Exploring hadron (meson) structure with **models** and **PDF sensitivity visualizations**

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PIEIC2018

arXiv:1803.02777

http://metapdf.hepforge.org/PDFSense/

… in part with Bo-Ting Wang, S. Doyle, J. Gao, T.-J. Hou, Pavel Nadolsky, and F. Olness

Seurat, ***The Lighthouse at Honfleur*** (1886) – The National Gallery.

outline: two distinct avenues to information on meson structure

I. Lattice

- historically, the first few PDF Mellin moments have been accessible to direct calculation on the Lattice

- Lattice-calculable quasi-distributions (previous talks) have made evaluation of PDFs' x dependence on the lattice possible

→ the earliest pioneering LQCD calculations of the **pion quasi-PDF** are presently forth-coming

this talk: model calculation in TJH, Phys. Rev. D**97** (2018) no.5, 054028.

→ Lattice practitioners require input regarding the detailed boost dependence and regions of validity for quasi-distribution calculations

II. PDF fits

- on the other hand, the most robust means of determining PDFs from experimental data is the technique of **QCD global analysis**

→ performing full QCD fits, especially in the absence of actual data is a time-consuming and arduous task

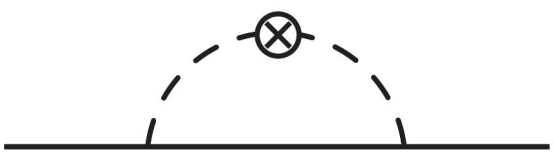
→ alternatively, a new technology **PDFSENSE** for quickly assessing the sensitivity of data to experimental (pseudo-)data might be applied to EIC-produced mesons or spectator tagging

arXiv:1803.02777

Part I: LaMET and the pion structure function

- via the **Sullivan process**, the structure of the interacting nucleon receives important contributions from pionic modes – the *pion cloud* :

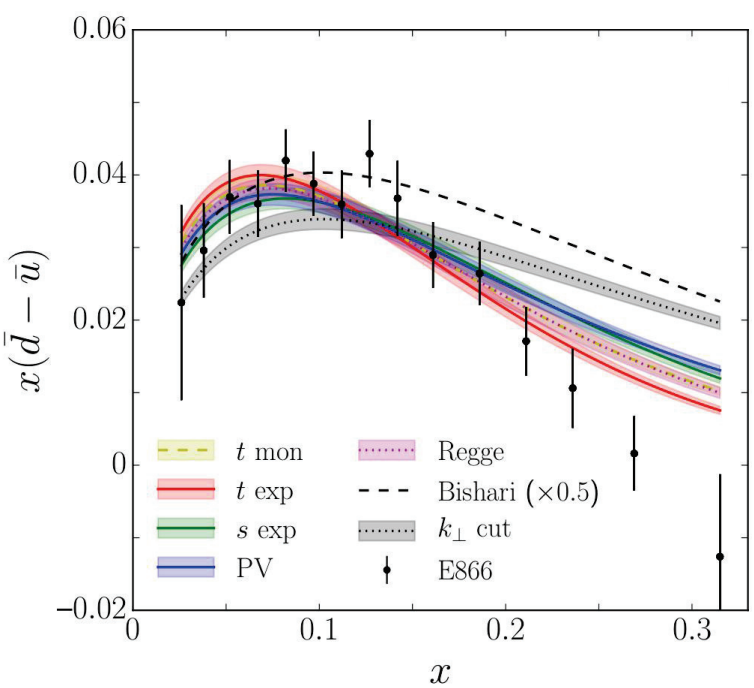
$$Z_{bare} \sim 1 - P_{N\pi} - P_{\Delta\pi} \qquad \begin{cases} P_{N\pi} \sim 0.20 - 0.25 \\ P_{\Delta\pi} \sim 0.05 - 0.10 \end{cases}$$



S. Theberge *et al.*, Phys. Rev. D**22**, 2838 (1980).

- this has implications for parton distributions; esp., light **flavor asymmetries**:

J. R. McKenney *et al.*, Phys. Rev. **D93**, 054011 (2016).



$$\bar{d} - \bar{u} = \left(f_{\pi^+ n} - \frac{2}{3} f_{\pi^- \Delta^{++}} \right) \otimes \bar{q}_v^{\pi}$$

hadronic splittings, à la χ_{PT}

pion valence PDF

- **can we better determine the pion structure func^n?**
  - new **measurements** (SeaQuest, JLab TDIS)
  - expanded QCD **global analyses**
  - direct calculation on the **Lattice**

formalism: LaMET and the pion structure function

- knowledge of the pion structure function is crucial to unraveling the nucleon's light quark sea (e.g., $\bar{d} - \bar{u}$); LaMET techniques may open this quantity to Lattice QCD

TJH, Phys. Rev. D**97** (2018) no.5, 054028.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-i\xi^- k^+} \langle p \left| \overline{\psi}(\xi^-)\gamma^+ \mathcal{U}(\xi^-,0)\psi(0) \right| p \rangle$$

...while matrix elements for lightlike correlations are not accessible on a Euclidean Lattice, *quasi-PDFs* are:

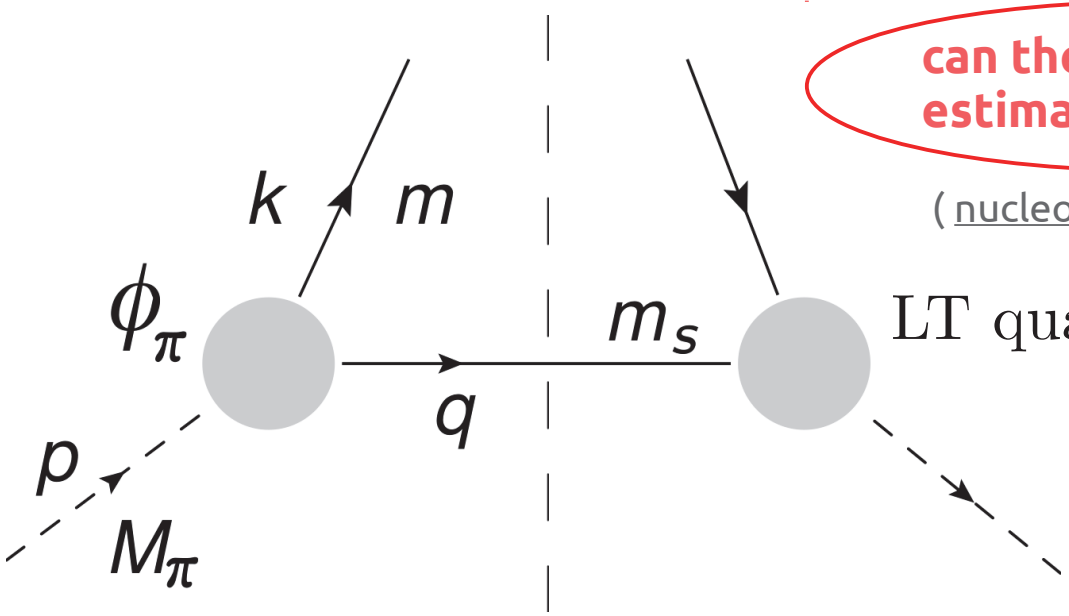
Ji, PRL**110**, 262002 (2013).

$$\widetilde{q}(x,\mu^2,p_z) = \int \frac{d\xi_z}{4\pi} e^{-i\xi_z k_z} \langle p \left| \overline{\psi}(\xi_z)\gamma^z \mathcal{U}(\xi_z,0)\psi(0) \right| p \rangle$$

these differ from the exact PDFs by power-suppressed corrections of order $\mathcal{O}\left(\frac{\Lambda^2}{p_z^2}, \frac{M^2}{p_z^2}\right)$

can these effects be estimated / controlled ?

(nucleon: Gamberg et al., PLB743 (2015) 112.)

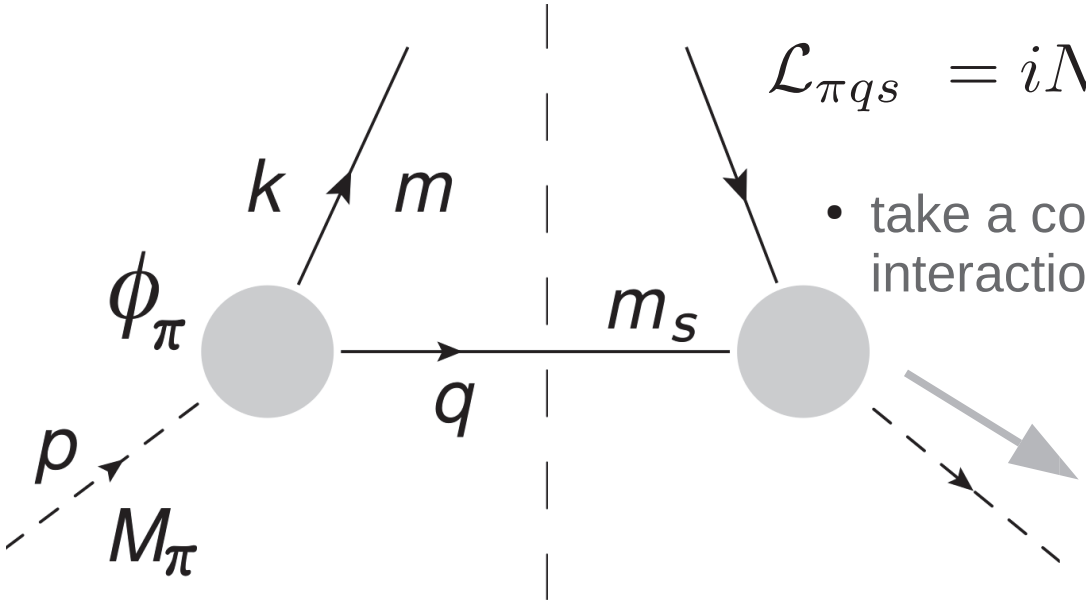


$$\text{LT quark}-\text{line cut} \longrightarrow \frac{\gamma^+}{2p^+}\,\delta\left(x - \frac{k^+}{p^+}\right)$$

$$\frac{\gamma^z}{2p_z}\,\delta\left(x - \frac{k_z}{p_z}\right)$$

the “exact” pion light-front PDF via a constituent quark model

- first evaluate the LF pion valence PDF using a minimal model that couples the pion to its constituent quarks



$$\mathcal{L}_{\pi q s} = i N_\pi^{1/2}\, \overline{\psi}_q \gamma_5 \varphi_\pi \psi_s + \text{h.c.}$$

- take a covariant vertex factor for the quark-pion interaction consistent with high x DSE results:

$$q_v(x) \sim (1-x)^2$$

$$\left[\phi_\pi(k^2)\right]^2 \equiv \left[\Lambda_\pi^2 \Big/ \left(k^2 - \Lambda_\pi^2\right)\right]^{3/2}$$

$$q_\pi^{\rm LF}(x) = \frac{N_\pi}{2\,(2\pi)^4} \int dk^+ dk^- d^2k_\perp \left(\frac{1}{2p^+}\right) \delta\left(x - \frac{k^+}{p^+}\right)$$

$$\times\; tr\Big(\gamma_5\,(\not{k}+m)\,\gamma^+(\not{k}+m)\gamma_5(-\not{q}+m_s)\Big)\; 2\pi\,\delta\big(q^2 - m_s^2\big) \left[\frac{\phi_\pi(k^2)}{(k^2-m^2)}\right]^2$$

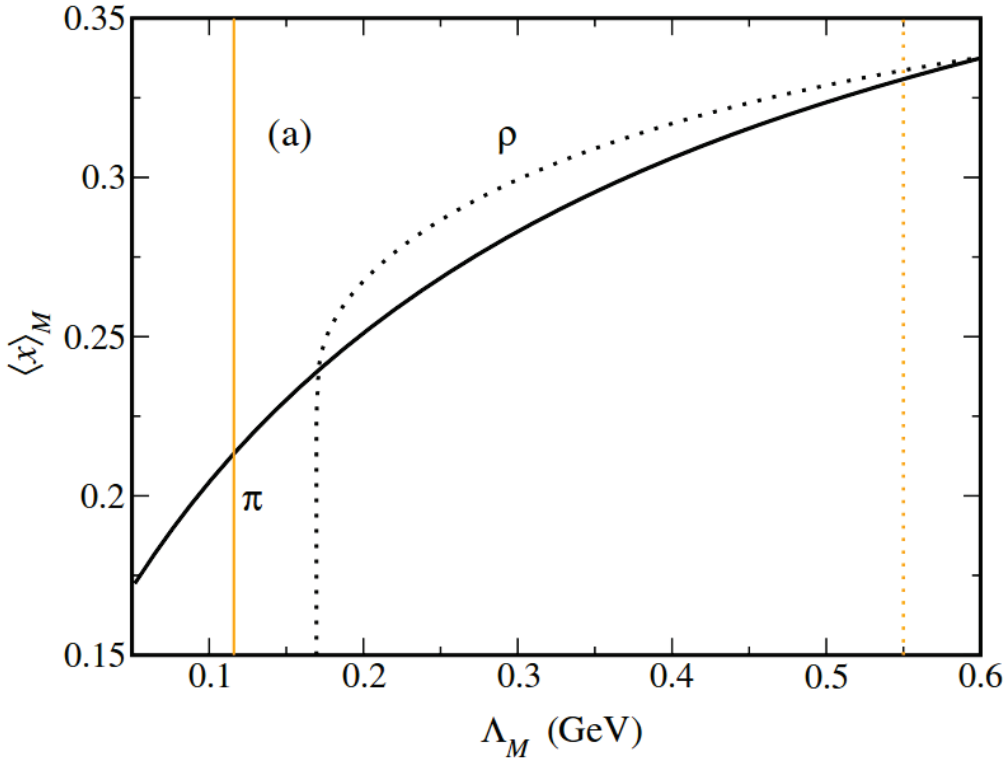
$$\boxed{q = p - k}$$

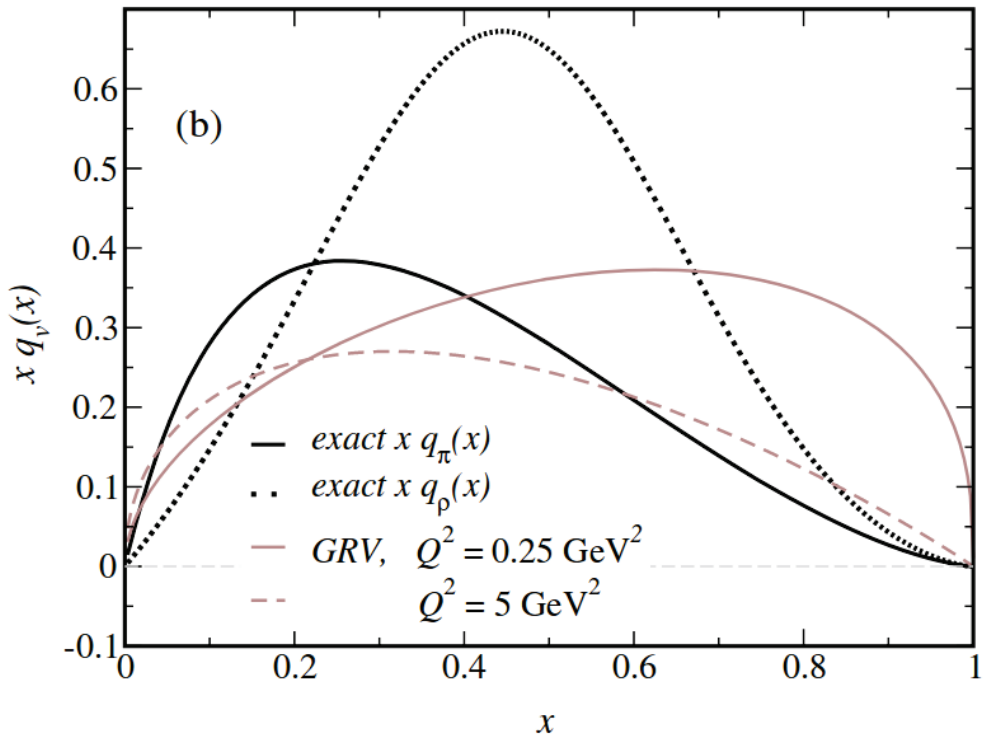
$$q_\pi^{\rm LF}(x) = \frac{N_\pi}{8\pi^2} \int \frac{dk_\perp^2}{x^2(1-x)} \left\{k_\perp^2 + \left(x m_s + (1-x)\,m\right)^2\right\} \left[\frac{\phi_\pi(t_\pi)}{(M_\pi^2 - \hat{s})}\right]^2$$

determining the pion SF model parameters

- for the pion, masses can be fixed to physical or constituent values:

$$M_\pi = 0.139\,\mathrm{GeV},\;\; m = M/3 \approx 0.33\,\mathrm{GeV}$$





$$\langle x \rangle_\pi \;=\; \int_0^1 dx\, x q_\pi^{\mathrm{LF}}(x) \;=\; 0.214(15)\left(^{+12}_{-9}\right)^{*} \;\longrightarrow\; \Lambda_\pi \;=\; 0.116\,\mathrm{GeV}$$

*LQCD 1st moment calculation: Abdel-Rehim et al., PRD**92**, 114513 (2015).

- the overall strength is set by a **normalization condition** such that the model is then completely determined

$$N_\pi \;=\; 1 \Big/ \int dx\, q_\pi^{\mathrm{LF}}(x)$$

the corresponding pion quasi-PDF may then be found:

$$\widetilde{q}_\pi(x,p_z) = \frac{N_\pi}{(2\pi)^4}\int dk^0 dk_z\, d^2k_\perp \left(\frac{1}{2p_z}\right)\delta\left(x-\frac{k_z}{p_z}\right)$$

$$\times\; tr\Big(\gamma_5\,(\not{k}+m)\,\gamma^z(\not{k}+m)\gamma_5(-\not{q}+m_s)\Big)\; 2\pi\,\delta\big(q^2-m_s^2\big)\left[\frac{\phi_\pi(k^2)}{(k^2-m^2)}\right]^2$$

- now, integrating delta functions introduces explicit dependence on p_z —

$$\delta\left(q^2-m_s^2\right) = \frac{1}{2\left(p^0-k^0\right)}\,\delta\left(p^0-k^0-\sqrt{m_s^2+k_\perp^2+(1-x)^2p_z^2}\right)$$

$$\widetilde{q}_\pi(x,p_z) = \frac{N_\pi}{4\pi^2}\int\frac{dk_\perp^2}{2(1-x)\mu_s}\left\{2x\left(mm_s+(\tilde{q}\cdot\tilde{k}_\pi)\right)+(m^2-\tilde{k}_\pi^2)\,(1-x)\right\}$$

$$\times\left[\frac{\phi_\pi(\tilde{k}_\pi^2)}{(M_\pi^2+m_s^2-m^2+2(1-x)\big(1-\mu_\pi\,\mu_s\big)}\right]^2$$

(the main result for the pion quasi-PDF)

$$\mu_\pi \equiv \sqrt{1+\frac{M_\pi^2}{p_z^2}}$$

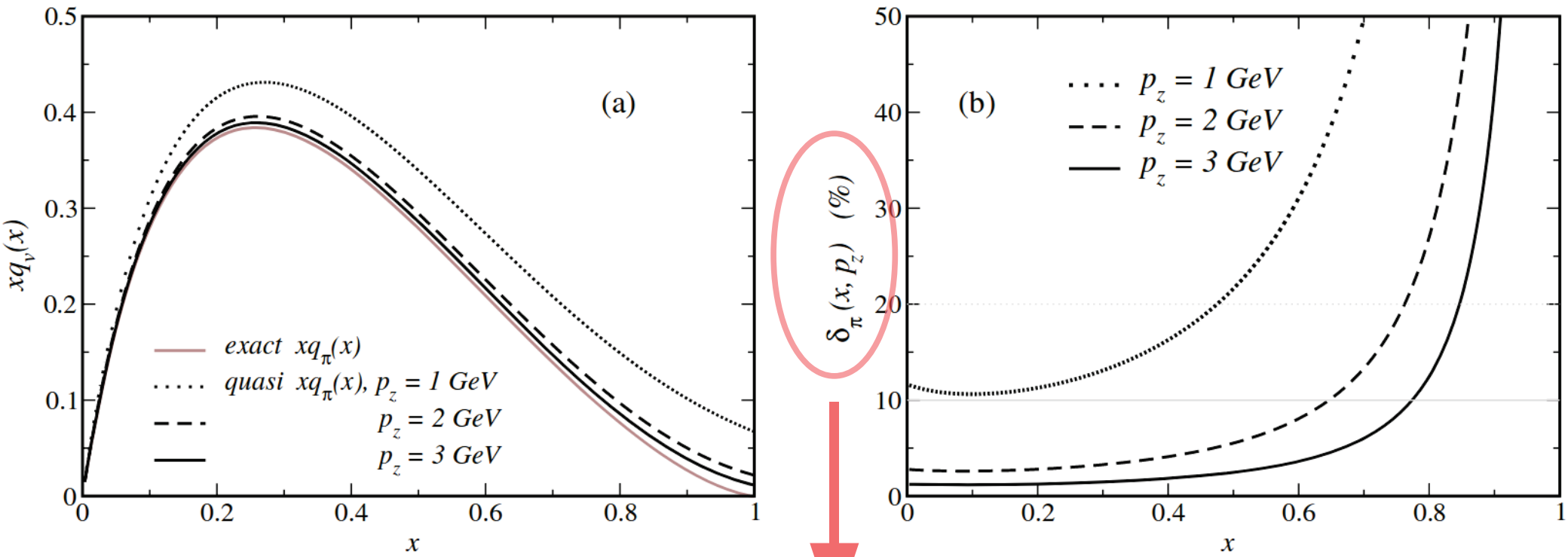
$$\mu_s \equiv \sqrt{1+\frac{m_s^2+k_\perp^2}{(1-x)^2p_z^2}}$$

$$\tilde{k}_\pi^2 = M_\pi^2+m_s^2+2\,(1-x)\big(1-\mu_\pi\,\mu_s\big)\,p_z^2$$

→ compare π quasi-/PDFs for several p_z

- we observe the expected behavior: at infinite boost, meson quasi-PDFs match onto the exact result,

$$\lim_{p_z \to \infty} \tilde{q}_M(x, p_z) = q_M^{\rm LF}(x)$$



- away from this limit, we compute the LaMET deviations from the LF PDF:

$$\delta_M(x, p_z) \equiv \frac{\tilde{q}_M(x, p_z)}{q_M^{\rm LF}(x)} - 1$$

→ even at fairly modest p_z these corrections can be $\lesssim 10\%$!

Part II: determining hadron PDFs from **QCD global fits** and visualizing the **sensitivity** of empirical data

- we want to determine from global analyses:

$$f_{q/p}(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-i\xi^- k^+} \langle p \left| \overline{\psi}(\xi^-)\gamma^+ \mathcal{U}(\xi^-,0)\psi(0) \right| p \rangle$$

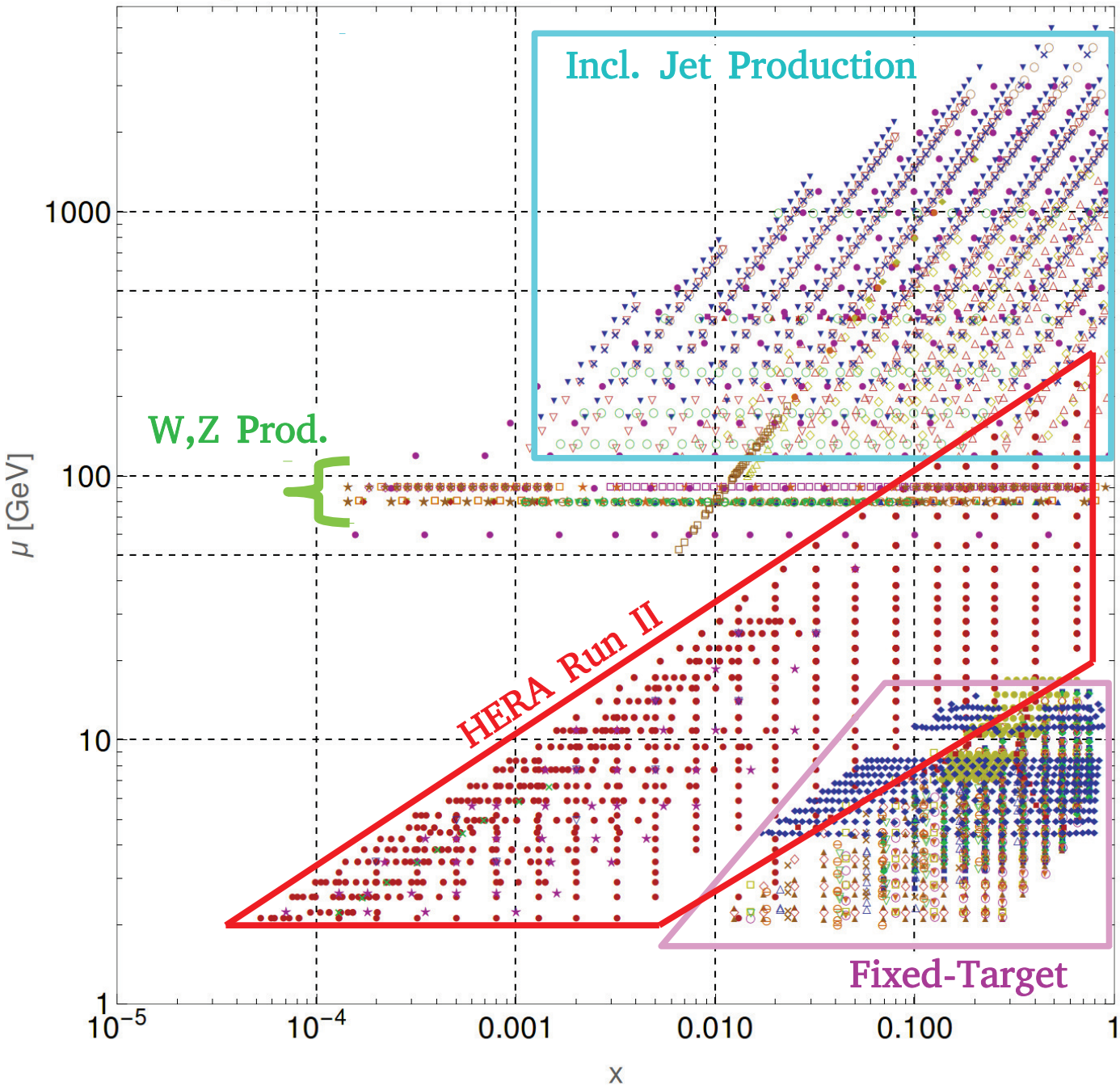
- in practice, this is achieved by leveraging our knowledge of pQCD and experimental data to constrain a flexible PDF parametrization:

$$f_{q/h}(x,Q_0^2) = a_{q_0} x^{a_{q_1}} (1-x)^{a_{q_2}} \; P[x, \{a_{qn-3}\}]$$

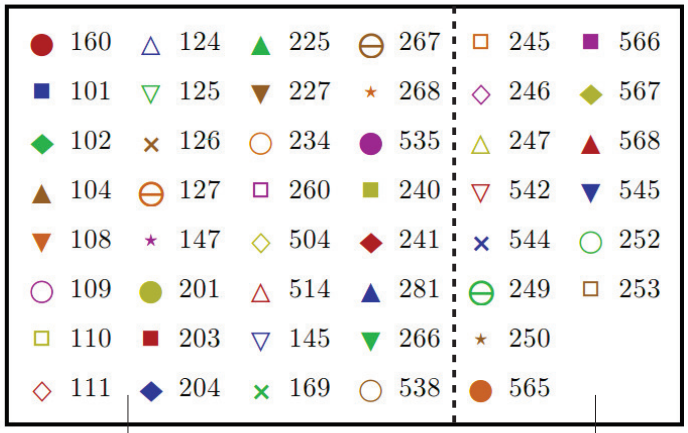
- this is, however, an extremely challenging undertaking!
  - global fits are time consuming/computationally expensive
  - **what are the highest impact data**? …and what theory is required?

- **might we instead identify data with the greatest sensitivity to PDF physics *before* fitting and direct global analyses accordingly?**

Experimental data in CTEQ-TEA set



the problem : modern global PDF analyses must contend with LARGE data sets involving many physical processes and channels!



individual experiments contributing to CT14 analysis of HERA Run II ("CT14H2")

new, unfitted datasets from, e.g., LHC at 7, 8 TeV

- Can we weigh the influence of datasets **WITHOUT** each time performing a full global analysis? **. . . we could then predict the impact of unfitted data and guide fits . . .**

the goal is to **quantify the strength of the constraints** placed on a particular set of PDFs by both individual and aggregated measurements *without direct fitting*

- for single-particle hadroproduction of gauge bosons at, e.g., LHC, factorization gives

$$\sigma(AB \to W/Z+X) = \sum_n \alpha_s^n(\mu_R^2) \sum_{a,b} \int dx_a dx_b$$

$$\times f_{a/A}(x_a, \mu^2)\, \hat{\sigma}^{(n)}_{ab\to W/Z+X}(\hat{s},\, \mu^2, \mu_R^2)\, f_{b/B}(x_b, \mu^2)$$

PDFs determined by fits to data; e.g., "CT14H2"

pQCD matrix elements – specified by theoretical formalism in a given fit

- *idea* : study the **statistical <u>correlation</u>** between PDFs and the quality of the fit at a measured data point(s); fit quality encoded in a (**T**heory) – (shifted **D**ata) ***residual*** :

$$r_i(\vec{a}) = \frac{1}{s_i}\left(T_i(\vec{a}) - D_{i,sh}(\vec{a})\right)$$

s_i : uncorrelated uncert.

$\vec{a}$: PDF parameters

a brief statistical aside

- the CTEQ-TEA global analysis relies on the **Hessian formalism** for its error treatment

$$\chi^2_E(\vec{a}) = \sum_{i=1}^{N_{pt}} r_i^2(\vec{a}) + \sum_{\alpha=1}^{N_\lambda} \overline{\lambda}_\alpha^2(\vec{a})$$

→ nuisance parameters to handle correlated errors

$$r_i(\vec{a}) = \frac{1}{s_i}\left(T_i(\vec{a}) - D_{i,sh}(\vec{a})\right)$$

these result in systematic shifts to data central values:

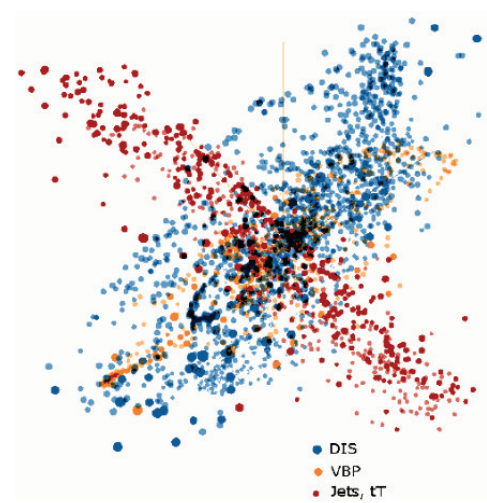
$$D_i \to D_{i,sh}(\vec{a}) = D_i - \sum_{\alpha=1}^{N_\lambda} \beta_{i\alpha}\overline{\lambda}_\alpha(\vec{a})$$

- a 56-dimensional parametric basis $\vec{a}$ is obtained by diagonalizing the Hessian matrix H determined from χ^2

→ use this basis to compute 56-component "normalized" residuals :

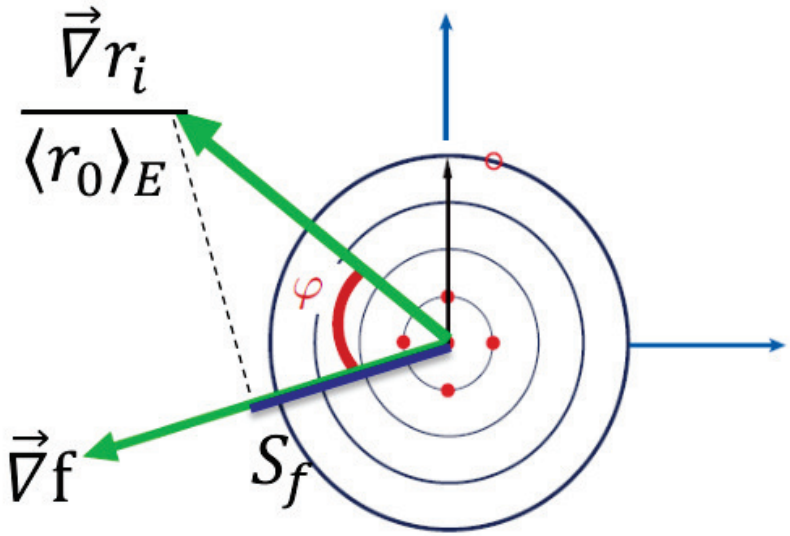
$$\delta^{\pm}_{i,l} \equiv \left(r_i(\vec{a}^{\pm}_l) - r_i(\vec{a}_0)\right) / \langle r_0 \rangle_E$$

where $\langle r_0 \rangle_E \equiv \sqrt{\frac{1}{N_{pt}} \sum_{i=1}^{N_{pt}} r_i^2(\vec{a}_0)}$



Correlation C_f and sensitivity S_f

The relation of data point i on the PDF dependence of f can be estimated by:



- $C_f \equiv \mathrm{Corr}[\rho_i(\vec{a})), f(\vec{a})] = cos\varphi$

$\vec{\rho}_i \equiv \vec{\nabla} r_i / \langle r_0 \rangle_E$ -- gradient of r_i normalized to the r.m.s. average residual in expt E;

$$\left(\vec{\nabla} r_i\right)_k = \left(r_i(\vec{a}_k^+) - r_i(\vec{a}_k^-)\right)/2$$

$$\mathrm{Corr}[X, Y] = \frac{1}{4\Delta X \Delta Y} \sum_{j=1}^{N} (X_j^+ - X_j^-)(Y_j^+ - Y_j^-)$$

C_f is **independent** of the experimental and PDF uncertainties. In the figures, take $|C_f| \gtrsim 0.7$ to indicate a large correlation.

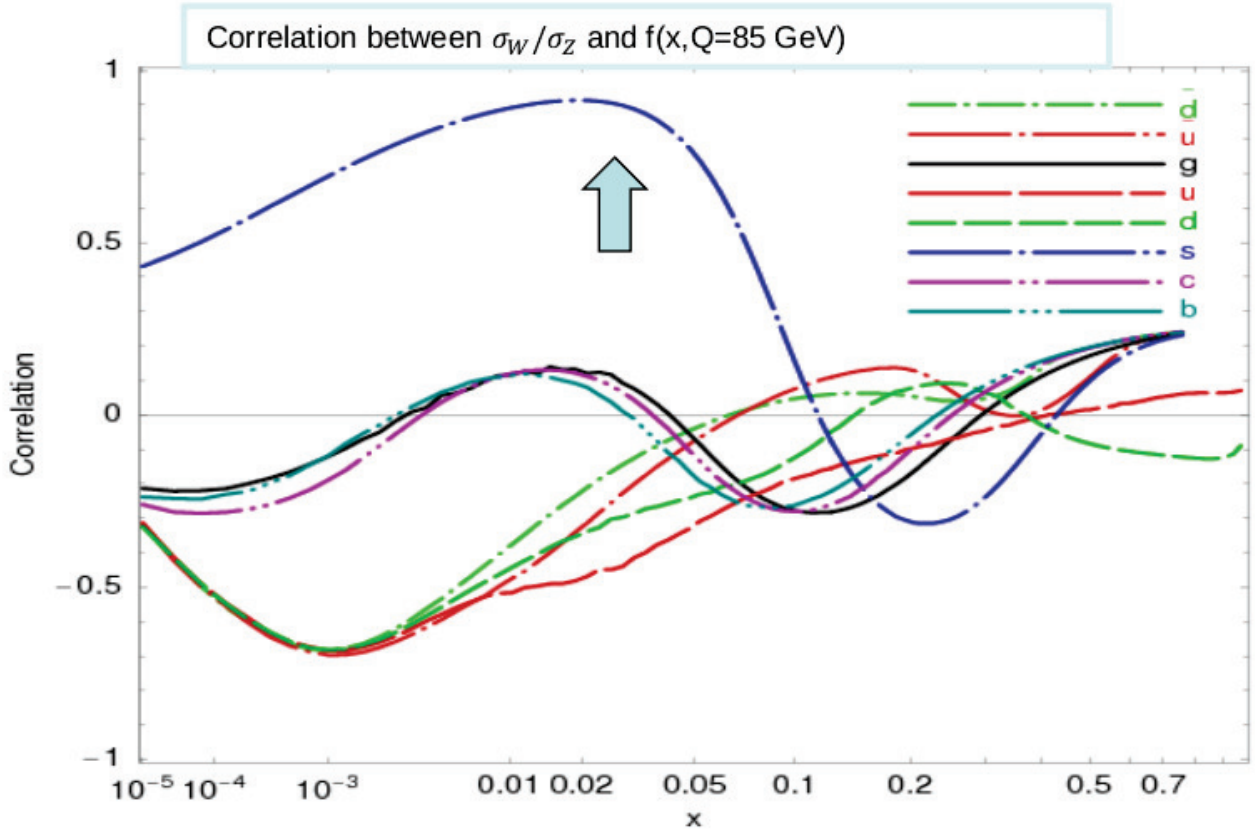
- $S_f \equiv |\vec{\rho}_i| cos\varphi = C_f \frac{\Delta r_i}{\langle r_0 \rangle_E}$ -- projection of $\vec{\rho}_i(\vec{a})$ on $\vec{\nabla} f$

S_f is proportional to $cos\varphi$ and the ratio of the PDF uncertainty to the experimental uncertainty. We can sum $|S_f|$.

In the figures, take $|S_f| > 0.25$ to be significant.

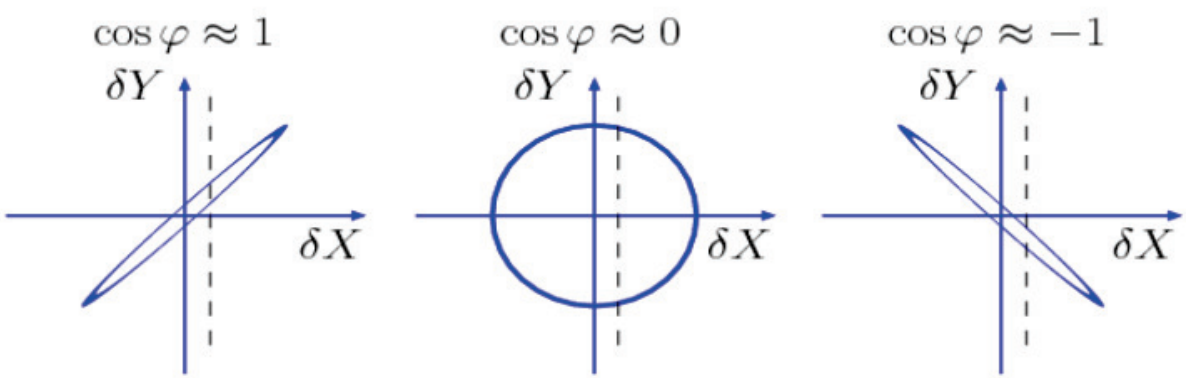
...we may turn to the Pearson correlations between PDFs and δ_i, but we first note

Correlations carry useful, but limited information



CTEQ6.6 [arXiv:0802.0007]: $\cos\varphi > 0.7$ shows that the ratio σ_W/σ_Z at the LHC must be sensitive to the strange PDF $s(x,Q)$

$\cos\varphi \approx \pm 1$ suggests that a measurement of X **may** impose tight constraints on Y



But, Corr[X,Y] between **theory** cross sections X and Y does not tell us about **experimental** uncertainties

2nd aside: **kinematical matchings**

- residual-PDF correlations and sensitivities are evaluated at parton-level kinematics determined according to leading-order matchings with physical scales in measurements

$$x_i \;:\; \text{parton mom. fraction}$$

$$\mu_i \;:\; \text{factorization scale}$$

deeply-inelastic scattering:

$$\mu_i \approx Q|_i\,,\;\; x_i \approx x_B|_i$$

hadron-hadron collisions:

$$AB \to CX \qquad \mu_i \approx Q|_i\,,\;\; x_i^{\pm} \approx \frac{Q}{\sqrt{s}}\exp(\pm y_C)\Big|_i$$

single-inclusive jet production: $Q = 2p_{Tj},\;\; y_C = y_j$

$t\bar{t}$ pair production: $Q = m_{t\bar{t}},\;\; y_C = y_{t\bar{t}}$

etc...

$d\sigma/dp_T^Z$ measurements: $Q = \sqrt{(p_T^Z)^2 + (M_Z)^2},\;\; y_C = y_Z$

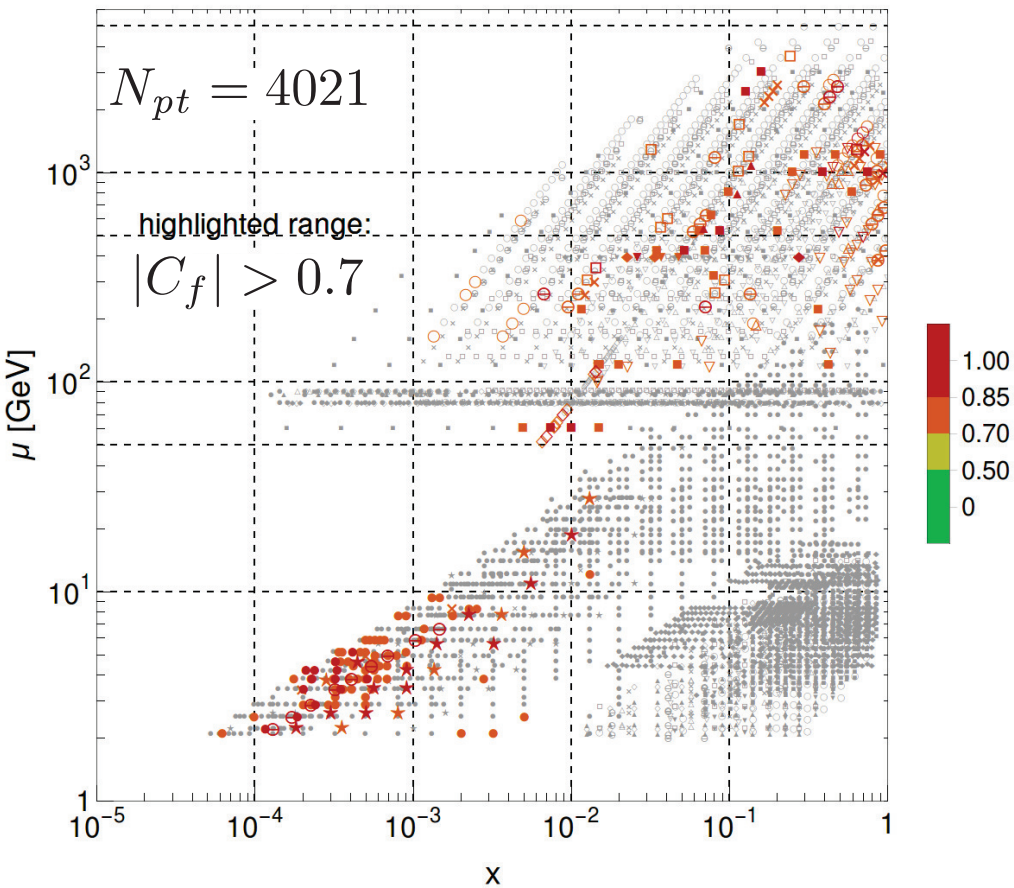
the sensitivity reveals a richer landscape than the correlation!

$$C_f(x_i, \mu_i) \equiv \mathrm{Corr}[f(x_i, \mu_i), r_i]$$

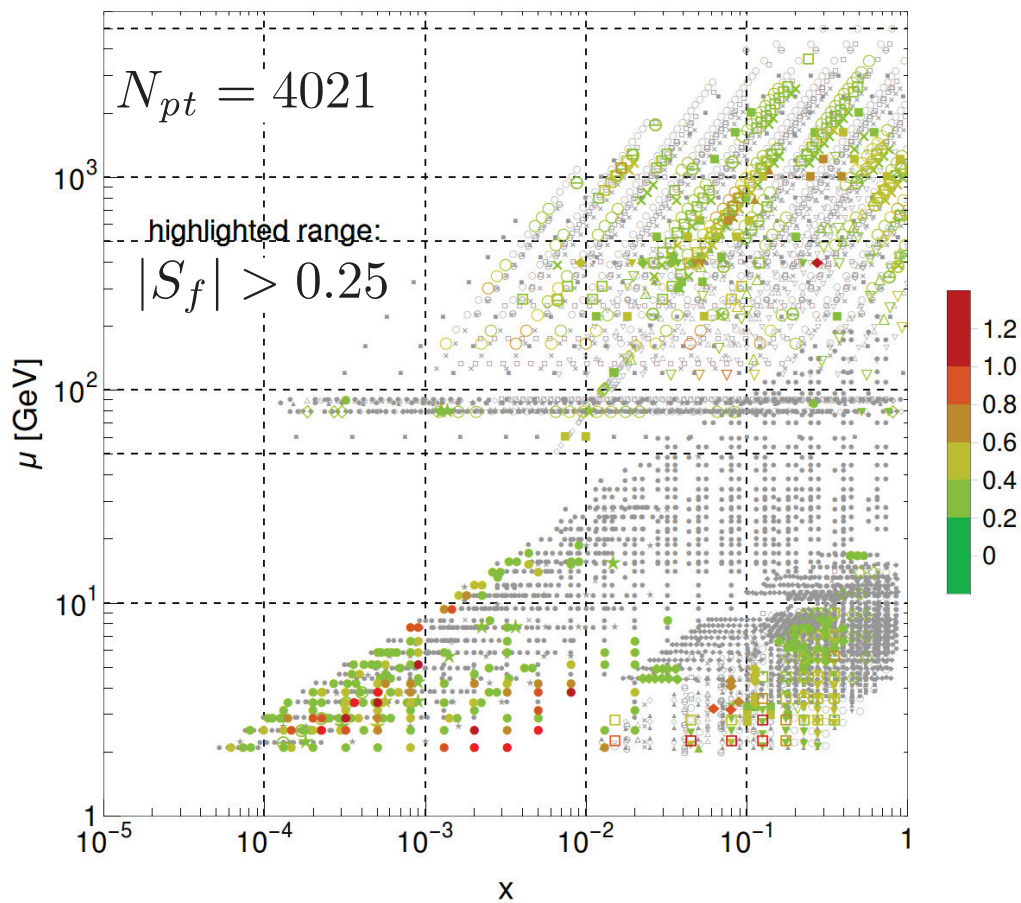
- broad outlays of the experimental parameter space are shown to be sensitive, but are **missed by the correlation**

$$S_f(x_i, \mu_i) \equiv \frac{\delta^{(\mathrm{PDF})} r_i}{\sqrt{\frac{1}{N}\sum_{i=1}^{N} r_i^2}} C_f(x_i, \mu_i)$$

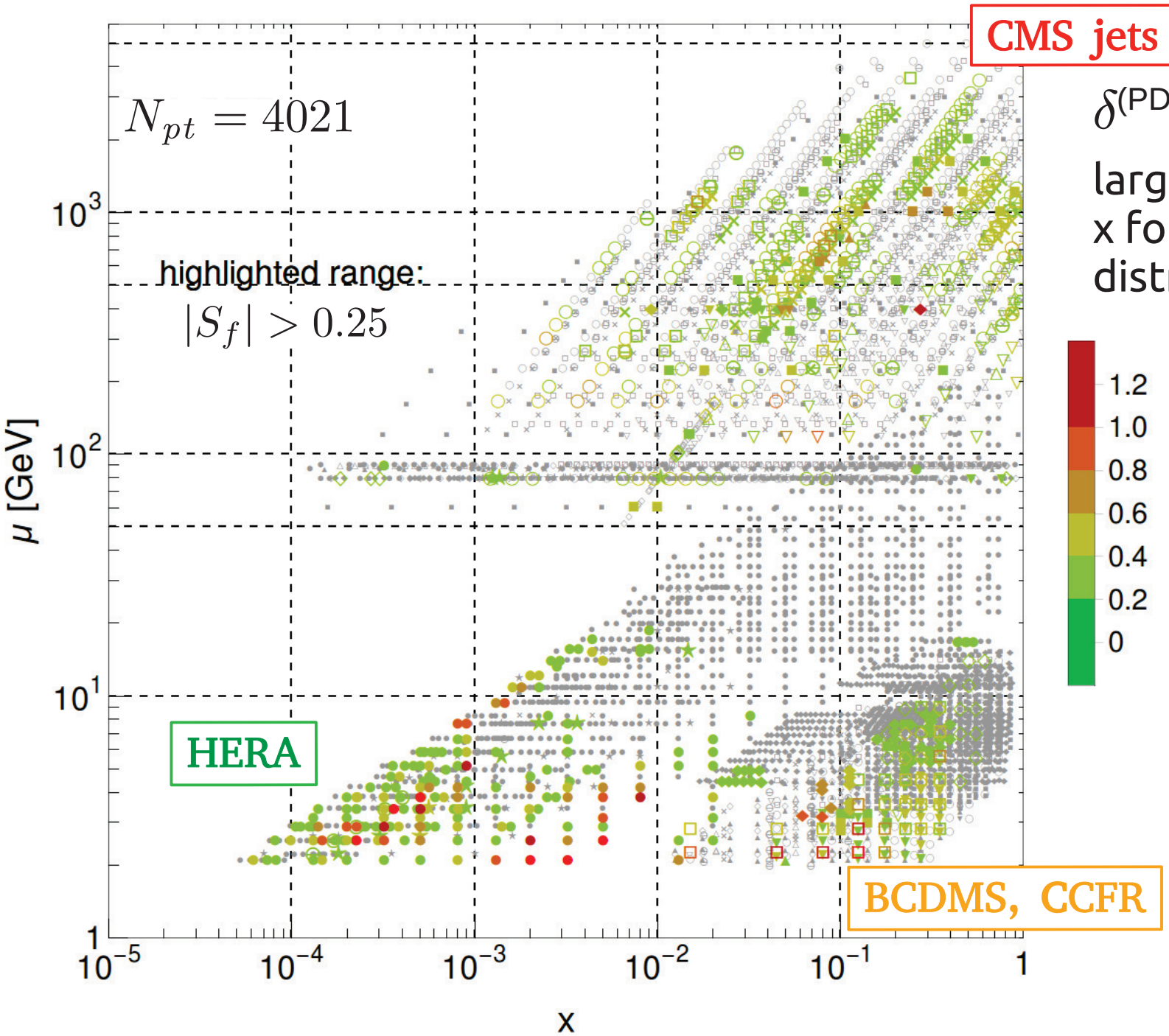
| C_f | for g(x,μ), CT14HERA2NNLO



| S_f | for g(x,μ), CT14HERA2NNLO



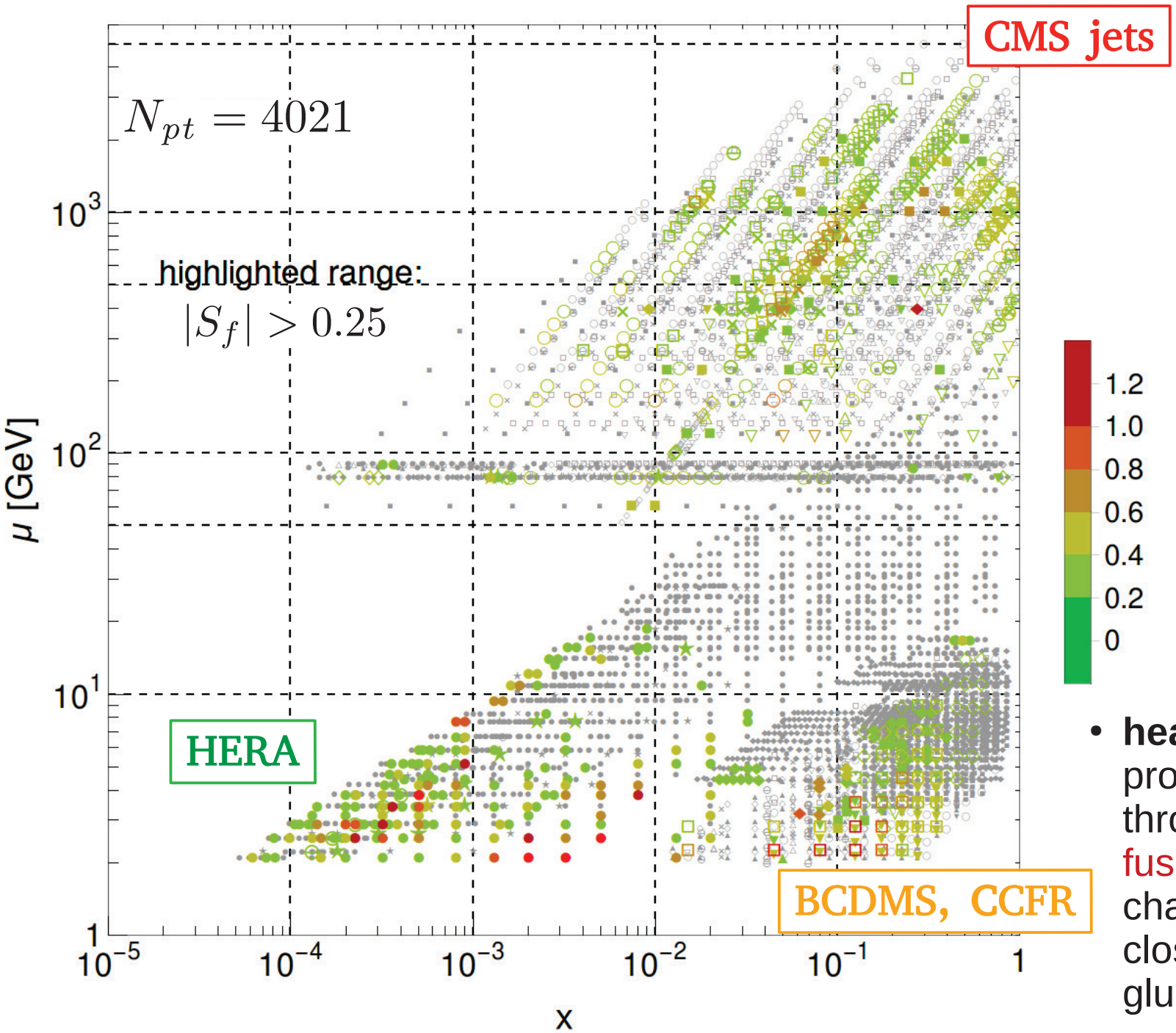
| S_f | for g(x,μ), CT14HERA2NNLO

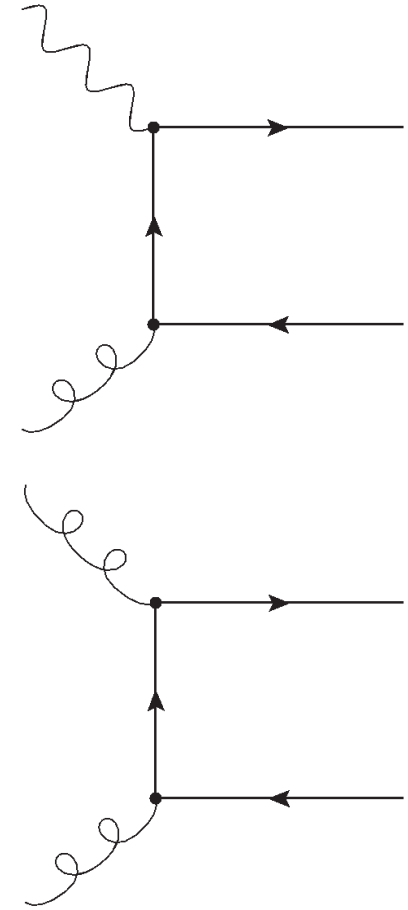


$\delta^{(\mathrm{PDF})} r_i$ relatively large at high and low x for the gluon distribution

- measurements at high/low x are therefore most sensitive; esp. Run 2 **HERA** points and **CMS** jet production at 7, 8 TeV

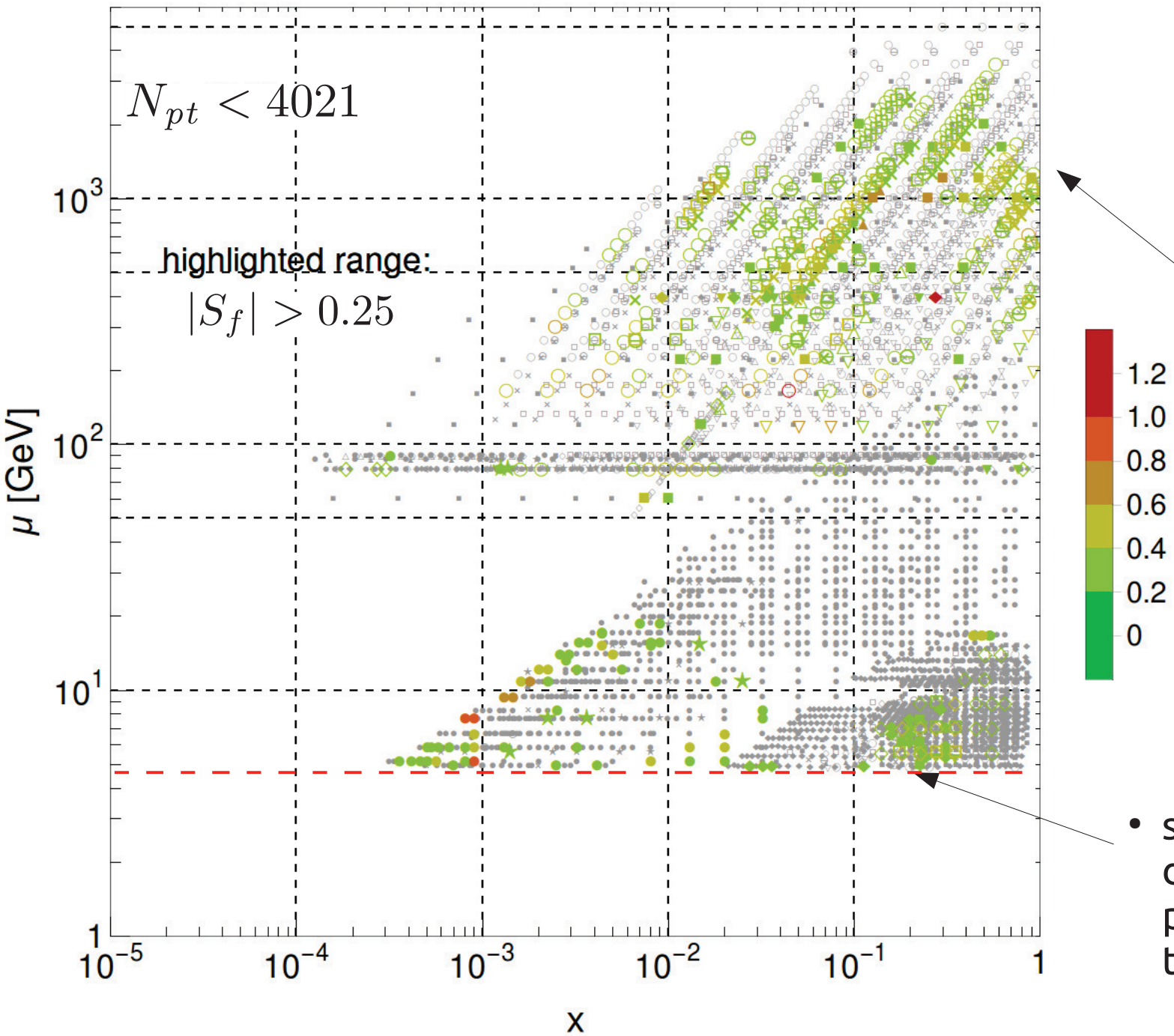
| S_f | for c(x,μ), CT14HERA2NNLO





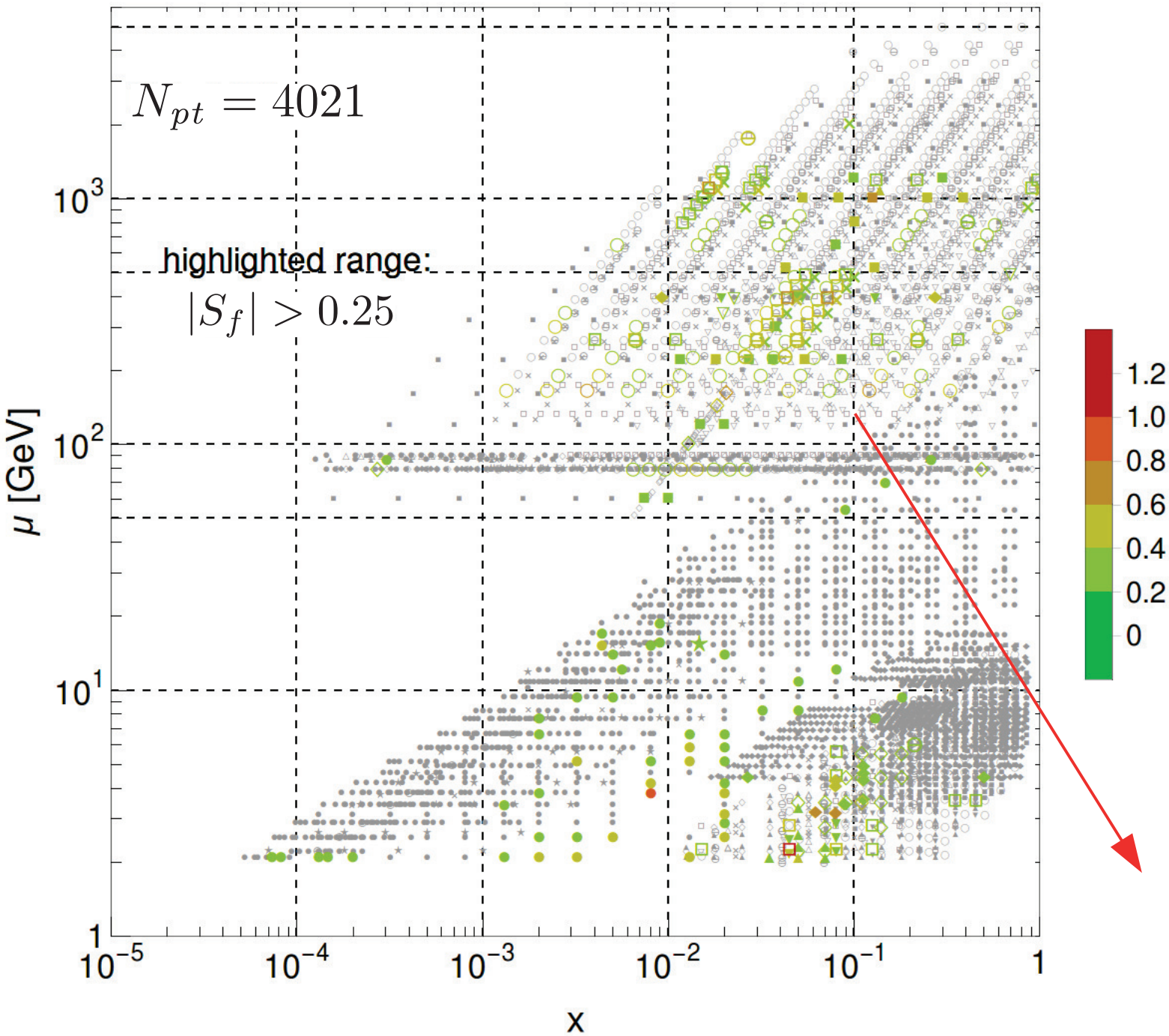
- **heavy quark** production proceeds through boson fusion diagrams: charm sensitivities closely track the gluon plot

| S_f | for b(x,μ), CT14HERA2NNLO



- sensitivities broadly follow the pattern set by the charm quark distributions
- especially strong sensitivity at large x from inclusive jet production: **7 TeV and 8 TeV CMS points**
- sensitivities are only defined above the parton-level threshold, $\mu > m_b$

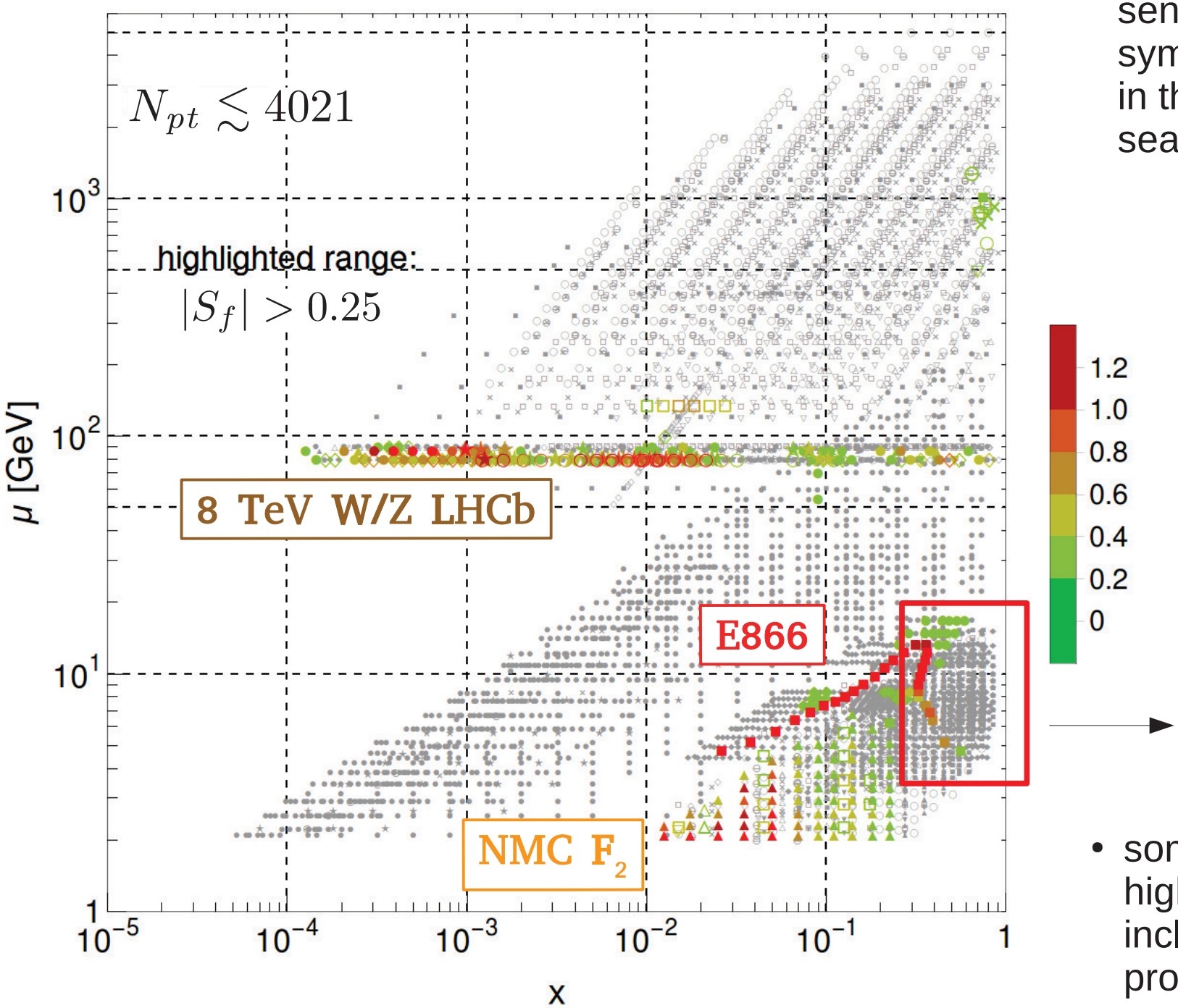
$|S_f|$ for σ_{H^0} 14 TeV, CT14HERA2NNLO



- several processes (high p_T Z prod., top prod.) have been suggested as providing leverage on the Higgs cross section

- in fact, we find **inclusive jet production** to have the broadest overall sensitivity!

$|S_f|$ for $\overline{d}/\overline{u}(x,\mu)$, CT14HERA2NNLO



- PDF ratio is sensitive to flavor symmetry breaking in the light quark sea
- the large E866 sensitivity *degrades* at larger x

this is a prime motivation for higher x DY measurements at **E906** (SeaQuest)

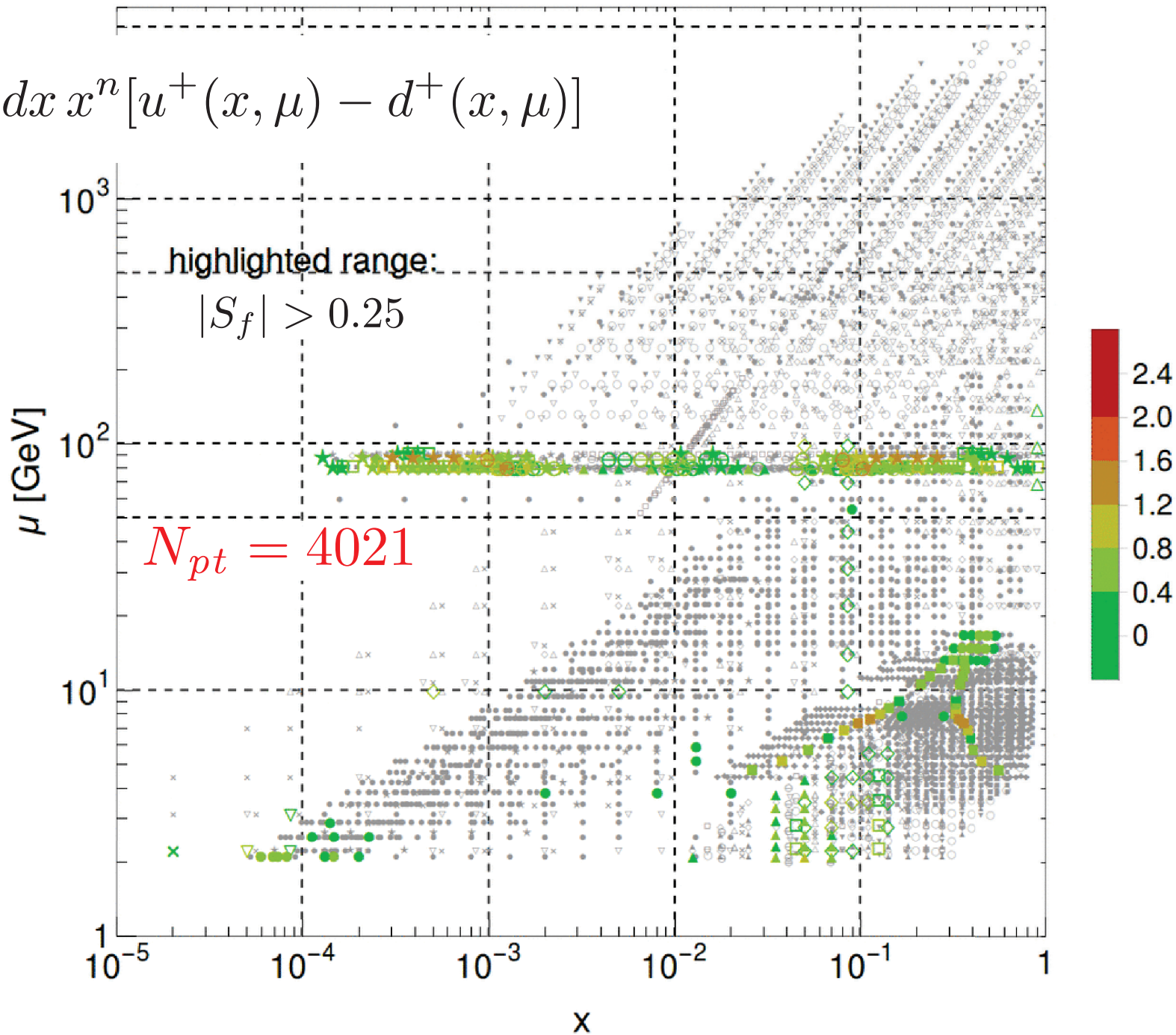
- some contribution at high x from CMS inclusive jet production

| S_f | for <x^−1>u+−d+, CT14HERA2NNLO

$$M^n_{u^+-d^+} = \int_0^1 dx\, x^n [u^+(x,\mu) - d^+(x,\mu)]$$

- 1st several PDF moments can be computed on the **lattice**; knowledge of sensitivities suggests where lattice calculations might constrain PDFs

- higher moments shift sensitivities *rightward*



Ranking tables

… to assess the impact of separate experiments

No.	Exp. ID	N_{pt}	$\sum_f \lvert S_f^E \rvert$	$\langle \sum_f \lvert S_f^E \rvert \rangle$	Rankings $\lvert S_{\bar{d}}^E \rvert$	$\langle \lvert S_{\bar{d}}^E \rvert \rangle$	$\lvert S_{\bar{u}}^E \rvert$	$\langle \lvert S_{\bar{u}}^E \rvert \rangle$	$\lvert S_g^E \rvert$	$\langle \lvert S_g^E \rvert \rangle$	$\lvert S_u^E \rvert$	$\langle \lvert S_u^E \rvert \rangle$	$\lvert S_d^E \rvert$	$\langle \lvert S_d^E \rvert \rangle$	$\lvert S_s^E \rvert$	$\langle \lvert S_s^E \rvert \rangle$
1	160	1120.	620.	**HERA**			**A**	3	**A**	3	**A**	3	B		C	
2	**545**	185	232.	**CMS jets 8**	3		C	3	B	2					C	3
3	111	86	218.	CCF3 F3p	**1**		C	**1**		3	B	**1**	C	2		
4	**542**	158	194.	**CMS jets 7**	3		C	3	B	2					C	3
5	101	337	184.	BCDMS F2p			C		C		B	3	C			
6	104	123	169.	NMC	2						C	2	B	2		
7	102	250	141.	BCDMS F2d					C	3	C	3	C	3		
8	109	96	115.	CDHSW	2		C	2		3	C	2	C	3		
9	201	119	113.	E605	2		C	2				3				

Experiments are listed in the descending order of the summed sensitivities to $\bar{d}, \bar{u}, g, u, d, s$

For each flavor, A and 1 indicate the strongest total sensitivity and strongest sensitivity per point

C and 3 indicate marginal sensitivities; low sensitivities are not shown

41	**247**	8	5.84	Z pT 7 TeV	3		3		3		3		3			
42	169	9	3.99	HERA F_L					2							
43	**567**	7	3.9	$t\bar{t}$					2							
44	227	11	3.7	CDF WASY (2005)									3			
45	**568**	5	3.4						2							
46	**566**	5	3.19	$t\bar{t}$					2							
47	145	10	1.14	HERA b												

Good per-point S_f , small N_{pt}

illustration: a high energy EIC, "**LHeC**"

- an electron-proton (or electron-ion) collider to achieve **high luminosities** $\gtrsim 1000$ times that of HERA

  → access a wide range of x, including $x \simeq 10^{-6}$

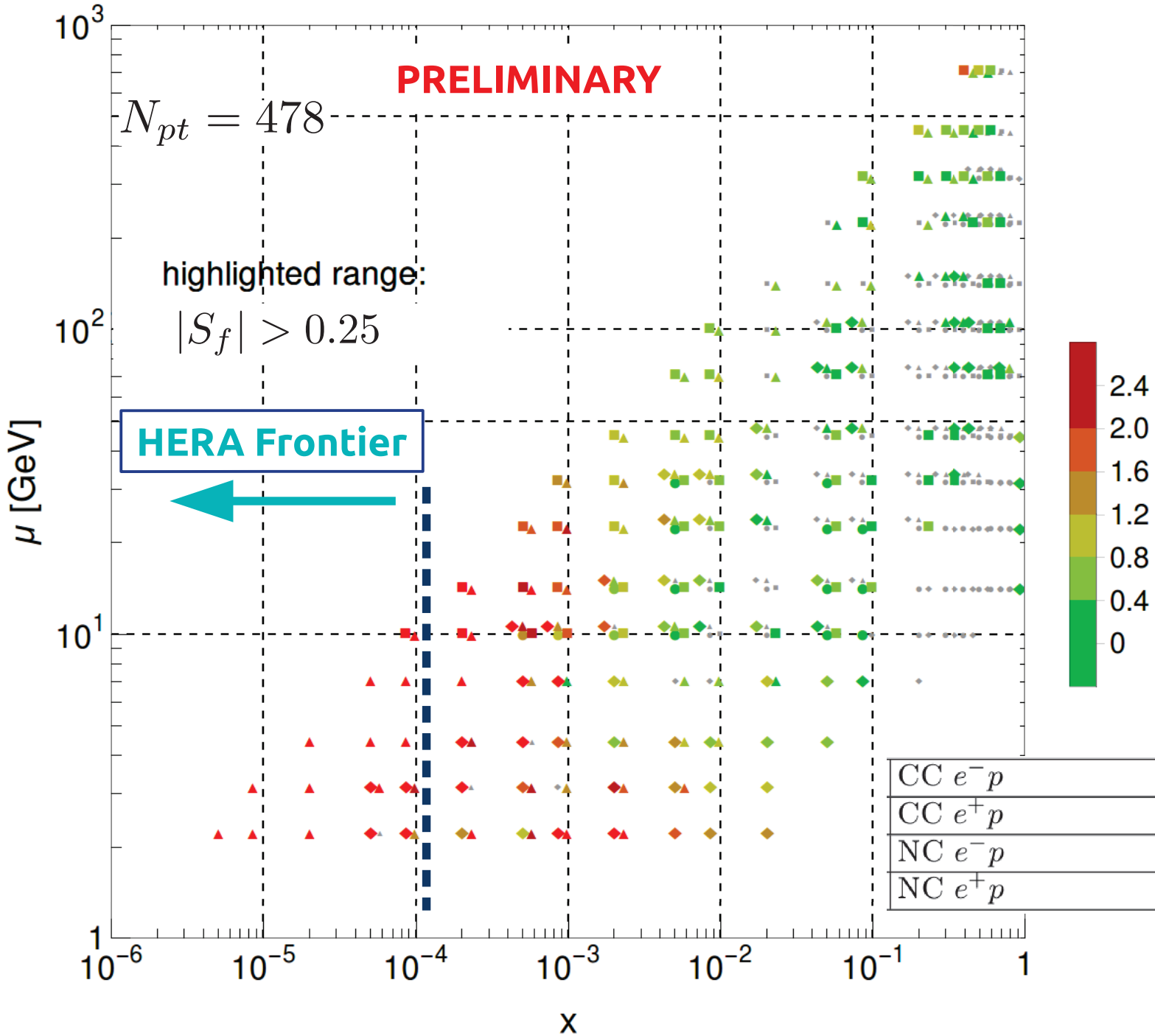
  → explore the dynamics of gluon saturation; greatly improve PDF precision; perform SM tests; and many other physics goals

- can perform a sensitivity analysis of Monte Carlo generated ep reduced NC/CC cross sections (Klein & Radescu, LHeC-Note-2013-002 PHY)

$$60\,\mathrm{GeV}\ e^{\pm}\ \mathrm{on}\ 1\ \mathrm{or}\ 7\,\mathrm{TeV}\ p$$

- to minimize the impact of large χ^2 of unfitted data (especially at low x), we study the sensitivities for **fluctuated data** – i.e., pseudodata randomly fluctuated about the CT14 prediction according to putative LHeC uncorrelated errors – based on **10 fb^{-1}** of data from a hypothetical **year of data-taking**

| S_f | for g(x,μ), CT14H2



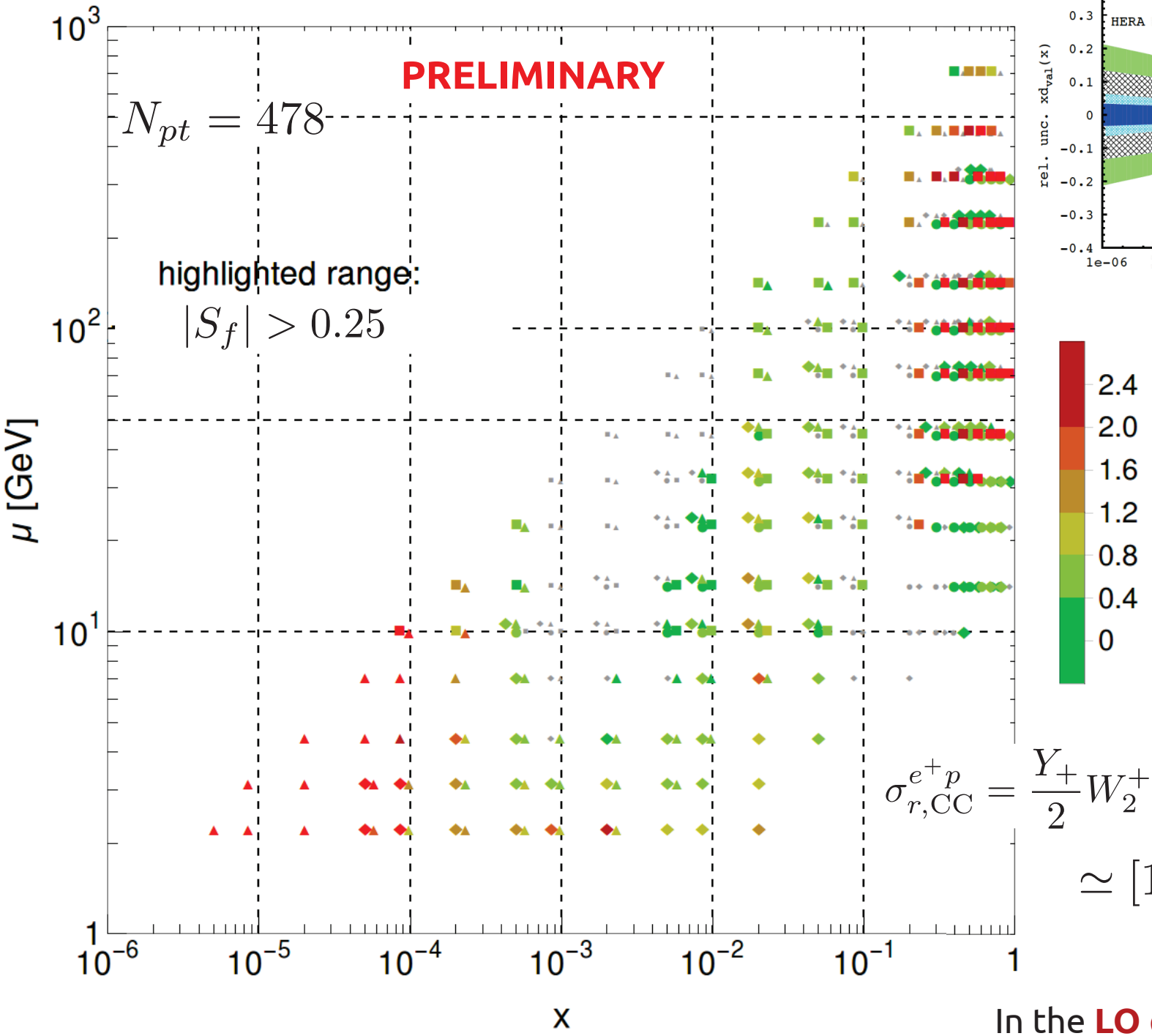
- extremely strong sensitivities along the frontier of the HERA Run II data, $x \lesssim 10^{-4}$

however, there are stringent constraints in general for $x \lesssim 10^{-2}$

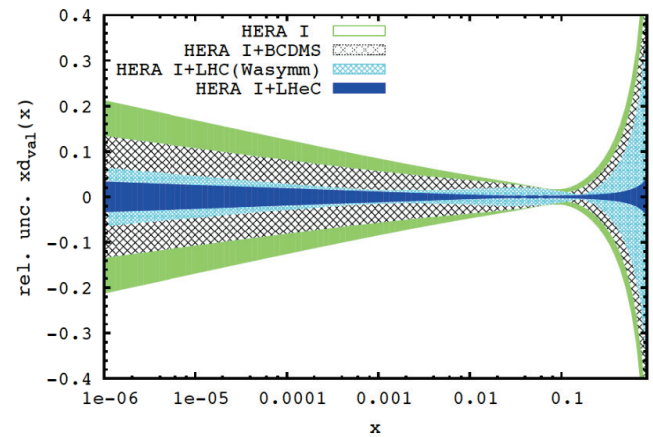
...and high x, μ

	N_d	
CC e^-p	93	circle
CC e^+p	109	square
NC e^-p	128	diamond
NC e^+p	148	triangle

| S_f | for d(x,μ), CT14H2



EIC Whitepaper:1206.2913



- The **strongest sensitivities** are realized for those regions with the **weakest current PDF constraints**; here, very high and very low x

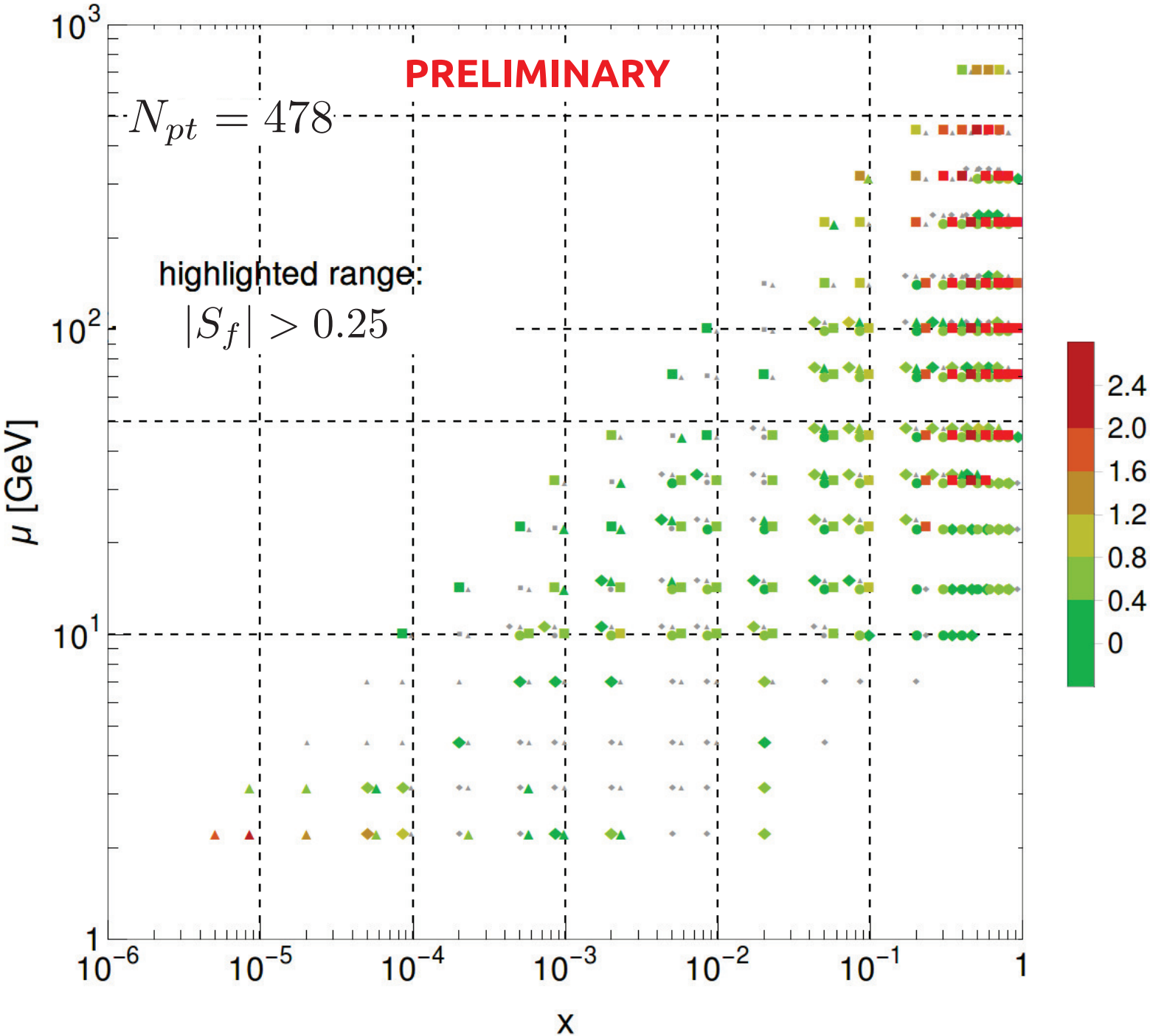
for $x \to 1$

$$\sigma^{e^+p}_{r,\mathrm{CC}} = \frac{Y_+}{2} W_2^+ \mp \frac{Y_-}{2} x W_3^+ - \frac{y^2}{2} W_L^+$$

$$\simeq [1-y]^2\, x(d+s)$$

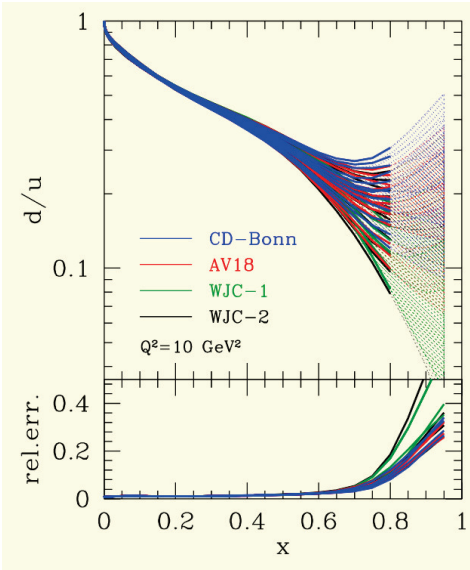
In the **LO quark-parton model**

| S_f | for d/u(x,mu), CT14H2



- LHeC's high luminosity may give it a reach to high enough x to help resolve the stubborn d/u question

…without a nuclear target…



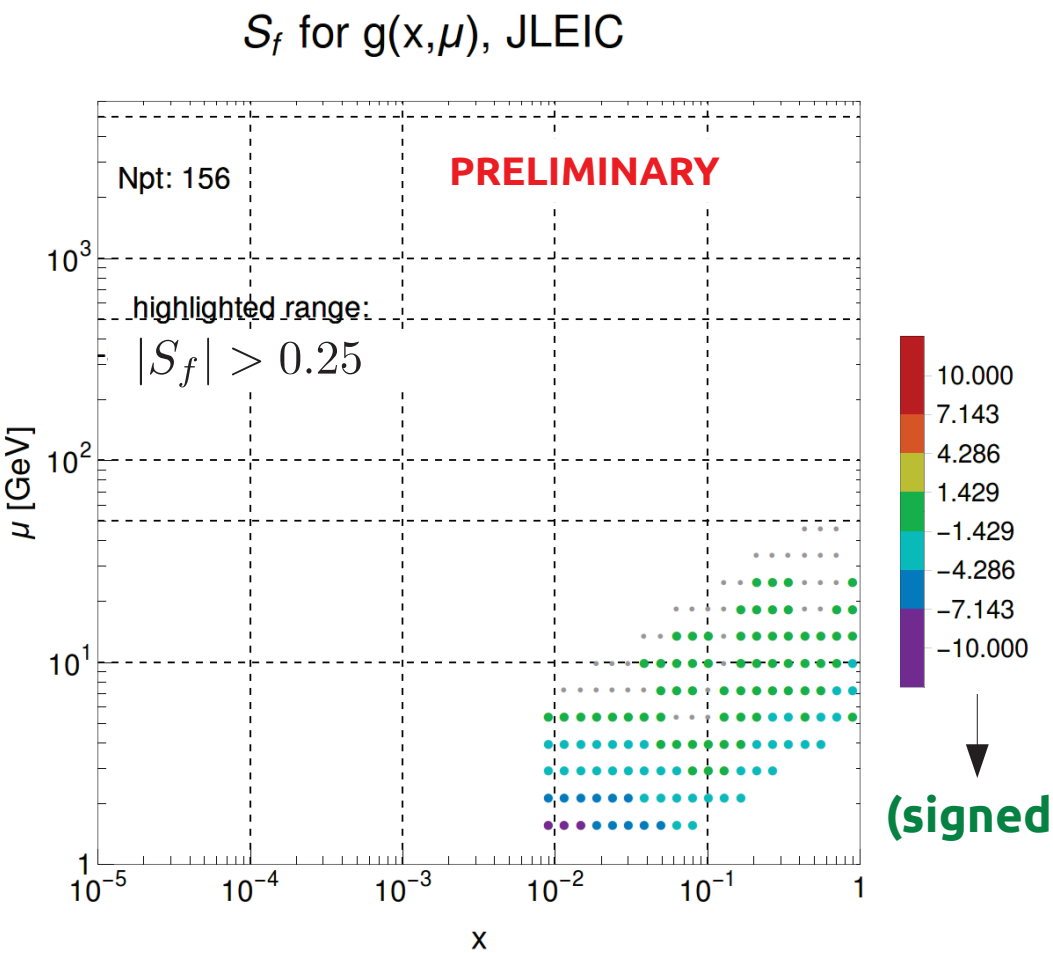
J.F. Owens, PAVI11

→ may apply these techniques to nucleon PDFs at **JL-EIC** !

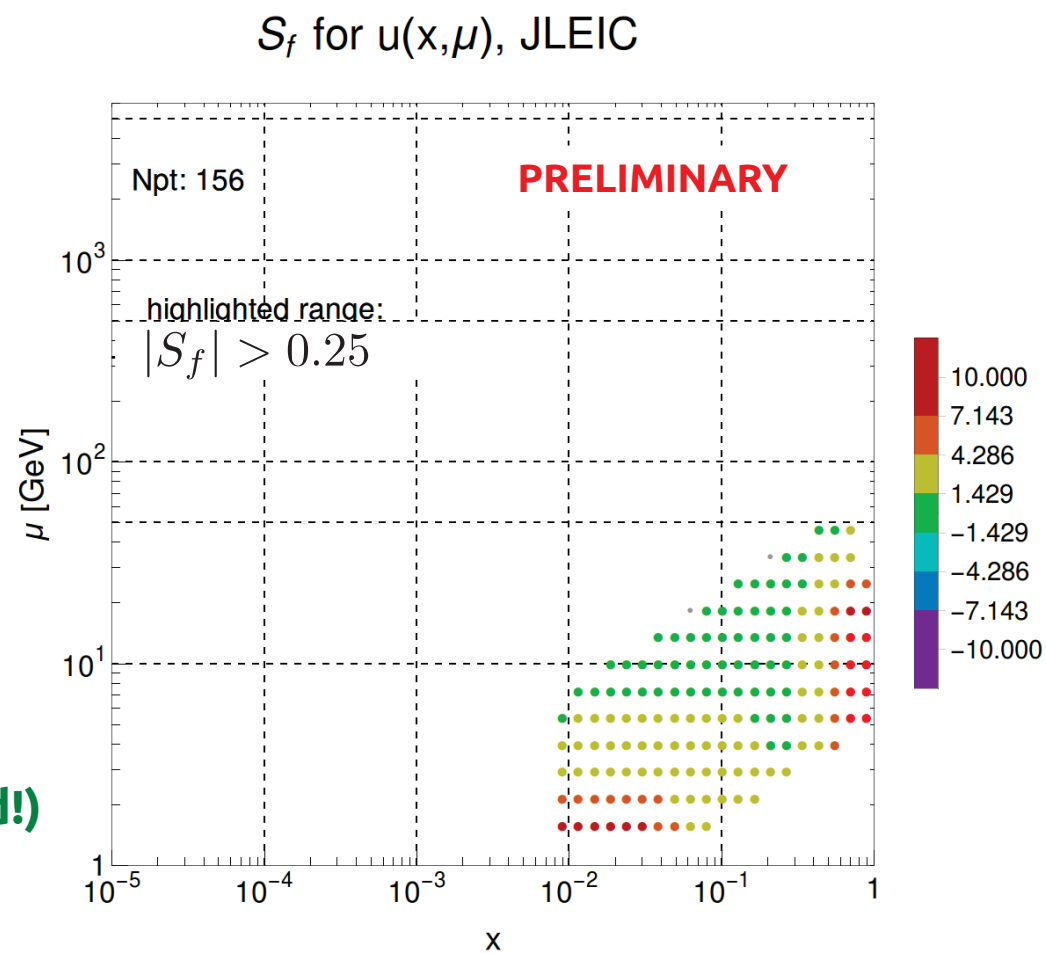
- **preliminarily**, we consider the highest energy NC pseudo-data

$$(10\,\mathrm{GeV}\ e^- \text{ on } 100\,\mathrm{GeV}\ p)$$

- NB: events were generated for a presumed decade-long run, L = **100 fb^{-1}**



(signed!)

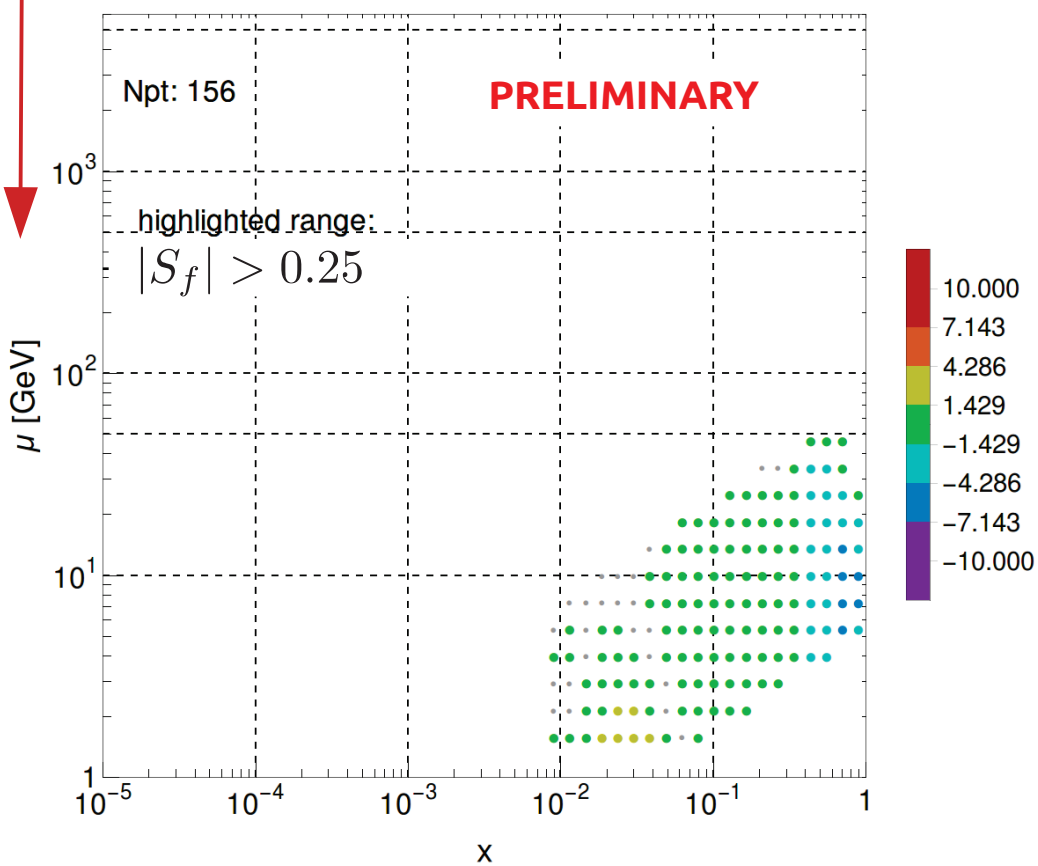


JL-EIC will have a powerful impact on both hadron structure and HEP phenomenology

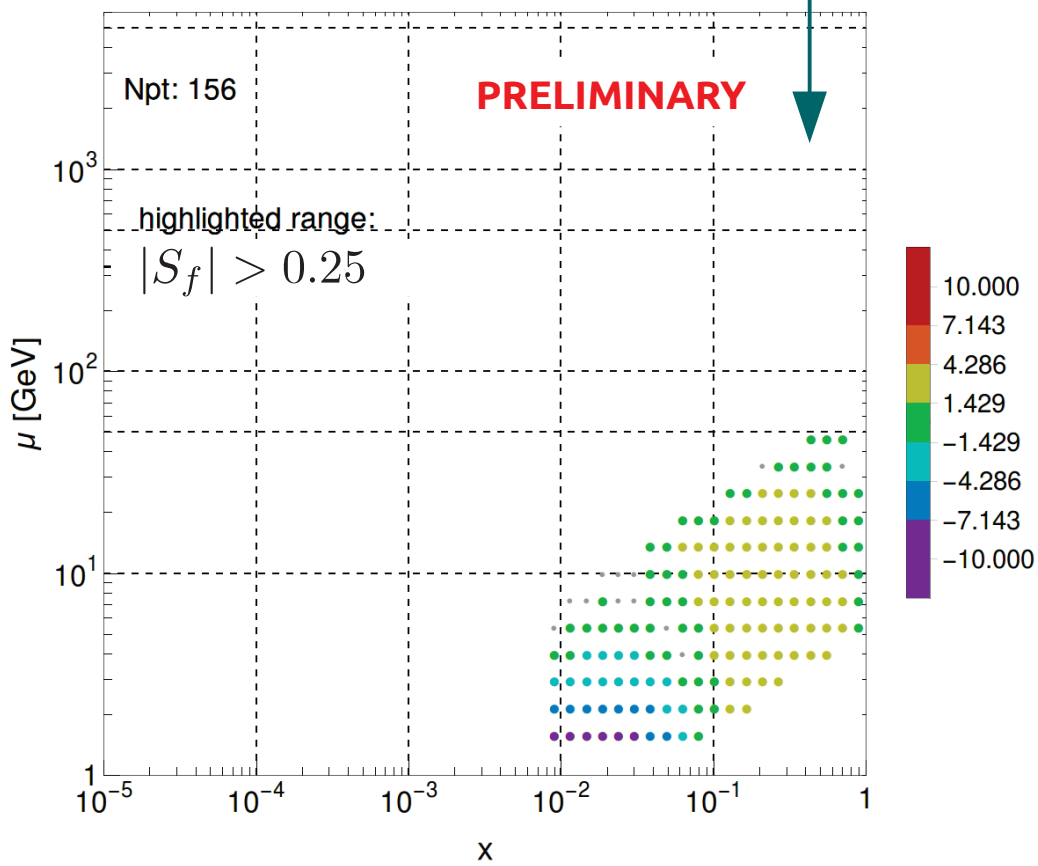
- preliminary analysis suggests very pronounced sensitivity to the high x behavior of $d(x,\mu),\ \ d/u(x,\mu)$

- concerning higher energy processes, JL-EIC will impose tight constraints on the **Higgs phenomenology**

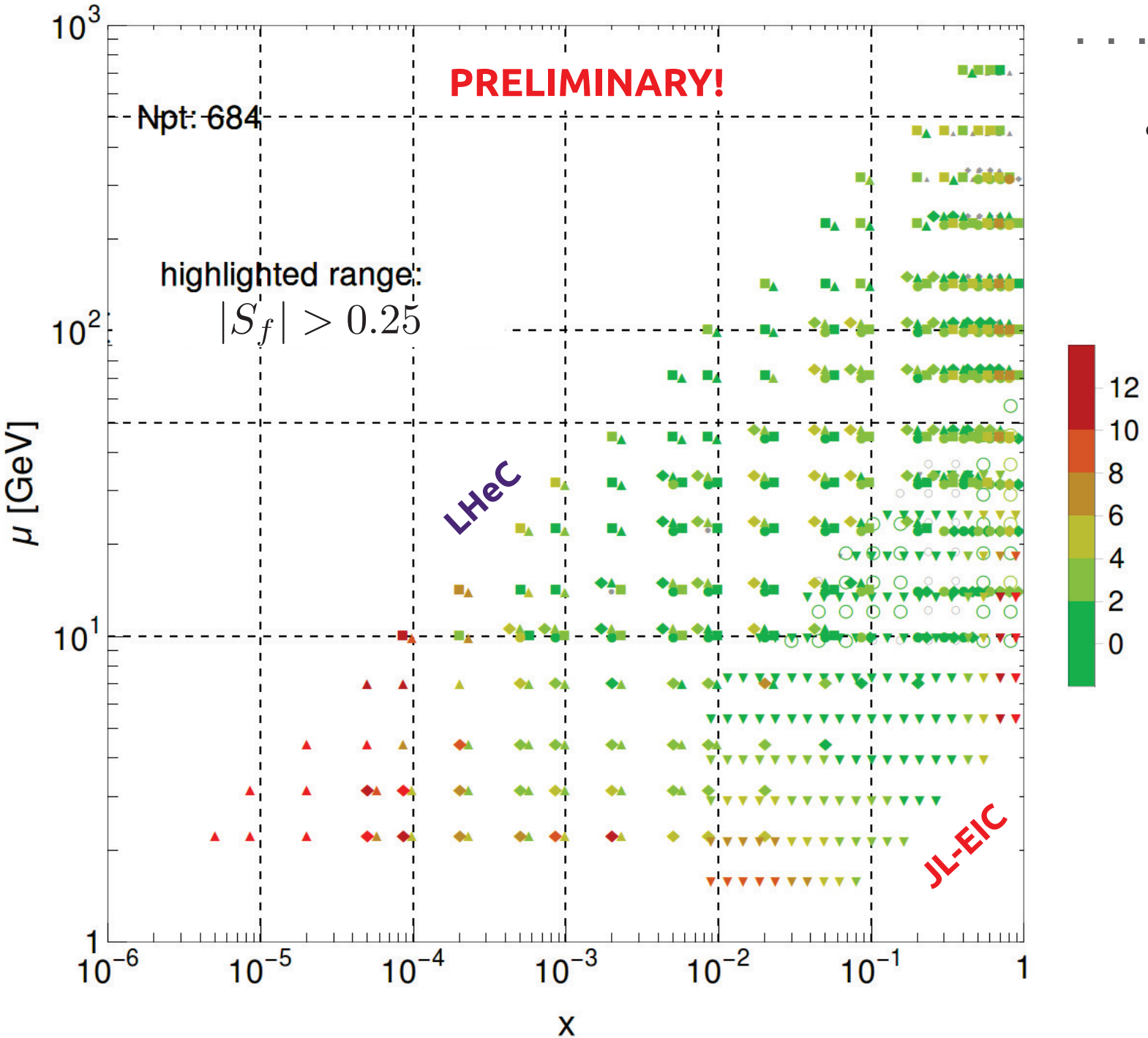
S_f for d/u(x,mu), JLEIC



S_f for sig(H0), 14 TeV, JLEIC



| S_f | for u(x,μ), CT14HERA2NNLO



IN PROGRESS

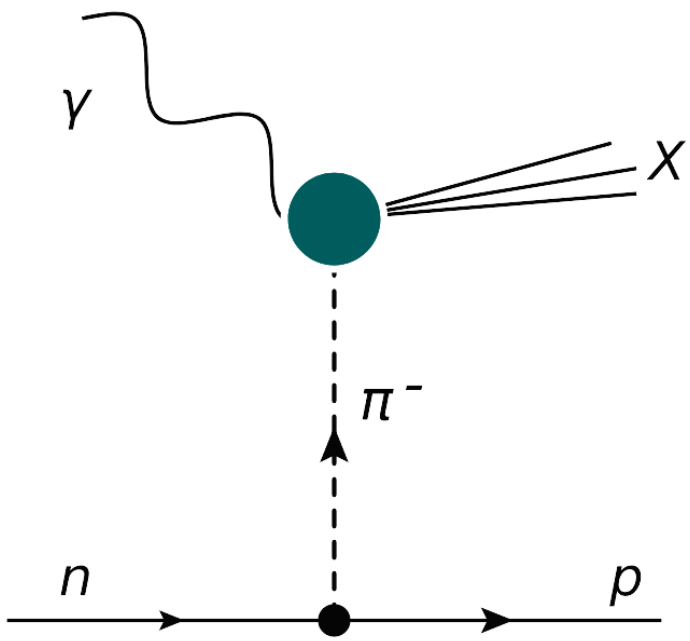
- **eventual goal:** direct, 'apples-to-apples' comparisons of the kinematical phase space coverage and sensitivity of proposed experimental programs – e.g., JL-EIC vs. LHeC, etc.

- in this illustration, we compare a presumed 10-year run of JL-EIC (in one bin of beam energy!) to **approximately** statistics-rescaled LHeC pseudo-data

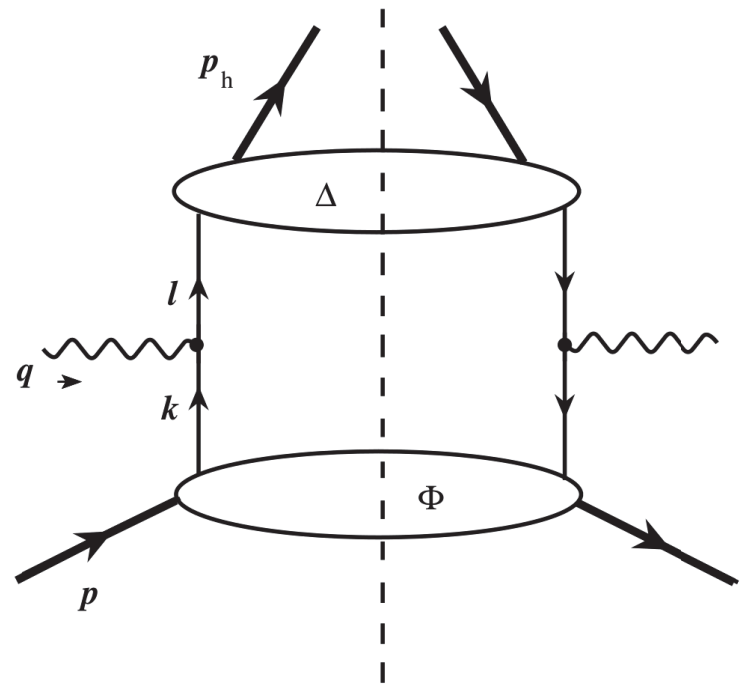
going forward: PDFSense for **meson structure at EIC**

- the PDFSense framework is of sufficient generality that the same approach might also be applied to EIC-relevant processes involving, e.g., $q_\pi(x, \mu^2)$

  - spectator-tagging for forward produced baryons (TDIS, leading neutron production):



  - standard semi-inclusive pion or kaon production in DIS:



- EIC projections for measurements of these processes may be constructed along the lines of the EW cross sections shown in previous slides; with this information, we might explore **optimal EIC run scenarios** for meson PDF extraction

conclusions and **future directions**

- LaMET techniques hold promise for computing the valence **quasi-distributions** of the pion, $\widetilde{q}_\pi(x)$, and *models can give guidance*

→ cf. the nucleon, the pion valence quasi-PDF may converge more rapidly to its exact LF distribution

- **PDFSENSE** allows a thorough exploration of the PDF impact of current and potential data ideal for investigating possible EIC scenarios

→ pseudodata sensitivities (**JL-EIC**, **LHeC**); only need estimated stat./sys. uncertainties in bins of definite x and Q^2

→ processes for meson structure: **semi-inclusive** production of light mesons (π, K); spectator tagging can be incorporated

- Moreover, we can compute sensitivities to many PDF-derived quantities: e.g., physical cross sections; PDF combinations; **lattice-calculable moments**, …

PDFLattice whitepaper: arXiv:1711.07916

→ **other physically interesting quantities** to motivate EIC