Overview on Jets, Energy Loss and Hadronization

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Session: Jets, Energy Loss, Hadronization, and Nuclear Structure

Focus on the connection to EIC environment
Jets

Definition:

- “footprints” or “trace” of quarks and gluons
- Inclusive cross section with limited phase space
- Well-defined at either hadron or parton level!

Sterman & Weinberg, PRL 1977
Parton energy loss leads to jet suppression

- Not well-defined since we never measure individual parton!
- Model dependent if we define it at the hadronic level

from X-N. Wang
h+h collision:
with a lot of energy
- $h+h$ collision:
  - with a lot of energy

- **Hadronization**
  - Emergence of hadrons, leptons, ...

- **Event with identified hadron**
  - cannot be calculated perturbatively!

S. Hoeche, 2018
**Hadronization**

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  - with a lot of energy

- **Hadronization**
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- No identified hadron – Jet cross sections – inclusive enough
  - No-perturbative fragmentation function(s)
High energy nuclear collisions

- **A-A:**
  - Nucleus
  - Sufficient energy to produce lots of pions

- **p-A:**
  - Proton
  - Final-state controlled probe!

- **e-A:**
  - Lepton
  - Initial-state controlled probe!
  - Color probe

$q^2$
Two theoretical approaches

- Analytic – with controllable approximation:
  - Rely on our understanding of physics
    - Make the right approximations, …
    - Give insight to what really happened, …
  - Motivate experimental measurement, …
  - Cannot capture the full story, …
Two theoretical approaches

- **Analytic – with controllable approximations**
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  - Cannot capture the full story, ...

- **Numerical – Monte Carlo Event Generator:**
  - Looking like a black box, ...
  - Output depends on what we put in (thinking of lattice QCD?), ...
    - Can in principle include all the right physics, including nuclei, medium effects, energy loss, ...
    - But, it is often working in progress, ...
  - Could give the real “event”, with experimental constraints, ...

I will focus on the more “analytic” approach, ...

Miss a lot of work for RHIC, LHC, ...
Jet definition – not unique!

Jet definition – how to combine particles into a jet

✧ Recombination algorithms (almost all e+e- cases):

- Recombination metric: \( y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2} \)
  \[ M_{ij}^2 = 2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij}) \]

- Combine particle pair \((i, j)\) with the smallest \(y_{ij}\):
  \((i, j) \rightarrow k\)  \(e.g.\) E scheme: \(p_k = p_i + p_j\)
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Cone jet algorithms (CDF, …, colliders):

- Cluster all particles into a cone of half angle \(R\) to form a jet:
- Recombination metric: 
  \(d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right) \frac{\Delta_{ij}^2}{R^2} \)
  \(\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2\)
- Classical choices:
  “\(k_T\) algorithm” \((p = 1)\), “anti-\(k_T\)” \((p = -1)\), …
- Require a minimum visible jet energy: \(E_{jet} > \epsilon\)
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  - “\(k_T\) algorithm” \((p = 1)\), “anti-\(k_T\)” \((p = -1)\), ...
  - Require a minimum visible jet energy: \(E_{jet} > \epsilon\)
  - Particle could be outside the “cone” \(\Rightarrow\) energy loss
Infrared safety for jet-definition

- For any observable with a phase space constraint, $\Gamma$,

$$
\frac{d\sigma}{d\sigma} (\Gamma) \equiv \frac{1}{2!} \int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2} \Gamma_2 (k_1, k_2) + \frac{1}{3!} \int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3} \Gamma_3 (k_1, k_2, k_3) + \ldots
$$

Where $\Gamma_n (k_1, k_2, \ldots, k_n)$ are constraint functions and invariant under interchange of $n$-particles.

- Conditions for IRS of $d\sigma (\Gamma)$:

$$
\Gamma_{n+1} (k_1, k_2, \ldots, (1 - \lambda) k_n^\mu, \lambda k_n^\mu) = \Gamma_n (k_1, k_2, \ldots, k_n^\mu) \quad \text{with} \quad 0 \leq \lambda \leq 1
$$

Physical meaning:

Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without the parton.

Special case: $\Gamma_n (k_1, k_2, \ldots, k_n) = 1$ for all $n \quad \Rightarrow \quad \sigma^{(\text{tot})}$
Incoherent/independent multiple scattering

- Weak quantum interference between scattering centers:

- Modify jet spectrum without changing the total rate

- Nuclear dependence from the scattering centers
  - number density of scattering centers
  - nature of the scattering centers – partonic or hadronic
  - momentum distribution and cut-off (new scale)
  - etc
Partonic multiple scattering

- Classical multiple scattering – cross section level:
  \[ d\sigma^{\text{Double}} : \sigma^{\text{single}}(p_{\text{in}}, p_1) \cdot \sigma^{\text{single}}(p_2, p_{\text{out}}) dp_1 dp_2 \delta(p_1 + p_2 + p_{\text{in}} - p_{\text{out}}) \]

- Parton level multiple scattering (incoherent/independent)

In massless pQCD, above

- Parton distribution at \( x=0 \) is ill-defined
- Pinch poles of \( k \) in above definition

\[ d\sigma^{\text{double}} \rightarrow \infty \text{ as } p_1 \text{ or } p_2 \rightarrow 0 \]

Introduce a cutoff

coherent multiple scattering – quantum interference
Coherent multiple scattering in QCD

- Quantum mechanical multiple scattering
  - Amplitude level

\[ \langle A | \phi^+(0)\phi^+(y_3)\phi(y_2)\phi(y_1) | A \rangle \]

\[ d\sigma^{\text{double}} \propto C^{\text{double}}(p_{in}, p_1, p_2, p_1', p_2', p_{out})T^{\text{double}}(p_1, p_2, p_1', p_2') \]
\[ \times dp_1 dp_2 dp_1' dp_2' \delta(p_1 + p_2 + p_1' + p_2' + p_{in} - p_{out}) \]

\[ T^{\text{double}}(p_1, p_2, p_1', p_2') \propto \langle A | \phi^+(0)\phi^+(y_3)\phi(y_2)\phi(y_1) | A \rangle dy_1 dy_2 dy_3 \]

Multiparton correlation, no probability interpretation!
Medium induced rescattering

- Basic leading order calculation:

\[ M_{1,0,0} = J(p)e^{ipx_0}(-i)\int \frac{d^2q_1}{(2\pi)^2} v(0,q_1)e^{-iq_1 \cdot b_1} \times \]
\[ \times (-2ig_s) \frac{\epsilon \cdot k}{k^2} e^{i\omega_0 z_1} c a_1 T_{a_1} . \]

\[ M_{1,1,0} = J(p)e^{ipx_0}(-i)\int \frac{d^2q_1}{(2\pi)^2} v(0,q_1)e^{-iq_1 \cdot b_1} \times \]
\[ \times 2ig_s \frac{\epsilon \cdot (k - q_1)}{(k - q_1)^2} e^{i(\omega_0 - \omega_1)z_1}(e^{i\omega_1 z_1} - e^{i\omega_1 z_0}) [c, a_1] T_{a_1} \]

Answer depends on:
- choice of source potential \( v(0, \vec{q}_1) \)
- scattering phases

Introduce approximation to make sense of the calculation
Color propagation and rescattering

- **With gluon radiation:**
  - BDMPS mechanism (2D S-eq)
  - Path integral formulation
  - Reaction operator approach

- **Without gluon radiation:**
  - Coherent multiple scattering:
    
    \[
    f(x, Q^2) \rightarrow \sum_{n=0}^{N} \frac{1}{n!} \left[ \frac{\xi^2}{Q^2} L \right]^n x^n \frac{d^n}{dx^n} f(x, Q^2)
    \approx \exp \left[ \Delta x \frac{d}{dx} \right] f(x, Q^2)
    = f(x(1 + \Delta), Q^2)
    \]

    \[
    \Delta = \frac{\xi^2}{Q^2} L \quad \xi^2 \propto \langle F^{+\alpha} F_{\alpha}^+ \rangle
    \]

  - Qiu, Vitev, 2004, …
  - Zakharov, 1996, …
  - Wiedemann, 2000, …
  - Gyulassy et al., 2000, …
Energy loss in QCD

- **BDMPS:**
  - **Transport coefficient:**  \( \hat{q} = \frac{\mu^2}{\lambda} \)
    - \( \mu \): Typical momentum transfer in single rescattering (\( \sim \) Debye mass \( M_D \sim gT \))
    - \( \lambda \): Mean free path of the radiated gluon
  - characterizes the scattering property of the medium
  - Energy loss is proportional to the \( \hat{q} \)
  - In static cold nuclear matter:
    \[
    - \Delta E_{\text{cold}} = \frac{\alpha_s C_R}{4} \hat{q} L^2
    \]
    Plus some logarithmic corrections
  - Relationship between energy loss and momentum broadening:
    \[
    - \frac{dE_{\text{cold}}}{dx} = \frac{\alpha_s N_c}{4} \langle p^2 \rangle
    \]
    Independent of parton flavors
  - Leading order pQCD calculation for cold nuclear matter:
    \[
    \hat{q} = \frac{4\pi^2 \alpha_s N_c}{N_c^2 - 1} \rho x G(x, Q^2) \big|_{x=x_s} \approx 0.02 \text{ GeV}^2/\text{fm}
    \]
How energy loss is implemented?

- **Inclusive single hadron production (analytic, ...):**
  
  - **Assumption:**
    
    Poisson approximation for independent multiple gluon emission
  
  - **Probability for fractional energy loss:**
    
    \[ P(\varepsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \prod_{i=1}^{n} \int d\omega_i \frac{dN(\omega_i)}{d\omega} \right] e^{-\int d\omega \frac{dN}{d\omega}} \delta \left( \varepsilon - \sum_{i=1}^{n} \frac{\omega_i}{E_{jet}} \right) \]
  
  - **Medium modified fragmentation functions:**
    
    \[ D_{h/f}^{\text{med}}(z, \mu^2) \equiv \int_{0}^{1} d\varepsilon P(\varepsilon) \frac{1}{1-\varepsilon} D_{h/f}^{\text{vac}} \left( \frac{z}{1-\varepsilon}, \mu^2 \right) \]
    
    \[ \rightarrow D_{h/f}^{\text{med}}(z, \mu^2) \quad \text{if} \quad P(\varepsilon) = \delta(\varepsilon) \]

- **Final-state medium effect – energy loss:**
  
    \[ D_{h/f}^{\text{vac}}(z, \mu^2) \rightarrow D_{h/f}^{\text{med}}(z, \mu^2) \]

Salgado, Wiedemann, PRL 89 (2002)
Semi-inclusive DIS

- Better controlled kinematics:
  \[ \nu = E - E' \quad z = \frac{p \cdot k}{p \cdot q} \]

- Compare with HERMES data:

Also see talk by W. Brooks

\[ \langle -\frac{dE}{dL} \rangle_{cold} \approx 0.5 - 0.6 \text{ GeV/fm} \]

Femtometer sized detector:

\[ \nu = \frac{Q^2}{2mx} \]

Control of \( \nu \) and medium length!
Semi-inclusive DIS @ EIC

- Femtometer sized detector:

\[ \nu = \frac{Q^2}{2mx} \]

- Control of \( \nu \) and medium length!

- Heavy quark energy loss:
  - Mass dependence of fragmentation

Also see talk by W. Brooks
Semi-inclusive DIS @ EIC

- Femtometer sized detector:
  - \[ \nu = \frac{Q^2}{2mx} \]
  - Control of \( \nu \) and medium length!

- Heavy quark energy loss:
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Need the collider energy of EIC and its control on parton kinematics
Jets – where does the lost energy go?

- Medium induced radiation:
  - Small angle in/near cone
  - Thermalize with the medium:
  - Broaden the jet

No suppression if the cone is bigger enough!
Radiation is gone!
Jet “cone” dependence!
Jet transverse shape Hadronization pattern, ...

Where does the lost energy go?

We do not know, since we did not keep track of every particles

What if we do keep track of every particles?

We should know the full event shape!
Event shapes are theoretically cleaner (more inclusive!):

- Thrust, as an example:

\[ T = \max_i \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \]

- Two jet configurations obtained in the limit:
  \[ T \rightarrow 1 \]

- Resummation of logarithms of (1-T), corresponds to a resummation of the jet veto logs

- Structure of resummation is simpler, no jet algorithm dependence
  (jet algorithm dependence begins at NNLO with two emissions)
The 1-jettiness could be a probe of nuclear multiple scattering, quantifying the medium induced radiation, without “cone”, ...
Tree-level 1-jettiness distribution

\[
\frac{d^3 \sigma}{d y d P_{JT} d \tau_1} \equiv \frac{d^3 \sigma (e^- + N_A \rightarrow J + X)}{d y d P_{JT} d \tau_1}
\]

- Tree-level distribution in 1-jettiness:

\[
\frac{d^3 \sigma^{(0)}}{d y d P_{JT} d \tau_1} = \sigma_0 \delta(\tau_1) \sum_q e_q^2 \frac{1}{A} f_{q/A}(x_A, \mu)
\]

Z. Kang, Mantry, Qiu, PRD, 2012
D. Kang, Lee, Stewart, PRD, 2013
Factorization

- Schematic form of factorization:

\[
\frac{d^3 \sigma}{dy dP_{JT} d\tau_1} \sim H \otimes B_A \otimes J \otimes S,
\]

Z. Kang, Mantry, Qiu, PRD, 2012
D. Kang, Lee, Stewart, PRD, 2013
Factorized cross section

- **Detailed form of factorization:**
  \[
  \frac{d^3\sigma}{dydP_{JT}d\tau_1} = \frac{\sigma_0}{A} \sum_{q,i} e_q^2 \int_0^1 dx \int ds_J \int dt_a 
  \times H(x A Q_e P_{JT} e^{-y}, \mu; \mu_H) \delta\left[x - \frac{e^y P_{JT}}{A(Q_e - e^{-y} P_{JT})}\right]
  \times J^q(s_J, \mu; \mu_J) B^q(x, t_a, \mu; \mu_B)
  \times S\left(\tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S\right),
  \]

- **Beam function matching onto the PDF:**
  \[
  B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} T^{qi}\left(\frac{x}{z}, t_a, \mu; \mu_B\right) f_{i/A}(z, \mu_B)
  \]

- **Tree-level matching:**
  \[
  B^q(x, t_a, \mu_B) = \delta(t_a) f_{q/A}(x, \mu_B)
  \]

Z. Kang, Mantry, Qiu, PRD, 2012
D. Kang, Lee, Stewart, PRD, 2013
1-jettiness distribution for various nuclei

Effect of nPDFs and smearing

\[ R_L(x, \mu) = \frac{\sum_q c_q^2 f_{q/A}^{EPS09}(x, \mu)}{\sum_q c_q^2 f_{q/p}(x, \mu)} \]

\[ R_x^A \]

NNLL resummation

\[ Q_e = 90 \text{ GeV} \]
\[ P_{JT} = 20 \text{ GeV} \]
\[ y = 0 \]
Summary

- Jet is a well-defined observable – not unique!
  - a “footprint”, or “living story” of an energetic quark/gluon

- QCD parton energy loss is a very interesting and rich topic
  - Color confinement and gluon self-interaction distinguish it from QED energy loss
  - Parton dE/dx is not a direct physical observable!
  - But, it does influence the observed hadronic cross section

- Jet’s transverse structure – measurable/calculable!
  - A time-dilated hadronization or color neutralization process

- Event shape based analysis can be a useful tool
  - Quantify gluon shower, medium induced radiation, …

- QCD dynamics is rich – EIC will help explore it!

Thanks!
Backup slides
Medium induced gluon production

- **Approximation:**
  \[
  \mathcal{M}_1 = -2ig_5e^{i\theta_0}e_\perp \cdot \{ \vec{H}_01c + \vec{B}_1e^{i\theta_0} [c, a_1] + \vec{C}_1e^{-i\theta_0(\omega_1 - \omega_0)} [c, a_1] \}.
  \]
  
  \[
  H = \frac{k}{k^2}, \quad C_{(i_1i_2...i_m)} = \frac{(k - q_{i_1} - q_{i_2} - ... - q_{i_m})}{(k - q_{i_1} - q_{i_2} - ... - q_{i_m})^2},
  \]
  
  \[
  B_i = H - C_i, \quad B_{(i_1i_2...i_m)(j_1j_2...i_n)} = C_{(i_1i_2...j_m)} - C_{(j_1j_2...j_n)}.
  \]

- **Induced gluon production:**
  \[
  \frac{dN_g^{(GB)}}{dyd^2k_\perp} = C_A \frac{\alpha_s}{\pi^2} \frac{q_1^2}{k_\perp^2(k - q_1)^2}.
  \]
  Where \( y = \ln 1/x \) is interpreted as rapidity.

- **Gluon production in a medium:**
  \[
  \left\langle \frac{dN_g^{BG}}{dyd^2k_T} \right\rangle_{q-transfer} = \int d^2q_T \left[ \frac{\mu^2}{\pi (q_T^2 + \mu^2)^2} \right] C_A \frac{\alpha_s(k_T)}{\pi^2} \frac{q_T^2}{k_T^2(k_T - q_T)^2}
  \]
  \[
  \sim C_A \frac{\alpha_s(k_T)}{\pi^2} \left( \frac{\mu^2}{k_T^2(k_T^2 + \mu^2)} \right)
  \]
  Introduce screen mass \( \mu \).