Mesons Structure in the Nuclear Medium

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25 de May, 2018

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- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**

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- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian:** $P^2 = P^+P^- - P_\perp^2$

Light-Front Coordinates

Four-Vector $\Longrightarrow$ $x^\mu = (x^0, x^1, x^2, x^3) \; = \; (x^+, x^-, x_\perp)$

$x^+ = t + z \qquad x^+ = x^0 + x^3 \quad \Longrightarrow$ **Time**

$x^- = t - z \qquad x^- = x^0 - x^3 \quad \Longrightarrow$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$p^+ = p^0 + p^3 \; , \; p^- = p^0 - p^3 \; , \; p^\perp = (p^1, p^2)$

Dirac Matrix and Electromagnetic Current

$$\gamma^+ = \gamma^0 + \gamma^3 \implies \textbf{Electr. Current} \quad J^+ = J^0 + J^3$$
$$\gamma^- = \gamma^0 - \gamma^3 \implies \textbf{Electr. Current} \quad J^- = J^0 - J^3$$
$$\gamma^\perp = (\gamma^1, \gamma^2) \implies \textbf{Electr. Current} \quad J^\perp = (J^1, J^2)$$

$$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, \vec{x_\perp} \implies p^+, p^-, \vec{p}_\perp$$

$p^- \implies$ **Light-Front Energy**

$$p^2 = p^+p^- - (\vec{p}_\perp)^2 \implies \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$

On-shell

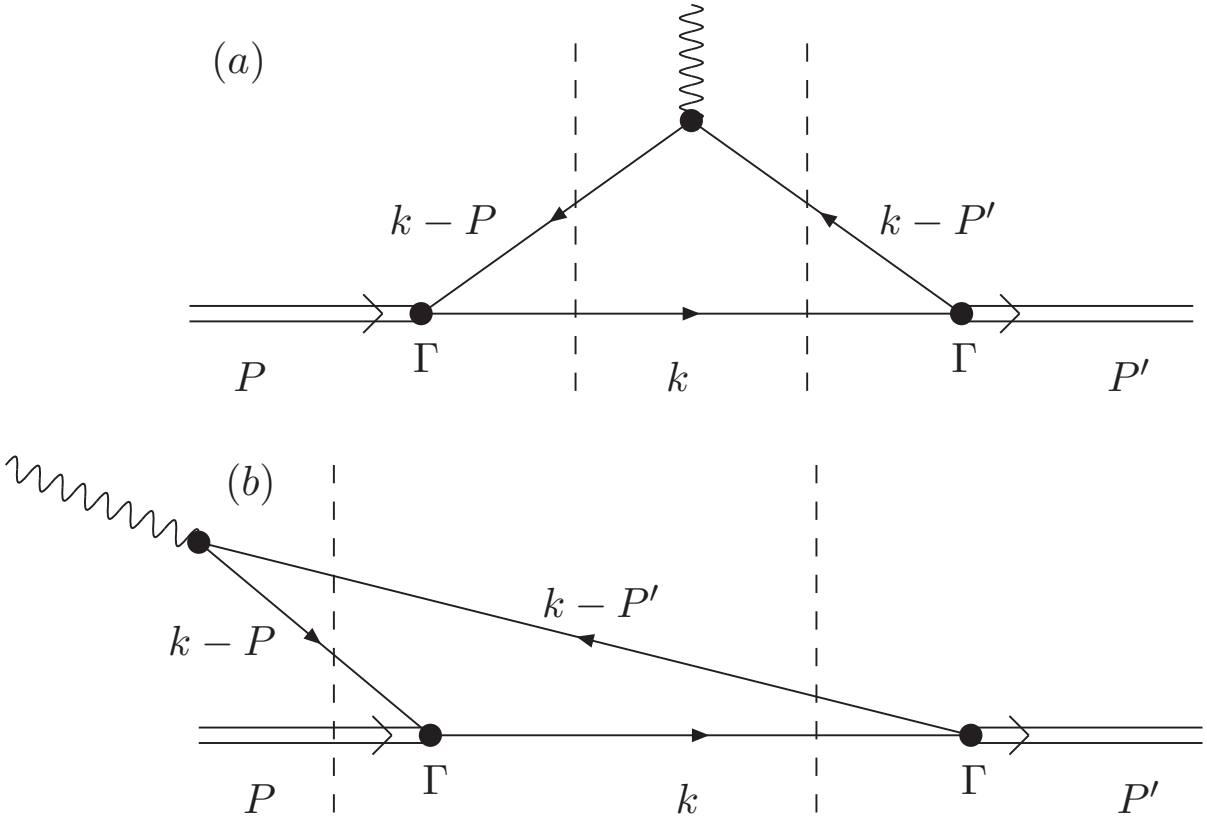
Bosons $\implies \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\implies \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- **Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky**
- **A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.**
- **An Introduction to Light-Front Dynamics for Pedestrians Avaroth Harindranath**

Light-Front book organizers: James Vary and Frank Wolz,(1997)



(a) ⇒ Valence Component of the Electromagnetic Current

(b) ⇒ Non-Valence Component of the Electromagnetic Current

Ref.: de Melo and Frederico, PRC (1997) , de Melo, Naus, Frederico and Sauer, PRC(1999)

Effective Lagrangian to Vertex $\pi \to q\bar{q}$

$$\mathcal{L}_I = -\imath \frac{m}{f_\pi} \vec{\pi} \cdot \overline{q} \gamma^5 \vec{\tau} q$$

- **Electromagnetic Current:** J_π^+

$$\begin{aligned} J^\mu &= -\imath 2e \frac{m^2}{f_\pi^2} N_c \int \frac{d^4k}{(2\pi)^4} Tr[S(k)\gamma^5 \\ &\times S(k-P')\gamma^\mu S(k-P)\gamma^5 \Lambda(k,P')\Lambda(k,P)] \end{aligned}$$

$S(p) = \frac{1}{\not{p} - m + \imath\epsilon}$

- **Vertex Function**
- Symmetric Vertex Function

$$\Lambda(k,P) = \frac{N}{(k^2 - m_R^2 + \imath\epsilon)} + \frac{N}{((P-k)^2 - m_R^2 + \imath\epsilon)}$$

- Ref. Nucl. Phys. **A 707** (2002) 399-424
- Nonsymmetric Vertex Function

$$\Lambda(k,P) = \frac{N}{((P-k)^2 - m_R^2 + \imath\epsilon)}$$

- J.P.B.C. de Melo, T. Frederico and H.L. Naus, Phy.Rev. **C59** (1999) 2278

- Frame

$$
\begin{aligned}
q^+ &= -q^- = \sqrt{-q^2}\sin\alpha \\
q_x &= \sqrt{-q^2}\cos\alpha, \;\; q_y = 0 \\
q^2 &= q^+q^- - (q_\perp)^2 \; .
\end{aligned}
$$

- Breit Frame $(\alpha = 0) \implies q^+ \to 0 \; , q^- = 0 \; ; \; \vec{q} \neq 0$
- $J_\pi^+ = J^0 + J^3 \implies$ No Pair Term Contribuition
- $J_\pi^- = J^0 - J^3 \implies$ Pair Term Contribuition
- **de Melo, Frederico, Pace and Salmé, NPA 707 (2002) 399**
- **de Melo, Frederico and Naus, PRC 59 (1999) 2278**

Wave Function, Pion // Kaon

$$\begin{aligned}\Psi(x,k_\perp,p^+,\vec{p}_\perp) &\propto \left[\frac{1}{(1-x)(m_{0^-}^2-\mathcal{M}^2(m_q^2,m_R^2))}\right.\\ &+ \left.\frac{1}{x(m_{0^-}^2-\mathcal{M}^2(m_R^2,m_{\bar{q}}^2))}\right]\frac{1}{m_{0^-}^2-\mathcal{M}^2(m_q^2,m_{\bar{q}}^2)},\\ &+ [q\leftrightarrow\bar{q}]\end{aligned}$$

here:

$$\mathcal{M}^2(m_a^2,m_b^2)=\frac{k_\perp^2+m_a^2}{x}+\frac{(p-k)_\perp^2+m_b^2}{(1-x)}-p_\perp^2$$

- **In the case of quarks mass** $\Rightarrow$ **Free Mass Operador**
- m_{0^-} $\Longleftrightarrow$ **Mass of the bound state**

- **Motivation: The present model work well (Symmetric Vertex)!**

Observables: Decay constant and charge radius						
	f_{0^-} (MeV)	r_{0^-}	m_u (π^+)	m_d (π^+)	m_d (K^+)	$m_{\bar{s}}$ (K^+)
Pion	**93.12** **101.85**	**0.736** **0.670**	**220** **250**	**220** **250**		
Kaon	**101.81** **113.74**	**0.754** **0.687**			**220** **250**	**440** **440**

$m_R = 600\ MeV$, **(all masses in MeV and radius in fm)**
Ex.(Pion): $f_\pi = 92.4 \pm 0.021\ MeV$, $r_\pi = 0.672 \pm 0.08\ fm$ **(PDG)**
Ex.(Kaon): $f_{k^+} = 110.38 \pm 0.1413\ MeV$, $r_{k^+} = 0.560 \pm 0.031$ **(PDG)**

- **Ref.: de Melo, Frederico, Pace and Salmè, NPA707, 399 (2002); ibid., Braz. J. Phys. 33, 301 (2003)**
- **Yabusaki, Ahmed, Paracha, de Melo, El-Bennich,PRD92 (2015) 034017.**

Quark Meson Coupling Model (QMC): Basic Ingredients

- **QMC Lagrangian:**

$$\mathcal{L} = \bar{\psi}[i\gamma \cdot \partial - m_N^*(\hat{\sigma}) - g_\omega \hat{\omega}^\mu \gamma_\mu]\psi + \mathcal{L}_{\mathbf{meson}}$$

- ψ, $\hat{\sigma}$ **and** $\hat{\omega}$**: Nucleon, Lorentz-scalar-isoscalar** σ**, and Lorentz-vector-isoscalar** ω **field operators,**
- σ**-Field Coupling Constant:**

$$m_N^*(\hat{\sigma}) = m_N - g_\sigma(\hat{\sigma})\hat{\sigma},$$

- **Free meson Lagrangian is:**

$$\mathcal{L}_{\text{meson}} = \frac{1}{2}(\partial_\mu \hat{\sigma} \partial^\mu \hat{\sigma} - m_\sigma^2 \hat{\sigma}^2) - \frac{1}{2}\partial_\mu \hat{\omega}_\nu (\partial^\mu \hat{\omega}^\nu - \partial^\nu \hat{\omega}^\mu) + \frac{1}{2} m_\omega^2 \hat{\omega}^\mu \hat{\omega}_\mu \ ,$$

- **Present work: Nuclear matter in Rest Frame**
- **Also: Symmetric Nuclear Matter Case: (Mean-field Approximation)**
- **Nucleon Fermi momentum k_F // scalar density, Conected Sigma-mean Field**

$$\rho\ (Baryon) = \frac{4}{(2\pi)^3}\int d\vec{k}\ \theta(k_F - |\vec{k}|) = \frac{2k_F^3}{3\pi^2},$$

$$\rho_s\ (Scalar) = \frac{4}{(2\pi)^3}\int d\vec{k}\ \theta(k_F - |\vec{k}|)\frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + \vec{k}^2}},$$

- $m_N^*(\sigma)$: **Effective nucleon mass at some density: with QMC model**

- **Dirac Equation:** **Light quark and antiquark**

$$\left[i\gamma\cdot\partial_x-(m_q-V_\sigma^q)\mp\gamma^0\left(V_\omega^q+\frac{1}{2}V_\rho^q\right)\right]\begin{pmatrix}\psi_u(x)\\ \psi_{\bar{u}}(x)\end{pmatrix}=0,$$
$$\left[i\gamma\cdot\partial_x-(m_q-V_\sigma^q)\mp\gamma^0\left(V_\omega^q-\frac{1}{2}V_\rho^q\right)\right]\begin{pmatrix}\psi_d(x)\\ \psi_{\bar{d}}(x)\end{pmatrix}=0,$$

- **Coulomb Interaction:** **Neglete**
- $SU(2)$ **Symmetry:** $\underline{m_q=m_u=m_d}$

- Bag radius in medium: R^*_ρ $\Rightarrow$ **Stability Condition for the mass of the Hadron**

Obs. Eigenenergies in units of $1/R^*_\rho$

$$\begin{aligned}
\begin{pmatrix} \epsilon_u \\ \epsilon_{\bar{u}} \end{pmatrix} &= \Omega^*_q \pm R^*_\rho \left(V^q_\omega + \frac{1}{2} V^q_\rho \right), \\
\begin{pmatrix} \epsilon_d \\ \epsilon_{\bar{d}} \end{pmatrix} &= \Omega^*_q \pm R^*_\rho \left(V^q_\omega - \frac{1}{2} V^q_\rho \right), \\
\epsilon_Q &= \epsilon_{\bar{Q}} = \Omega_Q .
\end{aligned}$$

- **Rho Meson Masse,** m^*_ρ**:**

$$m^*_h = \sum_{j=q,\bar{q}} \frac{n_j \Omega^*_j - z_\rho}{R^*_\rho} + \frac{4}{3}\pi R^{*3}_\rho B, \qquad \left.\frac{\partial m^*_\rho}{\partial R_\rho}\right|_{R_\rho = R^*_\rho} = 0, \tag{1}$$

- $\Omega^*_q = \Omega^*_{\bar{q}} = [x^2_q + (R^*_\rho m^*_q)^2]^{1/2}$, $m^*_q = m_q - g^q_\sigma \sigma$,
- x_q **Lowest bag eigenfrequencies**
- **QMC Review: K. Saito, K. Tsushima and A. W. Thomas**
  Prog. Part. Nucl. Phys. 58 (2007) 1

Table : The MIT bag model quantities and coupling constants, the parameter Z_N, bag constant B (in $B^{1/4}$), and the properties for symmetric nuclear matter at normal nuclear matter density $\rho_0 = 0.15$ fm^{-3}, for $m_q = 5,\ 220$ and 430 MeV. The effective nucleon mass, m_N^*, and the nuclear incompressibility, K, are quoted in MeV (the free nucleon bag radius used is $R_N = 0.8$ fm, the standard value in the QMC model.

m_q(MeV)	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	m_N^*	K	Z_N	$B^{1/4}$(MeV)
5	5.39	5.30	754.6	279.3	3.295	170
220	6.40	7.57	698.6	320.9	4.327	148
430	8.73	11.93	565.25	361.4	5.497	69.75

QMC (Quark Meson Coupling)* "plus" Light-Front

- **Hatree Mean Field Approximation:** $\Rightarrow \quad p^\mu \longrightarrow p^\mu + V^\mu$

- **Vector**

$$p^\mu \pm \delta_0^\mu V_\omega^q = \begin{cases} +\,, & \textbf{quark} \\ -\,, & \textbf{antiquark} \end{cases}$$

- **Scalar:** V_s

$\Rightarrow \quad m_q \longrightarrow m_q^* + V_s$ **here** $V_s = m_q - V_\sigma^q$

$\star$ **Ref.: K. Saito, K. Tsushima and A. W. Thomas, Progress Part. Phys. 58 (2007) 1**

- **Propagadors for Quarks in the Mediun**

$$S^*(p+V) = \frac{1}{\not{p} - \not{V} - m_q^* + \imath\epsilon}$$

- **Vertex $q\pi\bar{q}$ in Medium**

$$\Lambda^*(k+V,P) = \frac{C^*}{((k+V)^2 - m_R^2 + i\epsilon)} + \frac{C^*}{((P-k-V)^2 - m_R^2 + i\epsilon)}$$

- **Effective Lagrangian in the Medium**

$$\mathcal{L}_I = -ig^*\vec{\Phi}\cdot\overline{q}\gamma^5\vec{\tau}q\,\Lambda^*$$

⋆ **de Melo, K. Tsushima, B. El-Bennich, E. Rojas and T, Frederico, PRC90 (2014) 035201**

Valence Light-front wave function in the Medium

$$\Phi^*(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp) = \frac{P^+}{m_\pi^{*2} - M_0^2} \left[\frac{N^*}{(1-x)(m_\pi^{*2} - \mathcal{M}^2(m_q^{*2}, m_R^2))} + \frac{N^*}{x(m_\pi^{*2} - \mathcal{M}^2(m_R^2, m_q^{*2}))} \right]$$

- $x = k^+/P^+$, **with** $0 \leq x \leq 1$
- $\mathcal{M}^2(m_a^2, m_b^2) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(P-k)_\perp^2 + m_b^2}{1-x} - P_\perp^2$
- **Free Square Mass operador:** $M_0^2 = \mathcal{M}^2(m_q^{*2}, m_q^{*2})$.

- **Pion Electromagnetic Form Factor in the Breit, $q^+ = 0$:**

$$F_\pi^{*(WF)}(q^2) = \frac{1}{2\pi^3(P'^+ + P^+)} \int \frac{d^2k_\perp dk^+ \theta(k^+)\theta(P^+ - k^+)}{k^+(P^+ - k^+)(P'^+ - k^+)} \Phi^*(k^+, \vec{k}_\perp; P'^+, \tfrac{\vec{q}_\perp}{2})$$
$$\times \left(k^-_{\rm on} P^+ P'^+ - \tfrac{1}{2}\vec{k}_\perp \cdot \vec{q}_\perp (P^+ - P'^+) - \tfrac{1}{4}\, k^+ q_\perp^2 \right)$$
$$\times\, \Phi^*(k^+, \vec{k}_\perp; P^+, -\tfrac{\vec{q}_\perp}{2})$$

- $C^* \quad \Rightarrow \quad F_\pi^*(0) = 1$

- **Transverse momentum probability density**

$$f^*(k_\perp) = \frac{1}{4\pi^3 m_\pi^*}\int_0^{2\pi} d\phi \int_0^{P^+} \frac{dk^+ M_0^{*2}}{k^+(P^+ - k^+)}\Phi^{*2}(k^+, \vec{k}_\perp; m_\pi^*, \vec{0}),$$

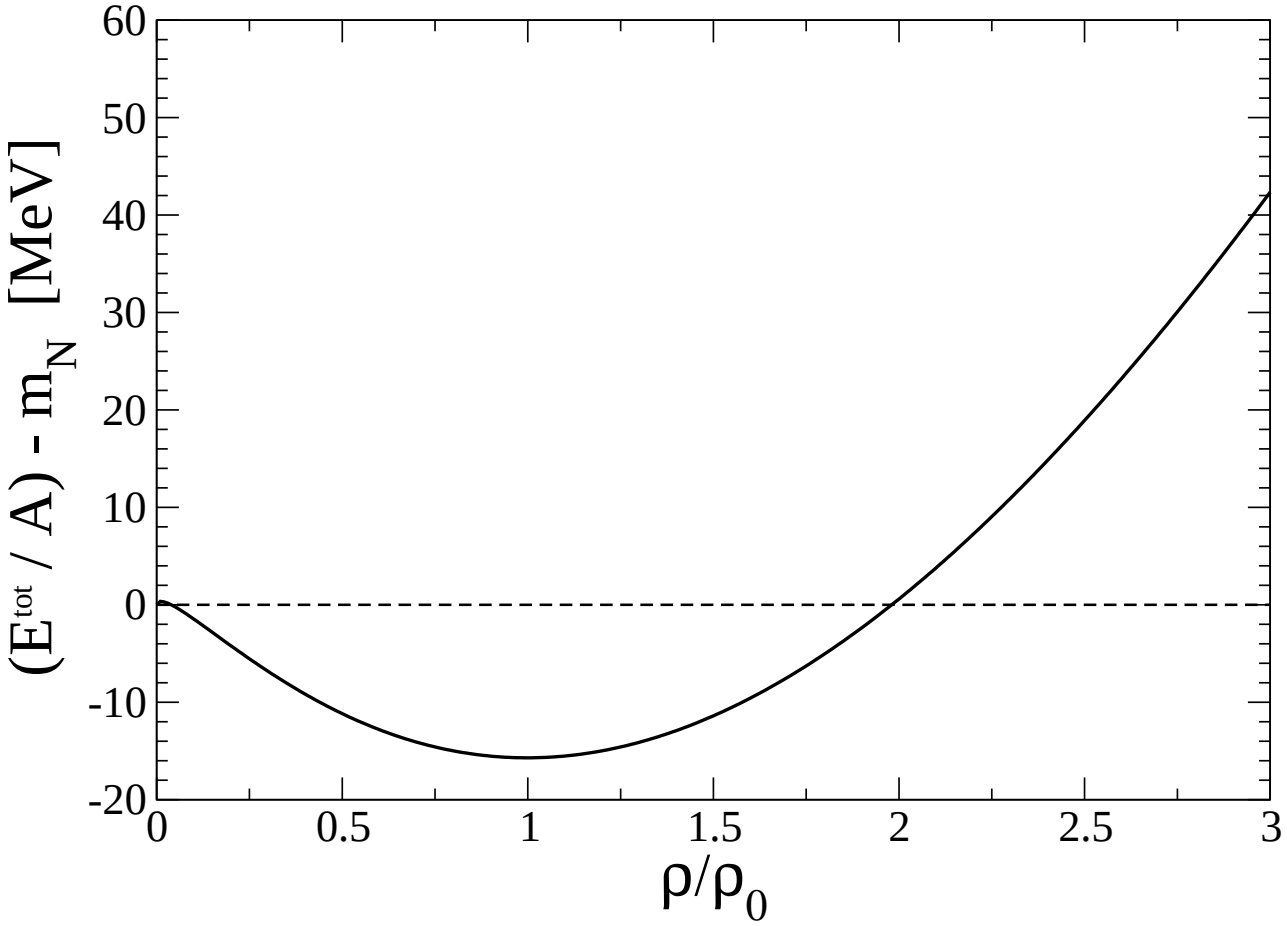
- **Integration of $f^*(k_\perp)$: Probability of the valence component in the pion**

$$\eta^* = \int_0^\infty dk_\perp k_\perp f^*(k_\perp).$$

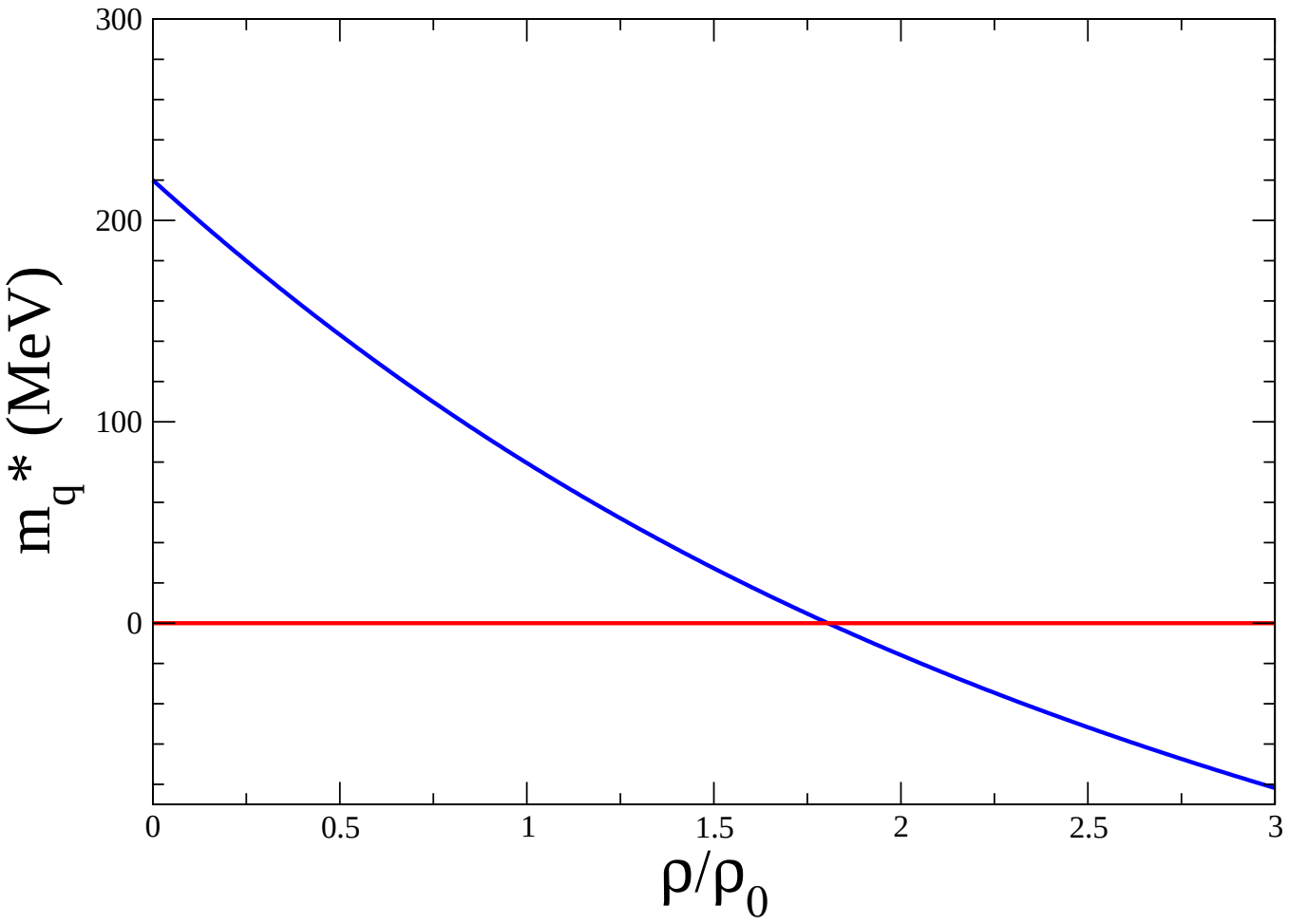
- **Pion decay constant**

$$P_\mu \langle 0(\rho)|A_i^\mu|\pi_j^*\rangle = i m_\pi^{*2} f_\pi^* \delta_{ij} \simeq i m_\pi^2 f_\pi^* \delta_{ij}.$$

Results: Quark Meson Coupling + Light-Front



- Symmetric Nuclear Matter - Biding Energy per Nucleon



- **Effective mass of constituent quarks, up and down**

Pion properties in medium. η^* is the probability of the valence component in the pion.

ρ/ρ_0	m_q^* [MeV]	f_π^* [MeV]	$< r_\pi^{*2} >^{1/2}$ [fm]	η^*
0.00	220	93.1	0.73	0.782
0.25	179.9	80.6	0.84	0.812
0.50	143.2	68.0	1.00	0.843
0.75	109.8	55.1	1.26	0.878
1.00	79.5	40.2	1.96	0.930

- **GMOR (Gell-Mann-Oakes-Renner) Relation:**

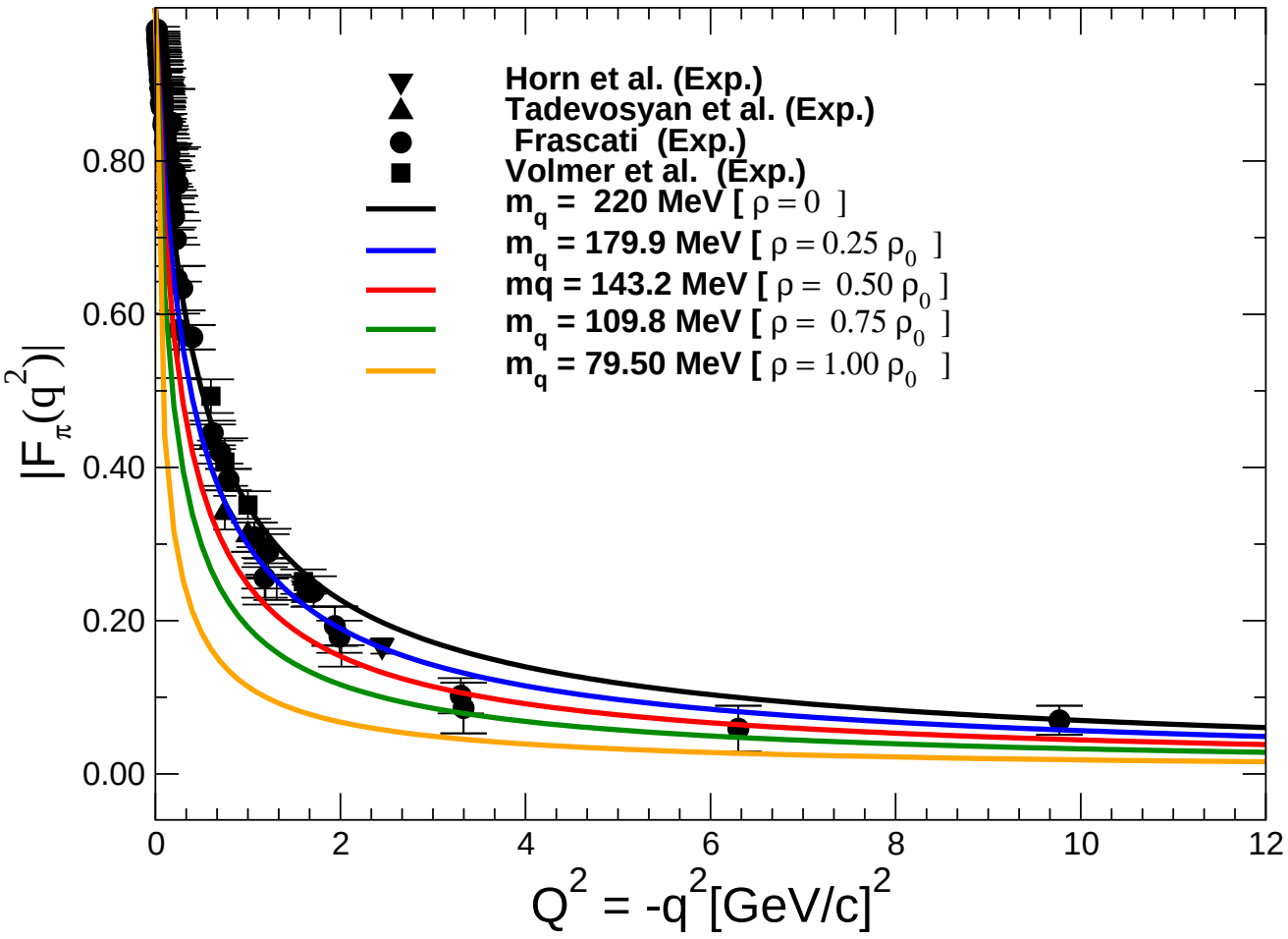
$$
\begin{aligned}
m_\pi^2 f_\pi^2 &= -2m_q < \overline{q}q >, \\
m_\pi^{*2} f_\pi^{*2} &= -2m_q^* < \overline{q}q >^*
\end{aligned}
$$

- **Vacuum quarks condensate in the medium**

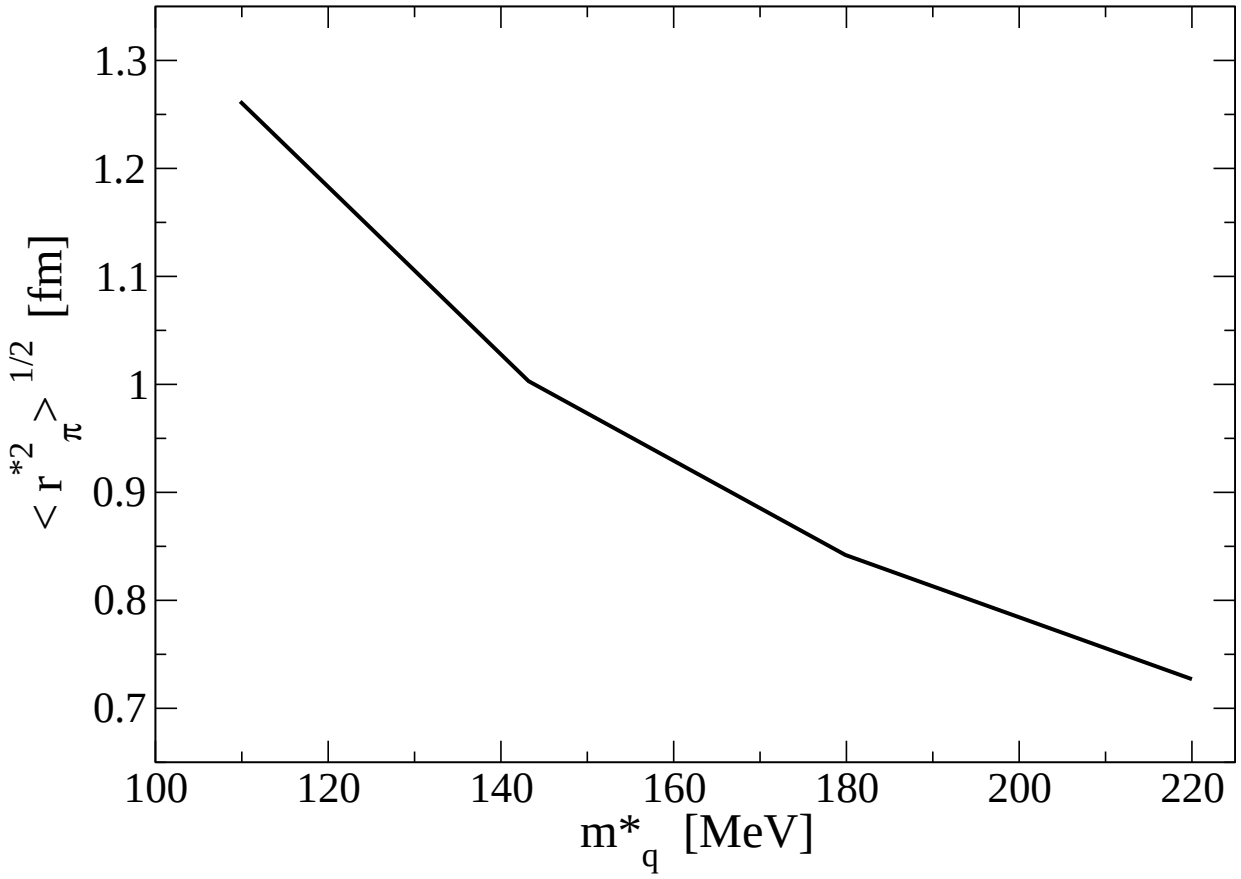
$$
\frac{< \overline{q}q >^*}{< \overline{q}q >} = \frac{m_q}{m_q^*} \frac{m_\pi^{*2} f_\pi^{*2}}{m_\pi^2 f_\pi^2} \simeq \frac{m_q}{m_q^*} \frac{f_\pi^{*2}}{f_\pi^2}
$$

- $\rho_0 = 0.15\ fm^{-3} \Longrightarrow \approx 0.52$ **(This work)**
- $\rho_0 = 0.17\ fm^{-3} \Longrightarrow \approx 0.67 \pm 0.06$

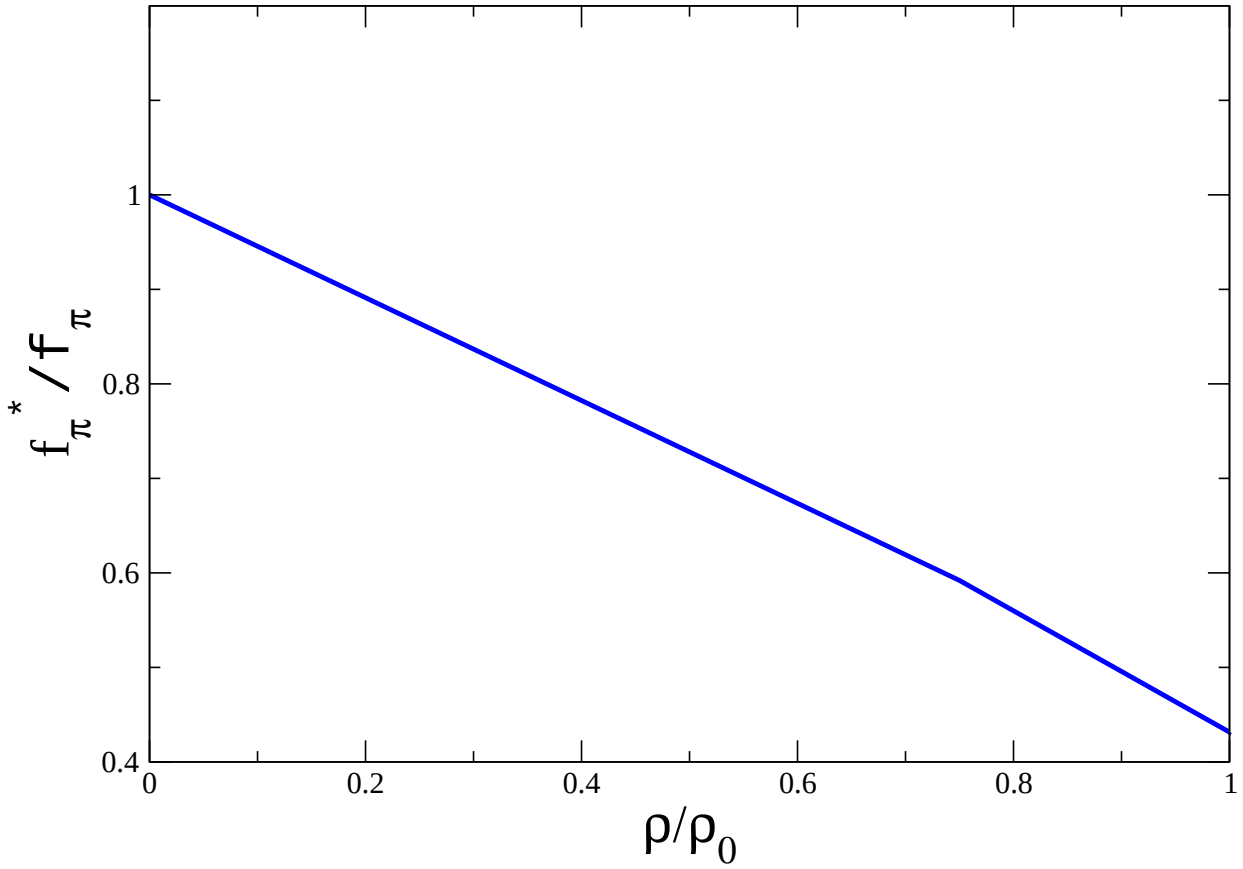
from Kienle & Yamazaki, Prog. Part. Nucl. Phys. 52 (2004) 85.



- **Exp. date in the Vacuum!!**



- **Pion Electromagnetic Radius**



- **Pion Decay Constant**

Kaon: First Results

- **Hartree mean field approximation**

$\Longrightarrow$ **Shift of the light-quark momentum**

- **Due to the vector potential:**

$$P^\mu \to P^{*\mu} = P^\mu + V^\mu = P^\mu + \delta_0^\mu V^0$$

- **Lorentz-scalar potential:**

$\to \; m_u^* = m_u + V_s$, $m_{\bar{s}} \to m_{\bar{s}}^* = m_{\bar{s}}$ **and** $(k^*)^\mu = k^\mu + \delta_0^\mu V^0$

- **Fermions propagadors**

$$S(p^*, m_u^*) = \frac{1}{(\not{p}^* - m_u^* + i\epsilon)}$$

$$S(p, m_s) = \frac{1}{(\not{p} - m_s + i\epsilon)}$$

- **G. Yabusaki, de Melo, W. de Paula, K. Tsushima and T. Frederico, Few Body Syst. 59 (2018) no.3, 37**

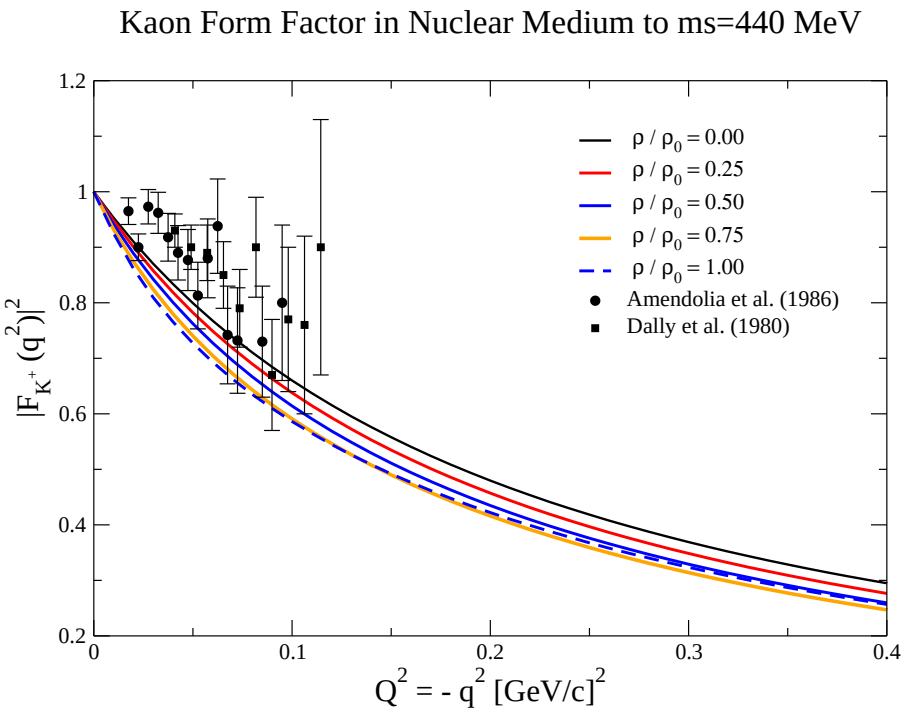
Kaon Results

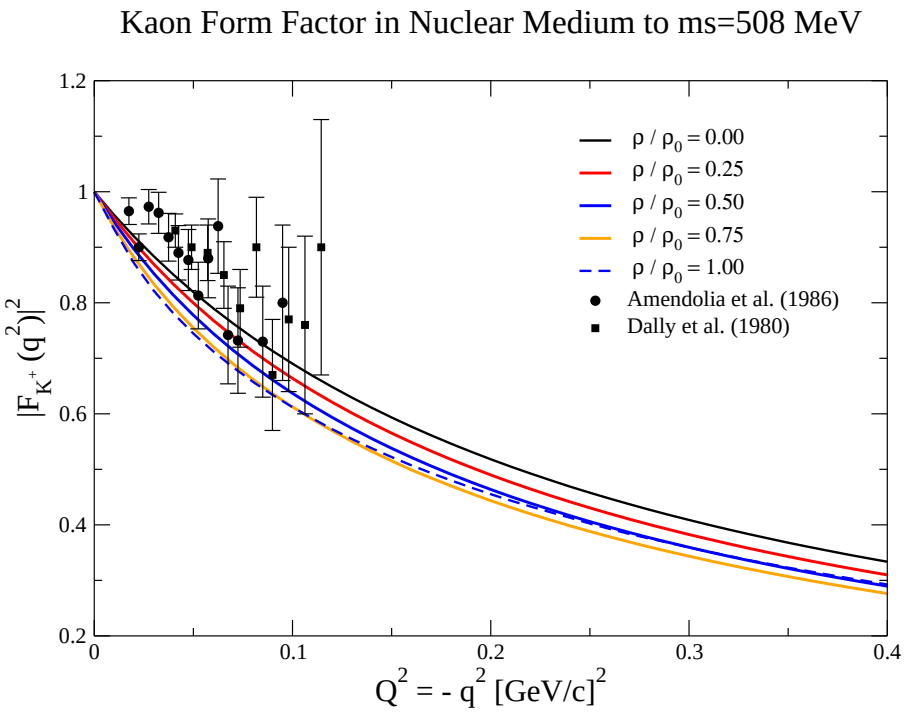
Table : **Parameters:** $m_{\bar{s}} = 0.220\ GeV$, $m_{\bar{s}} = 0.440\ GeV$ **and** $m_R = 0.600\ GeV$

ρ/ρ_0	$m_K^+\ [GeV]$	$m_u\ [GeV]$	$V\ [GeV]$	$< r_{K^+}^{*2} >^{1/2}\ [fm]$
0.00	0.494	0.220	0.000	0.755
0.25	0.472	0.180	0.029	0.798
0.50	0.452	0.143	0.058	0.856
0.75	0.435	0.110	0.087	0.927
1.00	0.419	0.079	0.117	1.014

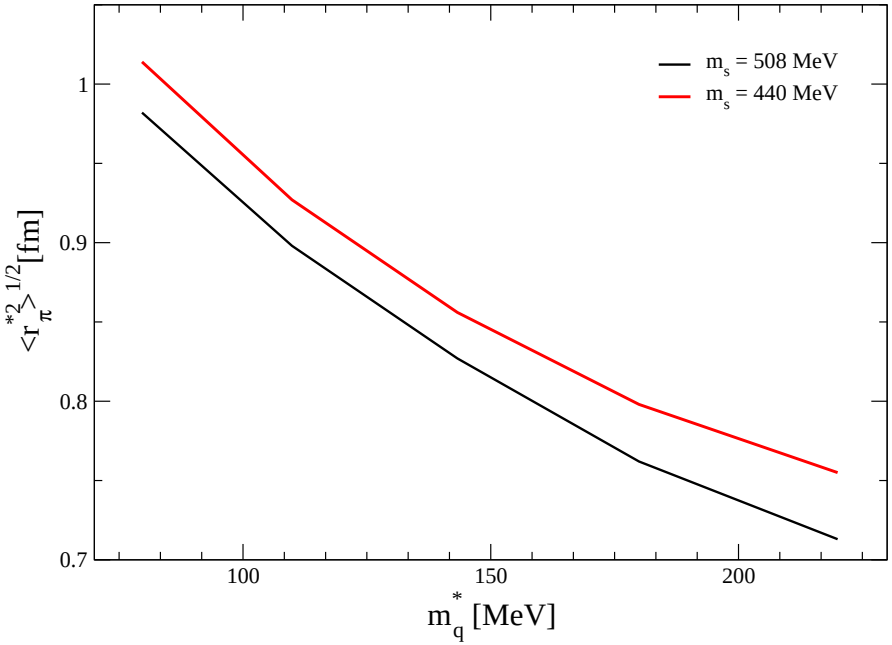
Table : **Parameters:** $m_{\bar{s}} = 0.220\ GeV$, $m_{\bar{s}} = 0.508\ GeV$ **and** $m_R = 0.600\ GeV$

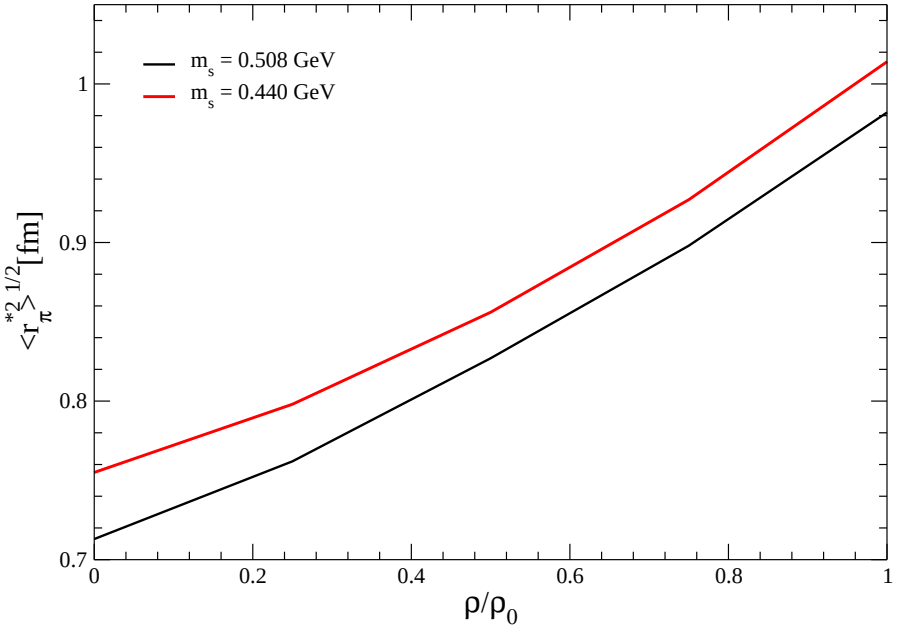
ρ/ρ_0	$m_K^+\ [GeV]$	$m_u\ [GeV]$	$V\ [GeV]$	$< r_{K^+}^{*2} >^{1/2}\ [fm]$
0.00	0.494	0.220	0.000	0.713
0.25	0.472	0.180	0.029	0.762
0.50	0.452	0.143	0.058	0.827
0.75	0.435	0.110	0.087	0.898
1.00	0.419	0.079	0.117	0.982





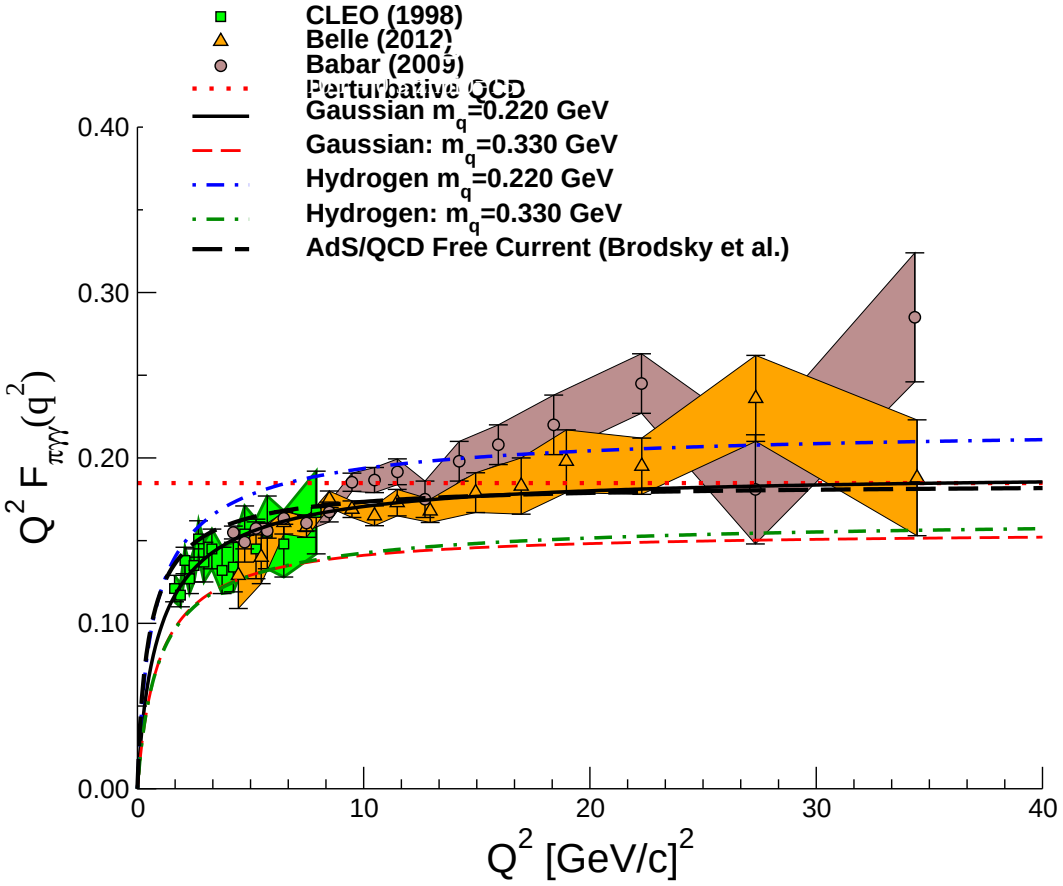
- $m_u = 220\ MeV$ **vacuum**





- $m_u = 220\ MeV$ **vacuum**

- **Hard Exclusive Pion Production**



$\Longrightarrow$ **Sensitive to Pion Quark Valence Distribution: Pion DA**

- **El-Bennich, de Melo, Frederico, Few Body Syst. 54 (2013) 1851-1863**

ibid. de Melo, El-Bennich, Frederico, Few Body Syst. 55 (2014) 373-379

Distribuition Amplitude

Def.: DA's

$$\phi^*_{ps}(x) = \frac{2\sqrt{6}}{f_{ps}} \int \frac{d^2\vec{k}_\perp}{(16\pi^3)} \Psi^*_{ps}(x, \vec{k}_\perp) \ .$$

- **Normalization:**

$$\int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \Psi^*(x, \vec{k}_\perp) \ = \ \frac{f^*_{ps}}{2\sqrt{6}}$$

- **Pion Asymptotic Wave Function**

$$\phi^{as}_\pi(x, \mu^2) \ \propto \ 6x(1-x)$$

- **de Melo, Tsushima and Ahmed, Phys. Lett. B 766, (2017) 125**
- **Tsushima,K. and de Melo, J., Few Body Syst. 58 (2017) 85**

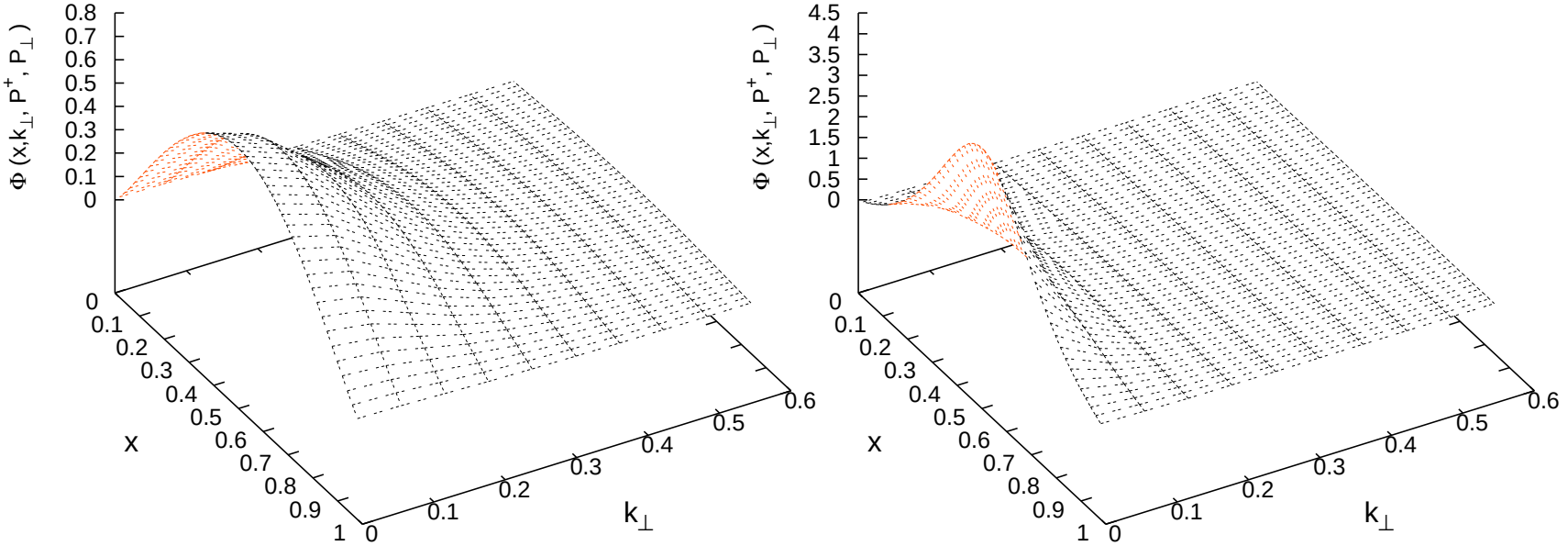


Figure : **Pion valence wave functions in vacuum ($\rho = 0$) [left panel] and in medium (ρ/ρ_0) [right panel] v.s. x and $k_\perp = |\vec{k}_\perp|$, where $P^+ = m_\pi = m_\pi^*$ and $P_\perp = |\vec{P}_\perp| = 0$. The wave functions are given in the units, $10^{-8}\times$(GeV)$^{-1}$. Notice that the differences in the vertical axis scales for the left and right panels.**

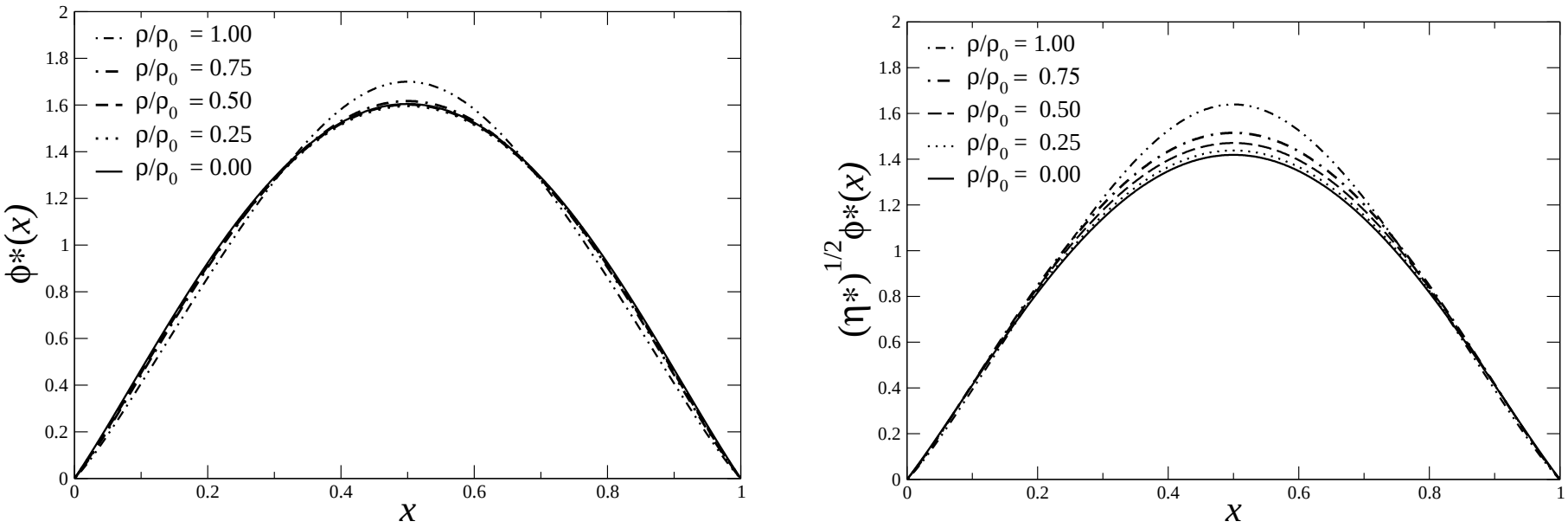


Figure : **Pion valence parton distribution functions (left panel). (Right) Effective pion valence parton distribution functions in vacuum and in medium, respectively multiplied by $\sqrt{\eta}$ and $\sqrt{\eta^*}$.**

General Electromagnetic Current: Spin-1

$$J^{\mu}_{\alpha\beta} = [F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_\alpha q_\beta}{2m_\rho^2}]p^\mu - F_3(q^2)(q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha) \ ,$$

- Polarization Vectors

$\epsilon^\mu_x = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$, $\epsilon^\mu_y = (0, 0, 1, 0)$, $\epsilon^\mu_z = (0, 0, 0, 1)$,
$\epsilon^{'\mu}_x = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$, $\epsilon^{'\mu}_y = \epsilon_y$, $\epsilon^{'\mu}_z = \epsilon_z$,
where $\eta = q^2/4m_\rho^2$

- Breit Frame:

$p^\mu_i = (p^0, -q_x/2, 0, 0)$ (Initial) where $p^0 = m_\rho\sqrt{1+\eta}$.
$p^\mu_f = (p^0, q_x/2, 0, 0)$ (Final)

$$J^{+}_{ji} = \imath \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon_j^{'\beta}\Gamma_\beta(k, k-p_f)(\not{k} - \not{p}_f + m)}{((k-p_i)^2 - m^2 + \imath\epsilon)(k^2 - m^2 + \imath\epsilon)}$$
$$\times \frac{\gamma^+(\not{k} - \not{p}_i + m)\epsilon_i^\alpha \Gamma_\alpha(k, k-p_i)(\not{k} + m)]\Lambda(k, p_f)\Lambda(k, p_i)}{((k-p_f)^2 - m^2 + \imath\epsilon}$$

- Regulator Function

$$\Lambda(k, p_{i(f)}) = N/((p-k)^2 - m_R + \imath\epsilon)^2$$

- ρ-Meson Vertex

$$\Gamma^\mu(k, p) = \gamma^\mu - \frac{m_\rho}{2}\frac{2k^\mu - p^\mu}{p.k + m_\rho m - \imath\epsilon}$$

- Mass Squared $\left(x = \frac{k^+}{P^+} \implies 0 < x < 1\right)$

$$M^2(m_a, m_b) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_b^2}{1 - x} - p_\perp^2$$

- Free Mass $M_0^2(m, m)$ and Function $M_R^2(m, m_R)$

The function M_R^2 is given by

$$M_R^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_R^2}{1 - x} - p_\perp^2$$

$$M_0^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m^2}{1 - x} - p_\perp^2$$

- Wave Function

$$\Phi_i(x,\vec{k}_\perp) = \frac{N^2}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2}\vec{\epsilon}_i.[\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2}+m}]$$

Refs.

- **Phy.Rev. C55 (1997) 2043 J.P.B. C. de Melo and T. Frederico**
- **Phy.Lett. B708 (2012) 87 J.P.B. C. de Melo and T. Frederico**
- **Few.Body.Syst. 52 (2012) 403, J.P.B. C. de Melo and T. Frederico**
- **Few Body Syst. 56, (2015) 509, C. S. Mello, A. N. da Silva, J. P. B. C. de Melo and T. Frederico**
- **Few Body Syst. 56, (2015) 503, J. P. B. C. de Melo, A. N. da Silva, C. S. Mello and T. Frederico**

- Instant-Form Spin Base

$$J^{+}_{ji} = \frac{1}{2}\begin{pmatrix} J^{+}_{xx} + J^{+}_{yy} & \sqrt{2}J^{+}_{zx} & J^{+}_{yy} - J^{+}_{xx} \\ -\sqrt{2}J^{+}_{zx} & 2J^{+}_{zz} & \sqrt{2}J^{+}_{zx} \\ J^{+}_{yy} - J^{+}_{xx} & -\sqrt{2}J^{+}_{zx} & J^{+}_{xx} + J^{+}_{yy} \end{pmatrix}$$

- Light-Front

$$I^{+}_{m'm} = \begin{pmatrix} I^{+}_{11} & I^{+}_{10} & I^{+}_{1-1} \\ -I^{+}_{10} & I^{+}_{00} & I^{+}_{10} \\ I^{+}_{1-1} & -I^{+}_{10} & I^{+}_{11} \end{pmatrix}$$

$$\Delta(q^2) = (1+2\eta)I^+_{11} + I^+_{1-1} - \sqrt{8\eta}I^+_{10} - I^+_{00} = (1+\eta)(J^+_{yy} - J^+_{zz}) = 0$$

- **Angular Condition: Violation !!**

$$q_x \Longrightarrow J^+_{yy} = J^+_{zz} \begin{cases} Parity \\ + \\ Rotations \end{cases}$$

$$\Delta(q^2) \neq 0$$

- **Ref:**
- **Sov. J. Nucl. Phys. 39 (1984) 198**

I.Grach and L.A. Kondratyku

- **Phy. Rev. Lett. 62 (1989) 387**

L.L. Frankfurt, I.Grach, L.A. Kondratyku and M. Strikman

Prescriptions

$$\begin{cases} FFS\ (Frederico, Frankfurt, Strikman) \\ GK\ (Grach, Kondratyku) \\ CCKP\ (Coester, Chung, Keister, Polyzou) \\ BH\ (Brodsky, Hiller) \end{cases}$$ **vs** **COVARIANT**

- **Breit Frame** $\Longrightarrow P^+ = P'^+$, $P^- = P'^-$, $\vec{P'}_\perp = -\vec{P}_\perp = \vec{q}/2$
- **B.F:** $q^+ = q^0 + q^3 = 0$
- J^+_ρ $\begin{cases} 4\ Current\ Matrix\ Elements \\ 3\ Form\ Factors\ \ G_0\ ,\ G_1\ and\ G_2 \end{cases}$

Inna Grach Prescription: I^{+}_{00}

$$
\begin{aligned}
G_0^{GK} &= \frac{1}{3}[(3-2\eta)I^{+}_{11} + 2\sqrt{2\eta}I^{+}_{10} + I^{+}_{1-1}] = \\
&\quad \frac{1}{3}[J^{+}_{xx} + \eta J^{+}_{zz}(2-\eta)J^{+}_{yy}] \\
G_1^{GK} &= 2[I^{+}_{11} - \frac{1}{\sqrt{2\eta}}I^{+}_{10}] = J^{+}_{yy} - J^{+}_{zz} - \frac{J^{+}_{zx}}{\sqrt{\eta}} \\
G_2^{GK} &= \frac{2\sqrt{2}}{3}[-\eta I^{+}_{11} + \sqrt{2\eta}I^{+}_{10} - I^{+}_{1-1}] = \\
&\quad \frac{\sqrt{2}}{3}[J^{+}_{xx} + J^{+}_{yy}(-1-\eta) + \eta J^{+}_{zz}] \ .
\end{aligned}
$$

CCKP

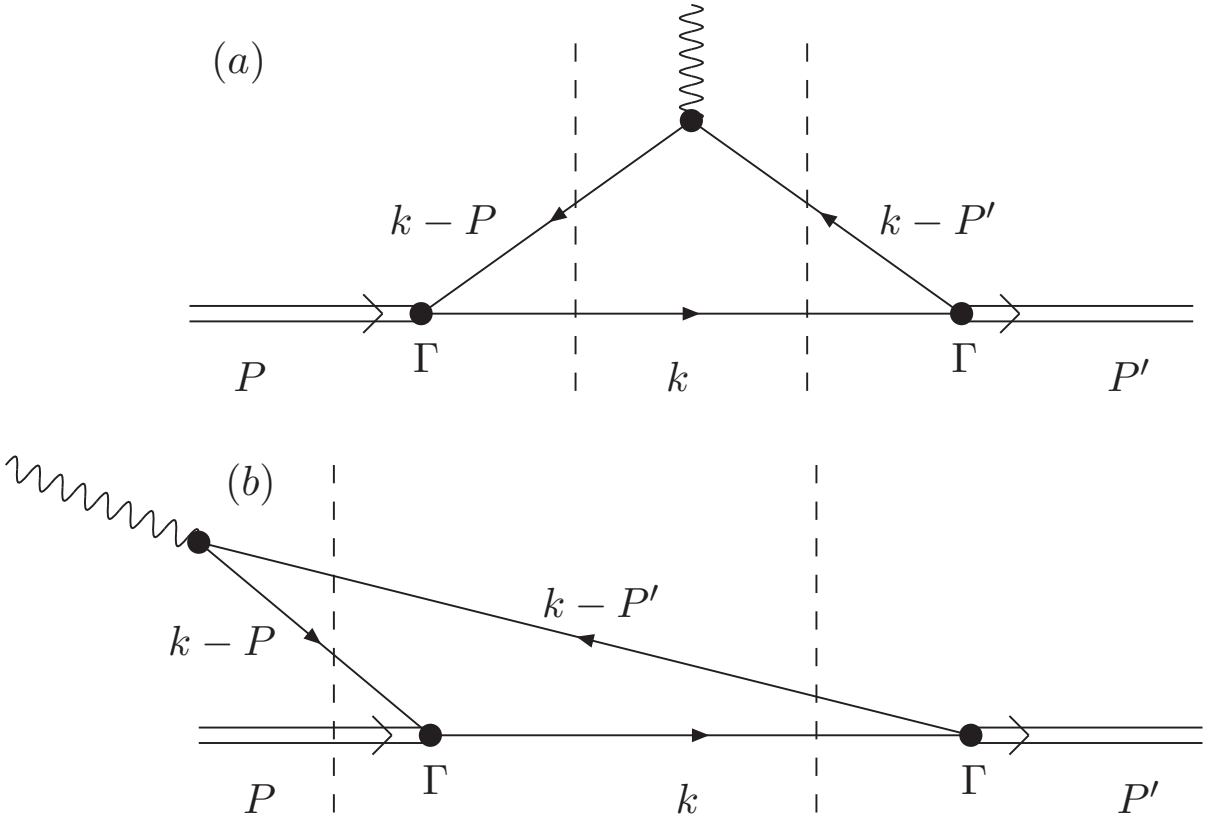
$$
\begin{aligned}
G_0^{CCKP} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}] \\
&= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}] \\
G_1^{CCKP} &= \frac{1}{(1+\eta)}[I_{11}^{+}+I_{00}^{+}-I_{1-1}^{+}-\frac{2(1-\eta)}{\sqrt{2\eta}}I_{10}^{+}] = -\frac{J_{zx}^{+}}{\sqrt{\eta}} \\
G_2^{CCKP} &= \frac{\sqrt{2}}{3(1+\eta)}[-\eta I_{11}^{+}-\eta I_{00}^{+}+2\sqrt{2\eta}I_{10}^{+}-(\eta+2)I_{1-1}^{+}] = \\
& \frac{\sqrt{2}}{3}[J_{xx}^{+}-J_{yy}^{+}]
\end{aligned}
$$

Brodsky-Hiller - (BH) - I_{11}^{+}

$$
\begin{aligned}
G_0^{BH} &= \frac{1}{3(1+2\eta)}[(3-2\eta)I_{00}^{+} + 8\sqrt{2\eta}I_{10}^{+} + 2(2\eta-1)I_{1-1}^{+}] \\
&= \frac{1}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)] \\
G_1^{BH} &= \frac{2}{(1+2\eta)}[I_{00}^{+} - I_{1-1}^{+} + \frac{(2\eta-1)}{\sqrt{2\eta}}I_{10}^{+}] \\
&= \frac{1}{(1+2\eta)}[\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}] \\
G_2^{BH} &= \frac{2\sqrt{2}}{3(1+2\eta)}[\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} - (\eta+1)I_{1-1}^{+}] \\
&= \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}]
\end{aligned}
$$

FFS

$$
\begin{aligned}
G_0^{FFS} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}] \\
&= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}] \\
G_1^{FFS} &= G_1^{CCKP} \\
G_2^{FFS} &= G_2^{CCKP}
\end{aligned}
$$



(a) ⇒ Valence Component of the Electromagnetic Current
(b) ⇒ Non-Valence Component of the Electromagnetic Current
Ref.: de Melo and Frederico, PRC (1997) , de Melo, Naus, Frederico and Sauer, PRC(1999)

Elimination / Zero Modes: **Vertex** $\Gamma(\gamma^\mu, \gamma^\nu)$

$$Tr[gg]_{ji} = Tr[\not{\epsilon}_f^\alpha(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\not{\epsilon}_i^\alpha(\not{k} + m)]$$

- **+Z (Pair Terms):** $\mathbf{Tr[gg]_{ji}^{+Z} = \frac{k^-}{2}\ R_{gg}}$
- **where:** $\mathbf{R_{gg} = Tr[\not{\epsilon}_f^\alpha(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\not{\epsilon}_i^\alpha\gamma^+]}$
- **Simplification:** $[\gamma^\mu, \gamma^\nu]$ **Dirac Trace:**

$$\begin{aligned} Tr[gg]_{xx}^{+Z} &= -\eta\ Tr[gg]_{zz}^{+Z} \\ Tr[gg]_{zx}^{+Z} &= -\sqrt{\eta}\ Tr[gg]_{zz}^{+Z} \\ Tr[gg]_{zz}^{+Z} &= R_{gg} \end{aligned}$$

Also:

$$Tr[gg]_{yy}^{+Z} = 4k^-(p^+ - k^+)^2$$

- **Pair Terms**

$$
\begin{aligned}
J_{xx}^{+Z} &= \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J_{xx}^{+Z}]}{[1][2][4][5][6][7]} \neq 0 \\
J_{zx}^{+Z} &= \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J_{zx}^{+Z}]}{[1][2][4][5][6][7]} \neq 0 \\
J_{zz}^{+Z} &= \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J_{zz}^{+Z}]}{[1][2][4][5][6][7]} \neq 0 \\
J_{yy}^{+Z} &= \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J_{yy}^{+Z}]}{[1][2][4][5][6][7]} = 0
\end{aligned}
$$

- **Basis $I^{+}_{m'm}$:**

$$
\begin{aligned}
I_{11}^{+Z} &= 0, \;\; I_{10}^{+Z} = 0 \\
I_{1-1}^{+Z} &= 0, \;\; I_{00}^{+Z} = (1+\eta)J_{zz}^{+} \neq 0
\end{aligned}
$$

- **Pair Term Contribution:** **only: I_{00}^{+Z} !!**
- **Inna Grach: Elimination** I_{00}^{+}

$$
\begin{aligned}
G_0^{GK\ (+Z)} &= \frac{1}{3}\left(J_{xx}^{(+Z)}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \\
& \frac{1}{3}\left(-\eta J_{zz}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = 0 \\
G_1^{GK\ (+Z)} &= \left(-J_{zz}^{+Z}[gg] - \frac{J_{zx}^{+Z}[gg]}{\sqrt{\eta}}\right) = \\
& -J_{zz}^{+Z}[gg] + \sqrt{\eta}\frac{J_{zz}^{+Z}[gg]}{\sqrt{\eta}} = 0 \\
G_2^{GK\ (+Z)} &= \frac{\sqrt{2}}{3}\left(J_{xx}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \frac{\sqrt{2}}{3}\left(-\eta J_{zz}^{+} + \eta J_{zz}^{+Z}\right) = 0
\end{aligned}
$$

- **Vertex (Others):**
- **Cross term with γ^μ and derivatives:**

$$\gamma^\mu \cdot \frac{m_\rho}{2}\left[\frac{2k^\mu - p^\mu}{p^r.k_r + m_\rho m_q - \imath\epsilon}\right]$$

- **Direct term with derivative couplings:**

$$\frac{m_\rho}{2}\left[\frac{2k^\mu - p^\mu}{p^r.k_r + m_\rho m_q - \imath\epsilon}\right] \cdot \frac{m_\rho}{2}\left[\frac{2k^\nu - p^\nu}{p^r.k_r + m_\rho m_q - \imath\epsilon}\right]$$

Resume/Results:

$I_{11}^{+Z} = 0$, $I_{10}^{+Z} = 0$, $I_{1-1}^{+Z} = 0$ **and** $I_{00}^{+Z} = (1+\eta)J_{zz}^{+Z}$ **with** $\lim_{\delta^+ \to 0_+} J_{zz}^{+Z} \neq 0$

Inna Grach Prescription Final Result

No Zero Modes or Pair Terms Contribution with Inna Grach prescp.!!

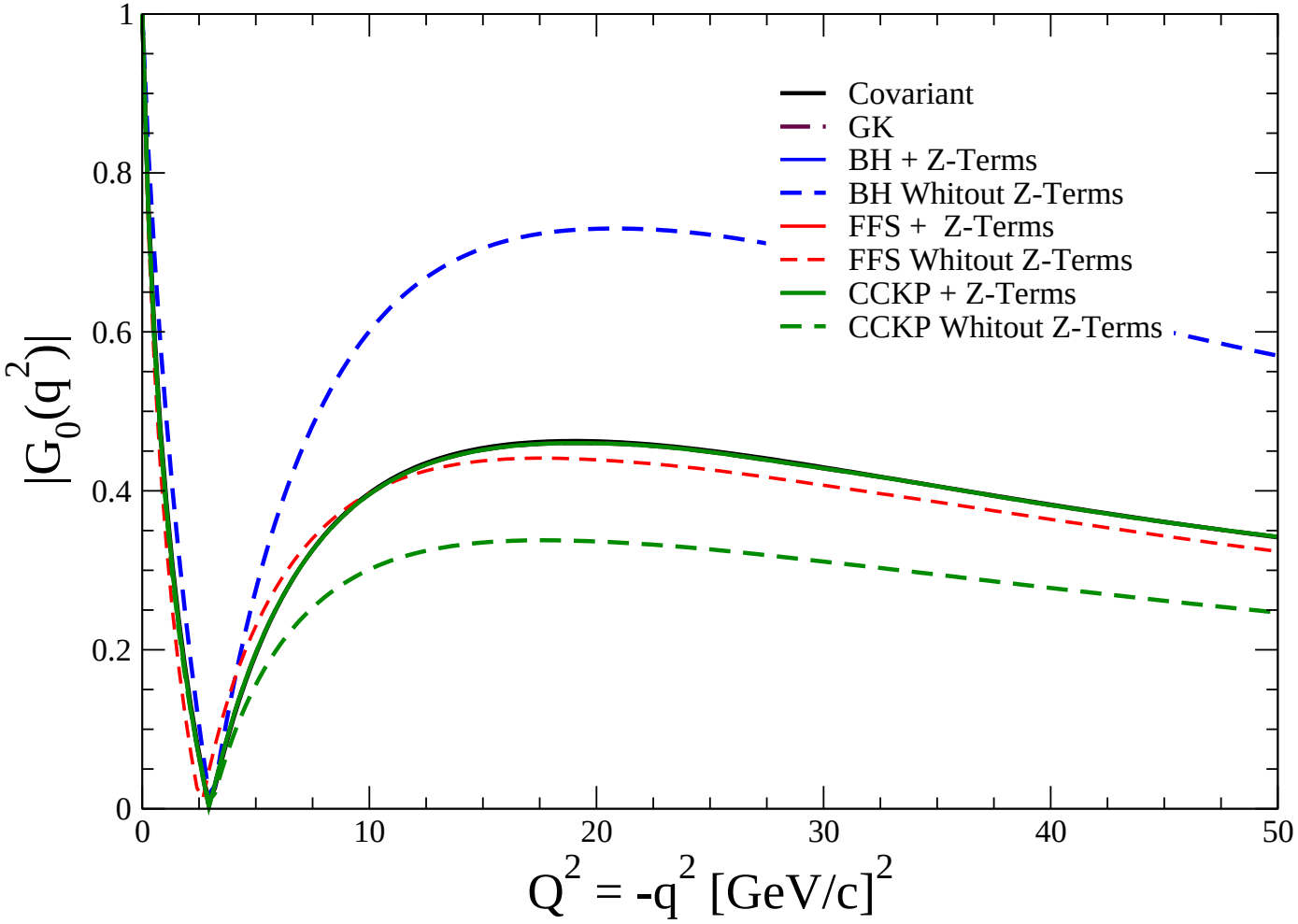
REf.:

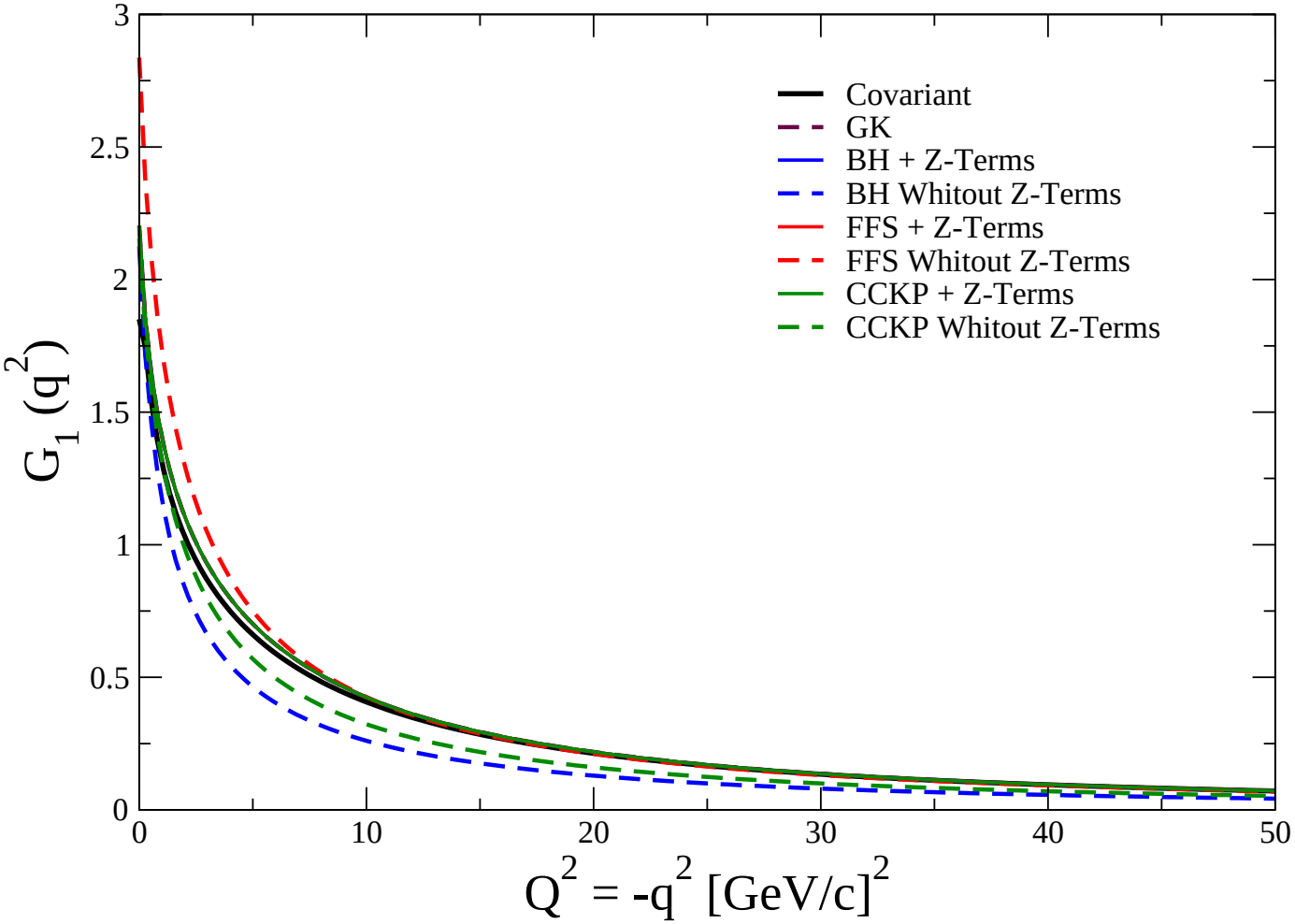
- **J.P.B.C. de Melo and T. Frederico**,**Phys. Lett. B708, (2012) 87**
- **J.P.B.C. de Melo and T. Frederico,Few Body Syst. 52 (2012) 403**
- **Similar Results are found by Ji, Bakker and Choi:**
- **Phy.Rev.D65 (2002) 116001**
- **Phy.Rev.D70 (2004) 053015**

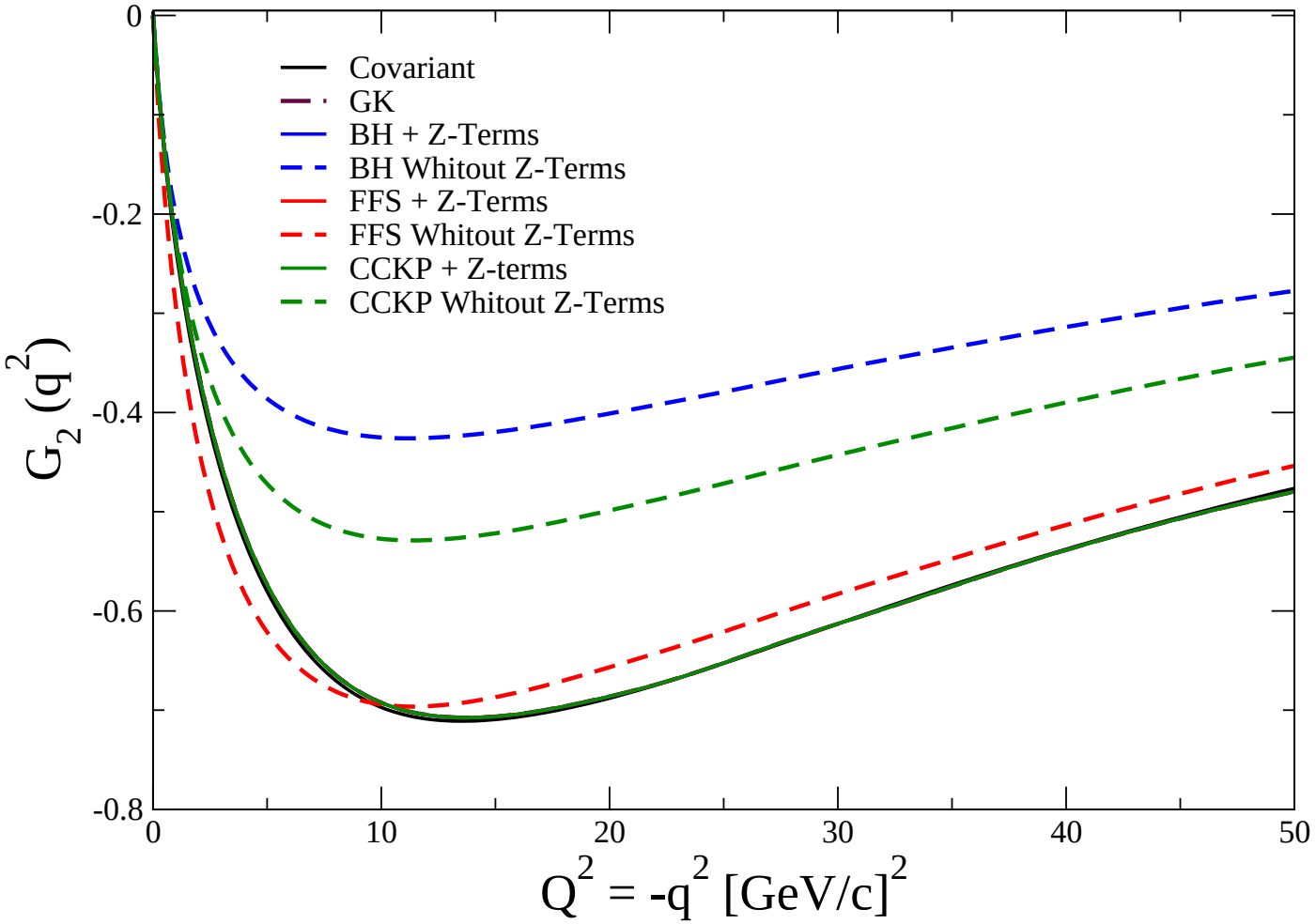
Observables

m_q / m_R $[GeV]$	f_ρ $[GeV]$	$< r_\rho^2 >$ $[fm^2]$	μ_ρ	Q_d $[e/m_\rho^2]$
0.430 / 3.0	0.154	0.267	2.20	-0.898
[1]	0.134 / 0.151	0.296	2.10	-0.910
[2]	0.130	0.312	2.11	-0.850
[3]	0.207	0.540	2.01	-0.410
[4]	-	-	2.11±0.10	-
[5]	-	-	2.1±0.5	-
PDG	0.152 ± 0.008			

[1] Phy.Rev.D65 (2002) 116001, B. Bakker, H. M. Choi and C. R. Ji ; / Phys.Rev. D89 (2014) 033011
[2] Phy.Rev.C83 (2011) 065206, H. L. Roberts, A. Bashir, L.X.G. Guerrero, C. Roberts,
[3] Phy.Rev.C77 (2008) 025203, M. S. Bhagwat and P. Maris
[4] ArXiv:1608.3472v1[hep-lat], E.V. Luscheva, O.E. Solojeva and O. V. Teyaev
[5] Int. J. Mod. Phys. A 18 & 19 (2015) 155014, D.G. Gudino and G. T. Sánchez


Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-Terms
CCKP Whitout Z-Terms
$|G_0(q^2)|$
$Q^2 = -q^2$ [GeV/c]^2
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50


Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-Terms
CCKP Whitout Z-Terms
$G_1(q^2)$
$Q^2 = -q^2$ [GeV/c]^2
0
0.5
1
1.5
2
2.5
3
0
10
20
30
40
50


Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-terms
CCKP Whitout Z-Terms
0
-0.2
-0.4
-0.6
-0.8
0
10
20
30
40
50
$G_2(q^2)$
$Q^2 = -q^2$ [GeV/c]^2

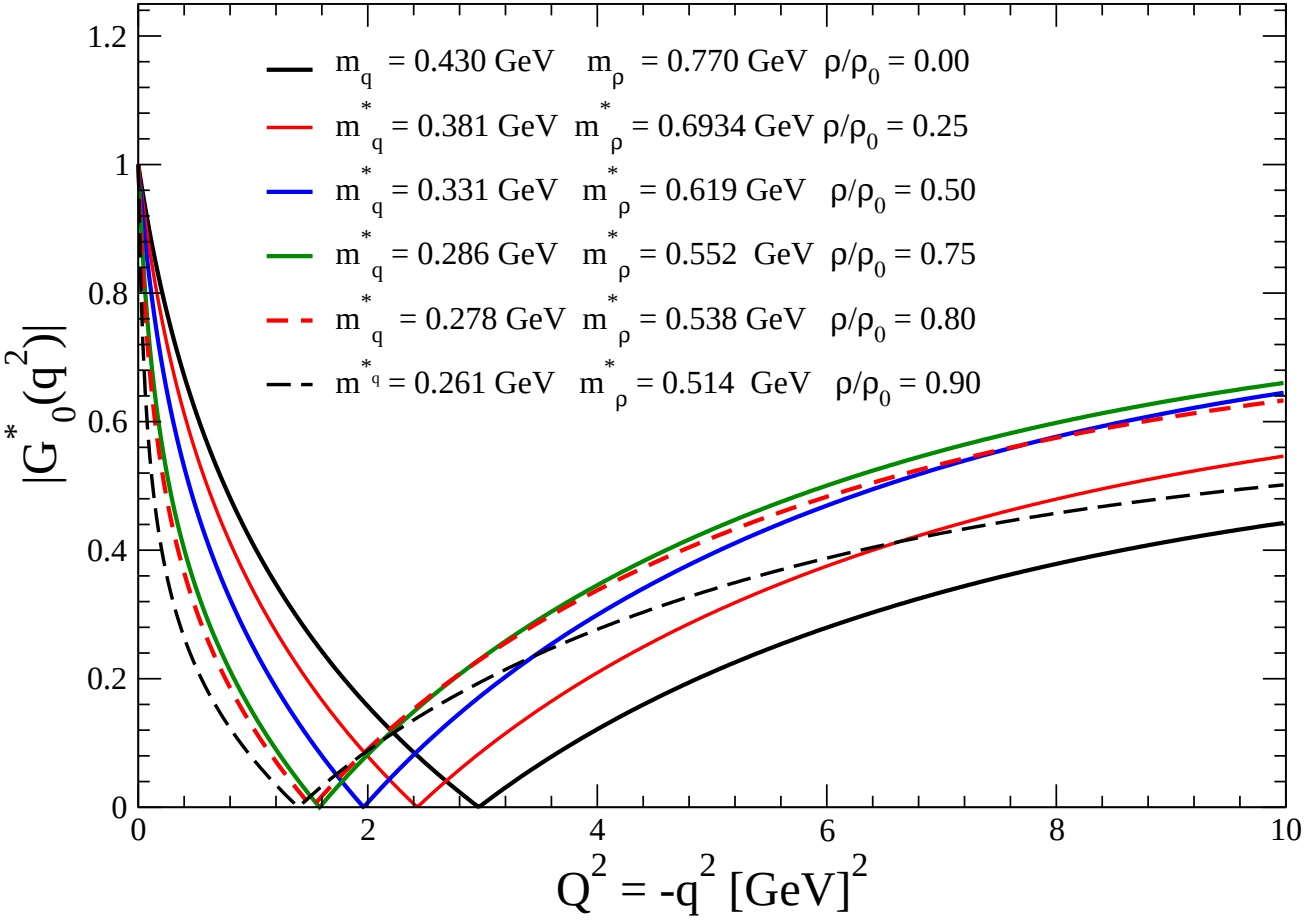
Results: Rho Meson in Medium

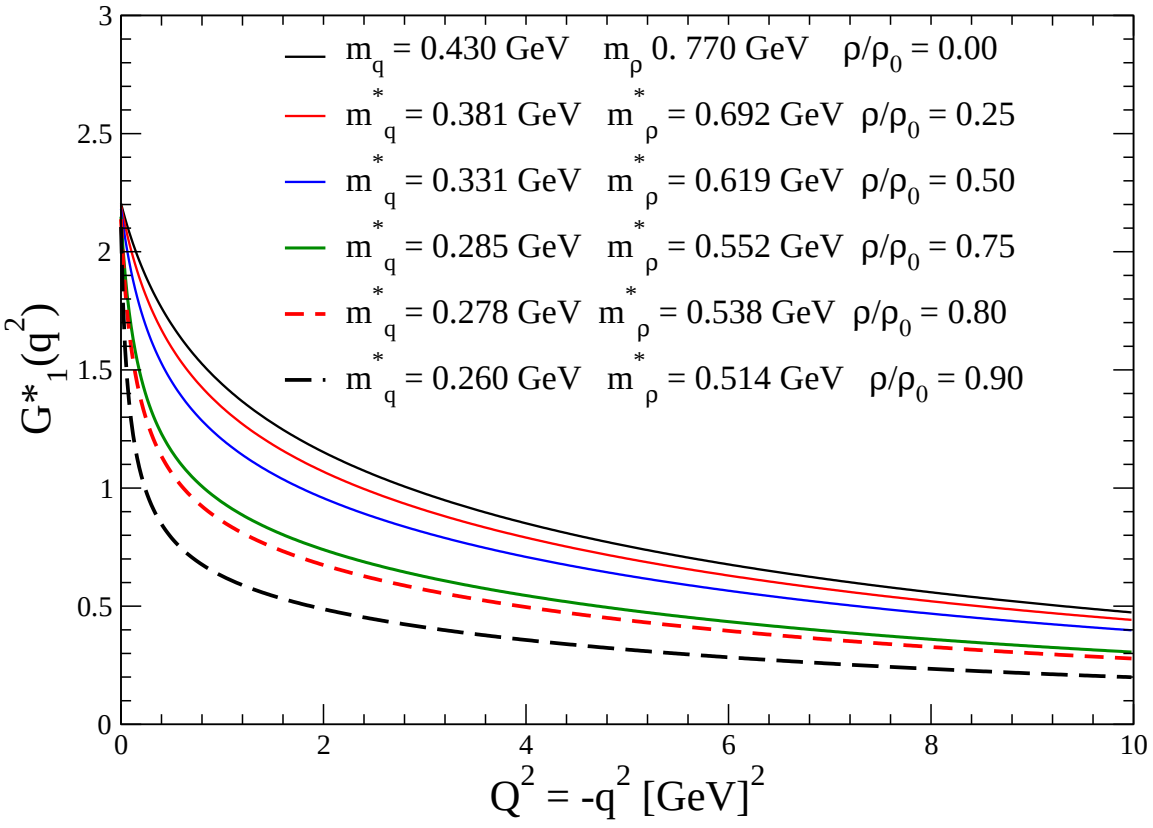
Table : **Observables**

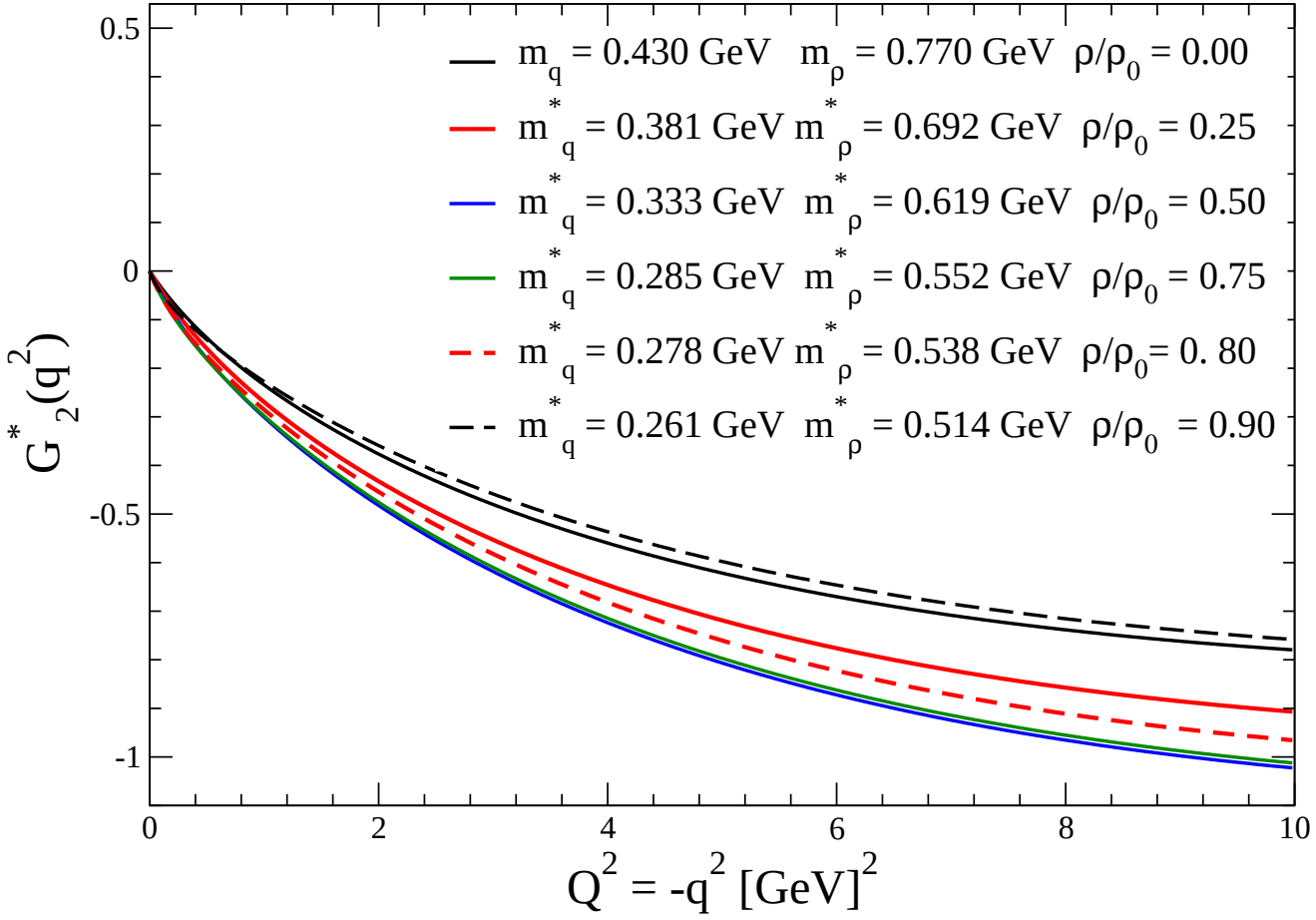
Units: Masses in [GeV], radius [fm^2], magnetic momentum $[e/2m_\rho]$**, quadrupole momentum** $[fm^2]$**.**

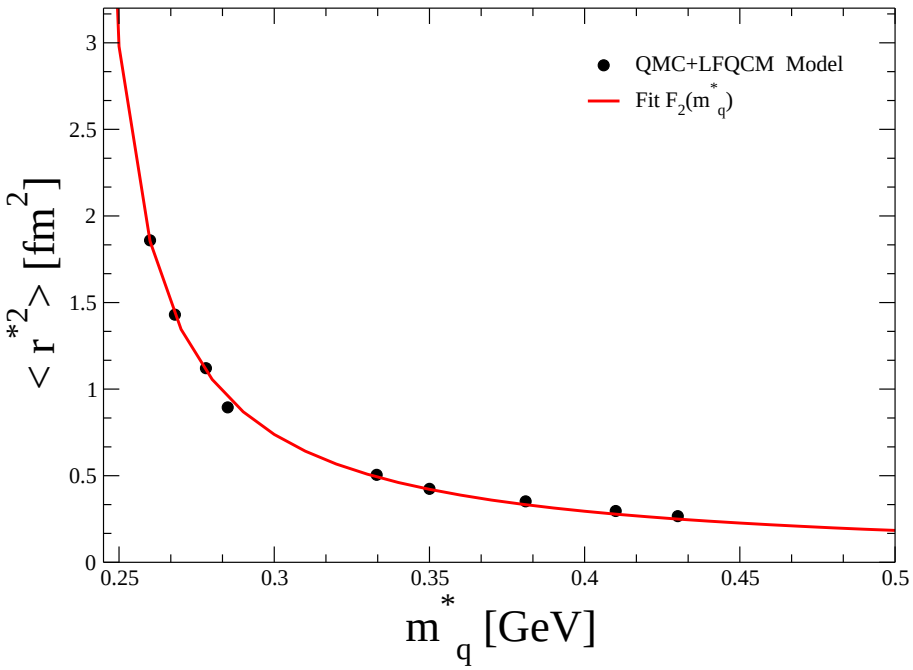
ρ/ρ_0	m_q^*	m_ρ^*	$< r_\rho^{*2} >$	f_ρ^* $[MeV]$	μ^*	Q_0^*
0.0	0.430	0.770	0.2667	153.657	2.20	-0.05895
0.10	0.410	0.738	0.2960	185.126	2.20	-0.06387
0.25	0.381	0.692	0.3520	166.147	2.19	-0.07214
0.40	0.350	0.648	0.4243	175.165	2.19	-0.081677
0.50	0.333	0.618	0.5053	177.694	2.18	-0.08840
0.75	0.285	0.552	0.8944	176.796	2.15	-0.09939
0.80	0.278	0.538	1.121	176.326	2.14	-0.10279
0.85	0.268	0.527	1.430	164.277	2.12	-011340
0.90	0.260	0.514	1.859	155.940	2.10	-0.11520
Exp.				152±8		

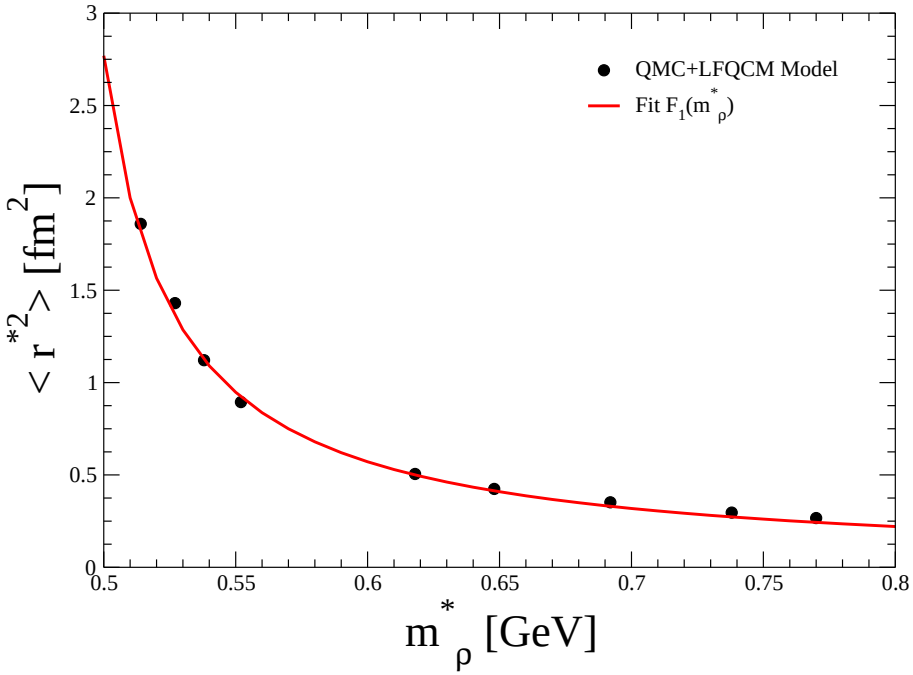
Ref.: Few-Body Systems (2017), de Melo, J. and Tsushima, K.


m_q = 0.430 GeV m_ρ = 0.770 GeV ρ/ρ_0 = 0.00
m*_q = 0.381 GeV m*_ρ = 0.6934 GeV ρ/ρ_0 = 0.25
m*_q = 0.331 GeV m*_ρ = 0.619 GeV ρ/ρ_0 = 0.50
m*_q = 0.286 GeV m*_ρ = 0.552 GeV ρ/ρ_0 = 0.75
m*_q = 0.278 GeV m*_ρ = 0.538 GeV ρ/ρ_0 = 0.80
m*_q = 0.261 GeV m*_ρ = 0.514 GeV ρ/ρ_0 = 0.90
|G*_0(q^2)|
Q^2 = -q^2 [GeV]^2
0
0.2
0.4
0.6
0.8
1
1.2
0
2
4
6
8
10


m_q = 0.430 GeV m_ρ 0. 770 GeV ρ/ρ_0 = 0.00
m*_q = 0.381 GeV m*_ρ = 0.692 GeV ρ/ρ_0 = 0.25
m*_q = 0.331 GeV m*_ρ = 0.619 GeV ρ/ρ_0 = 0.50
m*_q = 0.285 GeV m*_ρ = 0.552 GeV ρ/ρ_0 = 0.75
m*_q = 0.278 GeV m*_ρ = 0.538 GeV ρ/ρ_0 = 0.80
m*_q = 0.260 GeV m*_ρ = 0.514 GeV ρ/ρ_0 = 0.90
G*_1(q^2)
Q^2 = -q^2 [GeV]^2
0
0.5
1
1.5
2
2.5
3
0
2
4
6
8
10


m_q = 0.430 GeV m_ρ = 0.770 GeV ρ/ρ_0 = 0.00
m^*_q = 0.381 GeV m^*_ρ = 0.692 GeV ρ/ρ_0 = 0.25
m^*_q = 0.333 GeV m^*_ρ = 0.619 GeV ρ/ρ_0 = 0.50
m^*_q = 0.285 GeV m^*_ρ = 0.552 GeV ρ/ρ_0 = 0.75
m^*_q = 0.278 GeV m^*_ρ = 0.538 GeV ρ/ρ_0= 0. 80
m^*_q = 0.261 GeV m^*_ρ = 0.514 GeV ρ/ρ_0 = 0.90
$G^*_2(q^2)$
$Q^2 = -q^2$ [GeV]^2
0.5
0
-0.5
-1
0
2
4
6
8
10



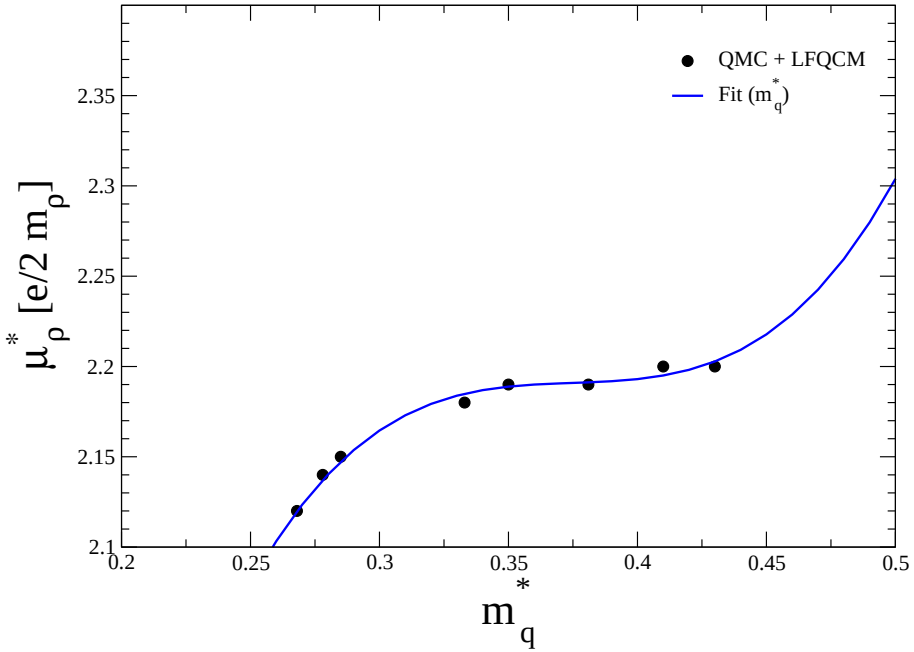


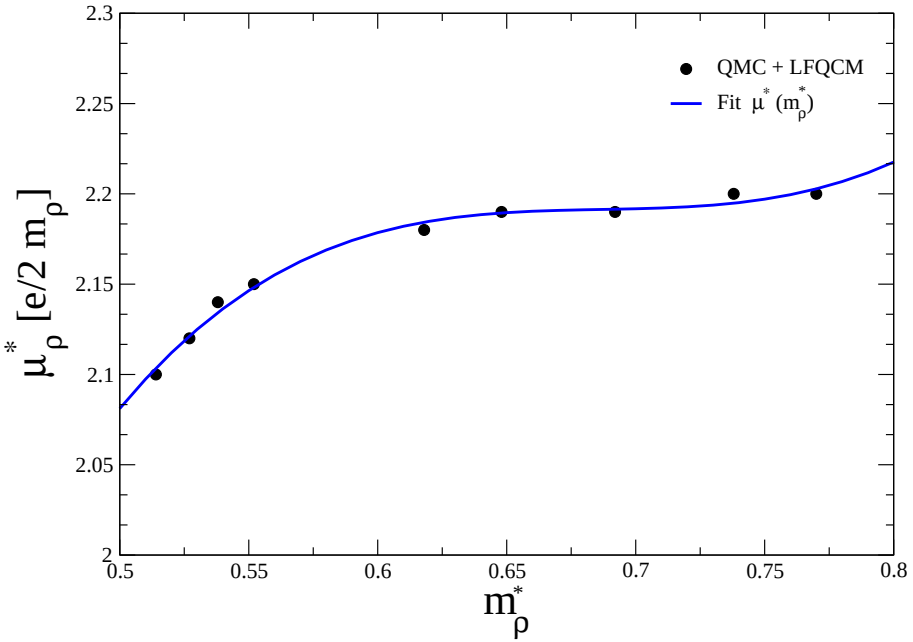
$$F_1(m_\rho^*) =< \ r^{*2}(m_\rho^*) \ > \ = \frac{a_0}{m_\rho^* - a_1} = \frac{0.0720}{m_\rho^* - 0.474}$$

where: $a_0 = [GeV][fm^2]$ **and** $a_1 = [GeV]$**.**

$$F_2(m_q^*) =< \ r^{*2}(m_q^*) \ > \ = \frac{b_0}{m_\rho^* - b_1} = \frac{0.049064}{m_q^* - 0.233531}$$

where: $b_0 = [GeV][fm^2]$ **and** $b_1 = [GeV]$**.**



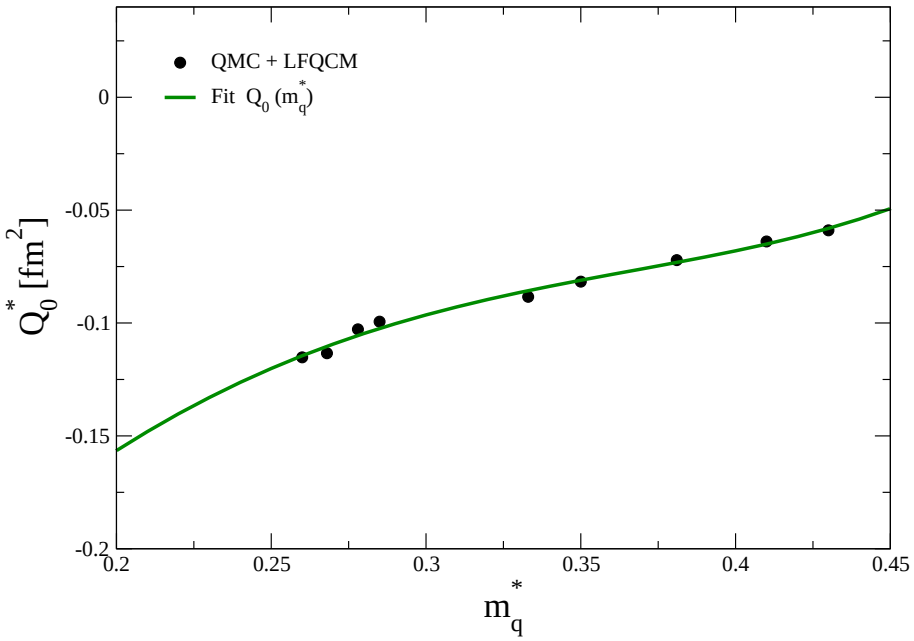


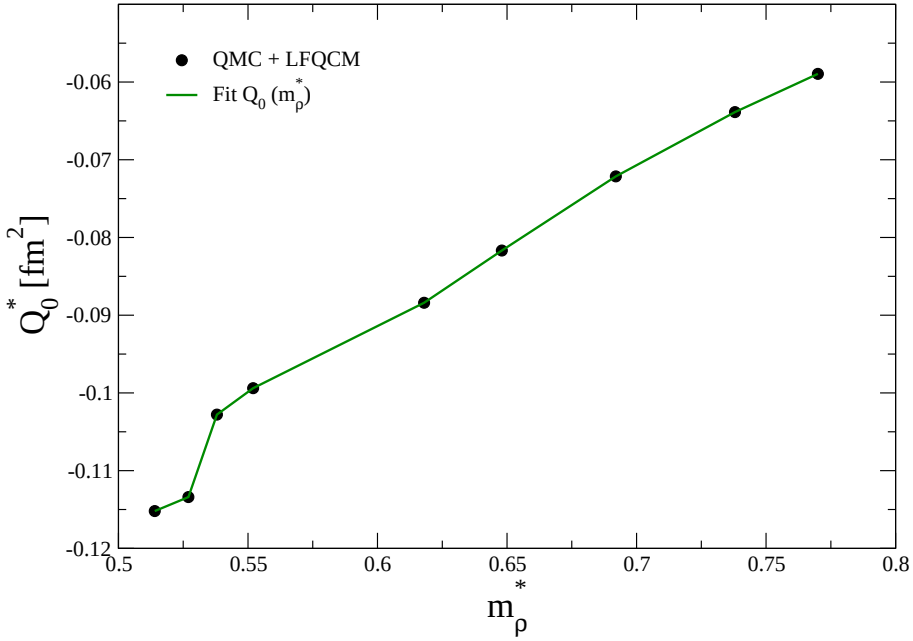
$$\mu_\rho^*(m_q^*) \;=\; a_0\; m_q^{*3} + \; a_1\; m_q^{*2} + \; a_2\; m_q^{*2} + a_3$$

here, $a_0 = 54.2853,\; a_1 = -61.0225,\; a_2 = 22.9146$ **and** $a_3 = -0.683494$**.**

$$\mu_\rho^*(m_\rho^*) \;=\; b_0\; m_q^{*3} + \; b_1\; m_q^{*2} + \; b_2\; m_q^* + b_3$$

here, $b_0 = 16.1392,\; b_1 = -33.2602,\; b_2 = 22.8736, b_3 = -3.05802$





$$Q^*_{0\rho}(m^*_q) \;=\; a_0\; m^{*5}_q + \;a_1\; m^{*4}_q + \;a_2\; m^{*3}_q + a_3 m^{*2}_q + a_4 m_\rho + a_5$$

here, $a_0 = 4.57347,\; a_1 = 2.90565,\; a_2 = -4.18667,\; a_3 = -5.39273,\; a_4 = 8.98056,\; a_5 = -1.46461$**.**

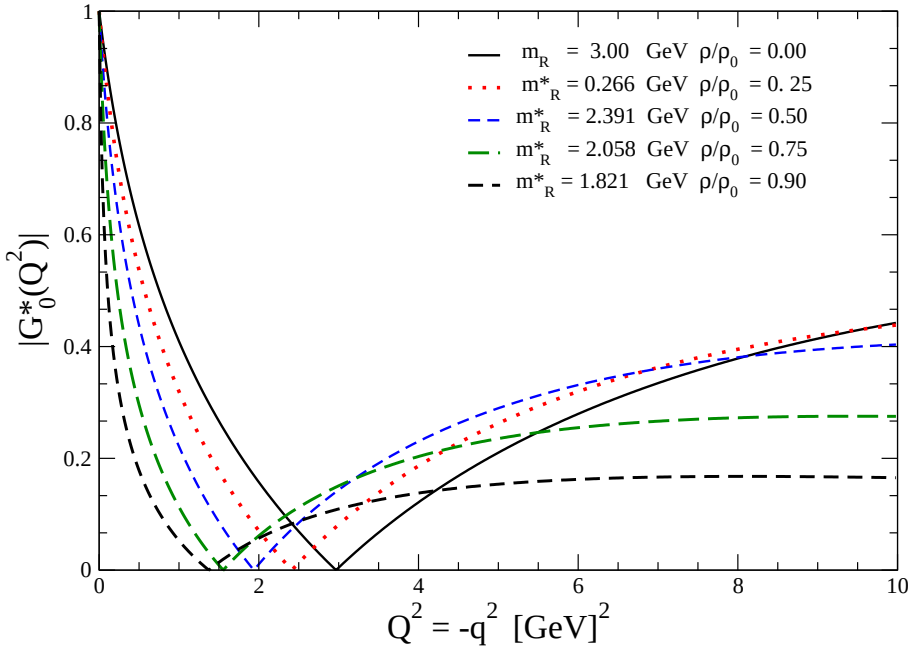
$$Q^*_{0\rho}(m^*_\rho) \;=\; b_0\; m^{*5}_q + \;b_1\; m^{*4}_q + \;b_2\; m^{*3}_q + b_3 m^{*2}_q + b_4 m_\rho + b_5$$

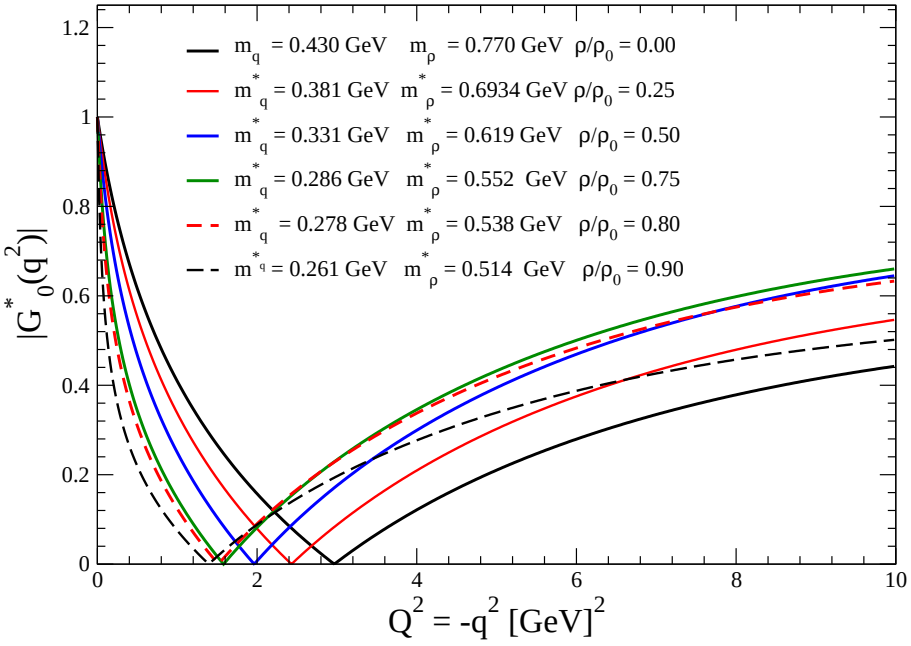
here, $a_0 = 0.0115492, a_1 = -0.256173, a_2 = -0.14145, a_3 = 0.510934, a_4 = -0.000000135009, a_5 = -0.211799$

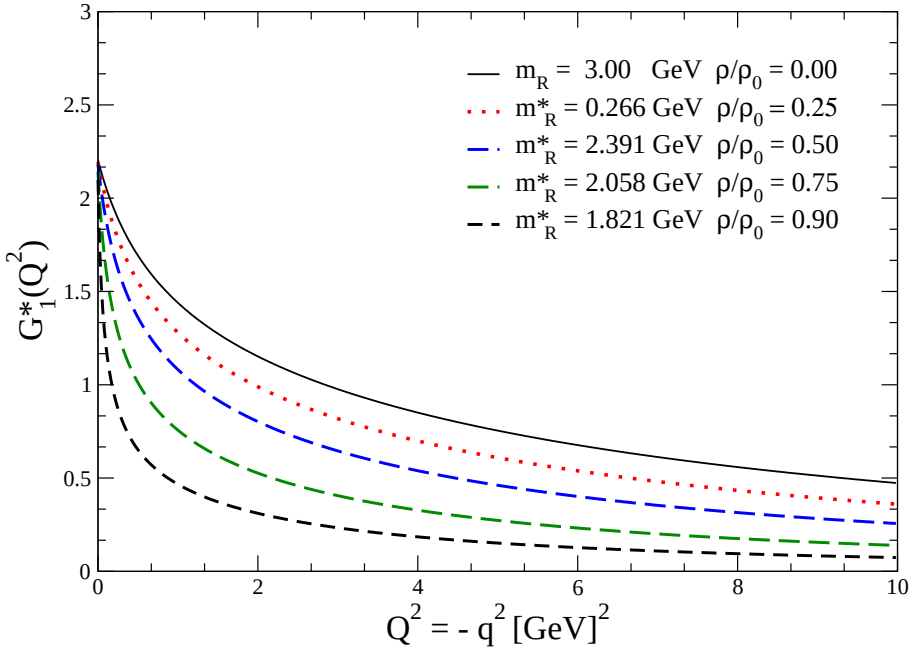
- **First Guest: M_R^* density dependence and Preliminary Results!**

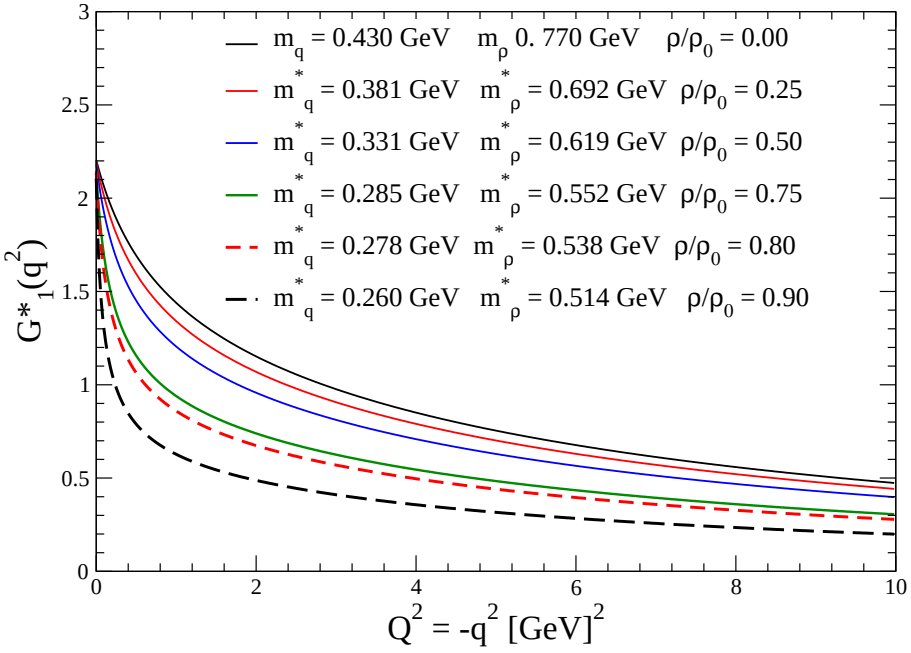
- **Brown-Rho scaling**

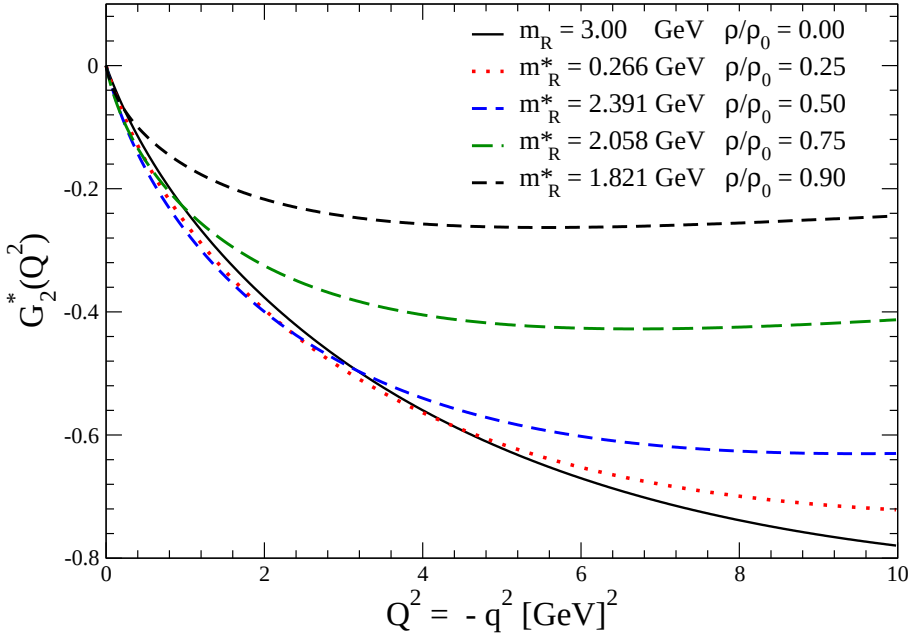
$$M_R^*(\rho/\rho_0) = M_R \left(\frac{m_q^*}{m_q} \right)$$

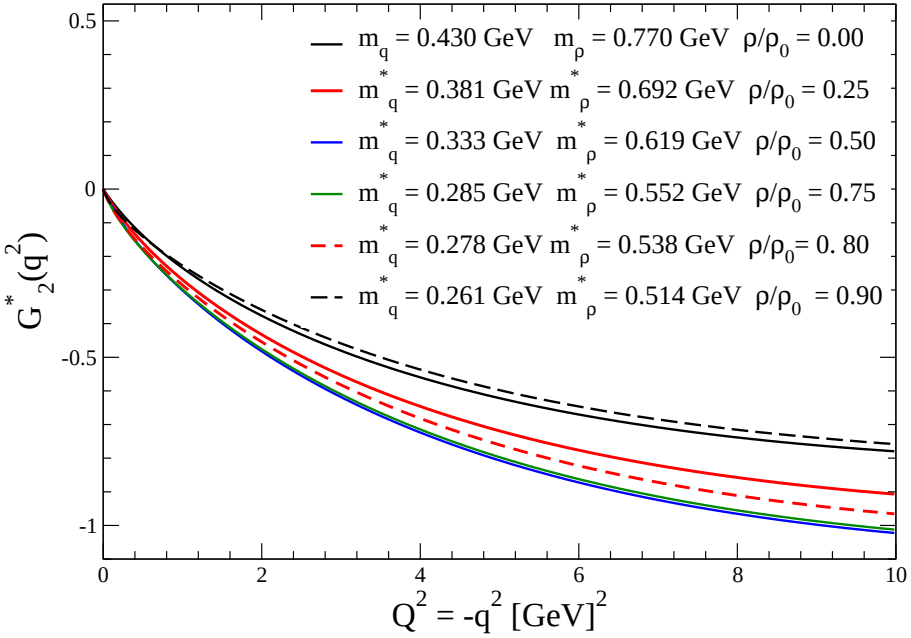

m_R = 3.00 GeV ρ/ρ_0 = 0.00
m*_R = 0.266 GeV ρ/ρ_0 = 0. 25
m*_R = 2.391 GeV ρ/ρ_0 = 0.50
m*_R = 2.058 GeV ρ/ρ_0 = 0.75
m*_R = 1.821 GeV ρ/ρ_0 = 0.90
|G*_0(Q^2)|
Q^2 = -q^2 [GeV]^2


m_q = 0.430 GeV m_ρ = 0.770 GeV ρ/ρ_0 = 0.00
m*_q = 0.381 GeV m*_ρ = 0.6934 GeV ρ/ρ_0 = 0.25
m*_q = 0.331 GeV m*_ρ = 0.619 GeV ρ/ρ_0 = 0.50
m*_q = 0.286 GeV m*_ρ = 0.552 GeV ρ/ρ_0 = 0.75
m*_q = 0.278 GeV m*_ρ = 0.538 GeV ρ/ρ_0 = 0.80
m*_q = 0.261 GeV m*_ρ = 0.514 GeV ρ/ρ_0 = 0.90
|G*_0(q^2)|
Q^2 = -q^2 [GeV]^2


m_R = 3.00 GeV ρ/ρ_0 = 0.00
m^*_R = 0.266 GeV ρ/ρ_0 = 0.25
m^*_R = 2.391 GeV ρ/ρ_0 = 0.50
m^*_R = 2.058 GeV ρ/ρ_0 = 0.75
m^*_R = 1.821 GeV ρ/ρ_0 = 0.90
$G^*_1(Q^2)$
$Q^2 = -q^2$ [GeV]^2


m_q = 0.430 GeV m_ρ 0. 770 GeV ρ/ρ_0 = 0.00
m^*_q = 0.381 GeV m^*_ρ = 0.692 GeV ρ/ρ_0 = 0.25
m^*_q = 0.331 GeV m^*_ρ = 0.619 GeV ρ/ρ_0 = 0.50
m^*_q = 0.285 GeV m^*_ρ = 0.552 GeV ρ/ρ_0 = 0.75
m^*_q = 0.278 GeV m^*_ρ = 0.538 GeV ρ/ρ_0 = 0.80
m^*_q = 0.260 GeV m^*_ρ = 0.514 GeV ρ/ρ_0 = 0.90
$G^*_1(q^2)$
$Q^2 = -q^2$ [GeV]^2


m_R = 3.00 GeV ρ/ρ_0 = 0.00
m*_R = 0.266 GeV ρ/ρ_0 = 0.25
m*_R = 2.391 GeV ρ/ρ_0 = 0.50
m*_R = 2.058 GeV ρ/ρ_0 = 0.75
m*_R = 1.821 GeV ρ/ρ_0 = 0.90
G*_2(Q^2)
Q^2 = - q^2 [GeV]^2


m_q = 0.430 GeV m_ρ = 0.770 GeV ρ/ρ_0 = 0.00
m*_q = 0.381 GeV m*_ρ = 0.692 GeV ρ/ρ_0 = 0.25
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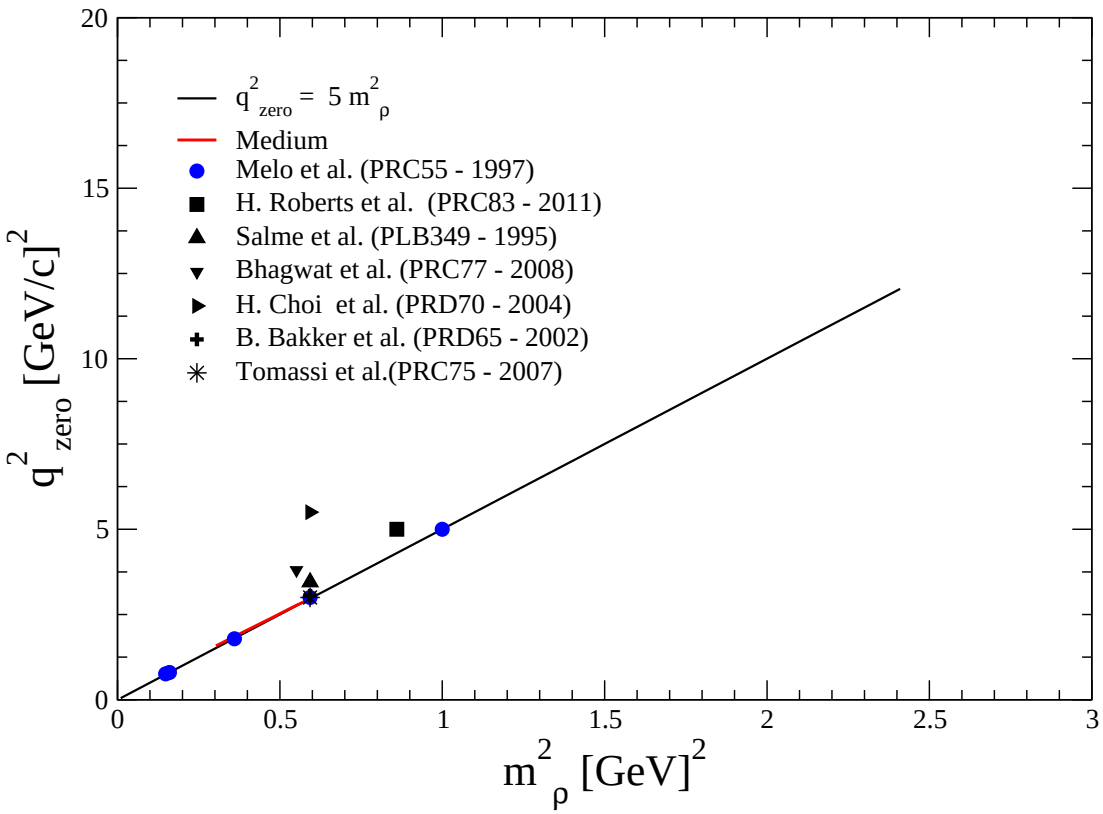
m*_q = 0.278 GeV m*_ρ = 0.538 GeV ρ/ρ_0 = 0. 80
m*_q = 0.261 GeV m*_ρ = 0.514 GeV ρ/ρ_0 = 0.90
G*_2(q^2)
Q^2 = -q^2 [GeV]^2

Zero of Charge Form Factor: $G_0(q^2_{zero}) = 0$



- Ref. **Few-Body Systems (2017), de Melo, J. and Tsushima, K.**
- **de Melo and Tsushima, K., arXiv:1802.06096 [hep-ph]**

- Light-Front $\Longrightarrow \begin{cases} \textit{Bound States} \\ \textit{Covariance} \end{cases}$
- Rotational Invariance Broken $\Longrightarrow k^-$ Problematic
- Electromagnetic Current:
- $\begin{cases} - \textbf{Present Work}: \ \mathbf{J}^+ \ \textbf{Component} \\ - \textbf{Future Works}: \ \mathbf{J}^- \ \textbf{and} \ \mathbf{J}_\perp \end{cases}$
- Pair Terms Contribution: $\Longrightarrow J^+$, J^- and $J_\perp$
- Take New Informations about Bound States

$\Longrightarrow$ • **kaon + another pseudo-scalar particles**

$\Longrightarrow$ • **Vector particles**

$\Longrightarrow$ • **Nucleon**

$\Longrightarrow$ • **Pion: Space-like and Time-like**

$\Longrightarrow$ • **Mesons decay's**

$\Longrightarrow$ • **Bound States / Bethe-Salpeter Amplitudes - Nakanishi Integral Representation**

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Thanks to the Organizers

Workshop on Pion and Kaon Structure at an Electron - Ion Collider - PIEIC 2018

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- **FAPESP , CNPq and CAPES**

Thanks (Obrigado)!!