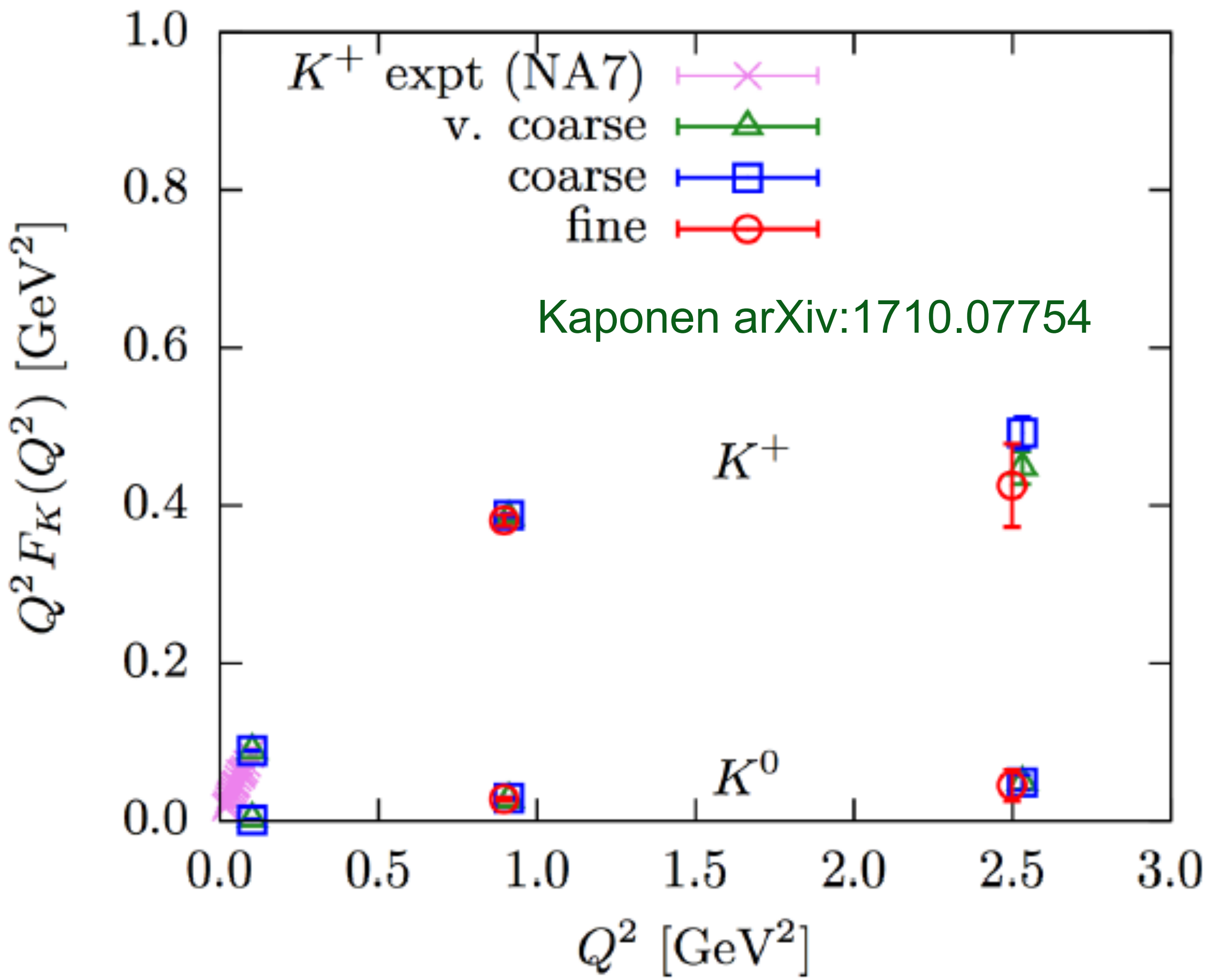
Form Factor at high Q^2



arXiv:1702.01513
arXiv:1710.07554

Form factor at high momenta achievable

Kaon Form Factor



$m_\pi \simeq 310\,\mathrm{MeV}$

Pion and Kaon PDFs

J. Karpie, C. Egerer, J.W. Qiu, B. Chakraborty, R. Edwards, K. Orginos, DGR, R. Sufian

PDFs

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use *Operator-Product-Expansion* to formulate in terms of *Mellin Moments*

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}\psi(0) \mid P\rangle$$

$$\downarrow$$

$$\langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\ldots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\ldots P_{\mu_n}a^{(n)}$$

- Moment Methods
  - Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
  - Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

KF Liu, SJ Dong, PRL72, 1790 (1994)

- Hadronic Tensor (HT) $W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq.z}\langle p \mid J_\mu(z)^\dagger J_\nu(0) \mid p\rangle$

$$C_4(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_f} e^{-i\vec{p}.\vec{x}_f} \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{q}.(\vec{x}_2-\vec{x}_1)}\langle N(\vec{x}_f,t_f)J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\bar{N}(\vec{0},t_0)\rangle$$

This is a ***four-point*** function.

- Quasi-PDF (qPDF) interpreted in LaMET (Large Momentum Effective Theory) was proposed by X.Ji **X. Ji, Phys. Rev. Lett. 110 (2013) 262002**

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > \\ + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

Quasi distributions approach light-cone distributions in limit of large *P*^{z}

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

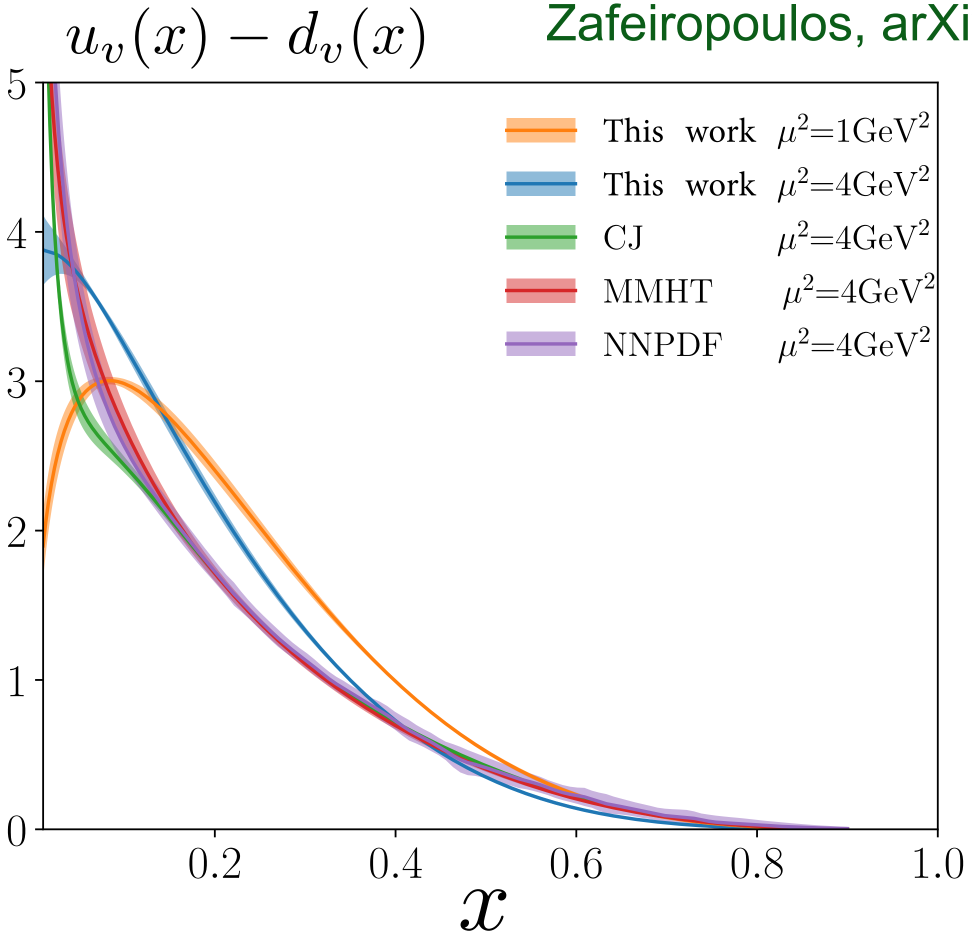
- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

A. Radyushkin, PLB767 (2017)

$$\mathcal{M}^\alpha(z,p) = \langle p \mid \bar{\psi}(z)\gamma^\alpha \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0) \mid p\rangle$$

Recent qPDF and pPDF Results

Orginos, Radyushkin, Karpie and Zafeiropoulos, arXiv:1706.05373



- Pseudo-PDF
- Quenched
- pion mass 600 MeV

Challenges/Questions

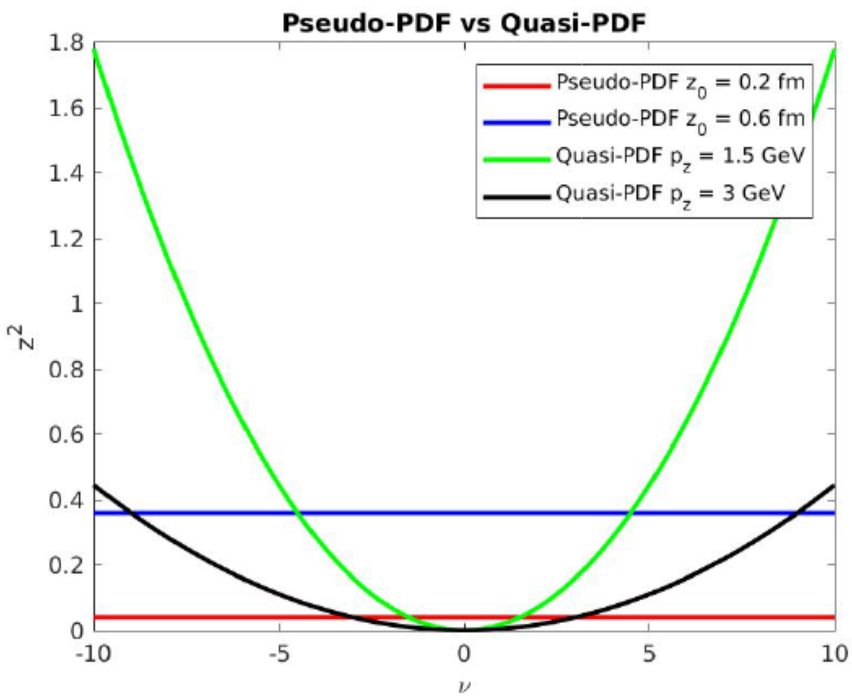
Relation between qPDF and pPDF approaches

– Both integrals of Ioffe-Time Distribution Function

– Should yield same PDF after matching and systematic controls

$$P(x, z_0^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} M(\nu, z_0^2)$$

$$Q(x, p_z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} M(\nu, \frac{\nu^2}{p_z^2})$$



Lattice Cross Sections

- Good "Lattice Cross Sections" (LCS) Ma and Qiu, Phys. Rev. Lett. 120 022003

$$\sigma_n(\omega, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P\rangle$$

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2\Lambda^2_{\rm QCD})$$

Calculated in LQCD

Structure function

Calculated in perturbation theory

Factorize in $\omega = P \cdot \xi,\ \xi^2 P^2$ providing $\xi << \frac{1}{\Lambda_{\rm QCD}}$

Momentum space

$$\tilde{\sigma}(\tilde{\omega}, q^2P^2) \equiv \int \frac{d^4\xi}{\xi^4} \sigma(P\cdot\xi, \xi^2, P^2)$$

$$\tilde{\omega} = 1/x_B$$

Lattice Cross Sections - II

- Quasi- and Pseudo-distributions particular case

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Wilson Line

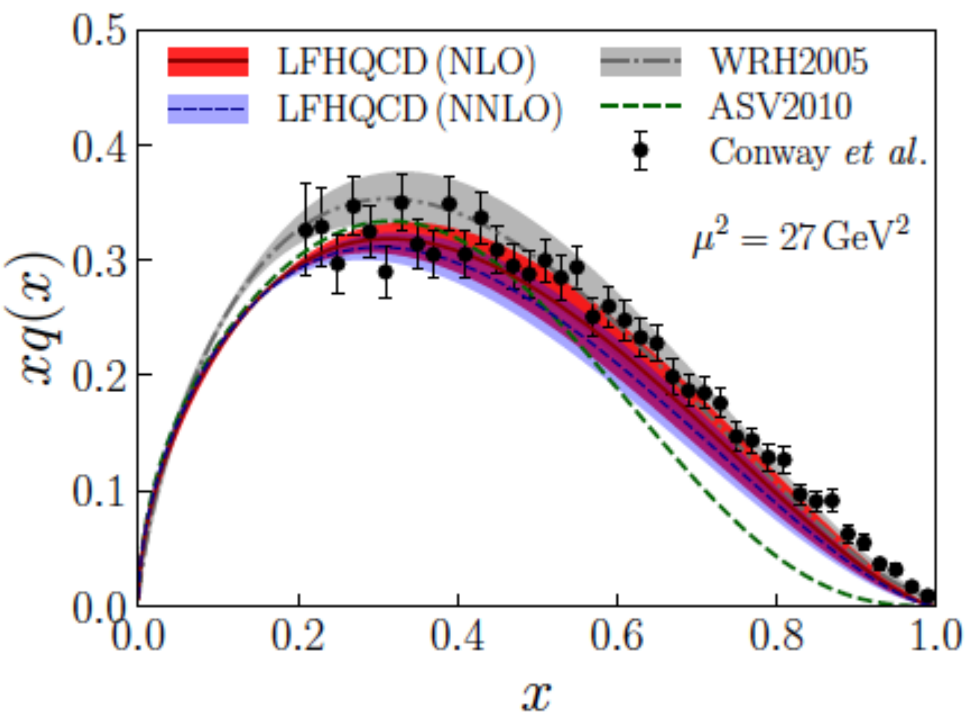
- Current-current correlators, e.g.

$$\begin{aligned}
\mathcal{O}_S(\xi) &= \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0) \\
\mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi\cdot\gamma\psi_{q'}](\xi)[\bar{\psi}_{q'} \xi\cdot\gamma\psi](0) \\
& \quad F_{\mu\rho}F_{\nu}^{\rho}
\end{aligned}$$

- Gauge-invariant
- Renormalization straightforward

Pion PDFs

- u distribution of FNAL E615 to leading order
- C12-15-006 at Hall A will look at structure of pion
- C12-15-006A at Hall A will look at structure of Kaon
- No free pion target



de Teramond, liu, Sufian, Dosch, Brodsky, Deur, PRL (2018)

Discrepancy in large-x behavior of pion distribution

Pion PDFs - II

- Pion less computationally demanding that nucleon
  - *Larger signal-to-noise ratio*

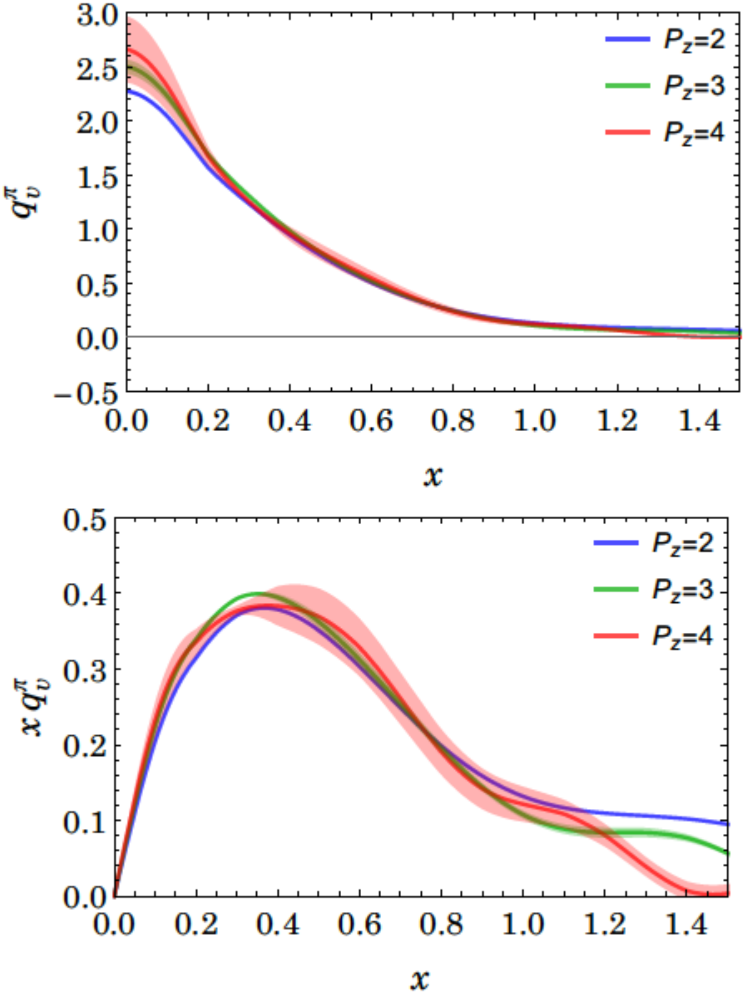
$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^{\dagger}(0,0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} \rightarrow e^{-E(\vec{p})t}$$

$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \rightarrow \begin{cases} e^{-m_\pi t} & \text{Mesons} \\ e^{-(3m_\pi/2)t} & \text{Baryons} \end{cases}$$

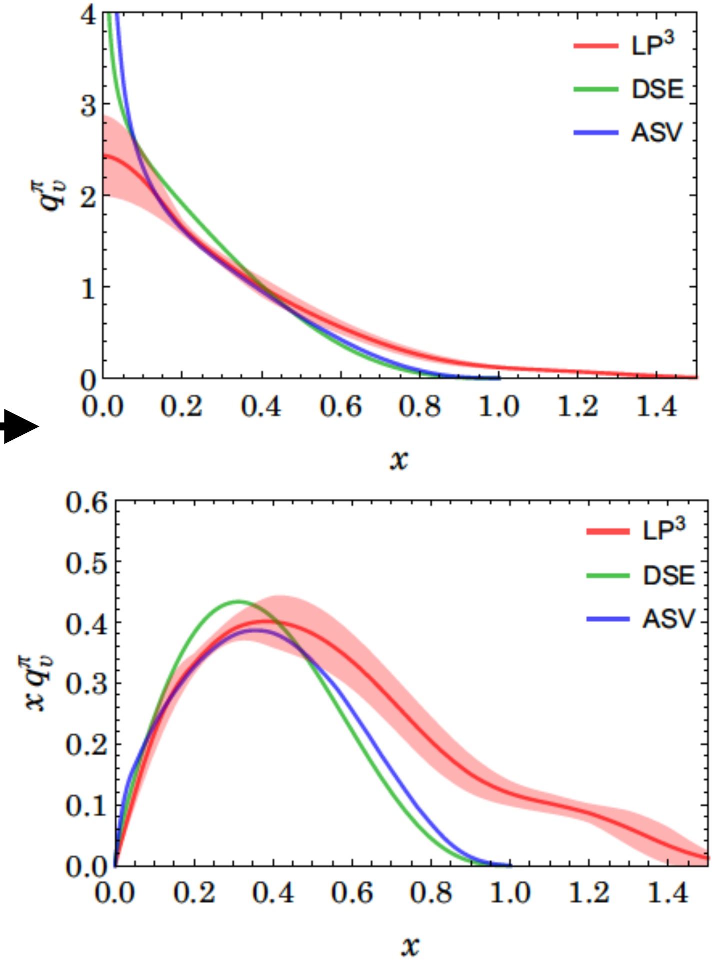
- Important constraint on systematic uncertainty is understanding operator renormalization
  - *Operator renormalization “independent” of external states*
- Admits simple computational methodology
  - *Coordinate-space currents computationally demanding in lattice QCD*

Quasi-Distribution of Pion

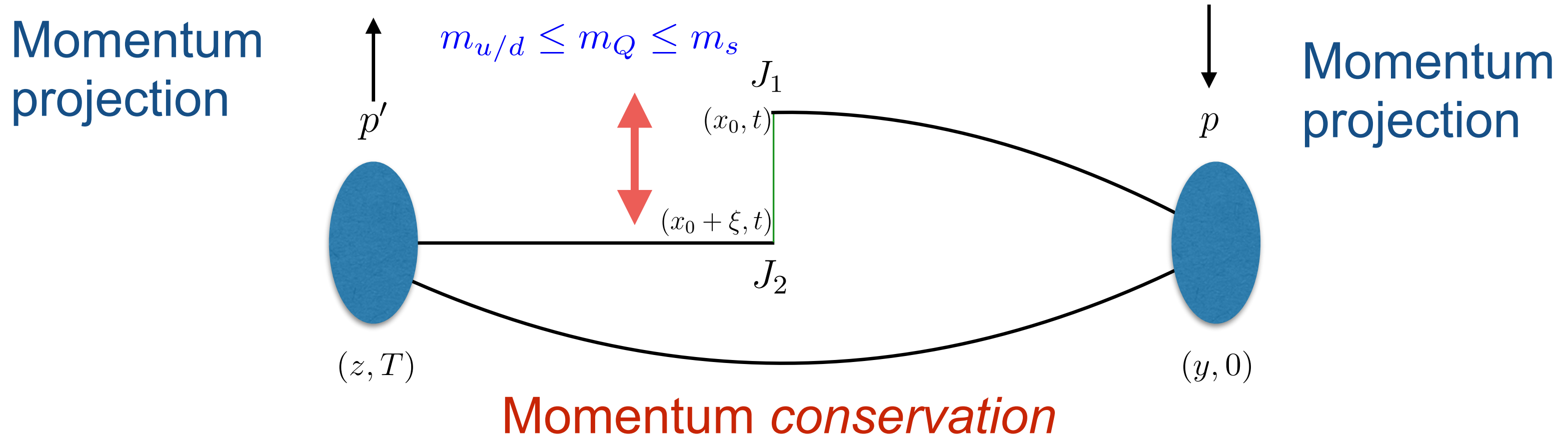
$m_\pi \simeq 300\,\mathrm{MeV}$



LP3, arXiv:1804.01483



Computational Setup



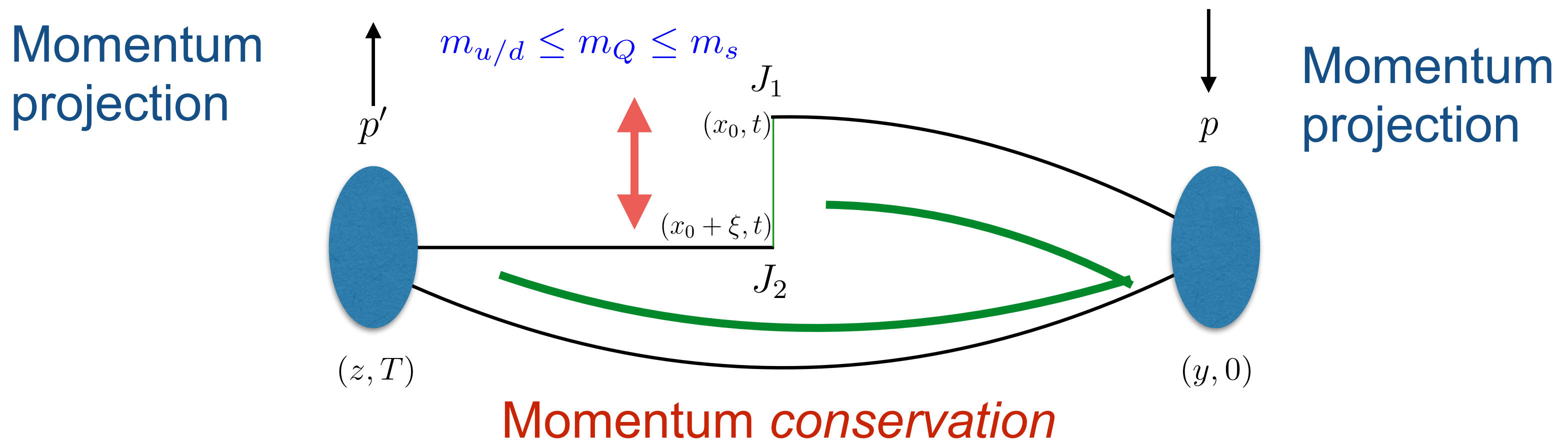
$$\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle =$$

$$= \sum_{y,z} e^{i(p'.z-p.y)}\langle \bar{q}\,\Gamma_\Pi\, q(z,T)\,\bar{Q}\,J_2\,Q(x_0+\xi,t)\,\bar{q}\,J_1\,q(x_0,t)\,\bar{q}\,\Gamma_\Pi\,q(y,0)\rangle$$

$$= \sum_{y,z} e^{i(p'.z-p.y)}\mathrm{Tr}[J_2\,G_Q(x_0+\xi,t;x_0,t)\,J_1\,G(x_0,t;y,0)\,\Gamma_\Pi G(y,0;z,T)\Gamma_\Pi\,G(z,T;x_0+\xi,t)]$$

Straightforward computational setup using sequential-source method:

Computational Setup



$$\langle \Pi(-p') | \mathcal{O}_{J_1}(x_0) \mathcal{O}_{J_2}(\xi) | \Pi(-p') \rangle =$$

$$= \sum_{y,z} e^{i(p'.z - p.y)} \langle \bar{q}\, \Gamma_\Pi\, q(z,T)\, \bar{Q}\, J_2\, Q(x_0+\xi, t)\, \bar{q}\, J_1\, q(x_0, t)\, \bar{q}\, \Gamma_\Pi\, q(y, 0) \rangle$$

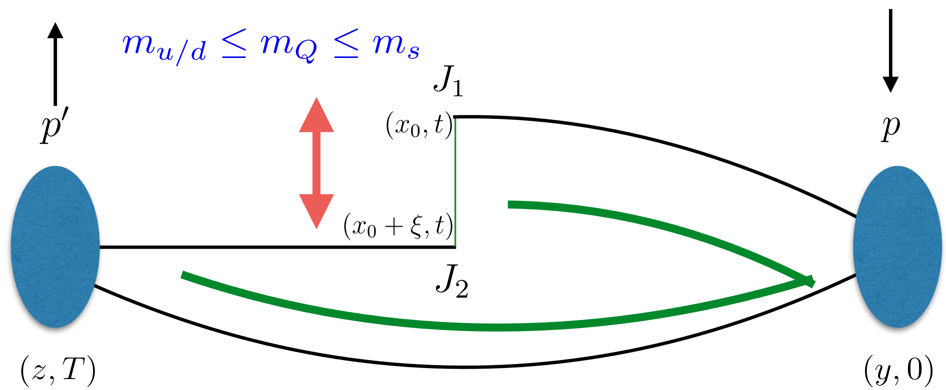
$$= \sum_{y,z} e^{i(p'.z - p.y)} \mathrm{Tr}[J_2\, G_Q(x_0+\xi, t; x_0, t)\, J_1\, G(x_0, t; y, 0)\, \Gamma_\Pi G(y, 0; z, T) \Gamma_\Pi\, G(z, T; x_0+\xi, t)]$$

Straightforward computational setup using sequential-source method:

$$D(Z, T; w) H(w; x_0, t) = \sum_y e^{-ip\cdot y} \Gamma_\Pi G(y, 0; x_0, t)$$

Computational Setup

Momentum projection



Momentum projection

Momentum *conservation*

$$\langle \Pi(-p') | \mathcal{O}_{J_1}(x_0) \mathcal{O}_{J_2}(\xi) | \Pi(-p') \rangle =$$

$$= \sum_{y,z} e^{i(p'.z - p.y)} \langle \bar{q}\, \Gamma_\Pi\, q(z,T)\, \bar{Q}\, J_2\, Q(x_0 + \xi, t)\, \bar{q}\, J_1\, q(x_0, t)\, \bar{q}\, \Gamma_\Pi\, q(y, 0) \rangle$$

$$= \sum_{y,z} e^{i(p'.z - p.y)} \mathrm{Tr}[J_2\, G_Q(x_0 + \xi, t; x_0, t)\, J_1\, G(x_0, t; y, 0)\, \Gamma_\Pi G(y, 0; z, T) \Gamma_\Pi\, G(z, T; x_0 + \xi, t)]$$

Straightforward computational setup using sequential-source method:

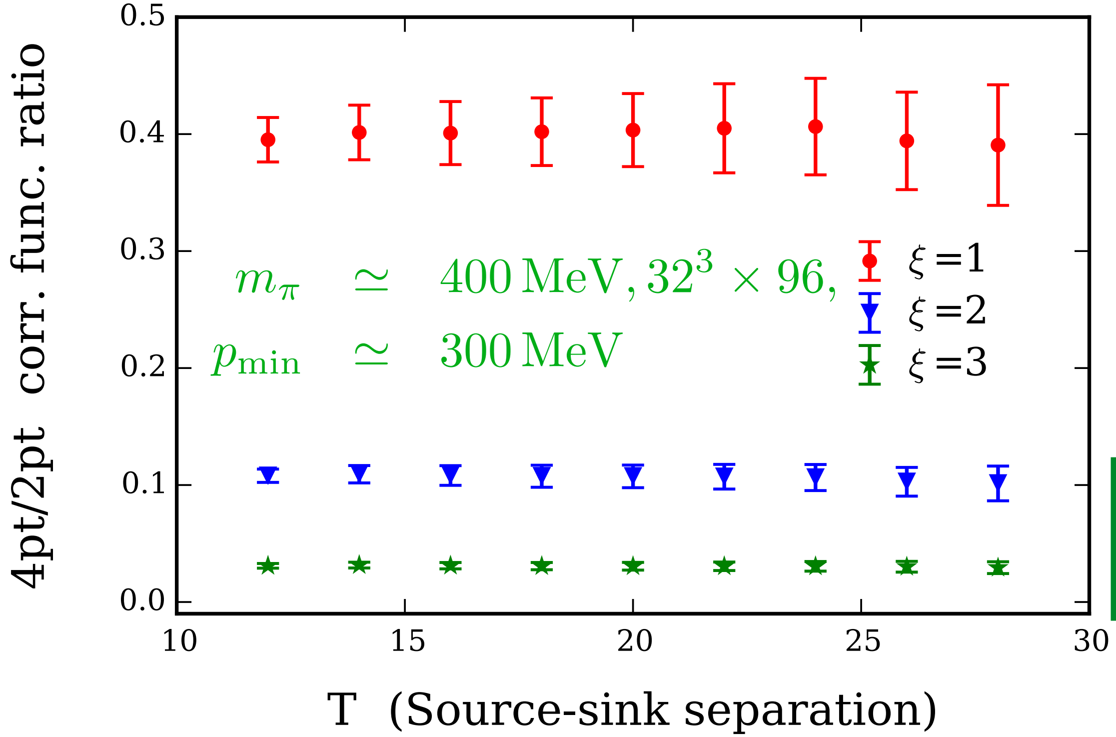
$$D(Z, T; w) H(w; x_0, t) = \sum_y e^{-ip\cdot y} \Gamma_\Pi G(y, 0; x_0, t)$$

$$D(s; w) \tilde{H}(w; x_0, t) = \sum_z e^{ip\cdot z} \Gamma_\Pi H(z, T; x_0, t)$$

Preliminary Results

- 2+1 Flavor clover-fermion action

$$a \simeq 0.12, 0.09\,\mathrm{fm}$$
$$m_\pi \simeq 400, 440\,\mathrm{MeV}$$

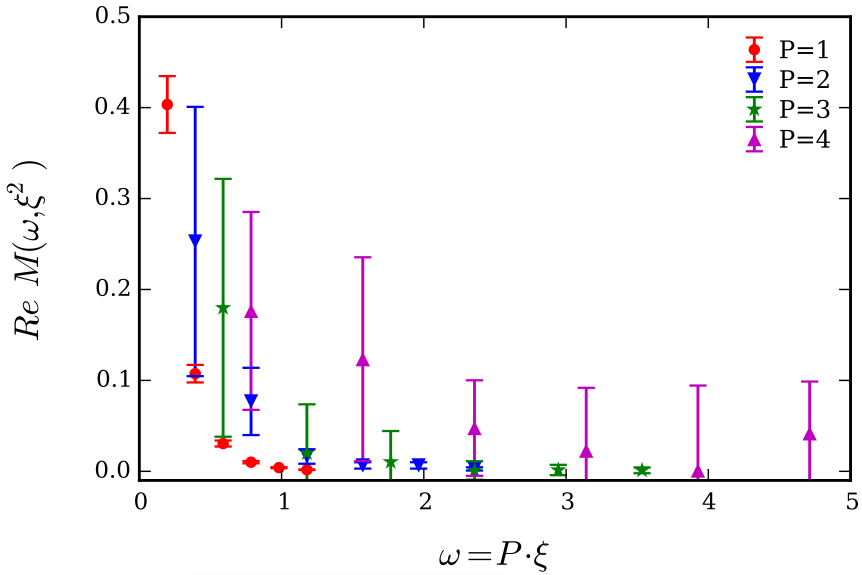


$$J_1 = V, J_2 = A$$

- 110 configurations
- Single source point for current J_1

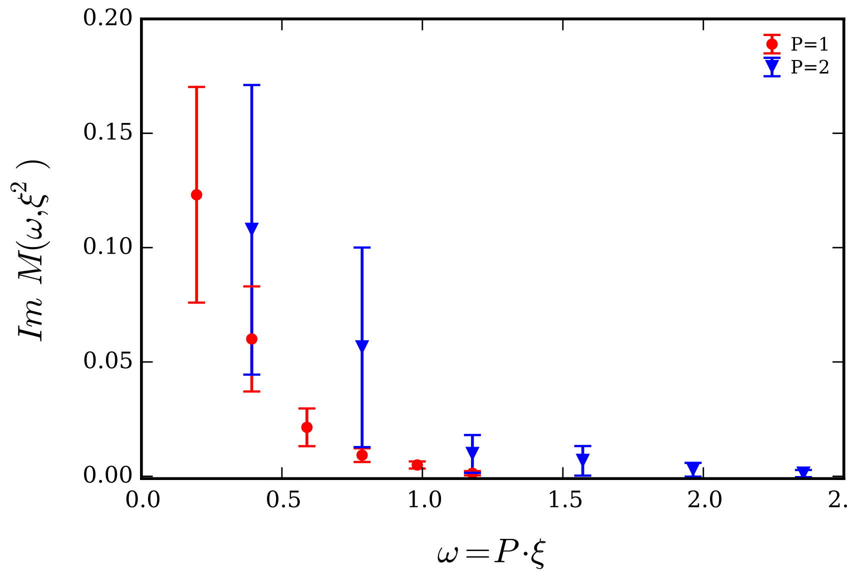
Clear isolation of pion matrix element

Preliminary Results - II



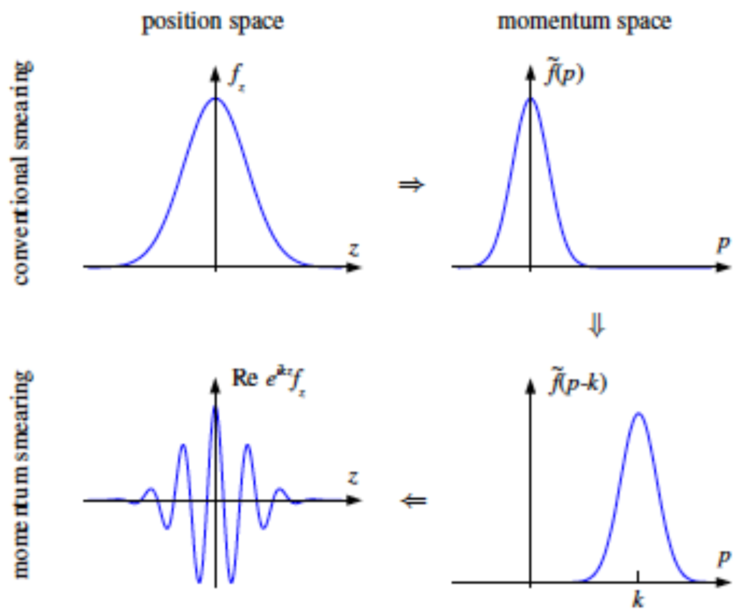
Clear signal in real part to p around 1 GeV

Imaginary part only to small values of p



Challenges/Questions

High spatial momentum and lattice systematics



Boosted interpolating operators

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

Inverse Problem - *common to all approaches*

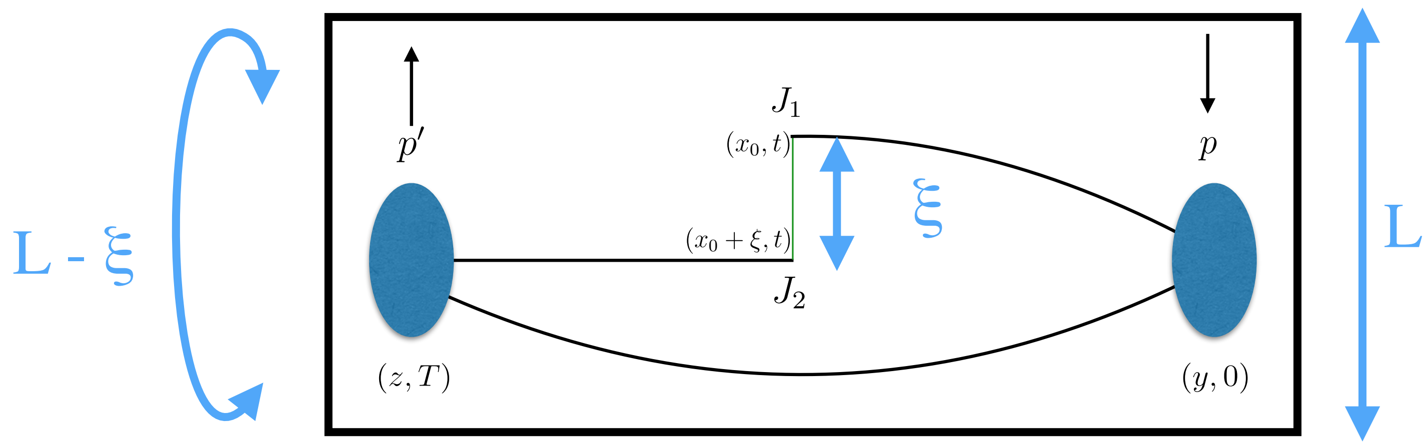
$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2\Lambda^2_{\rm QCD})$$

Extract PDF?

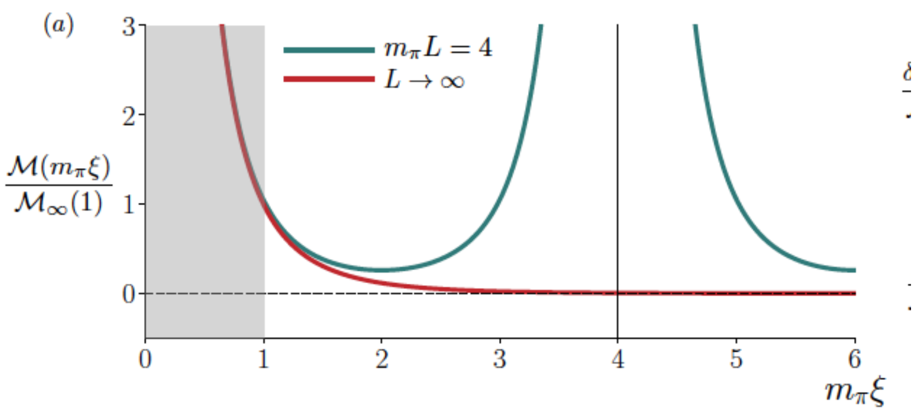
Calculate on Lattice

Calculate in PQCD

Finite Volume Effects



Briceno, Guerrero, Hansen and Monahan, arXiv:1805.01304



Typically $m_\pi L \simeq 4$

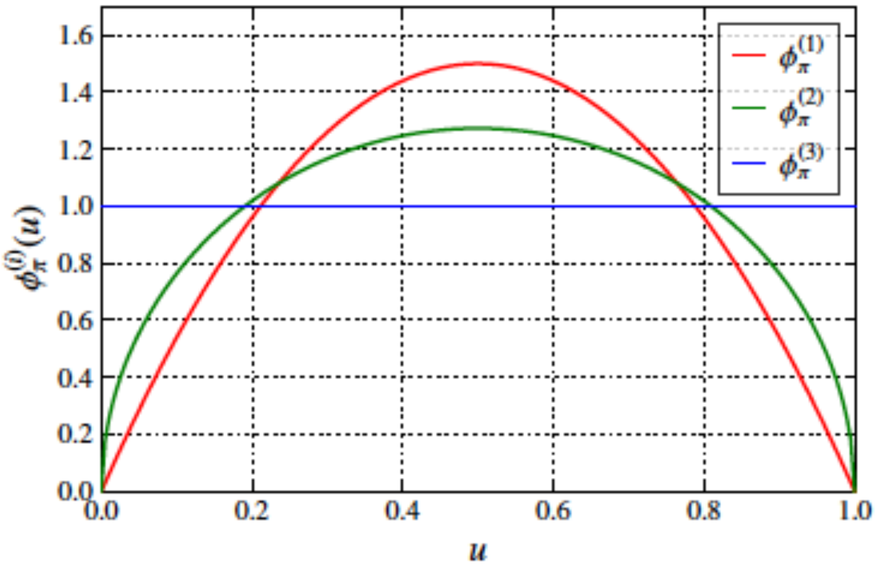
Future? $\begin{cases} \xi \text{ shortdistance} \\ m_\pi \to m_\pi^{\text{phys}} \end{cases}$

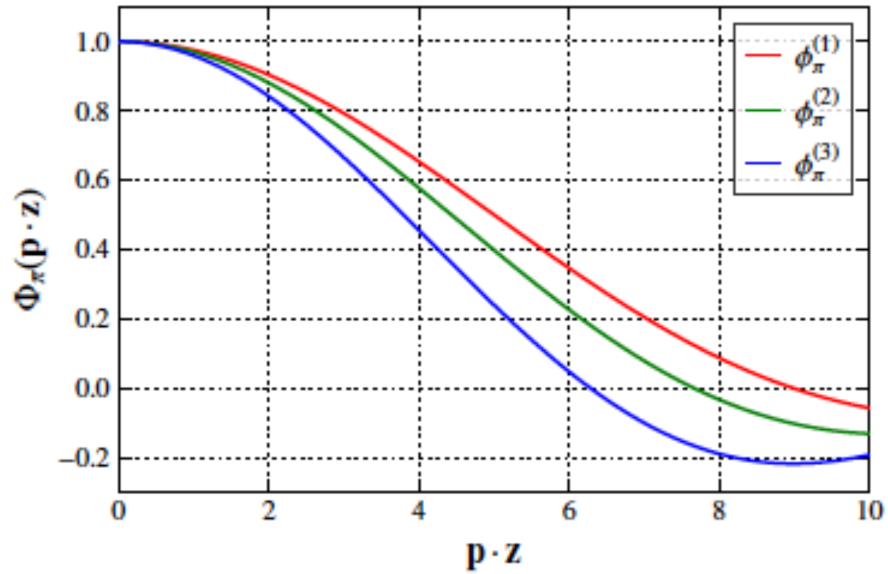
Pion Quark Distribution Amplitude

"Pseudo" quark distribution amplitudes

Bali et al., arXiv:1709.04325

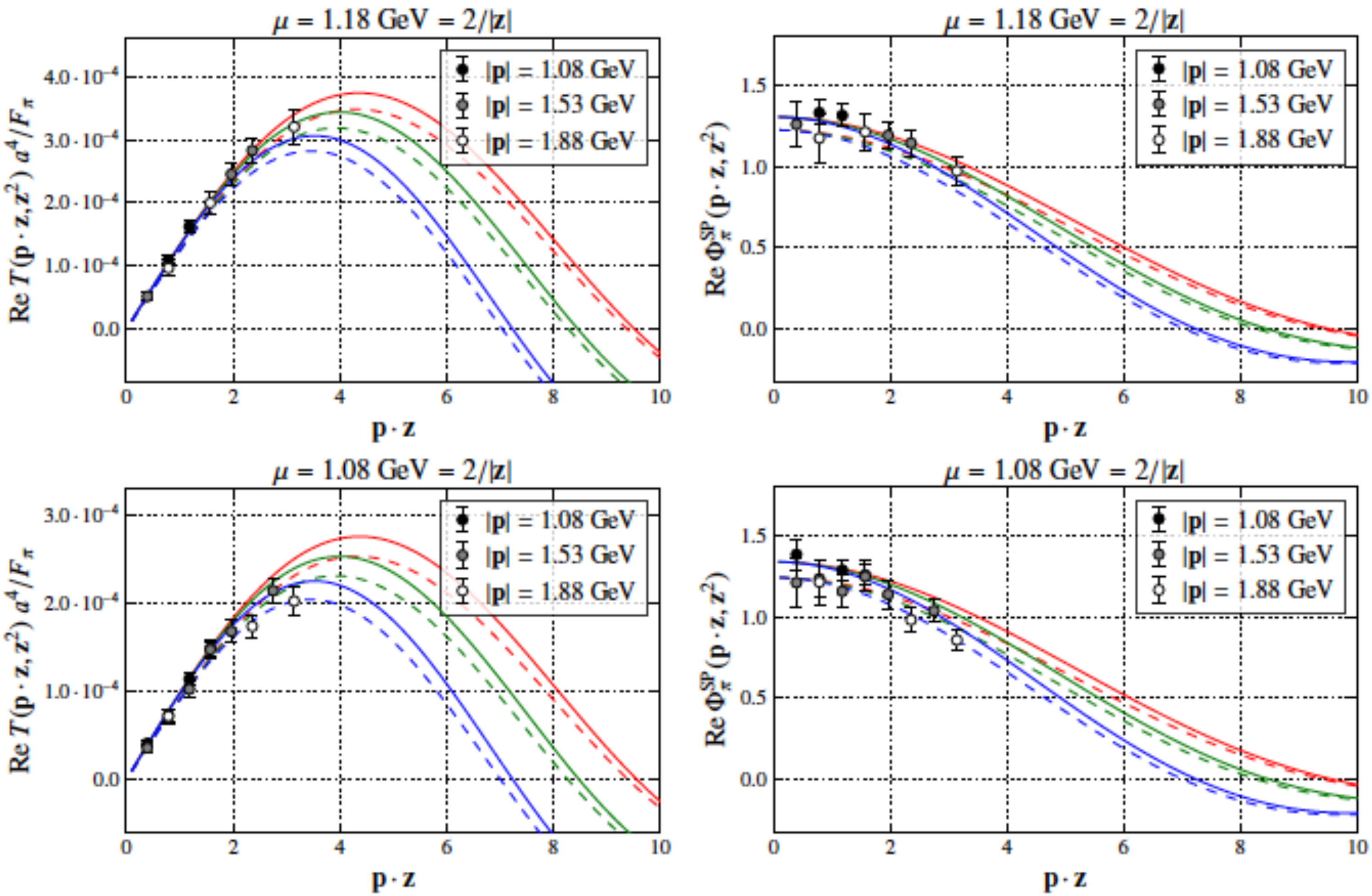
$$T(\omega, z^2) = \langle \pi(\vec{p} \mid [\bar{u}Q](z/2)[\bar{Q}u](-z/2) \mid 0 \rangle$$





Quark Distribution Pictures ⟷ Ioffe pseudo-time

Pion QDA - II



Fixed z^2

Summary

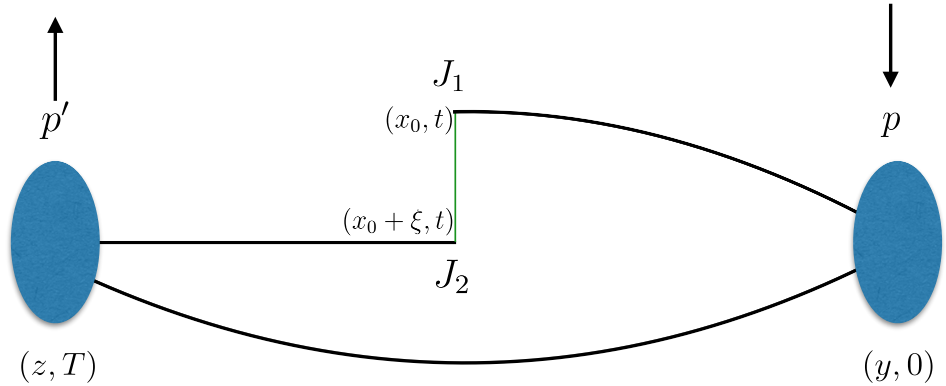
- Pion Form Factors at high momenta with reach comparable to 12 GeV at JLab
- Calculation of current-current correlators for pion *and kaon* in progress for variety of local operators
  - Important to understand finite-volume effects
  - Extending calculation to close-to-physical

$$m_\pi \simeq 170 \text{ MeV}$$
$$64^3 \times 128 \text{ Lattices}$$

- Variety of lattice cross sections - including pseudo PDFs - on same ensemble of lattices.

Outlook

Momentum projection



Momentum projection

Momentum *conservation*

TMDs ⟵ Compute for any ξ and p, p' ⟶ GPDs