Jefferson Lab

Pion and Kaon Structure at an EIC
Catholic University of America
May 25, 2018

Meson loop effects within nonlocal chiral EFT

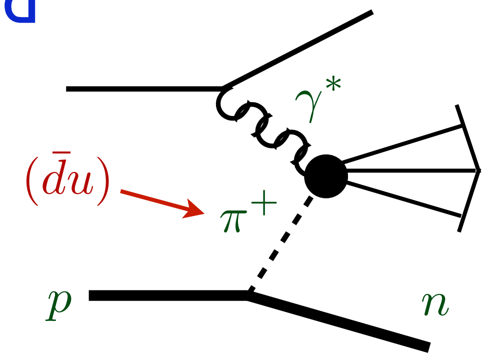
Wally Melnitchouk

in collaboration with

Yusupujiang Salamu, Ping Wang, Chueng Ji, Tony Thomas

Local chiral EFT

- A common and natural explanation for flavor asymmetries in the nucleon $(\bar{d} - \bar{u}\,,\, s - \bar{s}\,, \ldots)$ is a meson "cloud"



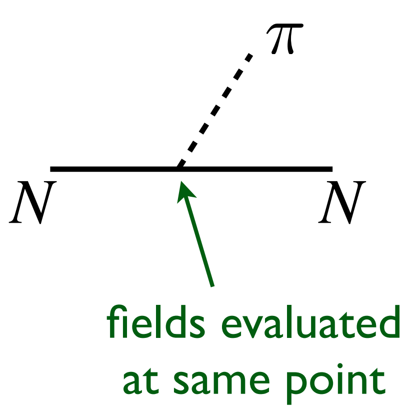
- Most efforts have been within low-energy models of QCD

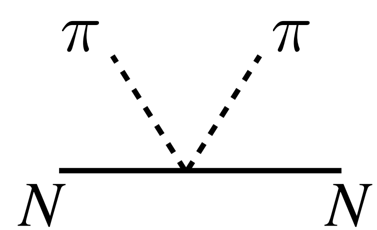
  → relatively successful phenomenology, but connection with QCD often unclear

- Rigorous connection with QCD established via chiral EFT

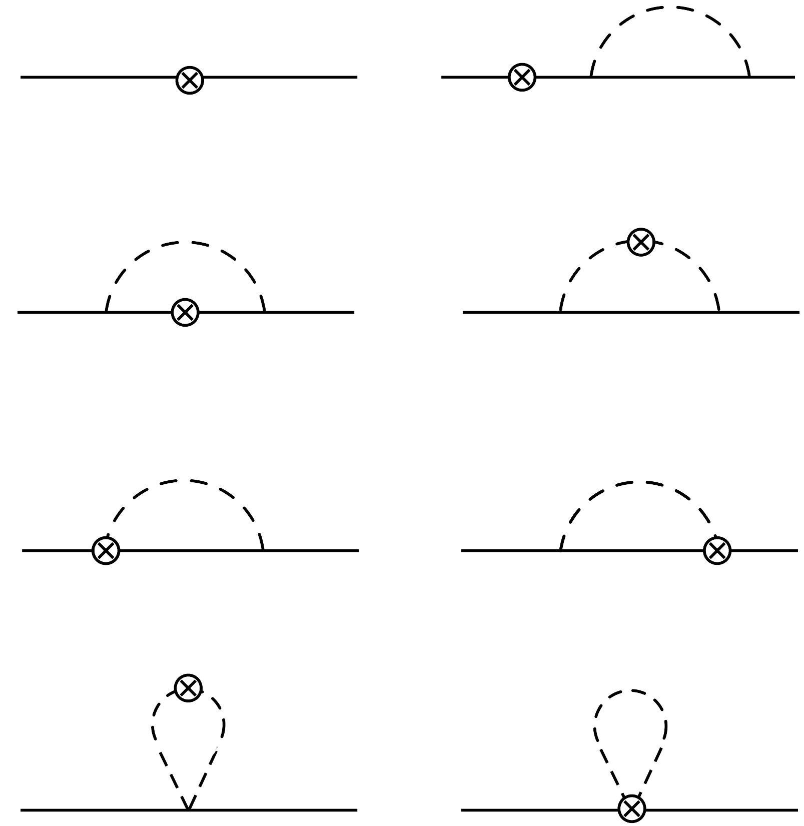
$$\mathcal{L}_{\rm eff} = \frac{g_A}{2f_\pi}\,\bar{\psi}_N \gamma^\mu \gamma_5\, \vec{\tau} \cdot \partial_\mu \vec{\pi}\, \psi_N - \frac{1}{(2f_\pi)^2}\,\bar{\psi}_N \gamma^\mu\, \vec{\tau} \cdot (\vec{\pi} \times \partial_\mu \vec{\pi})\, \psi_N \; + \; \ldots$$

Weinberg (1967)





- At leading order, pion-nucleon interaction includes pion rainbow, Kroll-Ruderman (needed for gauge invariance), and tadpole diagrams



- Matching of quark-level and hadron-level operators with same symmetries

$$\mathcal{O}_q^{\mu_1\cdots\mu_n} = \sum_h c_{q/h}^{(n)}\ \mathcal{O}_h^{\mu_1\cdots\mu_n}$$

$\longrightarrow$ yields convolution representation for PDFs

$$q(x) = \sum_h \int_x^1 \frac{dy}{y} f_h(y)\, q_v^h(x/y)$$

hadronic splitting functions

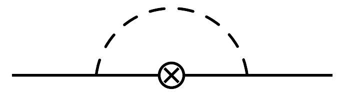
PDF in loop hadron

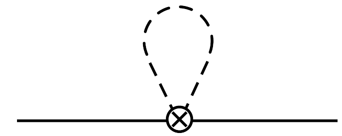
Parton distributions

- More specifically, contributions to quark PDF from different diagrams can be organized as:

wave function renormalization

rainbow

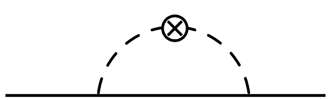




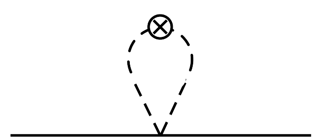
tadpole

$$q(x) = Z_2\, q_{_N}(x) + \left([f_N + f_{\rm tad}] \otimes q_{_N}\right)(x)$$

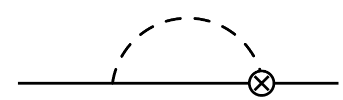
$$+ \left([f_\pi + f_{\rm bub}] \otimes q_\pi(x)\right) \; + \; \left(f_{\rm KR} \otimes \Delta q_{_N}\right)(x)$$



rainbow



bubble

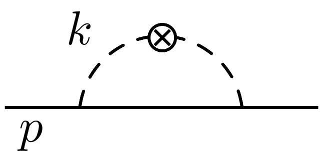


KR

(depends on N helicity PDF)

Chiral splitting functions

- Splitting functions for pion rainbow diagram



$$f_\pi^{(\text{rbw})}(y) = f^{(\text{on})}(y) + f^{(\delta)}(y)$$

has on-shell $(y = k^+/p^+ > 0)$ and $\delta(y)$ contributions!

$$f^{(\text{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y\,(k_\perp^2 + y^2 M^2)}{(1-y)^2 D_{\pi N}^2}\, \mathcal{F}^2$$

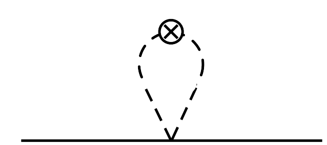
$$D_{\pi N} = -\frac{k_\perp^2 + y^2 M^2 + (1-y) m_\pi^2}{1-y}$$

$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)\ \mathcal{F}^2$$

$\mu = k^-$ cutoff

- Bubble diagram contributes only at $y = 0$ (hence $x = 0$)

$$f_\pi^{(\text{bub})}(y) = \frac{8}{g_A^2} f^{(\delta)}(y)$$



$\longrightarrow$ contributes to lowest moment, but not at $x > 0$

Salamu, Ji, WM, Wang (2015)

- Pion rainbow diagram with nucleon coupling

$$f_N^{(\mathrm{rbw})}(y) = f^{(\mathrm{on})}(y) + f^{(\mathrm{off})}(y) - f^{(\delta)}(y)$$

$$f^{(\mathrm{off})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{2y}{(1-y) D_{\pi N}} \mathcal{F}^2$$

additional off-shell contribution at $y > 0$

- Tadpole contribution also only at $y = 0$

$$f_\pi^{(\mathrm{tad})}(y) = -\, f_\pi^{(\mathrm{bub})}(y)$$

- KR diagram has off-shell and δ-function contributions

$$f_N^{(\mathrm{KR})}(y) = -f^{(\mathrm{off})}(y) + 2 f^{(\delta)}(y)$$

- Satisfy gauge-invariance relation *

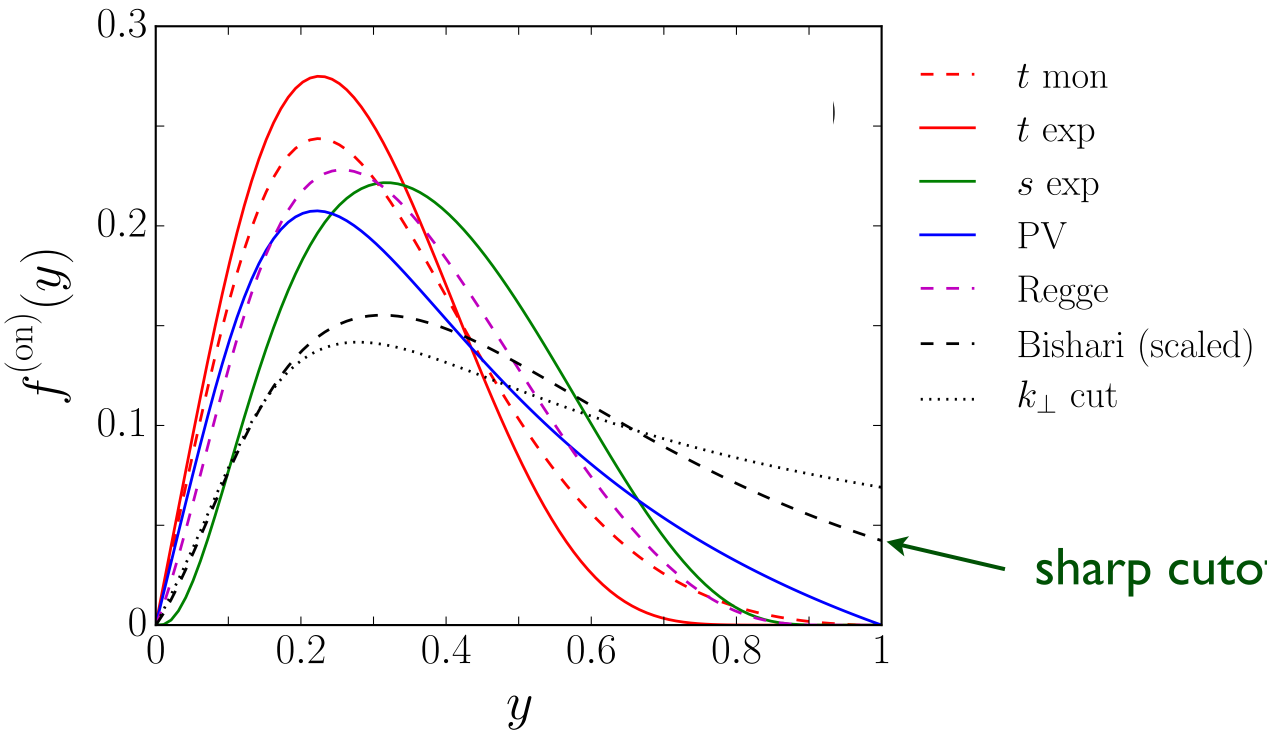
$$f_N^{(\mathrm{rbw})}(y) + f_N^{(\mathrm{KR})}(y) = f_\pi^{(\mathrm{rbw})}(y)$$

* for symmetry-preserving regulators

- For point-like nucleons and pions, integrals divergent

$\longrightarrow$ finite size of nucleon provides natural regularization scale

e.g. on-shell function



sharp cutoffs

$\mathcal{F} = \Theta(\Lambda^2 - k_\perp^2)$ $k_\perp$ cutoff

$\mathcal{F} = \left(\dfrac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t}\right)$ t monopole

$\mathcal{F} = \exp\left[(t - m_\pi^2)/\Lambda^2\right]$ t exponential

$\mathcal{F} = \exp\left[(M^2 - s)/\Lambda^2\right]$ s-dep. exponential

$\mathcal{F} = \left[1 - \dfrac{(t - m_\pi^2)^2}{(t - \Lambda^2)^2}\right]^{1/2}$ Pauli-Villars

$\mathcal{F} = y^{-\alpha_\pi(t)} \exp\left[(t - m_\pi^2)/\Lambda^2\right]$ Regge

E866 $\bar{d} - \bar{u}$ data can be fitted with range of regulators

$$\bar{d} - \bar{u} = \left[f_\pi^{(\text{rbw})} + f_\pi^{(\text{bub})} \right] \otimes \bar{q}_v^\pi$$

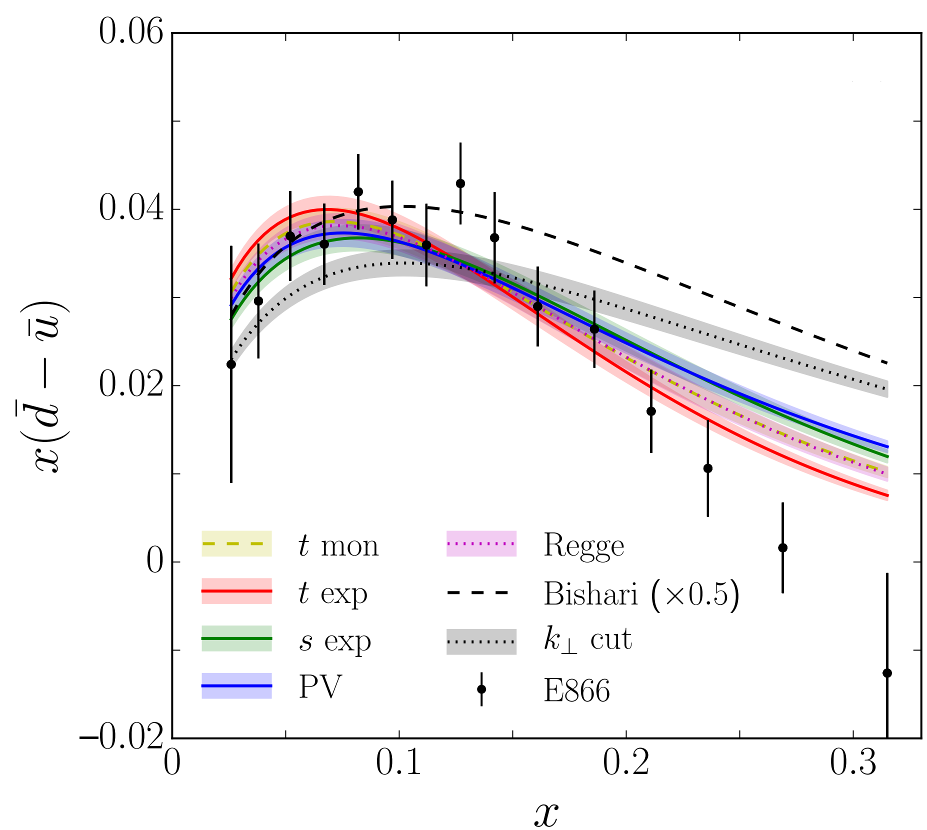
$\sim \; f^{(\text{on})} + f^{(\delta)}$ $\qquad \sim \; f^{(\delta)}$

→ only on-shell function contributes at $x > 0$

average pion "multiplicity"

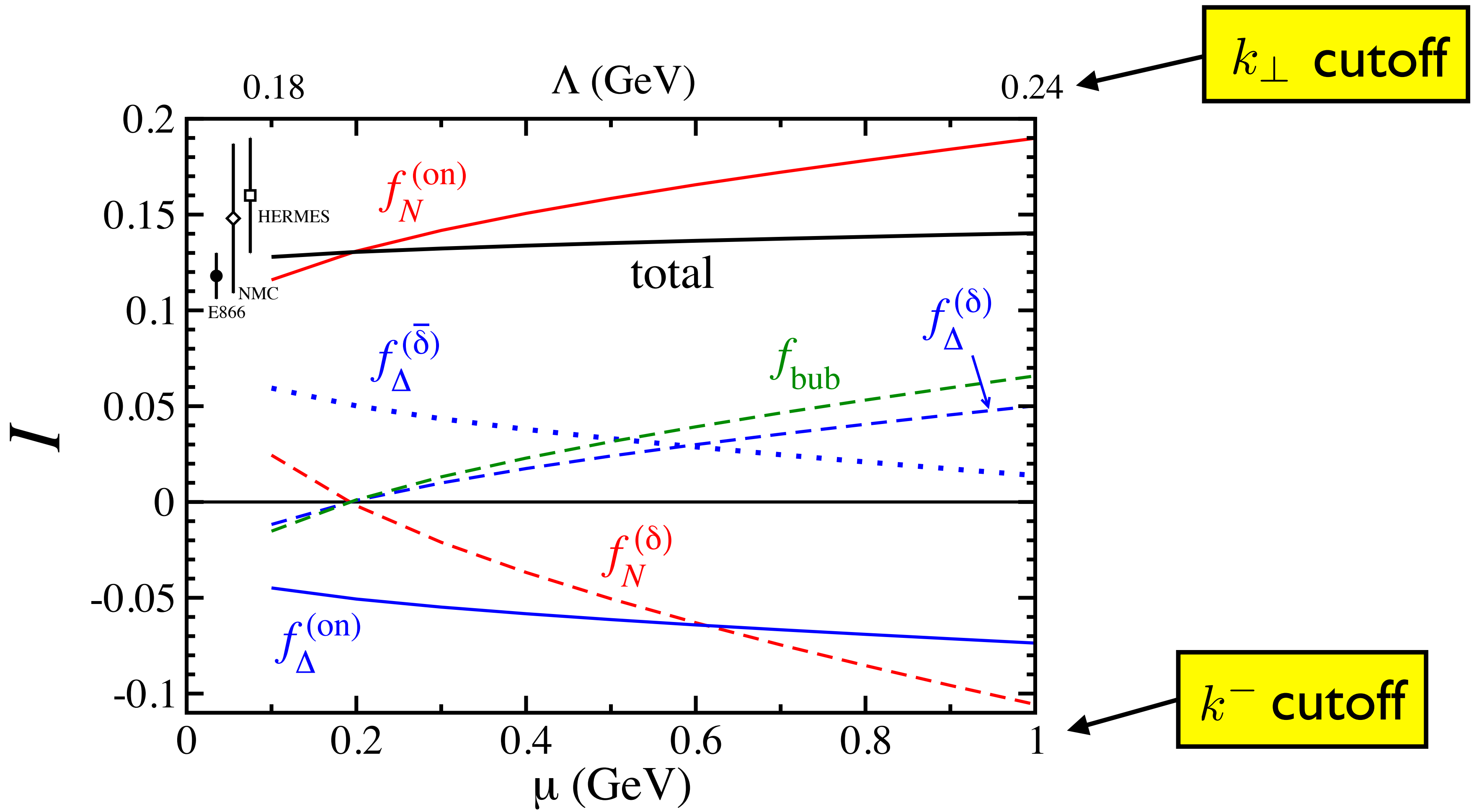
$$\langle n \rangle_{\pi N} = 3 \int_0^1 dy \, f^{(\text{on})}(y)$$

$$\sim \; 0.25 - 0.3$$



McKenney, Sato, WM, C. Ji (2016)

■ Integrated asymmetry $I = \int_0^1 dx(\bar{d} - \bar{u})$



Salamu, C. Ji, WM, Wang (2014)

⟶ N on-shell contribution $\approx$ total!

Effect on moments of PDFs

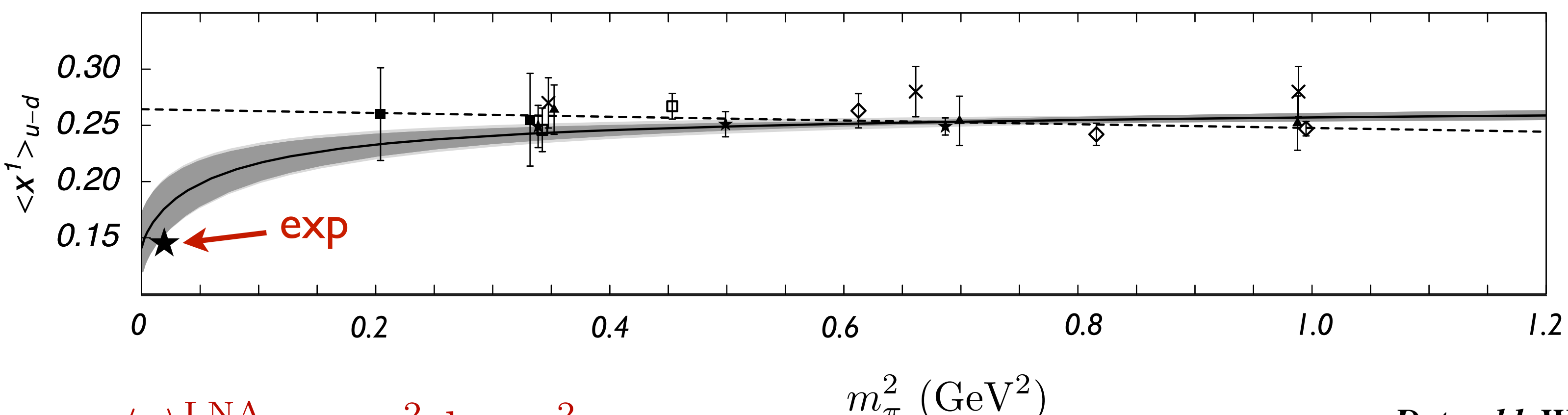
→ coefficients of leading nonanalytic (LNA) terms, reflecting infrared behavior, are *model-independent!*

→ QCD therefore *predicts* a nonzero asymmetry from π loops

$$\int_0^1 dx\,(\bar{d}-\bar{u}) = \frac{(3g_A^2-1)}{(4\pi f_\pi)^2}\, m_\pi^2 \log(m_\pi^2/\mu^2) \;+\; \text{analytic in } m_\pi^2$$

Thomas, WM, Steffens (2000)

→ nonanalytic behavior vital for understanding lattice data on PDF moments at low m_π



$$\langle x\rangle_{u-d}^{\rm LNA} \sim m_\pi^2 \log m_\pi^2$$

Detmold, WM, Thomas (2001)

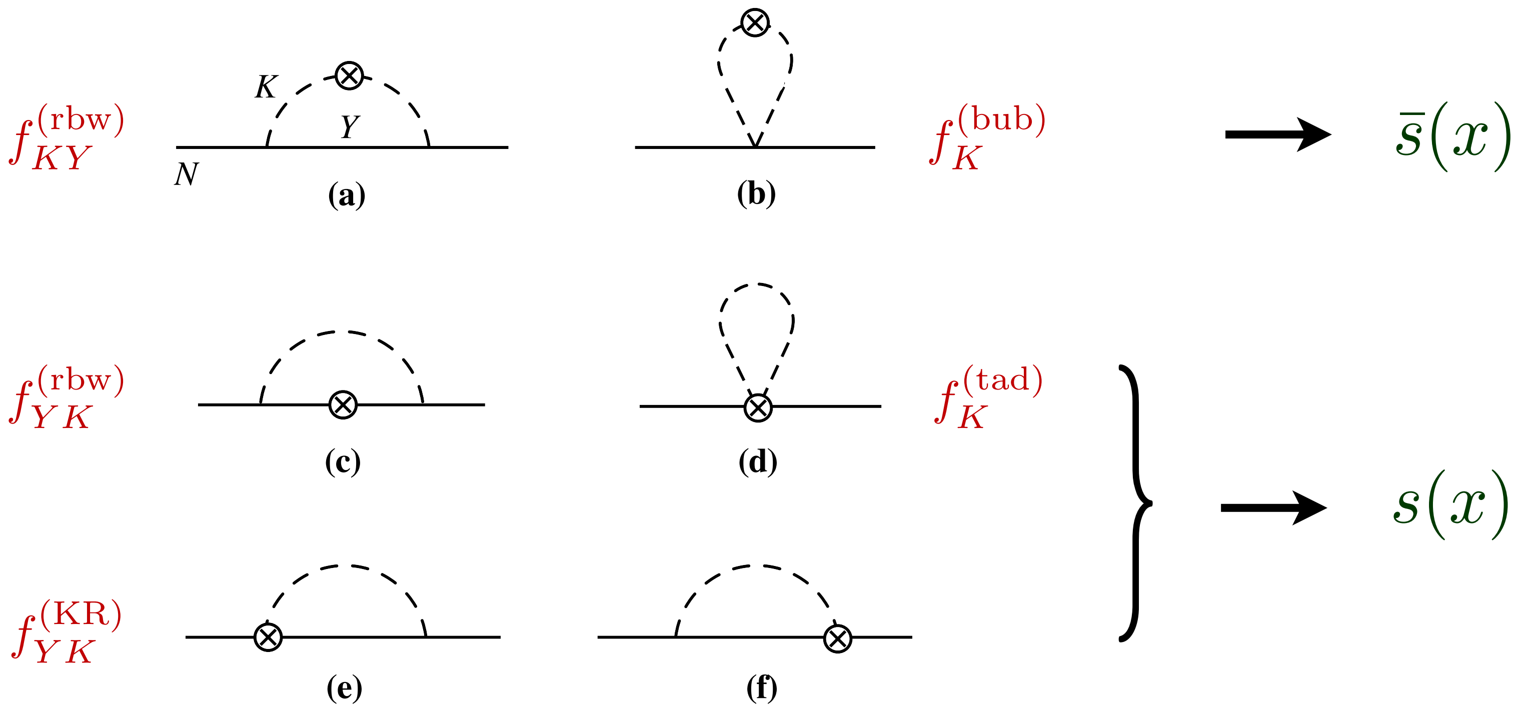
Strange quarks

- Some indication of strange–antistrange asymmetry from $\nu/\bar{\nu}$ DIS

$$S^{-} = \int_0^1 dx\, x(s-\bar{s}) \;= (2.0 \pm 1.4) \times 10^{-3}$$

NuTeV (2007)

- Chiral SU(3) effective theory analysis suggests natural mechanism for generating strange asymmetry



$\longrightarrow$ gauge invariance requires the relations

$$f_{YK}^{(\mathrm{rbw})} + f_{YK}^{(\mathrm{KR})} = f_{KY}^{(\mathrm{rbw})}$$

$$f_K^{(\mathrm{tad})} + f_K^{(\mathrm{bub})} = 0$$

Convolution representation

$$\bar{s} = \left(f_{KY}^{(\mathrm{rbw})} + f_{K}^{(\mathrm{bub})} \right) \otimes \bar{s}_K$$

$$s = \left(\bar{f}_{YK}^{(\mathrm{rbw})} \otimes s_Y + \bar{f}^{(\mathrm{KR})} \otimes s_Y^{(\mathrm{KR})} \right) + \bar{f}_K^{(\mathrm{tad})} \otimes s_K^{(\mathrm{tad})}$$

$\bar{f}(y) \equiv f(1-y)$ $\quad\quad$ $\sim \Delta u, \Delta d$ $\quad\quad$ $\sim u, d$

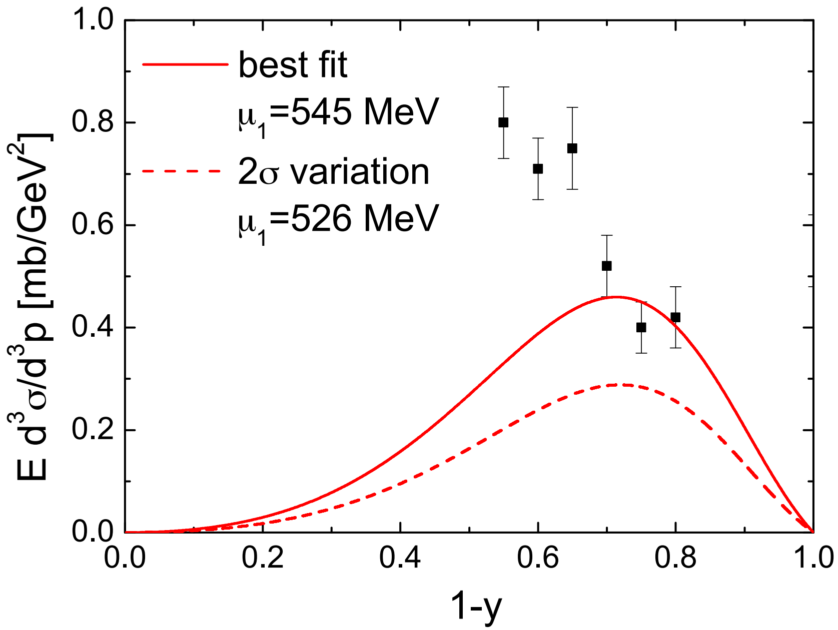
→ KY splitting functions regularized using 1 Pauli-Villars subtraction

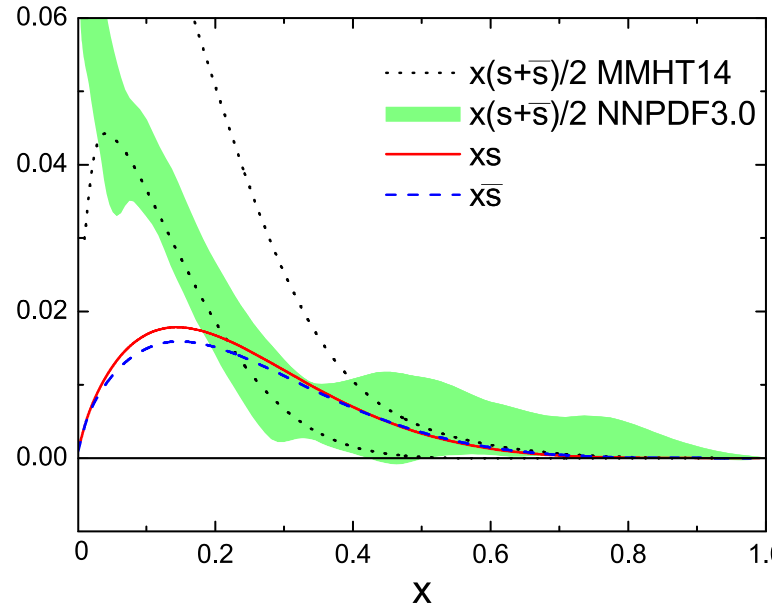
→ δ-function terms require 2 subtractions (parameters μ_1, μ_2)

→ since $f_K^{(\mathrm{tad})}(y) \sim \delta(y)$, tadpole term generates *valence-like* strange-quark PDF (nonzero at $x > 0$!) through convolution

$$\sim \; s_K^{(\mathrm{tad})}(x)$$

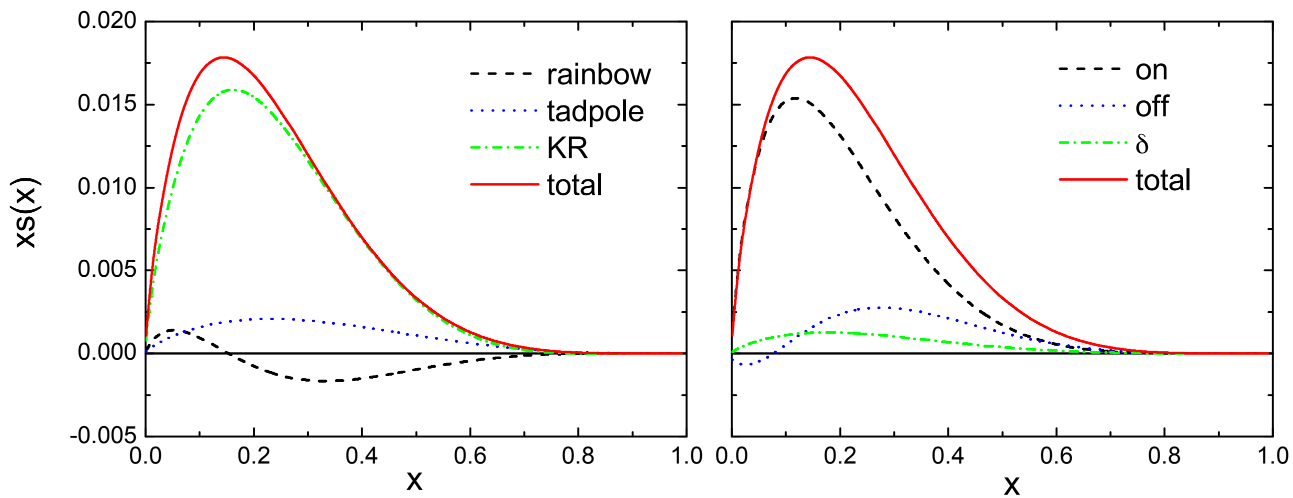
Constraints on cutoff parameters from $pp \to \Lambda X$ and total $(s+\bar{s})_{\rm loops} \le (s+\bar{s})_{\rm total}$





X. Wang, C. Ji, WM, Salamu, Thomas, P. Wang (2016)

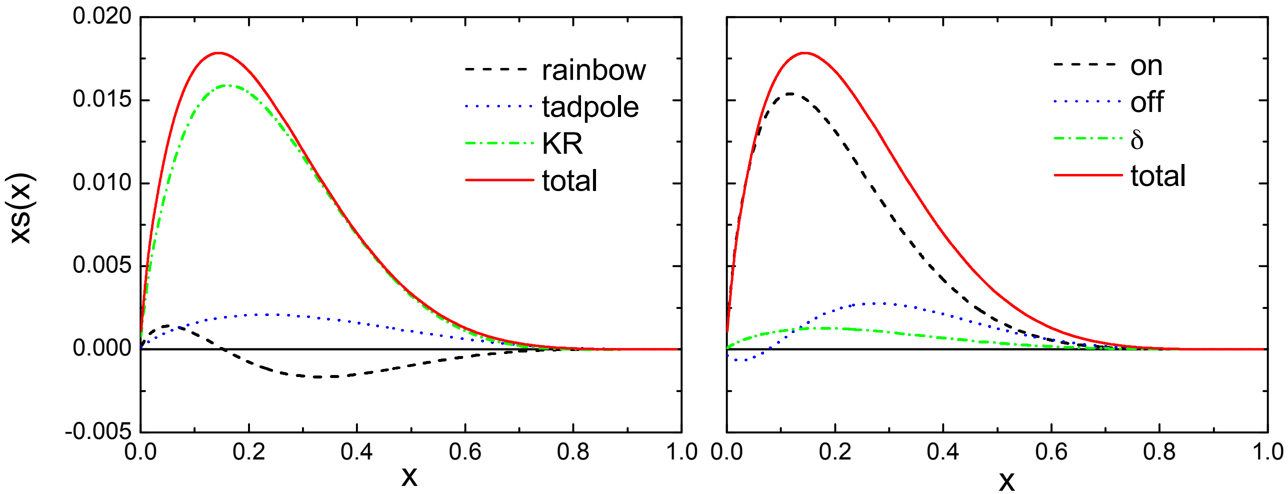
Breakdown into individual contributions to $s(x)$



$$
\begin{aligned}
s(x) &= \left(s^{(\mathrm{on})} + s^{(\mathrm{off})} + s^{(\delta)}\right)_{\mathrm{rbw}} + s^{(\delta)}_{\mathrm{tad}} + \left(s^{(\mathrm{off})} + s^{(\delta)}\right)_{\mathrm{KR}} \\
&= \underbrace{s^{(\mathrm{on})}_{\mathrm{rbw}}}_{\text{on-shell}} + \underbrace{s^{(\mathrm{off})}_{\mathrm{rbw}} + s^{(\mathrm{off})}_{\mathrm{KR}}}_{\text{off-shell}} + \underbrace{s^{(\delta)}_{\mathrm{rbw}} + s^{(\delta)}_{\mathrm{tad}} + s^{(\delta)}_{\mathrm{KR}}}_{\delta\text{-function}},
\end{aligned}
$$

$$
\begin{aligned}
\bar{s}(x) &= \left(\bar{s}^{(\mathrm{on})} + \bar{s}^{(\delta)}\right)_{\mathrm{rbw}} + \bar{s}^{(\delta)}_{\mathrm{bub}} \\
&= \underbrace{\bar{s}^{(\mathrm{on})}_{\mathrm{rbw}}}_{\text{on-shell}} + \underbrace{\bar{s}^{(\delta)}_{\mathrm{rbw}} + \bar{s}^{(\delta)}_{\mathrm{bub}}}_{\delta\text{-function}},
\end{aligned}
$$

Breakdown into individual contributions to $s(x)$

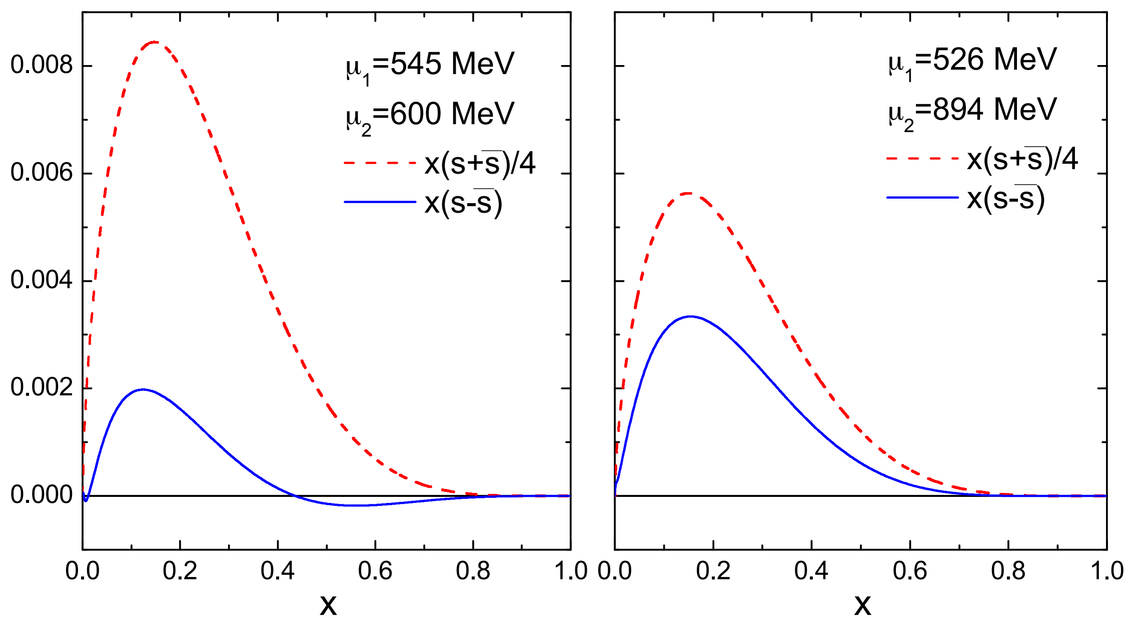


→ large cancellations between off-shell terms in rainbow & KR, and between δ-function terms in rainbow, KR and tadpole

→ total $s(x)$ well approximated by on-shell part of rainbow; total off-shell & δ-function terms small

→ explains phenomenological success of loop calculations in terms of on-shell rainbow only!

- Gives rise to small but (mostly) positive $s - \bar{s}$ distribution



→ x-weighted difference $S^- = (0.4 - 1.1) \times 10^{-3}$

→ presence of δ-function terms in $\bar{s}(x)$ means that integrals of s and $\bar{s}$ at $x > 0$ need not cancel!

X. Wang, C. Ji, WM, Salamu, Thomas, P. Wang (2016)

Regularization

- For point particles, regulator functions $\mathcal{F}$ for on-shell, off-shell and δ-function distributions (which could be different!) are unity

  $\longrightarrow$ UV divergent … need to suppress large-k contributions

- Not all regularization schemes preserve symmetries of the field theory (Lorentz invariance, gauge invariance, chiral symmetry)

  $\longrightarrow$ dimensional regularization, Pauli-Villars (example of finite-range regulator) known to preserve chiral and Lorentz symmetries

  $\longrightarrow$ naive application of (some) hadronic form factors can lead to problems with gauge invariance

- A solution which allows preservation of symmetries with form factors is to use *nonlocal* theory!

Nonlocal chiral EFT

- For interactions of finite-sized hadrons, it is natural to imagine interactions would not necessarily be at a single point, but smeared out over spacetime

→ generalize local chiral SU(3) Lagrangian …

$$
\begin{aligned}
\mathcal{L}^{(\mathrm{local})}(x) &= \bar{B}(x)(i\gamma^\mu \mathscr{D}_{\mu,x} - M_B)B(x) + \frac{C_{B\phi}}{f}\left[\bar{p}(x)\gamma^\mu\gamma^5 B(x)\,\mathscr{D}_{\mu,x}\phi(x) + \mathrm{h.c.}\right] \\
&+ \overline{T}_\mu(x)(i\gamma^{\mu\nu\alpha}\mathscr{D}_{\alpha,x} - M_T\gamma^{\mu\nu})\,T_\nu(x) + \frac{C_{T\phi}}{f}\left[\bar{p}(x)\Theta^{\mu\nu}T_\nu(x)\,\mathscr{D}_{\mu,x}\phi(x) + \mathrm{h.c.}\right] \\
&+ \frac{iC_{\phi\phi^\dagger}}{2f^2}\bar{p}(x)\gamma^\mu p(x)\left[\phi(x)(\mathscr{D}_{\mu,x}\phi)^\dagger(x) - \mathscr{D}_{\mu,x}\phi(x)\phi^\dagger(x)\right] \\
&+ \mathscr{D}_{\mu,x}\phi(x)(\mathscr{D}_{\mu,x}\phi)^\dagger(x) + \cdots
\end{aligned}
$$

covariant derivatives

$$
\begin{aligned}
\mathscr{D}_{\mu,x}B(x) &= [\partial_\mu - ie_B^q\,\mathscr{A}_\mu(x)]\,B(x), \\
\mathscr{D}_{\mu,x}T^\nu(x) &= [\partial_\mu - ie_T^q\,\mathscr{A}_\mu(x)]\,T^\nu(x), \\
\mathscr{D}_{\mu,x}\phi(x) &= [\partial_\mu - ie_\phi^q\,\mathscr{A}_\mu(x)]\,\phi(x),
\end{aligned}
$$

e.m. field

meson fields

$$
\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}
$$

octet baryon fields

$$
B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}
$$

decuplet baryon fields

$$T_\mu^{ijk} = \Delta, \Sigma^*, \Xi^*, \Omega$$

Nonlocal chiral EFT

- For interactions of finite-sized hadrons, it is natural to imagine interactions would not necessarily be at a single point, but smeared out over spacetime

→ … to nonlocal Lagrangian

$$\begin{aligned}
\mathcal{L}^{(\mathrm{nonloc})}(x) &= \bar{B}(x)(i\gamma^{\mu}\mathscr{D}_{\mu,x} - M_B)B(x) + \overline{T}_{\mu}(x)(i\gamma^{\mu\nu\alpha}\mathscr{D}_{\alpha,x} - M_T\gamma^{\mu\nu})T_{\nu}(x) \\
&+ \bar{p}(x)\left[\frac{C_{B\phi}}{f}\gamma^{\mu}\gamma^5 B(x) + \frac{C_{T\phi}}{f}\Theta^{\mu\nu}T_{\nu}(x)\right] \\
&\qquad \times \int d^4a\, \mathcal{G}^q_{\phi}(x, x+a)F(a)\ \mathscr{D}_{\mu,x+a}\phi(x+a) + \mathrm{h.c.} \\
&+ \frac{iC_{\phi\phi^{\dagger}}}{2f^2}\bar{p}(x)\gamma^{\mu}p(x)\int d^4a \int d^4b\ \mathcal{G}^q_{\phi}(x+b, x+a)F(a)F(b) \\
&\qquad \times \left[\phi(x+a)(\mathscr{D}_{\mu,x+b}\phi)^{\dagger}(x+b) - \mathscr{D}_{\mu,x+a}\phi(x+a)\phi^{\dagger}(x+b)\right] \\
&+ \mathscr{D}_{\mu,x}\phi(x)(\mathscr{D}_{\mu,x}\phi)^{\dagger}(x) + \cdots,
\end{aligned}$$

→ gauge link $\mathcal{G}^q_{\phi}(x,y) = \exp\left[-ie^q_{\phi}\int_x^y dz^{\mu}\mathscr{A}_{\mu}(z)\right]$ preserves local gauge invariance of fields

→ coordinate space meson-baryon vertex form factor $F(a)$ in Lagrangian

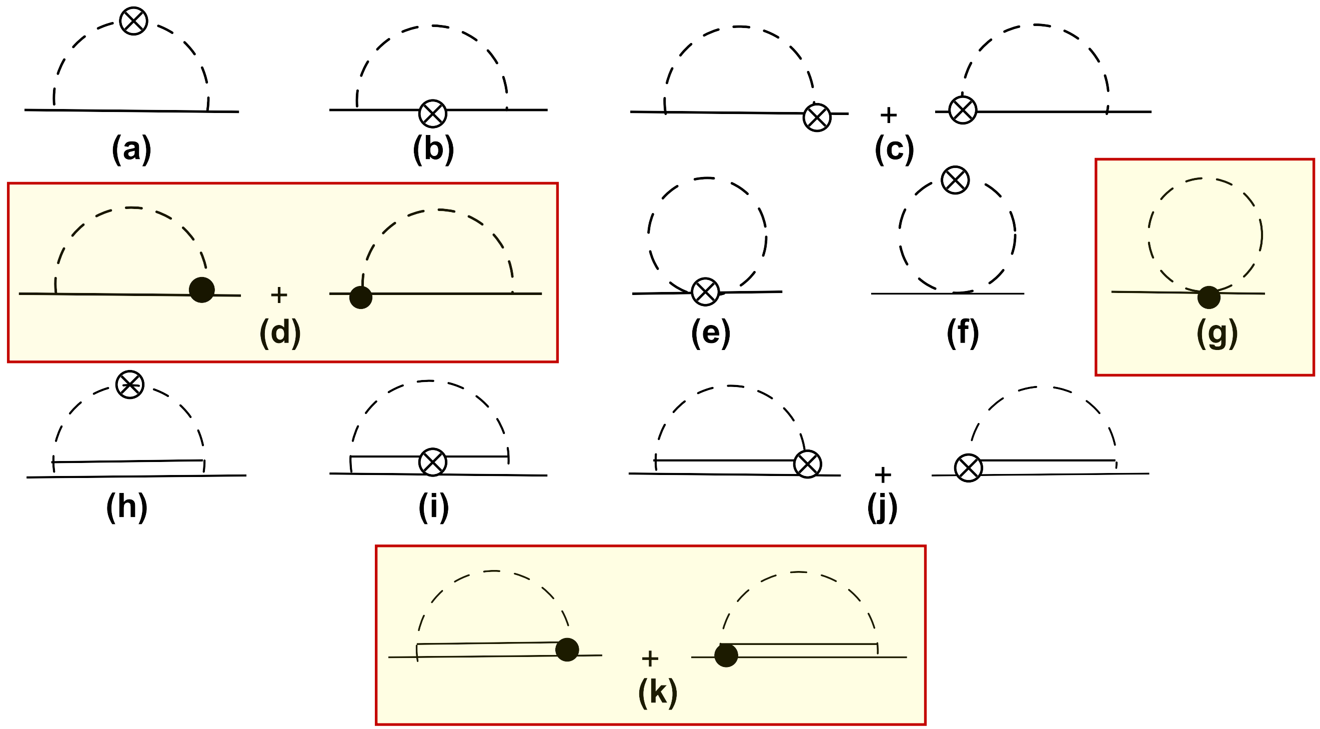
- Expand gauge link to lowest order

$$\mathcal{G}^q_\phi(x+b,x+a) = \exp\left[-ie^q_\phi\,(a-b)^\mu \int_0^1 dt\, \mathscr{A}_\mu\big(x+at+b(1-t)\big)\right]$$
$$= 1 \;+\; \delta\mathcal{G}^q_\phi \;+\; \cdots,$$

→ allows nonlocal Lagrangian to be written as sum of free and interacting parts, with latter consisting of nonlocal purely <u>hadronic</u>, <u>electromagnetic</u> (from 1st term), and <u>gauge-link</u> parts (from 2nd term)

$$\mathcal{L}^{(\rm nonloc)}_{\rm link}(x) = -ie^q_\phi\,\bar{p}(x)\left[\frac{C_{B\phi}}{f}\,\gamma^\rho\gamma^5 B(x) + \frac{C_{T\phi}}{f}\,\Theta^{\rho\nu}T_\nu(x)\right]$$
$$\times \int_0^1 dt \int d^4a\, F(a)\, a^\mu\, \partial_\rho\phi(x+a)\mathscr{A}_\mu(x+at) + \text{h.c.}$$
$$+\frac{e^q_\phi C_{\phi\phi^\dagger}}{2f^2}\,\bar{p}(x)\gamma^\rho p(x)\int_0^1 dt \int d^4a \int d^4b\, F(a)\,F(b)\,(a-b)^\mu$$
$$\times \left[\phi(x+a)\partial_\rho\phi^\dagger(x+b) - \partial_\rho\phi(x+a)\phi^\dagger(x+b)\right]\mathscr{A}_\mu\big(x+at+b(1-t)\big)$$

- Gauge link part of Lagrangian generates additional interactions specific to the nonlocal theory



→ there are also nonlocal contributions to all the other diagrams!

→ illustrate for case of a (k^2-dependent) dipole form factor

$$\widetilde{F}(k) = \left(\frac{\overline{\Lambda}}{D_\Lambda}\right)^2 \qquad D_\Lambda = k^2 - \Lambda^2 + i\varepsilon \qquad \overline{\Lambda}^2 \equiv \Lambda^2 - m_\phi^2$$

■ *e.g.* meson rainbow diagram

$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{C_{B\phi}^2 \overline{M}^2}{(4\pi f)^2}\left[f_B^{(\mathrm{on})}(y) + f_B^{(\delta)}(y) - \delta f_B^{(\delta)}(y)\right]$$

on-shell term

$$f_B^{(\mathrm{on})}(y) = \overline{\Lambda}^8 \int dk_\perp^2 \, \frac{y\left[k_\perp^2 + (yM+\Delta)^2\right]}{\bar{y}^2 D_{\phi B}^2 D_{\Lambda B}^4}$$

δ-function term

$$f_B^{(\delta)}(y) = -\frac{\overline{\Lambda}^8}{\overline{M}^2}\int dk_\perp^2 \int_0^1 dz \, \frac{z^3}{(k_\perp^2+\Omega)^4}\,\delta(y)$$
$$= \frac{1}{\overline{M}^2}\int dk_\perp^2 \left[\log\frac{\Omega_\phi}{\Omega_\Lambda} + \frac{\Lambda^2(11\,\Omega_\Lambda^2 - 7\,\Omega_\Lambda\Omega_\phi + 2\,\Omega_\phi^2)}{6\Omega_\Lambda^3}\right]\delta(y),$$

nonlocal δ-function contribution

$$\delta f_B^{(\delta)}(y) = -\frac{\overline{\Lambda}^8}{\overline{M}^2}\int dk_\perp^2 \int_0^1 dz \, \frac{z^4}{(k_\perp^2+\Omega)^4}\,\delta(y)$$
$$= \frac{1}{\overline{M}^2}\int dk_\perp^2 \left[-4\frac{\Omega_\phi}{\Lambda^2}\log\frac{\Omega_\phi}{\Omega_\Lambda} - \frac{3\Omega_\Lambda^3 + 13\Omega_\Lambda^2\Omega_\phi - 5\Omega_\Lambda\Omega_\phi^2 + \Omega_\phi^3}{3\Omega_\Lambda^3}\right]\delta(y)$$

$$\Omega_\phi = k_\perp^2 + m_\phi^2\,, \qquad \Omega_\Lambda = k_\perp^2 + \Lambda^2.$$

- $e.g.$ meson rainbow diagram

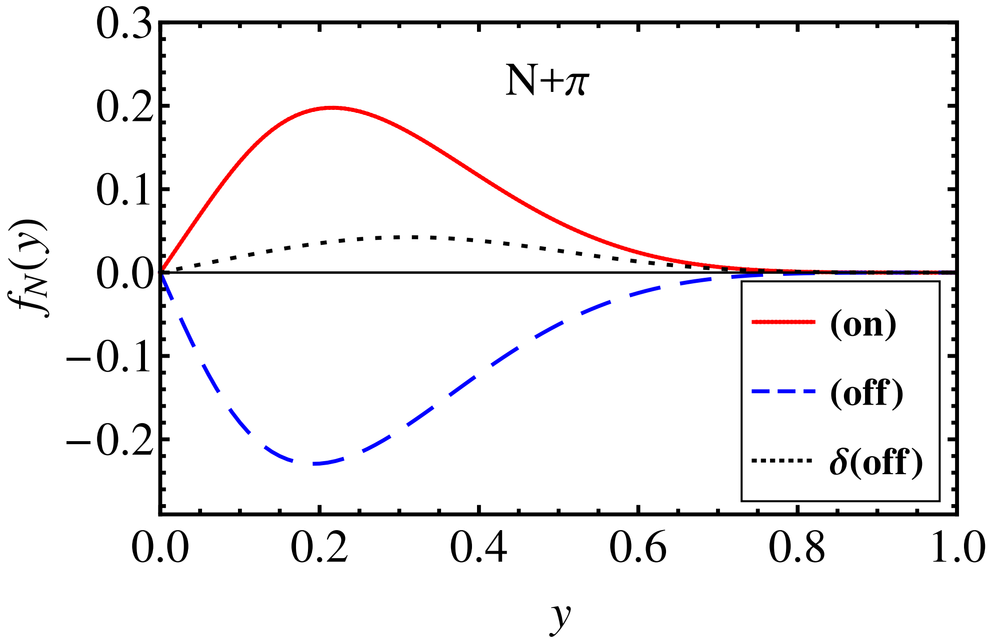
$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{C_{B\phi}^2 \overline{M}^2}{(4\pi f)^2}\left[f_B^{(\mathrm{on})}(y) + f_B^{(\delta)}(y) - \delta f_B^{(\delta)}(y)\right]$$

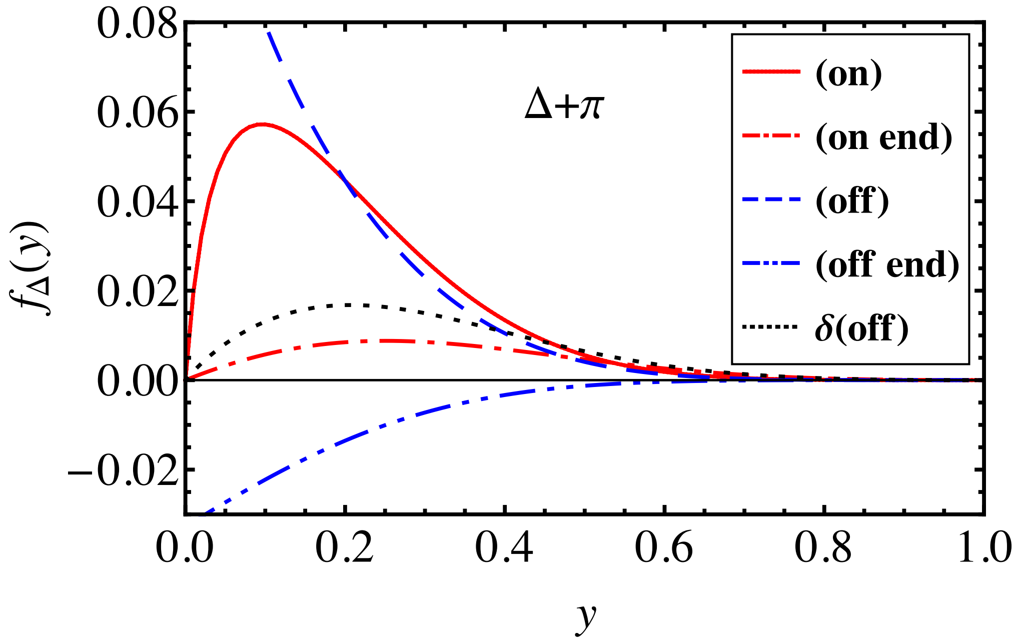
$\longrightarrow$ in $\Lambda \to \infty$ limit, $f_B^{(\mathrm{on})}$ and $f_B^{(\delta)}$ approach local limits;
purely nonlocal function $\delta f_B^{(\delta)}$ vanishes

- similarly for all other diagrams

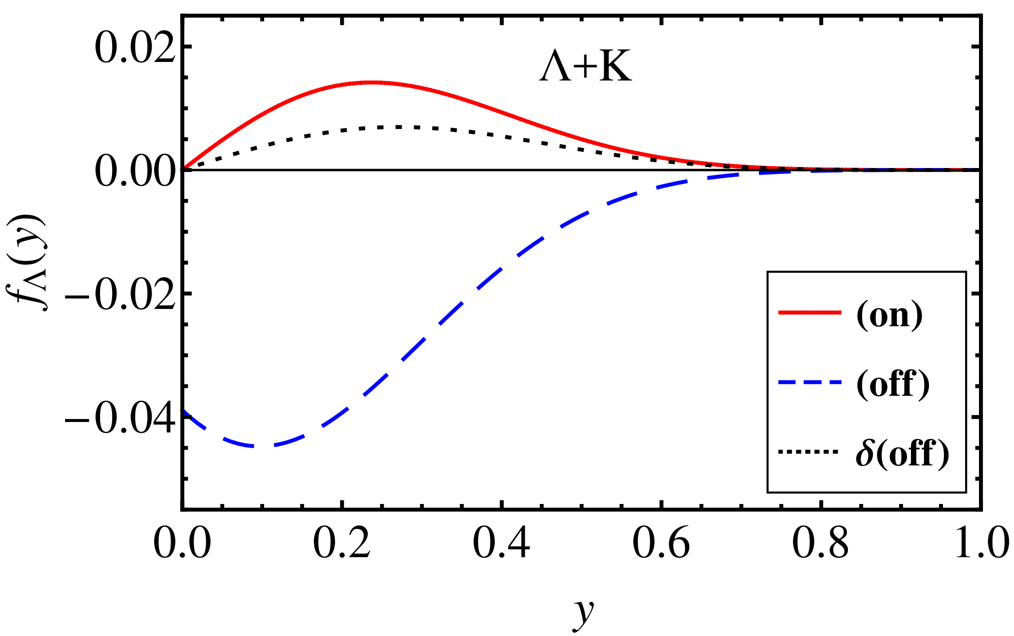
$\longrightarrow$ additional complications for decuplet diagrams,
with end-point contributions at $y = 1$

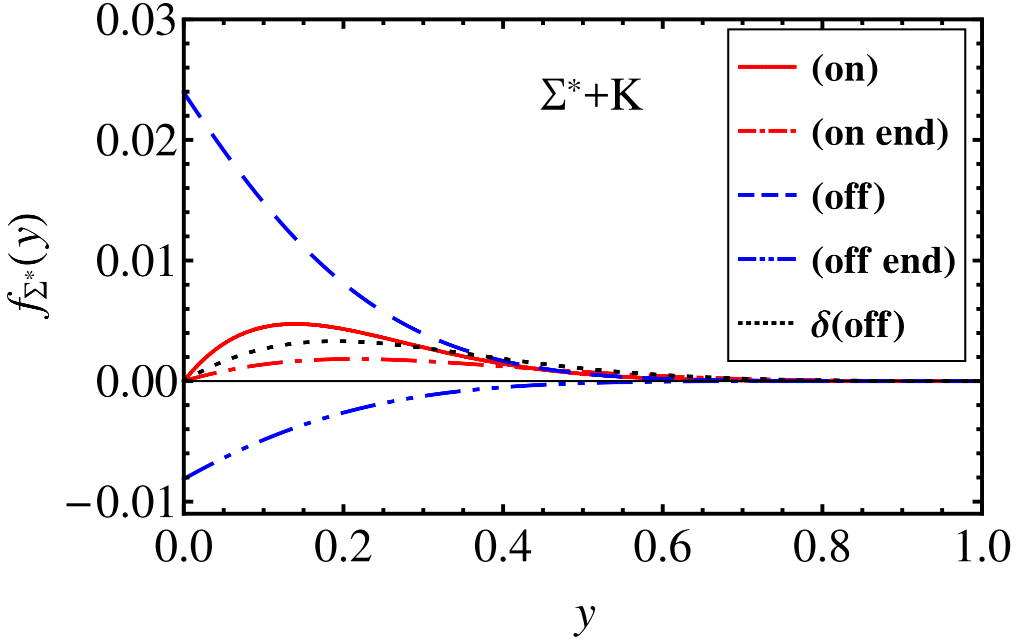
Numerical effects of nonlocality



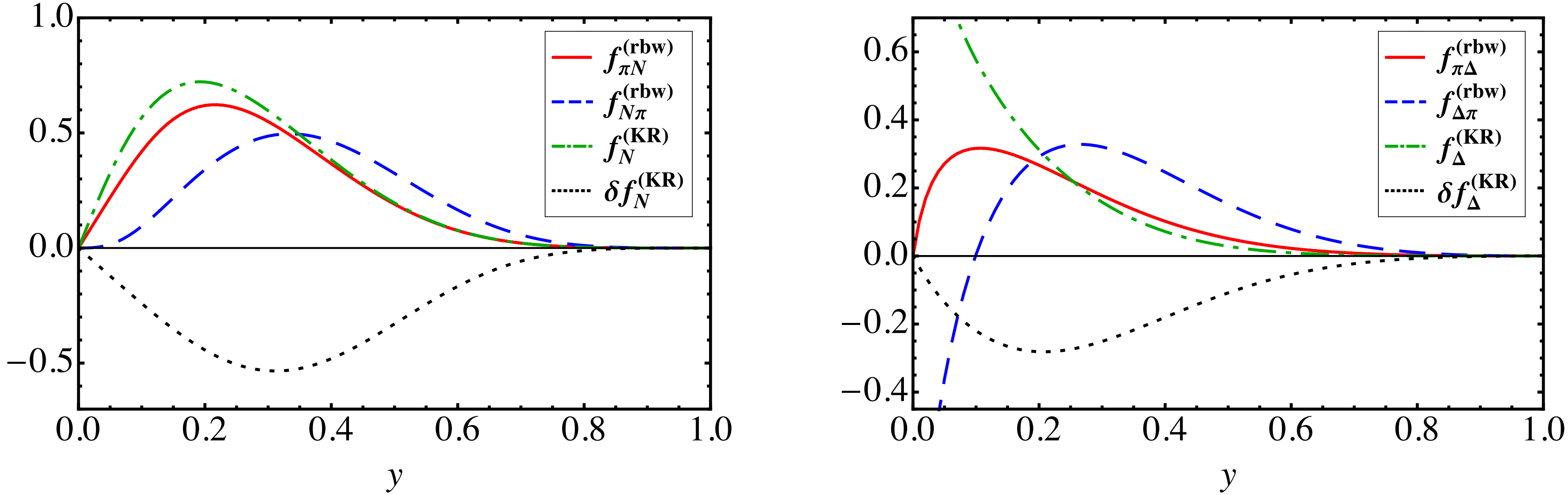


Numerical effects of nonlocality

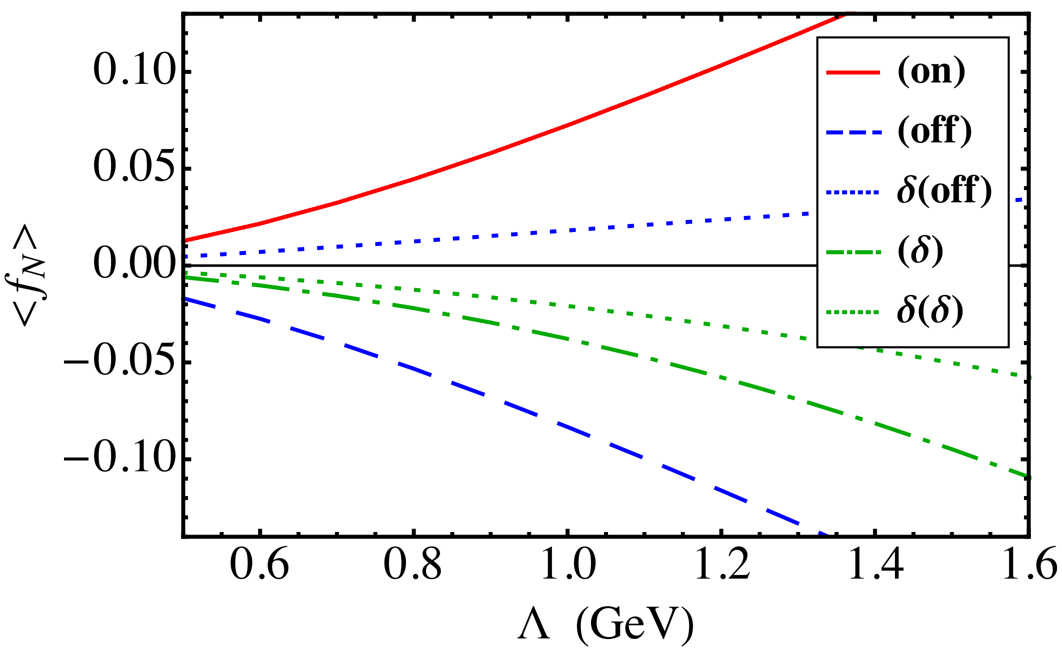


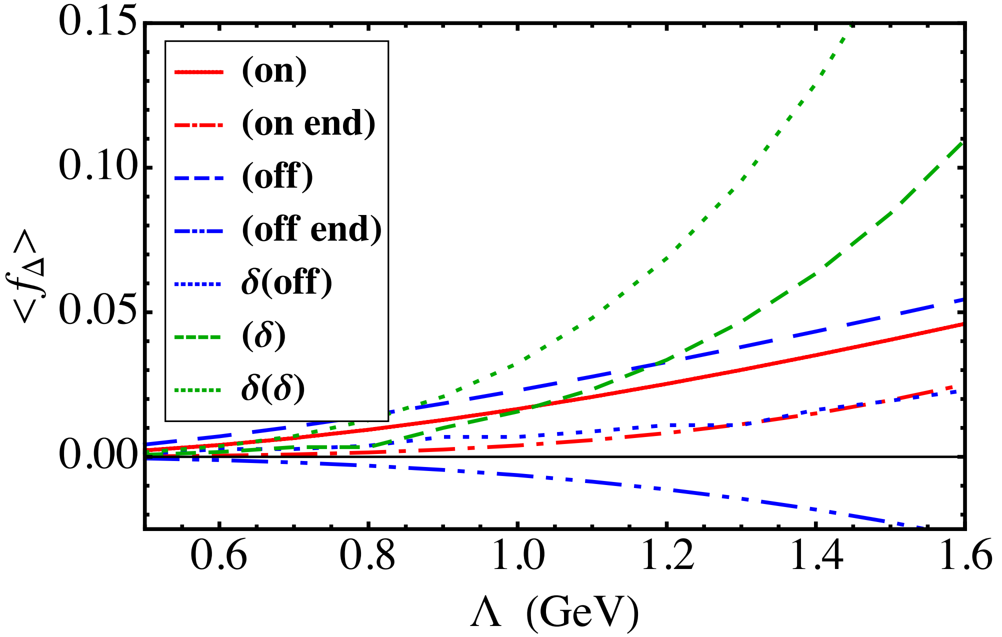


Numerical effects of nonlocality

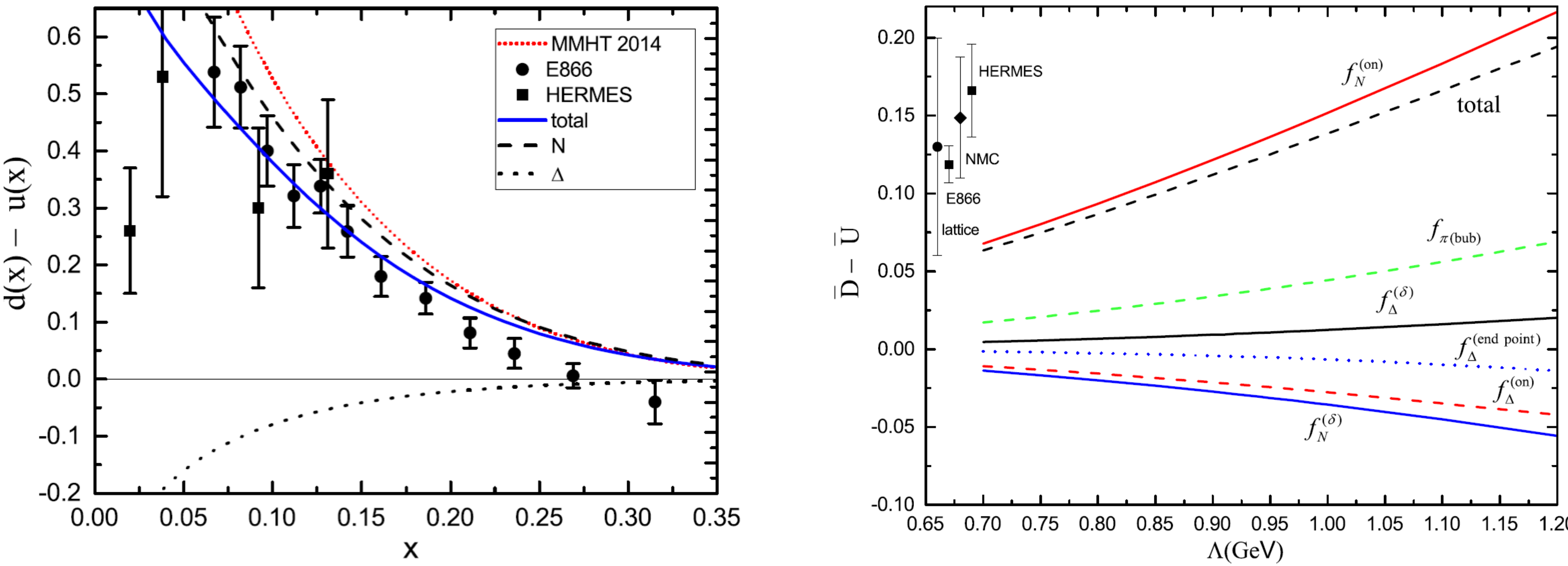


Numerical effects of nonlocality





Numerical effects of nonlocality



⟶ N on-shell contribution still ≈ total!

Outlook

- The future of chiral loops is “fuzzy”…