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Phenomenology of helicity TMD: status and challenges

Matteo Cerutti

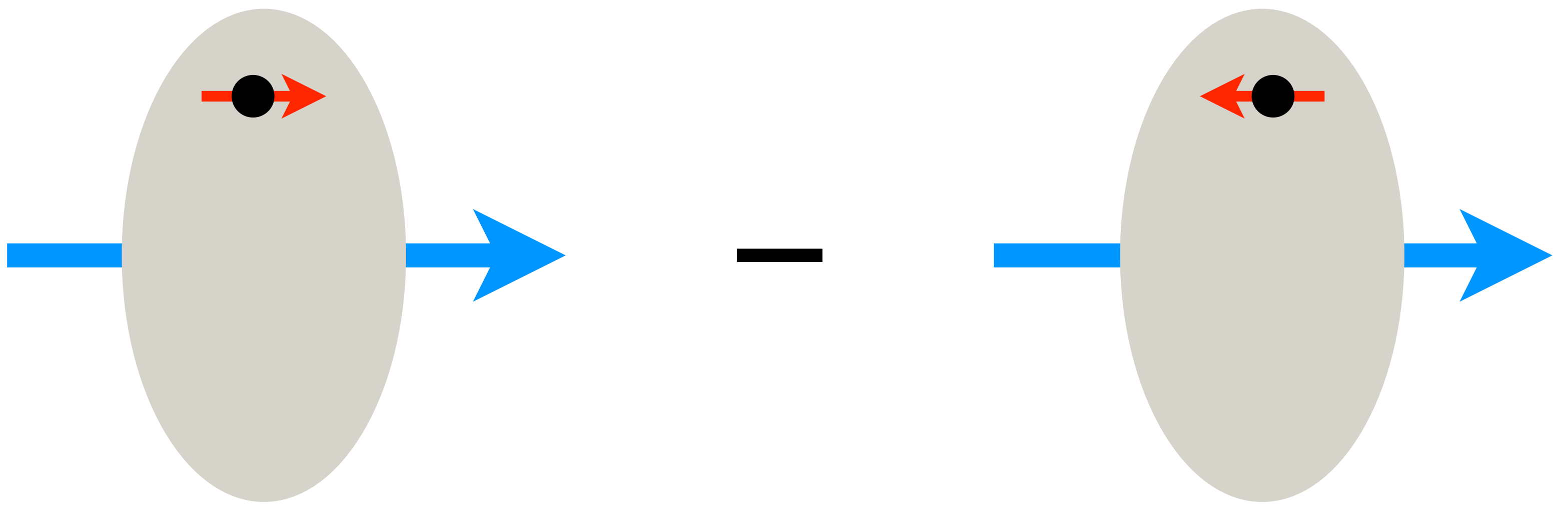
MAP Collaboration

A. Bacchetta, A. Bongallino, M. Radici, L. Rossi

arXiv: 2409.18078

Helicity distribution

$$g_1^q(x) = q^+ - q^-$$

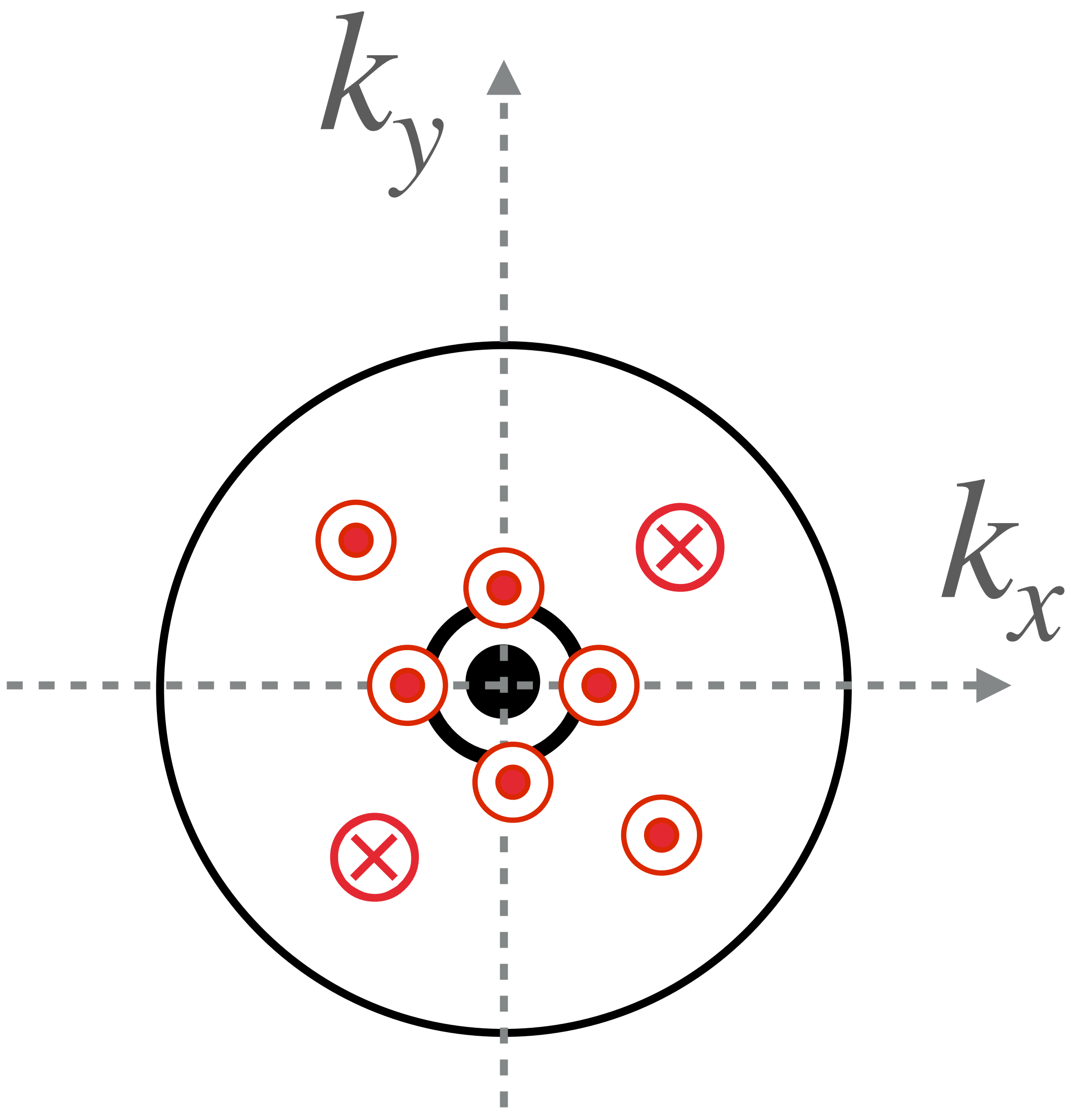


Quark Polarization

Nucleon Pol.		U	L	T
	U	$f_1(x)$		
	L		$g_1(x)$	
	T			$h_1(x)$

Helicity TMD distribution

$$g_1^q(x, k_\perp) = q^+ - q^-$$



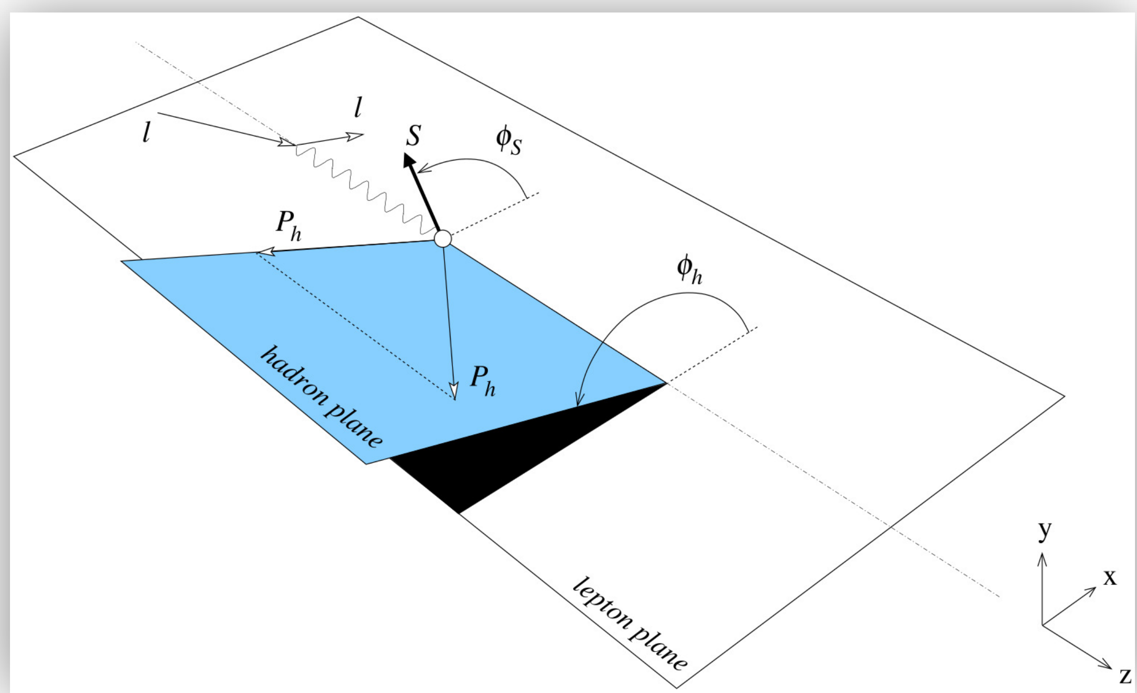
- How the polarization of the proton reflects on its internal structure in 3 dimensions?
- How the polarization of the quark **distorts** their transverse momentum?
- Do quarks with spin parallel to the proton's spin have smaller or larger transverse momentum?

Experimental Observables

Analysis of longitudinally polarized process

SIDIS

$$\ell^{\rightleftarrows}(l) + N^{\leftrightarrows}(P) \rightarrow \ell(l') + h(P_h) + X$$



DOUBLE SPIN ASYMMETRY

$$A_1 = \frac{d\sigma^{\rightarrow\leftarrow} - d\sigma^{\rightarrow\rightarrow} + d\sigma^{\leftarrow\rightarrow} - d\sigma^{\leftarrow\leftarrow}}{d\sigma^{\rightarrow\leftarrow} + d\sigma^{\rightarrow\rightarrow} + d\sigma^{\leftarrow\rightarrow} + d\sigma^{\leftarrow\leftarrow}}$$

A. Bacchetta et al., Phys.Rev.D 70 (2004), 117504

M. Diehl and S. Sapeta, Eur. Phys. J. C 41, 515 (2005)

Interpretation in terms of TMDs

TMD factorization

$$A_1(x, z, Q, |\boldsymbol{P}_{hT}|) = \frac{\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T|^2 J_0\left(\frac{|\boldsymbol{b}_T||\boldsymbol{P}_{hT}|}{z}\right) \hat{g}_1^a(x, |\boldsymbol{b}_T|^2, Q)\, \hat{D}_1^{a\to h}(z, |\boldsymbol{b}_T|^2, Q)}{\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T|^2 J_0\left(\frac{|\boldsymbol{b}_T||\boldsymbol{P}_{hT}|}{z}\right) \hat{f}_1^a(x, |\boldsymbol{b}_T|^2, Q)\, \hat{D}_1^{a\to h}(z, |\boldsymbol{b}_T|^2, Q)}$$

- Large energy scale $Q^2 \gg M^2$
- Small transverse momentum $q_T^2 \ll Q^2$

$\Longrightarrow$ Experimental observables in terms of universal objects

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unpolarized TMDs from MAP22 extraction

MAP Collaboration, Bacchetta et al., JHEP 10 (2022)

Parameterization

Collins, Soper, Sterman, Nucl. Phys. B (1985)
Collins, *Foundations of perturbative QCD* (2011)

The evolution of the TMDs follows the CSS approach **consistently**:

$$\hat{f}_1(x, |\boldsymbol{b}_T|^2, Q) = \left[C^f \otimes f_1\right](x, b_\star(|\boldsymbol{b}_T|^2))\, f_{NP}(x, |\boldsymbol{b}_T|^2, Q_0)\, e^{S(\mu_{b_\star}^2, Q^2)}\, e^{g_K(\boldsymbol{b}_T)\ln(Q^2/Q_0^2)}$$

Analogously for $D_1(z, |\boldsymbol{b}_T|^2, Q)$

$$\hat{g}_1(x, |\boldsymbol{b}_T|^2, Q) = \left[C^g \otimes g_1\right](x, b_\star(|\boldsymbol{b}_T|^2))\, g_{NP}(x, |\boldsymbol{b}_T|^2, Q_0)\, e^{S(\mu_{b_\star}^2, Q^2)}\, e^{g_K(\boldsymbol{b}_T)\ln(Q^2/Q_0^2)}$$

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MAP Collaboration, Bacchetta et al., JHEP 10 (2022)

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$$g_{NP}(x, \boldsymbol{k}_\perp^2, Q_0) = f_{NP}^{MAP22}(x, \boldsymbol{k}_\perp^2, Q_0)\, \frac{e^{-\frac{k_\perp^2}{\omega_1(x)}}}{k_{norm}(x)}$$

Simple Gaussian × MAP22

Positivity constraint

$$g_{NP}(x, \boldsymbol{k}_\perp^2, Q_0) = f_{NP}^{MAP22}(x, \boldsymbol{k}_\perp^2, Q_0)\, \frac{e^{-\frac{k_\perp^2}{\omega_1(x)}}}{k_{norm}(x)}$$

- Proportional to f_{NP}^{MAP22}
- x-dependent

$k_{\text{norm}}(x) \rightarrow \int d^2\boldsymbol{k}_\perp\, g_{NP} = 1$

$\omega_1(x) \rightarrow$ crucial to satisfy $|g_1| \leq f_1$

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$$\frac{g_1(x, \boldsymbol{k}_\perp^2, Q_0)}{f_1(x, \boldsymbol{k}_\perp^2, Q_0)} = \frac{g_1(x, Q_0)}{f_1(x, Q_0)} \frac{e^{-\frac{k_\perp^2}{\omega_1(x)}}}{k_{norm}(x)} \xrightarrow{|\boldsymbol{k}_\perp| \to 0} \infty$$

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$$\frac{g_1(x, Q_0)}{f_1(x, Q_0)} \frac{1}{k_{norm}(x)} \leq 1 \longrightarrow \omega_1(x) = f_{pos.}(x) + N_{1g}^2 \frac{(1-x)^{\alpha_{1g}^2} x^{\sigma_{1g}}}{(1-\hat{x})^{\alpha_{1g}^2} \hat{x}^{\sigma_{1g}}}$$

Phenomenology: STATUS

Airapetian et al. (HERMES), Phys. Rev. D (2019)

Experiment	$N_{\rm dat}$	$\chi^2_{\rm NLL}/N_{\rm dat}$	$\chi^2_{\rm NNLL}/N_{\rm dat}$
HERMES ($d \to \pi^+$)	47	1.34	1.30
HERMES ($d \to \pi^-$)	47	1.10	1.08
HERMES ($d \to K^+$)	46	1.26	1.25
HERMES ($d \to K^-$)	45	0.93	0.89
HERMES ($p \to \pi^+$)	53	1.17	1.21
HERMES ($p \to \pi^-$)	53	0.86	0.86
Total	291	1.11	1.09

- MAP22 kinematic cuts
- 291 fitted data points
- Perturbative order: **NLO**

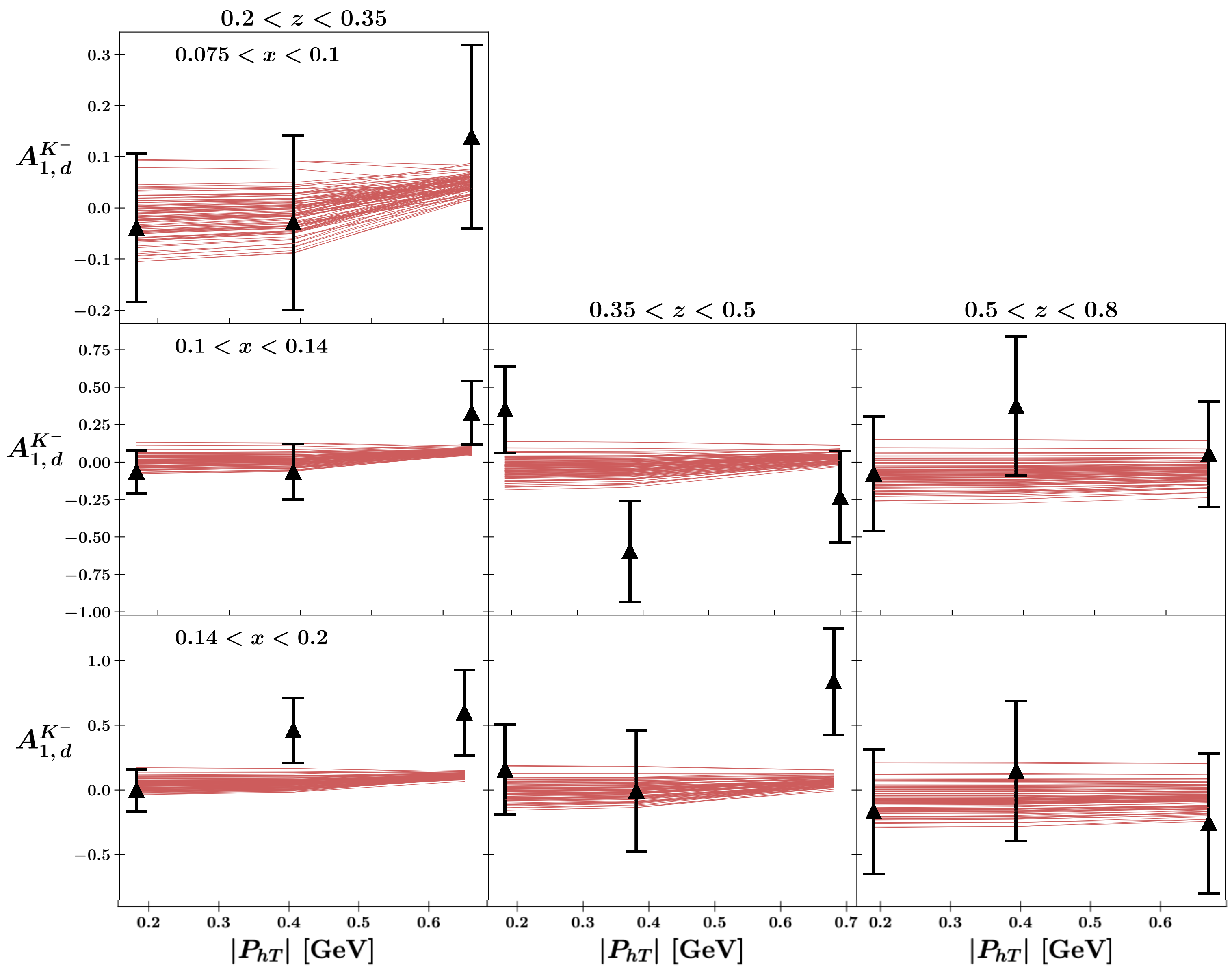
Highest possible

since C^g known up to NLO

Gutiérrez-Reyes et al., Phys. Lett. B (2017)

- Collinear PDFs: NNPDFPol, MMHT, DSS
- Perturbative accuracy: **NLL** & **N2LL**
- 3 fitted parameters
- Error analysis with bootstrap method

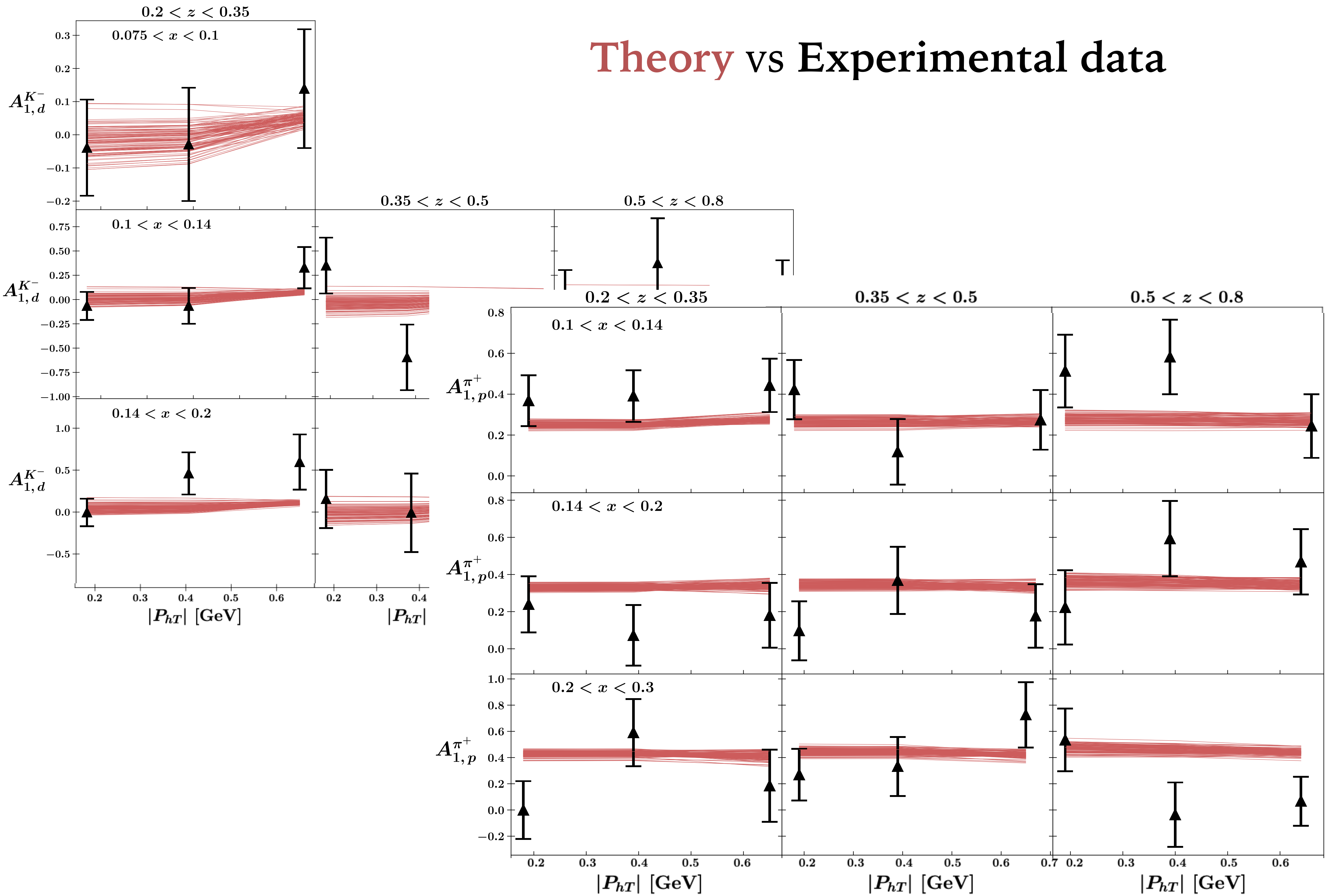
Phenomenology: STATUS



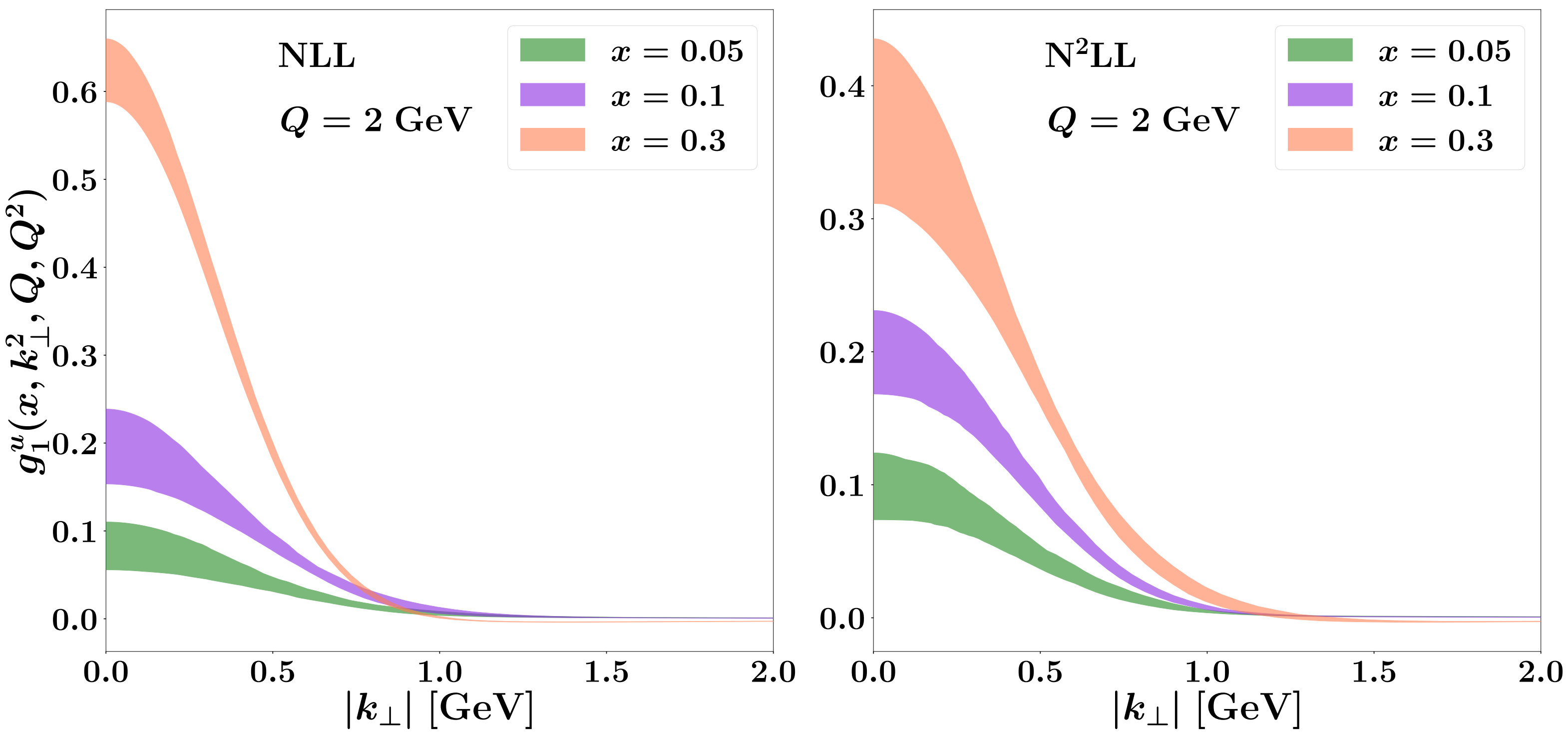
Experimental data

Phenomenology: STATUS

Theory vs Experimental data

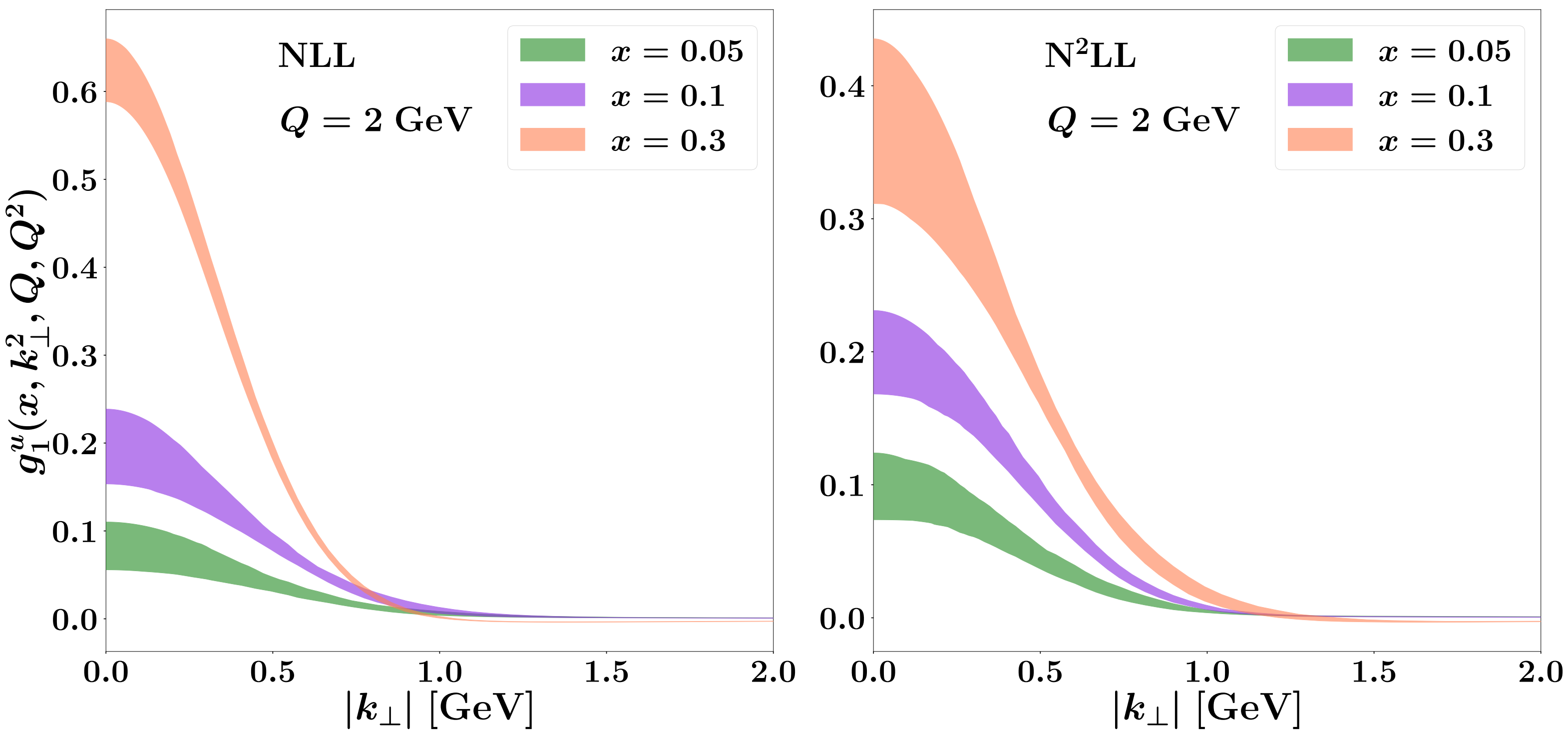


Phenomenology: STATUS



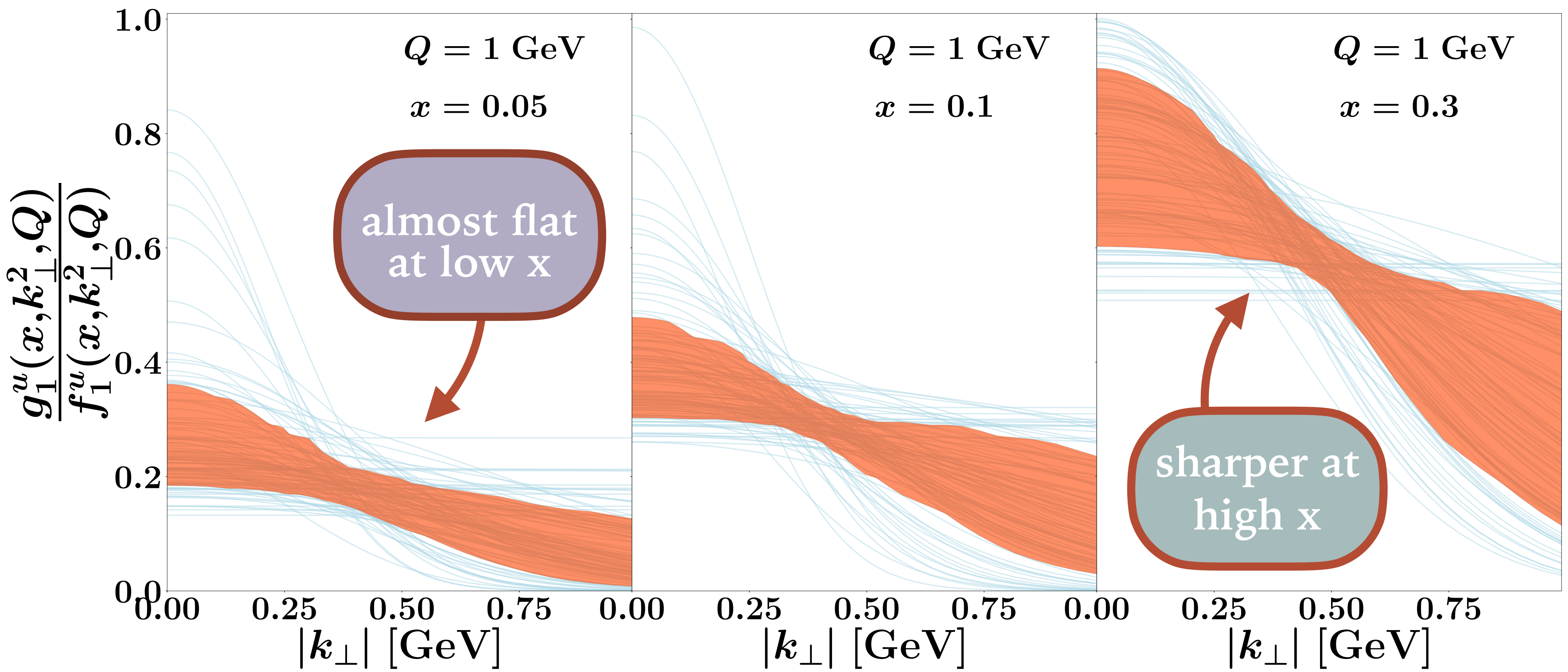
Extracted helicity TMDs

Phenomenology: STATUS

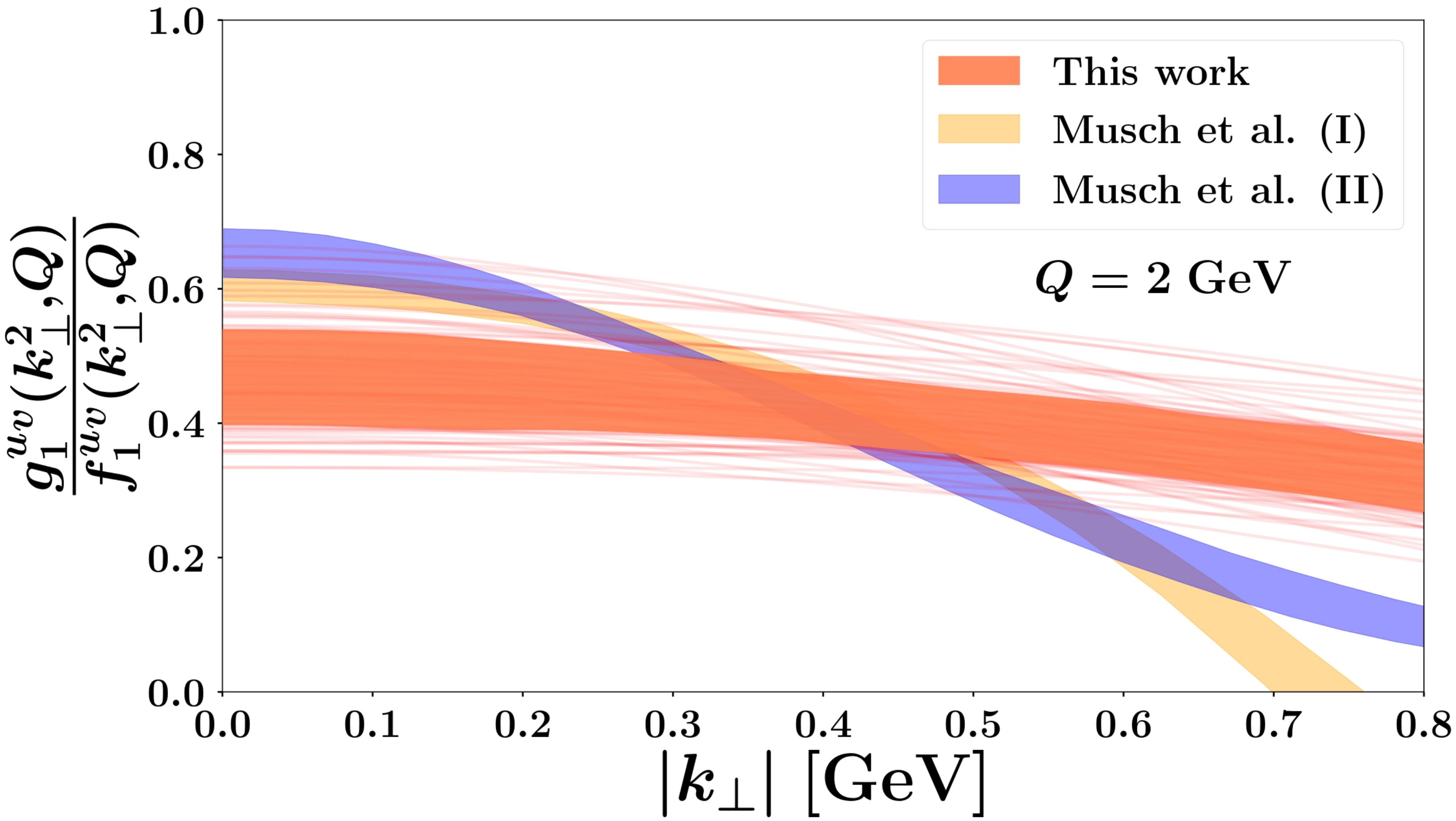


Extracted helicity TMDs

Helicity ratio



Phenomenology: STATUS



Comparison to lattice calculation

Musch et al., Phys. Rev. D (2011)

- g_1/f_1 TMD ratio for u_v integrated over x , at NNLL
- Yellow and blue bands correspond to two lattice predictions
- Milder slope but fair agreement

Phenomenology: CHALLENGES

- Not many constraints on fitted parameters

Parameters	N_{1g}	α_{1g}	σ_{1g}
NLL	0.70 ± 0.54	27.81 ± 27.70	0.42 ± 0.86
NNLL	0.87 ± 0.72	6.73 ± 6.58	3.04 ± 3.09

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NLL	0.70 ± 0.54	27.81 ± 27.70	0.42 ± 0.86
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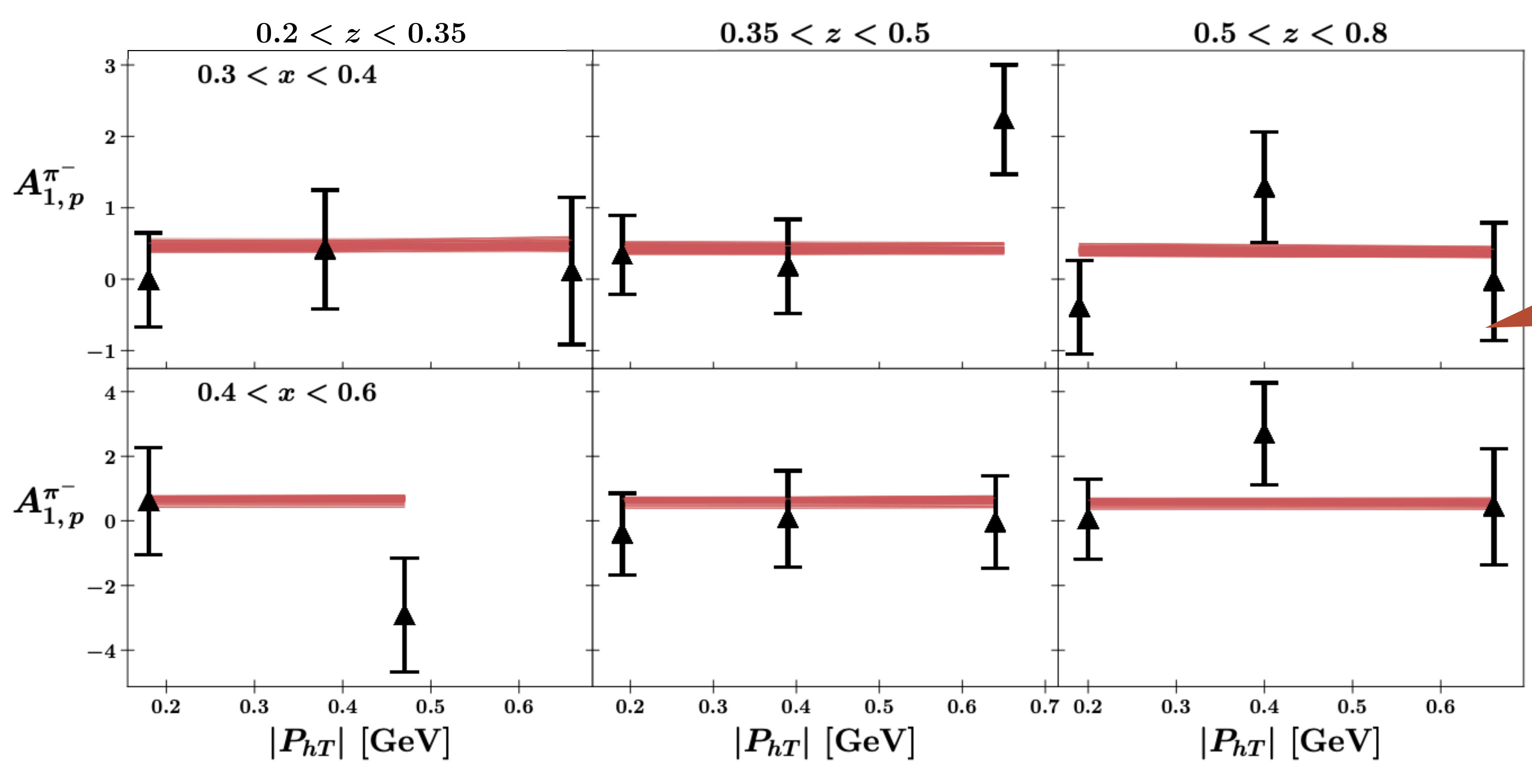
- Experimental asymmetries ≥ 1 in several bins



What does it happen at large x?

Need of **precise** data from JLab 22 GeV upgrade

Backup

MAP TMD fitting framework

https://github.com/MapCollaboration/NangaParbat

README.md

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

MAPTMD22 global fit: features

- Global analysis of Drell-Yan and SIDIS data sets: ***2031*** data points
- Perturbative accuracy: N^3LL^-
- ***Normalization prefactor*** for SIDIS observables
- Number of fitted parameters: ***21***
- Agreement with data: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

Structure of a TMD

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

Collinear extractions

b$_*$-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*}) \qquad : \mathrm{A}$$

Perturbative TMD at the initial scale

Perturbative

$$\times \exp\left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

Parameterization

Non-perturbative part of the TMD

J. Collins, "Foundation of Perturbative QCD"

MAP22: Perturbative accuracy

Resummation of large logs

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} R_{\text{N}^k\text{LL}}$$

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(\mu)}{4\pi}\right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n,2n-k)} \qquad L = \ln\left(\frac{\mu^2}{\mu_b^2}\right)$$

Accuracy	H and C	K and γ_F	γ_K	PDF/FF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL’	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL’	2	2	3	NNLO
N^{3}LL$^-$	**2**	**3**	**4**	**NNLO/NLO**
N^{3}LL	2	3	4	NNLO
N^{3}LL’	3	3	4	N^{3}LO

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)
TMD handbook, Boussarie, et al., 2023

MAP22: NP parametrization

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)
Bacchetta, Conti, Radici, PRD 78 (2008)
Pasquini, Cazzaniga, Boffi, PRD 78 (2008)
Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
Burkardt, Pasquini, EPJA (2016)
Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

11 parameters for TMD PDF

+ 1 for NP evolution + 9 for TMD FF

= 21 free parameters

MAP22: Kinematic cuts

Drell-Yan

Fixed-target low-energy DY

RHIC data

LHC and Tevatron data

$9 \lesssim Q \lesssim 11$ GeV excluded (Υ resonance)

$q_T|_{\max} = 0.2Q$

484 experimental points

SIDIS

HERMES data

COMPASS data

$Q > 1.3$ GeV

$0.2 < z < 0.7$

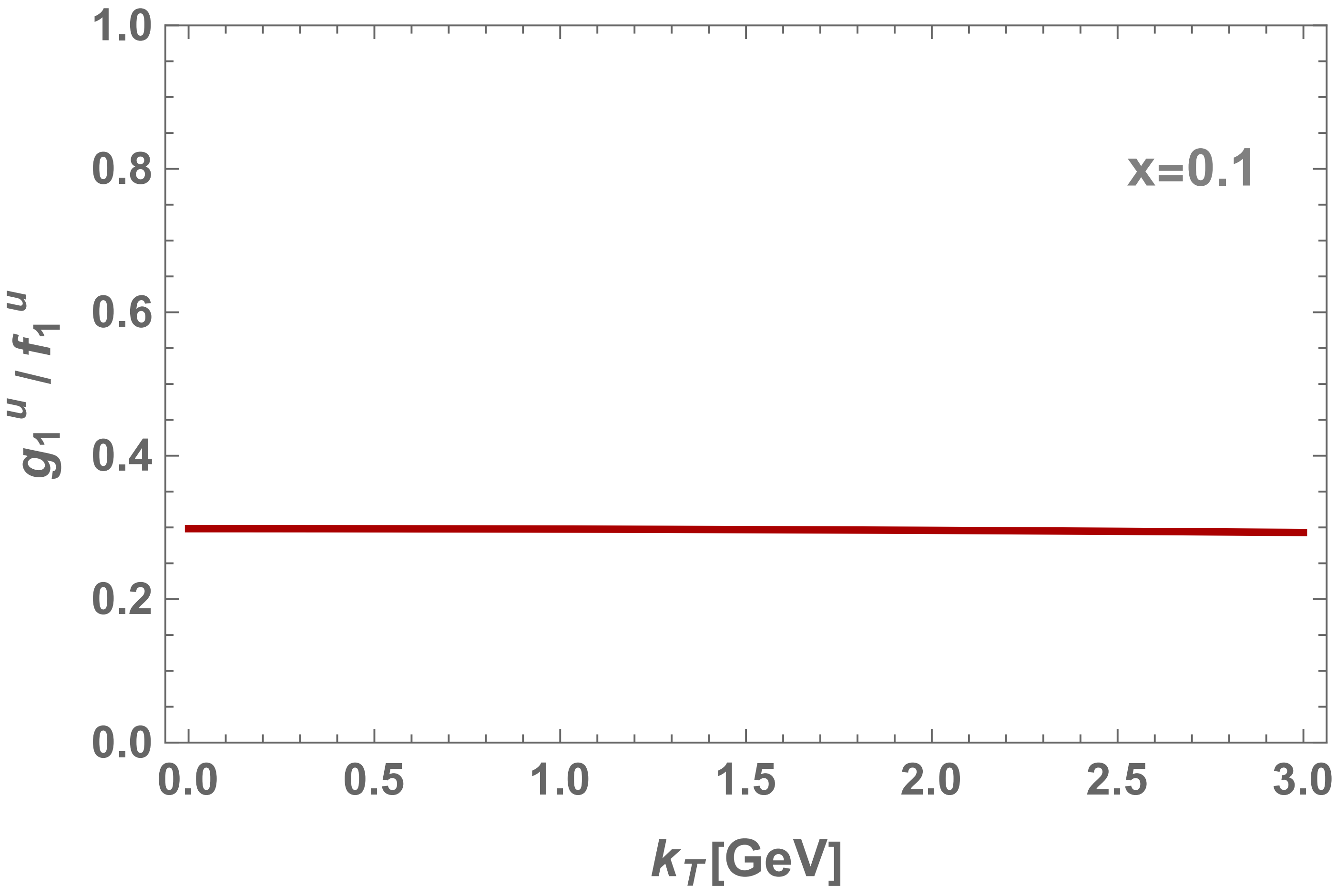
$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$

1547 experimental points

Role of Gaussian

$$g_{NP}(x, \boldsymbol{k}_\perp^2, Q_0) = f_{NP}^{MAP22}(x, \boldsymbol{k}_\perp^2, Q_0)\ \frac{e^{-\frac{k_\perp^2}{\omega_1(x)}}}{k_{norm}(x)}$$

$$\omega_1(x) \to \infty \implies g_1 \simeq f_1$$



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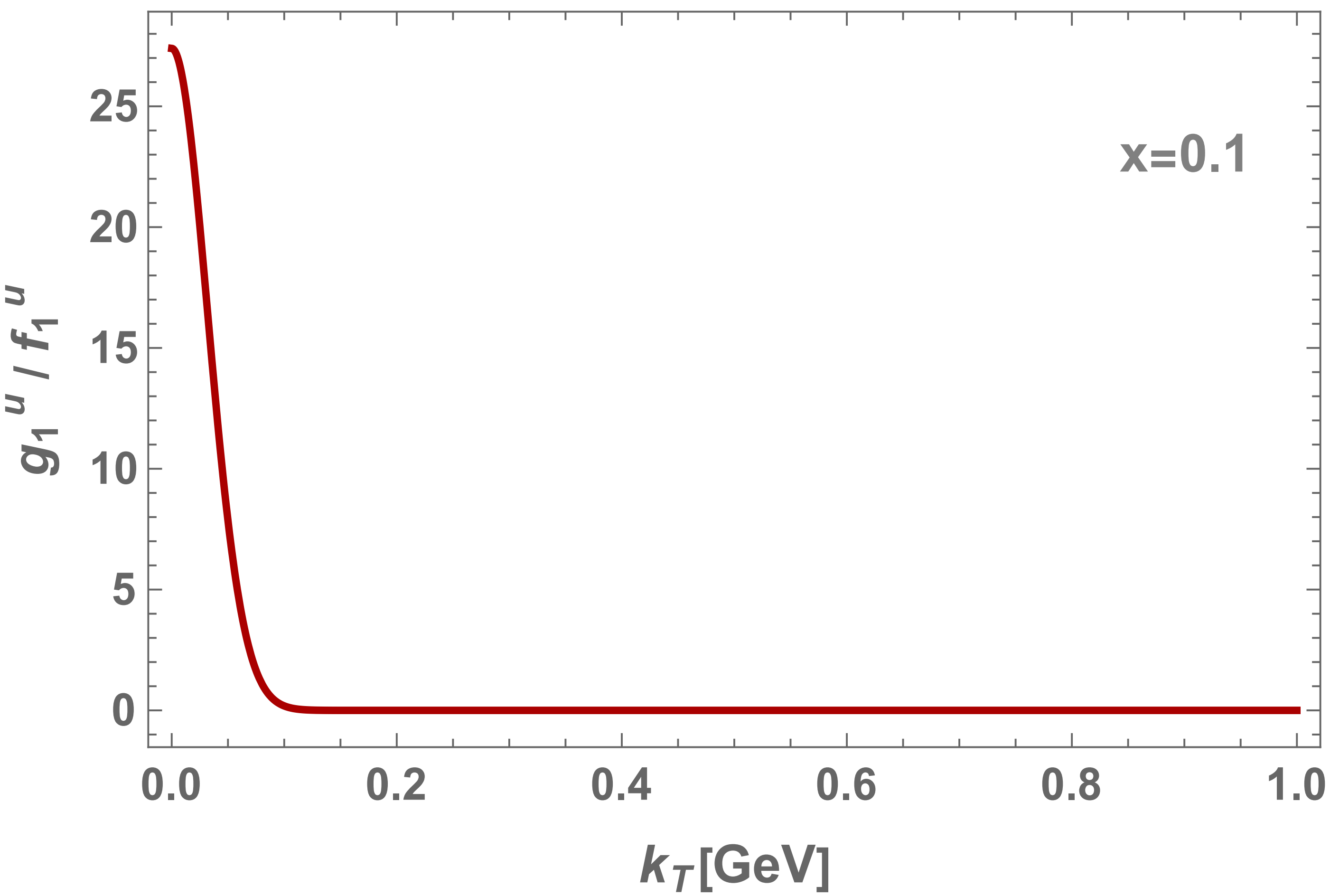
$\omega_1(x) \to 0 \implies$ Positivity broken at small kT



Positivity constraint

- $f_{pos.}(x)$ guarantees the positivity bound

  at TMD level

  $10^{-4} \leq x \leq 0.7$

$$f_{pos.}(x) \approx c + h^2 e^{-\frac{(x-\mu)^2}{\sigma^2}}$$

(interpolation function)

- $N_{1g}, \alpha_{1g}, \sigma_{1g}$ are the **free parameters** of the fit, $\hat{x} = 0.1$

$$\omega_1(x) = f_{pos.}(x) + N_{1g} \frac{(1-x)^{\alpha_{1g}^2} x^{\sigma_{1g}}}{(1-\hat{x})^{\alpha_{1g}^2} \hat{x}^{\sigma_{1g}}}$$

Propagation of errors

$f_1(x) \rightarrow$ MMHT2014 set, $D_1(z) \rightarrow$ DSS14, DSS17 sets

$g_1(x) \rightarrow$ **NNPDFpol1.1:** 100 MC members

↓

100 replicas of A_1 data points to be fitted

i-th replica of $g_1(x)$ and the extracted g_1 TMD associated with the **same replica** of unpolarized TMDs

↓

Uncertainty of extracted **collinear** PDF propagated onto **TMD**'s uncertainty