

TOMOGRAPHY OF PION AND KAON

—a view from Dyson-Schwinger equations

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Nonperturbative QCD

Nonperturbative QCD

Transverse momentum dependent distributions (TMD)

3-D tomography in the momentum space.

Generalized parton distributions (GPD)

3-D picture of hadrons in the mixed spatial-momentum space.



Nonperturbative QCD

QCD

$$\mathcal{L}_{\text{QCD}} = \bar{q}_i \gamma^\mu (i\partial_\mu - g_s t^a A_\mu^a - m_i) q_i - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f^{abc} A_\mu^b A_\nu^c \quad \alpha_s = \frac{g_s^2}{4\pi}$$

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Nonperturbative QCD methods

1. ADS/QCD (Holographic QCD)
2. Dyson-Schwinger equations.
3. Effective theories and models, e.g., ChiPT, NJL model...
4. Light front QCD.
5. Lattice QCD.
6. QCD sum rules.
- etc...

Transverse momentum dependent distributions (TMD)

3-D tomography in the momentum space.

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DSEs: QCD

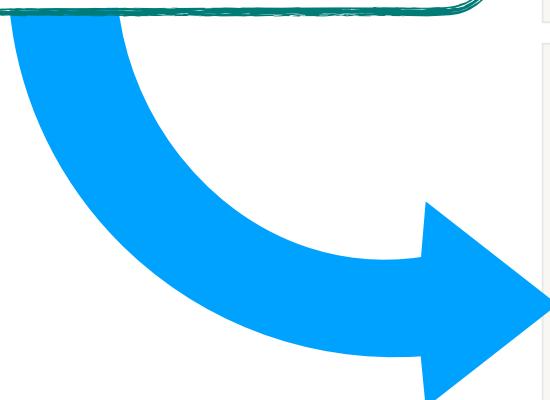
 **Dyson-Schwinger equations:** general relations between Green functions in quantum field theories.

Derivation:

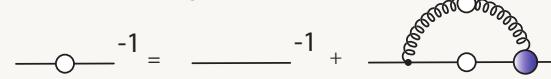
1. Quantum Field Theory
2. Path Integral formulation
3. Invariance of generating functional

$$\int \mathcal{D}\psi \frac{\delta}{\delta \psi_\alpha} e^{i[S + \int d^4y J_\alpha \psi_\alpha]} \Big|_{J_\alpha=0} = 0$$

(Non-perturbative)



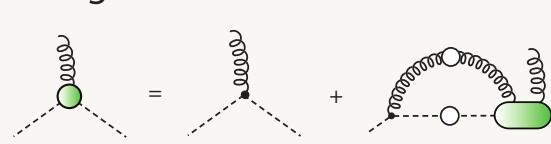
Quark propagator:



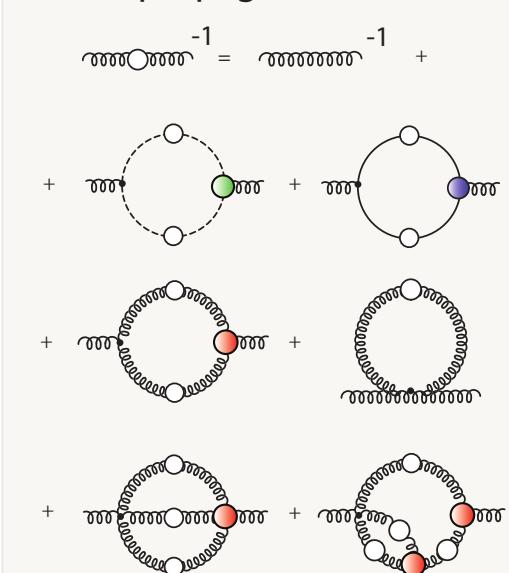
Ghost propagator:



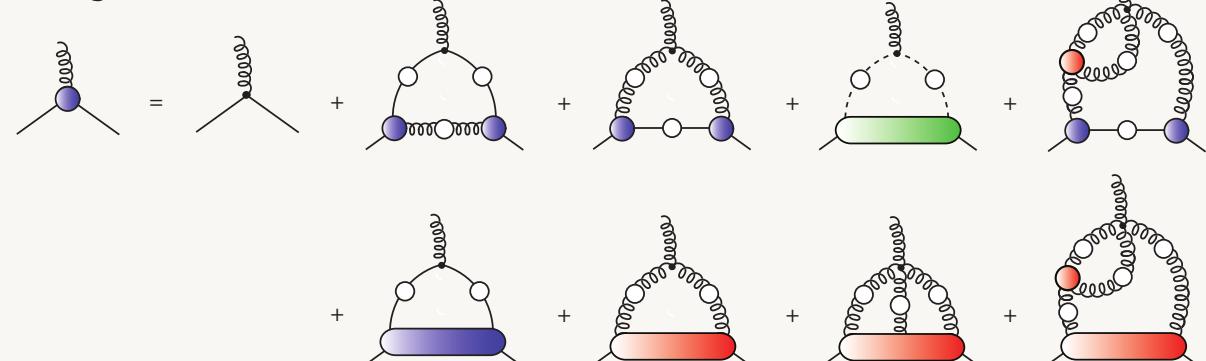
Ghost-gluon vertex:



Gluon propagator:

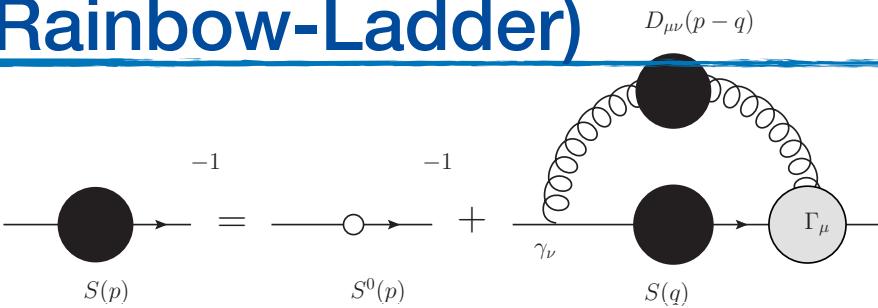


Quark-gluon vertex:

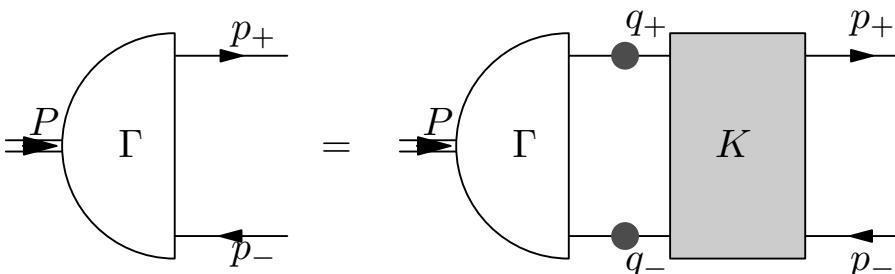


DSEs: Pion and Kaon (Rainbow-Ladder)

Quark's DSE:



Pion's BSE:



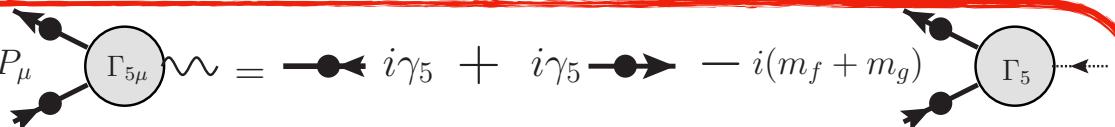
Rainbow-Ladder:

$$\Gamma_\mu^a = \gamma_\mu \lambda^a$$

$$K = D_{\mu\nu}^{ab} \quad \gamma_\mu \lambda^a \quad \gamma_\nu \lambda^b$$

(P. Maris, C.D. Roberts and P. C. Tandy, PLB1998)

Axial-Vector Ward-Takahashi Identity:



$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i \gamma_5 + i \gamma_5 S_g^{-1}(k_-) - i[m_f + m_g] \Gamma_5^{fg}(k; P),$$

DSEs: Pion and Kaon (Beyond Rainbow-Ladder)

Inhomogeneous BSE:

$$\text{When } P^2 \rightarrow -m_\pi^2, \Gamma_{5\mu}^j(k; P) \sim \frac{r_A P_\mu}{P^2 + m_\pi^2} \Gamma_\pi^j(k; P)$$

$$\Gamma_{5\mu}(k; P) = Z_2 \gamma_5 \gamma_\mu$$

$$- Z_2 \int_{dq} \mathcal{G}(k - q) D_{\rho\sigma}^{\text{free}}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S(q_+) \times \Gamma_{5\mu}(q; P) S(q_-) \frac{\lambda^a}{2} \tilde{\Gamma}_\beta(q_-, k_-) \\ + Z_1 \int_{dq} g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \times \frac{\lambda^a}{2} \Lambda_{5\mu\beta}(k, q; P) \quad (\text{Lei Chang and C.D. Roberts PRL2009})$$

Beyond RL

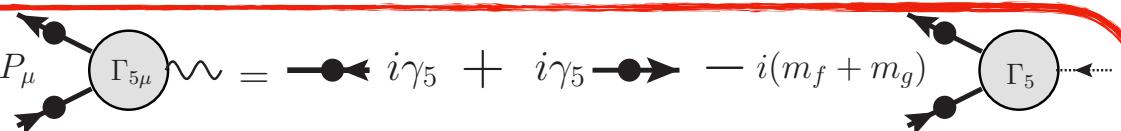
(Lei Chang, YX Liu and C.D. Roberts PRL2011,

$$\Gamma_\mu(p_1, p_2) = \Gamma_\mu^{\text{BC}}(p_1, p_2) + \Gamma_\mu^{\text{acm}}(p_1, p_2) \quad (\text{Lei Chang, C.D. Roberts PRC2012})$$

Constrain

$$2\Lambda_{5\beta(\mu)} = [\tilde{\Gamma}_\beta(q_+, k_+) + \gamma_5 \tilde{\Gamma}_\beta(q_-, k_-) \gamma_5] \times \frac{1}{S^{-1}(k_+) + S^{-1}(-k_-)} \Gamma_{5(\mu)}(k; P) \\ + \Gamma_{5(\mu)}(q; P) \frac{1}{S^{-1}(-q_+) + S^{-1}(q_-)} \times [\gamma_5 \tilde{\Gamma}_\beta(q_+, k_+) \gamma_5 + \tilde{\Gamma}_\beta(q_-, k_-)]$$

Axial-Vector Ward-Takahashi Identity:



$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) \\ - i[m_f + m_g] \Gamma_5^{fg}(k; P),$$

DSEs: $S(q)$ and $\Gamma(q; P)$ Parameterization

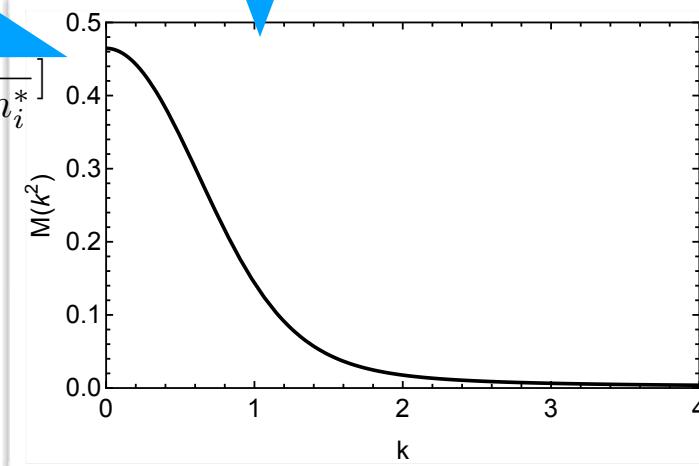
dressed quark propagator:

$$S(q) = \frac{Z(q^2)}{i\gamma \cdot q + M(q^2)}$$

$$= \sum_{i=1}^2 \left[\frac{Z_i}{i\gamma \cdot q + m_i} + \frac{Z_i^*}{i\gamma \cdot q + m_i^*} \right]$$

Complex conjugate poles, NO poles on the real axis, consistent with confinement!

Significantly enhanced mass function, signals the DCSB.



pion BS amplitude:

$$\Gamma_P(q; P) = \gamma_5 [iE_P(q; P) + \gamma \cdot P F_P(q; P) + \gamma \cdot q G_P(q; P) + \sigma_{\mu\nu} q_\mu P_\nu H_P(q; P)]$$

$$\mathcal{F}(k; P) = \mathcal{F}^i(k; P) + \mathcal{F}^u(k; P).$$

$$\mathcal{F}^i(k; P) = c_{\mathcal{F}}^i \int_{-1}^1 dz \rho_{\sigma_{\mathcal{F}}^i}(z) \left[a_{\mathcal{F}}^i \hat{\Delta}_{\Lambda_{\mathcal{F}}^i}^4(k_z^2) + (1 - a_{\mathcal{F}}^i) \hat{\Delta}_{\Lambda_{\mathcal{F}}^i}^5(k_z^2) \right],$$

$$\mathcal{F}^u(k; P) = c_{\mathcal{F}}^u \int_{-1}^1 dz \rho_{\sigma_{\mathcal{F}}^u}(z) \hat{\Delta}_{\Lambda_{\mathcal{F}}^u}(k_z^2),$$

correct power law in the UV part of BSA generated by one-gluon exchange

Nakanishi-like integral representation, allowing calculation on a much wider range of physical quantities.

$$\text{with } \hat{\Delta}_\Lambda(s) = \Lambda^2/(s + \Lambda^2), \quad k_z^2 = k^2 + z k \cdot P$$

(Lei Chang et al, PRL2013)



TMDs & GPDs: Light Front Approach

Fully Dressed Quark propagator & Pion Bethe-Salpeter amplitude



- Covariant approach:** Compute the triangle diagrams in terms of fully covariant propagators/vertices with appropriate truncations.
- Light-front approach:** Extract from pion's Bethe-Salpeter wave functions the LFWFs and calculate TMDs and GPDs using overlap representation.

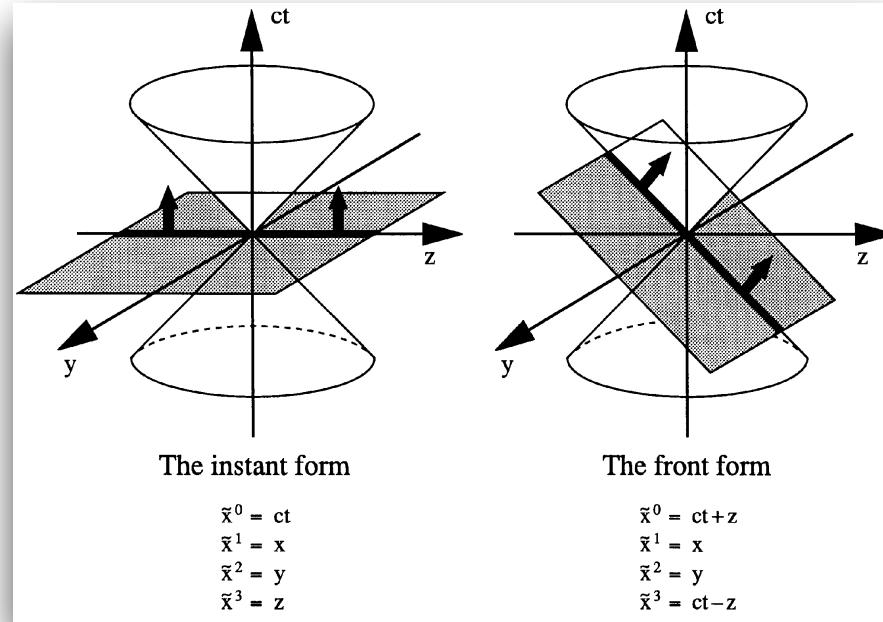


$$\text{TMD: } f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

$$\text{GPD: } H_\pi^q(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \langle p_2 | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \psi^q(\frac{z}{2}) | p_1 \rangle |_{z^+ = z_\perp = 0}$$



Light-front QCD



- 💡 A natural formalism in describing hard hadron scattering. The PDF, GPDs and TMDs are all defined with the front form.
- 💡 A relativistic description of bound systems in terms of quantum-mechanical-like wave functions, i.e., the light front wave functions (LFWFs). The LFWFs are boost invariant. They keep all the non-perturbative dynamical information of the hadron's internal structure.
- 💡 However, the calculation of LFWFs is numerically challenging for light front QCD in four space-time dimensions.

LFWFs & Bethe-Salpeter wave function

Leading Fock-space configuration:

LFWFs

$$|\pi^+(P)\rangle = |\pi^+(P)\rangle_{l_z=0} + |\pi^+(P)\rangle_{|l_z|=1}$$

$$|\pi^+(P)\rangle_{l_z=0} = i \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_0(x, k_\perp^2) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} [b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\downarrow j}^\dagger(\bar{x}, \bar{k}_\perp) - b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\uparrow j}^\dagger(\bar{x}, \bar{k}_\perp)] |0\rangle,$$

$$|\pi^+(P)\rangle_{|l_z|=1} = i \int \frac{d^2 k_\perp}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_1(x, k_\perp^2) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} [k_\perp^- b_{u\uparrow i}^\dagger(x, k_\perp) d_{d\uparrow j}^\dagger(\bar{x}, \bar{k}_\perp) + k_\perp^+ b_{u\downarrow i}^\dagger(x, k_\perp) d_{d\downarrow j}^\dagger(\bar{x}, \bar{k}_\perp)] |0\rangle,$$

Fourier transform of LFWF

Correlation function & LFWFs:

(M. Burkardt et al, PLB 2002)

$$\langle 0 | \bar{d}_+(0) \gamma^+ \gamma_5 u_+(\xi^-, \xi_\perp) | \pi^+(P) \rangle = i \sqrt{6} P^+ \psi_0(\xi^-, \xi_\perp),$$

$$\langle 0 | \bar{d}_+(0) \sigma^{+i} \gamma_5 u_+(\xi^-, \xi_\perp) | \pi^+(P) \rangle = -i \sqrt{6} P^+ \partial^i \psi_1(\xi^-, \xi_\perp).$$

BS wave function

$$\psi_0(x, k_\perp^2) = \sqrt{3} i \int \frac{d\hat{k}^2}{2\pi} \text{Tr}_D [\not{h} \gamma_5 \chi(k; P)] \delta(x P \cdot n - k \cdot n),$$

$$\psi_1(x, k_\perp^2) = \frac{\sqrt{3}}{\tilde{k}^2} \int \frac{d\hat{k}^2}{2\pi} \left[\sigma^{\mu\nu} n_\mu \tilde{k}_\nu \gamma_5 \chi(k; P) \right] \delta(x P \cdot n - k \cdot n),$$

where $\hat{k} = (k^0, \vec{k}^3)$, $\tilde{k} = (0, \vec{k}_\perp, 0)$ and $n^2 = 0$



LWFWs:

Reconstruction from moments

$$\langle x^m \rangle_{k_\perp^2} = \int_0^1 dx x^m \psi_0(x, k_\perp^2) \stackrel{E}{=} \frac{\sqrt{3}}{|P \cdot n|} \int \frac{d\hat{k}^2}{2\pi} \left(\frac{k \cdot n}{P \cdot n} \right)^m \text{Tr}_D[\not{n} \gamma_5 S(k) \Gamma(k; P) S(k - P)]$$

Direct computation

$$\int_0^1 dx x^m \psi_0(x, k_\perp^2) \stackrel{E}{=} \frac{\sqrt{3}}{|P \cdot n|} \int \frac{d\hat{k}^2}{2\pi} \left(\frac{k \cdot n}{P \cdot n} \right)^m \text{Tr}_D[\not{n} \gamma_5 S(k_-) \Gamma(k; P) S(k_+)]$$

$$\sum_\alpha \frac{z_\alpha}{i\cancel{p} + m_\alpha}$$

$$i\gamma_5 \sum_\beta \int_{-1}^1 dz \rho_\beta(z) \frac{U_\beta}{(k^2 + z k \cdot P + \Lambda_\beta^2)^\beta}$$

$$\sum_\gamma \frac{z_\gamma}{i\cancel{p} + m_\gamma}$$

- ↓
- Feynman parameterization
 - change of integral variables (Cedric Mezrag)

$$\int_0^1 dx' x'^m \int dy' dz' \psi'_0(x', y', z', k_\perp^2)$$

Conclusion:

$$\psi_0(x, k_\perp^2) = \int dy' dz' \psi'_0(x, y', z', k_\perp^2)$$



LFWFs: $\psi_0(x, k_T^2)$ & $\psi_1(x, k_T^2)$

- 📌 ψ_0 is broad concave in x and shrinks to the conformal limit $6x(1-x)$ as k_T increases.
- 📌 Strong support at IR of k_T , a consequence of the DCSB which generates significant strength in the infrared region of BS wave function.
- 📌 At UV in k_T , ψ_0 scale as $1/k_T^2$ and ψ_1 scale as $1/k_T^4$, as predicted by pQCD.
- 📌 The x and k_T dependence in the LFWFs are "un-factorizable", namely, the shape of LFWFs in x changes significantly as k_T changes.

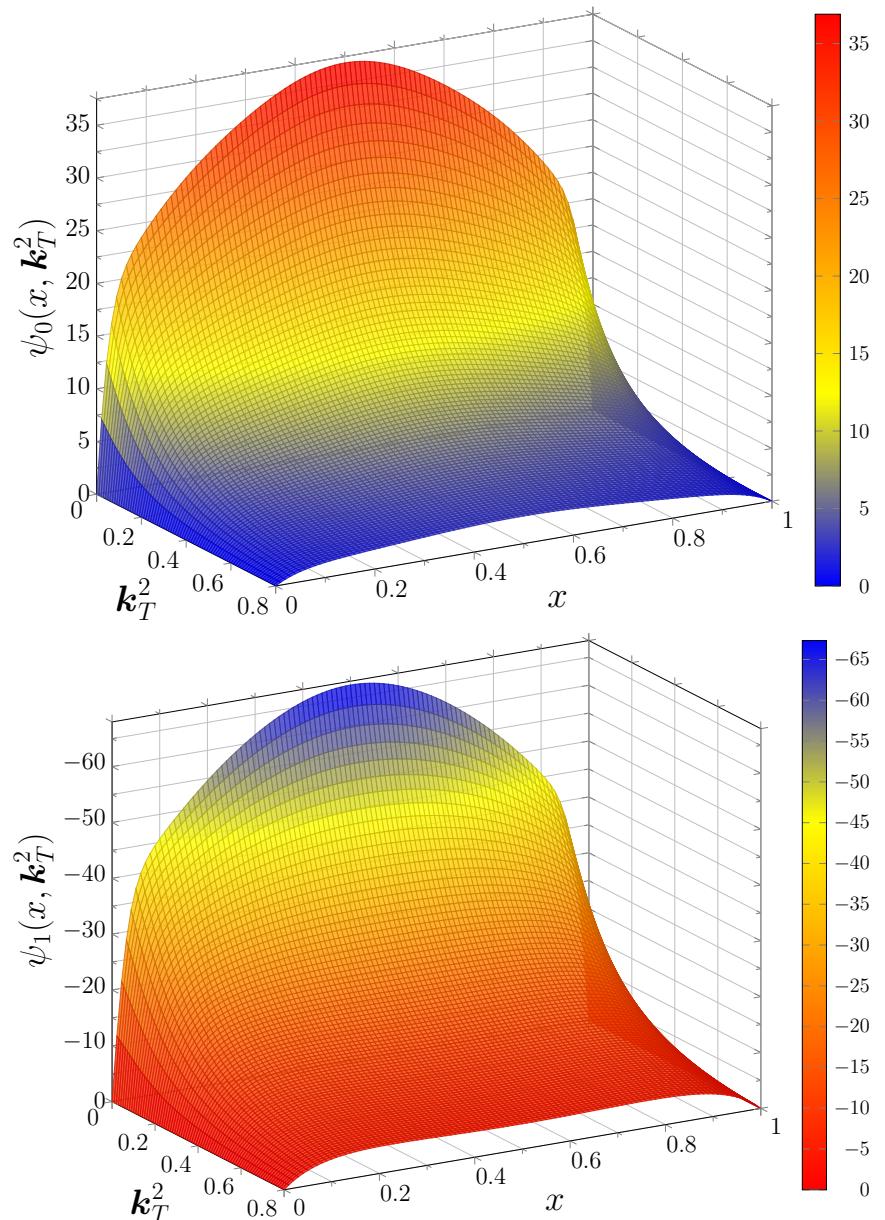


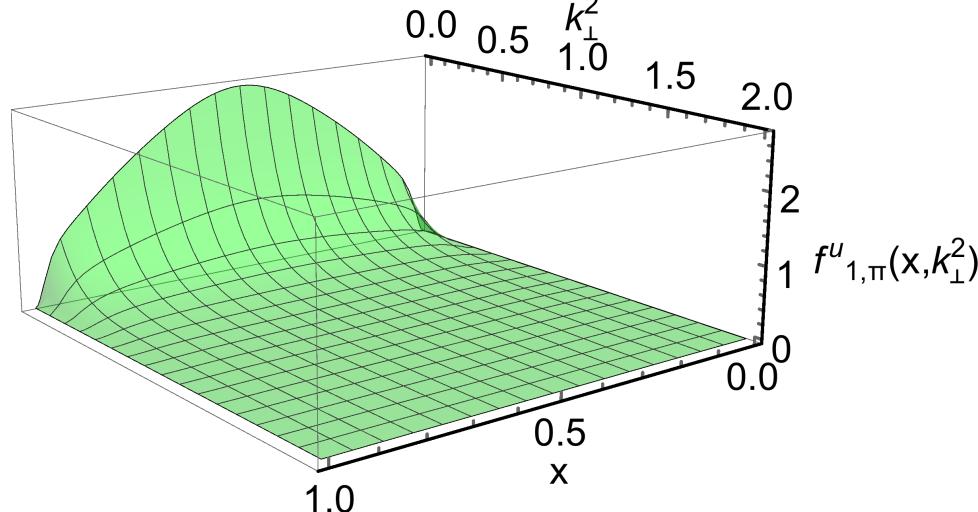
Figure 1. *Upper panel:* DSE result using the DCSB-improved kernel for the pion's $l_z = 0$ minimal ($\bar{q}q$) Fock-state LFWF. *Lower panel:* Analogous result for the pion's $|l_z| = 1$ minimal Fock-state LFWF. The LFWFs are given in units of GeV^{-2} and k_T^2 in GeV^2 .

TMD PDFs

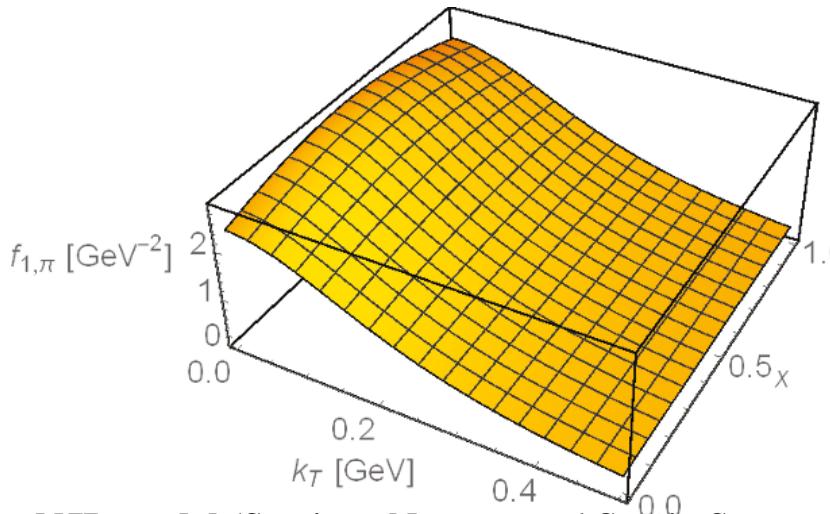
$$f_{1,\pi}(x, k_\perp^2) = \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \xi_\perp \cdot \mathbf{k}_\perp)} \langle \pi(P) | \bar{q}(0) \gamma^+ q(\xi^-, \xi_\perp) | \pi(P) \rangle.$$

★ $f_{1,\pi}(x, k_\perp^2) = |\psi_{\uparrow\downarrow}(x, k_\perp^2)|^2 + k_\perp^2 |\psi_{\uparrow\uparrow}(x, k_\perp^2)|^2$ (M. Burkardt et al, PLB 2002)

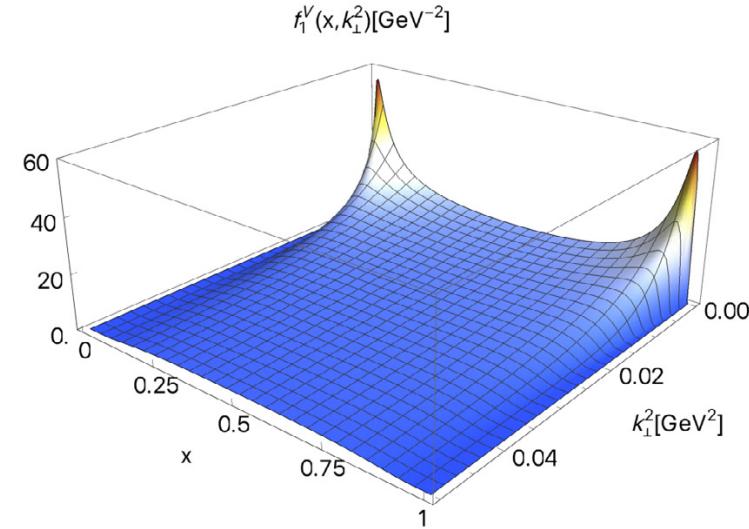
DSEs:



- 1. IR enhanced
- 2. UV power behavior
- 3. Unfactorizable x- and kT- dependence



NJL model (Santiago Noguera and Sergio Scopetta, PLB2017)



Holographic QCD(Alessandro Bacchetta, Sabrina Cotogno, Barbara Pasquini, PLB2017)

TMD: Gaussian Ansatz

Gaussian ansatz with non-constant $\langle k_\perp^2(x) \rangle_G$

$$f_{1,G}^q(x, k_\perp^2) = f_1^q(x, 0) \exp(-k_\perp^2 / \langle k_\perp^2(x) \rangle_G),$$

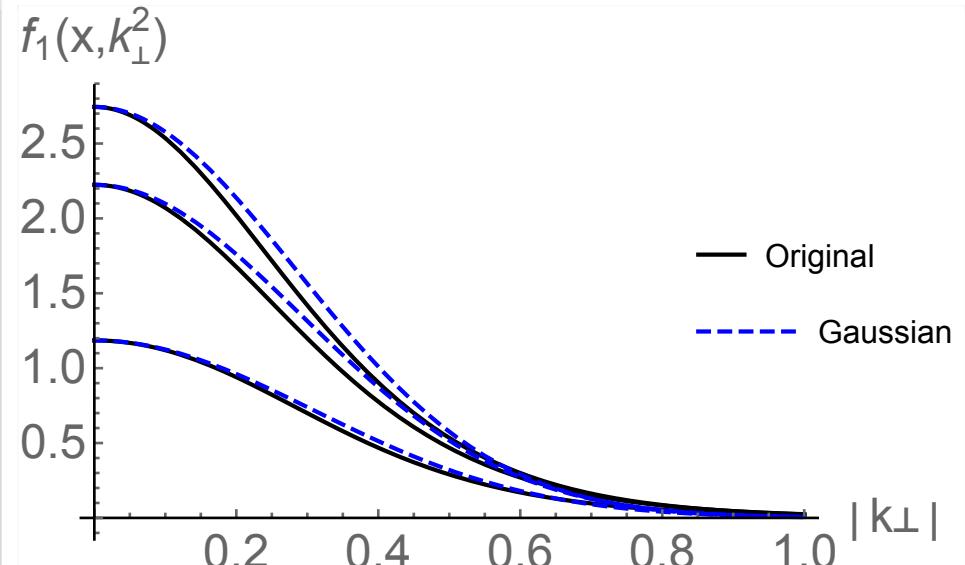
$$\langle k_\perp^2(x) \rangle_G = f^q(x) / (\pi f_1^q(x, 0)).$$



TMD: Gaussian Ansatz

Gaussian ansatz with non-constant $\langle k_\perp^2(x) \rangle_G$

$$f_{1,G}^q(x, k_\perp^2) = f_1^q(x, 0) \exp(-k_\perp^2 / \langle k_\perp^2(x) \rangle_G),$$
$$\langle k_\perp^2(x) \rangle_G = f_1^q(x) / (\pi f_1^q(x, 0)).$$



From up to bottom, $x=0.5, 0.3, 0.1$ respectively

- The Gaussian Ansatz approximates the profile our TMDs when k_\perp is not large.
- Within Gaussian ansatz $\langle k_\perp^2(x) \rangle_G$ varies about 15% when x in [0.1,0.9]. Separable k_\perp and x dependence works as an approximation.
- $\langle k_\perp^2 \rangle_G \approx 0.19(1)$ GeV 2 within the general estimation $[(0.3\text{GeV})^2, (0.5\text{GeV})^2]$. (Stanley Brodsky PRD2011).
- TMD evolution?

TMD evolution:

Renormalization group (RG) equation:

$$\mu^2 \frac{d}{d\mu^2} F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta) = \frac{1}{2} \gamma_F^f(\mu, \zeta) F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta)$$
$$\zeta \frac{d}{d\zeta} F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta) = -\mathcal{D}^f(\mu, \vec{b}) F_{f \leftarrow h}(x, \vec{b}; \mu, \zeta).$$

Anomalous Dimension

TMD PDF in the coordinate space

The scale μ is the standard RG scale, with the additional rapidity factorization scale ζ to regularize the light-cone divergence arising from Wilson lines. They were usually chosen to be the same order of scattering scale.

Solution:

$$F_{f \leftarrow h}(x, \vec{b}; \mu_f, \zeta_f) = \exp \left[\int_P \left(\gamma_F^f(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}^f(\mu, \vec{b}) \frac{d\zeta}{\zeta} \right) \right] F_{f \leftarrow h}(x, \vec{b}; \mu_i, \zeta_i)$$

TMD PDF at initial scale, non-perturbative input.

Contains a non-perturbative term of the anomalous dimension D at large b characterized by g^2

TMD evolution:

b*-prescription:

$$b^* = b_{\max} \left(\frac{1 - e^{-b_\perp^4/b_{\max}^4}}{1 - e^{-b_\perp^4/b_{\min}^4}} \right)^{1/4}$$

It saturates at the cut off b_{\max} where non-perturbative (small k_T) TMD PDF dominates. Matching the large and small b_T behavior.

Extraction of partonic transverse momentum distributions from semi-inclusive deep-inelastic scattering, Drell-Yan and Z-boson production (Alessandro Bacchetta et al, JHEP06(2017)081)

- HERMES and COMPASS, SIDIS, $10 \text{ GeV}^2 > Q^2 > 1.4 \text{ GeV}^2$
- E288 and E605, DY, $100 \text{ GeV}^2 > Q^2 > 16 \text{ GeV}^2$, $0.36 > x_F > 0$

ζ -prescription: $(\mu_i, \zeta_i) \rightarrow (\mu_i, \zeta_\mu)$ instead of $(\mu_i, \zeta_i) \rightarrow (\mu_f, \zeta_f)$ where $\mu^2 \frac{dF(x, \mathbf{b}; \mu, \zeta_\mu)}{d\mu^2} = 0$

Allows to minimize the impact of perturbative logarithms in a large range of scales and does not generate undesired power corrections.

Analysis of vector boson production within TMD factorization (Alexey Vladimirov et al, Eur. Phys. J. C (2018) 78:89)

- E288: Drell-Yan process, at $4 < Q < 14 \text{ GeV}$.
- CDF/D0: Z-boson production at $\sqrt{s} = 1.8, 1.96 \text{ TeV}$.
- ATLAS/CMS/LHCb: Z-boson production at $s = 7, 8, 13 \text{ TeV}$.
- ATLAS:Vector boson production outside the Z-peak ($46 < Q < 66$ and $116 < Q < 150 \text{ GeV}$) at $\sqrt{s} = 8 \text{ TeV}$.

arTeMiDe code

TMD evolution:

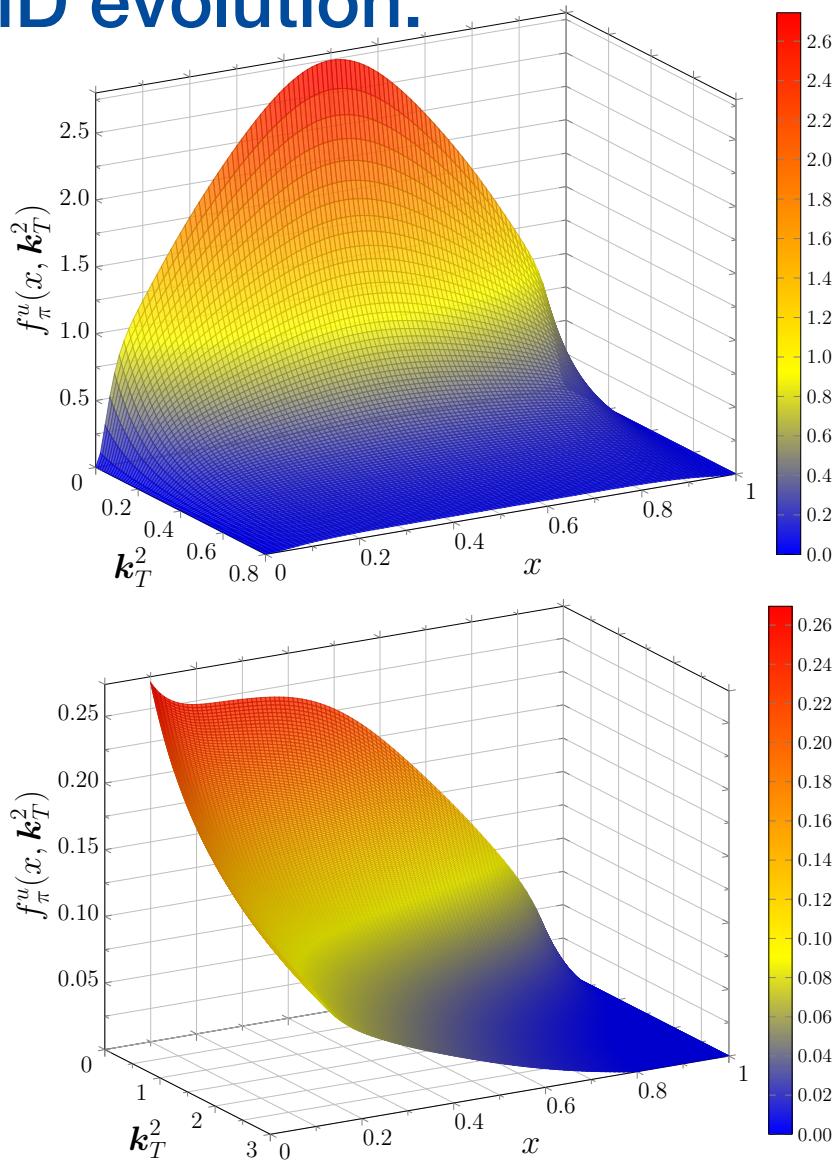
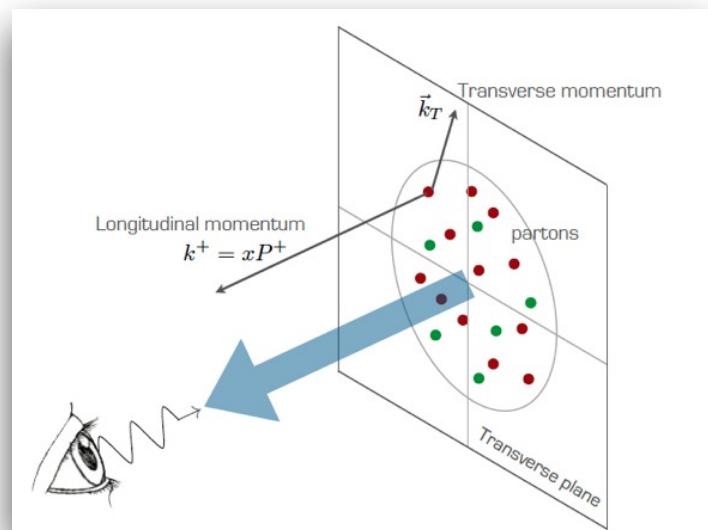


Figure 2. *Upper panel:* DSE result using the DCSB-improved kernel for the time-reversal even u -quark TMD of the pion, $f_\pi^u(x, \vec{k}_T^2)$, at the model scale of $\mu_0^2 = 0.52 \text{ GeV}^2$. *Lower panel:* Analogous result evolved to a scale of $\mu = 6 \text{ GeV}$ using TMD evolution with the b^* prescription and $g_2 = 0.09 \text{ GeV}$ [43]. The TMDs are given in units of GeV^{-2} and \vec{k}_T^2 in GeV^2 .



💡 Evolution has a significant effect, leading to approximately an order of magnitude of suppression at small k_T , and a broad tail at larger k_T .

💡 The evolved TMD PDF at smaller x is significantly broader than that at large x (Non-separable x and k_T dependence).

💡 Experiment?

Drell-Yan Process

Experiment (E615)

Transverse momentum dependence parameterized by function $P(q_T; x_F, m_{\mu\mu})$ (**DATA!**)

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} = \frac{d^2\sigma}{dx_\pi dx_N} P(q_T; x_F, m_{\mu\mu}).$$

$$q^0 = \frac{\sqrt{s}}{2}(x_\pi + x_N)$$

$$q^3 = \frac{\sqrt{3}}{2}(x_\pi - x_N)$$

"Experimental study of muon pairs produced by 252-GeV pions on tungsten", Conway, J.S. et al.
Phys.Rev. D39 (1989) 92-122.

Theory

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} \propto |q_T| F_{UU}^1(x_\pi, x_N, q_T)$$

(leading twist)

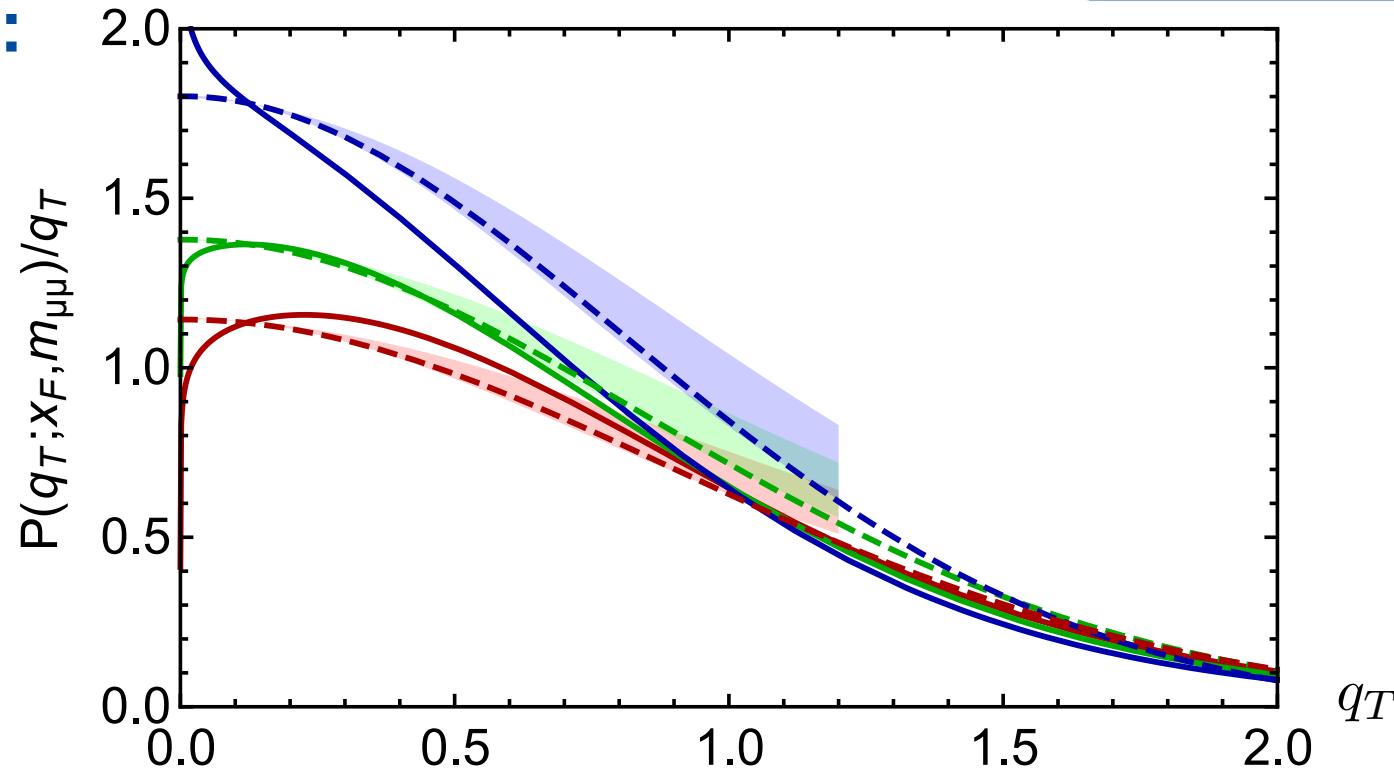
$$\text{TMD formalism: } F_{UU}^1(x_1, x_2, q_T) = \frac{1}{N_c} \sum_a e_a^2 \int d^2 k_{1\perp} d^2 k_{2\perp} \delta^{(2)}(\mathbf{q}_T - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) f_{1,\pi}^{\bar{a}}(x_1, \mathbf{k}_{1\perp}^2) f_{1,N}^a(x_2, \mathbf{k}_{2\perp}^2)$$

offer by DSEs&evolution

borrow from global analysis

$$\text{Examine: } P(q_T; x_F, m_{\mu\mu}) \propto |q_T| F_{UU}^1(q_T; x_F, \tau)$$

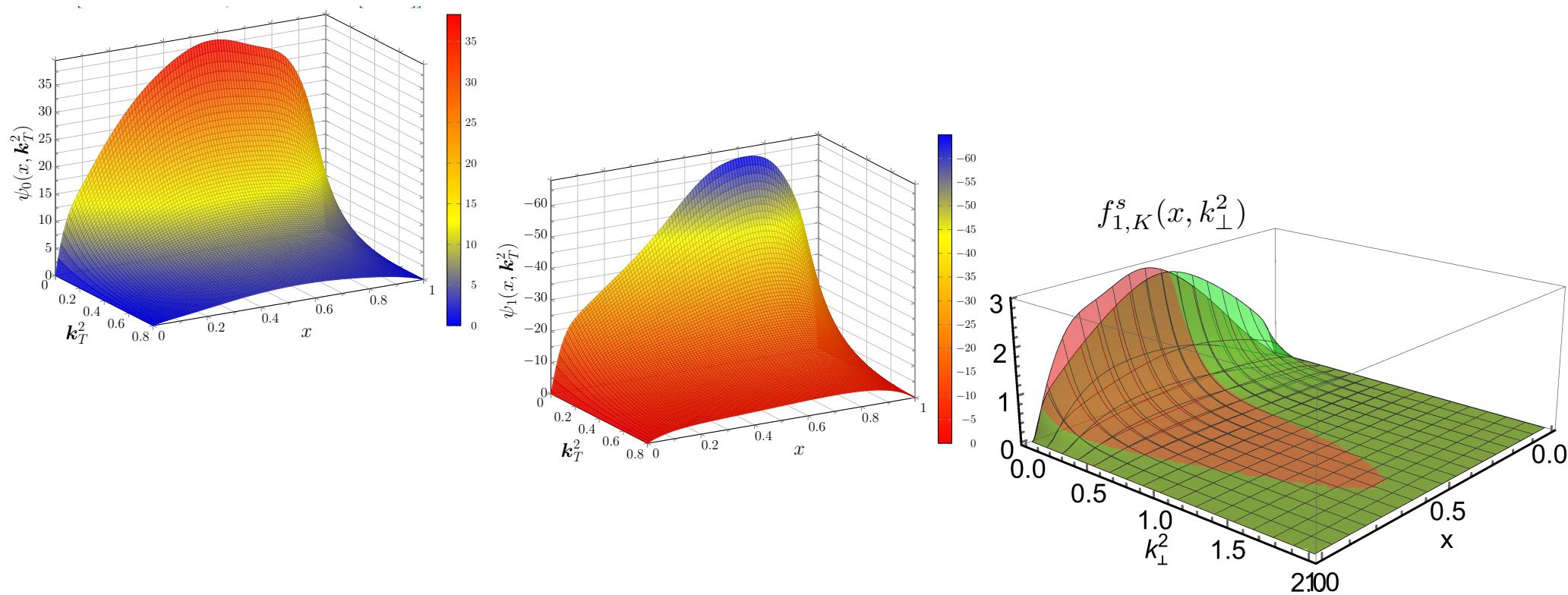
E615:



The fitting function $P(q_T; x_F, m_{\mu\mu})/q_T$ at $x_F = 0.0$ (red solid), 0.25 (green solid) and 0.5 (blue solid). The band colored bands are our results based on b^* -prescription, with upper boundary corresponding to $g_2 = 0.09$ and lower boundary for $g_0 = 0.0$. The dashed lines are obtained following ζ -prescription where g_2 is found to be consistent with zero at NNLL/NNLO.

- ➊ Our results using two evolution schemes generally agree with E615 measurement. In particular, when g_2 goes to zero as suggested by ζ -prescription at higher order. (The deviation is less than 10% for $x_F = 0$ and $x_F = 0.25$, and increases to 30% at most for $x_F = 0.5$.)
- ➋ Our calculation also shows the TMD formalism becomes less valid as x_F goes larger, where Berger- Brodsky effect emerges and parton distribution amplitude plays a role.

Kaon: LFWFs & TMD



- Non-symmetric in $x=0.5$, skewed with s quark carrying more longitudinal momentum fraction.
- The width of transverse momentum increases by about 10%, $m_s/m_u=20$ gets masked by DCSB effect.
- $f_{1;K}^{\bar{u}}(x, k_\perp^2) = f_{1;K}^s(1 - x, k_\perp^2)$ for momentum conservation. Some flavor dependence in the their k_T dependences.

GPD:

• Pion and Kaon GPD:

$$H_\pi^q(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2 | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \psi^q(\frac{z}{2}) | p_1 \rangle |_{z^+=z_\perp=0}$$

• Overlap representation, 2 particle and DGLAP region ($1>=x>=\xi$):

$$\begin{aligned} H_{\pi^+}^u(x, \xi, t) \Big|_{\xi \leq x} &= \\ \int \frac{d^2 \mathbf{k}_\perp}{16 \pi^3} &\left[\Psi_{l=0}^* \left(\frac{x-\xi}{1-\xi}, \hat{\mathbf{k}}_\perp \right) \Psi_{l=0} \left(\frac{x+\xi}{1+\xi}, \tilde{\mathbf{k}}_\perp \right) \right. \\ &+ \hat{\mathbf{k}}_\perp \cdot \tilde{\mathbf{k}}_\perp \left. \Psi_{l=1}^* \left(\frac{x-\xi}{1-\xi}, \hat{\mathbf{k}}_\perp \right) \Psi_{l=1} \left(\frac{x+\xi}{1+\xi}, \tilde{\mathbf{k}}_\perp \right) \right] \end{aligned}$$

with $\hat{\mathbf{k}}_\perp = \mathbf{k}_\perp + \frac{1-x}{1-\xi} \frac{\Delta_\perp}{2}$ and $\tilde{\mathbf{k}}_\perp = \mathbf{k}_\perp - \frac{1-x}{1+\xi} \frac{\Delta_\perp}{2}$,

Overlap representation of ERBL region requires 4-particle Fock state LFWF. See "N. Chouika, C. Mezrag, et al Eur. Phys. J. C (2017) 77:906" for an alternative technique.

• Impact Parameter dependent GPD:

$$q(x, b_T) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i \Delta_T \cdot b_T} H(x, 0, -\Delta_T^2)$$

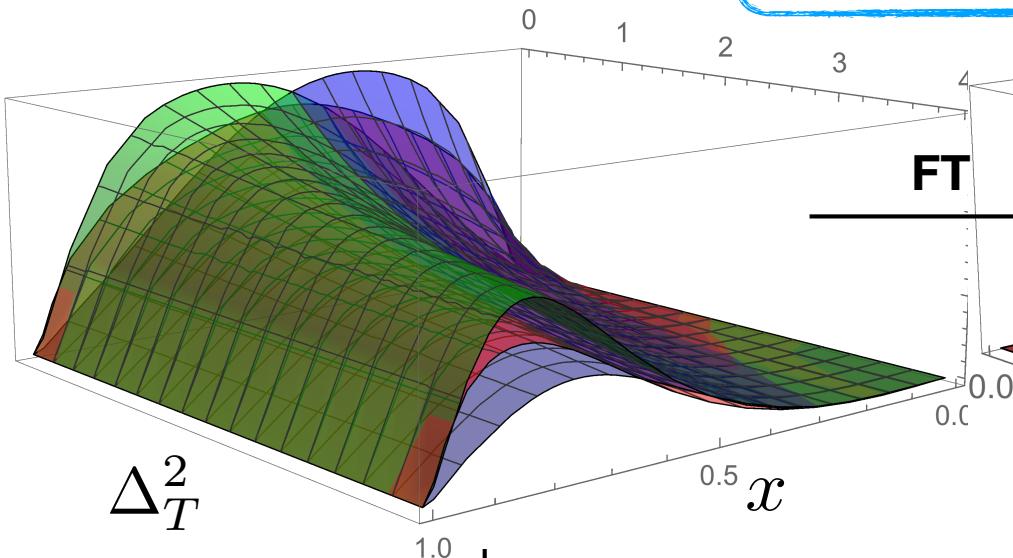
x: longitudinal momentum fraction carried by quark

b_T: transverse separation between the parton and hadron's center of transverse momentum.

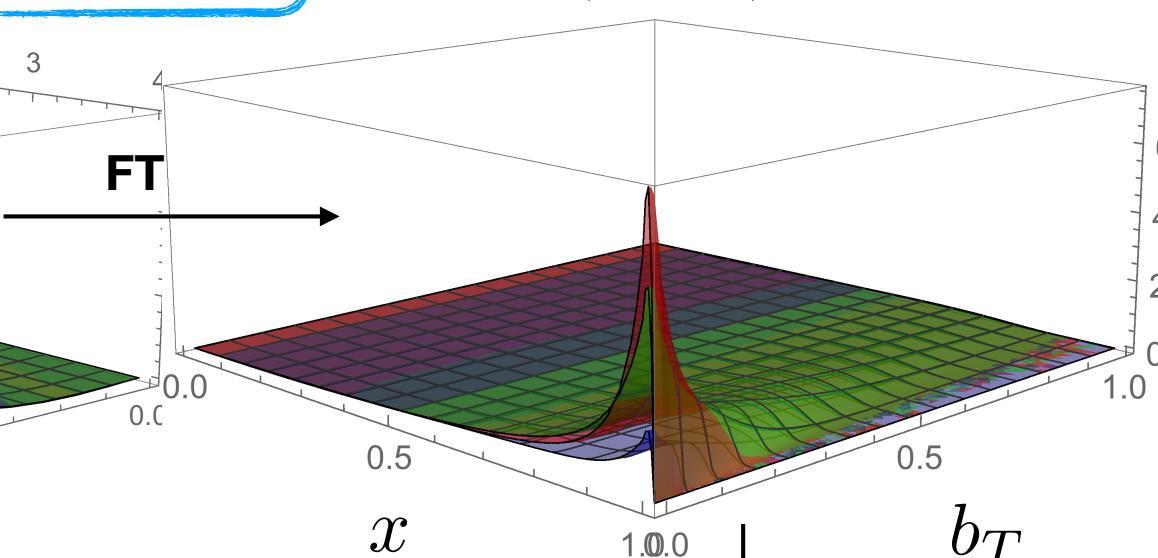
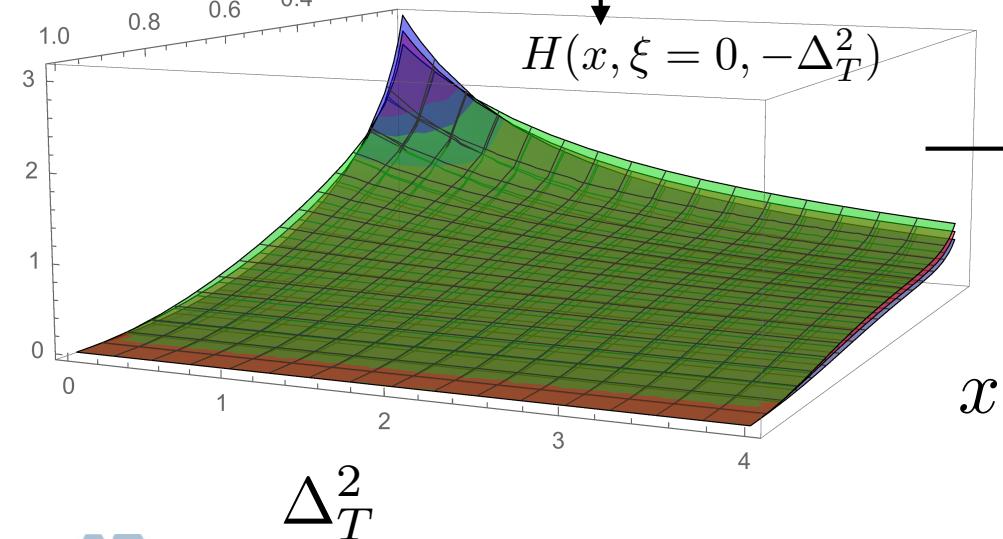
GPD

$$H(x, \xi = 0, -\Delta_T^2)$$

Red: u in pion
 Green: s in kaon
 Blue: u in kaon



DGLAP (4GeV)

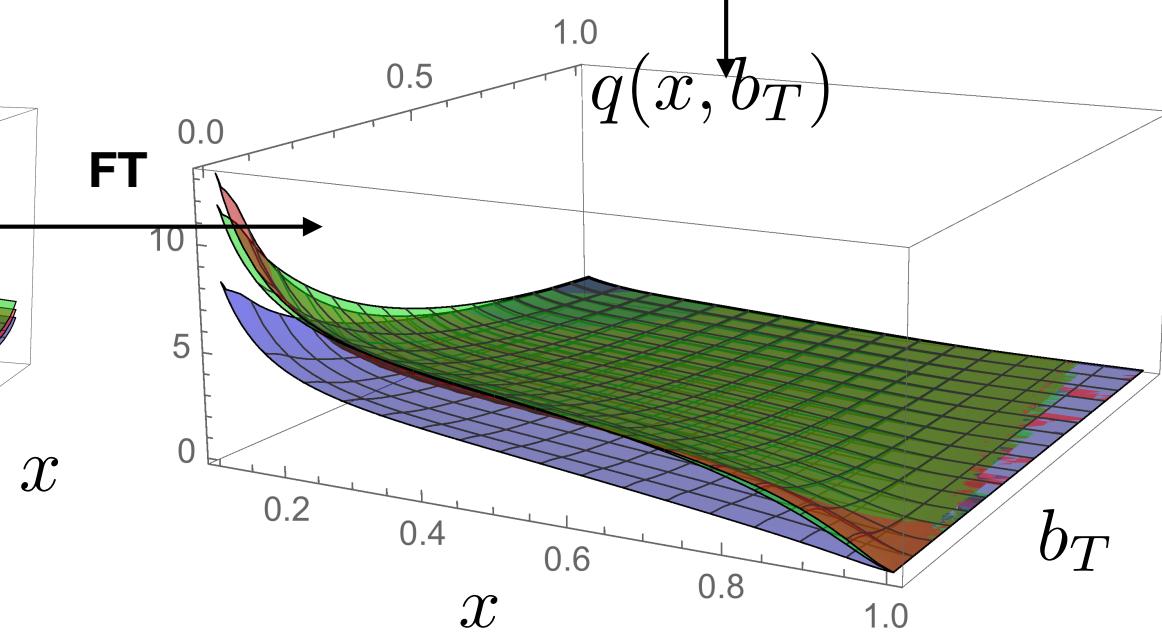


FT

FT

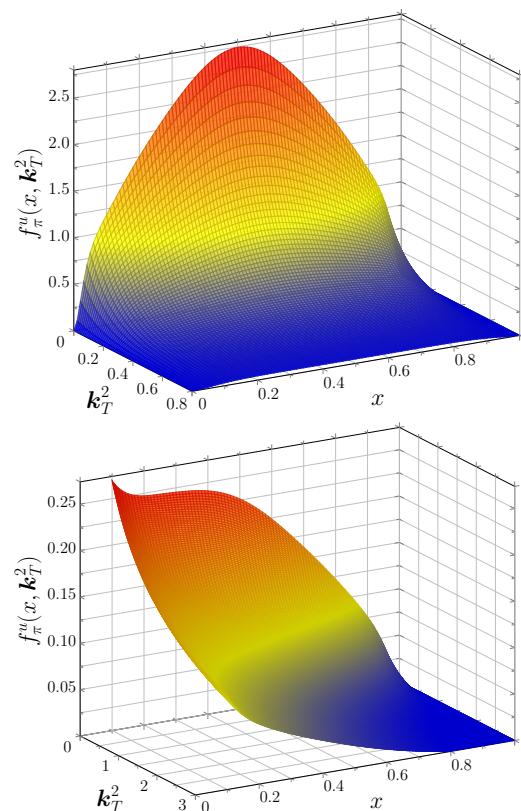
$$q(x, b_T)$$

DGLAP (4GeV)

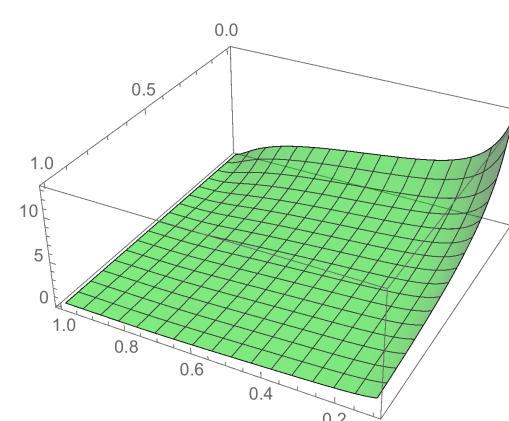
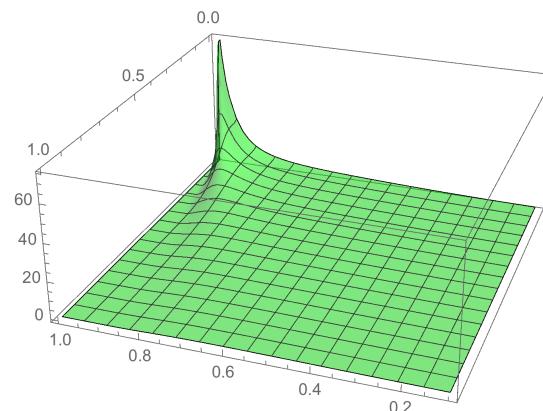


Summary

TMD



IPD GPD



Toward Nucleon

➊ Unpolarized parton distribution function
(Kyle Bednar, ICC and PCT. Phys.Lett. B782 (2018) 675-681)

➋ Parton distribution amplitude (Cedric Mezrag, Jorge Segovia et al, arXiv: 1711.09101)

➌ Electro-magnetic form factor (Gernot Eichmann, Phys.Rev. C77 (2008) 042202)

Outlook

➍ T-odd TMD PDF: careful treatment on the Wilson line; inequality relations to be satisfied.

➎ Higher Fock state for mesons.

➏ Nucleon tomography: advances in covariant approach; synergies with light-front QCD.



Thank
You