Dispersive Analysis of the Primakoff Reaction $\gamma K \to K\pi$

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7^{th} KLF Collaboration meeting

HISKP (Theorie)
Bonn University

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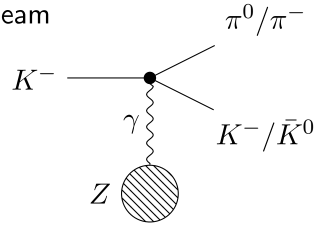
[Dax, DS and Kubis; Eur. Phys. J. C **81** (2021) 221]

Motivation

- pion production in the Coulomb field of a heavy nucleus
- $\gamma^{(*)}\pi \to \pi\pi$ investigated [Hoferichter et al., 2012, 2017], [Niehus et al., 2021]
- COMPASS++ experiment planned (+ OKA experiment)
- upgrade from pion beam to charged-kaon beam



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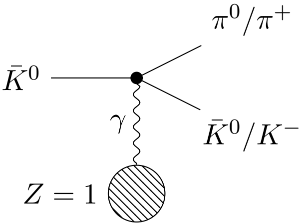

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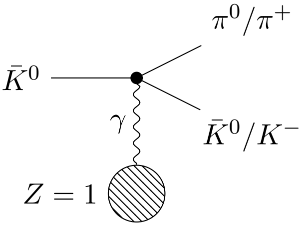
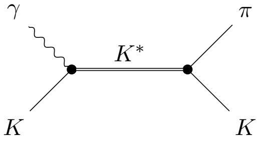
- K_L-facility with neutral-kaon beam
- proton target $\to$ weaker Coulomb field
- combine knowledge about
  - resonances ($K^*(892)$) at higher energies (radiative couplings)
  - chiral anomaly at $s = t = u = 0$

$$F_{KK\pi\gamma} = \frac{e}{4\pi^2 F_\pi^3}$$





Chiral anomaly

- quantum effect, model-independent prediction for QCD at low energies
- Wess, Zumino, Witten [1971,1983] for processes with odd intrinsic parity
- prime example: $\pi^0 \to \gamma\gamma$
- additionally three-pseudoscalar-photon processes
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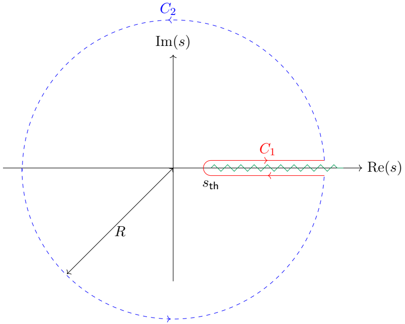
  $\gamma\pi \to \pi\pi$ and $\gamma K \to K\pi$
- prediction at zero energies $s = t = u = 0$ and zero masses
- only depend on e, F_π (and F_K)
- leading ChPT prediction, low-energy theorems

Dispersive formalism

- analyticity ($\simeq$ causality) & Cauchy's integral formula

$$f(s) = \frac{1}{2\pi i} \oint_{\partial U} \frac{f(s')}{s' - s} \mathrm{d}s'$$



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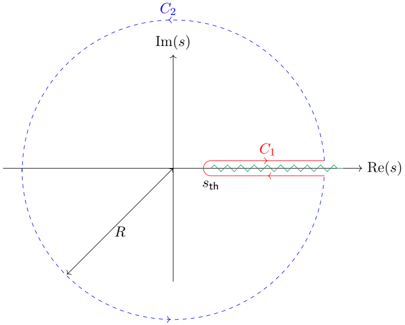

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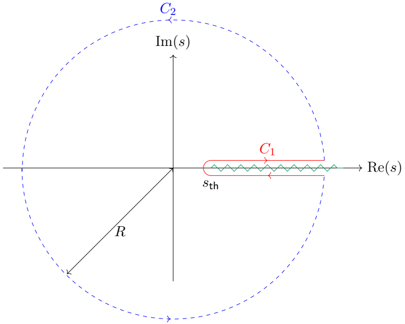

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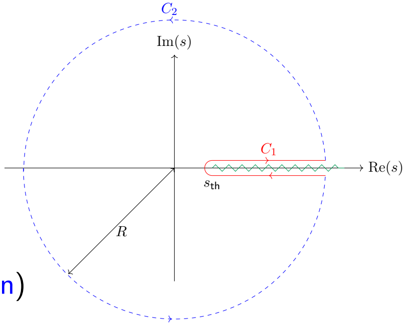
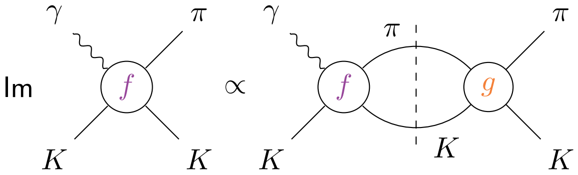


- unitarity relation ($\simeq$ prob. conservation)

$$\mathrm{Im}\, f(s) \propto f(s)\cdot g^*(s)$$



- obeys Watson's final state theorem [Watson, 1954]

Homogeneous Omnès problem

- $K\pi$ P-wave phase shift from [Peláez and Rodas, 2016] $\rightarrow$ no S wave
- $I = 1/2$ phase shift contains $K^*(892)$, $K^*(1410)$ and $K^*(1680)$
- very well constrained up to the $K\eta$ threshold
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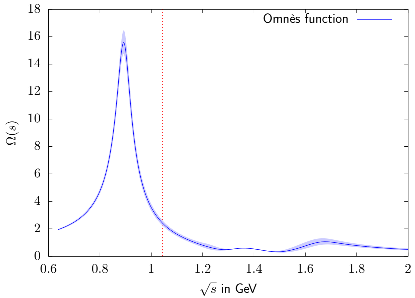

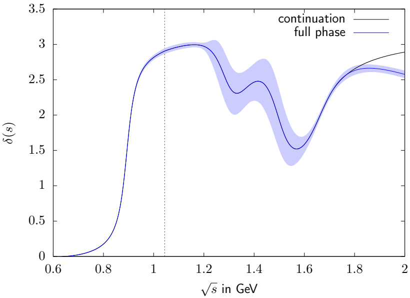

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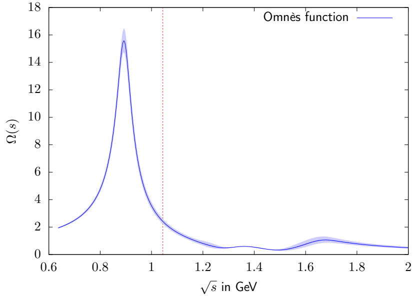

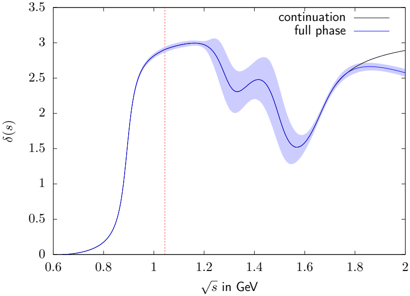

- $I = 3/2$ phase shift is $|\delta(s)| < 3^\circ$ for $s < (1.74\,\text{GeV})^2$
- approximate it with $\delta(s) = 0 \Rightarrow \Omega(s) = 1$

Reconstruction theorem

- separate kinematic prefactor $\mathcal{M} = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}p_1^{\nu}p_2^{\alpha}p_0^{\beta}\mathcal{F}(s,t,u)$
- decompose scalar amplitude $\mathcal{F}(s,t,u)$ using isospin and $s \leftrightarrow u$ symmetry into single variable amplitudes
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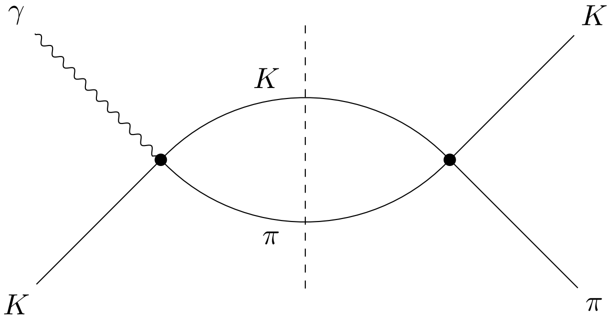

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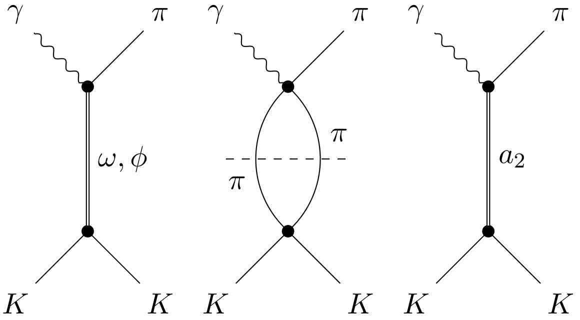

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- dispersion integral containing $\gamma\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ amplitudes
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- D-wave a_2 resonance via tensor meson dominance

Inhomogeneous Omnès problem

- use reconstruction theorems

$$f(s) = \mathcal{F}(s) + \widehat{\mathcal{F}}(s)$$

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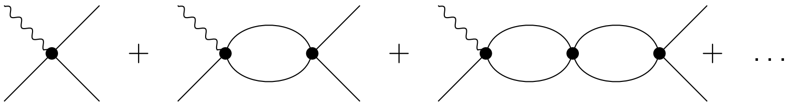
- representation of homogeneous solution $\Omega(s)$



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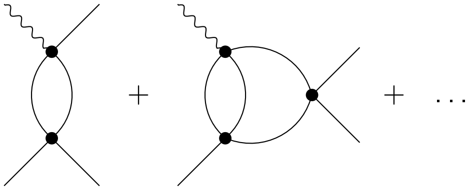

Basis functions

- solution depends on subtraction polynomials linearly
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- reduces computational effort dramatically
- fit/matching can be done using the basis functions
- can be provided for experimental analysis

Subtraction schemes

- how many free parameters are there?
- we find

$$\mathcal{F}^{(i)}(s) = \Omega(s)\left(P^{(i)}_{n-1}(s) + \frac{s^n}{\pi}\int_{s_{\text{th}}}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\widehat{\mathcal{F}}^{(i)}(s')\sin(\delta^{(i)}(s'))}{|\Omega(s')|(s'-s)}\right)$$

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- predict cross-section via available input of anomaly and coupling
- future goal: extract these quantities from data!

Matching: anomaly

- calculate chiral Wess–Zumino–Witten [1971,1983] anomaly

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- ω, ϕ amplitude: $\mathcal{G}^{(+)}(0) = 7.8(8)$ GeV^{-3} uncertainty dominated by $\omega \to K\bar{K}$ coupling

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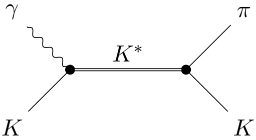

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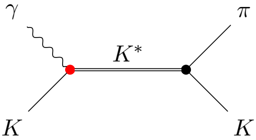

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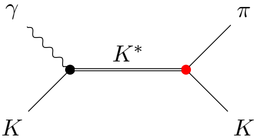
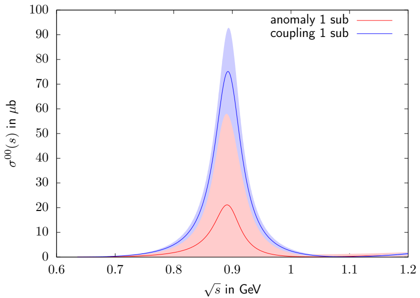
- $K\pi$ coupling and K^* pole position from Roy–Steiner analysis [Peláez, Rodas, Ruiz de Elvira, 2019]



Matching: anomaly or coupling

- minimal subtraction scheme
- fully determined by the anomaly or coupling



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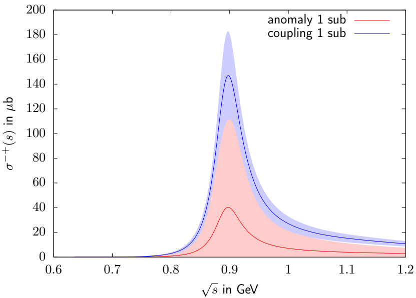
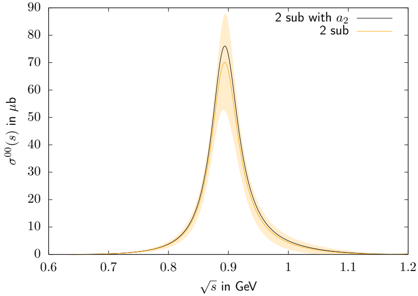


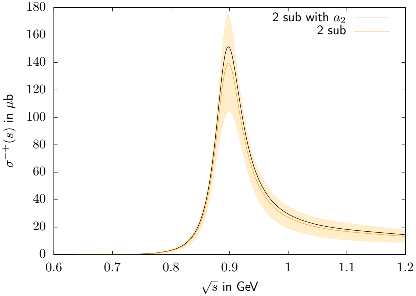
$$\gamma\bar{K}^0 \to K^-\pi^+$$

Matching: anomaly and coupling

- twice subtracted scheme with and without a_2 resonance



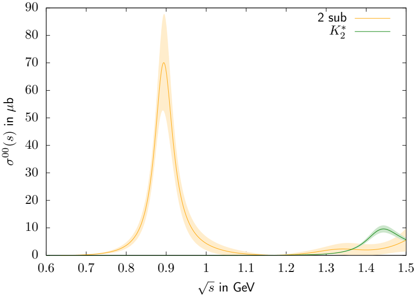
$\gamma \bar{K}^0 \to \bar{K}^0 \pi^0$



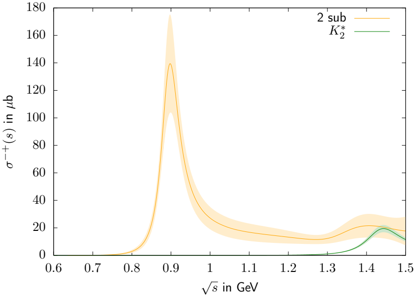
$\gamma \bar{K}^0 \to K^- \pi^+$

D-wave $K_2^*(1430)$

- use Lagrangians [Ecker and Zauner, 2007], [Plenter and Kubis, 2015]
- for neutral channel the radiative coupling is only an upper limit $\Gamma_{K_2^* \to K^0\gamma} < 5.4$ keV [Alavi-Harati et al. (KTeV), 2001]



$\gamma\bar{K}^0 \to \bar{K}^0\pi^0$



$\gamma\bar{K}^0 \to K^-\pi^+$

- D-wave relevant in charged channels above 1.35 GeV [Bacho, 2021]

Summary and outlook

- constructed a dispersive solution for the Primakoff reaction $\gamma K \to K\pi$ for all charge configurations
- input: fixed t-channels and $K\pi$ phase shift
- using the basis functions a fit to COMPASS++, KLF (or OKA) data is possible to determine the free parameters
- matching: use radiative couplings and chiral anomaly to predict the free parameters
- future goal: extract these quantities using a fit to data