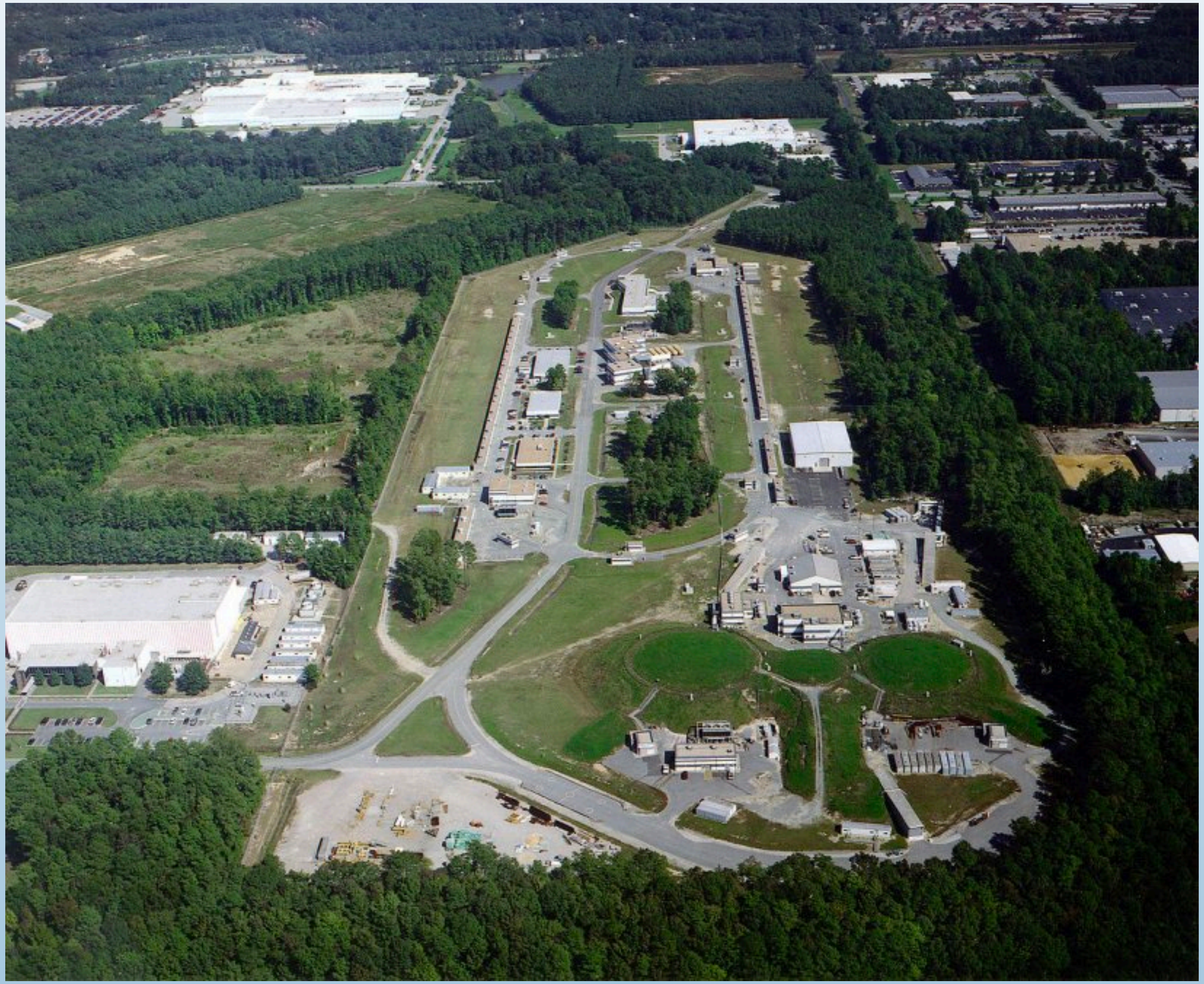
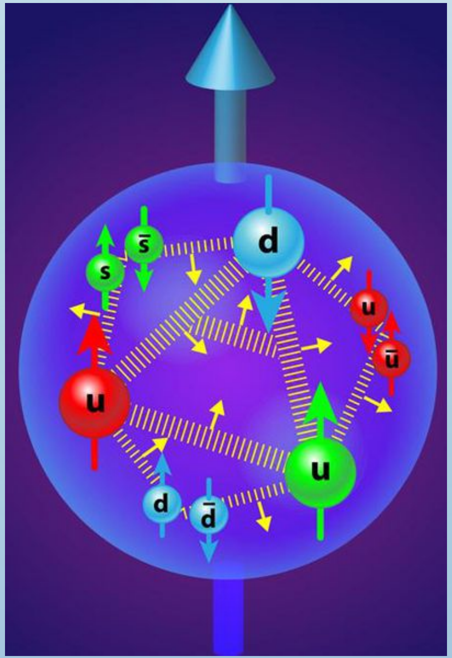
Impact of High x JLab Data on Transversity PDFs and Tensor Charges

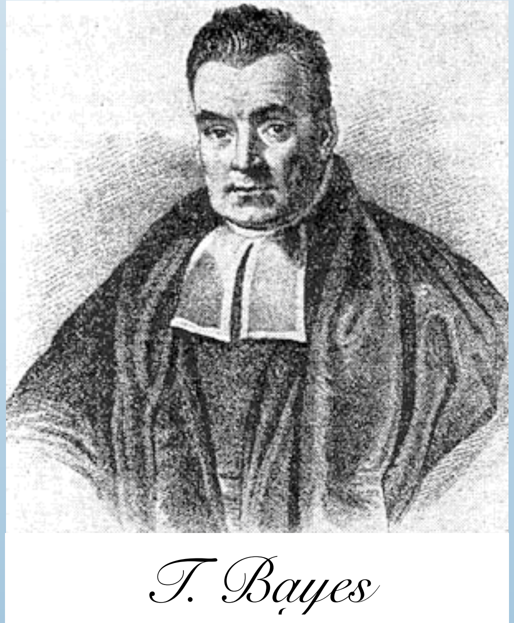
Christopher Cocuzza

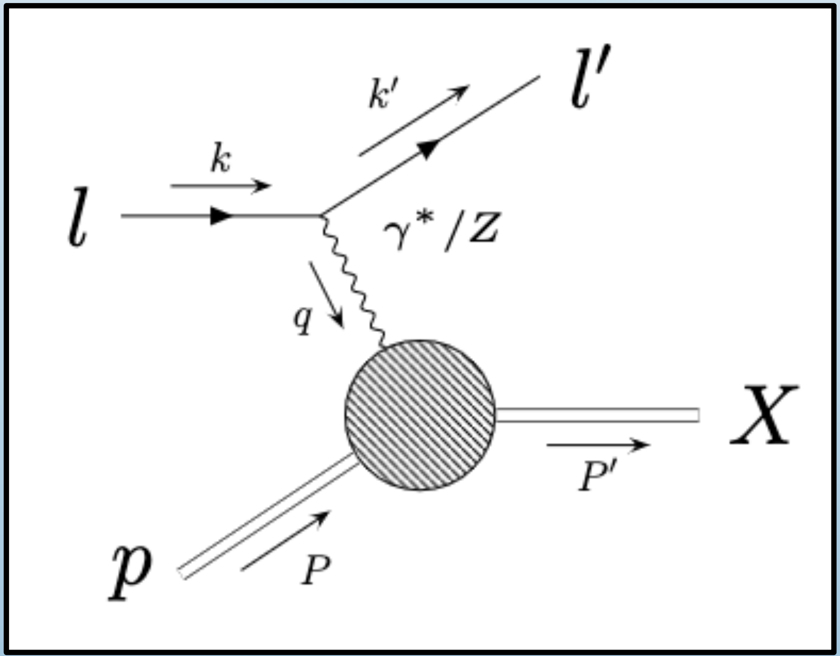


www.jlab.org/theory/jam

December 2, 2024




T. Bayes

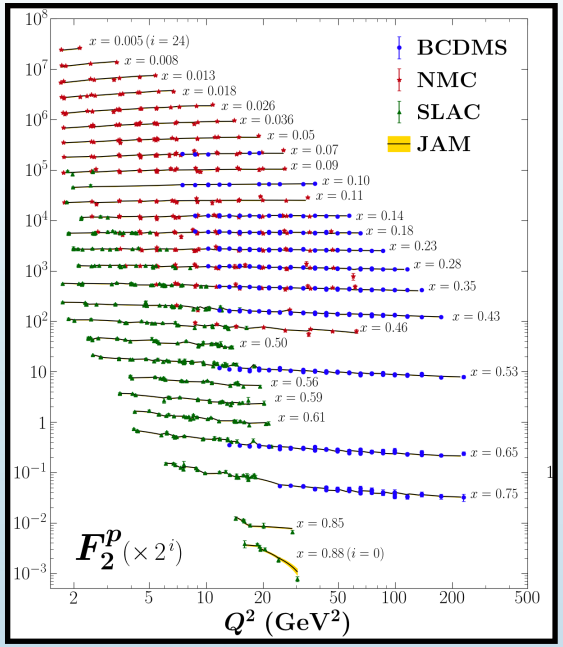

l
k
k'
l'
γ* /Z
q
X
P'
p
P

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

χ^2 Minimization

$$\mathcal{L}\left(\boldsymbol{a}, \text{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \text{data}\right)\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$



Hadron Structure

$$\frac{d}{d\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

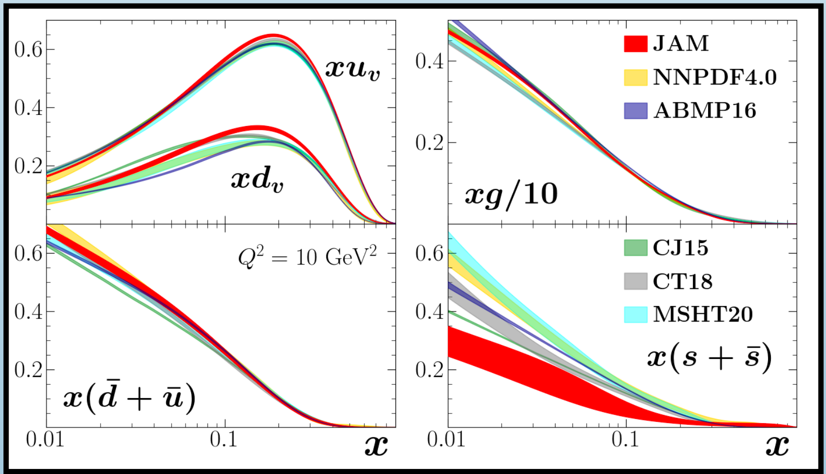
Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration



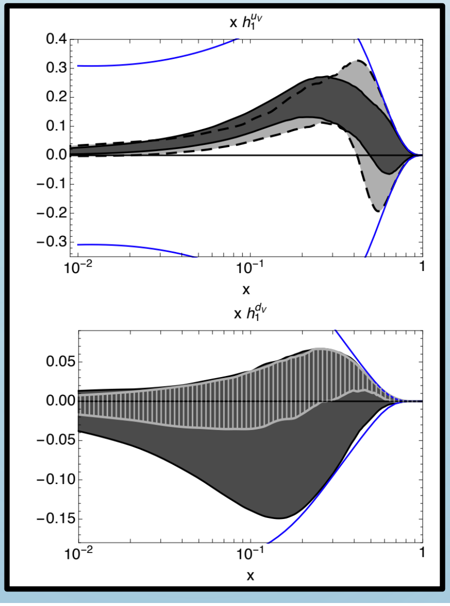
Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

Approaches to Extract Transversity

Dihadron Frag.

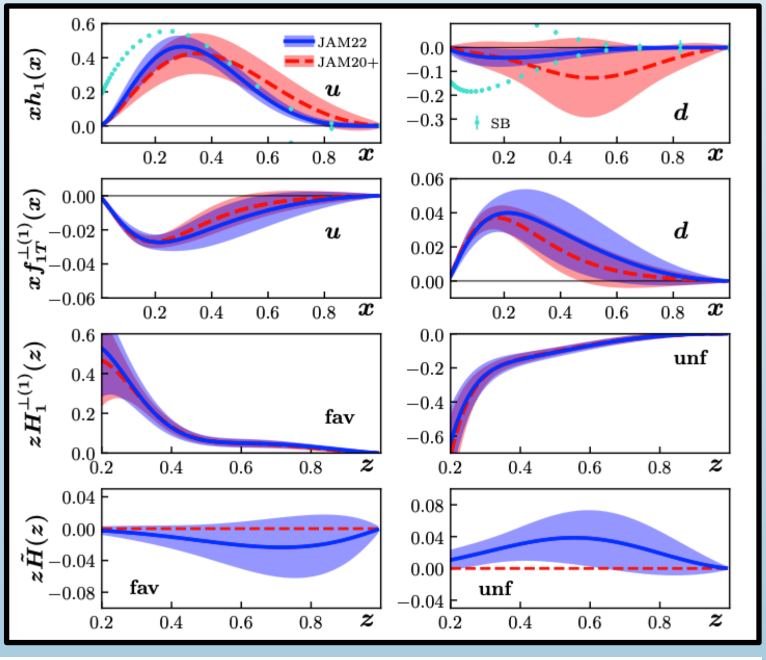
- Radici + Bacchetta (RB18)
- Benel + Courtoy + Ferro-Hernandez (2020)



M. Radici and A. Bacchetta, Phys. Rev. Lett. **120**, no. 19, 192001 (2018)

TMD + Collinear Twist-3

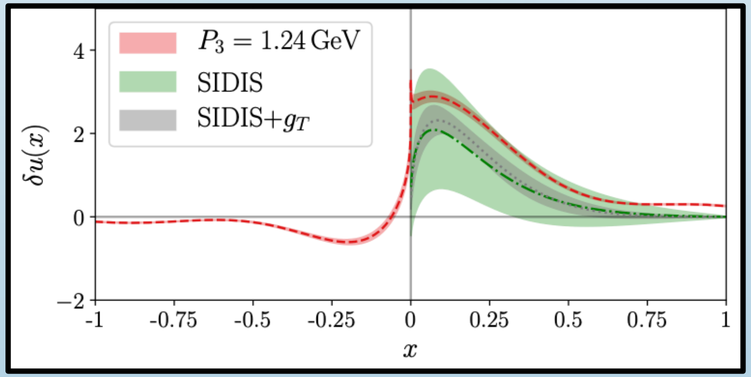
- JAM3D



L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

Lattice QCD

- ETMC Collaboration
- PNDME Collaboration
- LHPC Collaboration



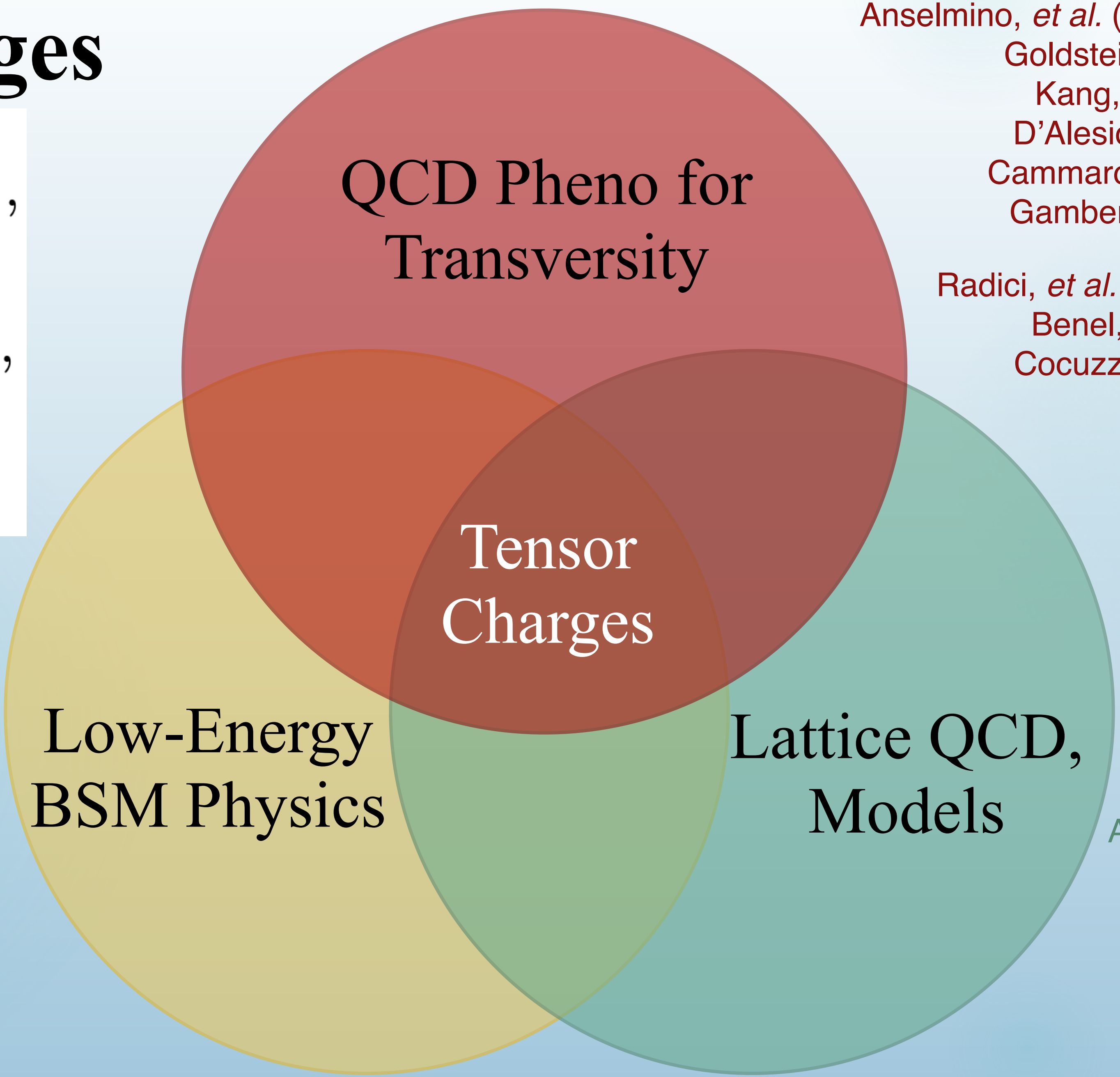
C. Alexandrou *et al.*, Phys. Rev. D **104**, no. 5, 054503 (2021)

Tensor Charges

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$



Anselmino, *et al.* (2007, 2009, 2013, 2015);
Goldstein, *et al.* (2014);
Kang, *et al.* (2016);
D'Alesio, *et al.* (2020);
Cammarota, *et al.* (2020);
Gamberg, *et al.* (2022)

Radici, *et al.* (2013, 2015, 2018);
Benel, *et al.* (2020);
Cocuzza, *et al.* (2023)

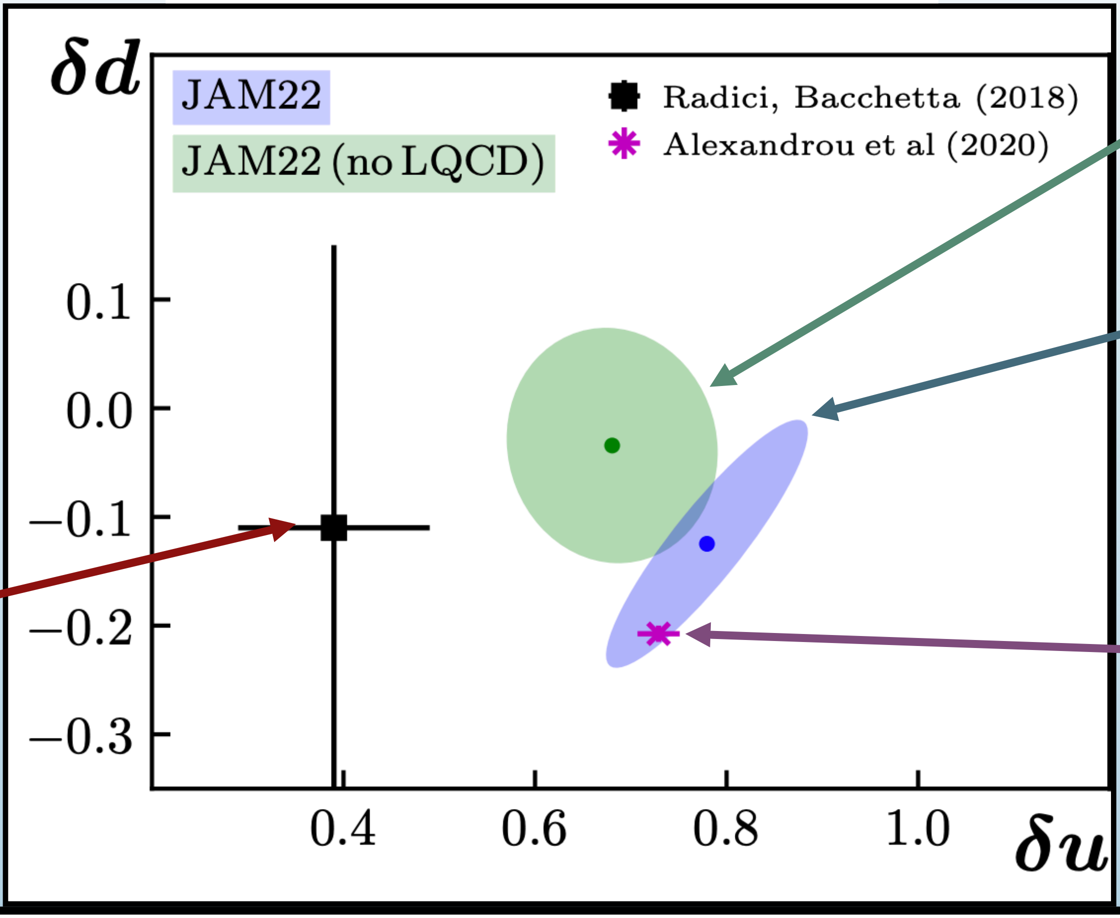
He, Ji (1995);
Barone, *et al.* (1997);
Schweitzer, *et al.* (2001);
Gamberg, Goldstein (2001);
Pasquini, *et al.* (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, *et al.* (2018);
Yamanaka, *et al.* (2018);
Hasan, *et al.* (2019);
Alexandrou, *et al.* (2019, 2023);
Yamanaka, *et al.* (2013);
Pitschmann, *et al.* (2015);
Xu, *et al.* (2015);
Wang, *et al.* (2018);
Liu, *et al.* (2019)

Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, *et al.* (2006);
Cirigliano, *et al.* (2013);
Courtoy, *et al.* (2015);
Yamanaka, *et al.* (2017);
Liu, *et al.* (2018);
Gonzalez-Alonso, *et al.* (2019)

Figure taken from D. Pitonyak

The Transverse Spin Puzzle?

L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)



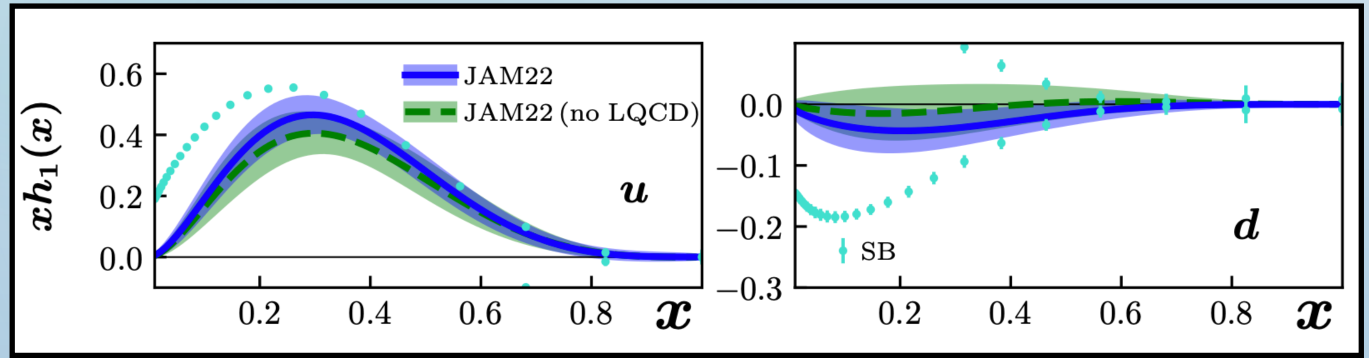
JAM3D (no LQCD)

JAM3D (w/ LQCD)

RB18

Lattice (ETMC)

Large disagreements between three approaches…
Can this be solved?



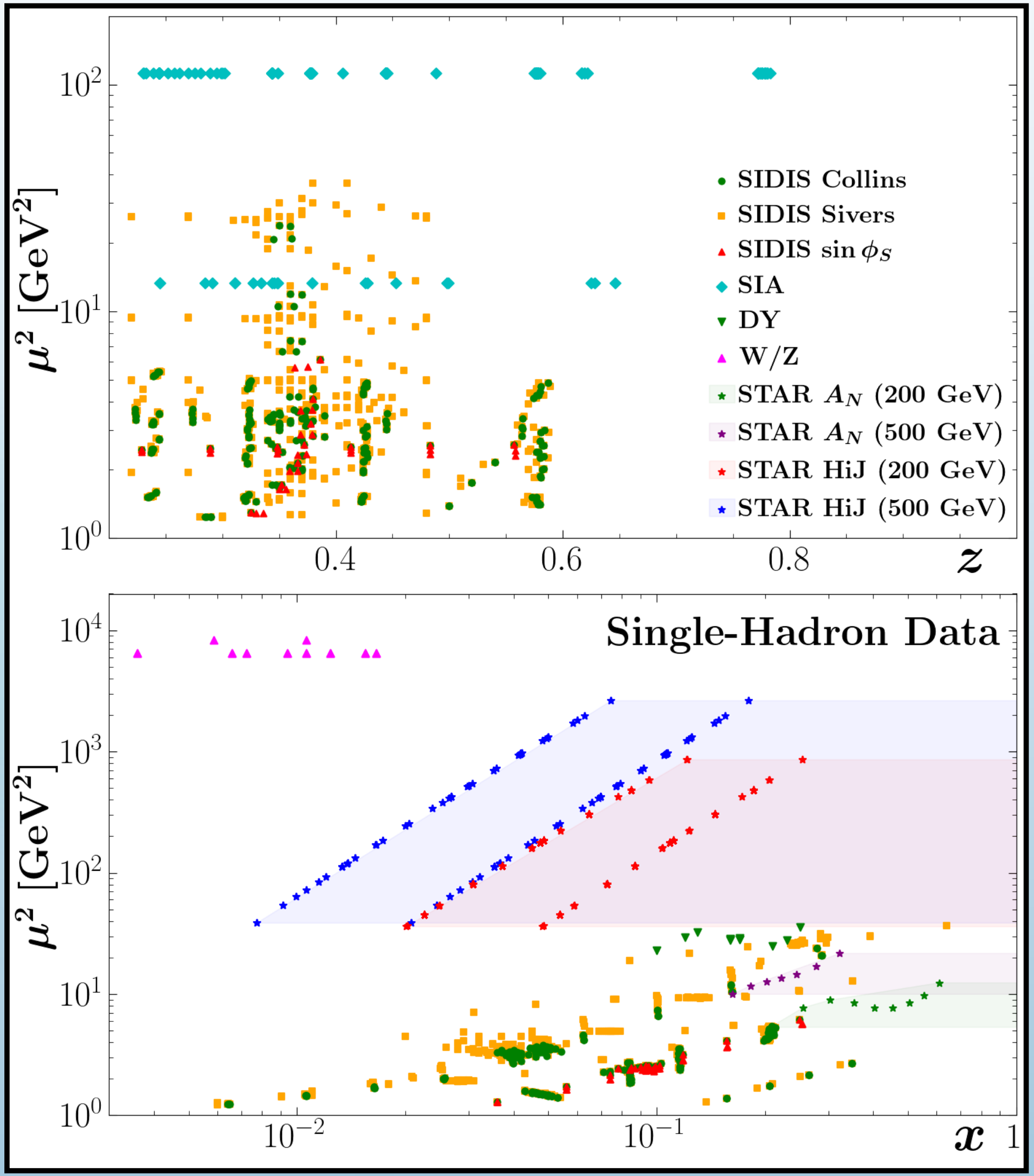
Updated QCD global analysis of single transverse-spin asymmetries: Extracting $\tilde{H}$, and the role of the Soffer bound and lattice QCD

Jefferson Lab Angular Momentum (JAM) and Jefferson Lab Angular Momentum Collaborations • Leonard Gamberg (Penn State U., Berks-Lehigh Valley) Show All(6)

May 2, 2022

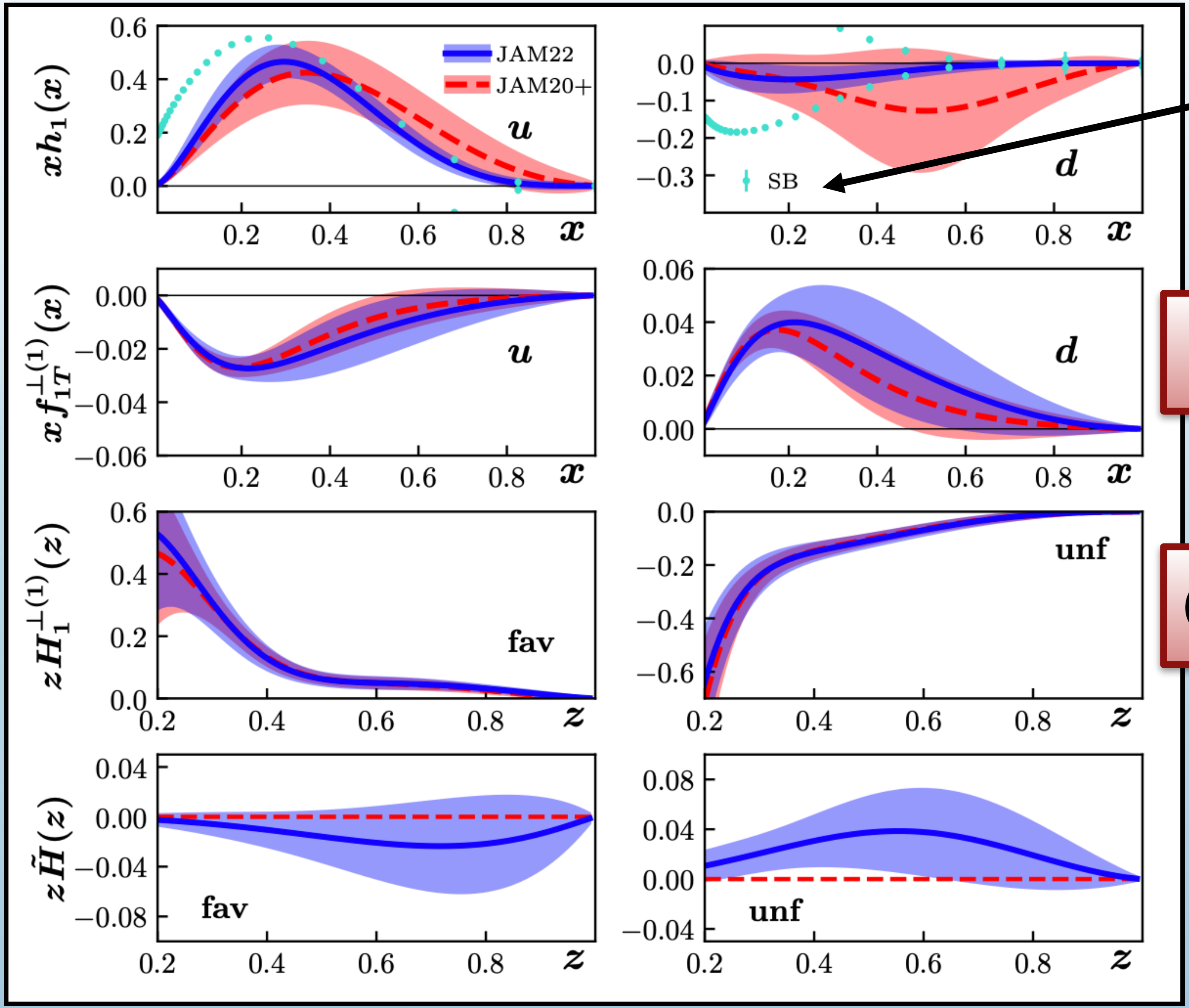
27 pages
Published in: *Phys.Rev.D* 106 (2022) 3, 034014

JAM3D Data



SIA	BaBar, Belle, BESIII	176 points
SIDIS (p, D)	COMPASS, HERMES	368 points
Drell-Yan	COMPASS	12 points
W/Z Production	STAR	17 points
AN	STAR	60 points

JAM3D Functions



Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

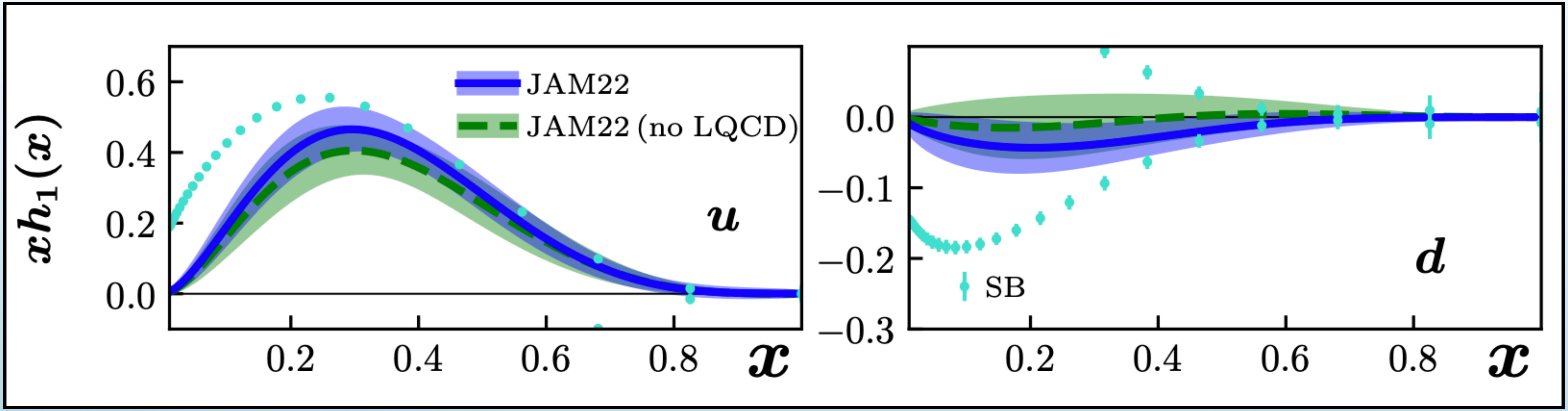
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Sivers

Collins

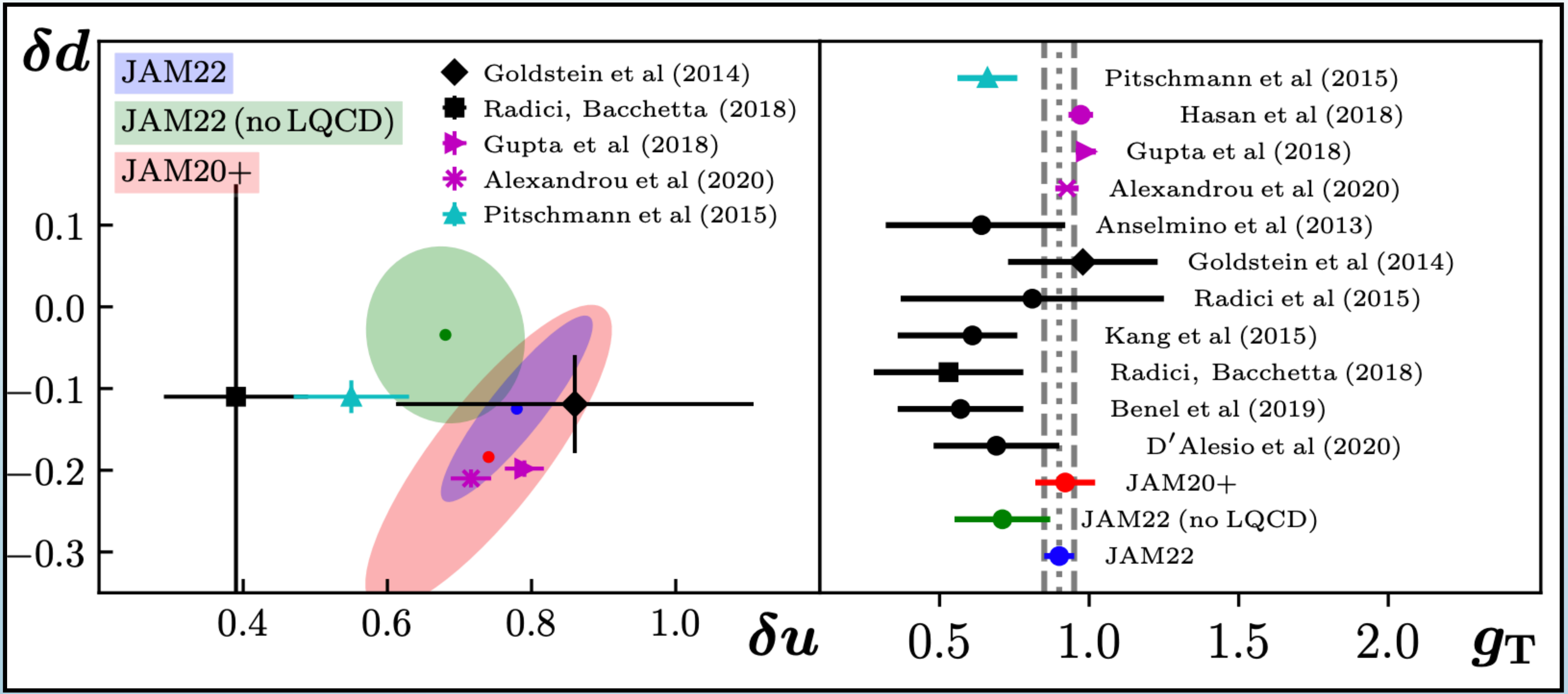
Transversity PDFs extracted alongside Collins and Sivers functions

JAM3D PDFs

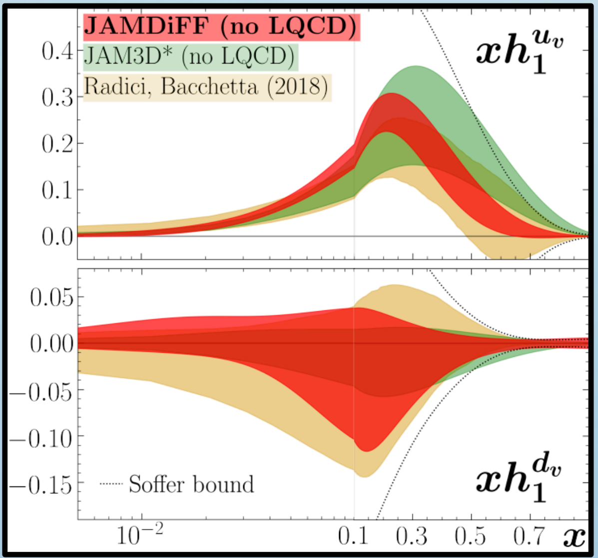


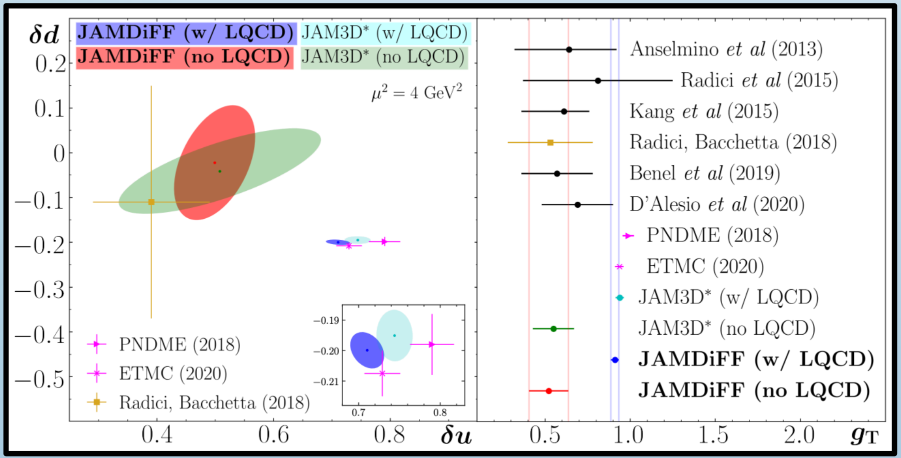
Fit was done with and without LQCD data included

JAM3D Tensor Charges



Showed compatibility between TMD data and LQCD calculations





First simultaneous global QCD analysis of dihadron fragmentation functions and transversity parton distribution functions

Jefferson Lab Angular Momentum (JAM) Collaboration • C. Cocuzza (Temple U.) Show All(6)

Aug 28, 2023

34 pages

Published in: *Phys.Rev.D* 109 (2024) 3, 034024

Transversity Distributions and Tensor Charges of the Nucleon: Extraction from Dihadron Production and Their Universal Nature

JAM Collaboration • C. Cocuzza (Temple U.) Show All(6)

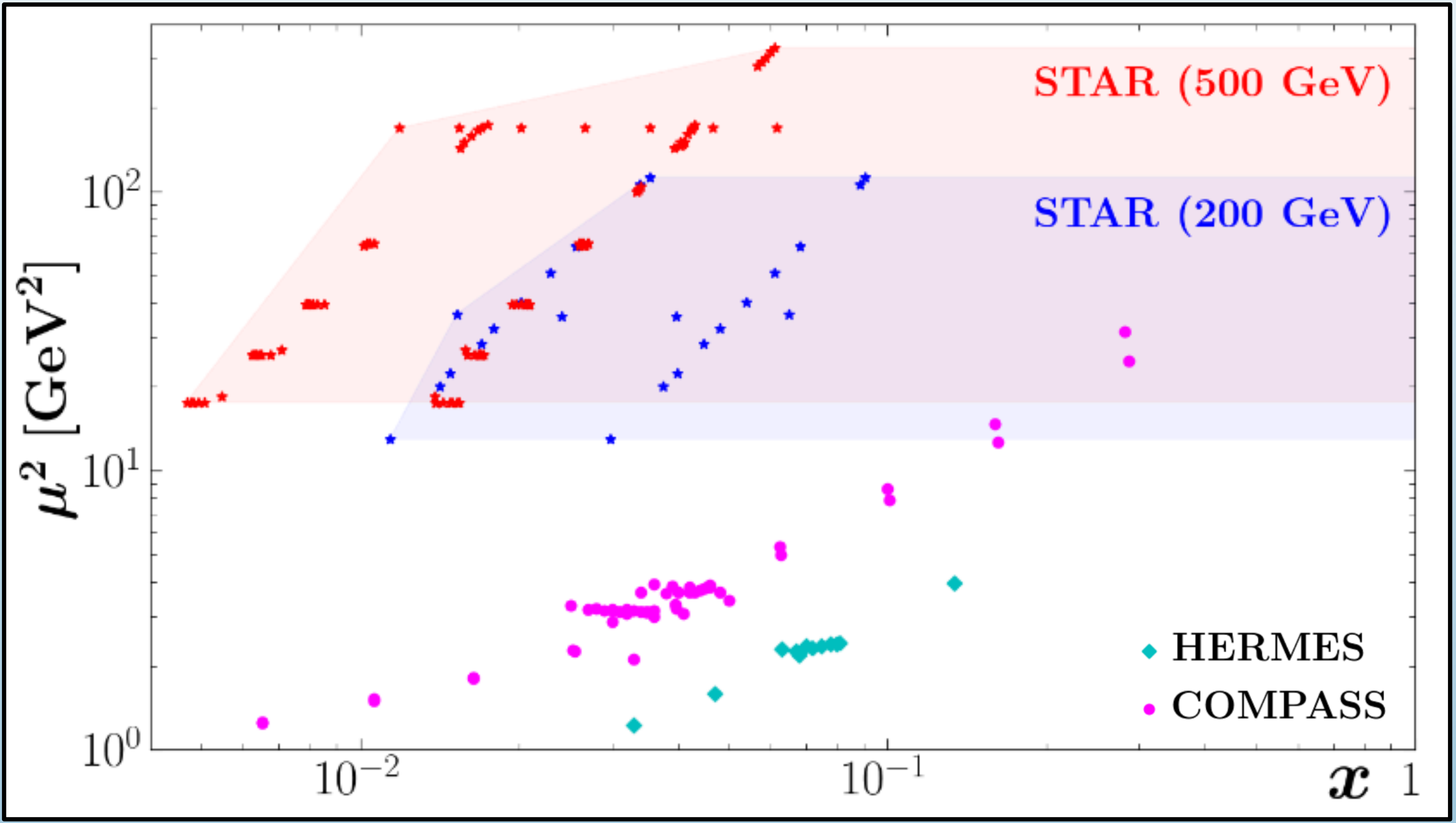
Jun 22, 2023

7 pages

Published in: *Phys.Rev.Lett.* 132 (2024) 9, 091901

JAMDiFF Data

SIDIS (p, D)	COMPASS, HERMES	64 points
Proton-Proton	STAR	269 points



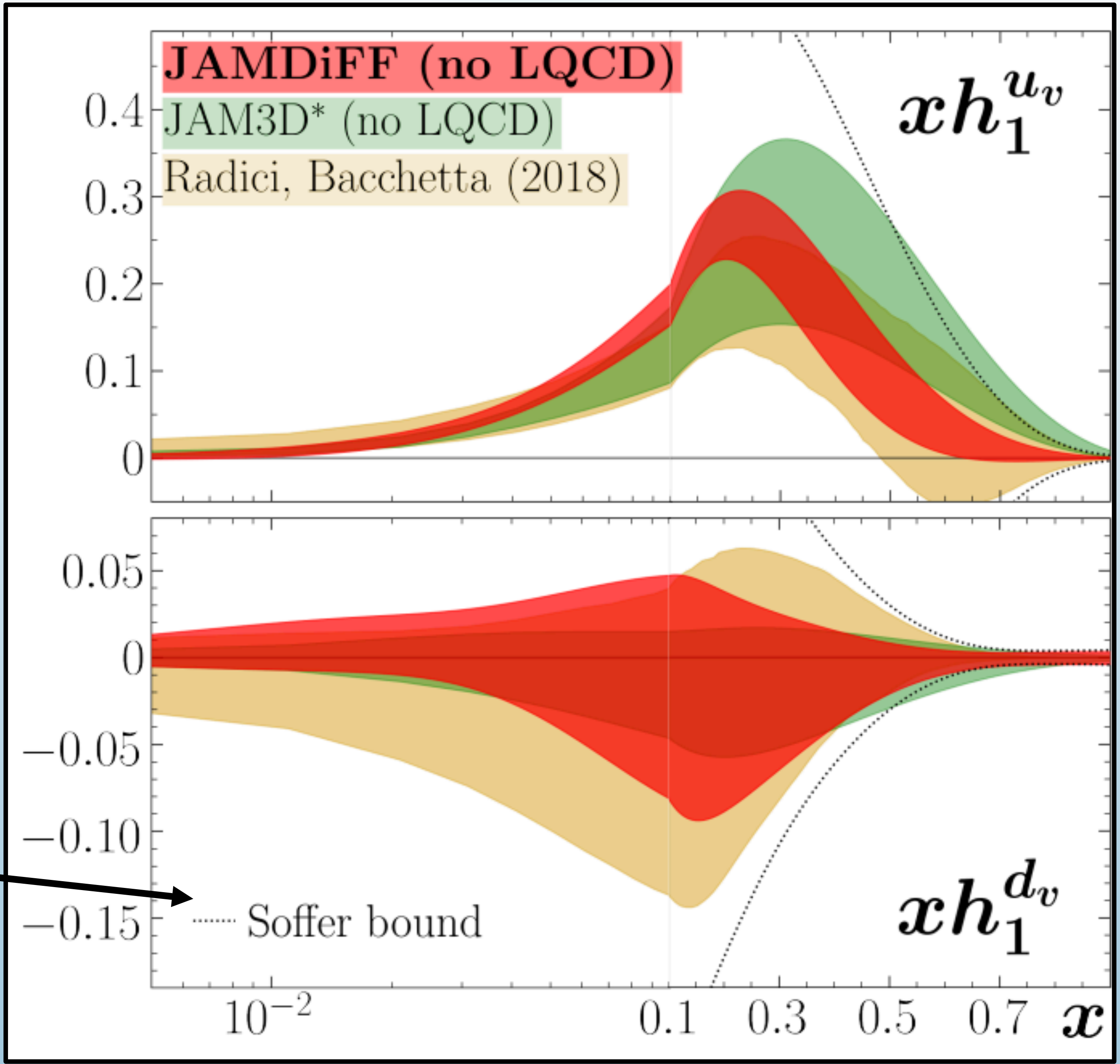
JAMDiFF PDFs

JAM3D* = JAM3D-22 (no LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 23)

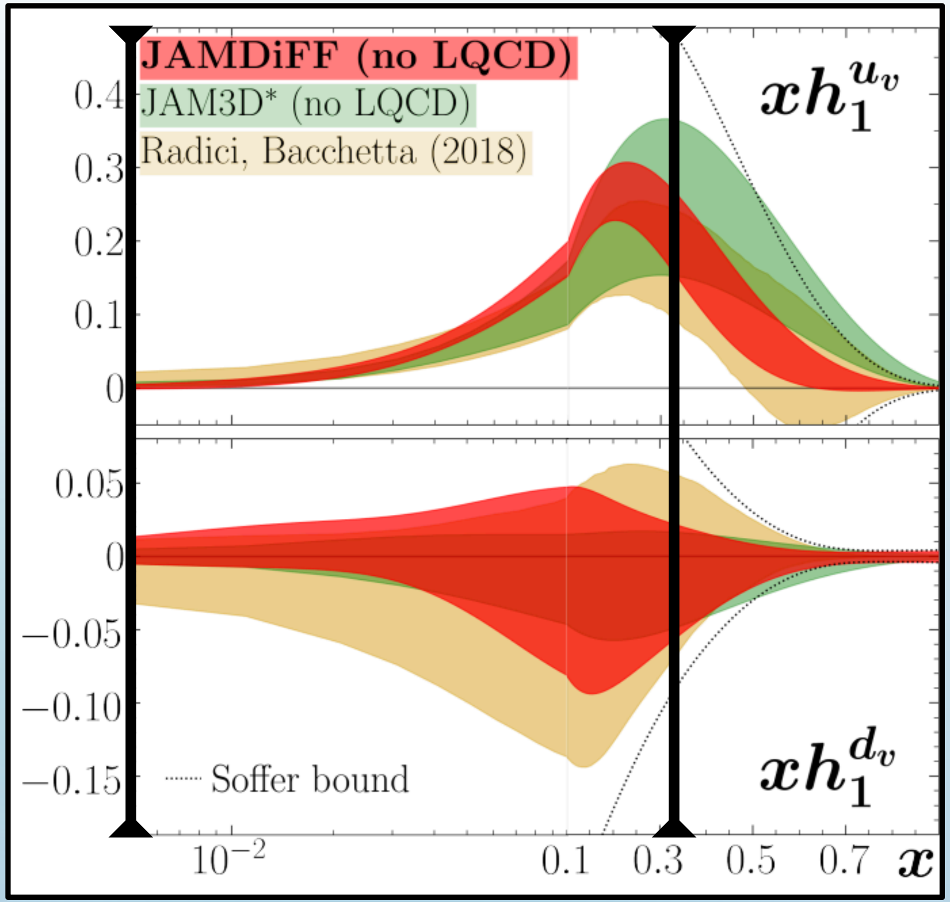
Agreement between all three analyses within errors

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



Controlling Extrapolation



Measured Region

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Large $x \gtrsim 0.3$

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

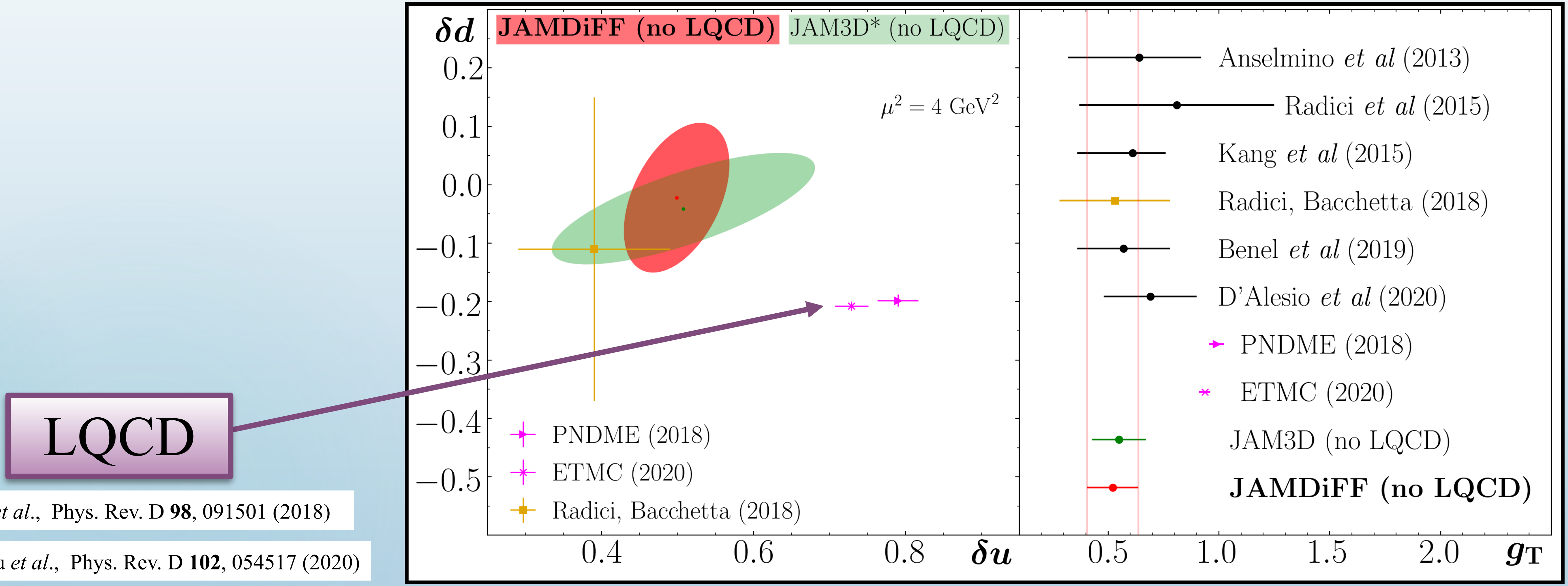
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Small $x \lesssim 0.005$

$$h_1^q \xrightarrow[x \to 0]{} x^{\alpha_q} \qquad \alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 0.17 \pm 0.085$$

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **99**, 054033 (2019)

JAMDiFF Tensor Charges

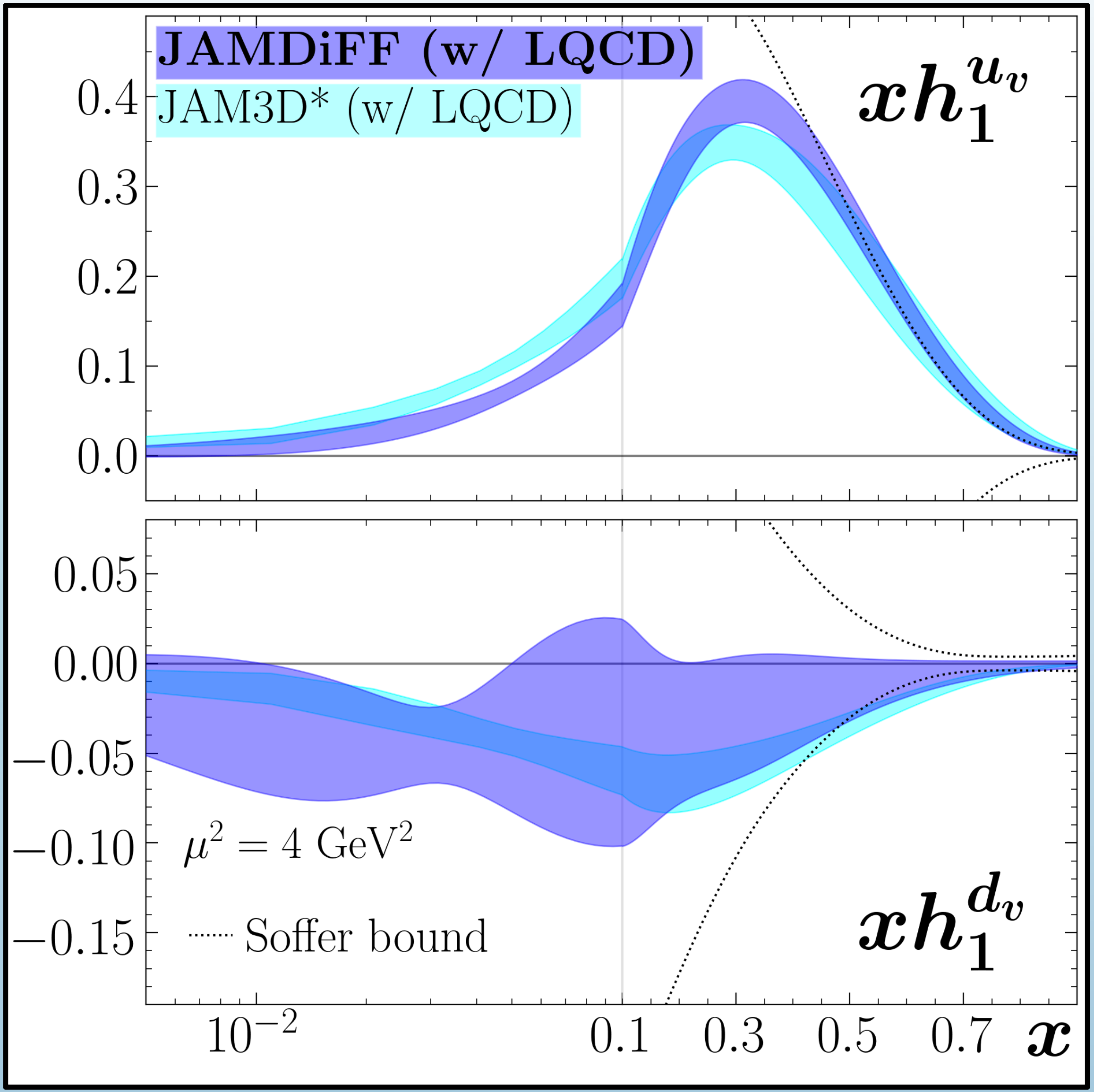


R. Gupta *et al.*, Phys. Rev. D **98**, 091501 (2018)

C. Alexandrou *et al.*, Phys. Rev. D **102**, 054517 (2020)

Consistent with RB18 and JAM3D* (no LQCD).
What happens if we include LQCD in the fit?

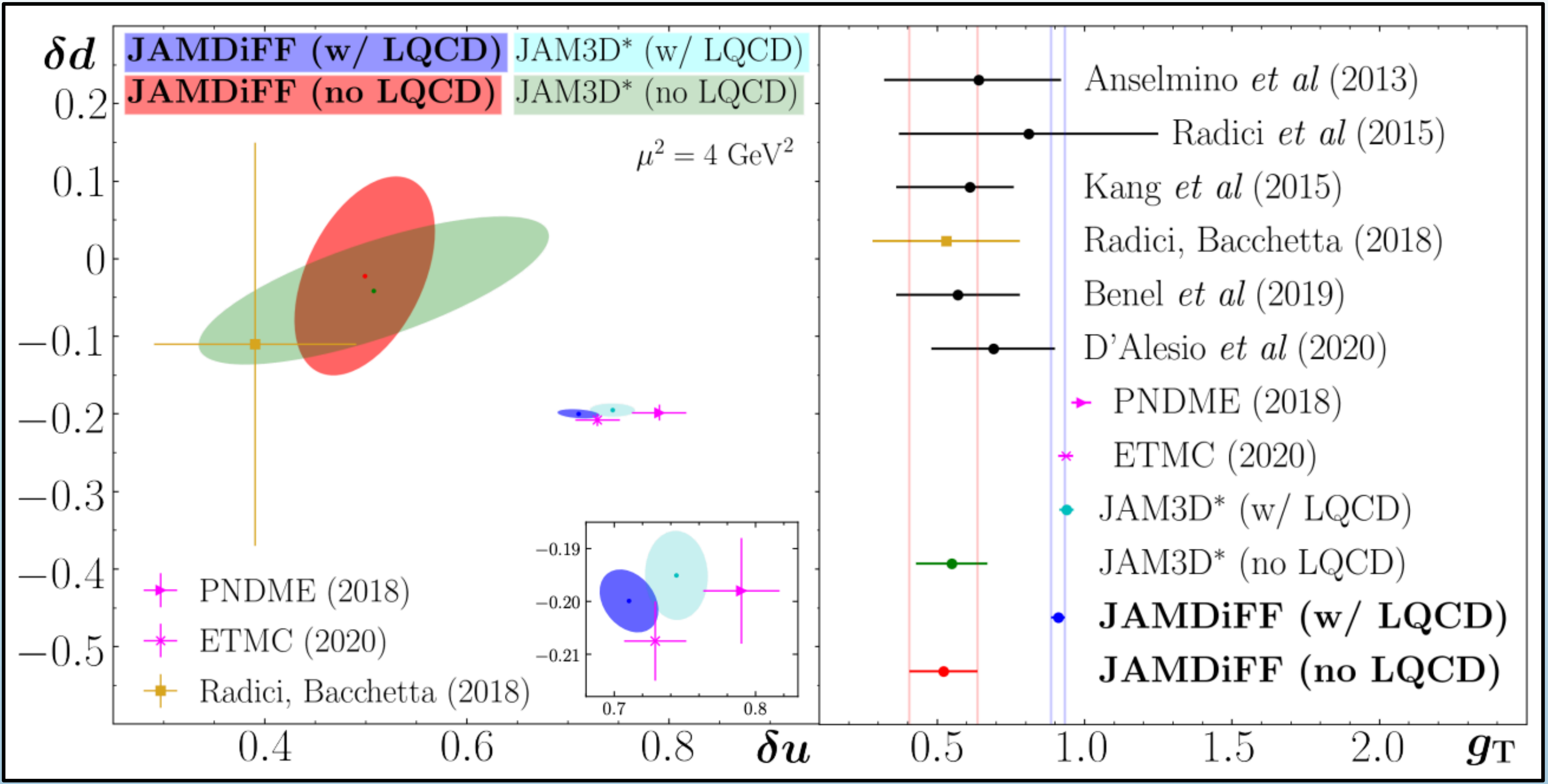
Transversity PDFs (w/ LQCD)



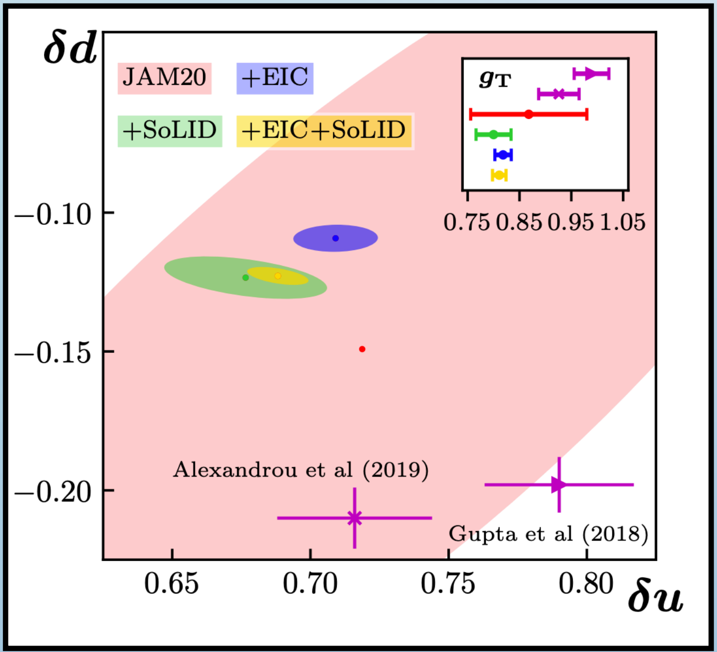
JAM3D* = JAM3D-22 (w/ LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 23)
+ $\delta u, \delta d$ from ETMC & PNDME
(instead of g_T from ETMC)

JAMDiFF (w/ LQCD) and JAM3D* (w/ LQCD) largely agree

Tensor Charges (w/ LQCD)



1. Introduction
2. DiFF Extraction Results
3. TMD Extraction Results
4. **Impact of Jefferson Lab Data**
5. Conclusions and Outlook



Electron-Ion Collider impact study on the tensor charge of the nucleon

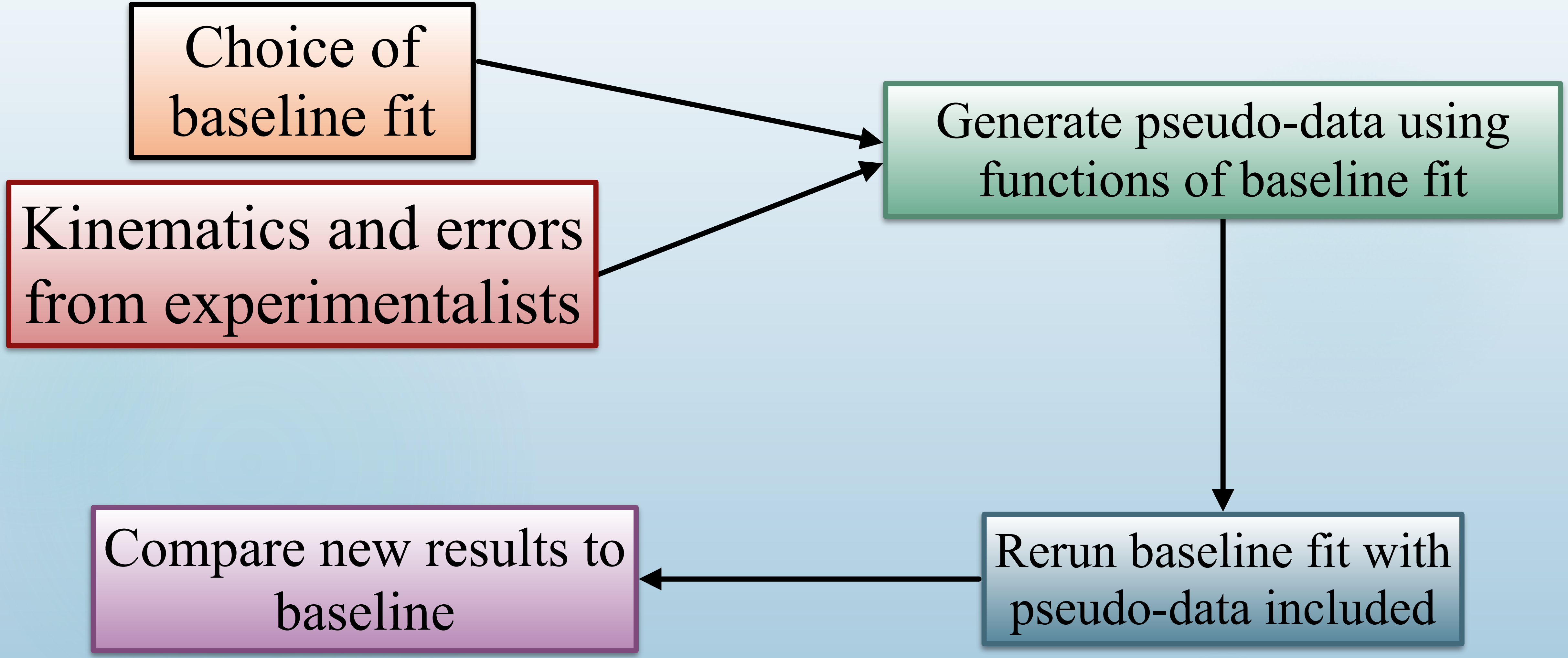
Leonard Gamberg (Penn State U., Berks-Lehigh Valley), Zhong-Bo Kang (UCLA), Daniel Pitonyak (Lebanon Valley Coll.), Alexei Prokudin (Jefferson Lab), Nobuo Sato (Jefferson Lab) Show All(6)

Jan 15, 2021

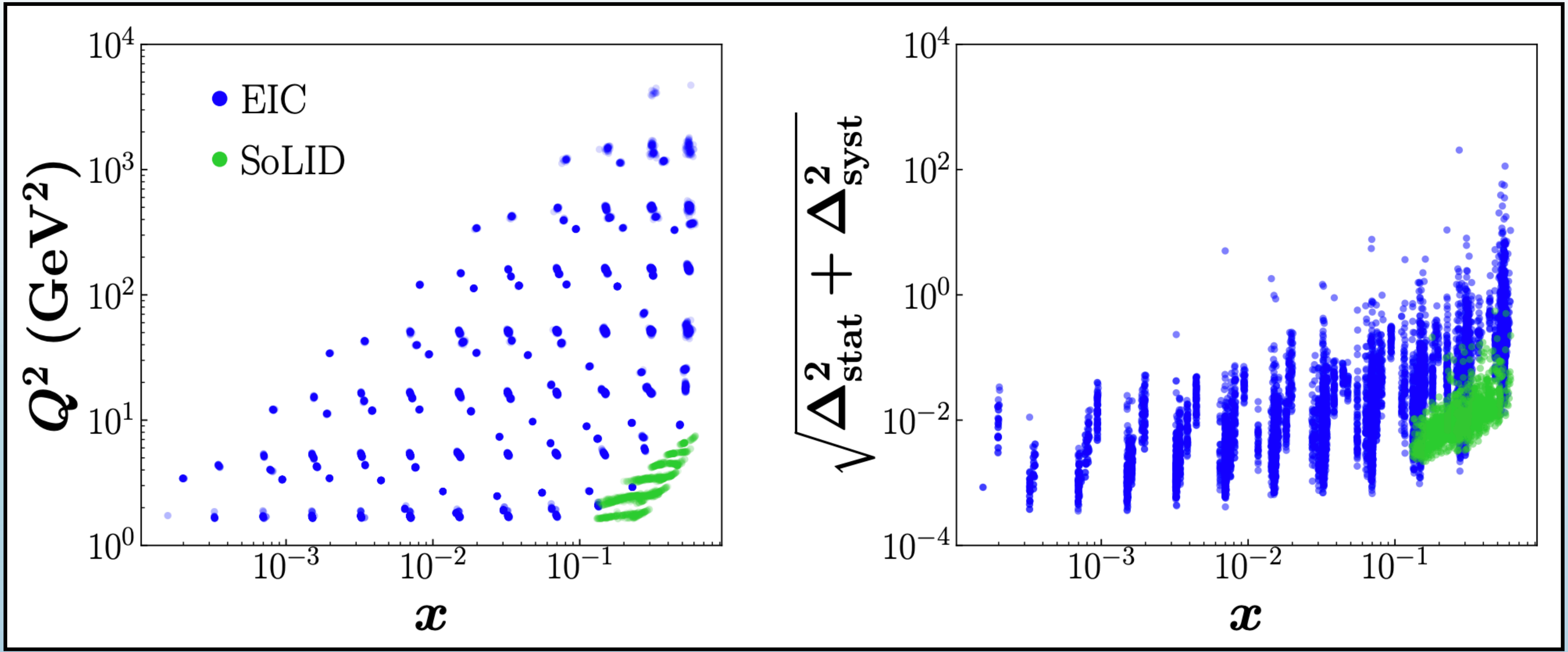
12 pages

Published in: *Phys.Lett.B* 816 (2021) 136255

JAM Pseudo-Data Approach

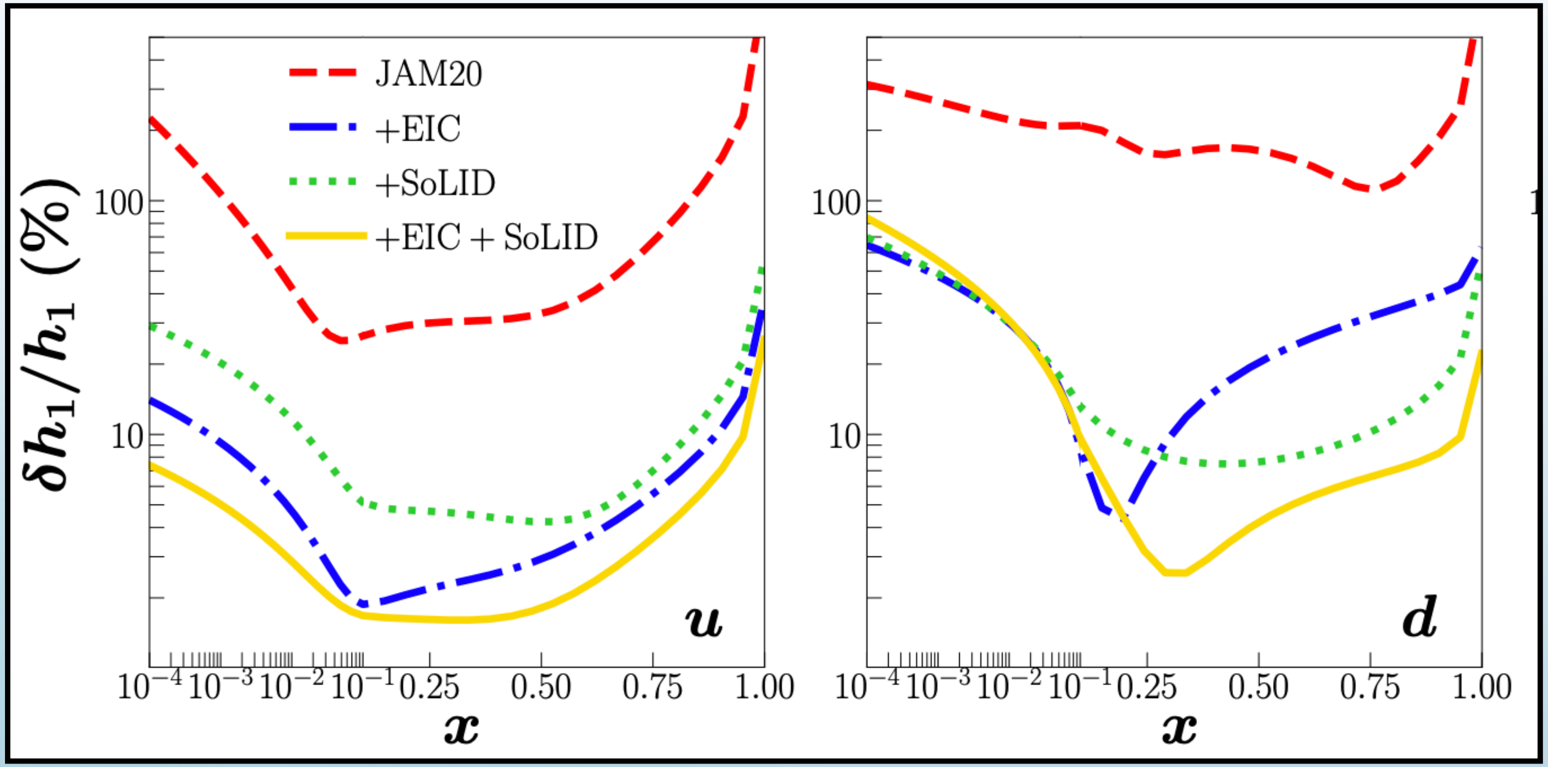


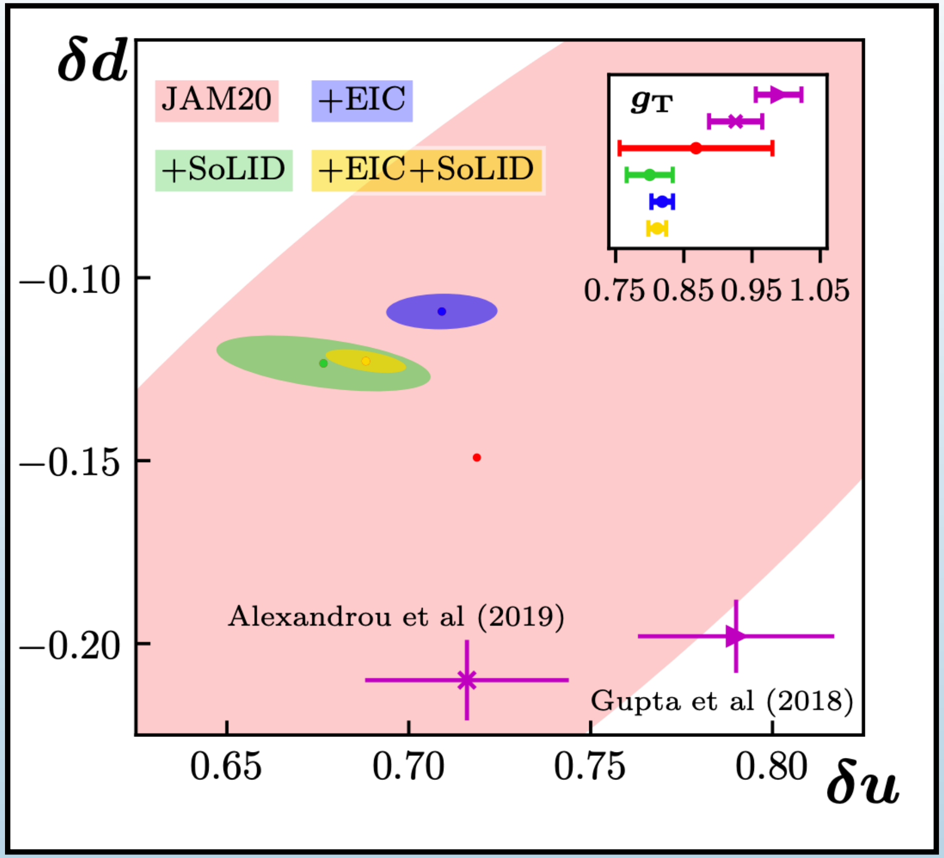
JAM3D Impact (SoLID)



SoLID data is for helium (neutron) target, covers $0.1 < x < 0.6$

JAM3D Impact (SoLID)





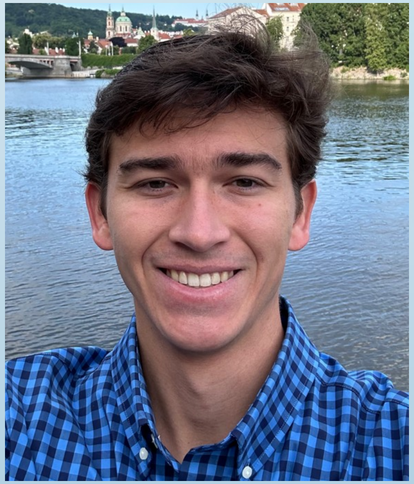
SoLID provides strong constraints on up and down PDFs

Greatly reduces errors on tensor charges

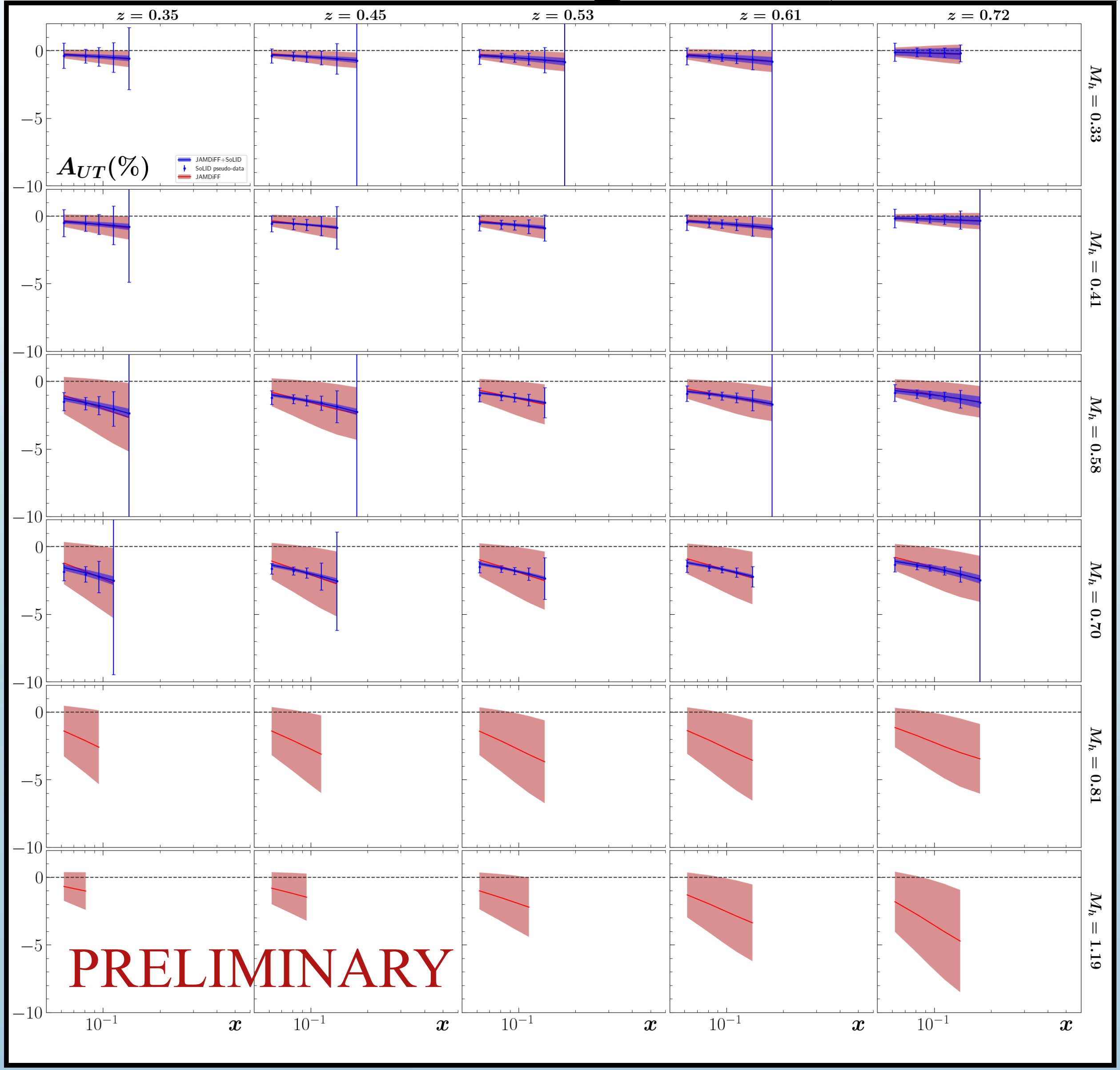
PRELIMINARY JAMDiFF Impact Study

Thanks to **Yorgo Sawaya** for running impact analysis in JAMDiFF framework

Thanks to **Matthew McEneaney** for producing pseudo-data



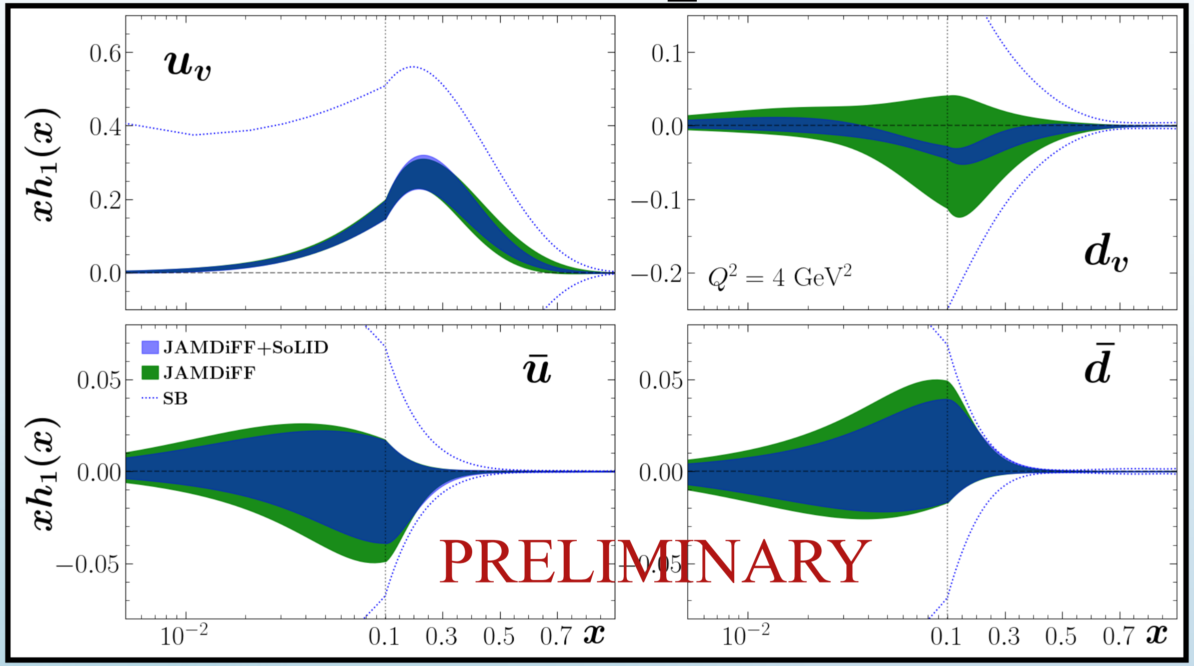
JAMDiFF Impact (SoLID)

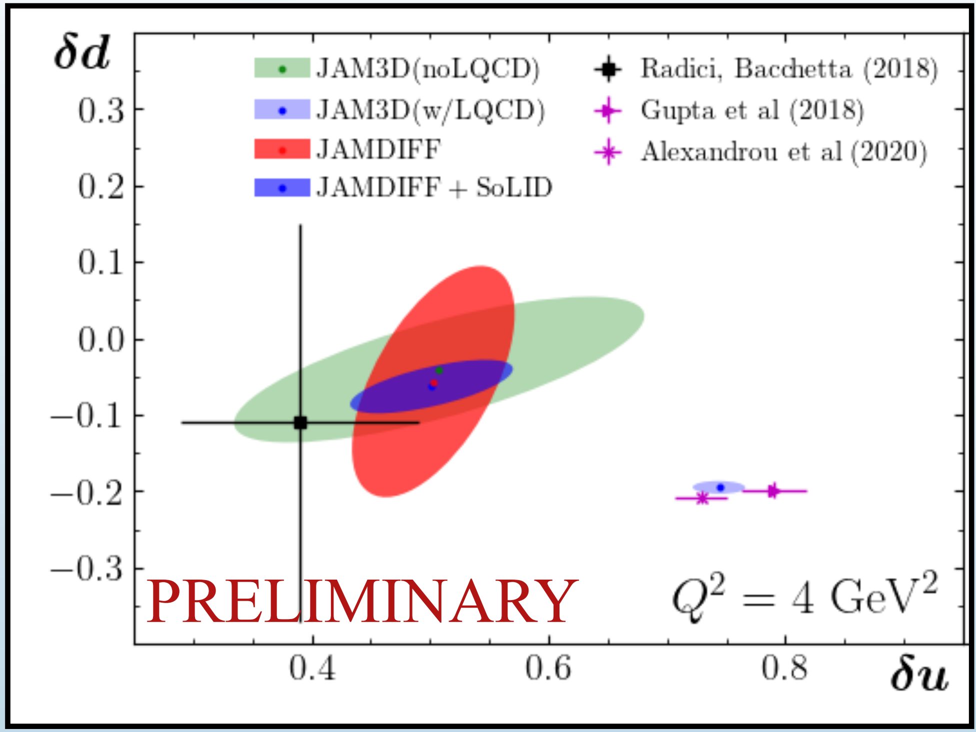


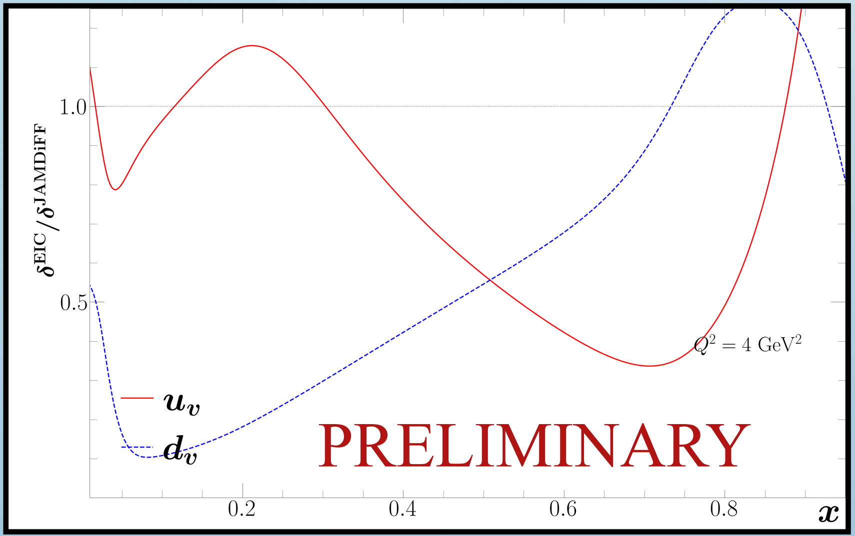
SoLID data is for helium (neutron), sensitive mostly to down quark transversity PDF

Note that all pseudo-data is generated using the (no LQCD) result of JAMDiFF!

JAMDiFF Impact (SoLID)

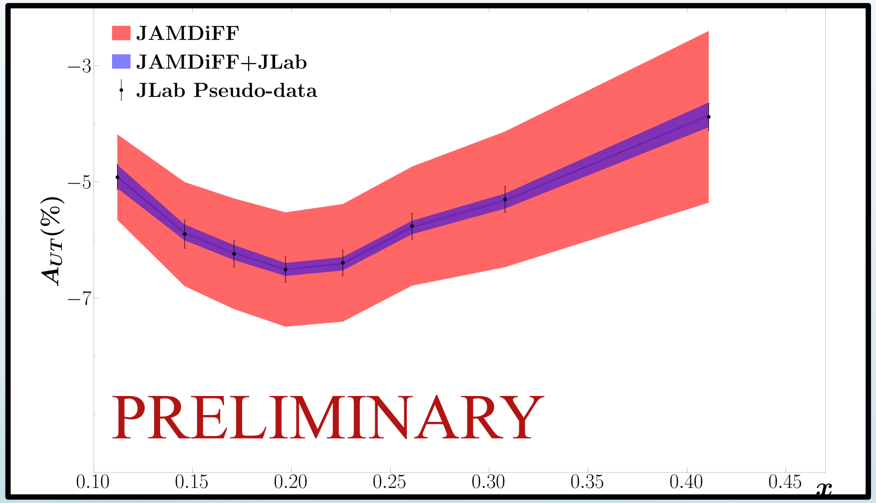


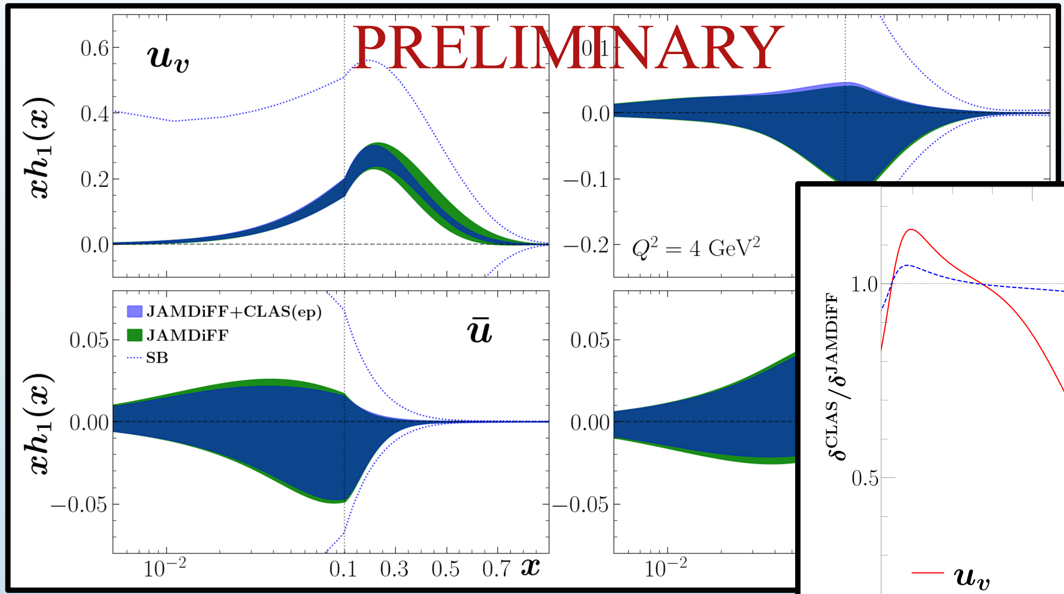


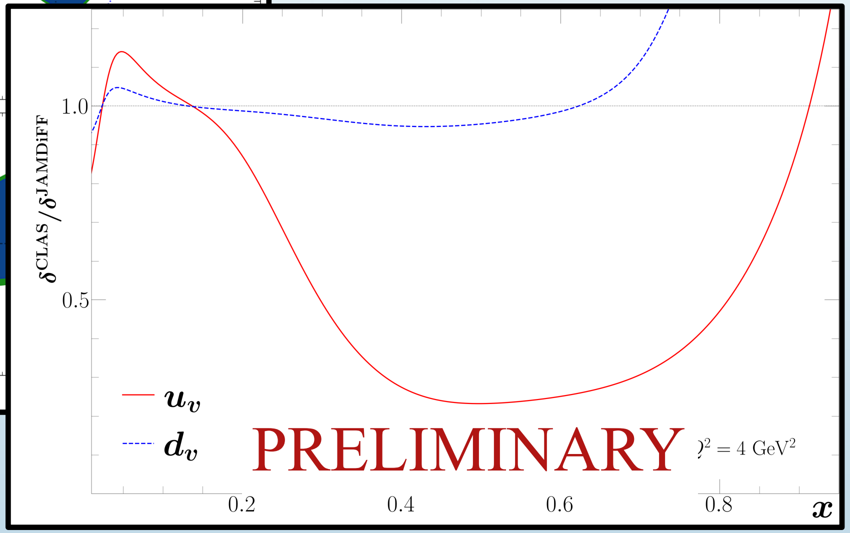


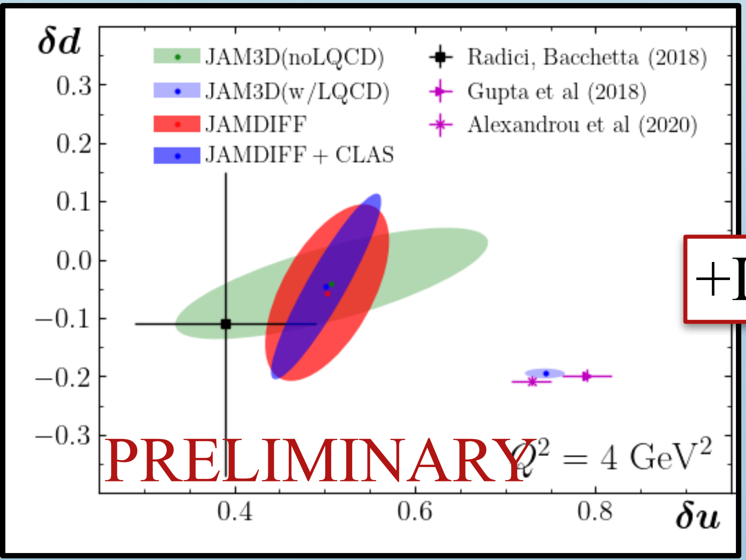
SoLID provides very strong experimental constraints on the down quark!

JAMDiFF Impact (CLAS12 proton)

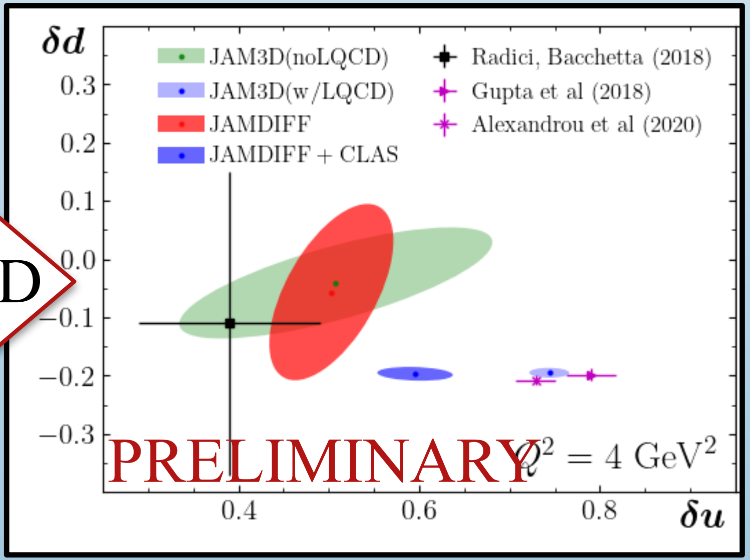






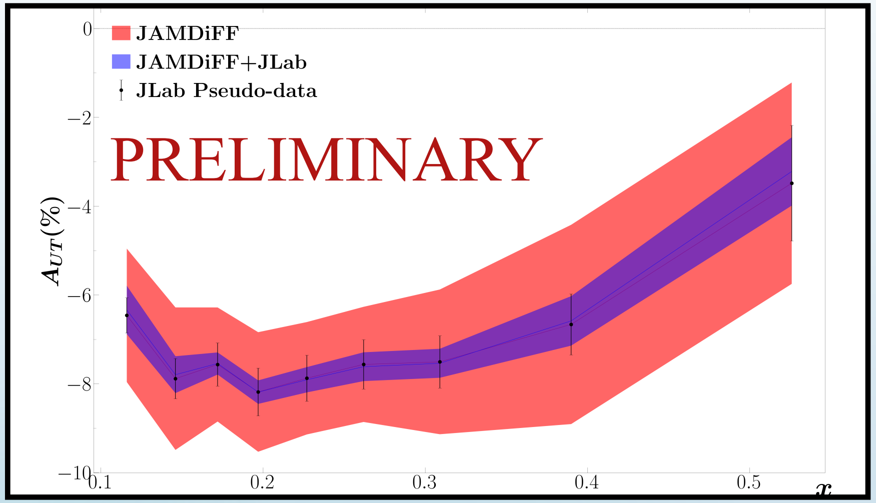


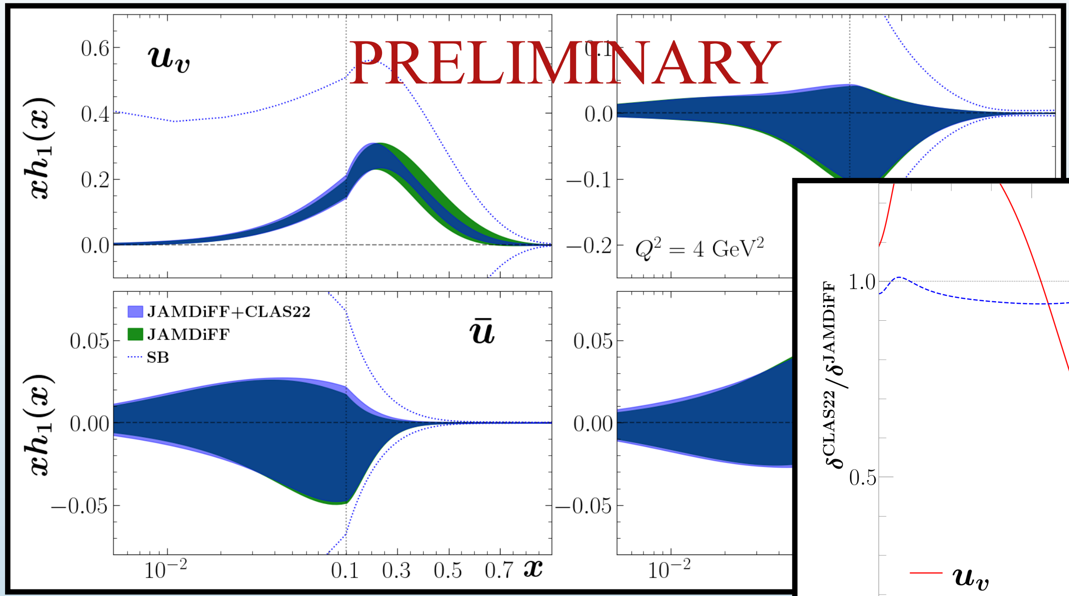
+LQCD

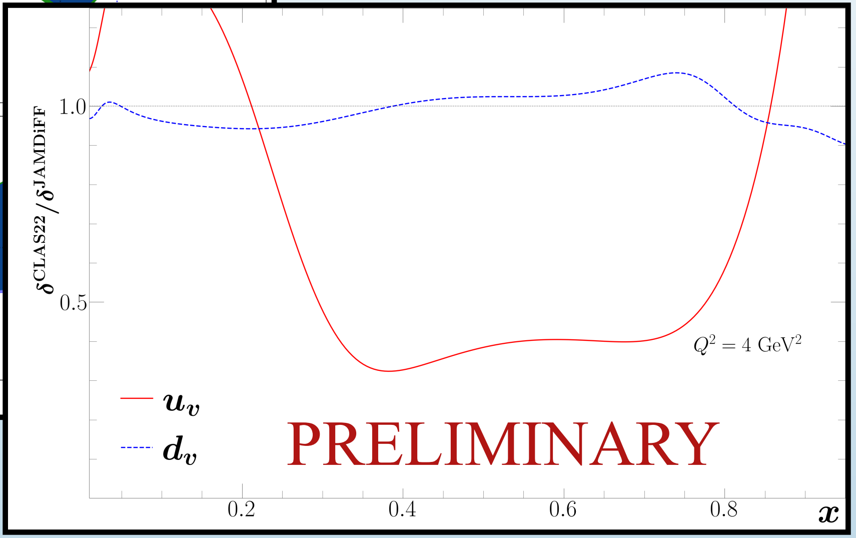


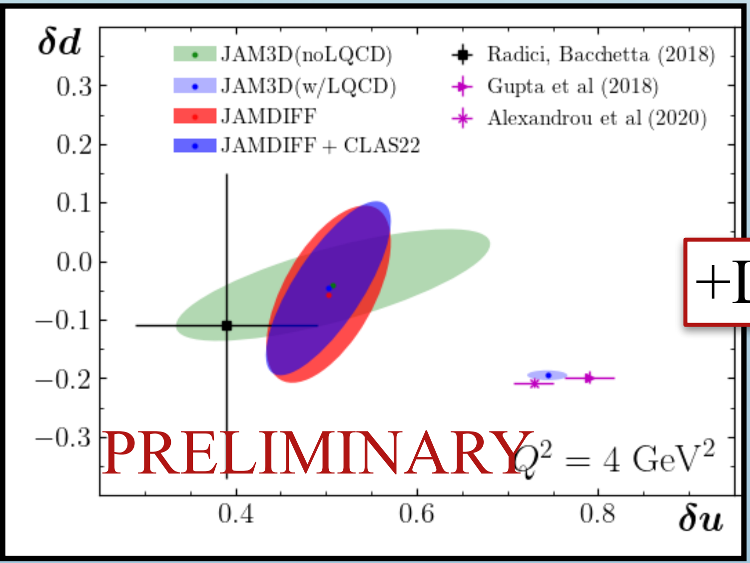
CLAS12 provides strong test of agreement with LQCD!

JAMDiFF Impact (CLAS22 proton)

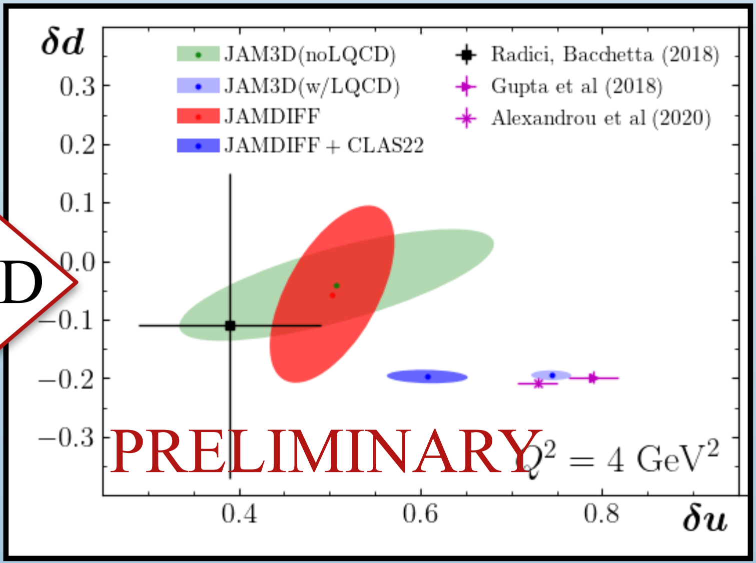








+LQCD



CLAS22 accesses new kinematics and tests higher twists, TMCs, etc.

JAMDiFF Impact (CLAS proton)

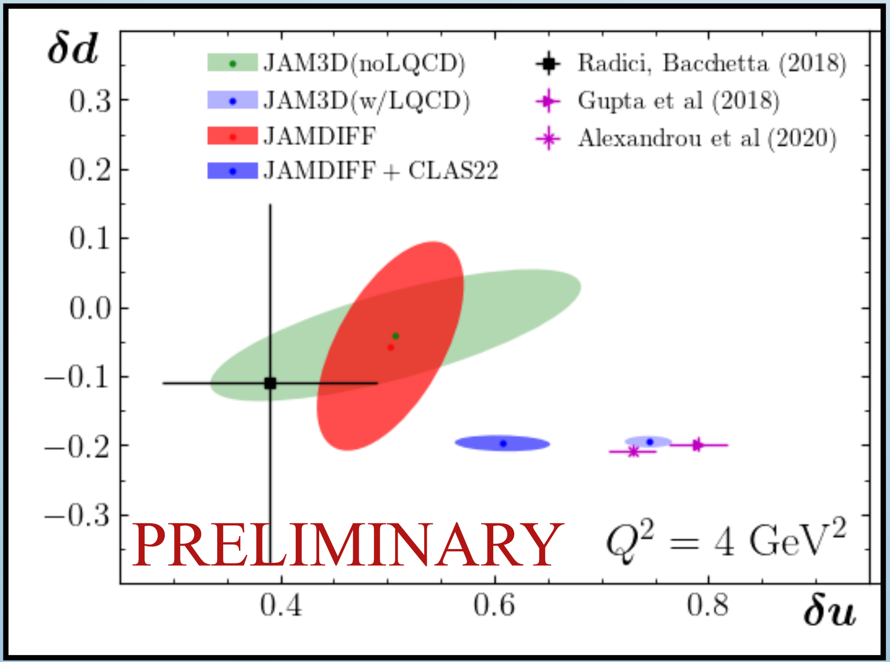
CLAS12

x	Q2	S	z	Mh	y
0.113	1.61	20.8	0.429	0.724	0.715
0.145	1.86	20.8	0.457	0.686	0.644
0.171	2.11	20.8	0.462	0.668	0.619
0.198	2.33	20.8	0.482	0.668	0.591
0.226	2.62	20.8	0.49	0.673	0.582
0.263	2.98	20.8	0.479	0.6	0.569
0.309	3.32	20.8	0.488	0.58	0.539
0.385	4.15	20.8	0.465	0.546	0.541
0.509	5.77	20.8	0.42	0.486	0.569

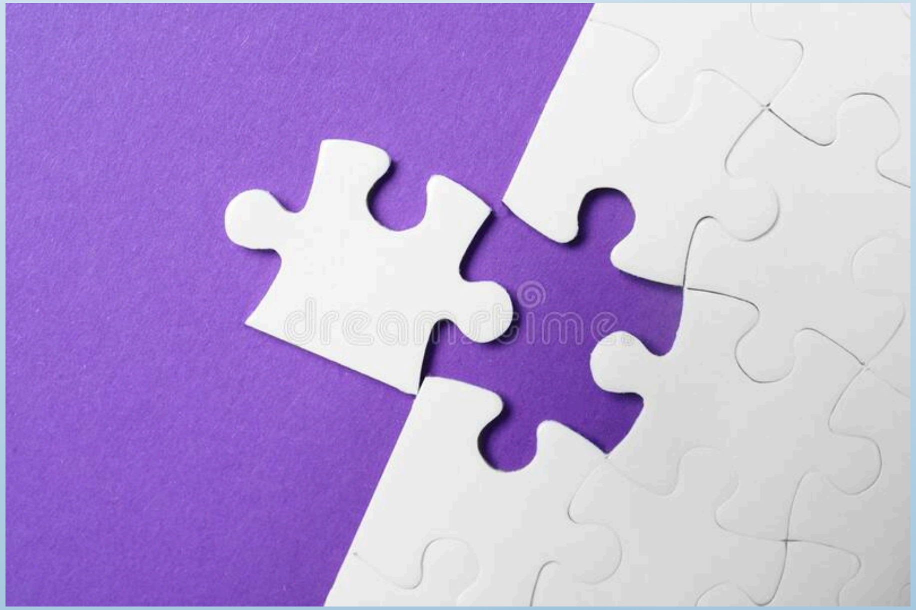
CLAS22

x	Q2	S	z	Mh	y
0.116	3.43	42.2	0.352	0.806	0.716
0.146	4.06	42.2	0.383	0.811	0.673
0.172	4.57	42.2	0.396	0.781	0.643
0.197	4.94	42.2	0.411	0.783	0.607
0.227	5.41	42.2	0.42	0.764	0.577
0.262	5.76	42.2	0.423	0.719	0.532
0.309	6.6	42.2	0.441	0.701	0.517
0.39	7.81	42.2	0.465	0.67	0.485
0.526	9.62	42.2	0.472	0.561	0.443

Similar x coverage, but CLAS22 has Q^2(and thus W^2) about twice as large

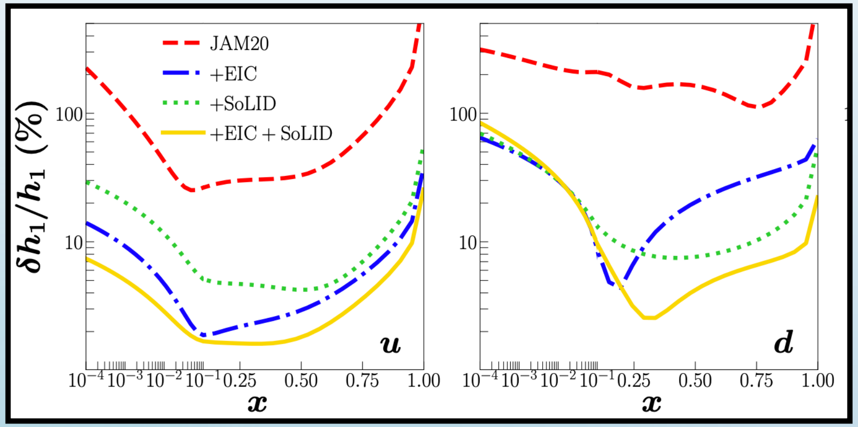


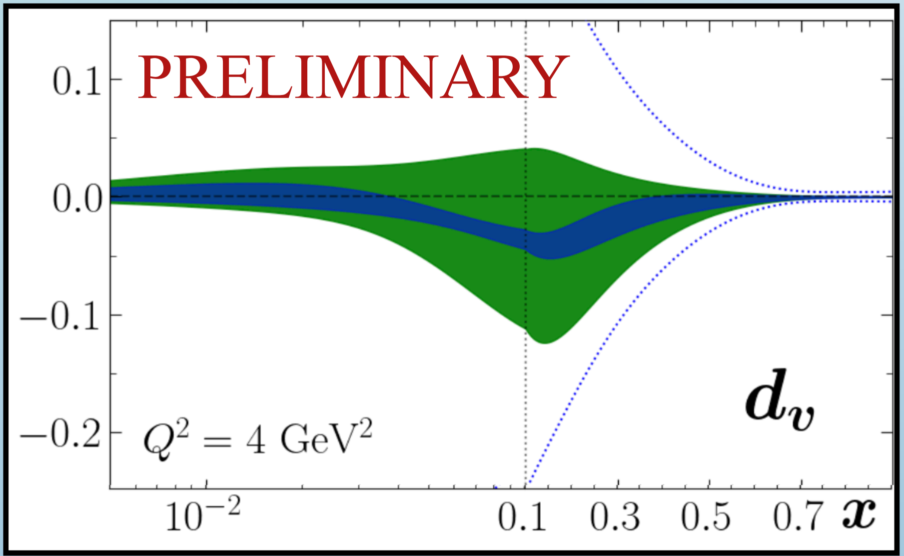
δd
JAM3D(noLQCD)
JAM3D(w/LQCD)
JAMDIFF
JAMDIFF + CLAS22
Radici, Bacchetta (2018)
Gupta et al (2018)
Alexandrou et al (2020)
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
PRELIMINARY
Q² = 4 GeV²
0.4
0.6
0.8
δu



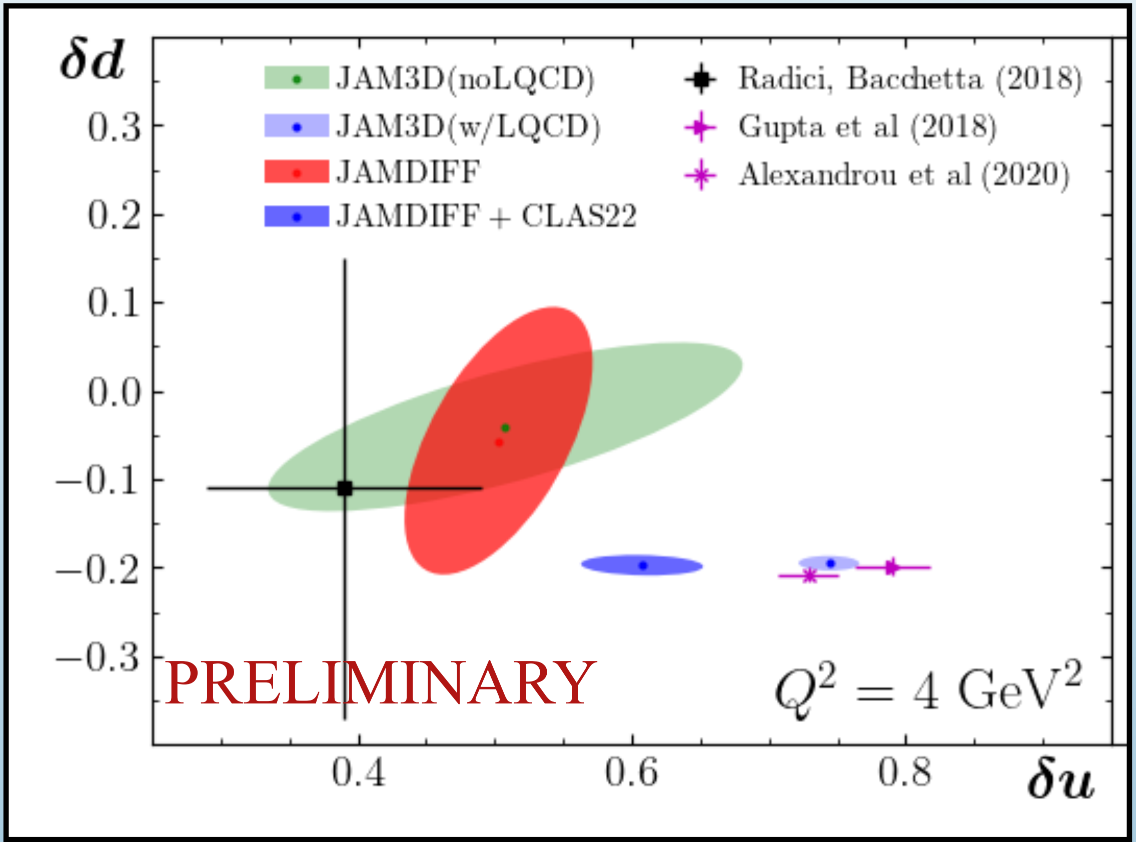
Conclusions and Outlook

SoLID neutron data would provide strongest experimental constraints on down quark





CLAS22/CLAS12 could solve the transverse spin puzzle for δu!



Collaboration

Andreas Metz

Nobuo Sato

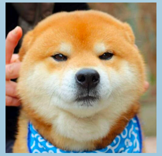
Daniel Pitonyak

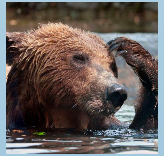
Alexey Prokudin

Yorgo Sawaya

Ralf Seidl

Thank you to Yiyu Zhou and
Patrick Barry for helpful discussions



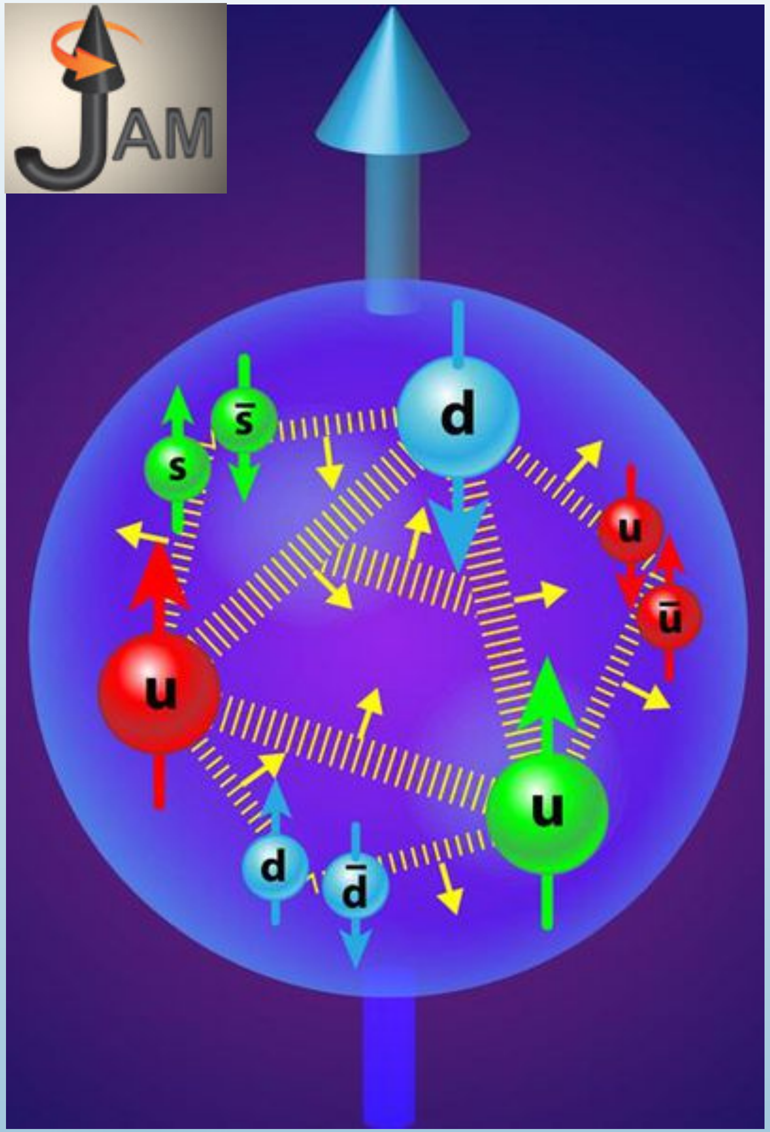


Extra Slides

JAM Collaboration

3-dimensional structure of nucleons:

- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

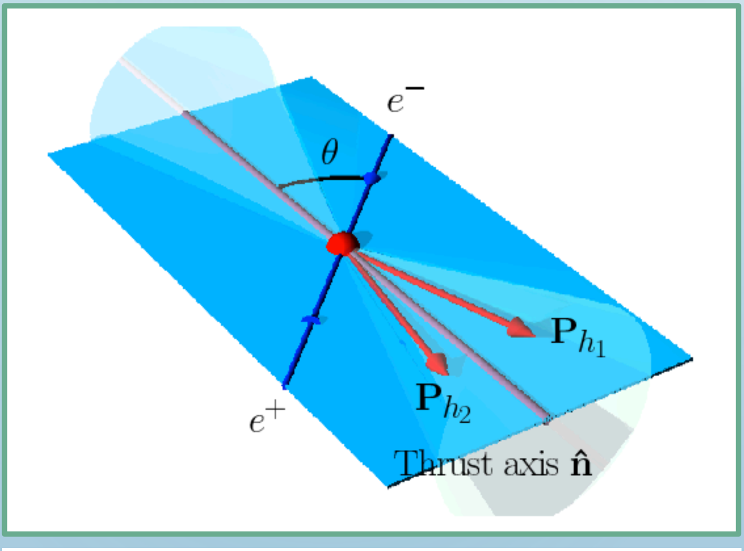


- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference

JAM Global Analysis in the collinear DiFF Approach

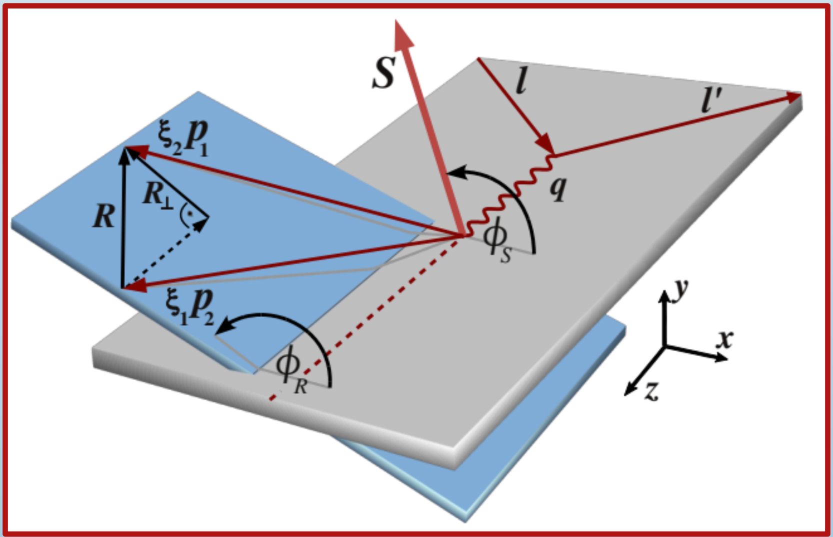
First *simultaneous* extraction of $\pi^+\pi^-$ DiFFs (D_1^q), IFFs ($H_1^{\sphericalangle,q}$), and transversity PDFs (h_1^q) at LO

Semi-Inclusive Annihilation



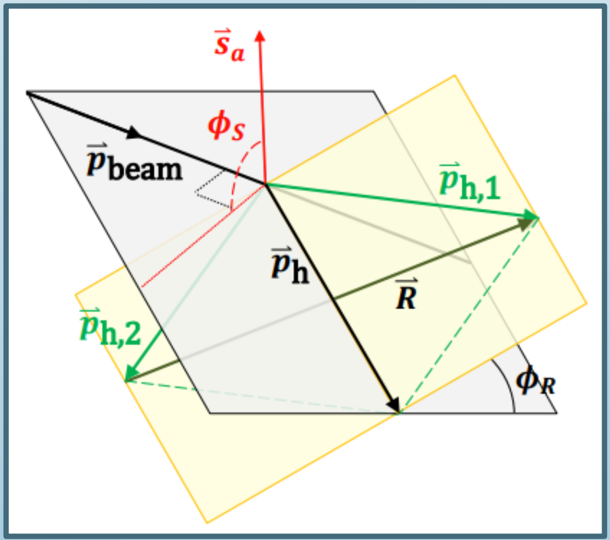
R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

Semi-Inclusive Deep Inelastic Scattering



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

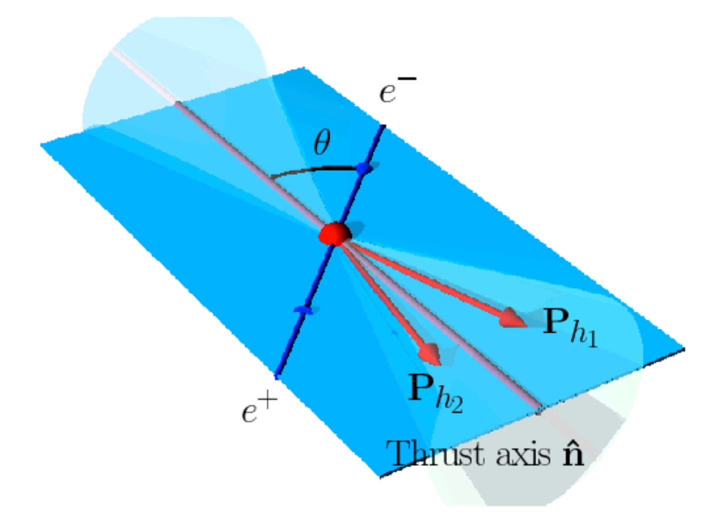
Proton-Proton Collisions



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

Observables for DiFFs

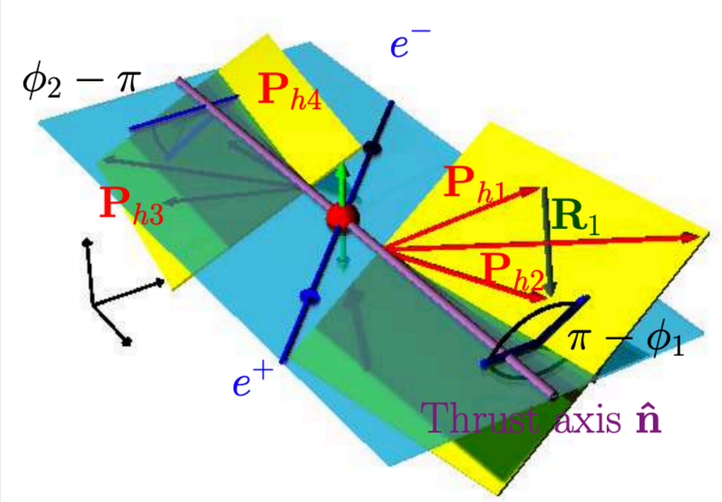
SIA Cross Section



R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

$$\frac{d\sigma}{dz\,dM_h} = \frac{4\pi\alpha_{em}^2}{s} \sum_q e_q^2 D_1^q(z, M_h)$$

SIA Artru-Collins Asymmetry



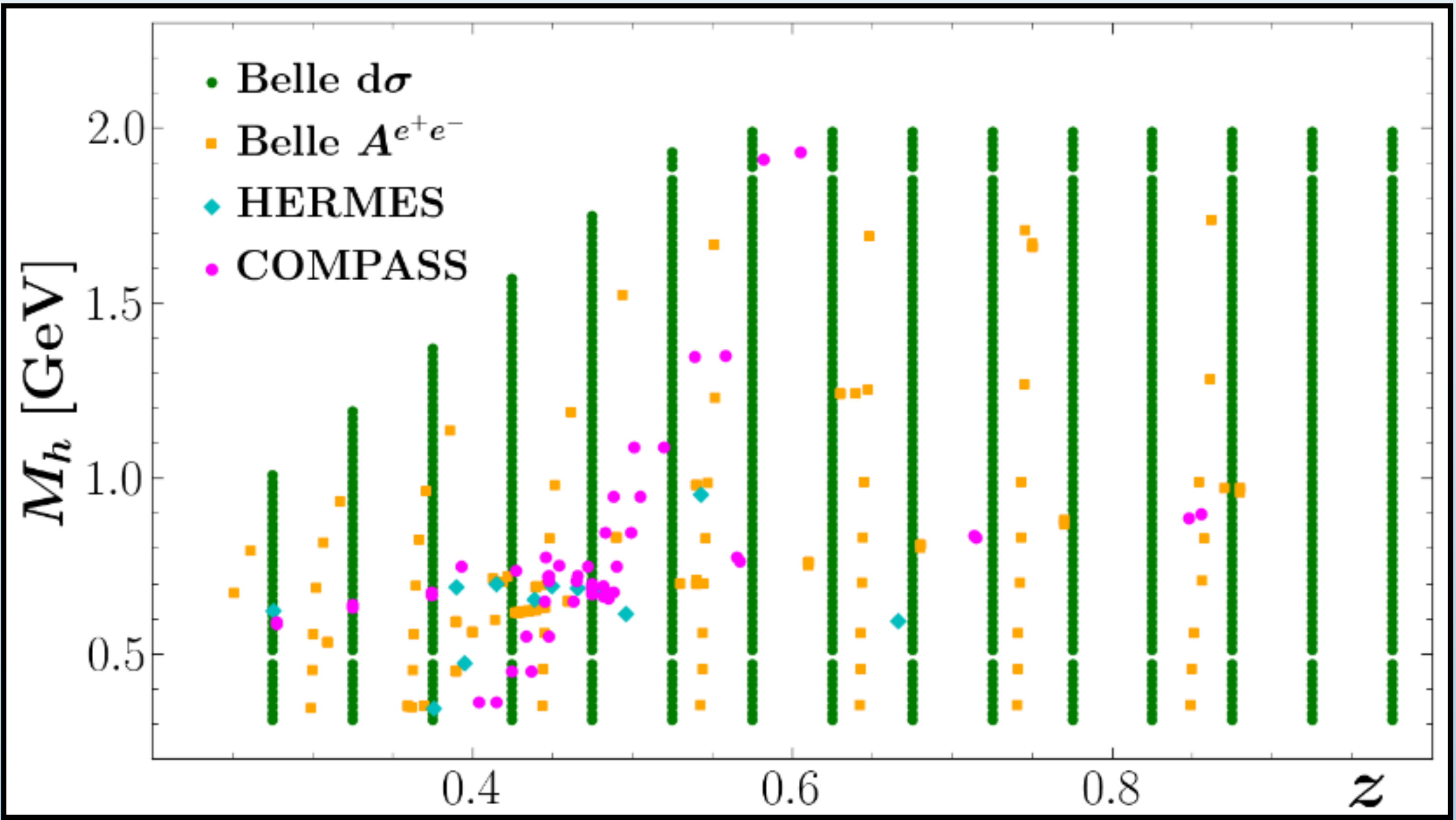
A. Vossen *et al.*, Phys. Rev. Lett. **107**, 072004 (2011)

$$A^{e^+e^-}(z, M_h, \bar{z}, \overline{M}_h) = \frac{\sin^2\theta \sum_q e_q^2 H_1^{\sphericalangle,q}(z, M_h) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta) \sum_q e_q^2 D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

Data for DiFFs

SIA cross section	Belle	1094 points
SIA Artru-Collins	Belle	183 points

$\pi^+\pi^-$ DiFFs



$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

5 independent functions (w/ D_1^g)

[supplement with PYTHIA data]

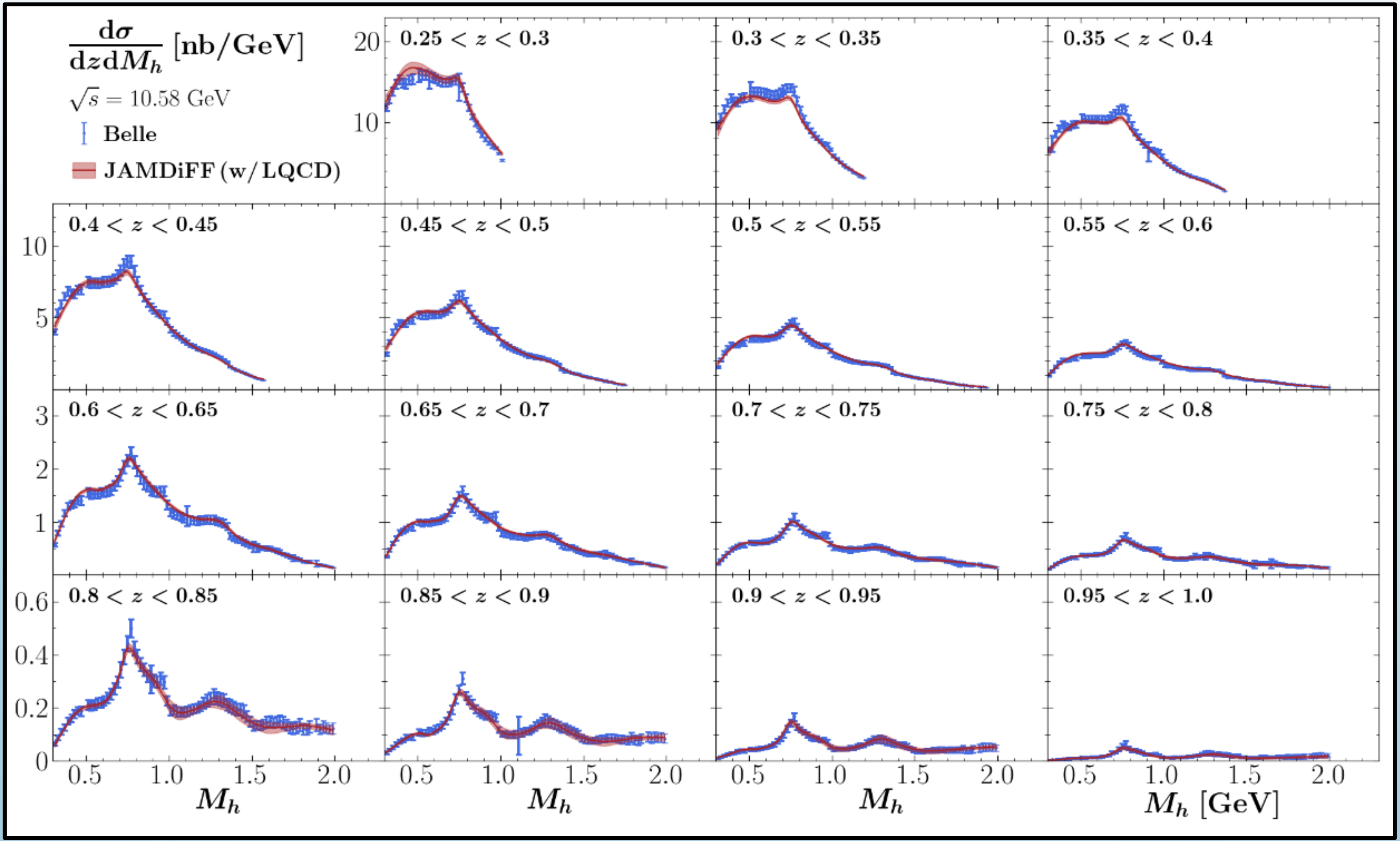
$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$

$$H_1^{\sphericalangle,s} = -H_1^{\sphericalangle,\bar{s}} = H_1^{\sphericalangle,c} = -H_1^{\sphericalangle,\bar{c}} = 0,$$

1 independent function

A. Courtoy et al., Phys. Rev. D **85**, 114023 (2012)

Quality of Fit (Unpolarized Cross Section)



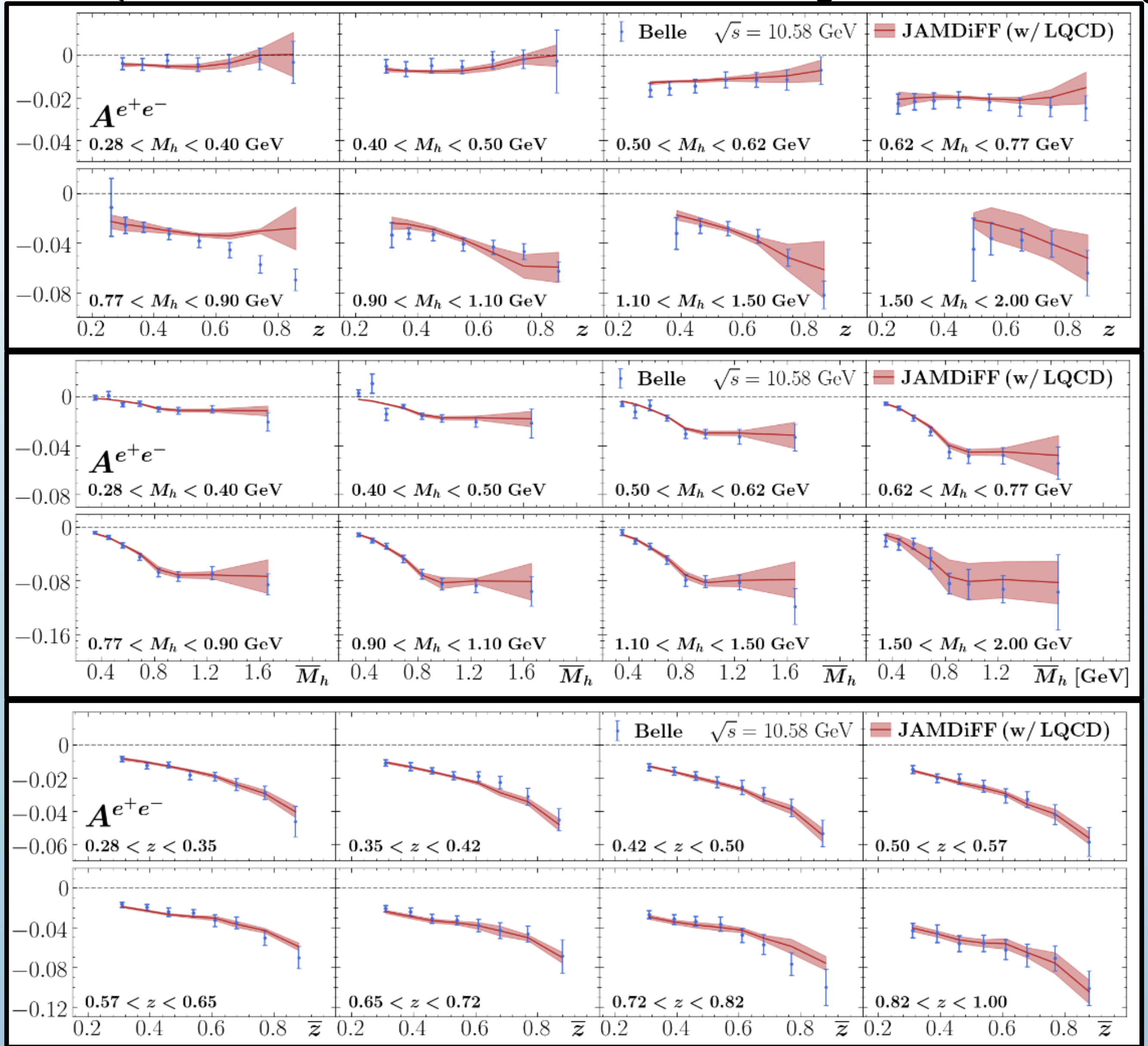
R. Seidl *et al.*, Phys. Rev. D **96**, 032005 (2017)

Quality of Fit (Artru-Collins Asymmetry)

(z, M_h) binning

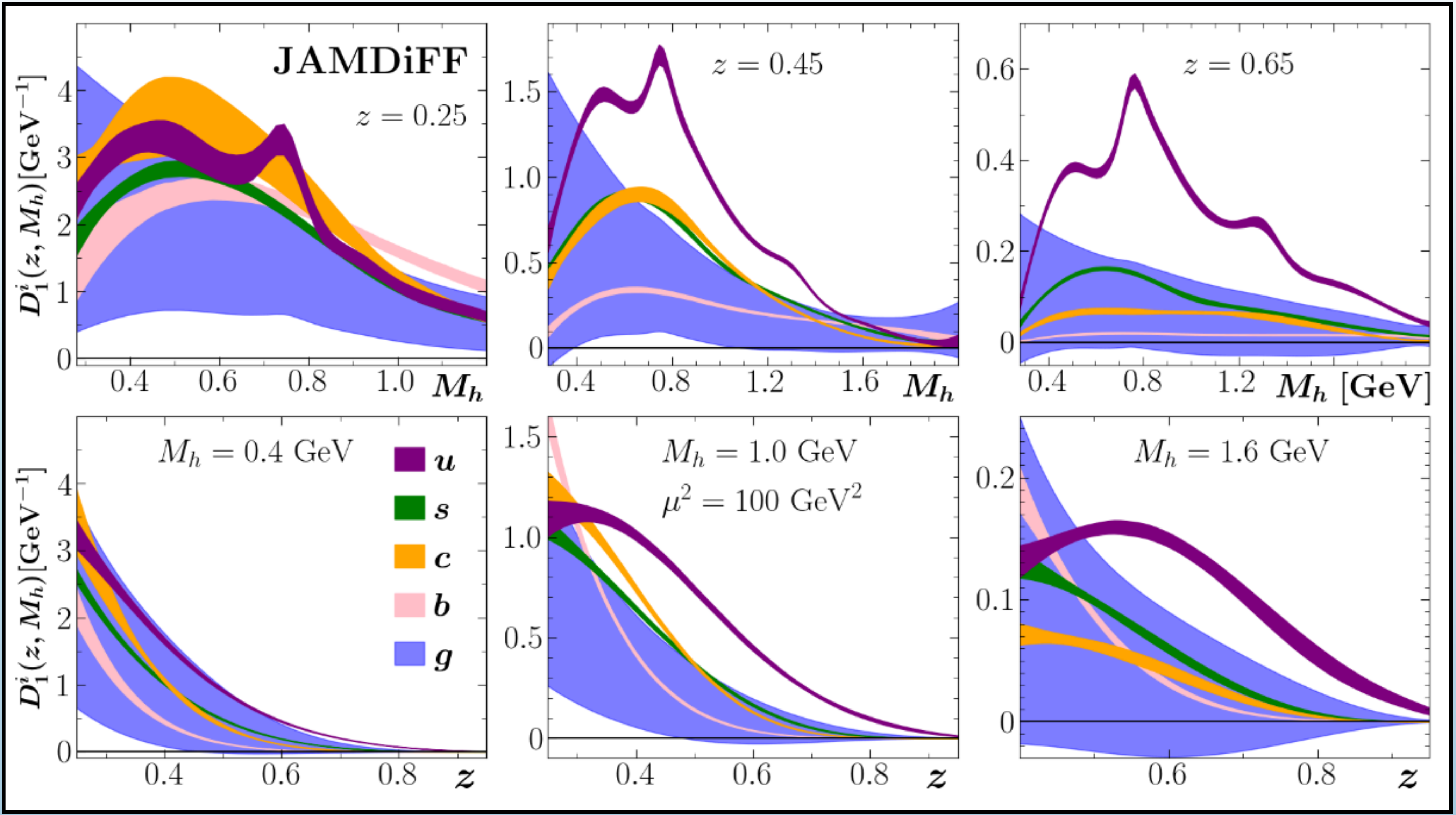
$(M_h, \overline{M}_h)$ binning

$(z, \bar{z})$ binning



A. Vossen *et al.*,
Phys. Rev. Lett. **107**, 072004 (2011)

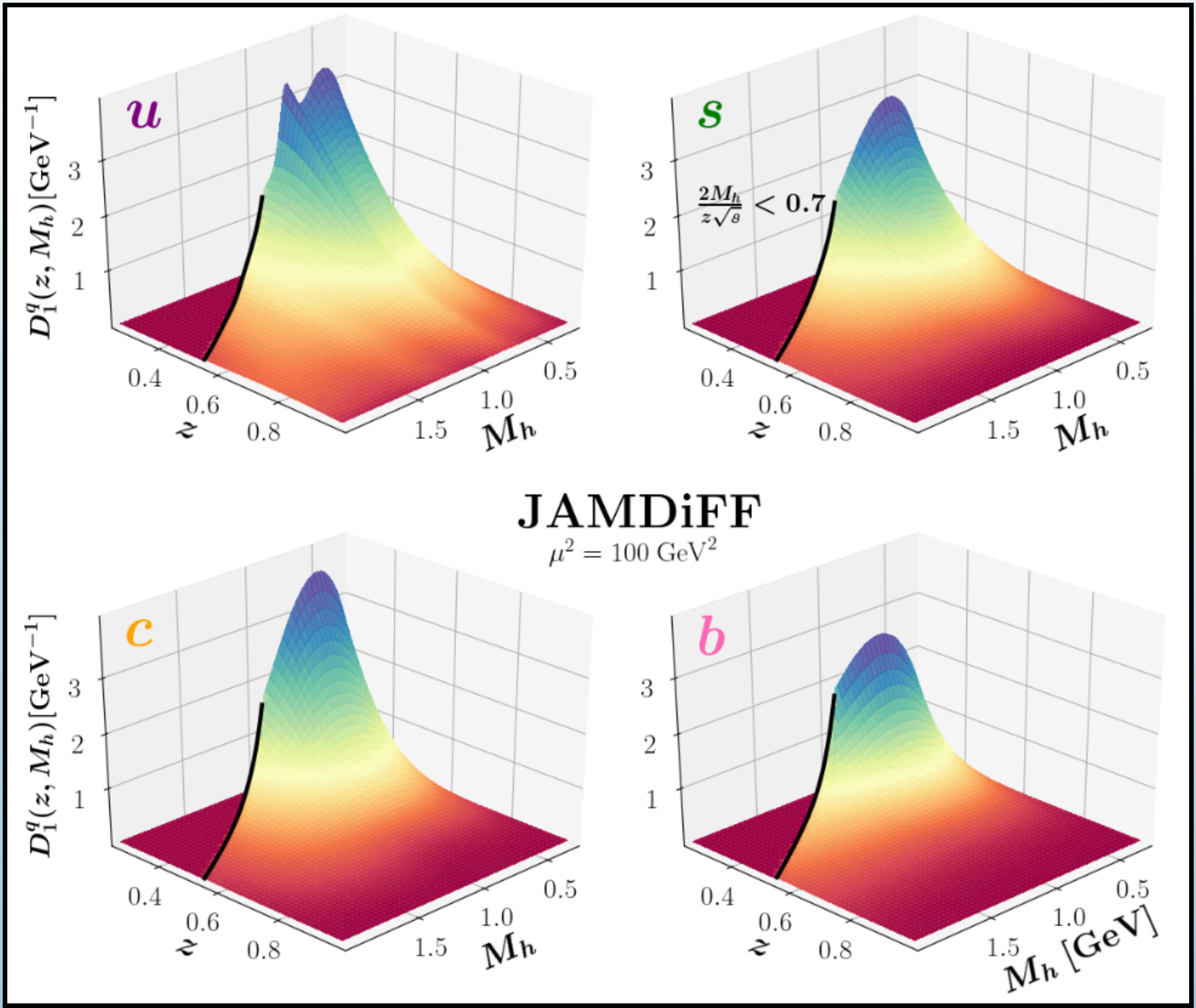
Extracted DiFFs



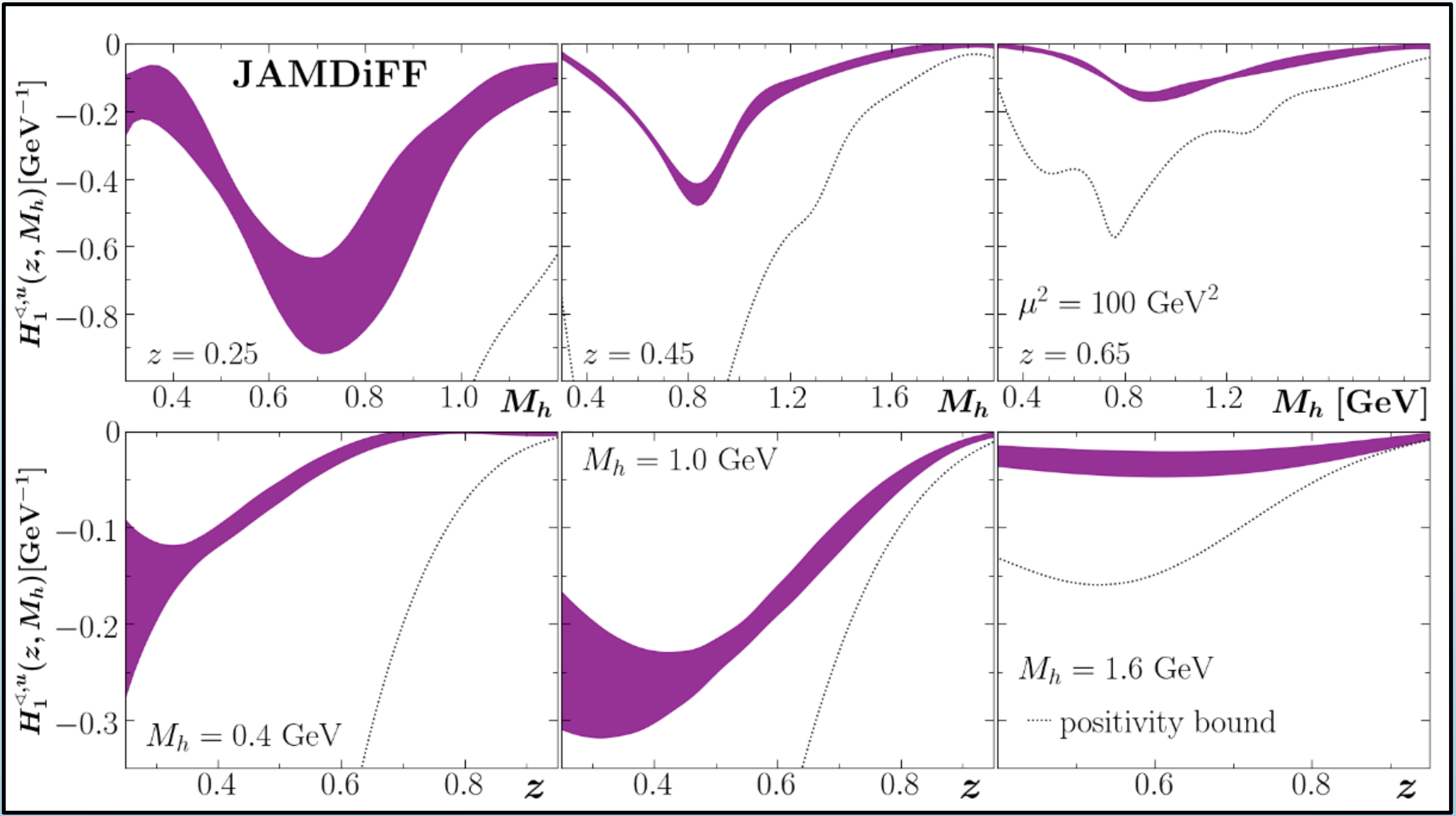
Bound: $D_1^q > 0$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

Extracted DiFFs (3D)



Extracted IFFs



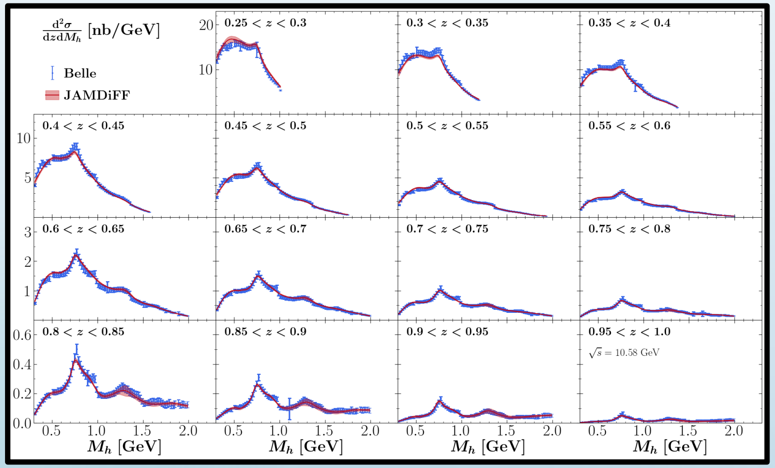
Bound:

$$|H_1^{\sphericalangle,q}| < D_1^q$$

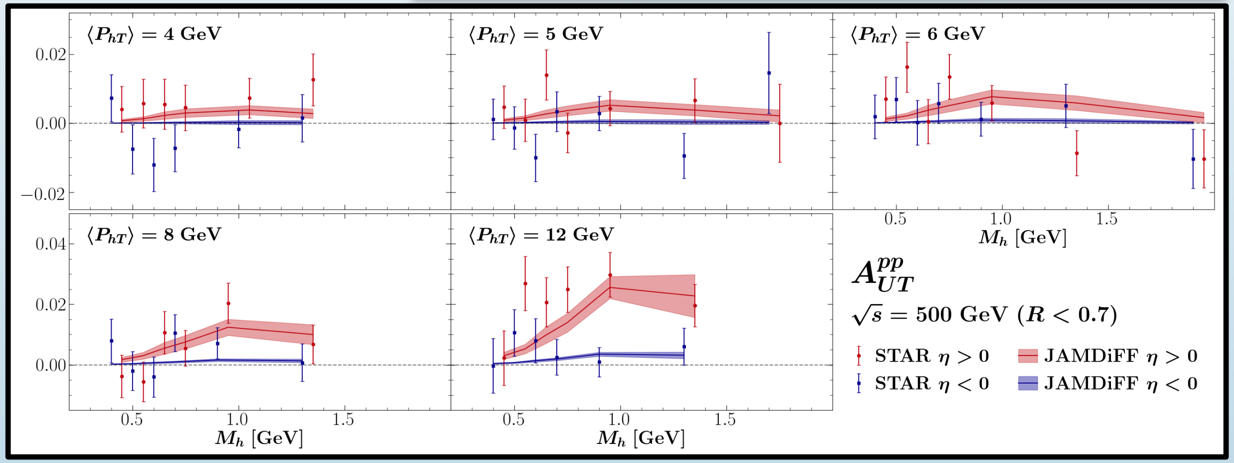
A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

Comprehensive Analysis of DiFFs and Transversity

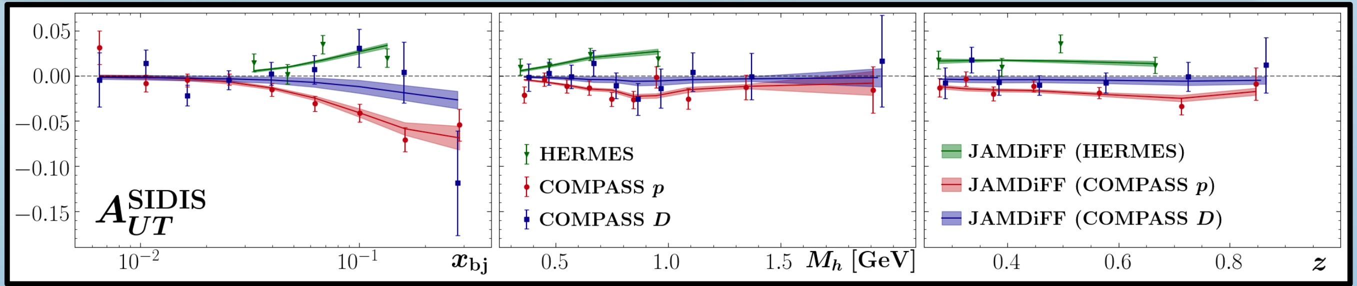
First inclusion of Belle cross section data



First inclusion of 500 GeV STAR data



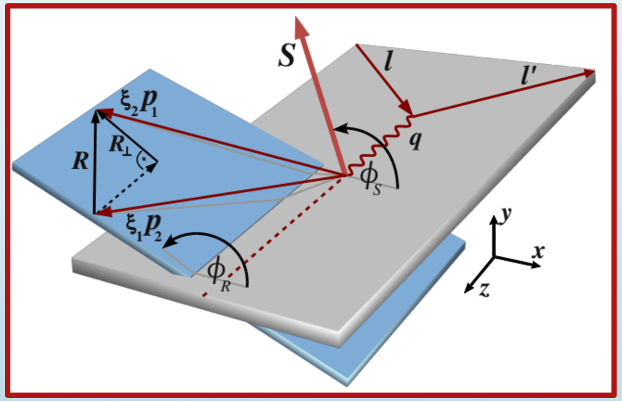
Utilized all binnings for Artru-Collins and SIDIS asymmetries



First simultaneous analysis of DiFFs and transversity PDFs

Observables for Transversity PDFs

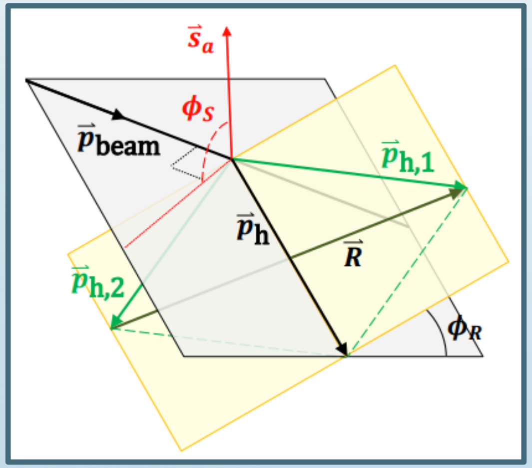
SIDIS asymmetry (*p* and *D*)



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

$$A_{UT}^{\text{SIDIS}} = c(y)\frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z,M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z,M_h)}$$

pp Asymmetry



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

$$A_{UT}^{pp} = \frac{\mathscr{H}(M_h, P_{hT}, \eta)}{\mathscr{D}(M_h, P_{hT}, \eta)}$$

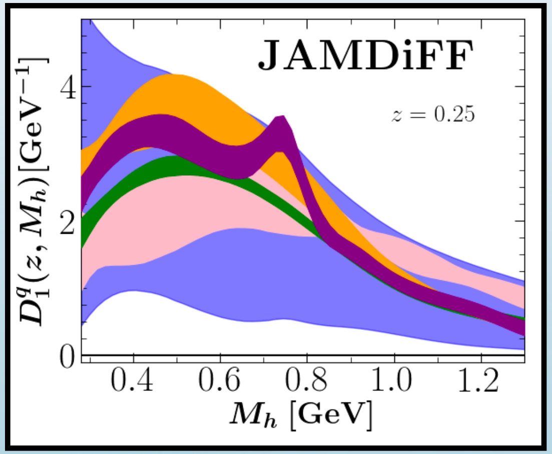
$$\mathscr{H}(M_h, P_{hT}, \eta) = 2P_{hT}\sum_i \sum_{a,b,c}\int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, h_1^b(x_b) \frac{\mathrm{d}\Delta\hat{\sigma}_{ab^\uparrow\to c^\uparrow d}}{\mathrm{d}\hat{t}} H_1^{\sphericalangle,c}(z, M_h)$$

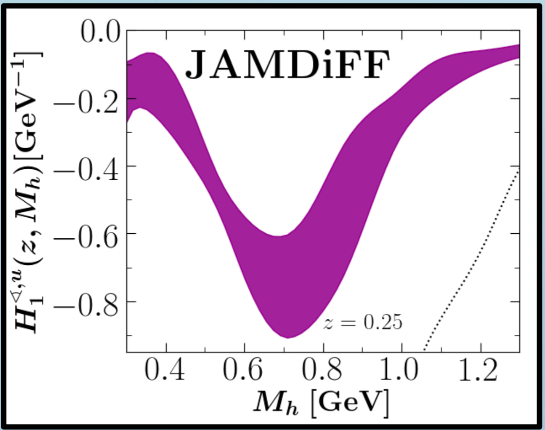
$$\mathscr{D}(M_h, P_{hT}, \eta) = 2P_{hT}\sum_i \sum_{a,b,c}\int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, f_1^b(x_b) \frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}\hat{t}} D_1^c(z, M_h)$$

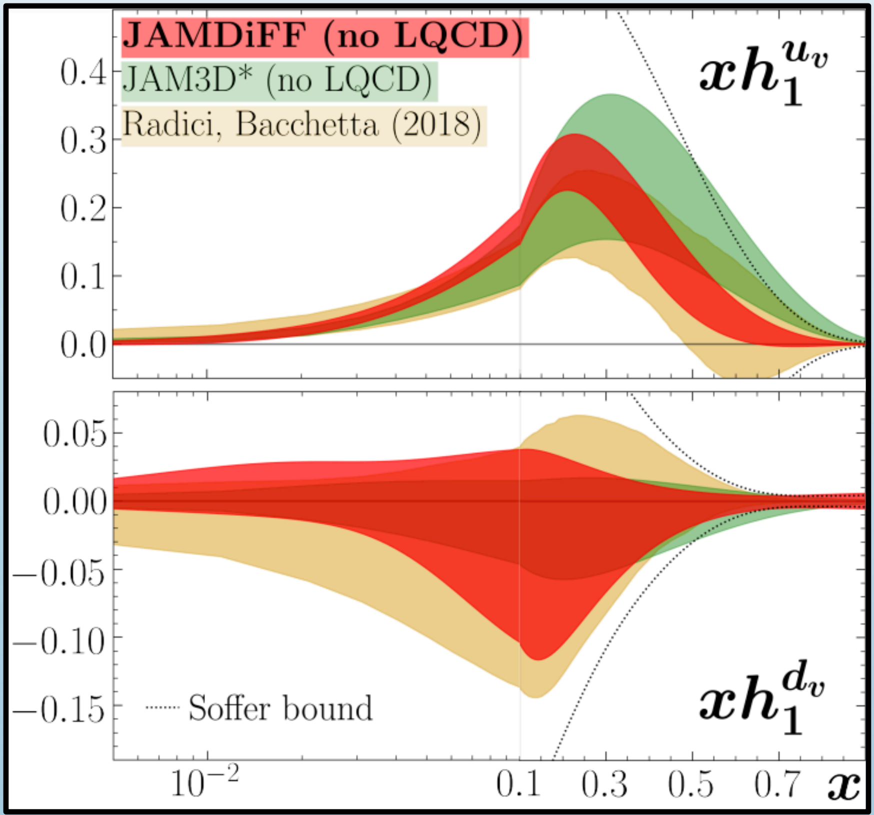
Conclusions

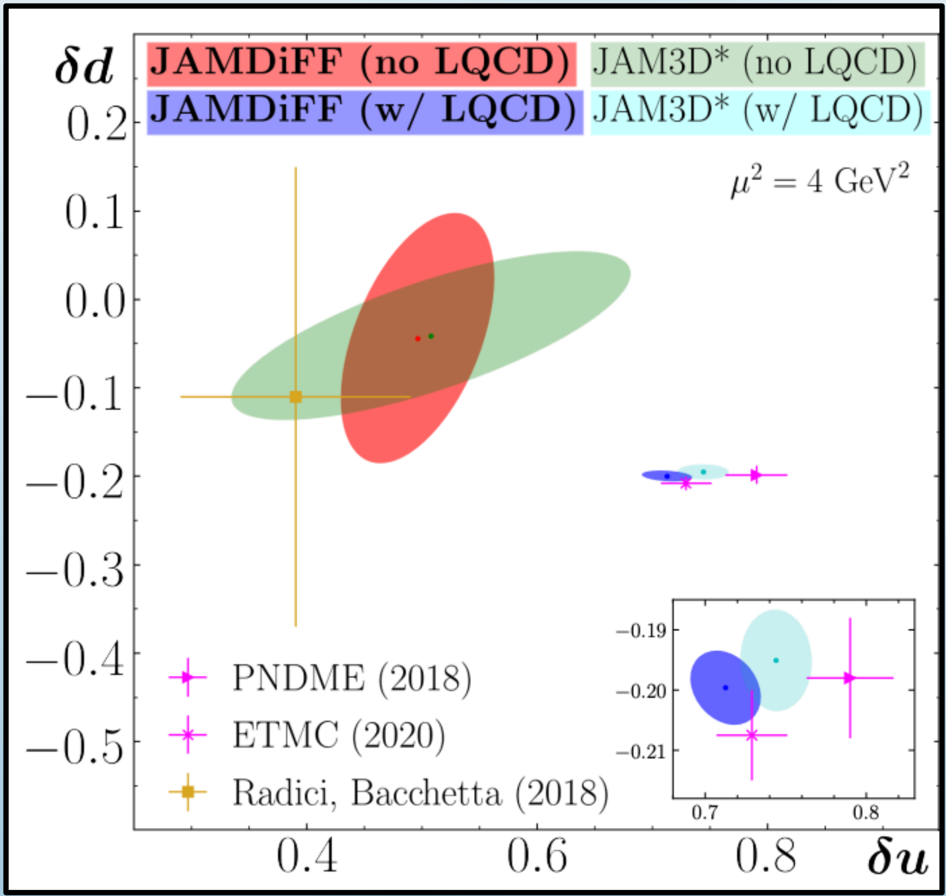
Simultaneous extraction of DiFFs and transversity PDFs

Universality of all available information on transversity









Outlook

More data from RHIC
Proton-proton cross section

SIDIS multiplicities
from COMPASS

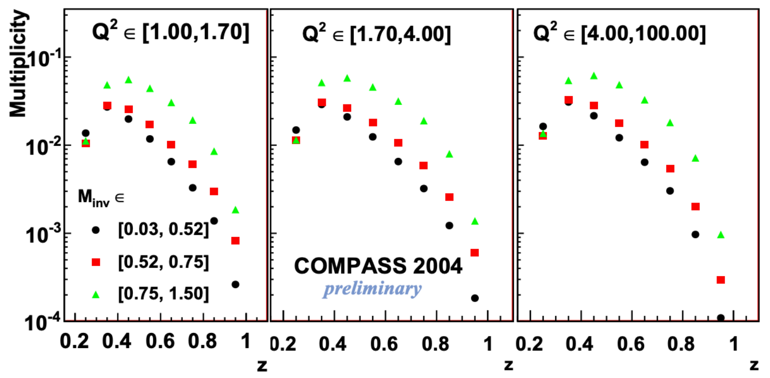
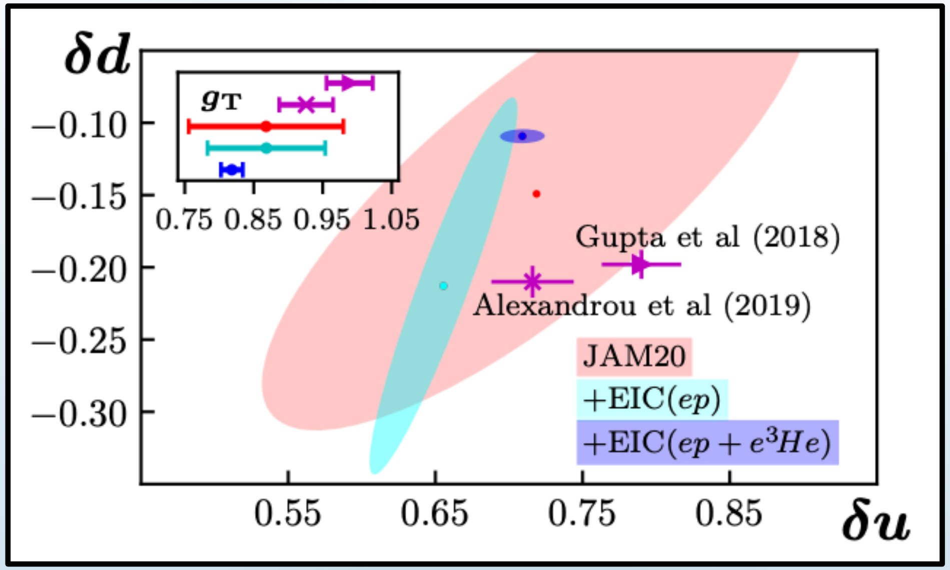


Figure 4: h^+h^- multiplicities measured versus z, M_{inv} and Q^2.

N. Makke, Phys. Part. Nucl. **45**, 138-140 (2014)

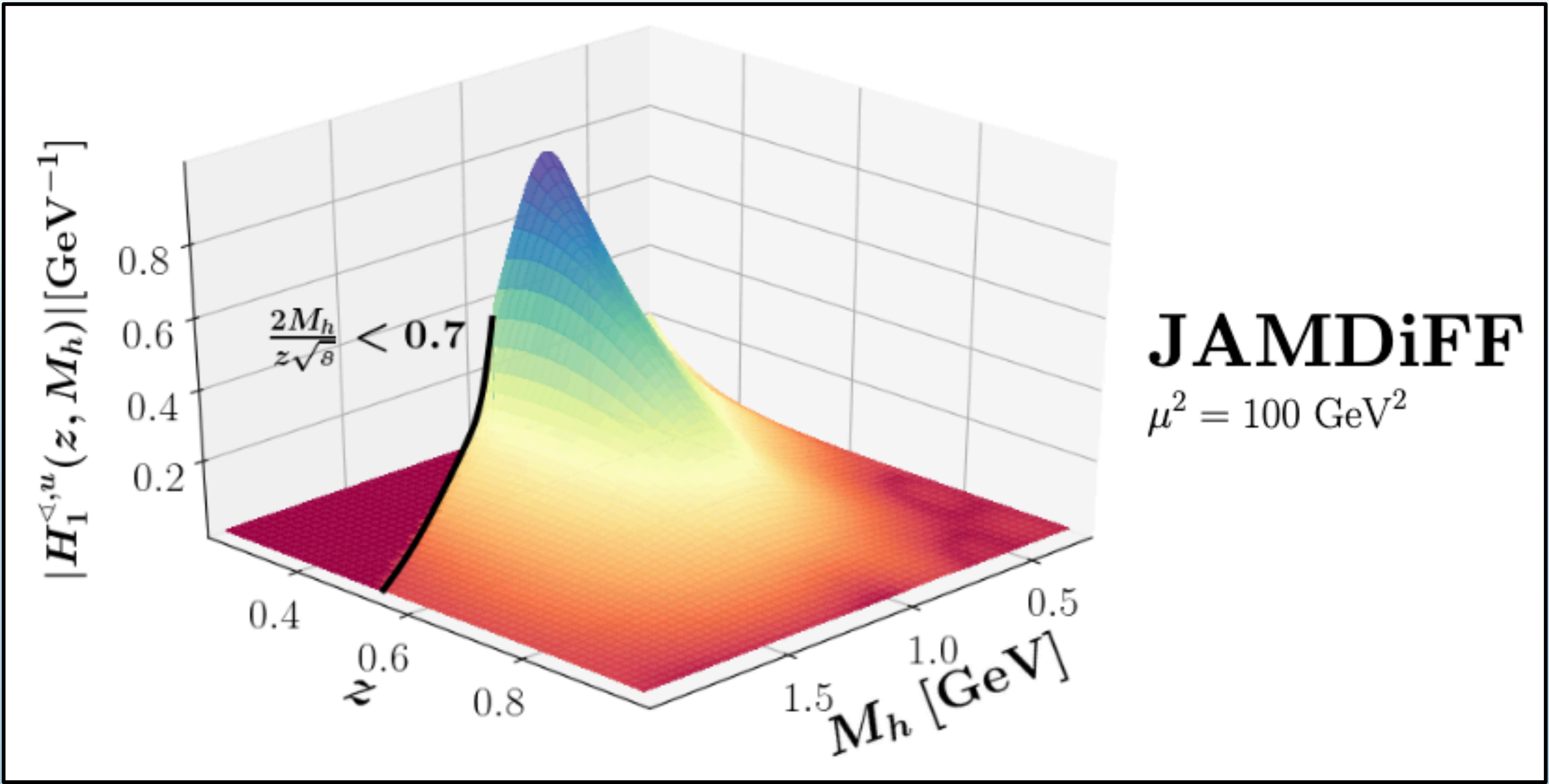
L. Gamberg *et al.*, Phys. Lett. B **816**, 136255 (2021)



EIC can provide new information

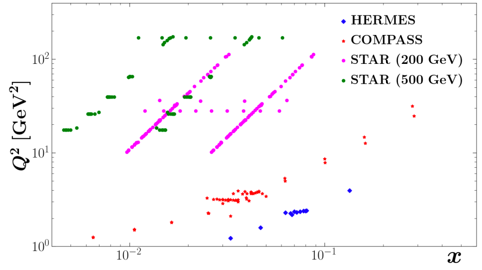
Simultaneous fit of DiFF channel + TMD channel + Lattice QCD

Extracted IFFs (3D)



Experiment + Lattice + Theory

EXPERIMENT
(measured region)



THEORY
(unmeasured regions)

$$|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$$

$$\alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

LATTICE
(full moments)

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Presently, trivial to find compatibility between any two

Only meaningful when all three are included

Quality of Fit

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
HERMES [72]	12	1.13	1.10
COMPASS (p) [71]	26	1.24	0.75
COMPASS (D) [71]	26	0.78	0.76
STAR (2015) [94]	24	1.47	1.67
STAR (2018) [64]	106	1.20	1.04
ETMC δu [28]	1	0.71	—
ETMC δd [28]	1	1.02	—
PNDME δu [25]	1	8.68	—
PNDME δd [25]	1	0.04	—
Total χ^2_{red} (N_{dat})		**1.01** (1475)	**0.98** (1471)

Physical Pion Mass

$N_f = 2 + 1 + 1$

Use δu and δd instead of g_T

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = N x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j\left(\frac{x}{z},\mu\right)$$

Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M\, \tilde{f}_j(N,\mu_0) \tilde{f}_l(M,\mu_0)$$
$$\otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0) \right]$$

Quality of Fit

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
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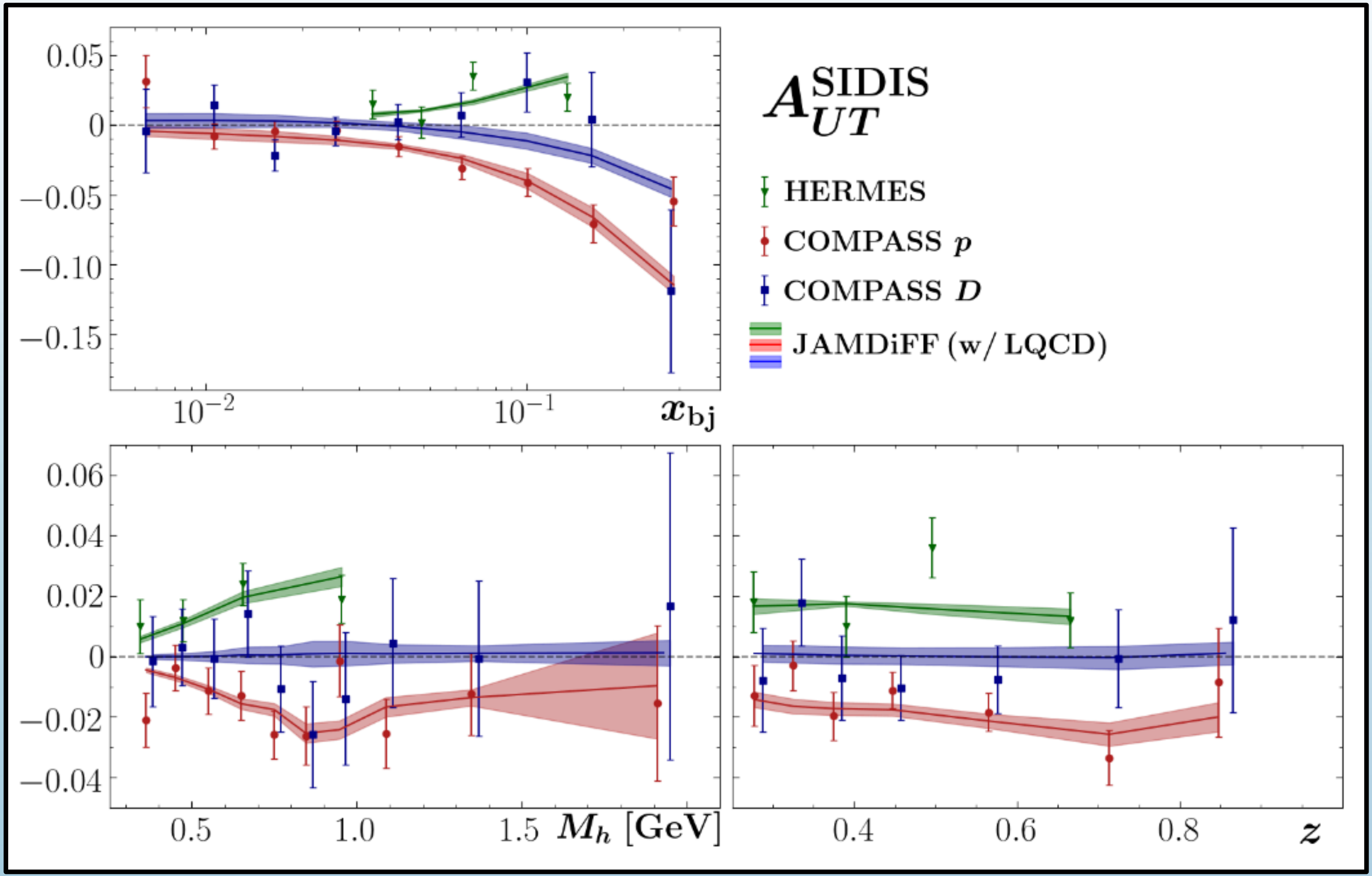
PNDME δd [25]	1	0.04	—
Total χ^2_{red} (N_{dat})		**1.01** (1475)	**0.98** (1471)

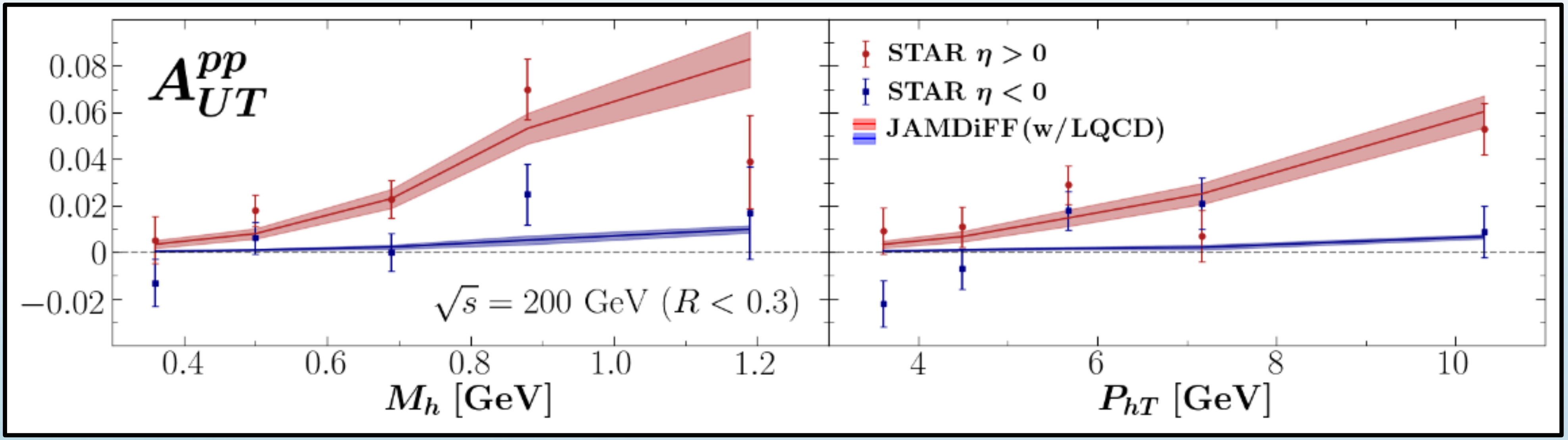
Quality of Fit (SIDIS)

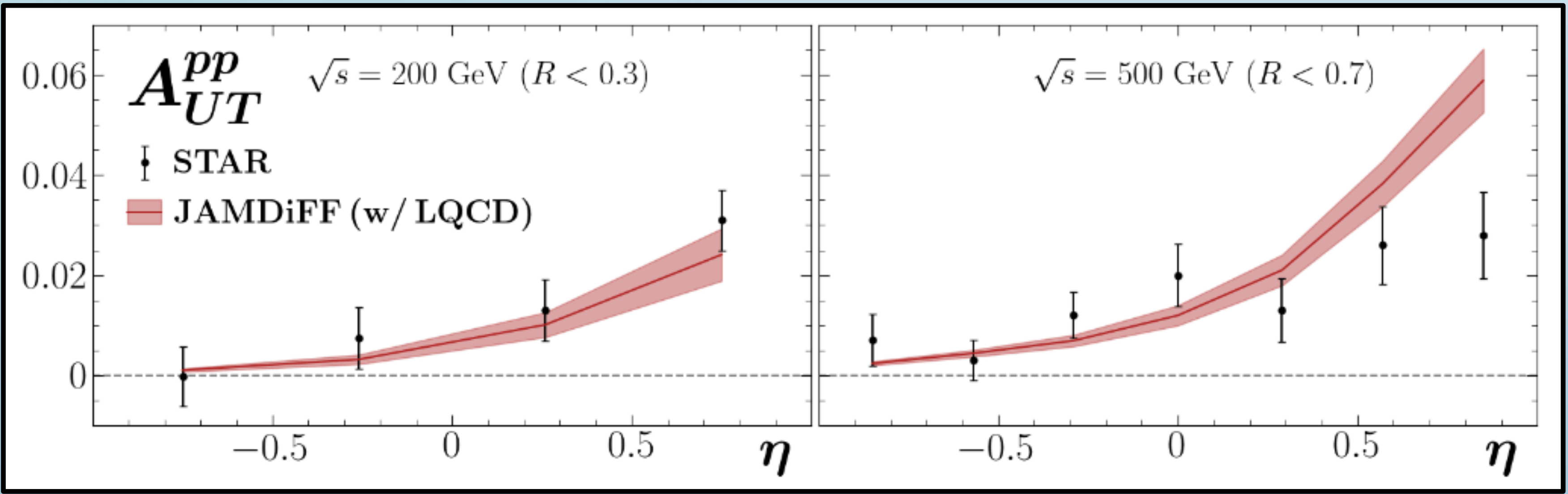


A. Airapetian *et al.*, JHEP **06**, 017 (2008)

COMPASS, arXiv:hep-ph/2301.02013 (2023)

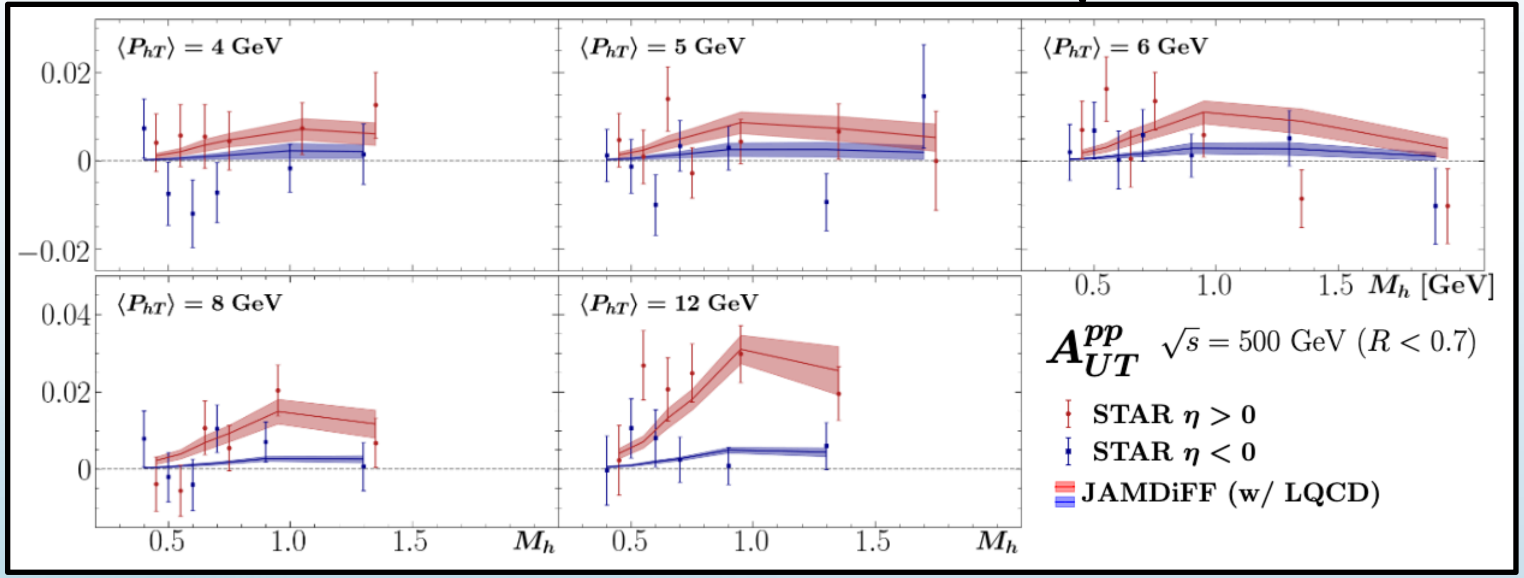
Quality of Fit (STAR $\sqrt{s} = 200$ GeV)

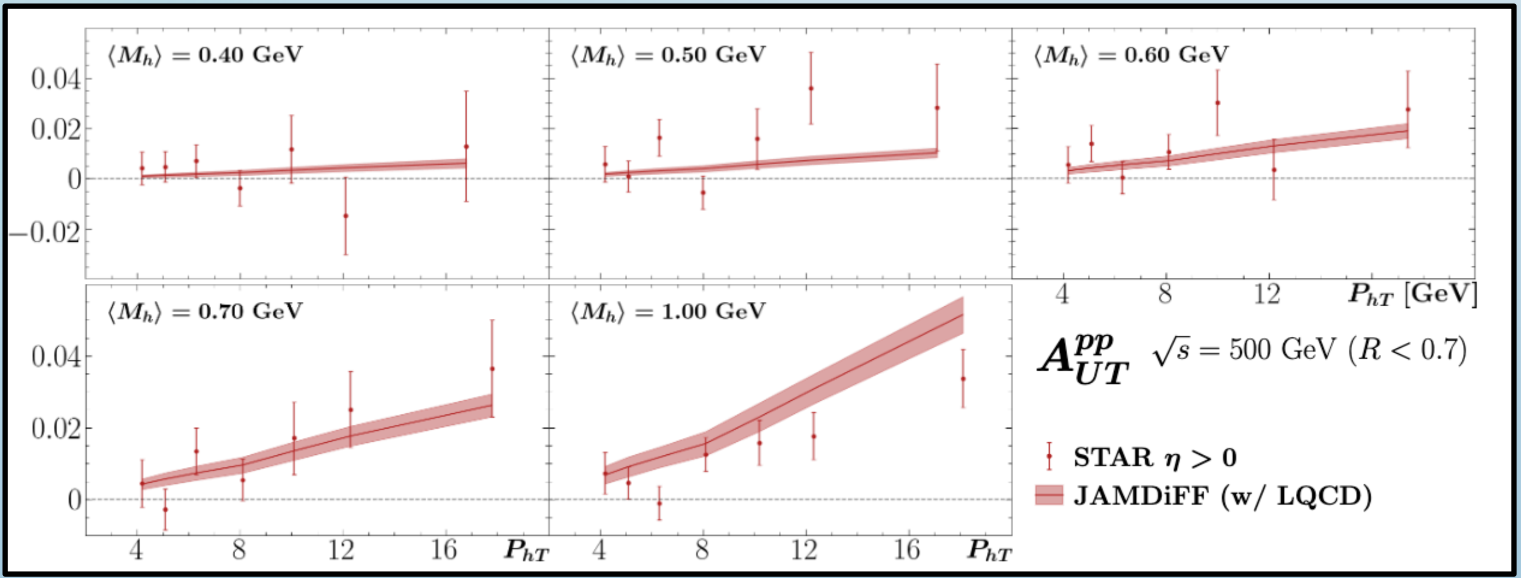




L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 24501 (2015)

Quality of Fit (STAR $\sqrt{s}$ = 500 GeV)





L. Adamczyk *et al.*, Phys. Rev. B **780**, 332-339 (2018)

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Now that the observables have been calculated…

Data

Theory

Normalization

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

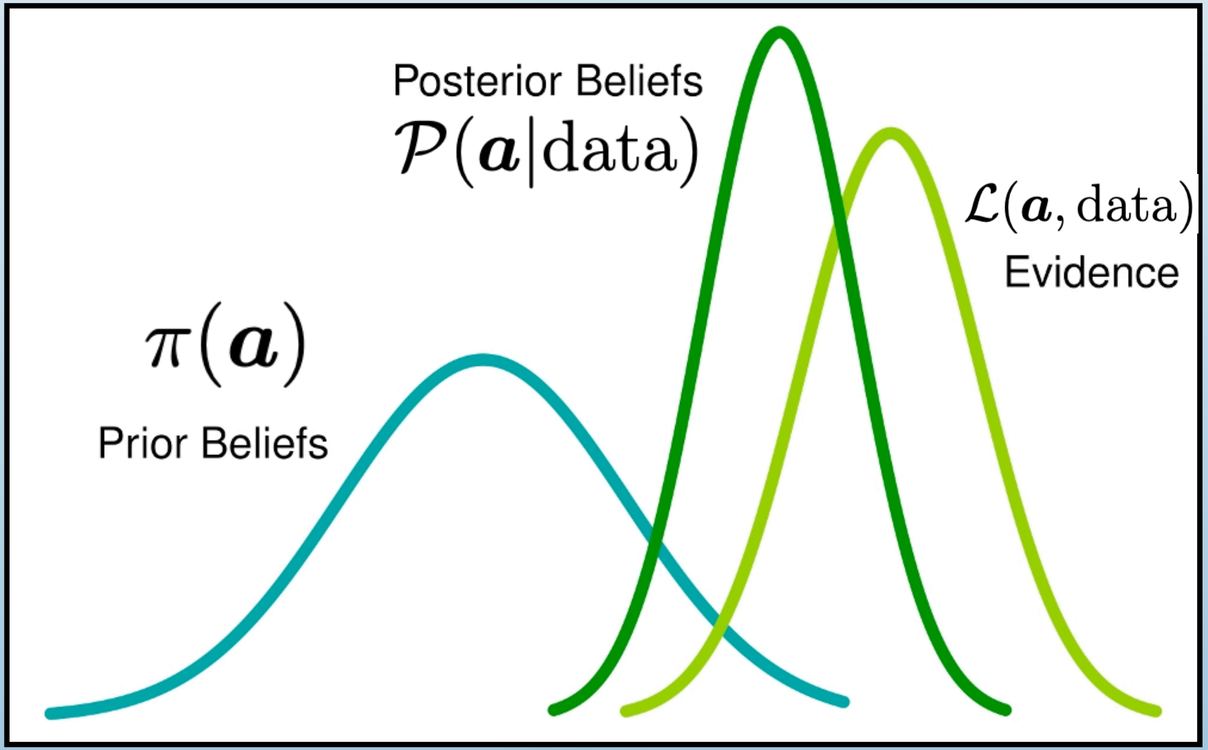
Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

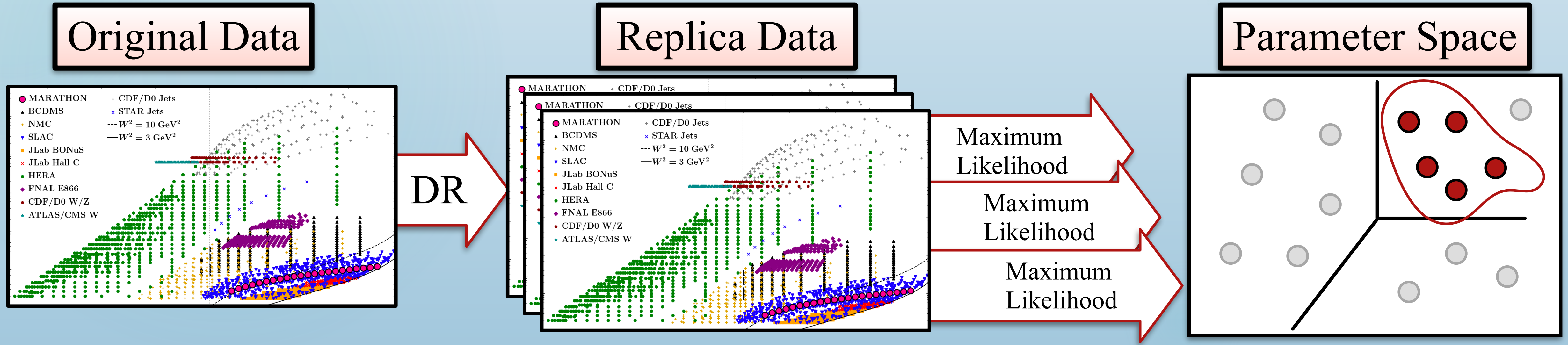
Bayes' Theorem

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



Pseudo-Data → $$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$ ← Uncorrelated Uncertainties

↑ Data



For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \; \rho(\boldsymbol{a}\,|\,data) \; O(\boldsymbol{a})$$

$$V[O] = \int d^n a \; \rho(\boldsymbol{a}\,|\,data) \; \left[O(\boldsymbol{a}) - E[O]\right]^2$$

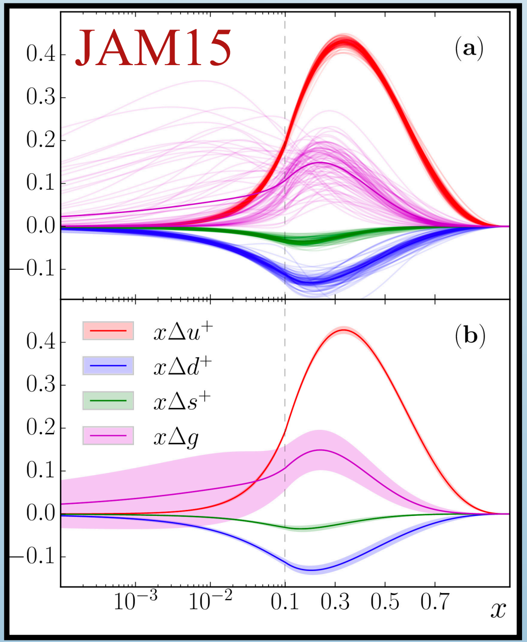
Exact, but $n = \mathcal{O}(100)$!

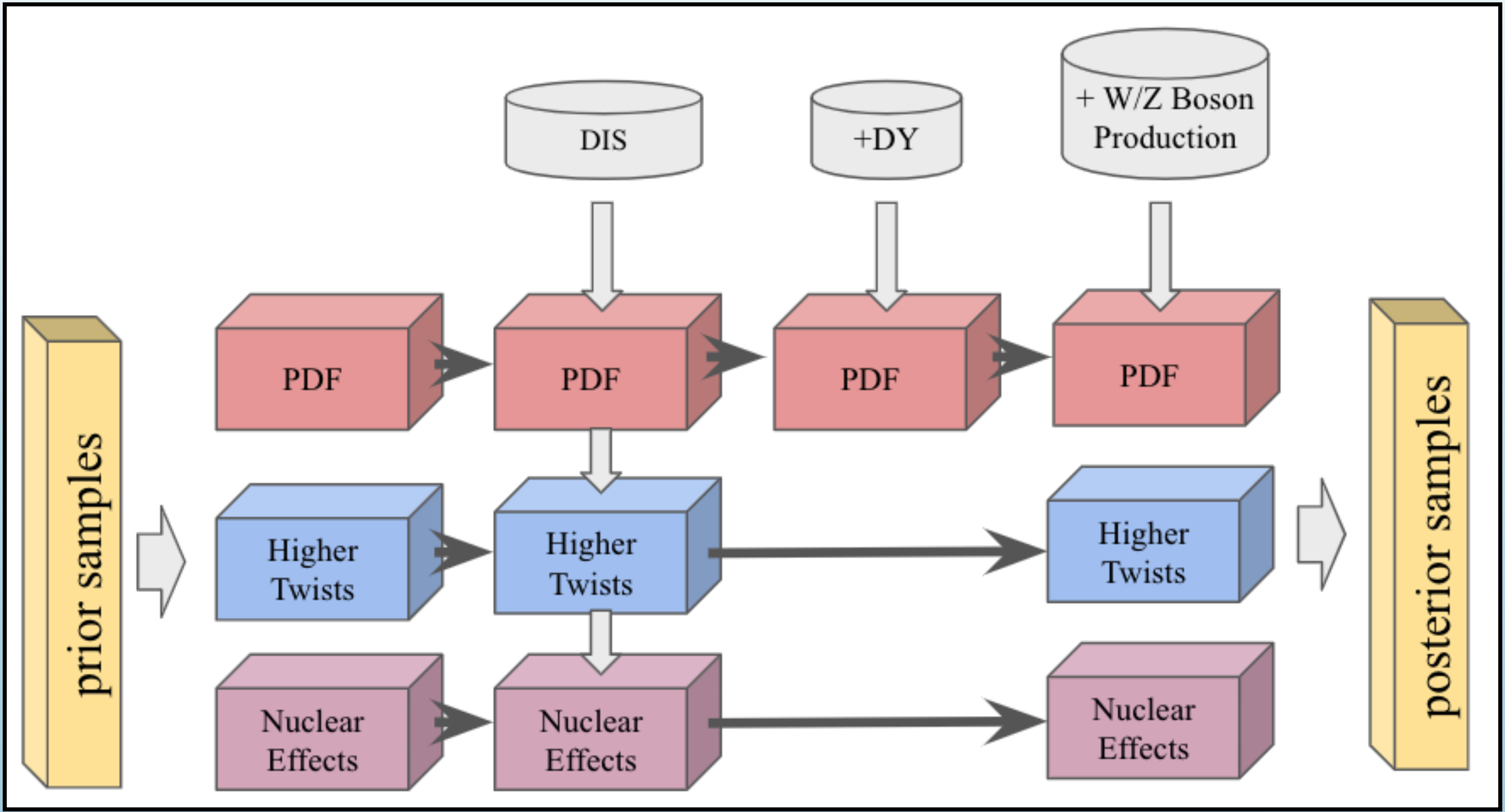
Build an MC ensemble

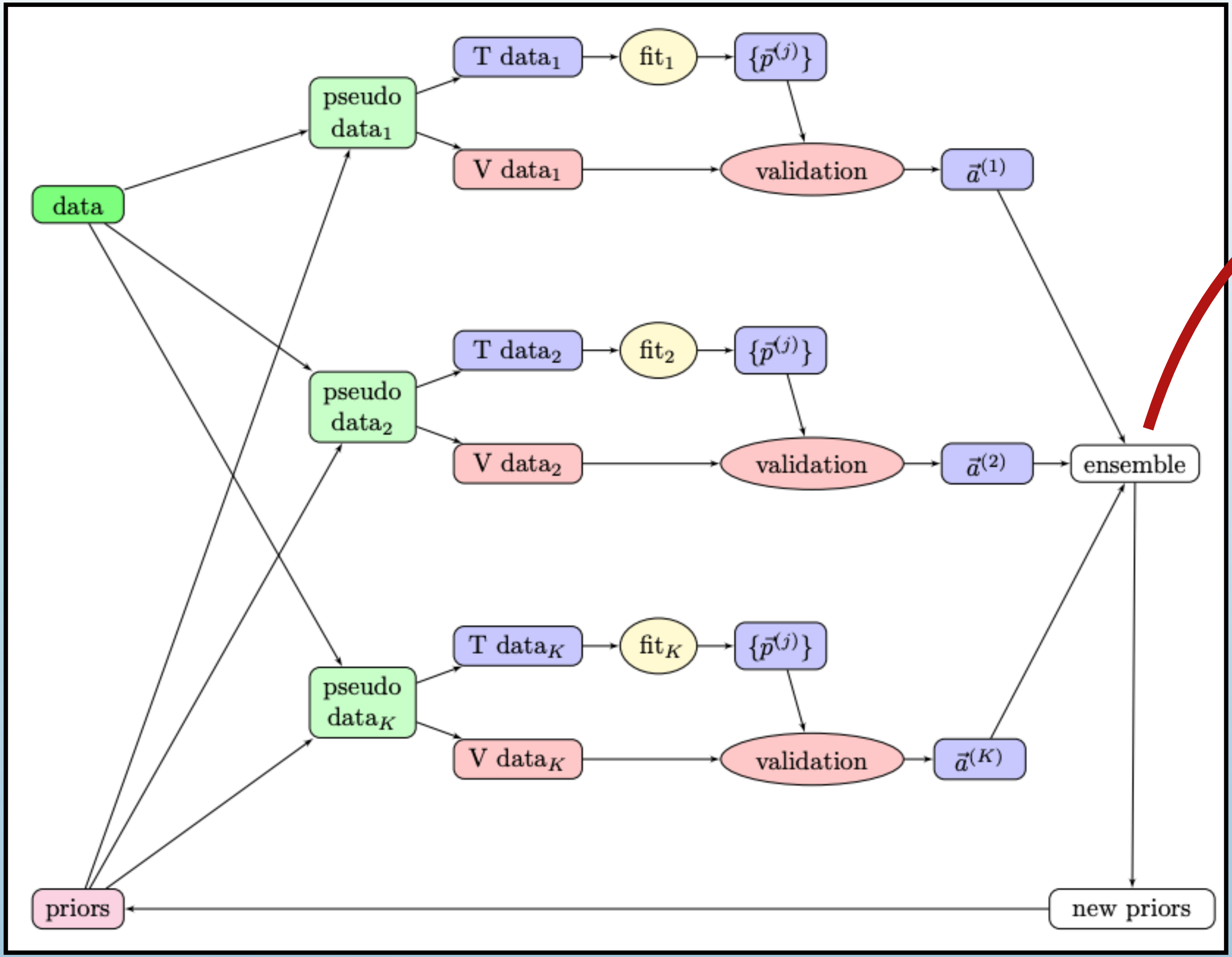
$$E[O] \approx \frac{1}{N} \sum_k O(\boldsymbol{a}_k)$$

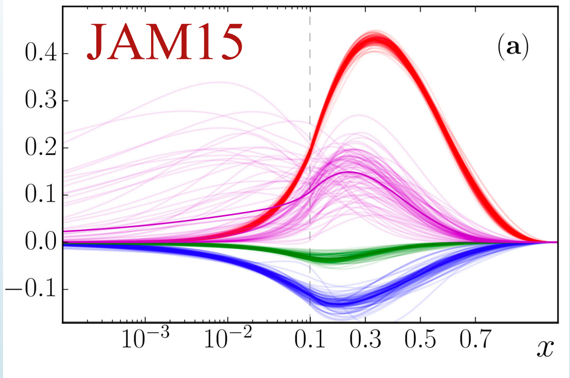
$$V[O] \approx \frac{1}{N} \sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$

Average over k sets of the parameters (replicas)



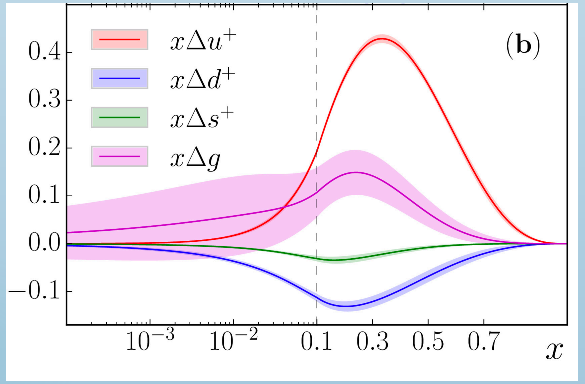

DIS
+DY
+ W/Z Boson Production
prior samples
PDF
PDF
PDF
PDF
Higher Twists
Higher Twists
Higher Twists
Nuclear Effects
Nuclear Effects
Nuclear Effects
posterior samples



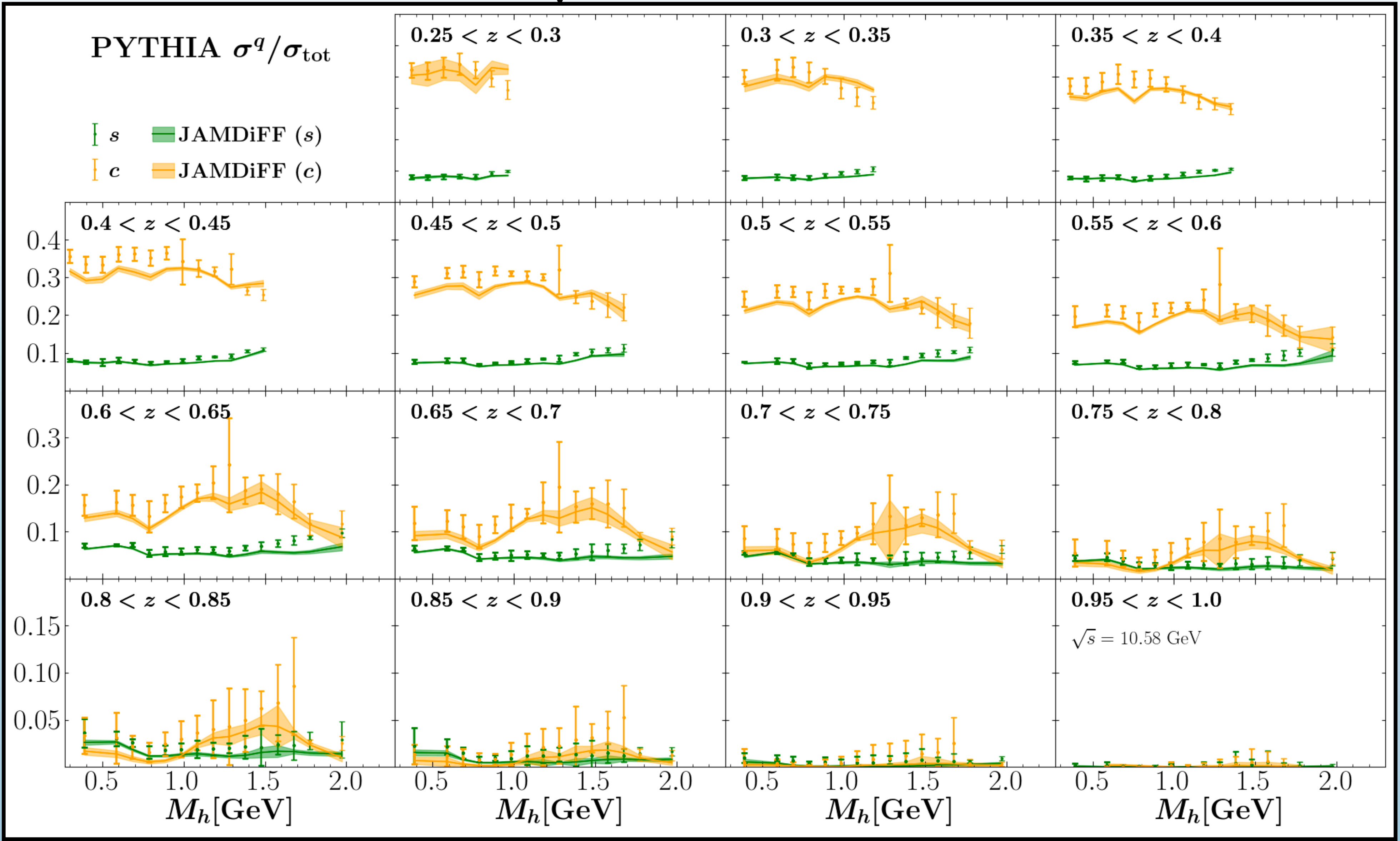


$$E[O] \approx \frac{1}{N}\sum_k O(\boldsymbol{a}_k)$$

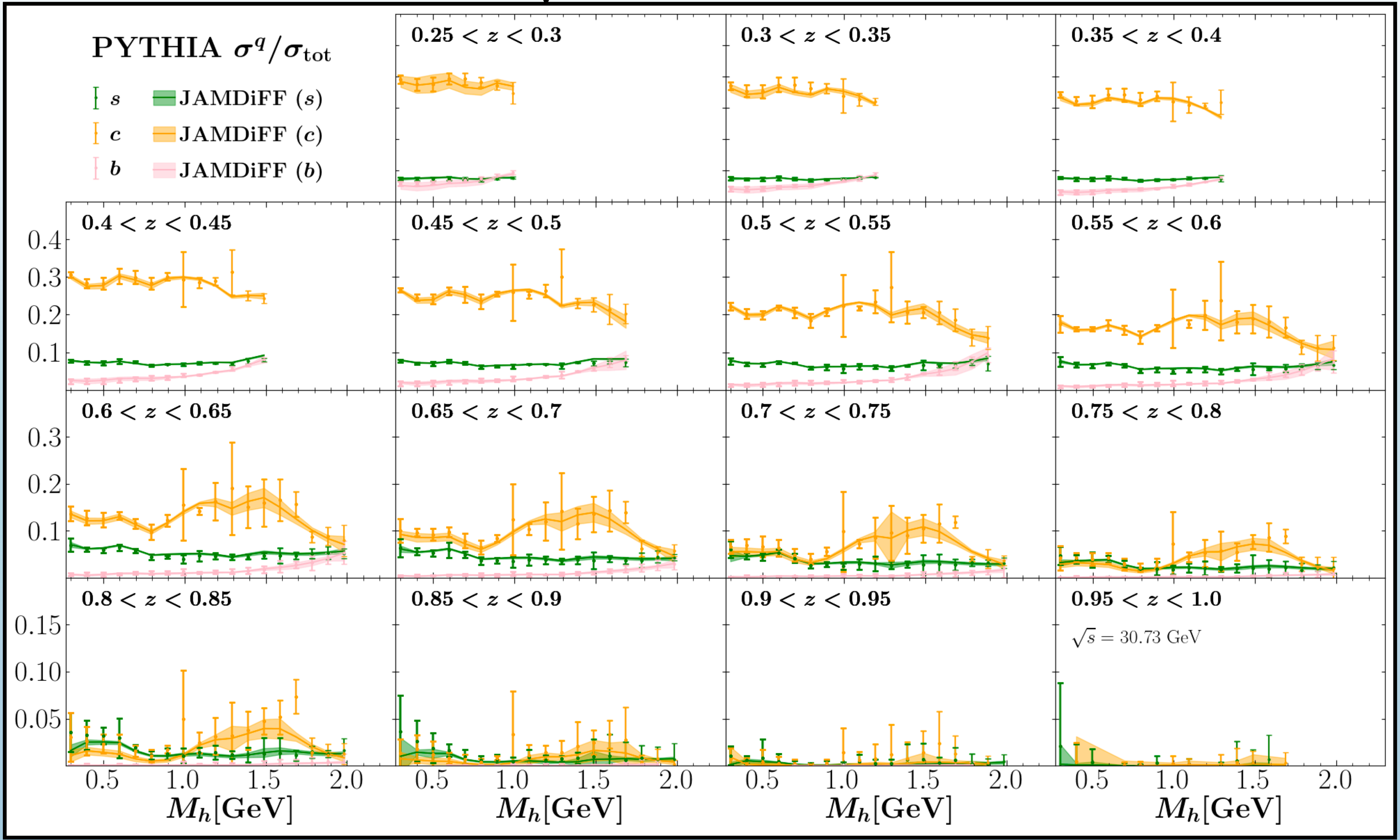
$$V[O] \approx \frac{1}{N}\sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$



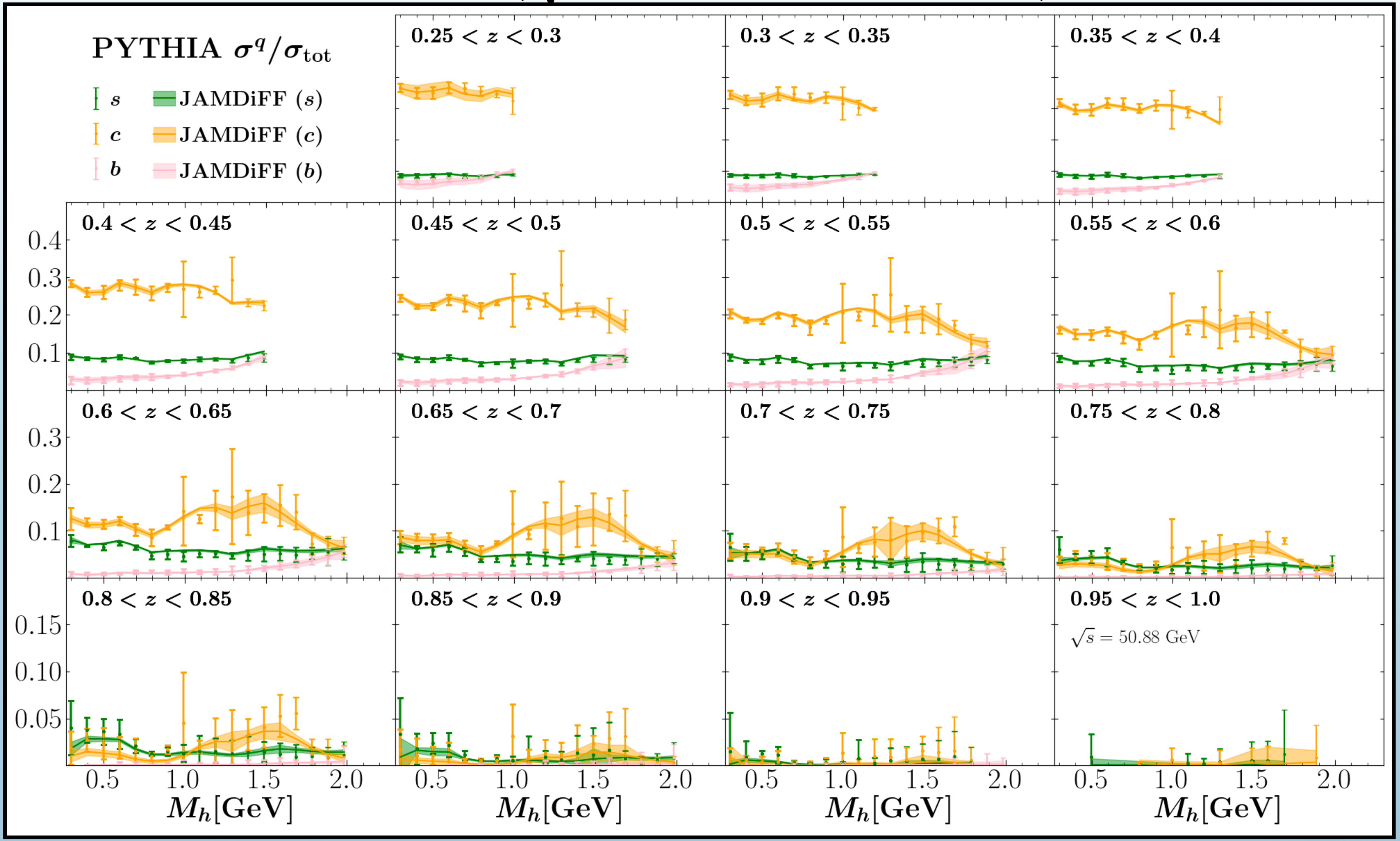
PYTHIA data ($\sqrt{s} = 10.58$ GeV)



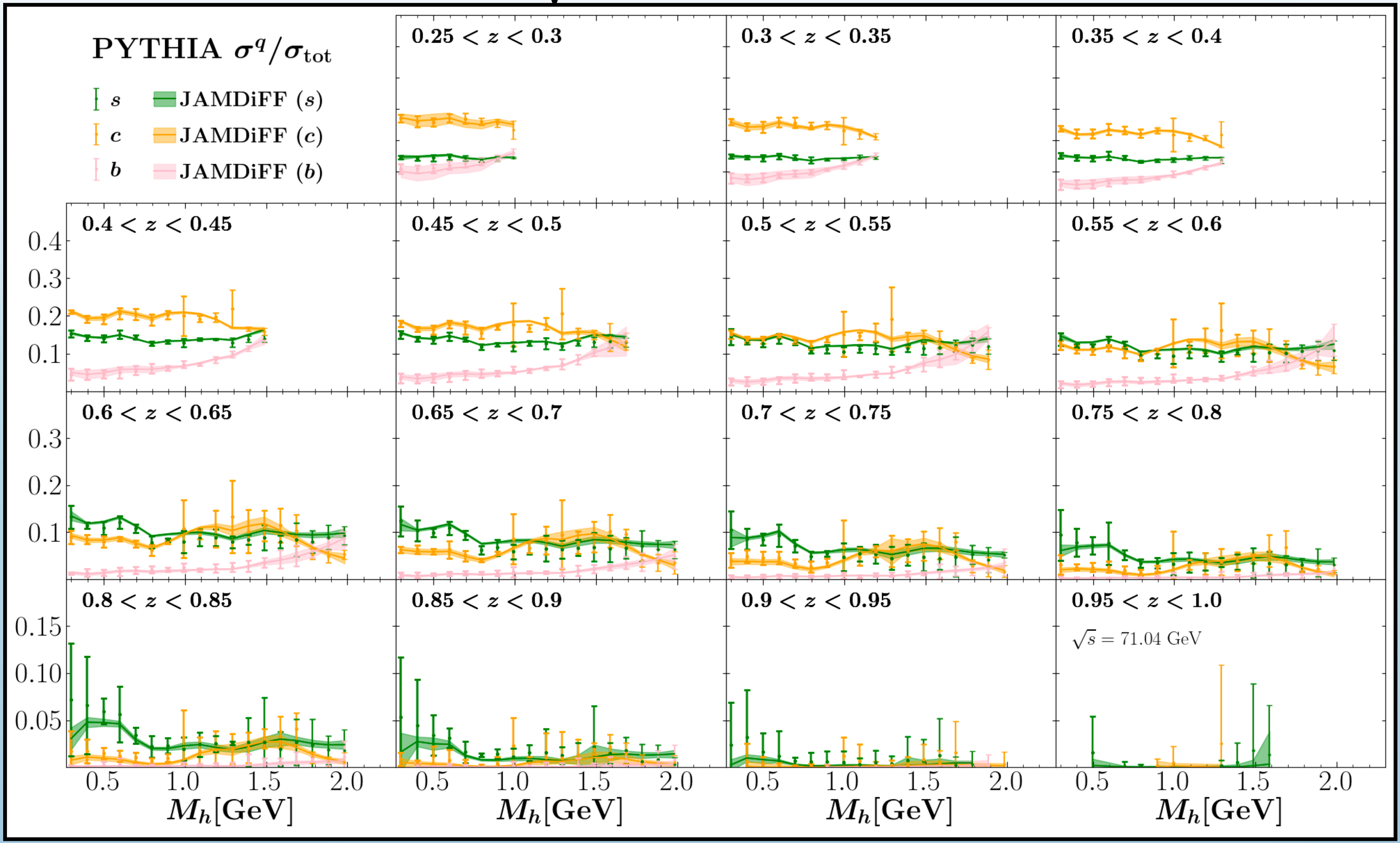
PYTHIA data ($\sqrt{s} = 30.73$ GeV)



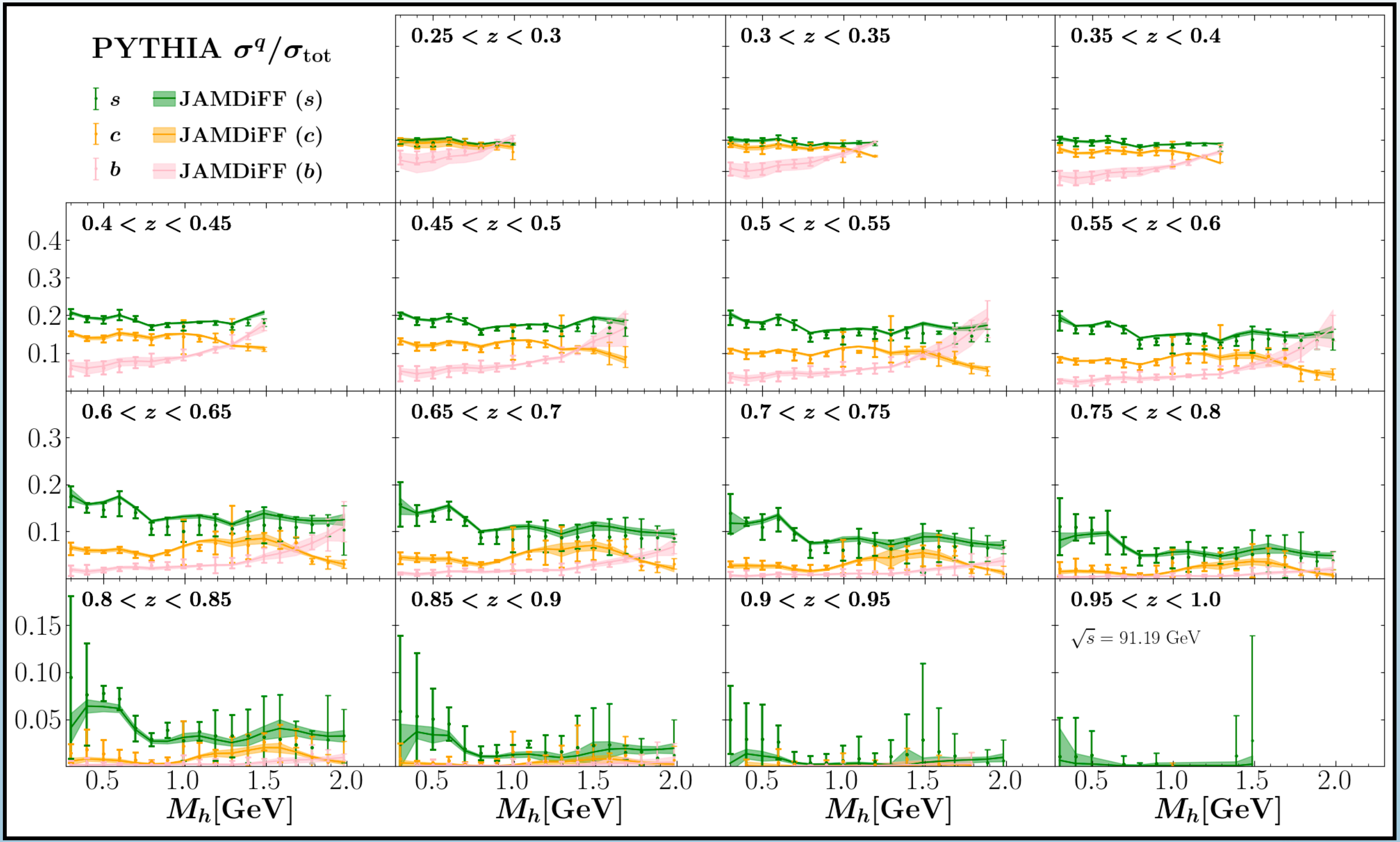
PYTHIA data ($\sqrt{s} = 50.88$ GeV)



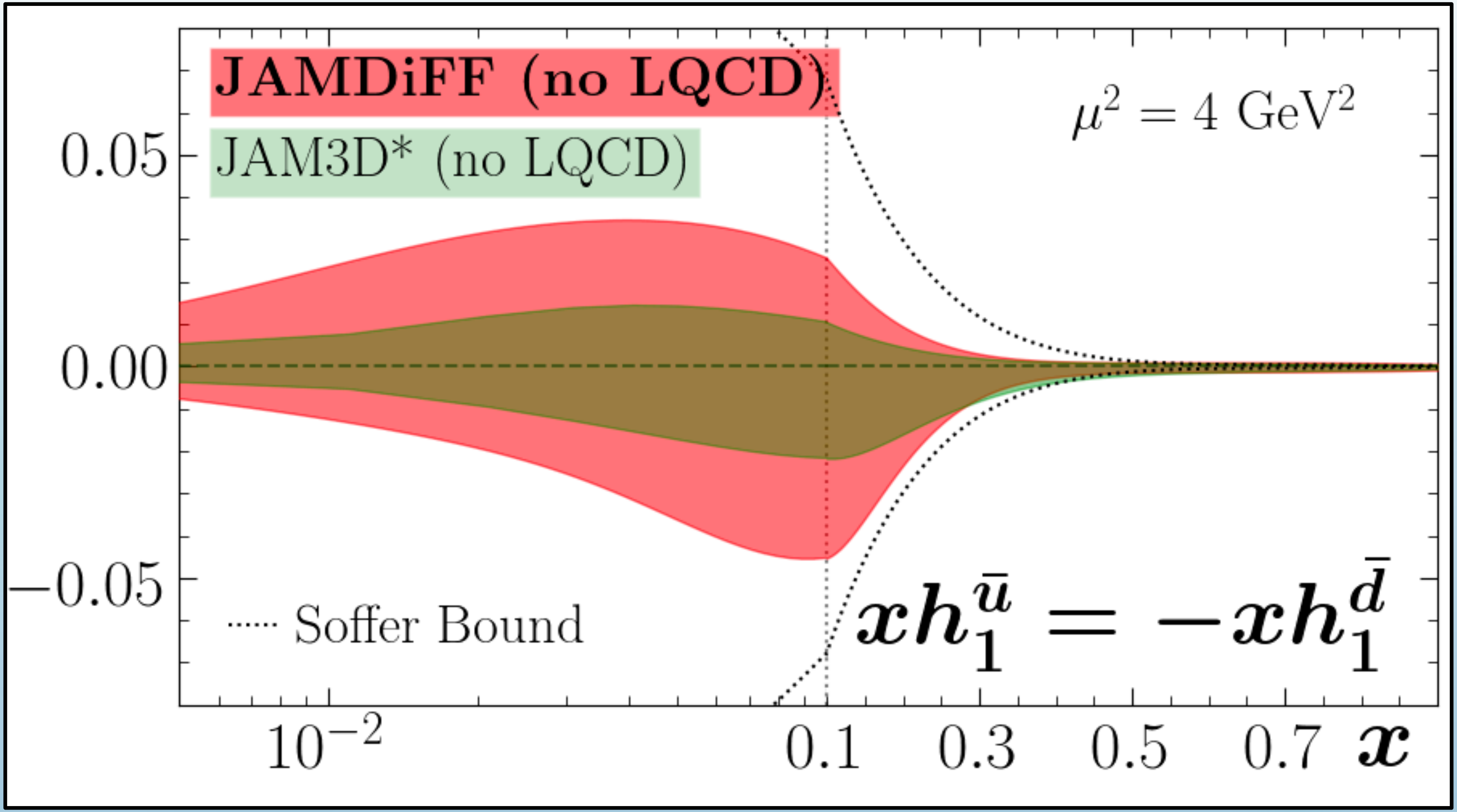
PYTHIA data ($\sqrt{s} = 71.04$ GeV)



PYTHIA data ($\sqrt{s} = 91.19$ GeV)



Transversity PDFs (antiquarks)



DiFF Parameterization

$$\mathbf{M}_h^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00] \text{ GeV}.$$

$$D_1^q(z, \mathbf{M}_h^{q,i}) = \sum_{j=1,2,3} \frac{N_{ij}^q}{\mathcal{M}_{ij}^q} z^{\alpha_{ij}^q} (1-z)^{\beta_{ij}^q},$$

204 parameters for D_1
48 parameters for $H_1^{\sphericalangle}$

PDF Parameterization

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

$$f(x,\mu_0^2) = \frac{N}{\mathcal{M}} x^\alpha (1-x)^\beta (1+\gamma\sqrt{x}+\eta x),$$

15 parameters for h_1

Tensor Charge Numbers

Fit	δu	δd	g_T
no LQCD	0.50(7)	-0.04(14)	0.54(12)
w/ LQCD	0.71(2)	-0.200(6)	0.91(2)