Measurements of α_s with JLab@22 GeV

A. Deur
Jefferson Lab

- Measurement of $\alpha_s(M_z)$
- Measurement of $\alpha_s(Q)$ for $1 < Q^2 < 22$ GeV^2

Importance of measuring $\alpha_s(M_z)$

- α_s: most important quantity of QCD, key parameter of the Standard Model, but (by far) the least known fundamental coupling: $\Delta\alpha_s/\alpha_s \simeq 10^{-2}$ ($\Delta\alpha/\alpha \simeq 10^{-10}$, $\Delta G_F/G_F \simeq 10^{-6}$, $\Delta G_N/G_N \simeq 10^{-5}$)
- Large efforts ongoing to reduce $\Delta\alpha_s/\alpha_s$ (Snowmass 2022, arXiv:2203.08271)
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  $\Rightarrow$ Strategy: combine many independent measurements with larger uncertainties.

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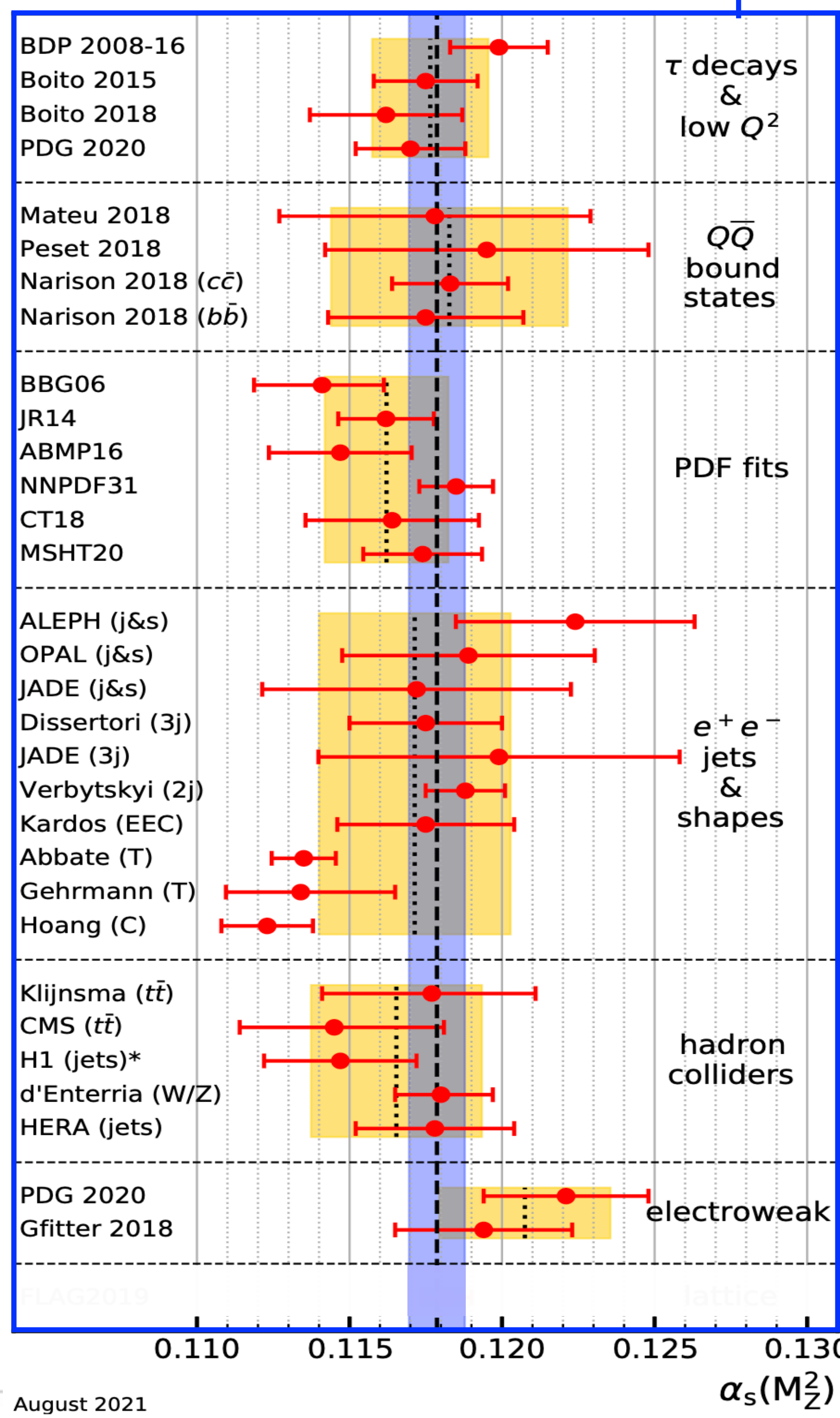

Fig. from Part. Data Group.

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Details given in talk at JLab@22 GeV Workshop, Jan. 2023

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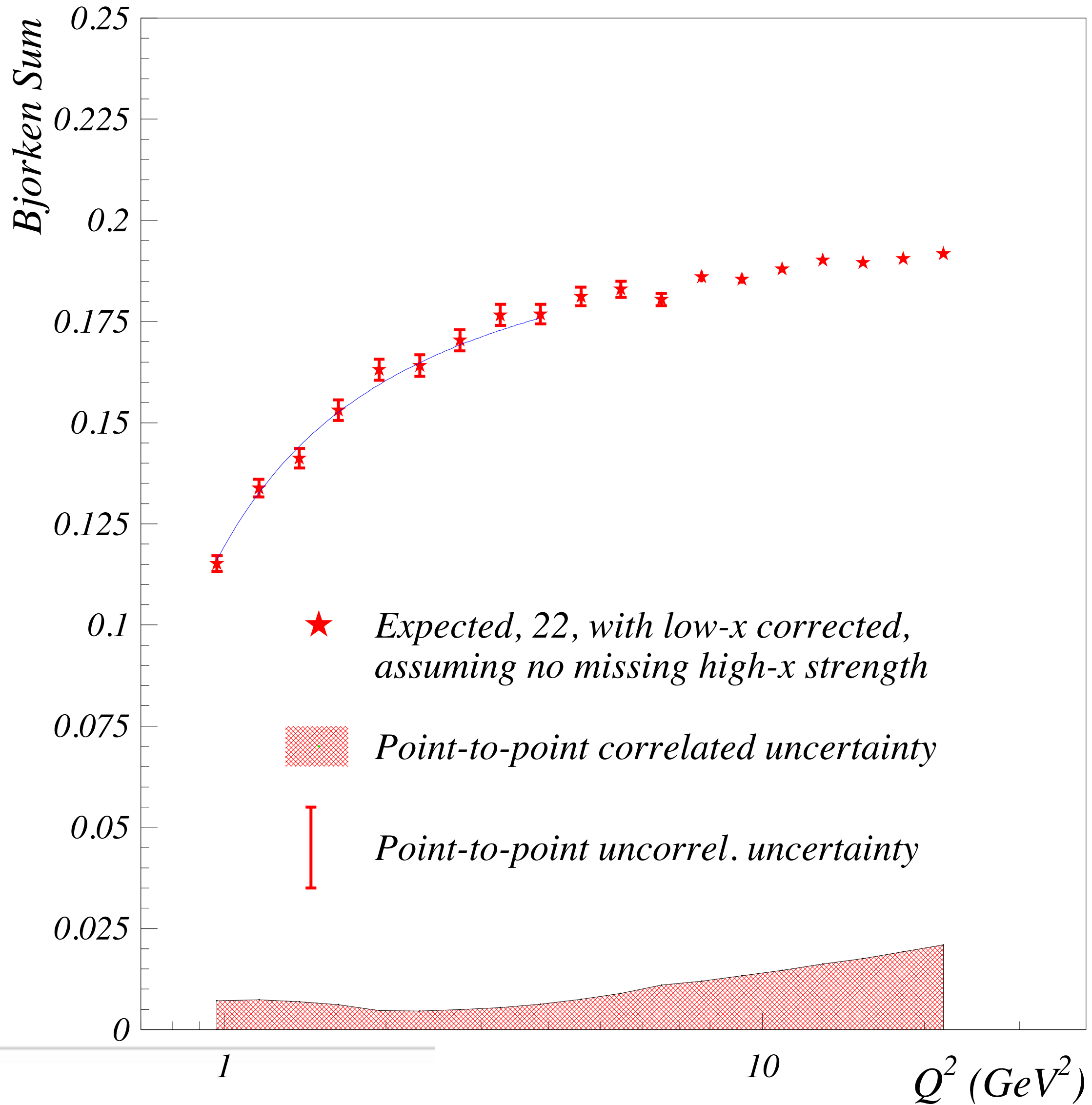

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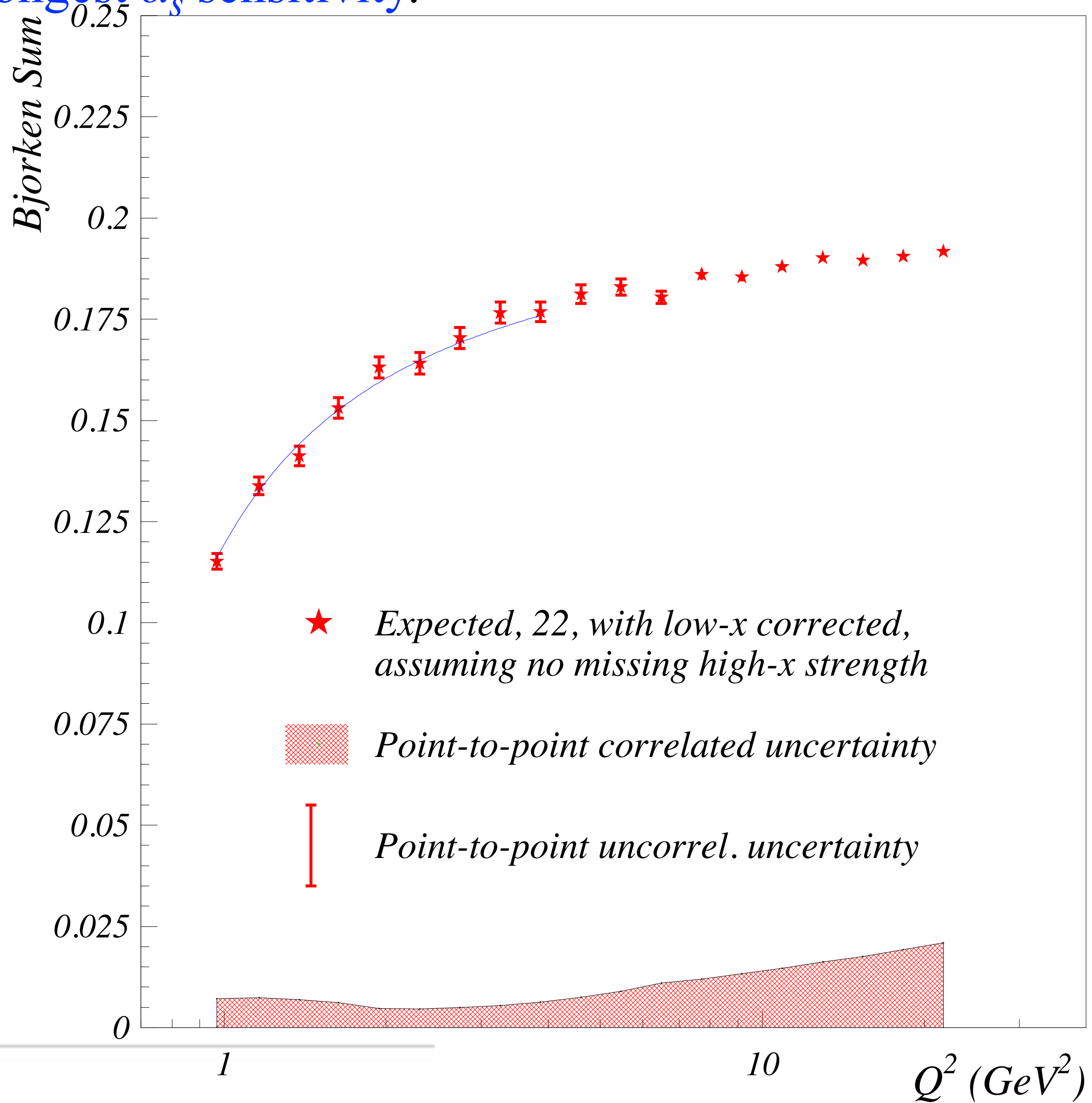

Jefferson Lab
Thomas Jefferson National Accelerator Facility
Exploring the Nature of Matter

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- Negligible statistical uncertainties (inclusive data obtained concurrently with exclusive data more demanding in stats).
- With polarized NH_3 and ^3He targets: 5% systematics (experimental only, i.e., not counting low-x uncertainty)
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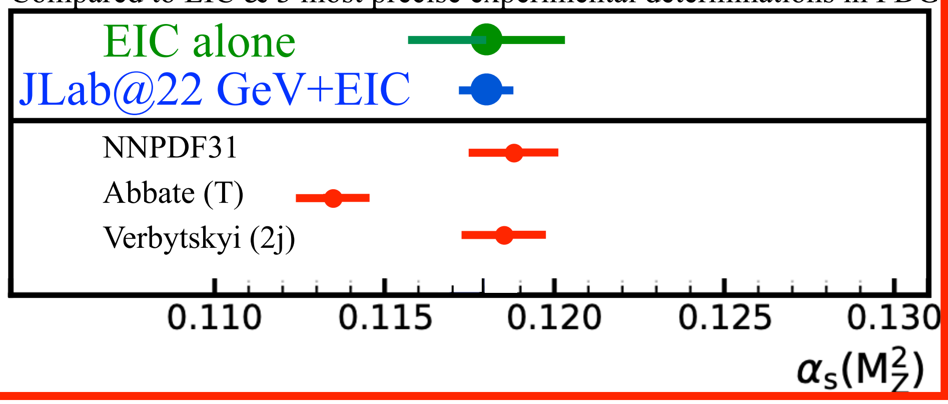
Compared to EIC & 3 most precise experimental determinations in PDG



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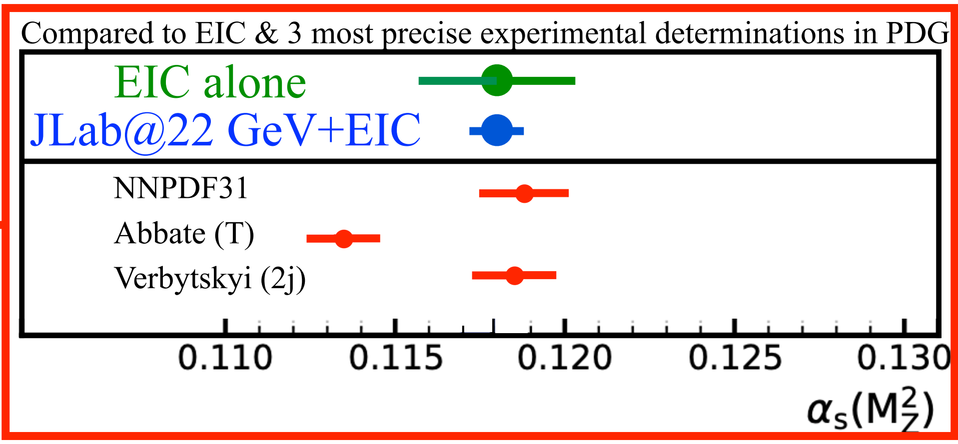

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- One extraction from Lab@22 GeV can yield α_s with greater accuracy than world data combined. It is just one possibility to access α_s with JLab@22 GeV. Others, e.g., global fits of (un)polarized PDFs may also provide competitive determinations.

Measuring α_s

Two possibilities to extract α_s from the Bjorken sum rule:

- Previous slides: Measurement of **Q^2-dependence** of $\Gamma_1^{p-n}(Q^2)$.
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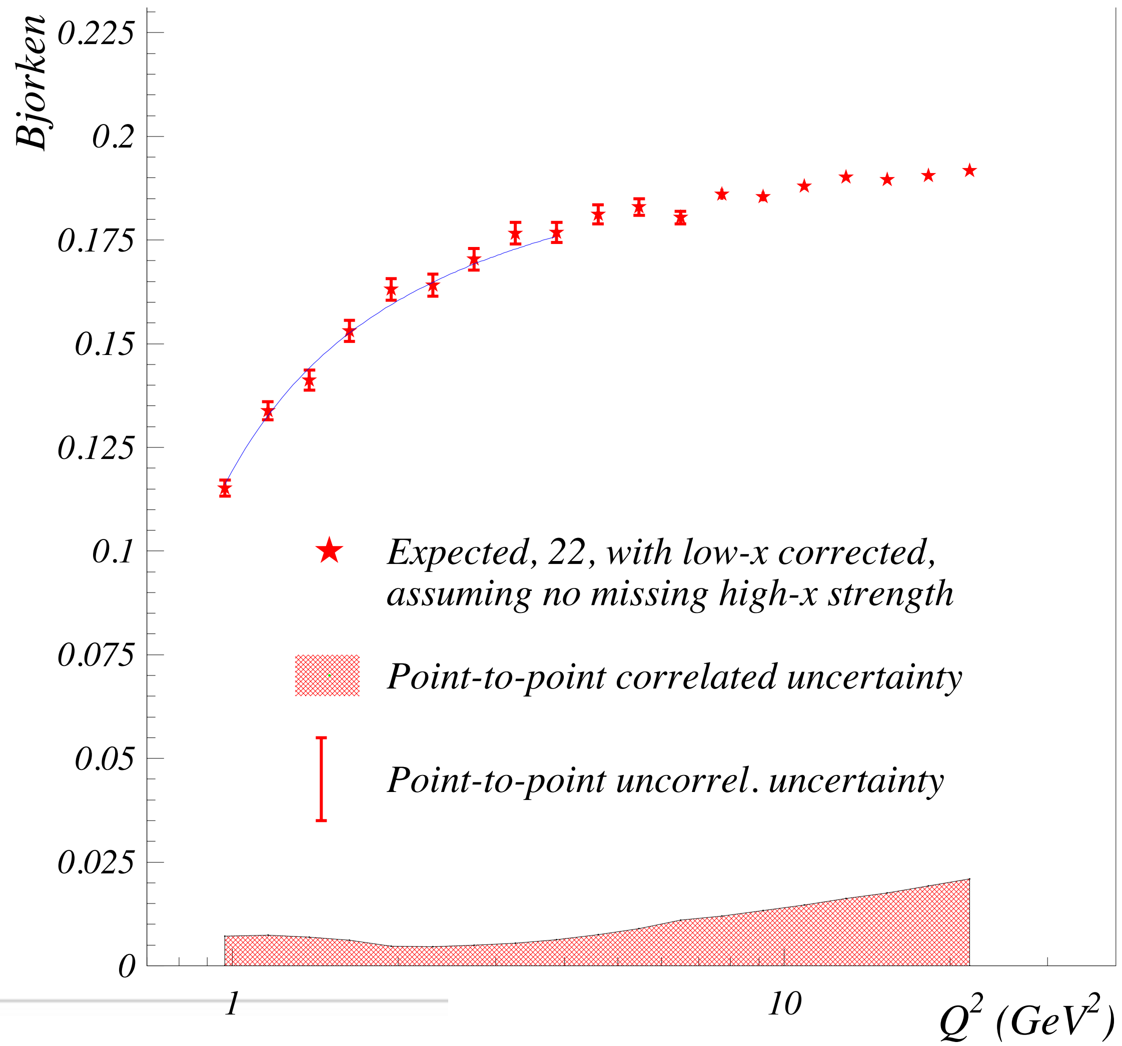

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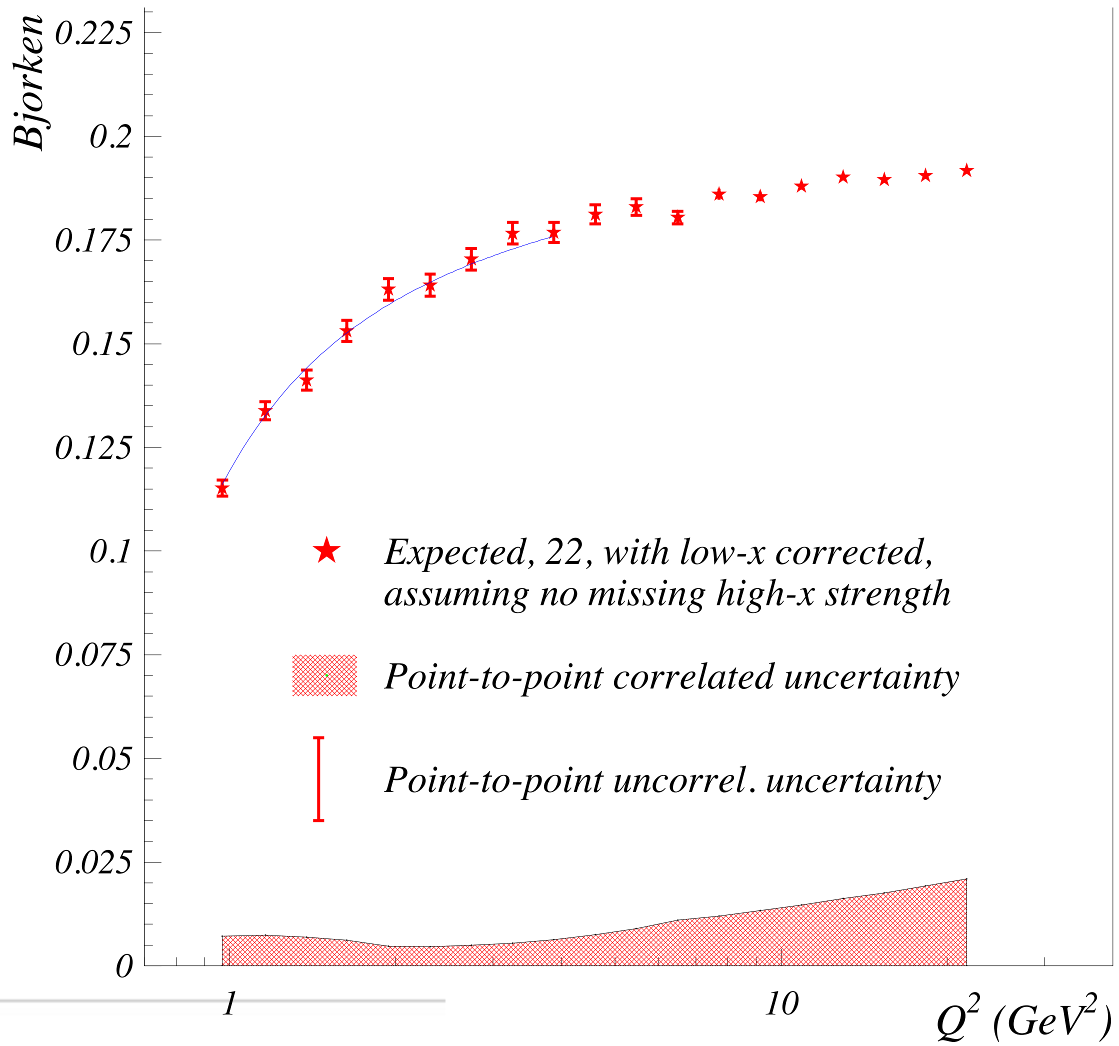
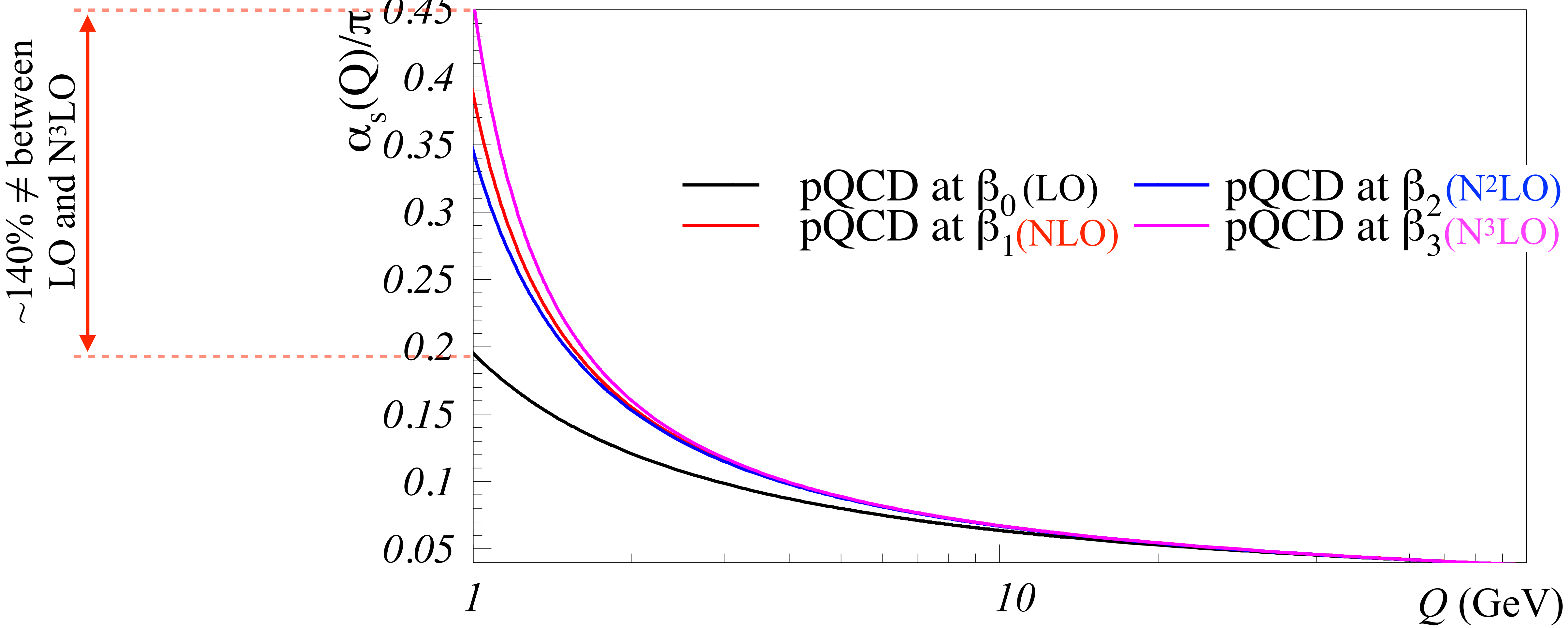


⇒Sensitivity to high-order QCD loops not yet been measured

Measuring $\alpha_s(Q)$



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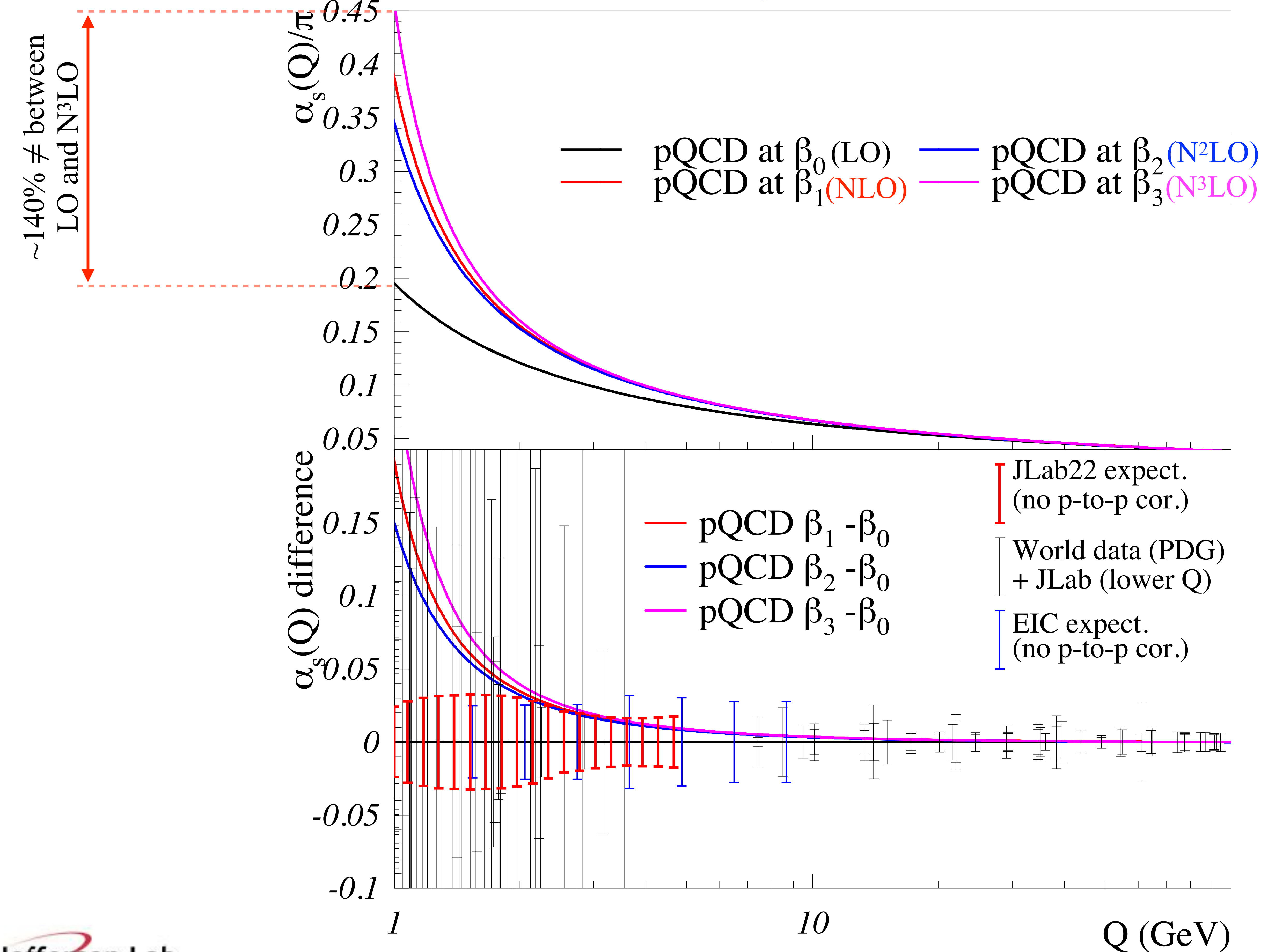

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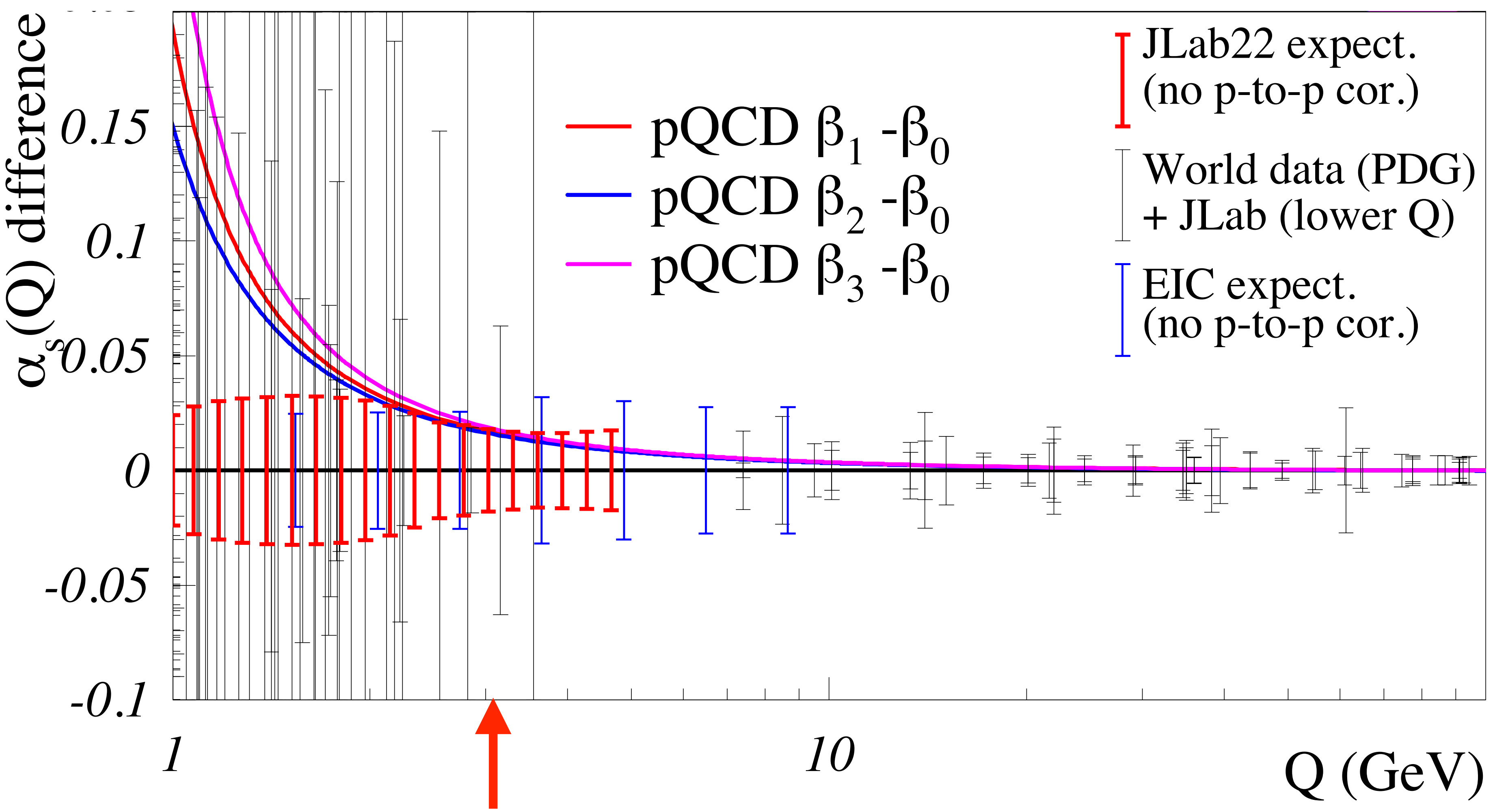

$Q^2 < 5.4$ GeV^2: start being sensitive to N^{n}LO

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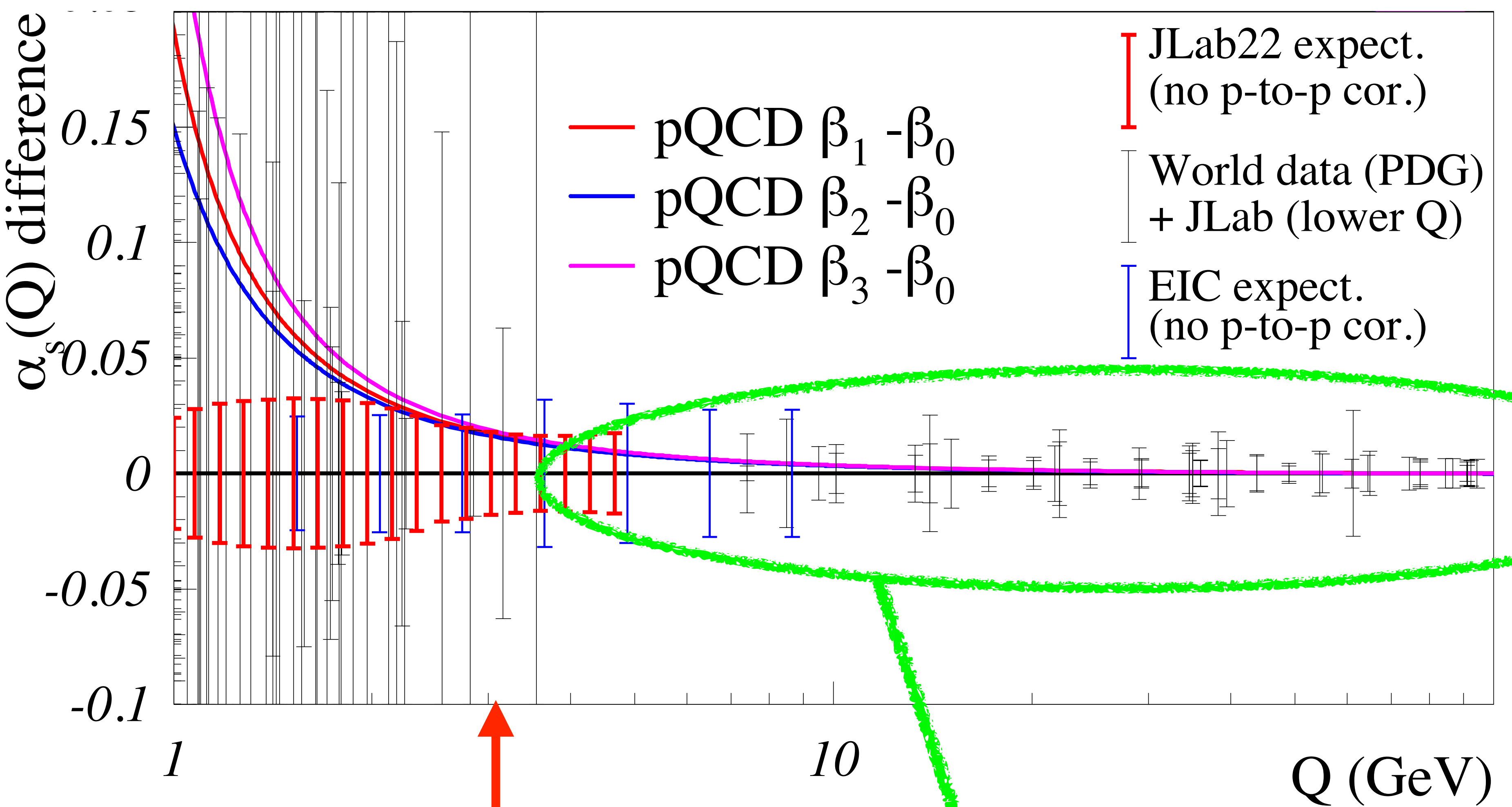

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Unique direct sensitivity to N^{n}LO.

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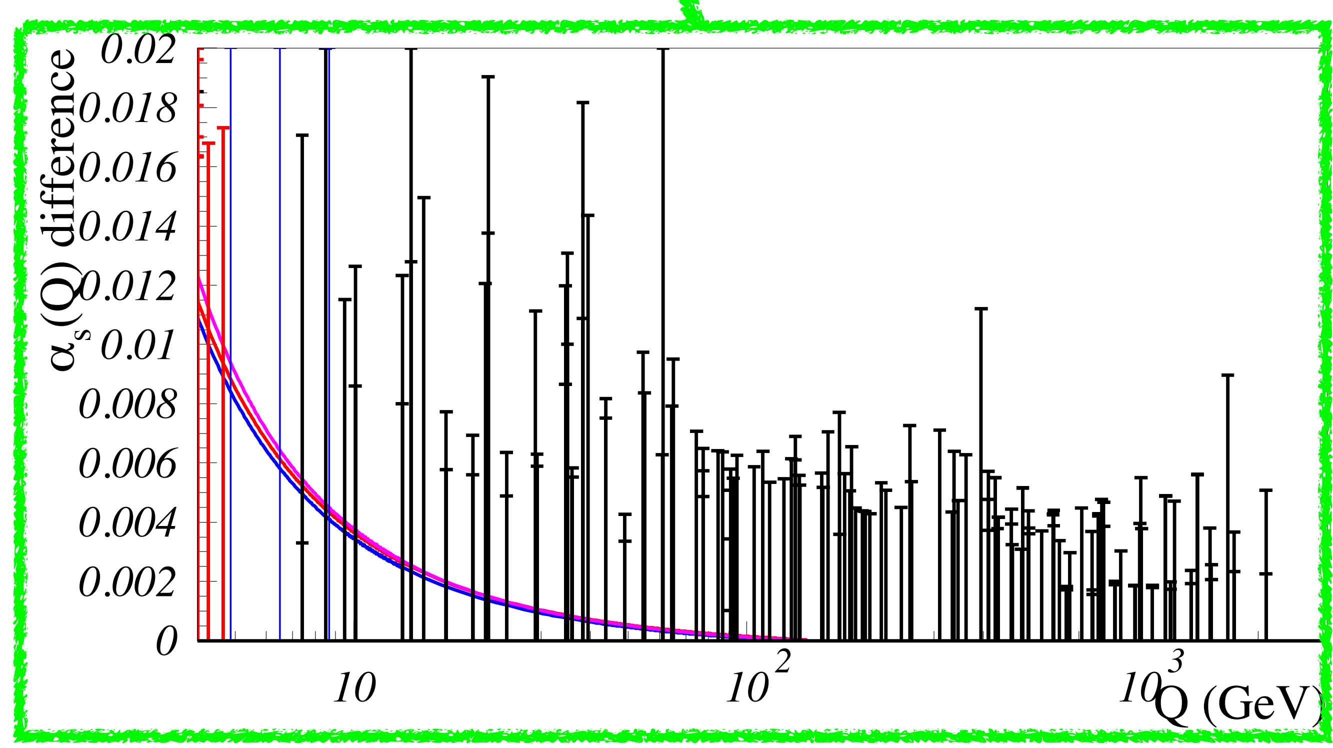

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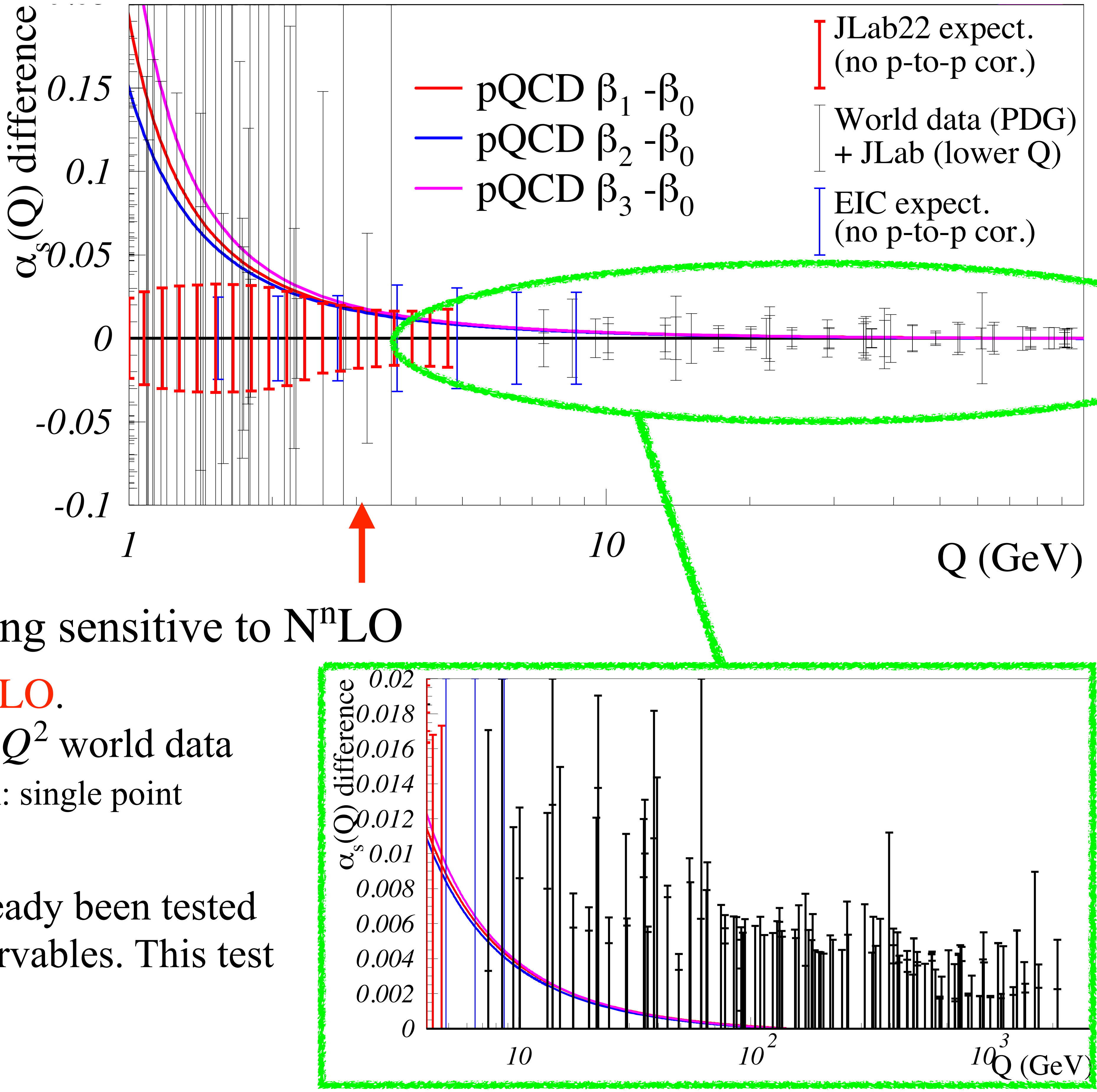

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pQCD Q^2-dependence has already been tested beyond LO using various observables. This test isolates loop effects.

Back-up slides

Bjorken sum rule

$$\Gamma_1^{p-n} \equiv \int g_1^{p-n} dx = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 - 175.7 \left(\frac{\alpha_s}{\pi}\right)^4 - \sim 893 \left(\frac{\alpha_s}{\pi}\right)^5 \right]$$

$$+ \frac{M^2}{Q^2} \left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s) \right] + \ldots$$

Nucleon's First spin structure function

Nucleon axial charge. (Value of $\Gamma_1^{p-n}(Q^2)$ in the $Q^2 \to \infty$ limit)

pQCD radiative corrections ($\overline{MS}$ Scheme.)

Non-perturbative $1/Q^{2n}$ power corrections. (+rad. corr.)

$\Rightarrow$ Two possibilities to extract $\alpha_s(M_Z)$:

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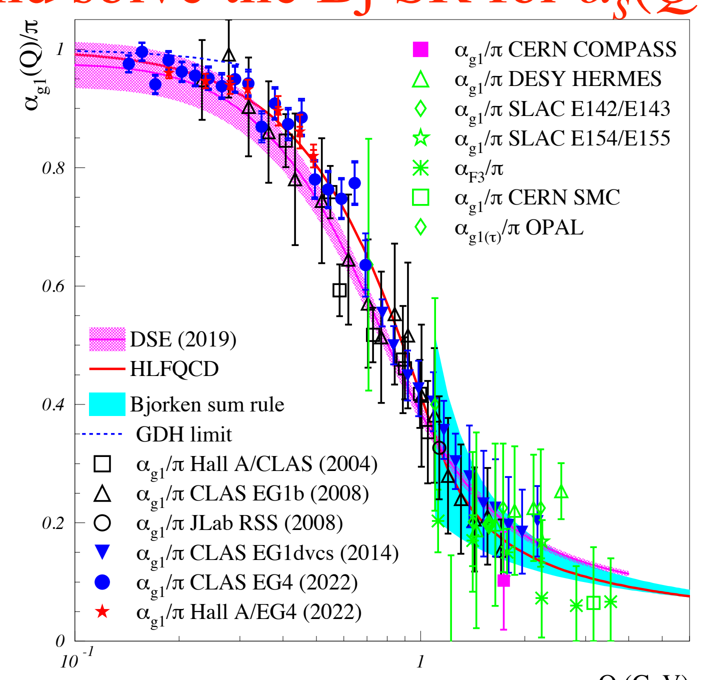

The Bj SR allows to extract $\alpha_s(Q^2)$ at all scale!

- Poor systematic accuracy, typically $\Delta\alpha_s/\alpha_s$ ~10% at high energy $\Rightarrow$ Not competitive.

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- Measurement of Q^2**-dependence** of $\Gamma_1^{p-n}(Q^2)$.
  - Need Γ_1^{p-n} at several Q^2 points. Only one (or a few) value of α_s.
  - Good accuracy: 1990's CERN/SLAC data yielded: $\alpha_s(M_Z)=0.120\pm0.009$

Altarelli, Ball, Forte, Ridolfi, Nucl.Phys. B496 337 (1997)

Bjorken sum rule at JLab@22 GeV

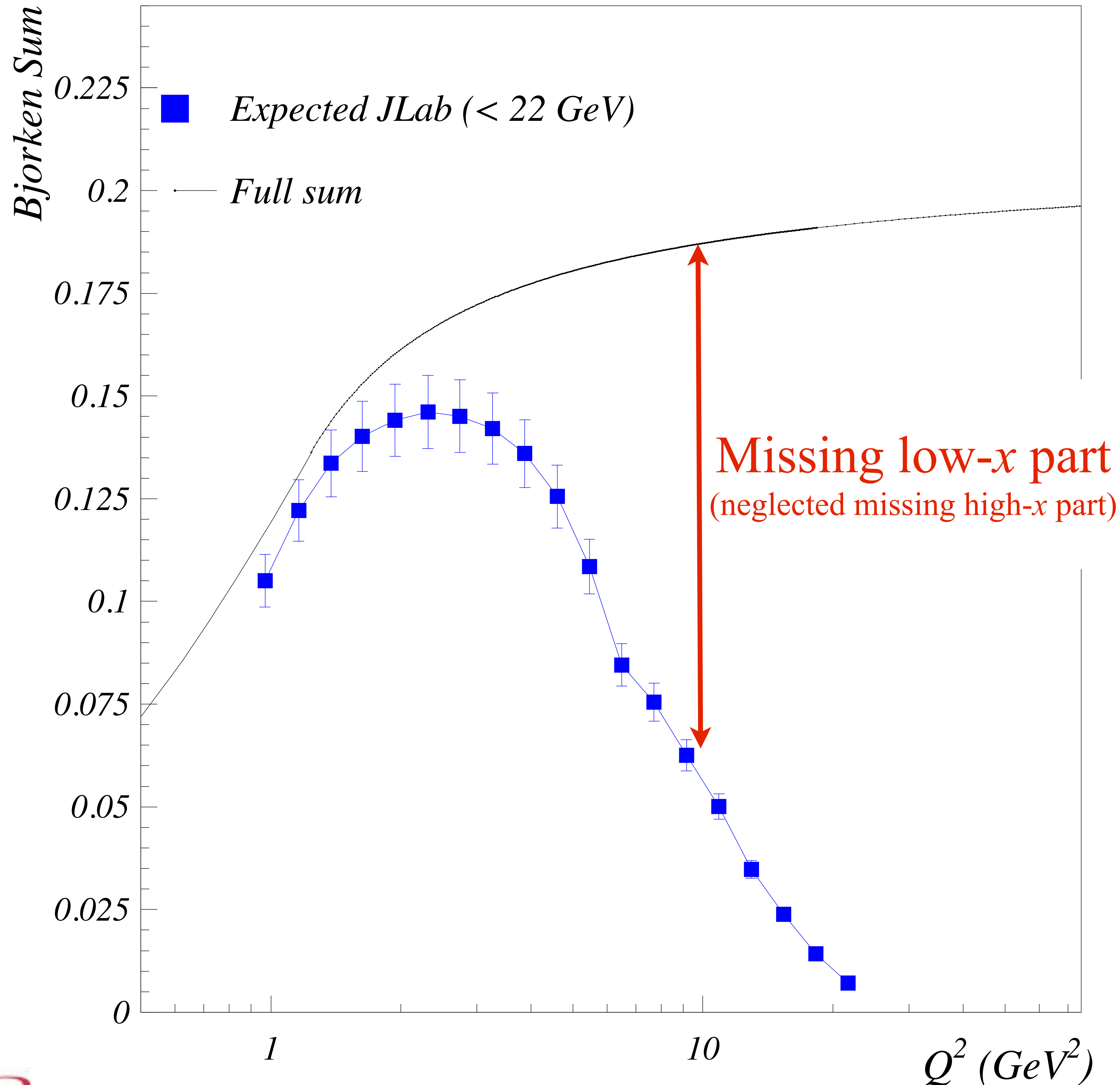
- Statistical uncertainties are expected to be negligible:
  - JLab is a high-luminosity facility;
  - A JLab@22 GeV program would include polarized DVCS and TMD experiments. Those imply long running times compared to those needed for inclusive data gathering;
  - High precision data already available from 6 GeV and 12 GeV for the lower Q^2 bins and moderate x.

- Looking at the 6 GeV CLAS EG1dvcs data, required statistics for DVCS and TMD experiments imply statistical uncertainties < 0.1% on the Bjorken sum. For the present exercise we will use 0.1% on all Q^2-points with Q^2-bin sizes increasing exponentially with Q^2.

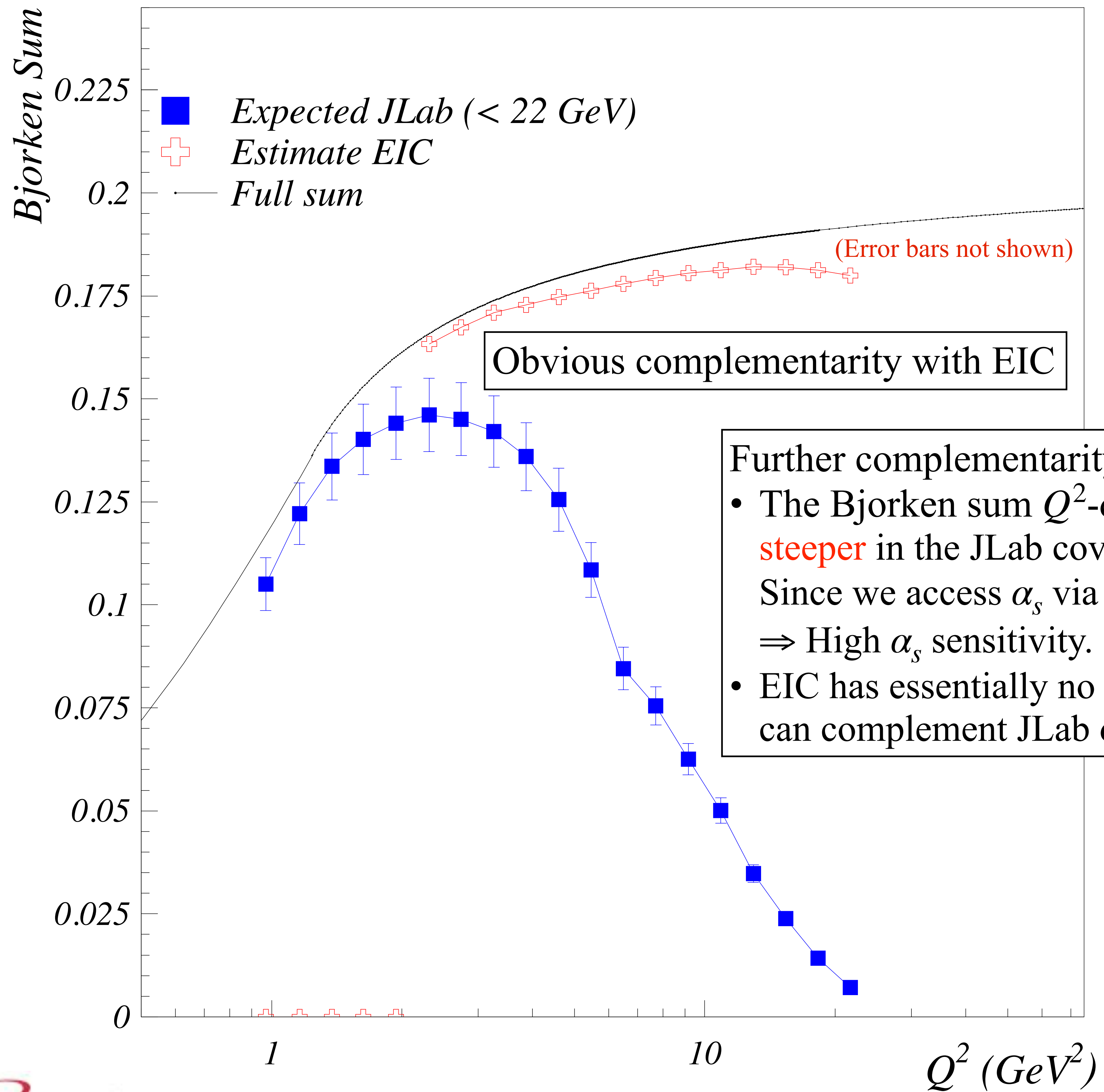
- Use 6% for experimental systematics (i.e. not including the uncertainty on unmeasured low-x).
  - Nuclear corrections:
    - D: negligible assuming we can tag the ~spectator proton
    - ^{3}He: 2% (5% on n, which contribute to 1/3 to the Bjorken sum: 5%/3≃2%)
  - Polarimetries: Assume ΔP_e-ΔP_N = 3%.
  - Radiative corrections: 1%
  - F_1 to form g_1 from A_1: 2%
  - g_2 contribution to longitudinal asym: Negligible, assuming it will be measured.
  - Dilution/purity:
    - Bjorken sum from P & D: 4%
    - Bjorken sum from P & ^{3}He: 3%
  - Contamination from particle miss-identification: Assumed negligible.
  - Detector/trigger efficiencies, acceptance, beam currents: Neglected (asym).

Adding in quadrature: ~5%

Under these assumptions:



Comparison with EIC



Obvious complementarity with EIC

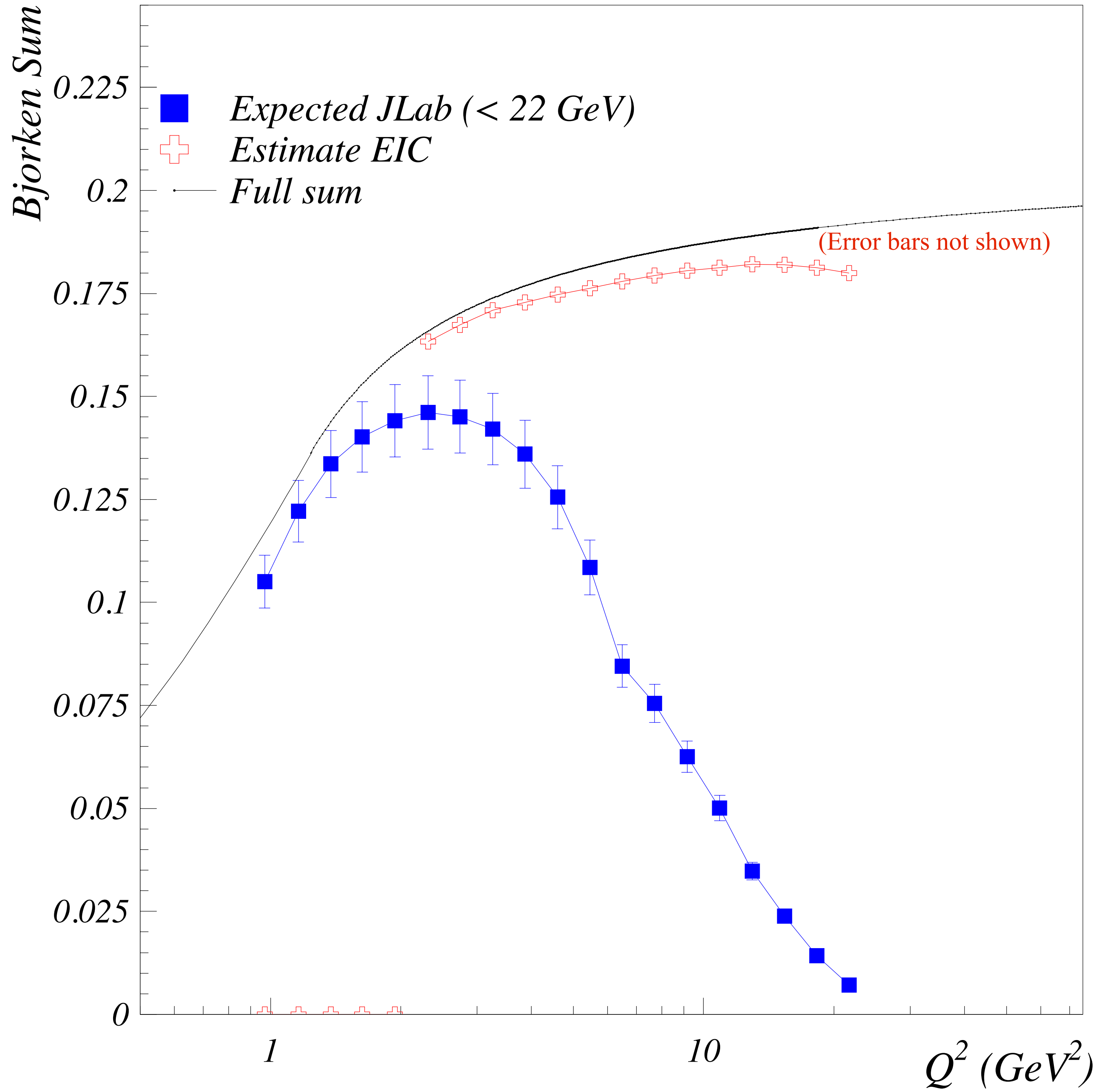
Further complementarity:

- The Bjorken sum Q^2-dependence is up to 50 times steeper in the JLab covered range than that of EIC. Since we access α_s via relative Q^2-dependence $\Rightarrow$ High α_s sensitivity.
- EIC has essentially no unmeasured low-x issue and can complement JLab data.

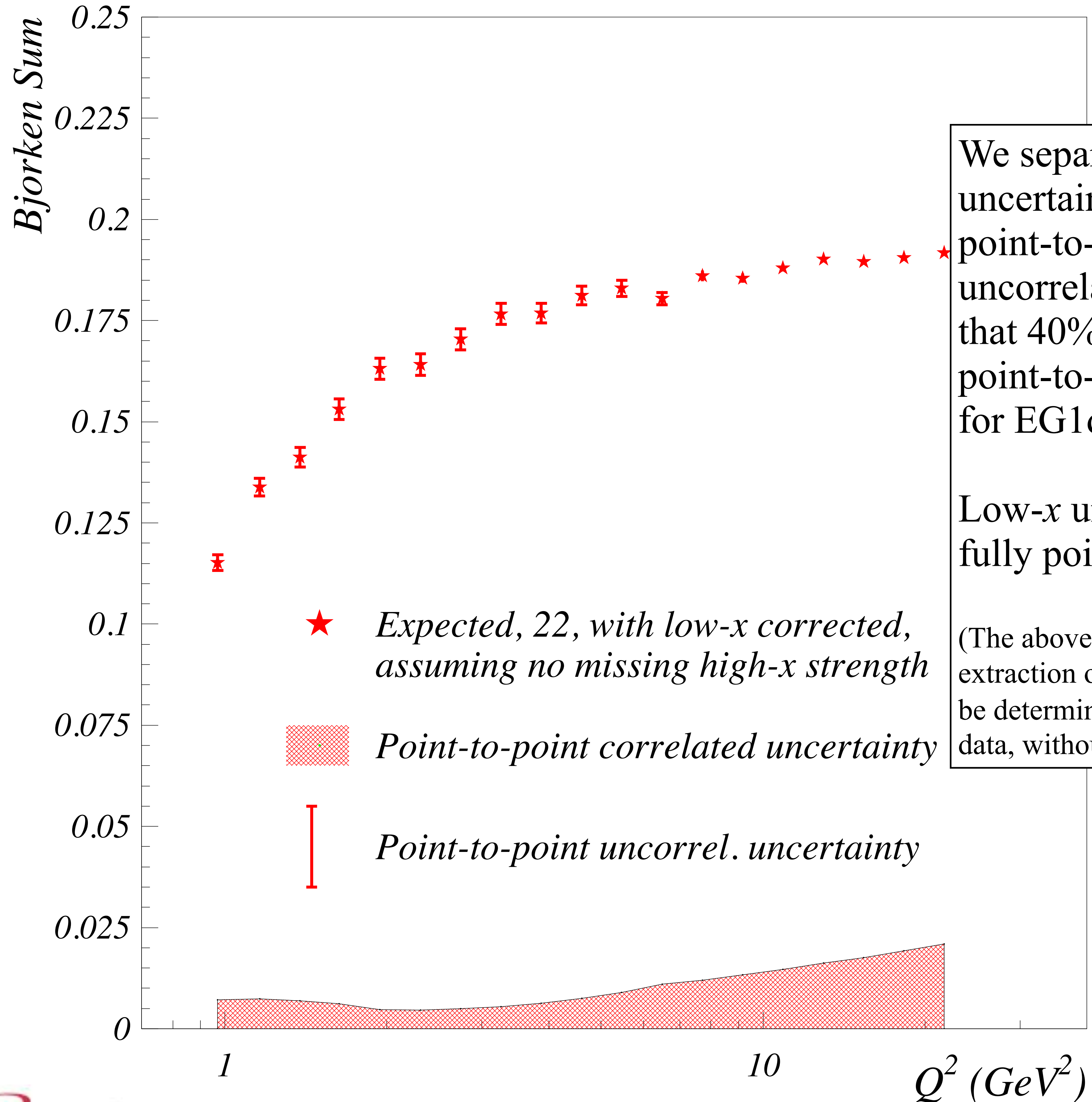
Low-x uncertainty

- For the Q^2 bins covered by EIC, global fits will be available up to the lowest x covered by EIC.
  $\Rightarrow$ assume 10% uncertainty on that missing (for the JLab measurement) low-x part.
  Assume 100% for the very small-x contribution not covered by EIC.

- For the 5 lowest Q^2 bins not covered by EIC:
  - Bin #5 close to the EIC coverage $\Rightarrow$ Constrained extrapolation, assume 20% uncertainty on missing low-x part.
  - Bin #4, assume 40% uncertainty, Bin #3, assume 60%, Bin #2, assume 80%, Bin #1, assume 100%.



Bjorken sum rule at JLab@22 GeV (meas.+low-x)

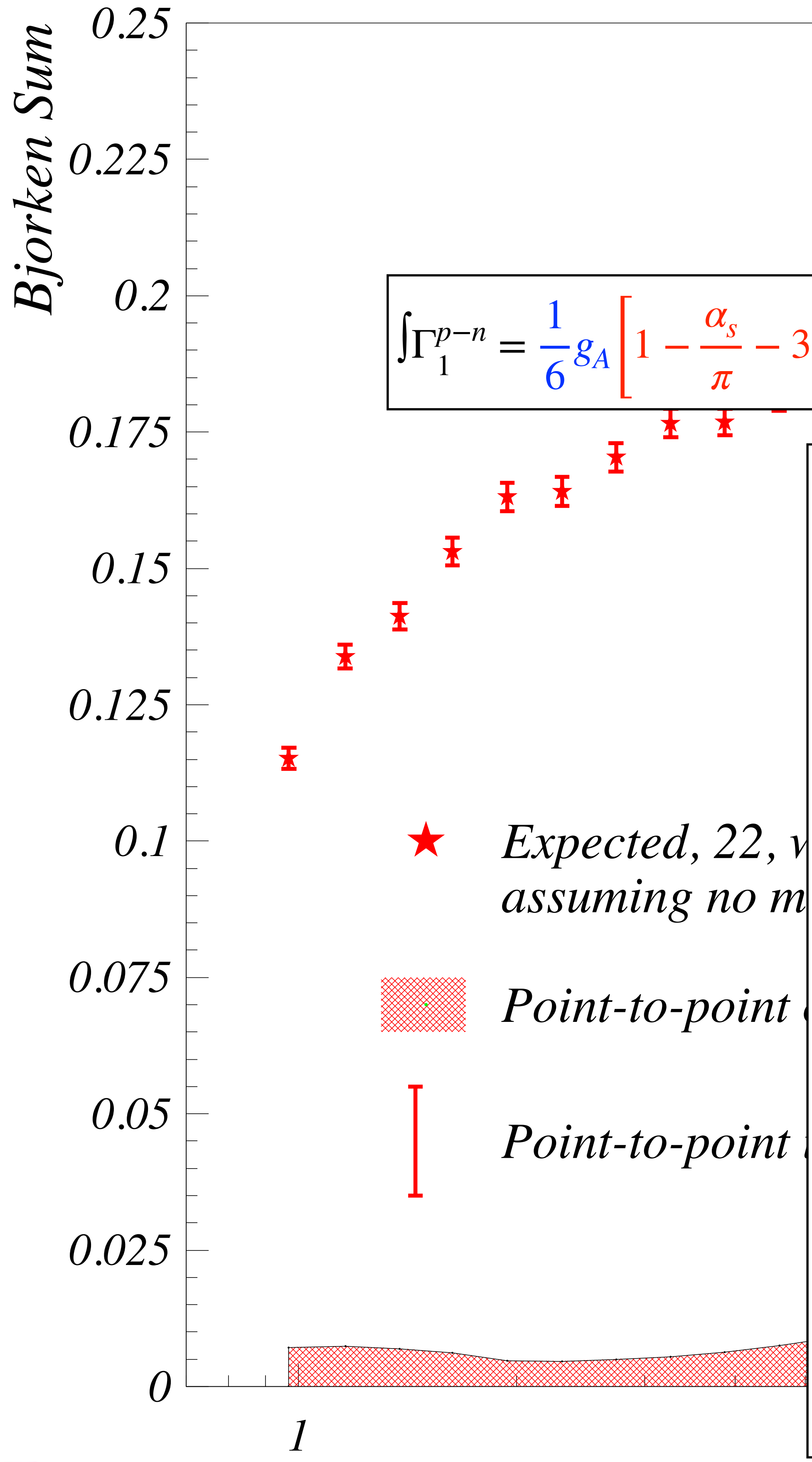


We separate the total <u>experimental</u> uncertainty (i.e. excluding the low-x error) in point-to-point correlated and uncorrelated contributions, assuming that 40% of the total uncertainty is point-to-point correlated (as obtained for EG1dvcs Bjorken sum analysis).

Low-x uncertainty is assumed to be fully point-to-point correlated.

(The above assumptions are not crucial for the extraction of α_s. Also, the proper separation would be determined from analysis of the actual 22 GeV data, without assumption.)

Extraction of $\alpha_s(M_Z)$



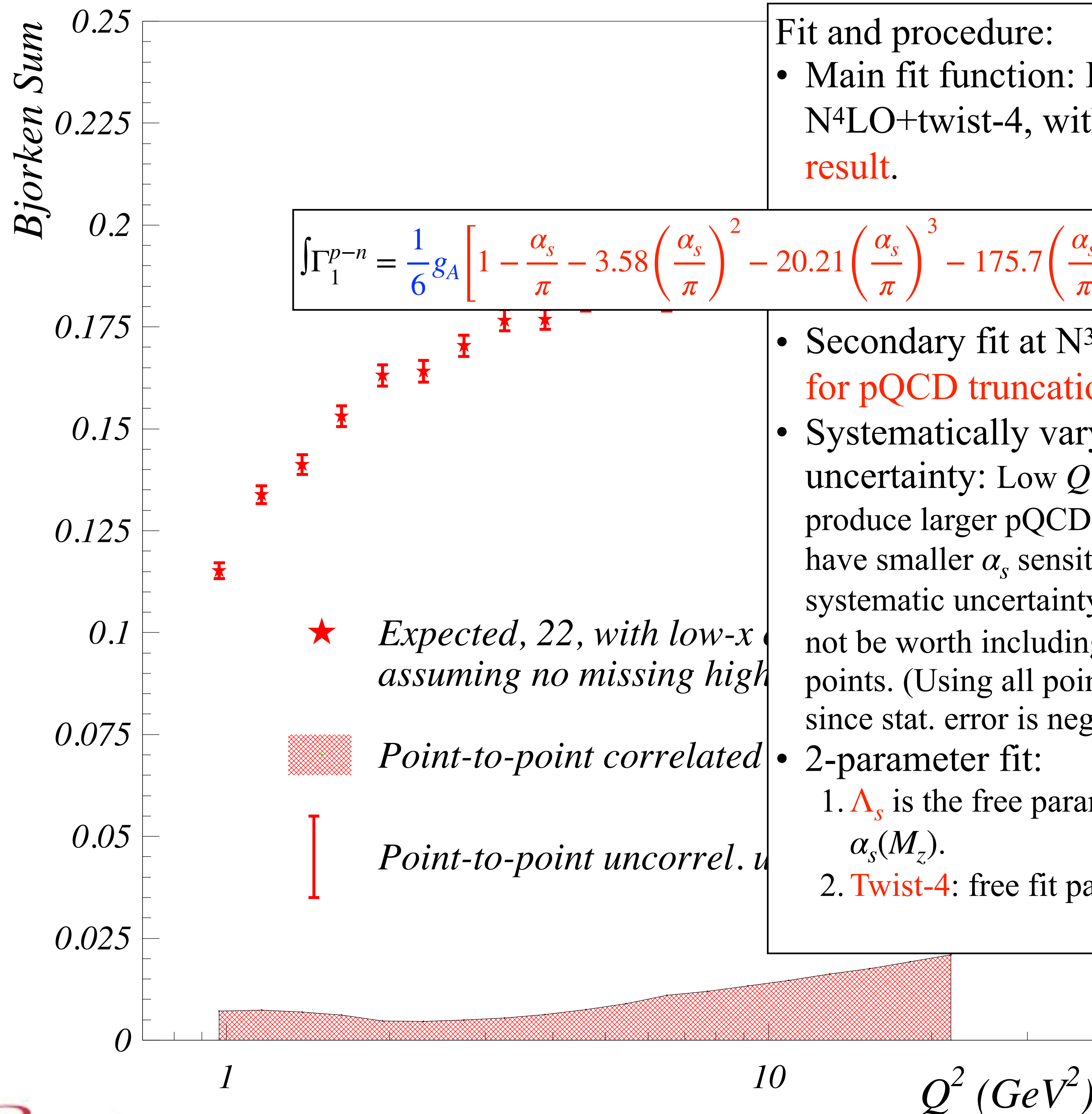
Fit and procedure:

- Main fit function: Bjorken sum approximant at N⁴LO+twist-4, with α_s at 4-loop (i.e. β_3), for main result.

$$\int\Gamma_1^{p-n} = \frac{1}{6}g_A\left[1-\frac{\alpha_s}{\pi}-3.58\left(\frac{\alpha_s}{\pi}\right)^2-20.21\left(\frac{\alpha_s}{\pi}\right)^3-175.7\left(\frac{\alpha_s}{\pi}\right)^4\right]+\frac{M^2}{Q^2}\left[a_2(\alpha_s)+4d_2(\alpha_s)+4f_2(\alpha_s)\right]$$

$$\begin{aligned}
\alpha_s^{\overline{\rm MS}}(Q) = \frac{4\pi}{\beta_0\ln(Q^2/\Lambda_s^2)}\Bigg[1 &- \frac{\beta_1}{\beta_0^2}\frac{\ln(\ln(Q^2/\Lambda_s^2))}{\ln(Q^2/\Lambda_s^2)}+\\
&\frac{\beta_1^2}{\beta_0^4\ln^2(Q^2/\Lambda_s^2)}\left(\ln^2(\ln(Q^2/\Lambda_s^2))-\ln(\ln(Q^2/\Lambda_s^2))-1+\frac{\beta_2\beta_0}{\beta_1^2}\right)+\\
&\frac{\beta_1^3}{\beta_0^6 ln^3(Q^2/\Lambda_s^2)}\Bigg(-\ln^3(\ln(Q^2/\Lambda_s^2))+\frac{5}{2}\ln^2(\ln(Q^2/\Lambda_s^2))+2\ln(\ln(Q^2/\Lambda_s^2))-\\
&\frac{1}{2}-3\frac{\beta_2\beta_0}{\beta_1^2}\ln(\ln(Q^2/\Lambda_s^2))+\frac{\beta_3\beta_0^2}{2\beta_1^3}\Bigg)+\frac{\beta_1^4}{\beta_0^8 ln^4(Q^2/\Lambda_s^2)}\Bigg(\ln^4(\ln(Q^2/\Lambda_s^2))-\\
&\frac{13}{3}\ln^3(\ln(Q^2/\Lambda_s^2))-\frac{3}{2}\ln^2(\ln(Q^2/\Lambda_s^2))+4\ln(\ln(Q^2/\Lambda_s^2))+\frac{7}{6}+\\
&\frac{7}{6}+\frac{3\beta_2\beta_0}{\beta_1^2}\left(2\ln^2(\ln(Q^2/\Lambda_s^2))-\ln(\ln(Q^2/\Lambda_s^2))-1\right)-\\
&\frac{\beta_3\beta_0^2}{\beta_1^3}\left(2\ln(\ln(Q^2/\Lambda_s^2))+\frac{1}{6}\right)\Bigg]
\end{aligned}$$

Extraction of $\alpha_s(M_Z)$



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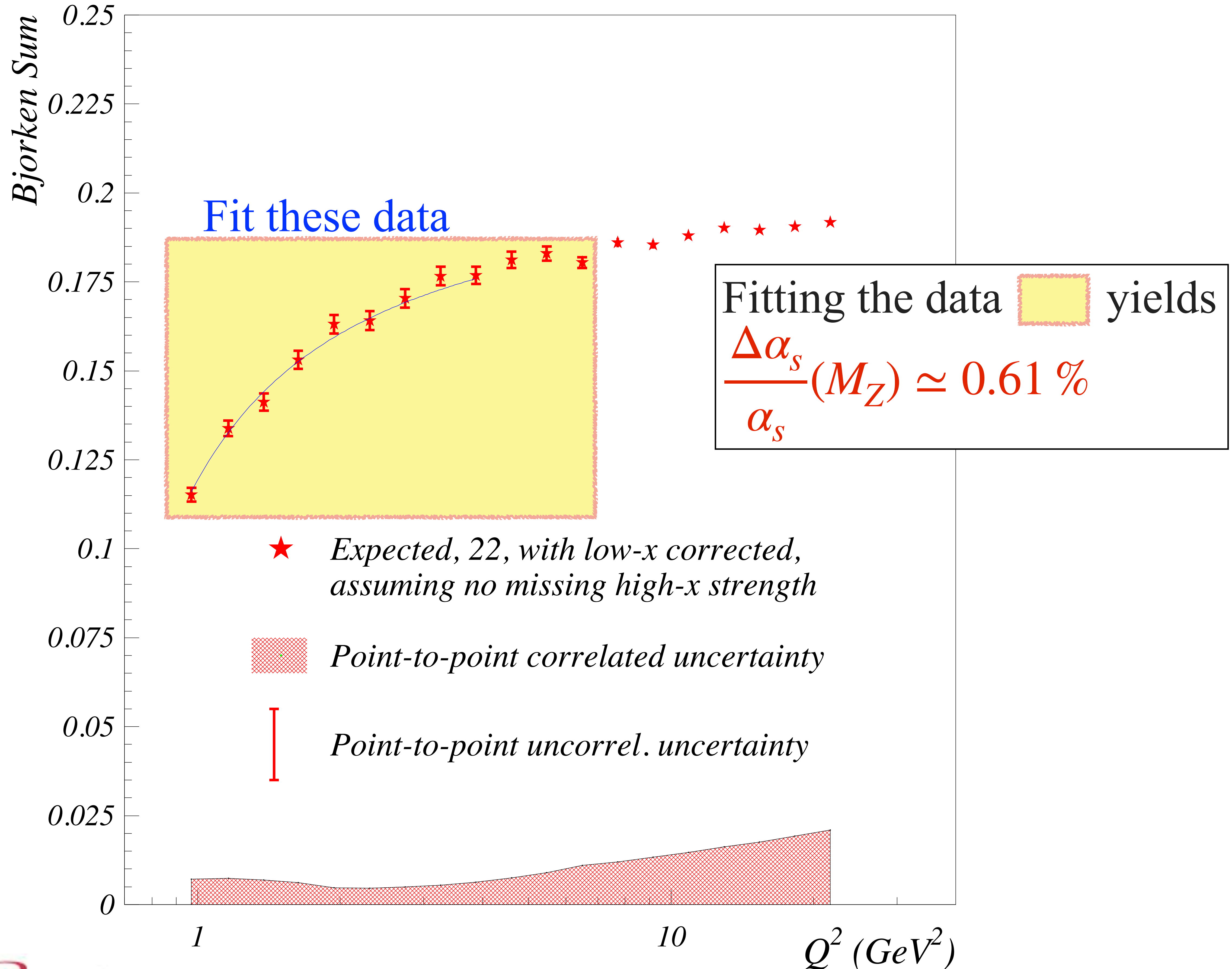
$$\int \Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4\right] + \frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right]$$

- Secondary fit at N³LO+twist-4 and α_s at 3-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2 range to minimize total uncertainty: Low Q^2 points have high α_s sensitivity but produce larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity and larger experimental systematic uncertainty but smaller pQCD error. ⇒May not be worth including the lowest and/or highest Q^2 points. (Using all points for statistics sake is not worth it, since stat. error is negligible.)
- 2-parameter fit:
  1. Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
  2. Twist-4: free fit parameter.

Extraction of $\alpha_s(M_Z)$



Jefferson Lab
Thomas Jefferson National Accelerator Facility
Exploring the Nature of Matter