

The proton within Minkowski-space dynamics

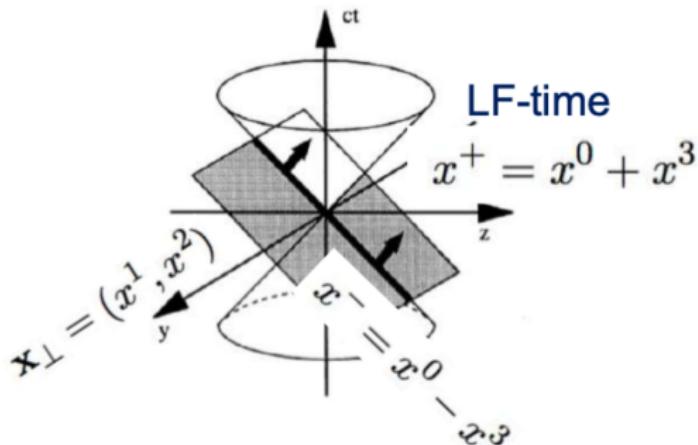
Tobias Frederico & Emanuel Ydrefors

Instituto Tecnologico de Aeronautica (TF) & Institute of Modern Physics, Chinese Academy of Sciences (EY)

References: PRD 104 (2021) 114012, arXiv: 2211.10959 [hep-ph]

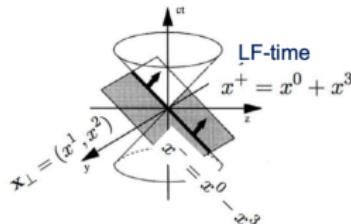
Meson Structure WG meeting
Jan 19, 2023 9:00AM ET

Light-front dynamics



- Dynamical system is characterized by ten generators of the Poincarè group: energy, momentum, angular momentum and boosts. (LFD: 7 kinematical, IF: 6)
- Conventional form (instant form): dynamical variables refer to physical conditions at some instant time, e.g. $x^0 = 0$. But, other choices are possible. In the Light-front (LF) dynamics refer to conditions on a front $x^+ = t + z = 0$. So, commutation relations defined at equal LF time ($x^+ = 0$).
- LF variables: $x^\pm = t \pm z$ and similarly for the momenta.
- Projection to the LF: eliminates the relative LF time between the particles, i.e. integration over the relative momentum k^- ($x^+ = 0$), the four-dimensional space reduced to a three-dimensional one (k^+, \vec{k}_\perp).

Light-front wave function



- It allows a Fock space expansion of a state vector in terms of contribution with well-defined particle-number;

$$|p\rangle = |n=3\rangle + |n=4\rangle + \dots \quad (1)$$

where each term has an associated boost-invariant wave function Ψ_n with probability

$$P_n = \left\{ \prod_{i=1}^n \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \delta \left(1 - \sum_{i=1}^n x_i \right) \delta \left(\sum_{i=1}^n \vec{k}_{i\perp} \right) |\Psi_n(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, \dots)|^2 \quad (2)$$

In Eq. (1), the leading contribution is referred to as valence component.

- Using the Fock space expansion one can derive a Schroedinger like equation:

$$H_{LC}|\Psi\rangle = M^2|\Psi\rangle, \quad H_{LC} = P^+ P^-, \quad (3)$$

with P^+ and \vec{P}_\perp diagonal and P^- a functional. In practice Fock-expansion is truncated.

Observables

- Electromagnetic form factors
- The parton distribution function, $f_1(x_1)$, i.e. probability distribution for a quark having a momentum fraction. Extracted from inclusive deep inelastic scattering, only scattered lepton detected.
- Transverse momentum distribution. Dependence on both momentum fraction x and transverse one \vec{k}_\perp . Associated with semi-inclusive deeply inelastic scattering (SIDIS), also high-momentum hadron detected.
- Double parton scattering cross section enters the double parton distribution function (DPDF) [1]:

$$D(x_1, x_2, \vec{\eta}_\perp) = \sum_{n=3}^{\infty} D_n(x_1, x_2, \vec{q}_\perp) = \sum_{n=3}^{\infty} \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \frac{d^2 k_{2\perp}}{(2\pi)^2} \left\{ \prod_{i \neq 1,2} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \\ \times \delta \left(1 - \sum_{i=1}^n x_i \right) \delta \left(\sum_{i=1}^n \vec{k}_{i\perp} \right) \Psi_n^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp, x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp, \dots) \Psi_n(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, \dots), \quad (4)$$

[1] B. Blok et al, PRD 83 (2011) 071501 (R).

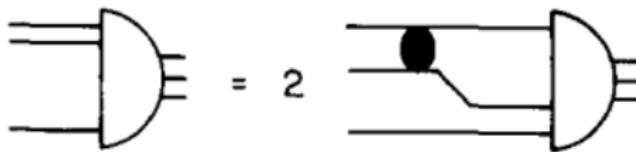
Proton model

W. Araújo, JPBC de Melo, TF, Phys. Rev. C 52 (1995) 2733

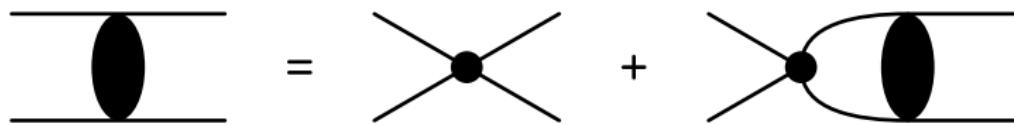
E. Ydrefors & TF, PRD 104 (2021) 114012, arXiv:2211.10959 [hep-ph]

- Fock basis truncated to valence order;
- spin degree-of-freedom is neglected;
- q-q transition amplitude: has a pole - s-wave diquark;
- no-confinement;
- zero-range q-q interaction (effective low-energy model).

Proton LF three-body model



- Three spinless particles of mass m . Spectator + pair of interacting particles. Factor of two due to symmetry of wave function with respect to exchange of the particles.



- In the present work a zero-range interaction with four-leg-vertex $i\lambda$ used. Then, for the two-body amplitude (see figure)

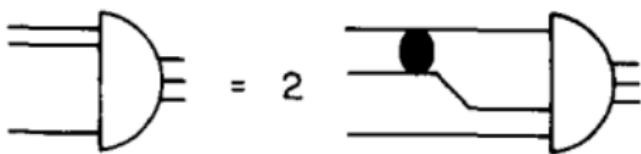
$$i\mathcal{F}(M_{12}^2) = i\lambda + (i\lambda)^2 \mathcal{B} + (i\lambda)^3 \mathcal{B}^2 + \dots = \frac{1}{(i\lambda)^{-1} - \mathcal{B}(M_{12}^2)} \quad (5)$$

with the bubble diagram

$$\mathcal{B}(M_{12}^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{i}{(k^2 - m^2 + i\epsilon)} \frac{i}{[(k - P)^2 - m^2 + i\epsilon]} \quad (6)$$

where $M_{12}^2 = P^2$. Bubble diagram regularized by assuming the diquark pole.

Proton Valence LF integral equation



- The valence three-body LF equation given by [1, 2]:

$$\Gamma(x, k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3} \int_0^{1-x} \frac{dx'}{x'(1-x-x')} \int_0^\infty d^2 k'_\perp \left[\frac{1}{M_0^2 - M_N^2} - \frac{1}{M_0^2 + \mu^2} \right] \Gamma(x', k'_\perp) \quad (7)$$

where μ is a cut-off, k_\perp transverse momentum and x momentum fraction of spectator. Furthermore, the squared free three-body mass

$$M_0^2 = (k_\perp^2 + m^2)/x' + (k_\perp^2 + m^2)/x + ((k'_\perp + k_\perp)^2 + m^2)/(1-x-x') \quad (8)$$

- The three-body valence LF wave function is given by

$$\Psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}) = \frac{\Gamma(x_1, \vec{k}_{1\perp}) + \Gamma(x_2, \vec{k}_{2\perp}) + \Gamma(x_3, \vec{k}_{3\perp})}{\sqrt{x_1 x_2 x_3} (M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}))}, \quad (9)$$

where due to momentum conservation: $x_3 = 1 - x_2 - x_3$ and $\vec{k}_{3\perp} = -\vec{k}_{1\perp} - \vec{k}_{2\perp}$.

[1] T. Frederico, PLB 282 (1992) 409

[2] J. Carbonell and V.A. Karmanov, PRC 67 (2003) 037001

Proton electromagnetic form factor

- The valence contribution to the Dirac form factor is obtained from the matrix element of γ^+ . In the frame $q^+ = 0$ and $q^2 = -Q^2 = -q_\perp^2$ it is given by

$$F_1(Q^2) = \left\{ \prod_{i=1}^3 \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \delta \left(1 - \sum_{i=1}^3 x_i \right) \delta \left(\sum_{i=1}^3 \vec{k}_{i\perp}^f \right) \Psi_3^\dagger(x_1, \vec{k}_{1\perp}^f, \dots) \Psi_3(x_1, \vec{k}_{1\perp}^i, \dots), \quad (10)$$

where $Q^2 = \vec{q}_\perp \cdot \vec{q}_\perp$ and the magnitudes of the momenta read

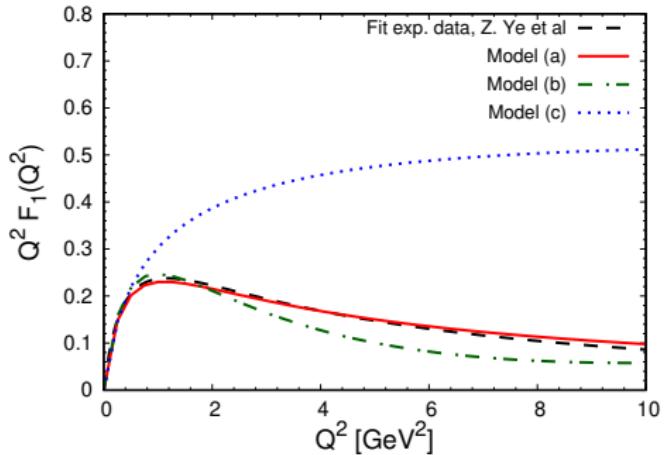
$$\left| \vec{k}_{i\perp}^{f(i)} \right|^2 = \left| \vec{k}_{i\perp} \pm \frac{\vec{q}_\perp}{2} x_i \right|^2 = \vec{k}_{i\perp}^2 + \frac{Q^2}{4} x_i^2 \pm \vec{k}_{i\perp} \cdot \vec{q}_\perp x_i \quad (i = 1, 2), \quad (11)$$

and

$$\begin{aligned} \left| \vec{k}_{3\perp}^{f(i)} \right|^2 &= \left| \pm \frac{\vec{q}_\perp}{2} (x_3 - 1) - \vec{k}_{1\perp} - \vec{k}_{2\perp} \right|^2 = \\ &(1 - x_3)^2 \frac{Q^2}{4} \pm (1 - x_3) \vec{q}_\perp \cdot (\vec{k}_{1\perp} + \vec{k}_{2\perp}) + (\vec{k}_{1\perp} + \vec{k}_{2\perp})^2. \end{aligned} \quad (12)$$

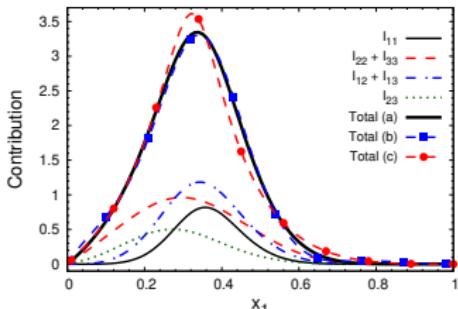
Proton Dirac FF

Model	m [MeV]	$a m$	μ/m	M_{dq} [MeV]	Ref.
(a)	366	2.70	1.00	644	arXiv: 2211.10959 [hep-ph]
(b)	362	3.60	∞	682	PRD 104 (2021) 114012
(c)	317	-1.84	∞	-	PRD 104 (2021) 114012



- In figure $Q^2 F(Q^2)$ for the three models compared with fit to exp. data by Z. Ye et al [1].
- Fair agreement with exp. data for $Q^2 < 5 \text{ GeV}^2$ but for larger values of Q^2 they deviate, presumably due to lack of a finite-range interaction.

Parton distribution function at model scale



- The single parton distribution function (PDF), is the integrand of the form factor at $Q^2 = 0$, i.e.

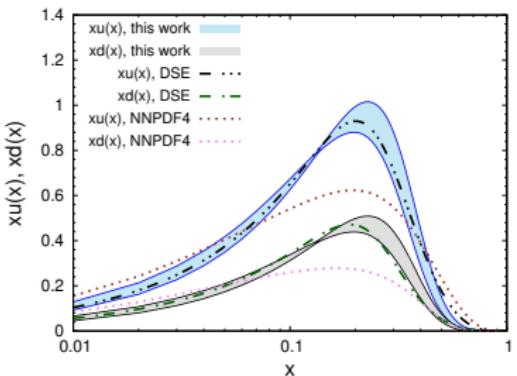
$$f_1(x_1) = \frac{1}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2 k_{1\perp} d^2 k_{2\perp} |\Psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp})|^2 = I_{11} + I_{22} + I_{33} + I_{12} + I_{13} + I_{23}. \quad (13)$$

with the Faddeev contributions

$$\begin{aligned} I_{ii} &= \frac{1}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2 k_{1\perp} d^2 k_{2\perp} \frac{\Gamma^2(x_i, \vec{k}_{i\perp})}{x_1 x_2 x_3 (M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}))^2} \\ I_{ij} &= \frac{2}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2 k_{1\perp} d^2 k_{2\perp} \frac{\Gamma(x_i, \vec{k}_{i\perp}) \Gamma(x_j, \vec{k}_{j\perp})}{x_1 x_2 x_3 (M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}))^2}; \quad i \neq j. \end{aligned} \quad (14)$$

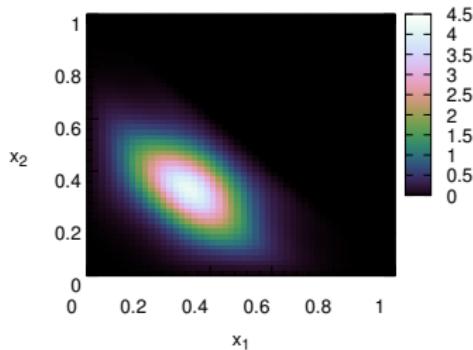
- The PDF at model scale is peaked around $x = 1/3$ and quite narrow. None of the Faddeev contributions are negligible.

Proton PDF $Q = 3.097$ GeV



- Colored areas: Computed u and d-quark xpdfs at $Q = 3.097$ GeV with the areas corresponding to the uncertainty in initial scale $Q_0 = 0.330 \pm 0.03$ GeV.
- Dash-dotted lines: Results from quark-diquark by Y. Lu et al [1]. Reasonable agreement. Disagreement at large x probably due to the use of contact interaction in our model.
- Dotted lines: Results from the NNPDF 4.0 global fit. None of the models agree well with these results.
- A few remarks:
 - Model of this work and the one by Y. Lu et al, are both quark-diquark models, but the latter one has also axial-vector diquark and a more realistic quark-quark interaction.

[1] PRD 104, 094036 (2021), [2] arXiv:2109.02653 [hep-ph]

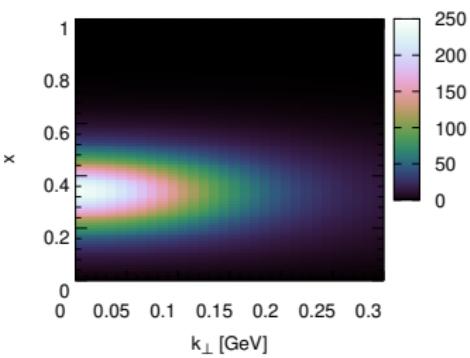
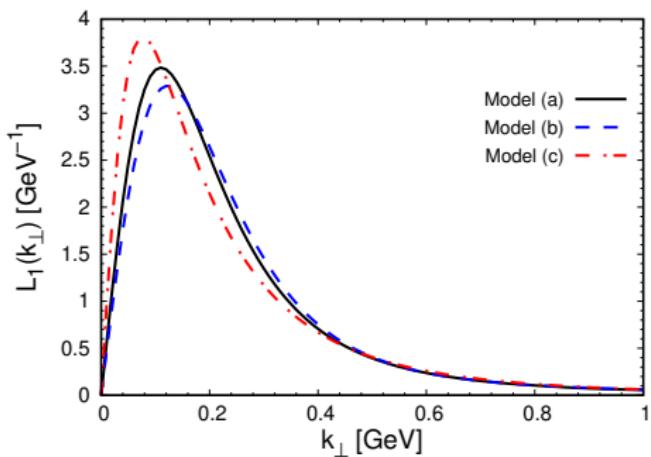


- The valence double parton distribution function (DPDF) is given by

$$D_3(x_1, x_2; \vec{\eta}_\perp) = \frac{1}{(2\pi)^6} \int d^2 k_{1\perp} d^2 k_{2\perp} \times \Psi_3^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp; x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp; x_3, \vec{k}_{3\perp}) \Psi_3(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}; x_3, \vec{k}_{3\perp}). \quad (15)$$

- Fourier transform of $D_3(x_1, x_2, \vec{\eta}_\perp)$ in $\vec{\eta}_\perp$ gives the probability of finding the quarks 1 and 2 with momentum fractions x_1 and x_2 at a relative distance $\vec{\eta}_\perp$ within the proton.
- In the figure is shown results for $\eta_\perp = 0$, showing a distribution centered around $x_1 = x_2 = 1/3$.

Transverse momentum distribution



$$L_1(k_\perp) = k_\perp \int_0^1 dx \tilde{f}(k_\perp, x) \quad (16)$$

with

$$\tilde{f}_1(k_\perp, x) = \int_0^1 dx_1 \delta(x - x_1) \int \frac{dk_{1\perp}}{(2\pi)^2} \delta(k_\perp - k_{1\perp}) \int_0^{2\pi} d\theta_1 \int \frac{d^2 k_{2\perp}}{(2\pi)^2} \int_0^{1-x} dx_2 |\Psi_3(\{x, \vec{k}_\perp\})|^2, \quad (17)$$

Pion and Proton unpolarized leading-twist TMD

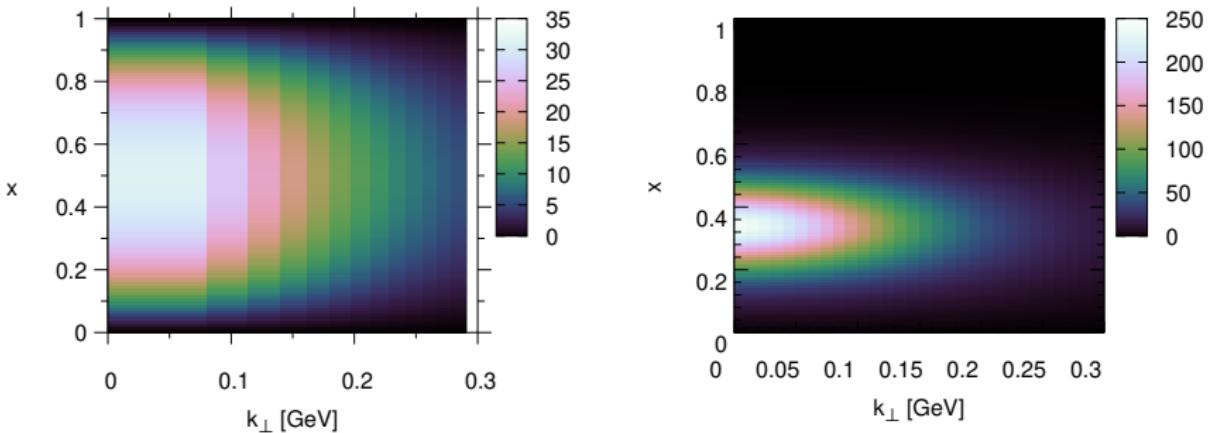


Figure: Leading twist unpolarized TMDs at the hadron scale. Left frame: Pion from Minkowski space Bethe-Salpeter equation model with constituent quarks, massive one-gluon exchange and quark-gluon form factor [1]. Right frame: Proton from a Light-front model with constituent quarks and a scalar diquark [2].

[1] W. de Paula, E. Ydrefors, J.H. Nogueira Alvarenga, T. Frederico, G. Salm e, PRD 105 (2022) L071505, and in preparation.

[2] E. Ydrefors, T. Frederico PRD 104 (2021) 114012; and arXiv: 2211.10959 [hep-ph].

Pion and Proton unpolarized PDFs

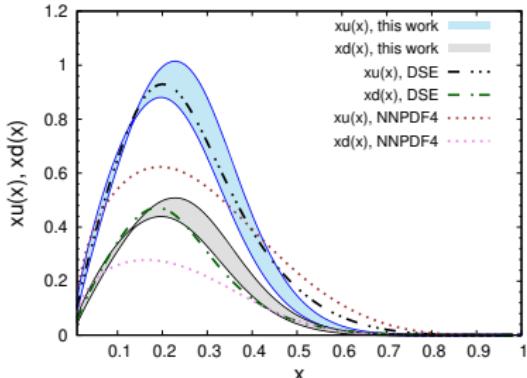
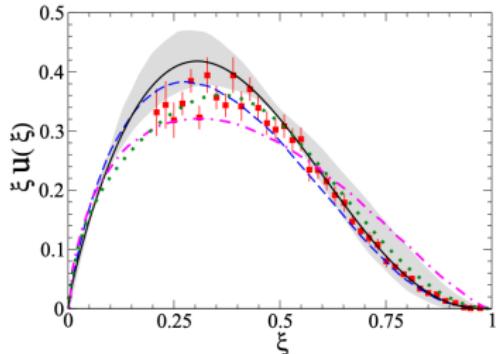


Figure: Unpolarized PDF. Left frame: Pion PDF at 5.2 GeV. Solid line: Minkowski space Bethe-Salpeter equation model with constituent quarks, massive one-gluon exchange and quark-gluon form factor from Ref. [1]; Dashed line: DSE calculation from Fig. 5 of Ref. [2]; Dash-dotted line: DSE calculation with dressed quark-photon vertex from Ref. [3]; Dotted line: BLFQ (Basis Light-Front Quantization) from Ref. [4]. Shaded area: Lattice QCD calculation extracted via Mellin moments from Ref. [5]. Red full circles: E615 Collaboration experimental data with soft-gluon resummation [6] evolved to 5.2 GeV. Right frame: Proton PDF at 3.097 GeV obtained with a Light-front model with constituent quarks and a scalar diquark from Ref. [7] blue and gray bars; Dashed-dot-dot from DSE [8]; Dotted lines NNPDF4.

- [1] W. de Paula, E. Ydrefors, J.H. Nogueira Alvarenga, T. Frederico, G. Salmè, PRD 105 (2022) L071505, and in preparation.
- [2] Z. F. Cui, M. Ding, J. M. Morgado, K. Raya, D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, J. Rodríguez-Quintero, and S.M. Schmidt, EPJA 58 (2022) 10.
- [3] K. D. Bednar, I. C. Cloët, and P. C. Tandy, PRL 124 (2020) 042002.
- [4] J. Lan, K. Fu, C. Mondal, X. Zhao, and J. P. Vary (BLFQ), PLB 825 (2022) 136890.
- [5] C. Alexandrou, S. Bacchio, I. Cloët, M. Constantinou, K. Hadjyianakou, G. Koutsou, and C. Lauer (ETM), PRD104 (2021) 054504.
- [6] M. Aicher, A. Schäfer, and W. Vogelsang, PRL 105 (2010) 252003.
- [7] E. Ydrefors, T. Frederico PRD 104 (2021) 114012; and arXiv: 2211.10959 [hep-ph].
- [8] Y. Lu, L. Chang, K. Raya, C. D. Roberts, J. Rodríguez-Quintero, PLB 830 (2022) 137130.

The present:

- Model for the proton valence state in Minkowski space - effective low-energy interaction - IR enhancement;
- no-confinement;
- Spin degree freedom neglected but not relevant for the PDF;

The future...

- spin dof;
- confinement;
- go beyond the valence approximation;
- observables sensitive to spin dof;
- help is needed...