Global Analysis of TMD and Collinear Twist-3 Observables

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Outline

- TMD and collinear twist-3 (CT3) functions
- Sivers and Collins effects & A_N in pp collisions
- Toward a global analysis of transverse spin observables
- Summary

TMD and Collinear Twist-3 Functions

TMD PDFs (x, k_T)

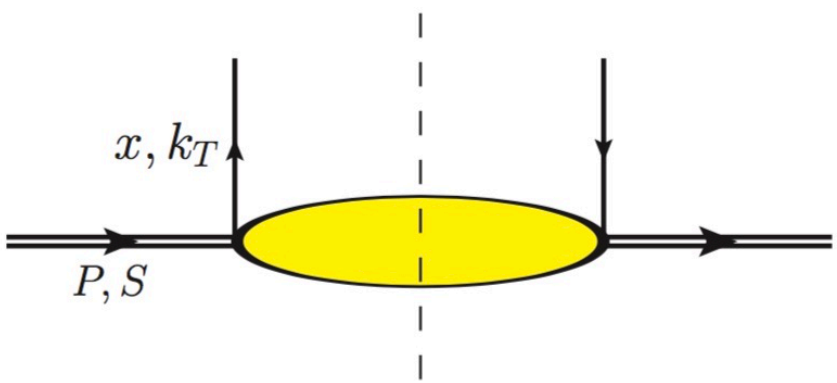
H pol. \ q pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

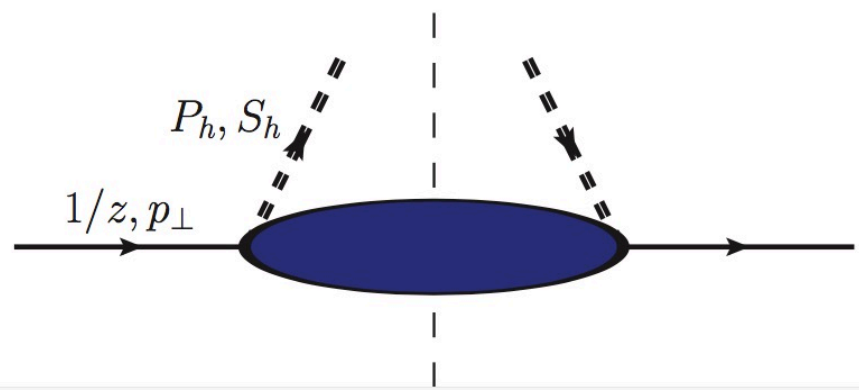
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

TMD FFs ($z, p_\perp$)

H pol. \ q pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))





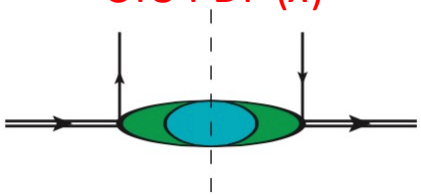
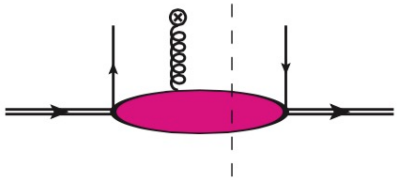
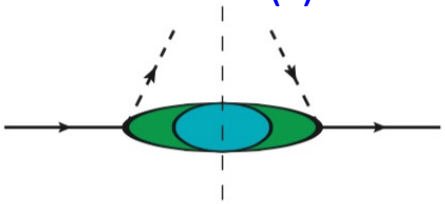
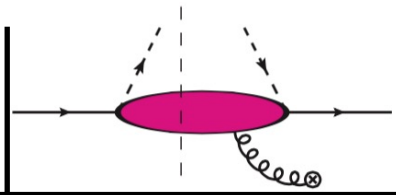
Collinear PDFs (x)

H pol. \ q pol.	U	L	T
U	f_1 unpolarized		
L		g_1 helicity	
T			h_1 transversity

Collinear FFs (z)

H pol. \ q pol.	U	L	T
U	D_1		
L		G_1	
T			H_1

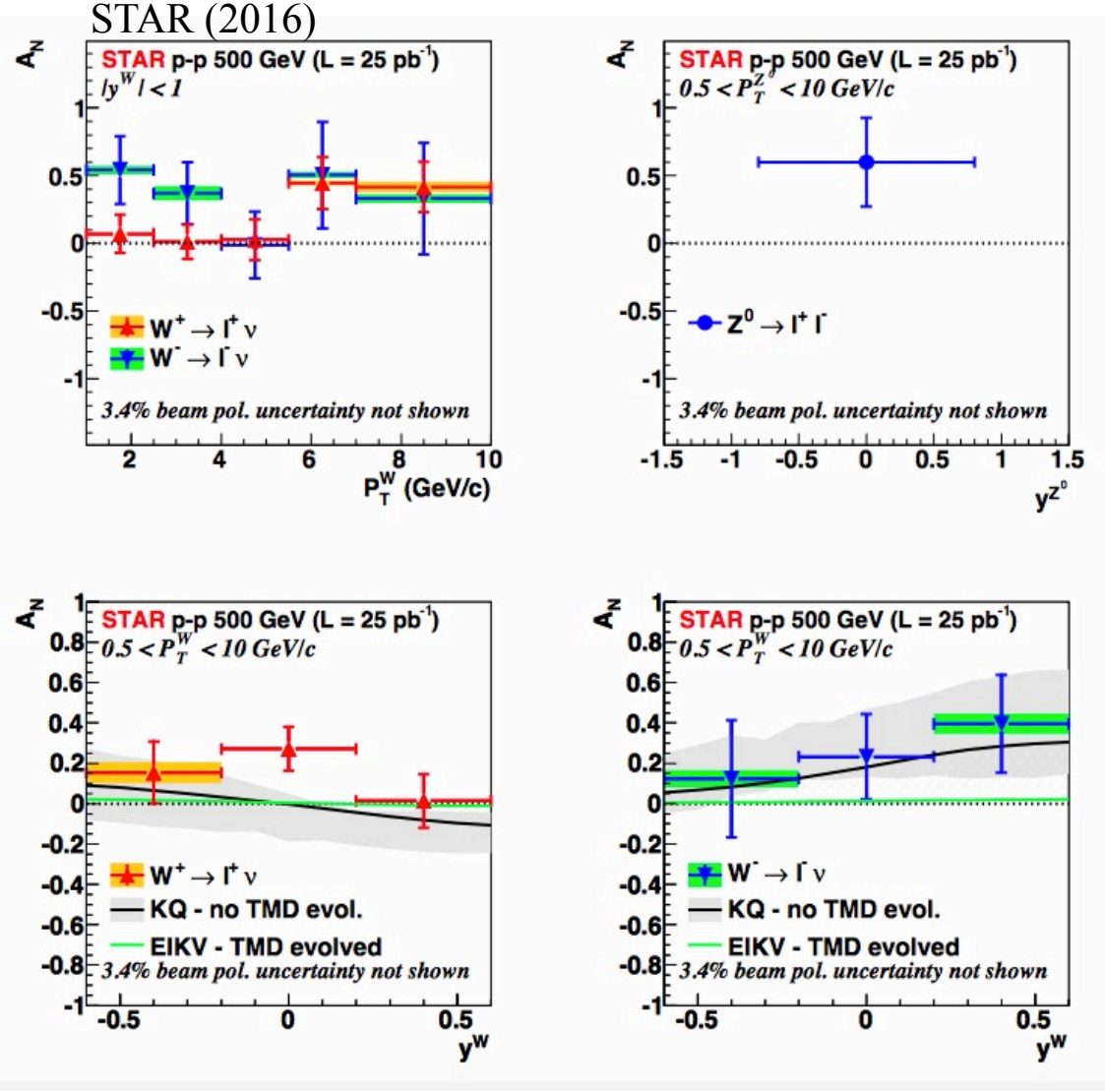
Integrate TMDs over k_T (or $p_\perp$) ➔ collinear PDFs and FFs

Hadron Pol.	CT3 PDF (x)		CT3 PDF (x, x_1)	CT3 FF (z)		CT3 FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

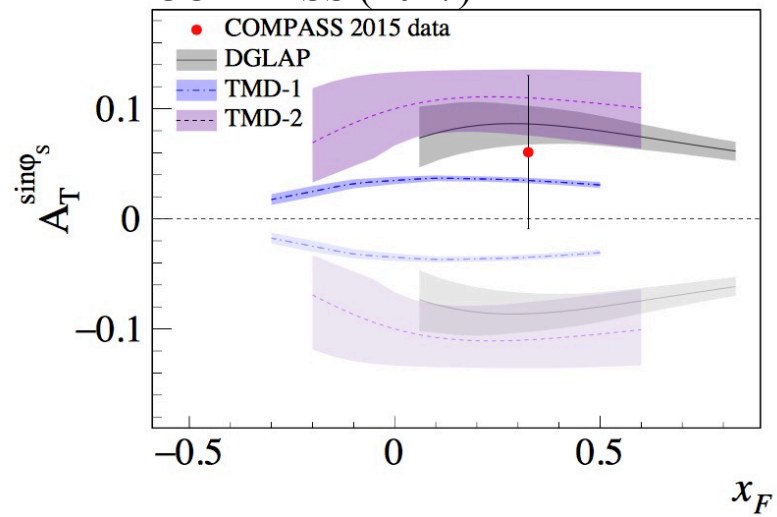
Sivers and Collins Effects & A_N in pp Collisions

Drell-Yan Sivers effect

STAR (2016)

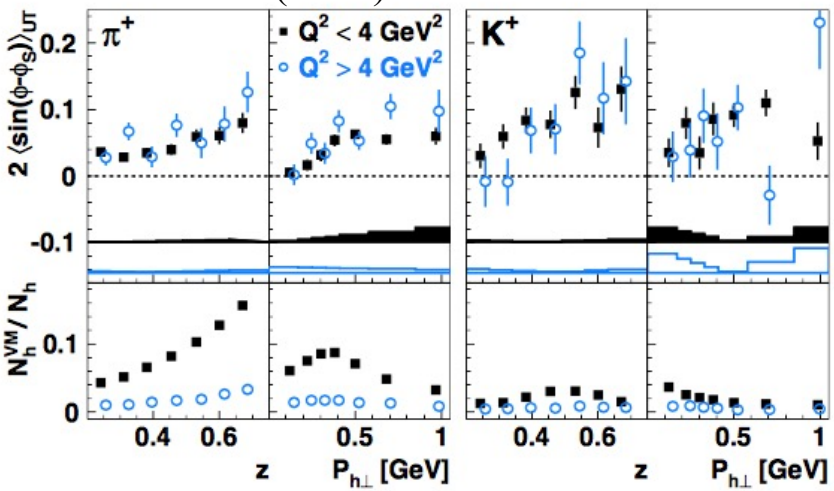


COMPASS (2017)

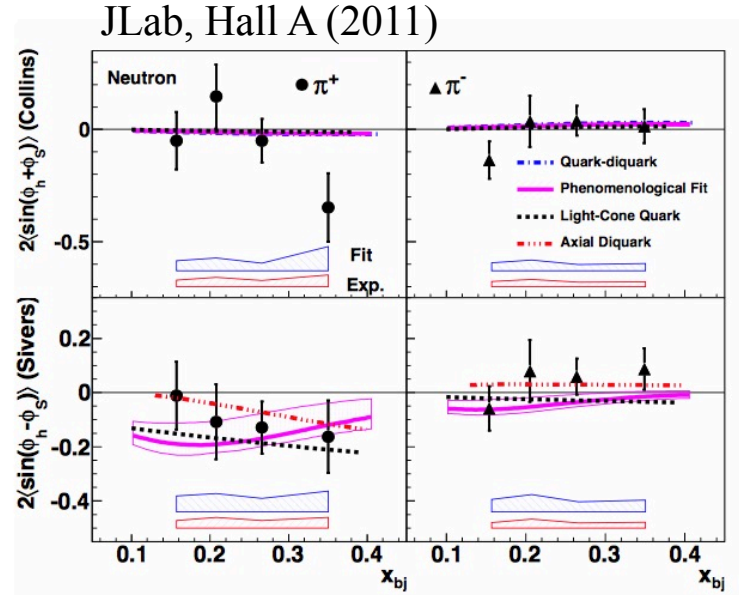


SIDIS Sivers effect ($\sin(\phi_h - \phi_s)$)

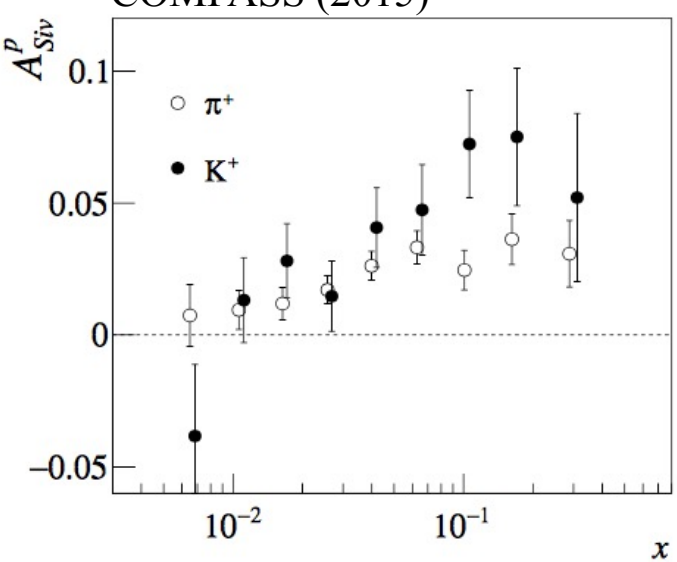
HERMES (2009)



JLab, Hall A (2011)

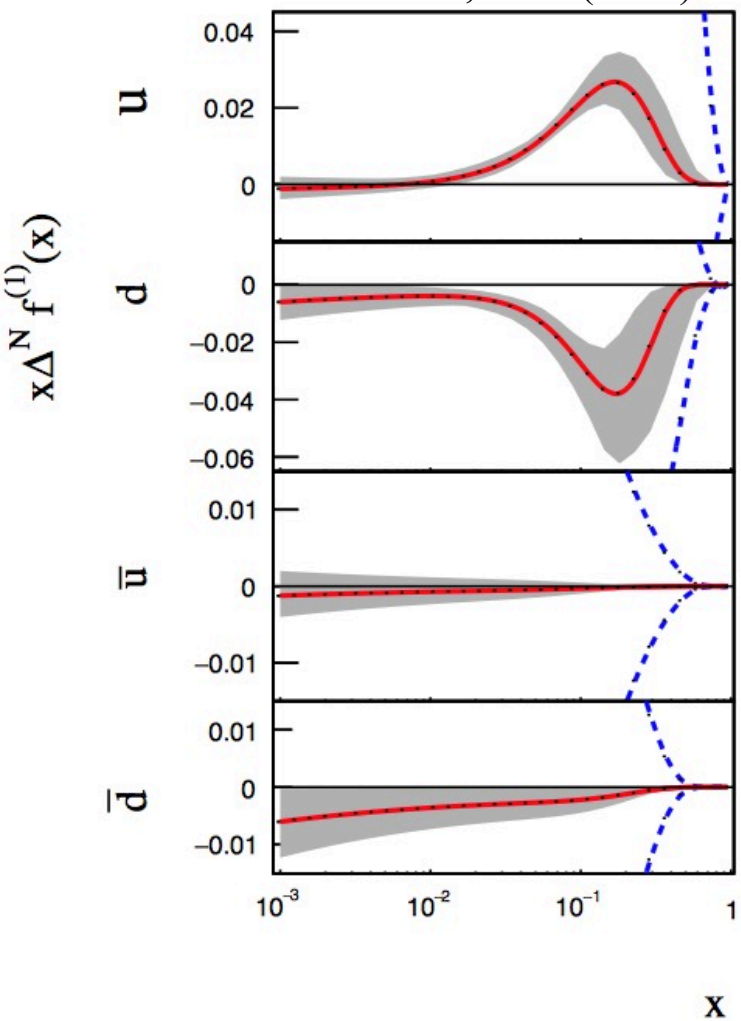


COMPASS (2015)

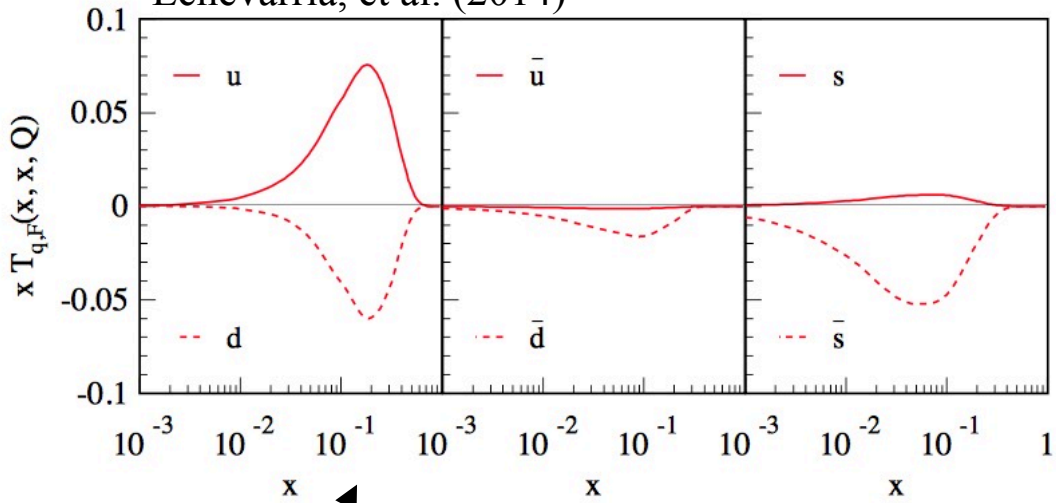


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M} f_{1T}^{\perp} D_1\right]$$

Anselmino, et al. (2017)



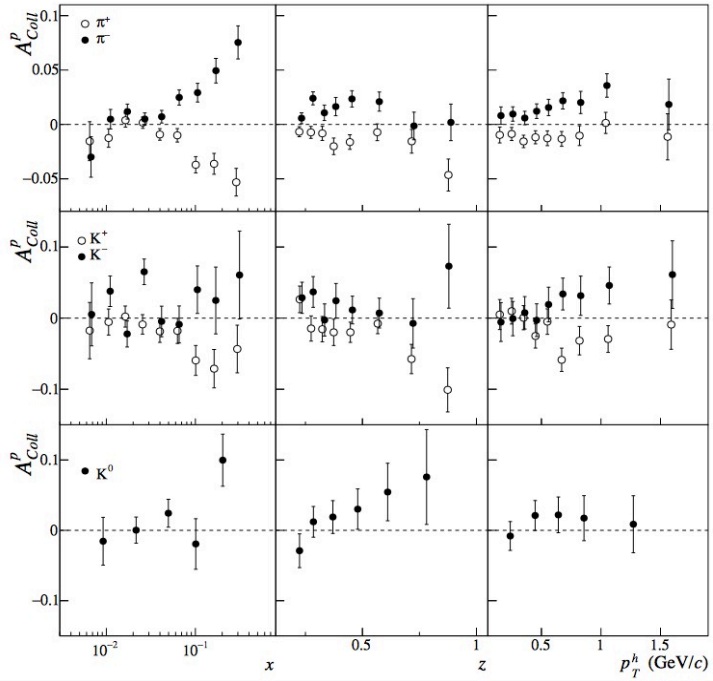
Echevarria, et al. (2014)



TMDs in Collins-Soper-Sterman (CSS) evolution formalism

SIDIS Collins effect ($\sin(\phi_h + \phi_s)$)

COMPASS (2015)

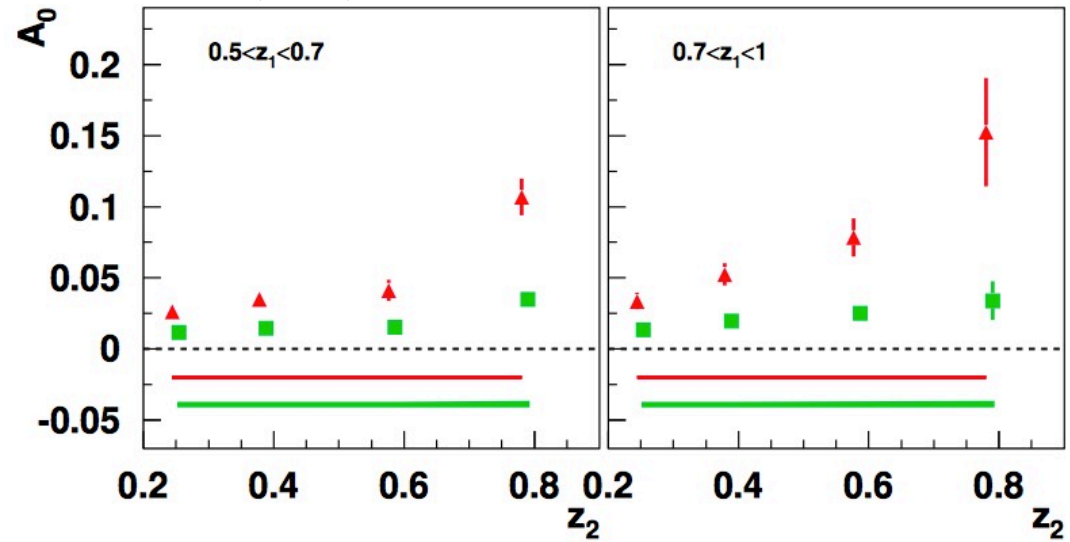


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\boldsymbol{H_1^\perp}\right]$$

e^+e^- Collins effect ($\cos(2\phi_0)$)

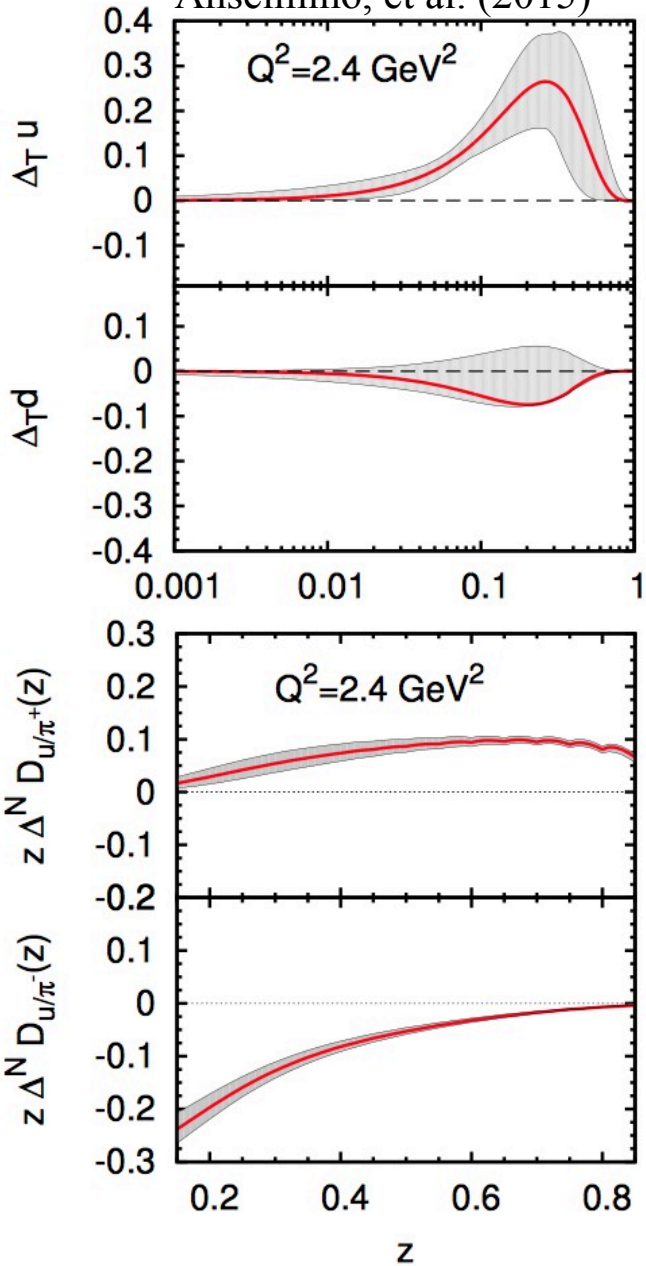
Belle (2008)



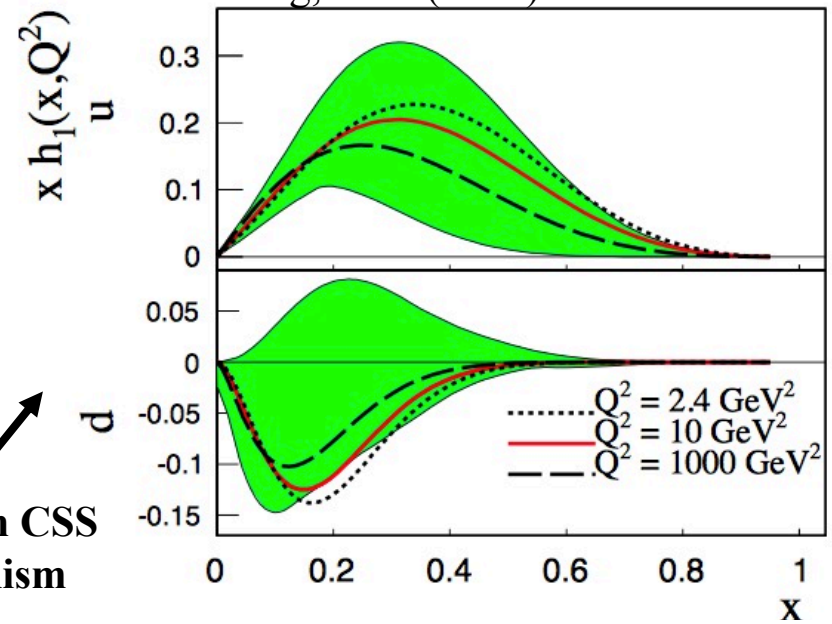
Also data from BaBar (2014) and BESIII (2016)

$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

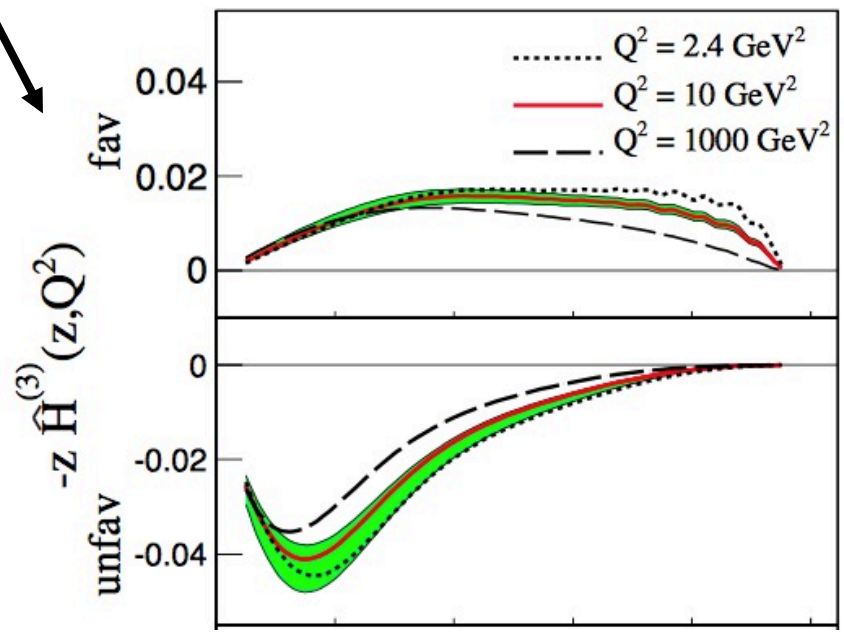
Anselmino, et al. (2015)



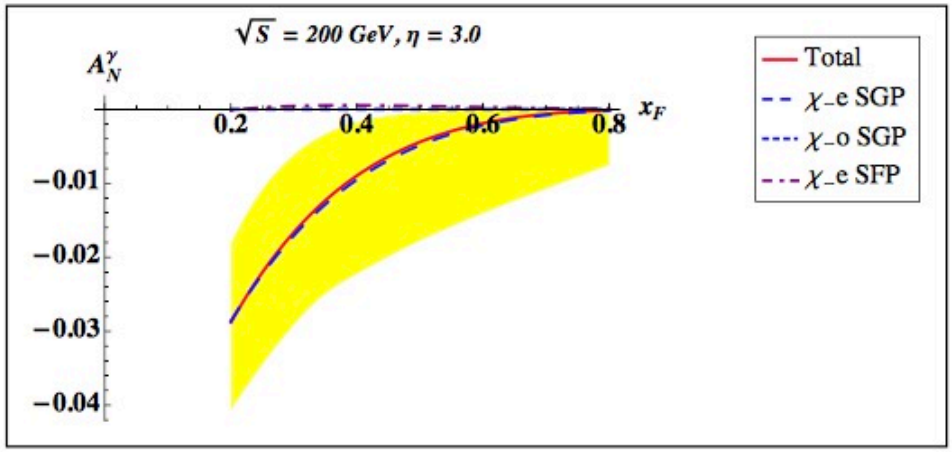
Kang, et al. (2016)

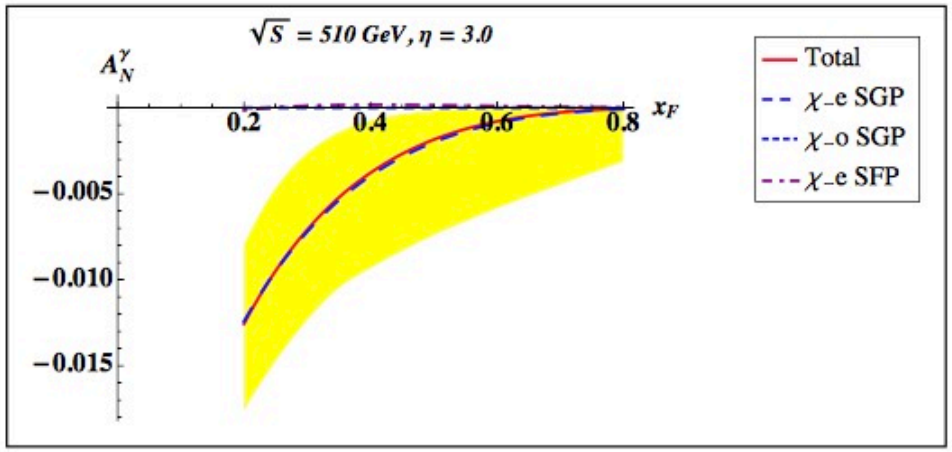


TMDs in CSS formalism



A_N in $pp \to \gamma X$





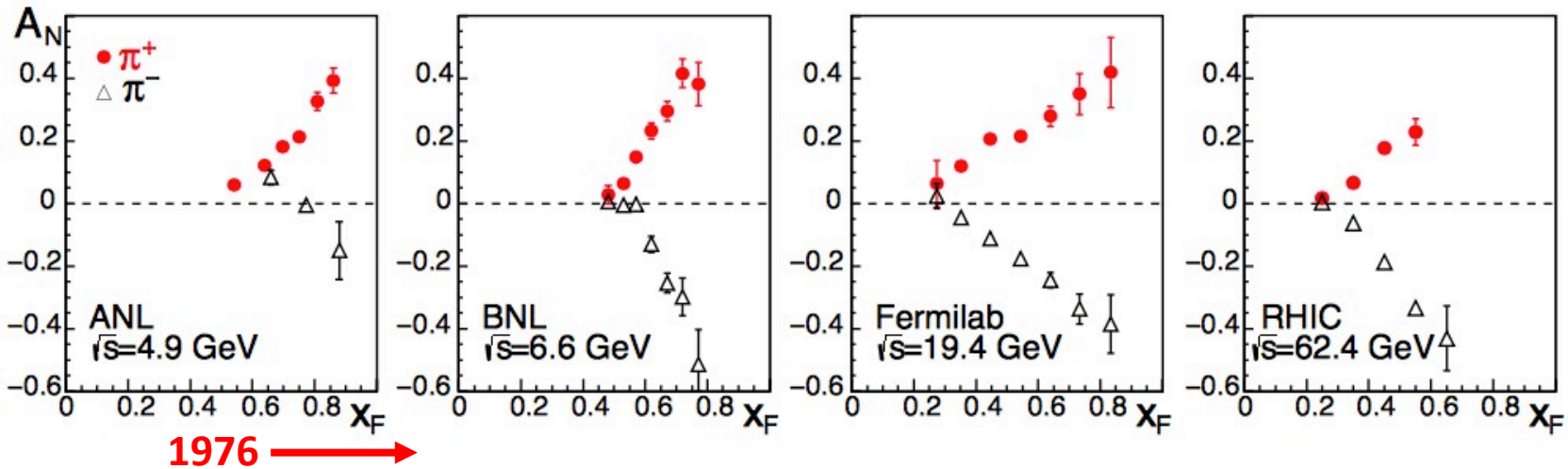
(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
(See also Gamberg, Kang, Prokudin (2013))

Qiu-Sterman term is the main cause of A_N in $pp \to \gamma X$ → Test of the process dependence of the Sivers function

$$d\Delta\sigma^\pi \sim H \otimes f_1 \otimes F_{FT}(x, x)$$

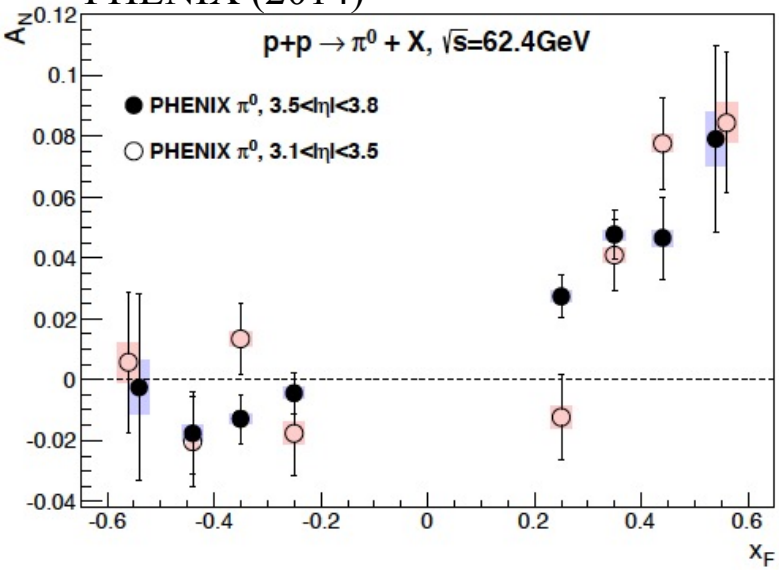
Qiu-Sterman function

A_N in $pp \to \pi X$ – PUZZLE FOR 40+ YEARS!

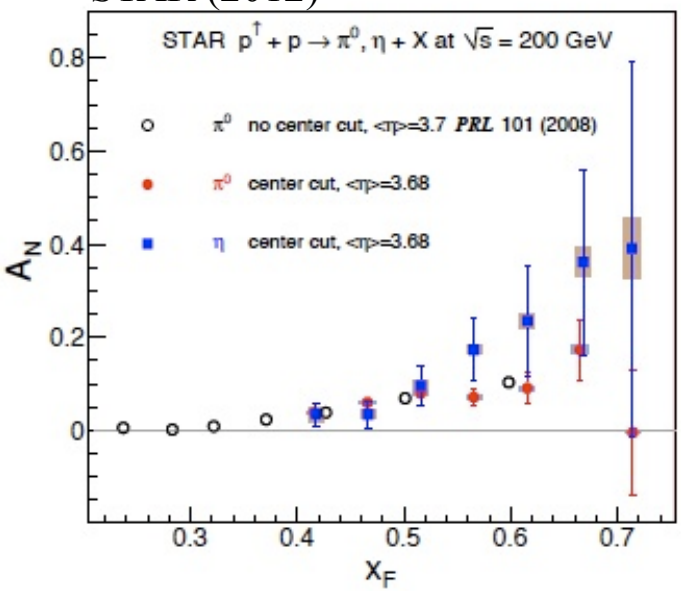


A_N in $pp \to \pi X$ – PUZZLE FOR 40+ YEARS!

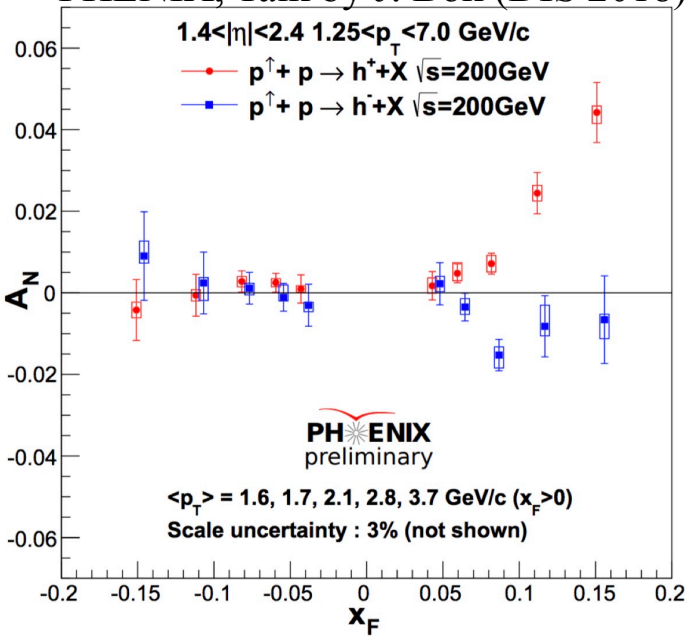
PHENIX (2014)



STAR (2012)



PHENIX, Talk by J. Bok (DIS 2018)



$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^{1} \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^{1} \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}} \right) \frac{1}{x} \left[T_{a,F}(x,x) - x \left(\frac{d}{dx} T_{a,F}(x,x) \right) \right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

$$\boxed{F_{FT} \sim T_F}$$

(Qiu and Sterman (1999), Kouvaris, et al. (2006))

For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $p^{\uparrow}p \to \pi X$

$$\cancel{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes F_{F^{\prime}T^{\prime}}(x, x)}$$

(Kang, Qiu, Vogelsang, Yuan (2011); Kang and Prokudin (2012);
Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou (2012))

$$\cancel{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes F_{FT}(x,x)}$$

$$d\Delta\sigma^{\pi} \sim h_1 \otimes S \otimes \left(H_1^{\perp(1)}, H, \int \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\ \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t} - x\hat{u})}$$
$$\times\ h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z \frac{dH_1^{\perp(1),c}(z)}{dz} \right] S^i_{H_1^\perp} + \frac{1}{z} H^c(z)\, S^i_H \right.$$
$$\left. + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{c,\Im}(z,z_1)\, S^i_{\hat{H}_{FU}} \right\}$$

(Metz and DP - PLB **723** (2013))

We now believe the TSSAs in $p^{\uparrow}p \to \pi X$ are due fragmentation effects as the partons form pions in the final state

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + \boxed{2z \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)}$$

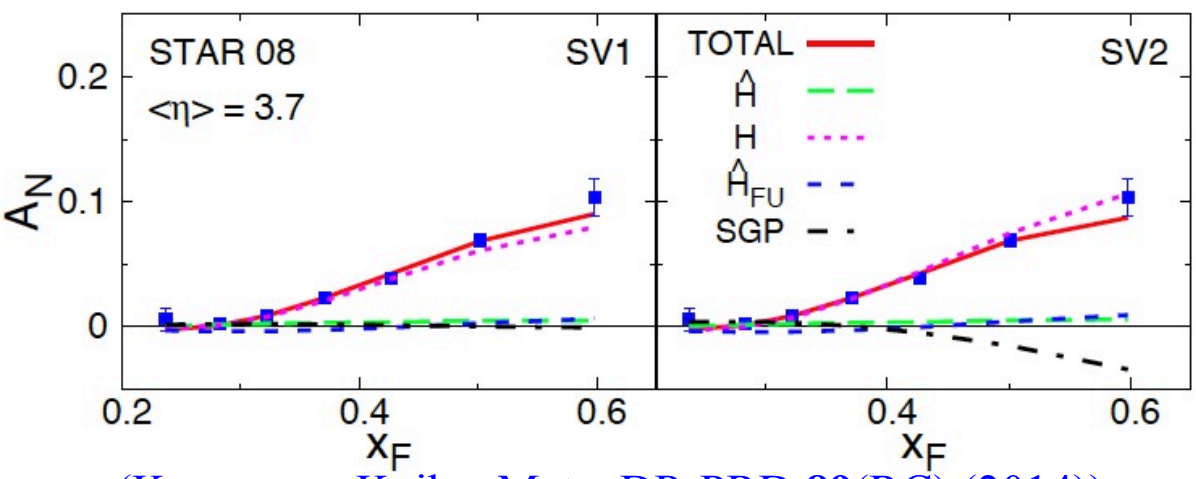
QCD e.o.m. relation (EOMR)

$$\equiv \tilde{H}^q(z)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \tilde{\boldsymbol{H}}, \int \frac{dz_1}{z_1^2} \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

Also included the Qiu-Sterman term $\pi \boldsymbol{F_{FT}(x,x)} = \boldsymbol{f_{1T}^{\perp(1)}(x)}$



Fragmentation term is the main cause of A_N in $pp \to \pi X$

(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^{\infty} \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

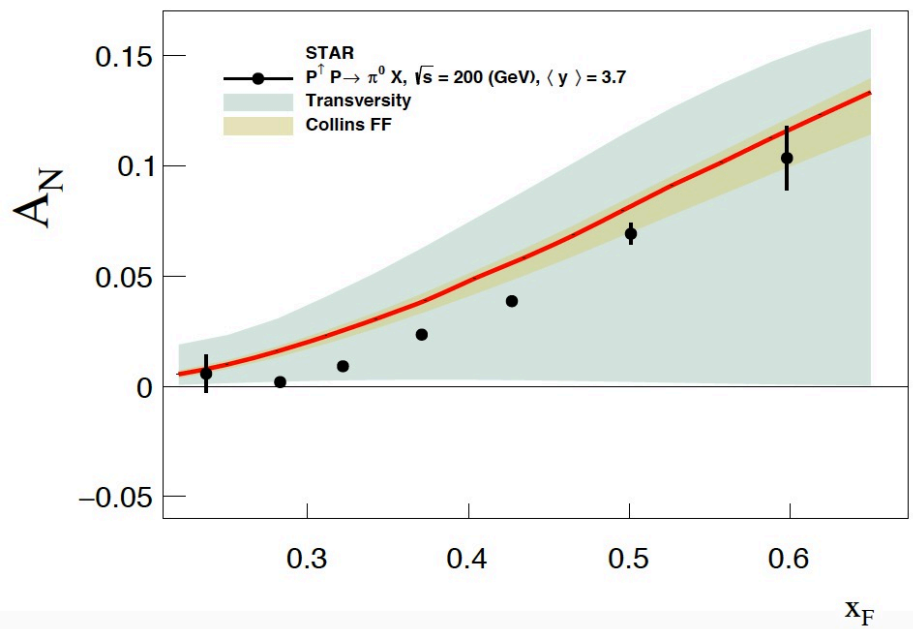
$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\ \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}}$$
$$\times\ h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z \frac{dH_1^{\perp(1),c}(z)}{dz} \right] \tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z) \right] \tilde{S}^i_H \right\}$$

where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(H_1^{\perp(1)}, \tilde{H}, \int \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

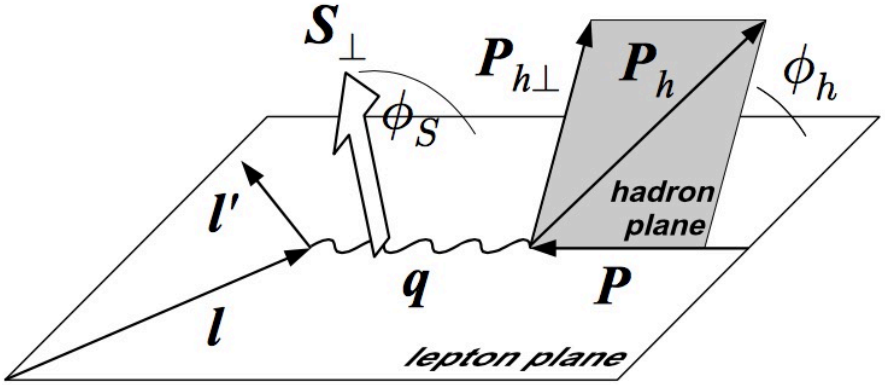
$$d\Delta\sigma^{\pi} \sim \boxed{\boldsymbol{h_1} \otimes \tilde{S} \otimes \left(H_1^{\perp(1)} \right.}, \left. \tilde{H} \right)$$



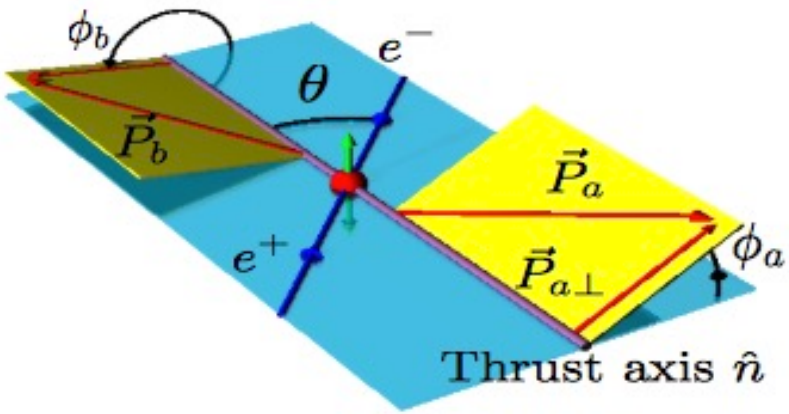
Fragmentation term is the main cause of A_N in $pp \to \pi X$

The ***A_N data from RHIC*** can be used along with ***measurements from SoLID at JLab*** to constrain ***transversity at large x!***

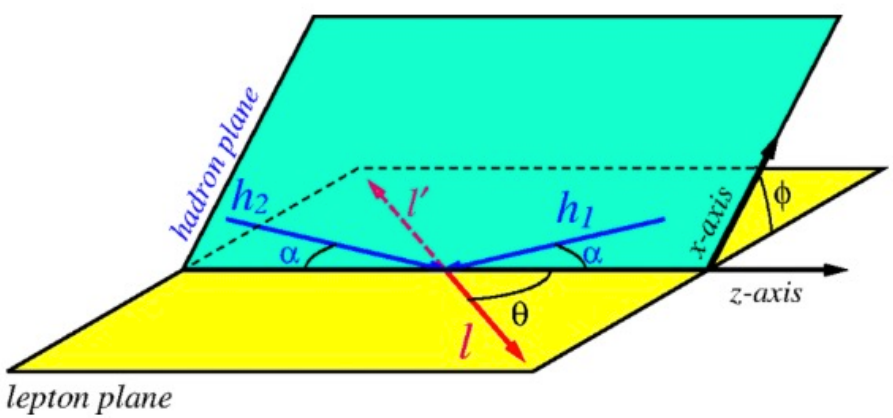
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



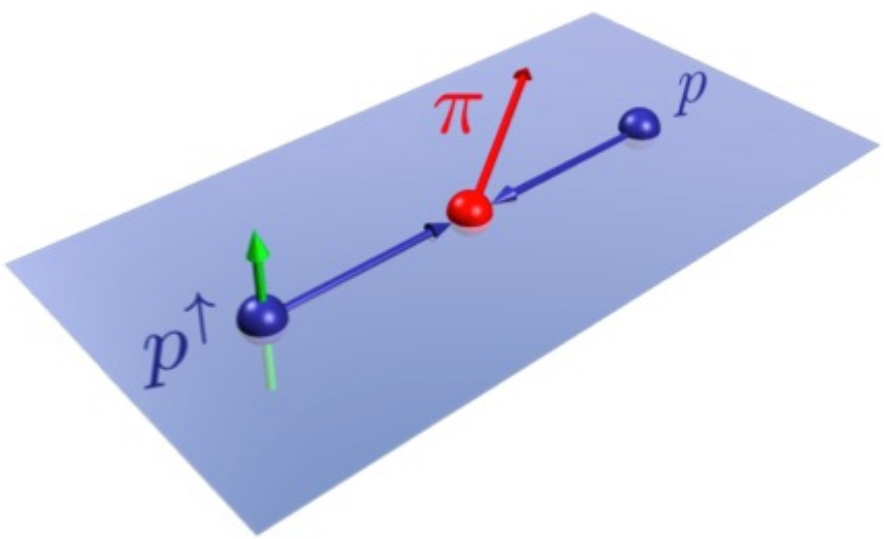
Sivers ~ $\sin(\phi_h - \phi_s)$, Collins ~ $\sin(\phi_h + \phi_s)$, …



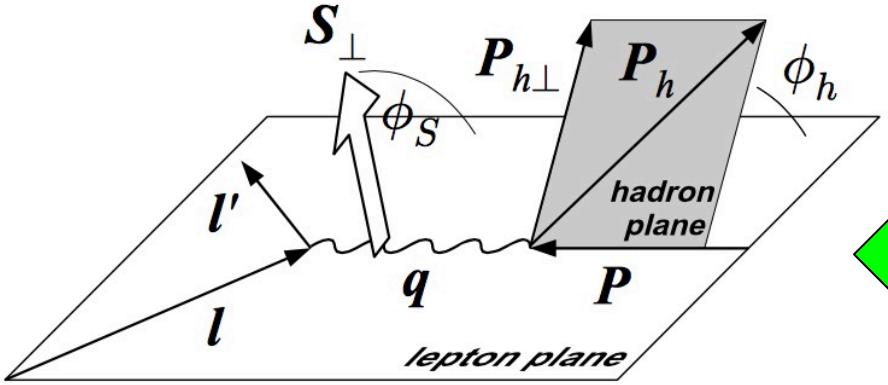
Collins ~ $\cos(\phi_a + \phi_b)$, …



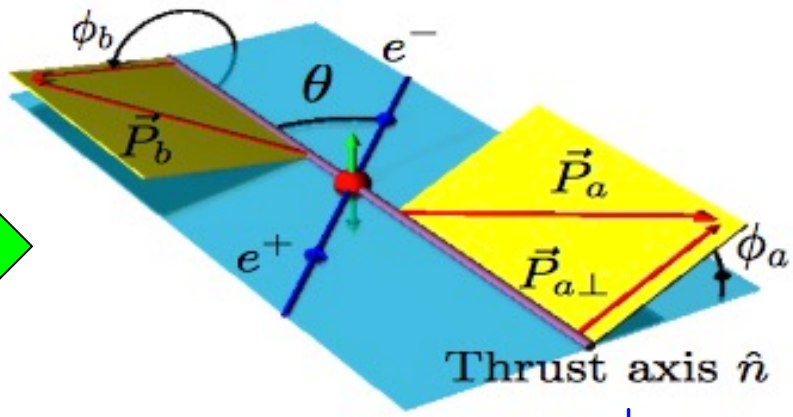
Sivers ~ $\sin(\phi_s)$ (lepton pair) / Sivers ~ $\cos(\phi_{W/Z})$ (boson)



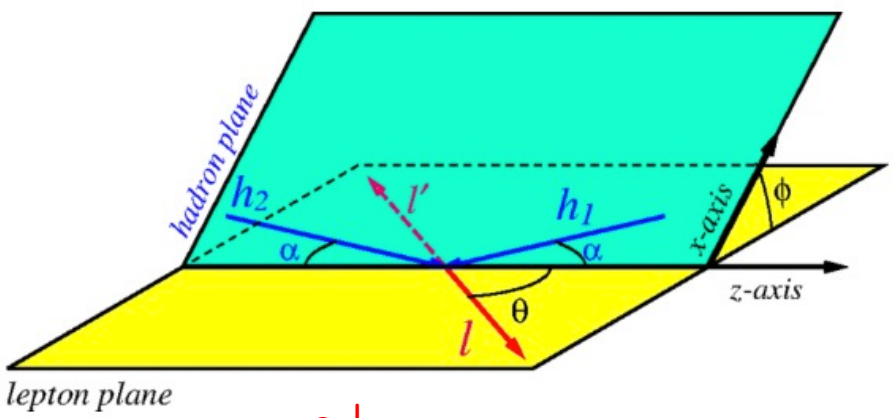
$A_N \sim d\sigma_L - d\sigma_R$



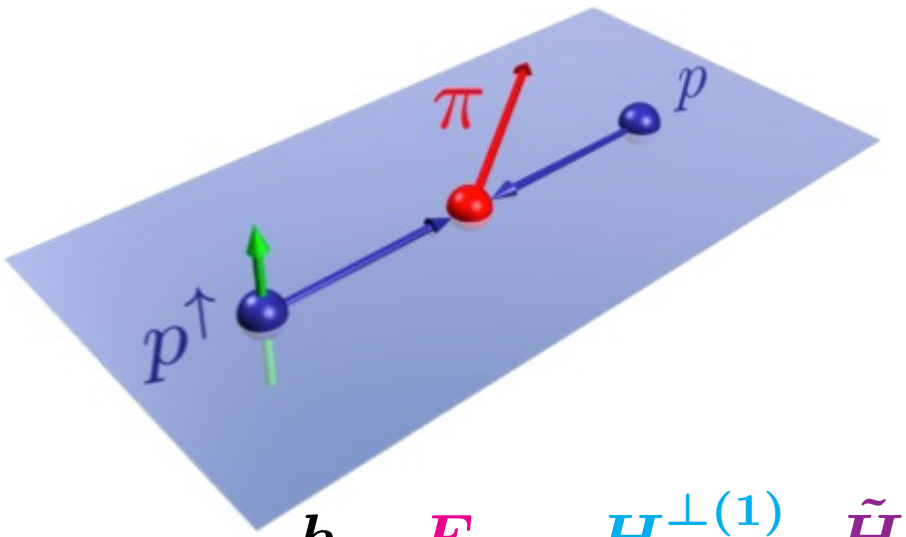
h_1, $f_{1T}^\perp$, $H_1^\perp$



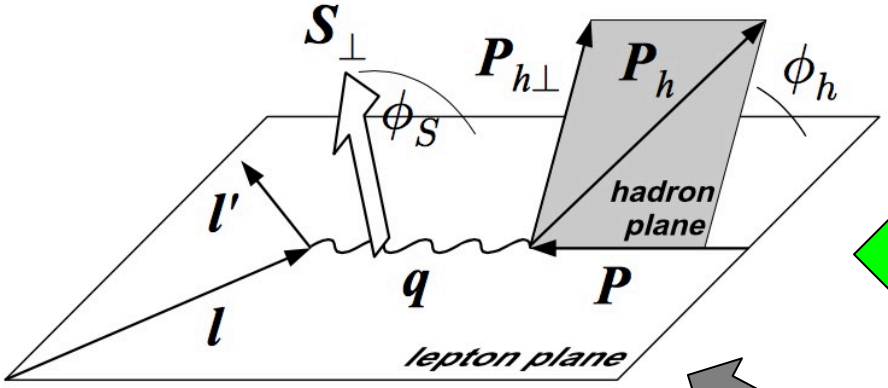
$H_1^\perp$



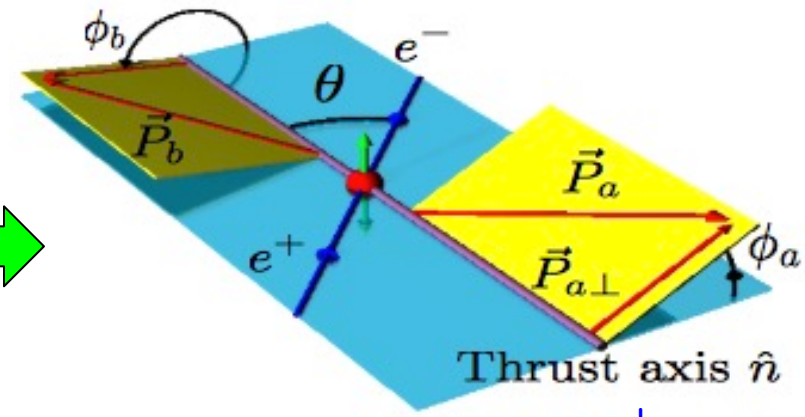
$f_{1T}^\perp$



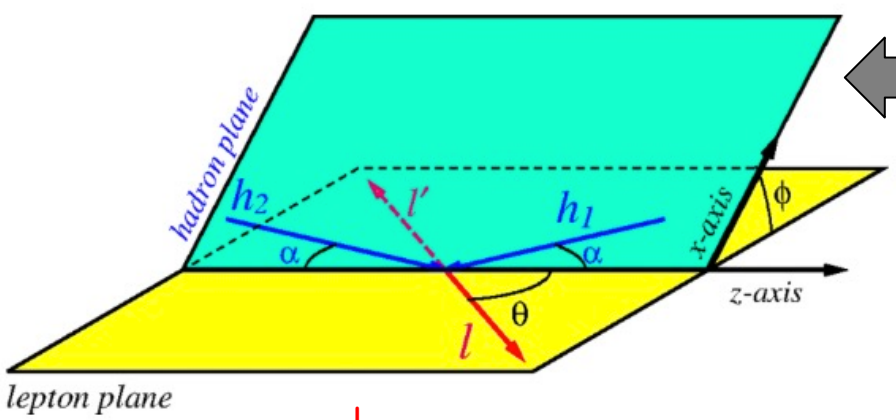
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



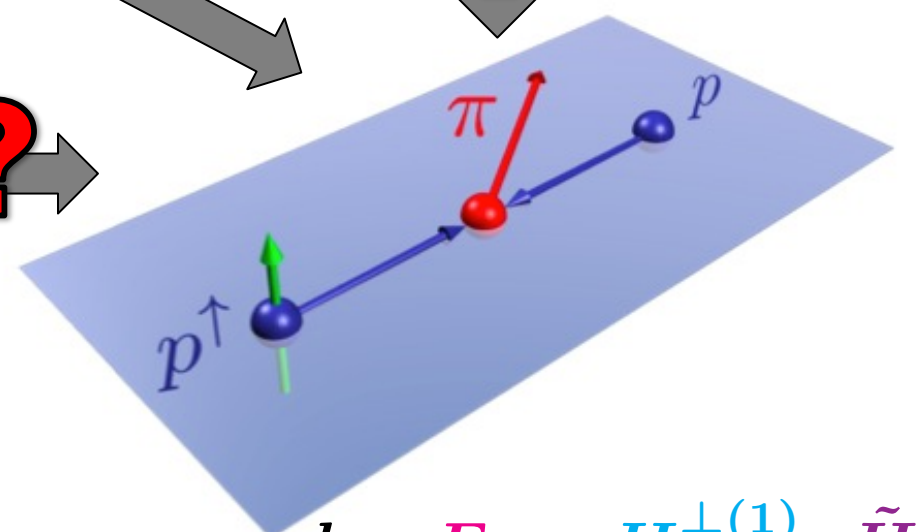
h_1, $f_{1T}^\perp$, $H_1^\perp$



$H_1^\perp$

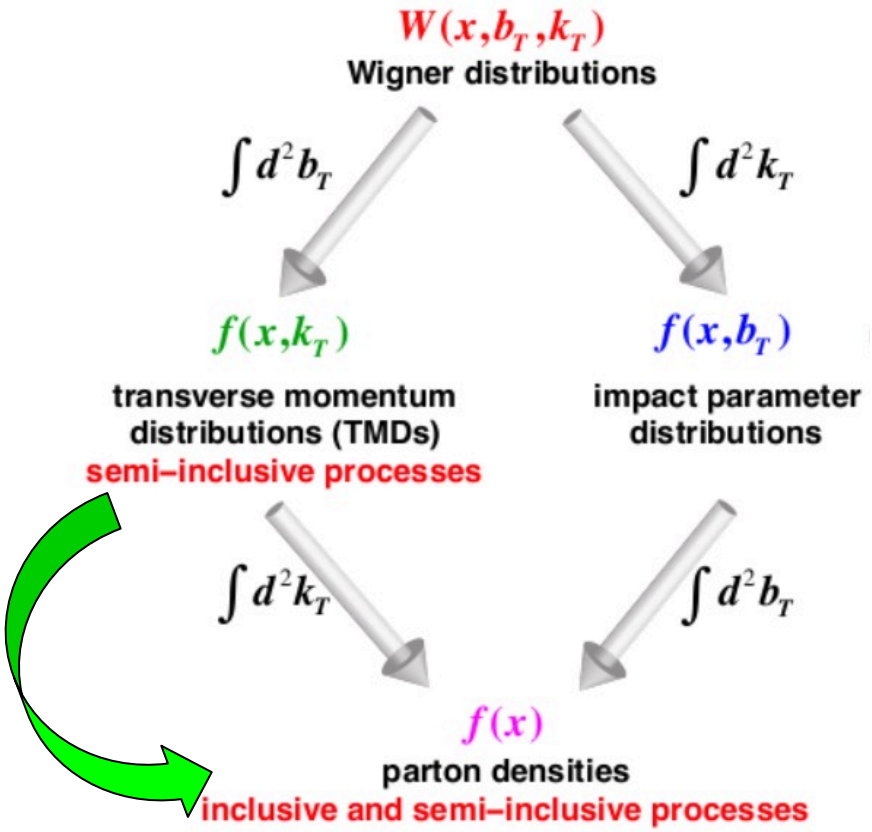


$f_{1T}^\perp$



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Figure from EIC Whitepaper



One naively expects that we can obtain collinear functions by integrating TMDs over k_T

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

"b-space" correlator

$$\tilde{\Phi}^{[\gamma^+]}(x,\vec{b}_T;Q^2,\mu_Q) = \tilde{f}_1(x,b_T;Q^2,\mu_Q) - iM\epsilon^{ij}b_T^i S_T^j\left[-\frac{1}{M^2}\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T;Q^2,\mu_Q)\right]$$

Boer, Gamberg, Musch, Prokudin (2011)

$$\equiv \tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q)$$

$$\tilde{f}_1(x,b_T;Q^2,\mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes f_1(\hat{x};\mu_{b_*})\right)$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_1}(b_T,Q)\right]$$

Collins (2011); ...

$$\tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q) \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1,\hat{x}_2,b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes F_{FT}(\hat{x}_1,\hat{x}_2;\mu_{b_*})\right)$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T,Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

Takes into account "complications" of QCD (e.g., parton re-scattering and gluon radiation)

$$\int d^2k_T\, f_1(x, k_T; Q^2, \mu_Q) = \tilde{f}_1(x, b_T \to 0; Q^2, \mu_Q) = 0\,!$$

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q) \equiv f_{1T}^{\perp(1)}(x; Q^2, \mu_Q) = \tilde{f}_{1T}^{\perp(1)}(x, b_T \to 0; Q^2, \mu_Q) = 0\,!$$

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

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(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

TMDs lose their physical interpretation in the "Original CSS" formalism!

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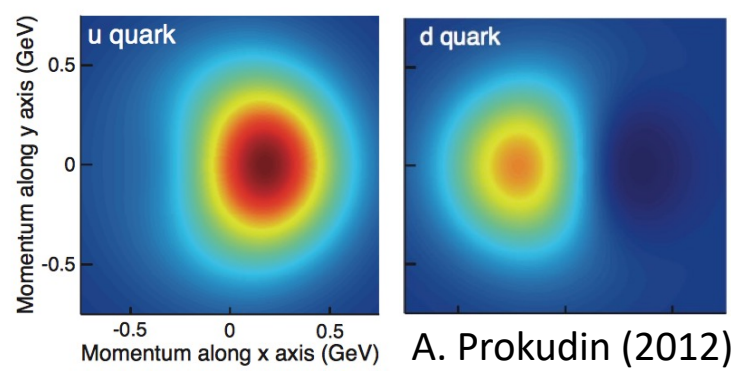
TMDs lose their physical interpretation in the "Original CSS" formalism!

$$\langle k_T^i(x)\rangle_{UT} = \int d^2k_T\, k_T^i \left(-\frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x, k_T) \right)$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target



A. Prokudin (2012)

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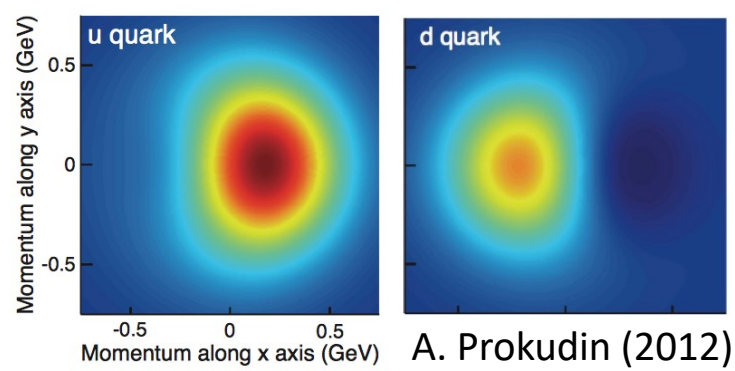


A. Prokudin (2012)

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$$\Rightarrow \quad \mu_{b_*} \to \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \quad \text{so} \quad \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

$$\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes f_1(\hat{x}; \bar{\mu})\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{f}_1(x, b_T; Q^2, \mu_Q) - iM\epsilon^{ij} b_T^i S_T^j \tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)$$

b_T -> $b_c(b_T)$

NO b_T -> $b_c(b_T)$ replacement – kinematic factor NOT associated with the scale evolution

b_T -> $b_c(b_T)$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$$\Rightarrow \mu_{b_*} \to \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \text{ so } \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

$$\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes f_1(\hat{x}; \bar{\mu})\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T, b_c(b_T); Q^2, \mu_Q) = \tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) - iM\epsilon^{ij} b_T^i S_T^j \tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)$$

$$\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_{1T}^\perp}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes F_{FT}(\hat{x}_1, \hat{x}_2; \bar{\mu}\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_c(b_T), Q)\right]$$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2\vec{k}_T\, f_1(x, k_T; Q^2, \mu_Q; C_5) = \tilde{f}_1(x, b_c(0); Q^2, \mu_Q) = f_1(x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, D_1(z, p_T; Q^2, \mu_Q; C_5) = \tilde{D}_1(z, b_c(0); Q^2, \mu_Q) = D_1(z; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \tilde{f}_{1T}^{\perp(1)}(x, b_c(0); Q^2, \mu_Q) = \pi\, F_{FT}(x, x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2 M_h^2}\, H_1^{\perp}(z, p_T; Q^2, \mu_Q; C_5) = \tilde{H}_1^{\perp(1)}(z, b_c(0); Q^2, \mu_Q) = H_1^{\perp(1)}(z; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the "Improved CSS" formalism we recover the relations one expects from the "naïve" operator definitions of the functions

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2\vec{k}_T\, f_1(x, k_T; Q^2, \mu_Q; C_5) = \tilde{f}_1(x, b_c(0); Q^2, \mu_Q) = f_1(x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, D_1(z, p_T; Q^2, \mu_Q; C_5) = \tilde{D}_1(z, b_c(0); Q^2, \mu_Q) = D_1(z; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \tilde{f}_{1T}^{\perp(1)}(x, b_c(0); Q^2, \mu_Q) = \pi\, F_{FT}(x, x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2 M_h^2}\, H_1^{\perp}(z, p_T; Q^2, \mu_Q; C_5) = \tilde{H}_1^{\perp(1)}(z, b_c(0); Q^2, \mu_Q) = H_1^{\perp(1)}(z; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the "Improved CSS" formalism we recover the relations one expects from the "naïve" operator definitions of the functions

The "Improved CSS" formalism (approximately) restores the physical interpretation of TMDs!

(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))

$$\int d^2\vec{k}_T \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \pi \boxed{F_{FT}(x, x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T k_T^i \int \frac{db^-}{2\pi} \int \frac{d^2b_T}{(2\pi)^2} e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T} \langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

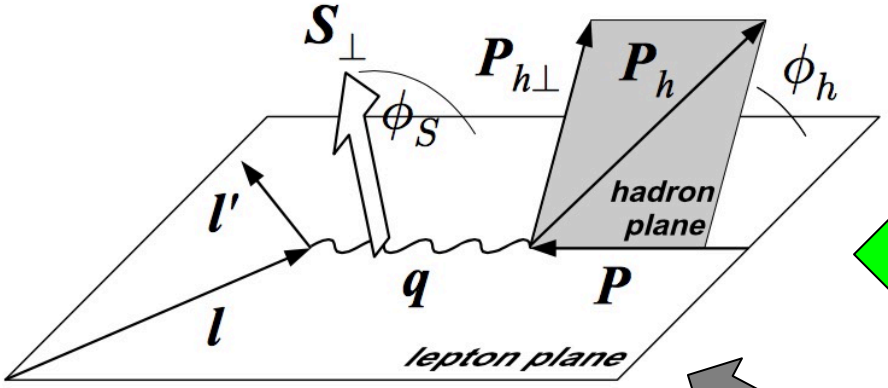
$$= \frac{1}{2}\int \frac{db^- dy^-}{4\pi} e^{ixP^+b^-} \langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$= -\pi M\epsilon^{ij}S_T^j F_{FT}(x,x;\mu)$$

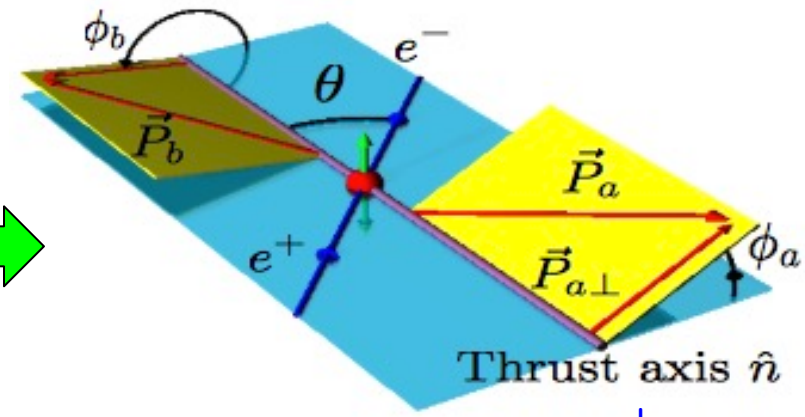
Recall also the Burkardt sum rule $\sum_{a=q,\bar{q},g}\int_0^1 dx\, F_{FT}^a(x,x) = 0$

The Qiu-Sterman function can fundamentally be understood as an avg. TM, and the first k_T-moment of the Sivers function (using "Improved CSS") retains this interpretation at LO

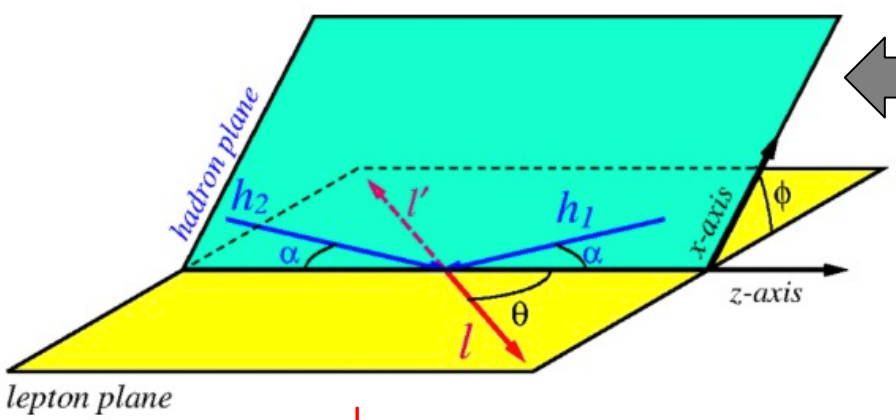
(Gamberg, Metz, DP, Prokudin, PLB **781** (2018))



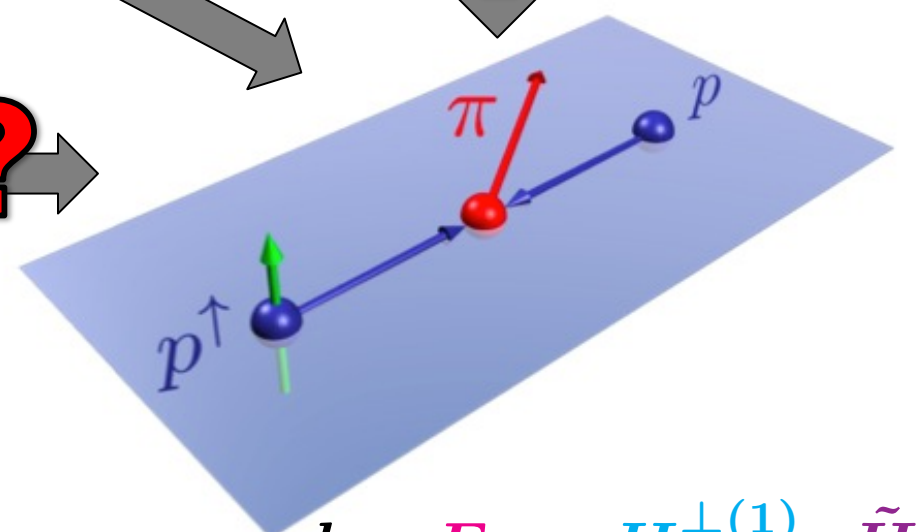
h_1, $f_{1T}^\perp$, $H_1^\perp$



$H_1^\perp$



$f_{1T}^\perp$



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Toward a Global Analysis of Transverse Spin Observables

Recall the current phenomenology of TMD observables...

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim \boxed{F_{FT}(x, x; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_T, Q)\right]$$

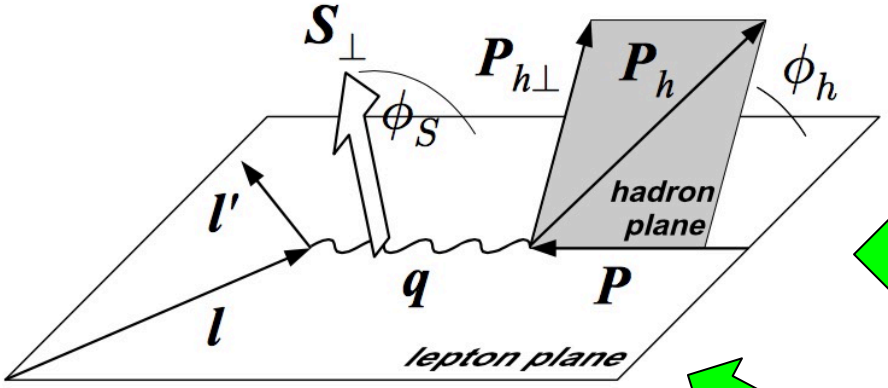
$$\boxed{g_{f_{1T}^\perp}(x, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim \boxed{H_1^{\perp(1)}(z; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)\right]$$

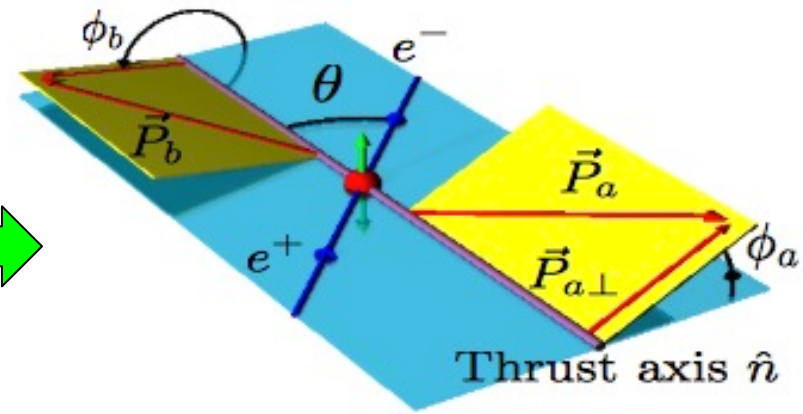
$$\boxed{g_{H_1^\perp}(z, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

The **CT3 functions** (along with the NP *g*-functions) are what get extracted in analyses of TSSAs in ***TMD processes*** that use CSS evolution!

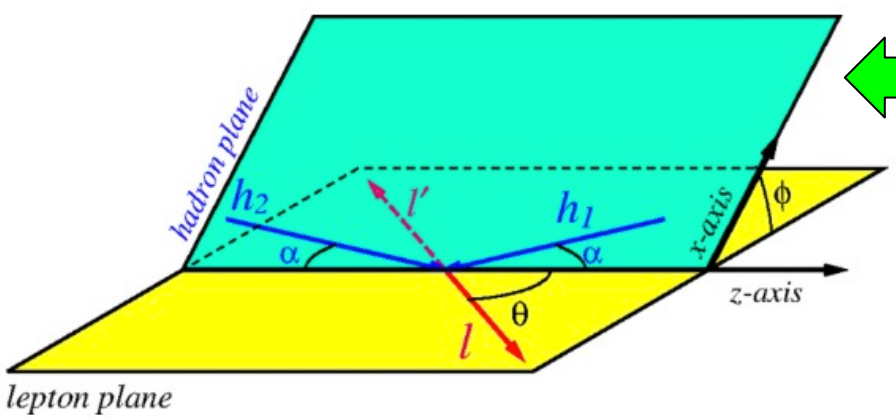
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



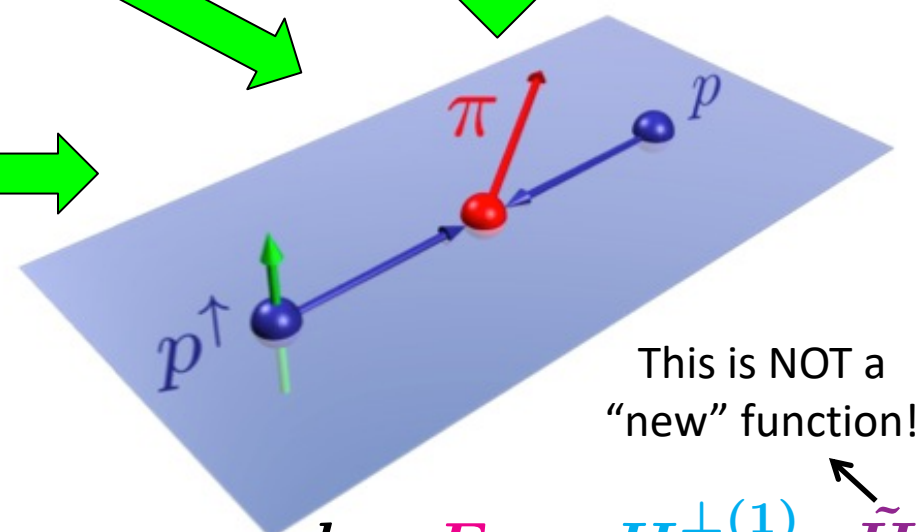
h_1, F_{FT}, $H_1^{\perp(1)}$



$H_1^{\perp(1)}$



F_{FT}

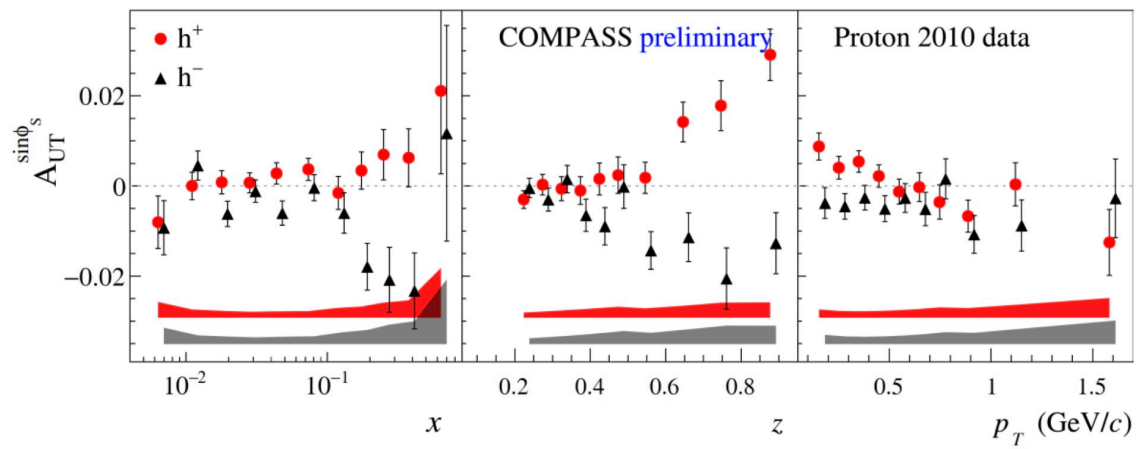


This is NOT a "new" function!

h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T (Mulders, Tangerman (1996); Bacchetta, et al. (2007))

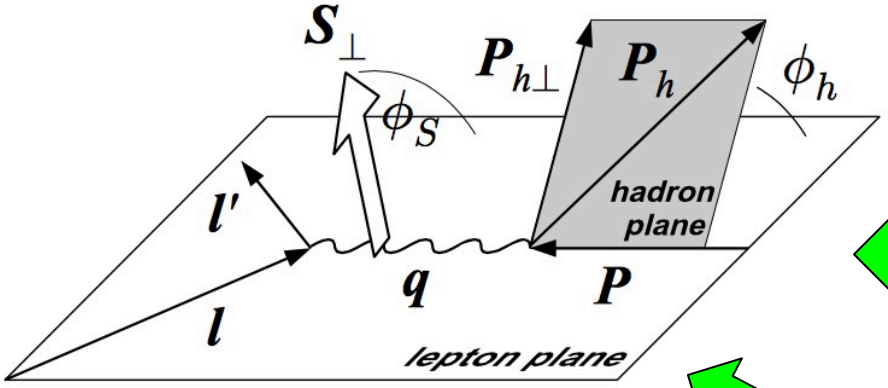
$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}^a(z)}{z}$$



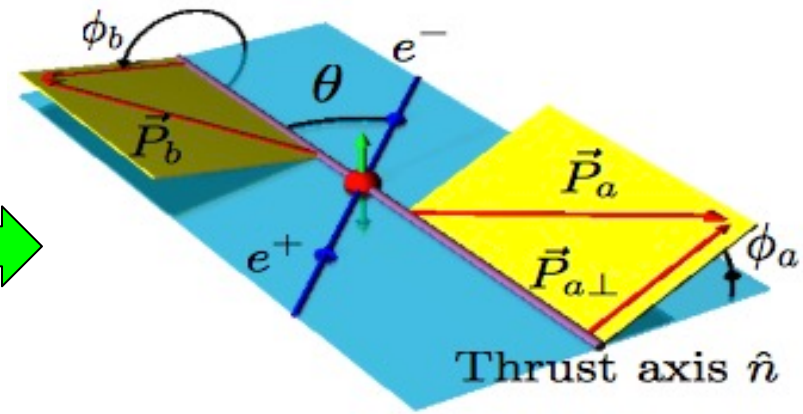
$A_{UT}^{\sin\phi_S}$ in $e^+e^- \rightarrow h_1 h_2 X$ integrated over q_T (Boer, Jakob, Mulders (1997))

$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}(z_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

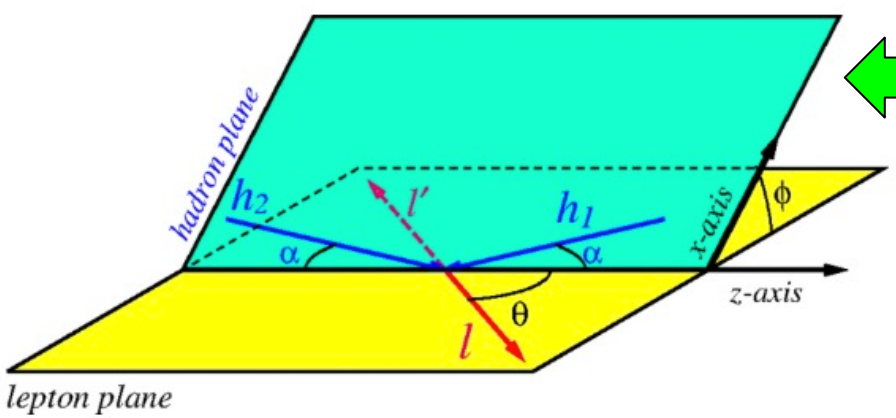
And also the TMD version of these (and other) observables (but with many more terms)



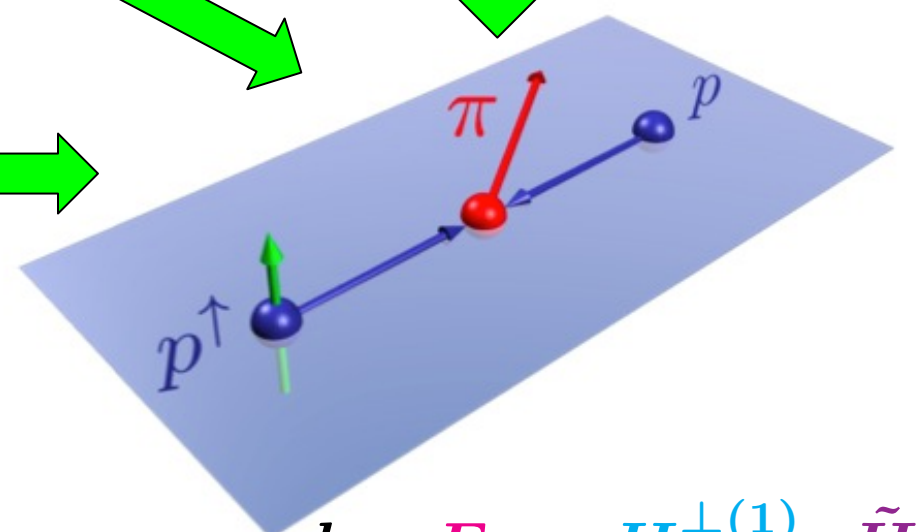
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

- What follows are *very preliminary* results of a global fit of
  1) Collins effect in e^+e^-
  2) Collins effect in SIDIS
  3) (Integrated) $A_{UT}^{\sin\phi_s}$ in SIDIS
  4) A_N in proton-proton collisions

- The plots only show the results of a single max likelihood fit. Final results will eventually include Monte Carlo (MC) sampling to determine error bands. For now, we use a simple Gaussian ansatz for TMDs.

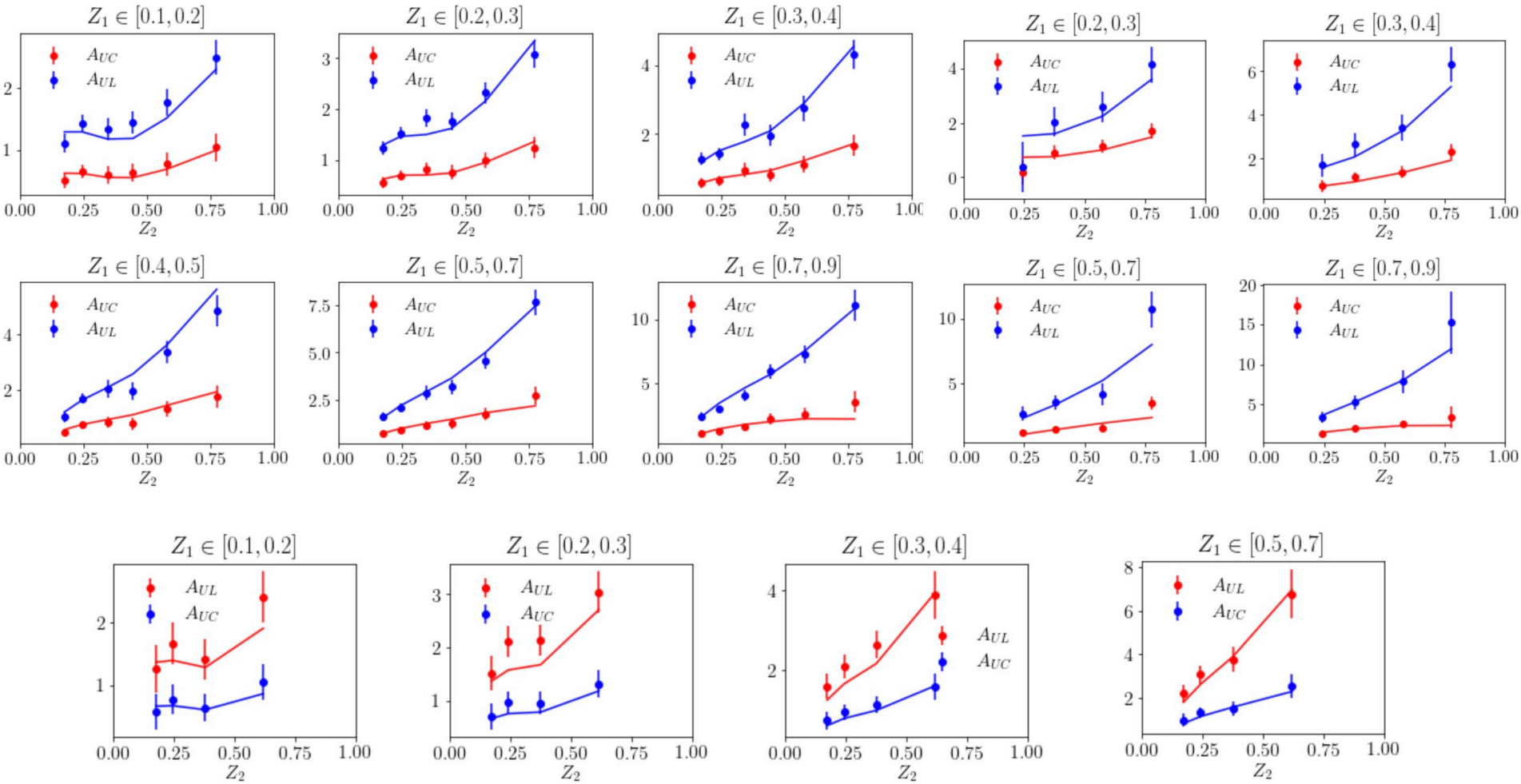
- We have found solutions for the relevant non-perturbative functions (including $\tilde{H}$!) that describe simultaneously a non-trivial amount of observables.

- Large errors in the (transversely polarized) deuteron SIDIS data make flavor separation subject to significant correlations which can only be estimated by MC – an EIC can hopefully deliver more accurate data.

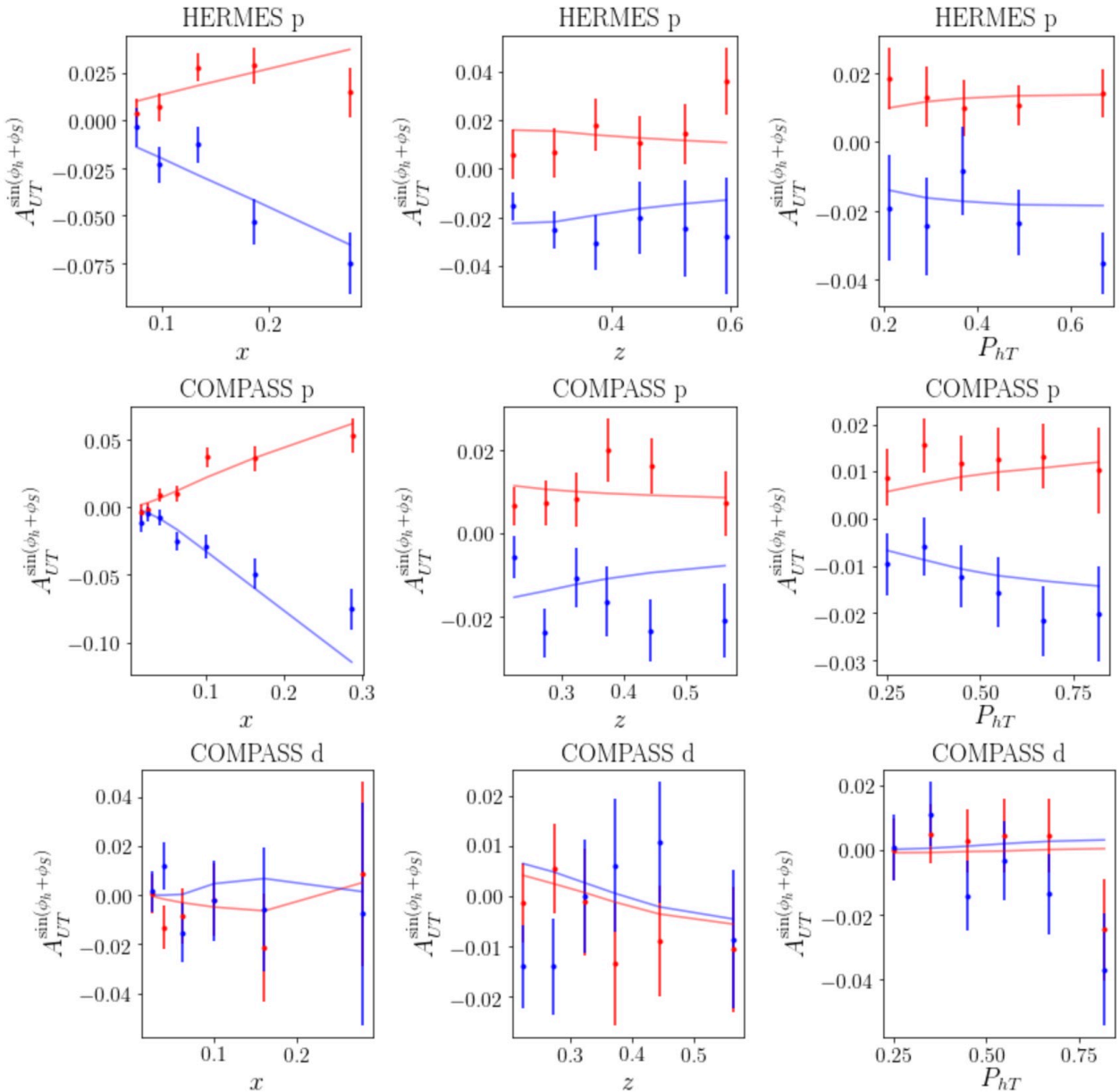
(Gamberg, Kang, DP, Prokudin, Sato, …, on-going work)

Collins effect e^+e^-

A_{UC} A_{UL}

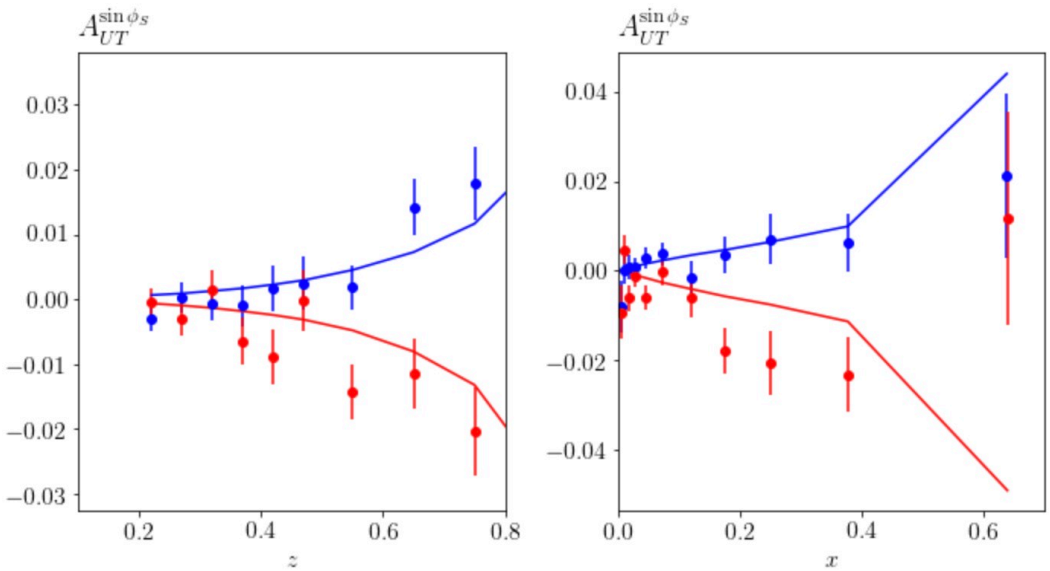


Collins effect SIDIS



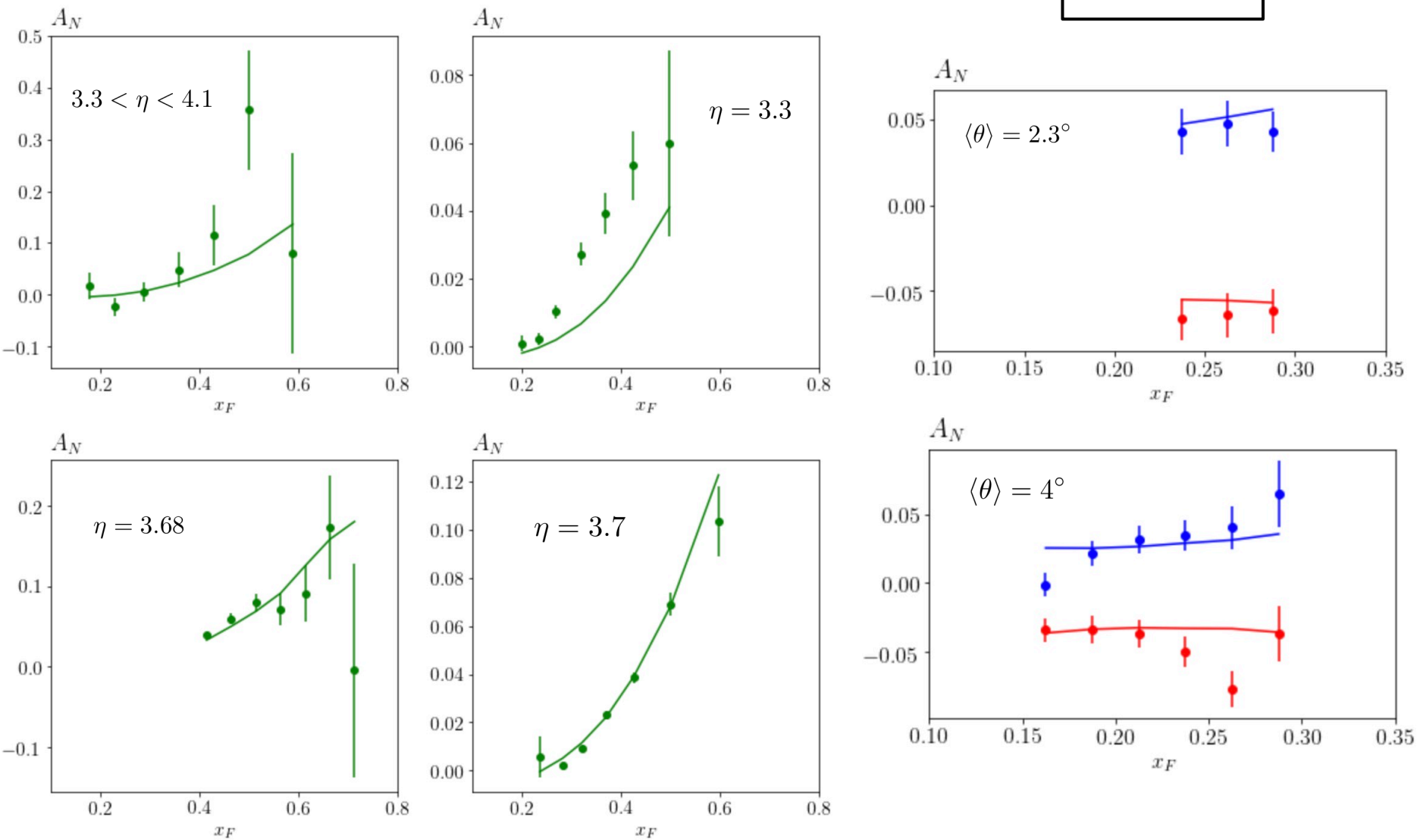
π^+ π^-

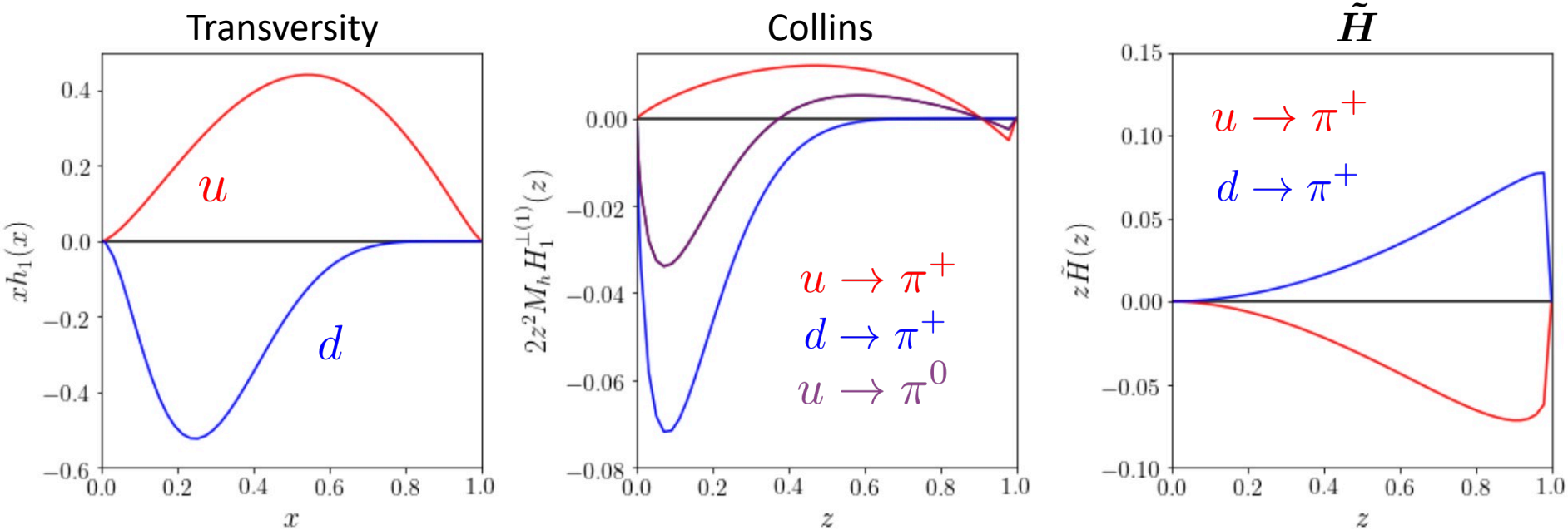
$A_{UT}^{\sin\phi_S}$ in SIDIS



h^+ h^-

A_N in *pp*


π^+ π^- π^0
A_N
$3.3 < \eta < 4.1$
$\eta = 3.3$
$\eta = 3.68$
$\eta = 3.7$
$\langle\theta\rangle = 2.3°$
$\langle\theta\rangle = 4°$
x_F


Transversity
$xh_1(x)$
x
u
d
Collins
$2z^2 M_h H_1^{\perp(1)}(z)$
z
$u \to \pi^+$
$d \to \pi^+$
$u \to \pi^0$
$\tilde{H}$
$z\tilde{H}(z)$
z
$u \to \pi^+$
$d \to \pi^+$

Summary

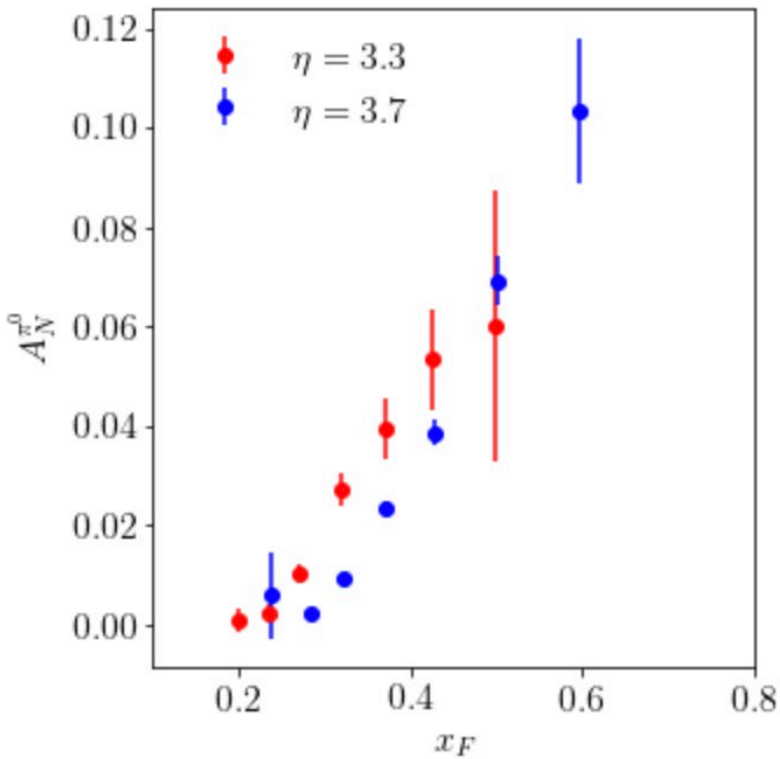
- TMD and collinear functions are highly interconnected, especially for reactions involving transverse spin, and we should treat both types of observables on the same footing.

- The current TMD formalism using "Improved CSS" allows one to rigorously connect these two different types of functions, and at LO we can restore the physical interpretation of (integrated) TMDs.

- A global analysis can be performed of TMD (Sivers and Collins effects) *AND* collinear twist-3 (A_N in *pp*, $A_{UT}^{\sin\phi_s}$ in SIDIS) transverse-spin observables.

- In addition to the Sivers and Collins effects that will be measured at a future EIC (with improved statistics needed for deuterium), we must also include measurements of A_N in electron-nucleon collisions.

Back-up Slides



There is an *increase* in A_N with P_T for $P_T < 2$ GeV. Need to see if evolution effects can account for this.