Highlights of the Spectroscopy of Hyperons and Cascade Baryons

Annika Thiel¹ and Eberhard Klempt¹

¹Helmholtz–Institut für Strahlen– und Kernphysik, Universität Bonn, 53115 Bonn, Germany

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Abstract. We highlight topical issues in hyperon spectroscopy that will be accessible when a secondary beam of neutral kaons is created in Hall D at Jefferson Lab as proposed to the JLab Program Advisory Committee. The new beam will have a flux of $10^4 K_L/s$. The reaction products will be analyzed in the GlueX experimental setup. We point out which physical questions need to be answered and suggest a number of novel experiments. In particular we suggest to measure the presently disputed SU(3) structure of the $\Lambda(1405)$, to search for Pentaquarks mit exotic quantum numbers and to search for baryons belonging to the SU(6) 20-plet representation.

1 Introduction

Our present knowledge on the spectrum of Λ and Σ hyperons and of cascade baryons Ξ still relies on experimental data taken in 1960s and 1970s. The masses, widths and decay properties of light baryons carrying strangeness S = -1 or -2 derived from early analyses are collected in the bi-annual Review of Particle Physics [1]. Recent reanalyses of the old data have cleaned up the spectrum slightly and reported evidence for some new Λ and Σ hyperons [2,3,4,5,6,7,8].

The search for new hyperon resonances and to confirm less-known ones is important to establish the multiplet structure of excited baryons. In the sector of N and Δ resonances the first excitation band is completely known and well established, most states in the second excitation band are at least seen – even though some of them with fair evidence only –, ten states (out of 45) in the third band are known, and N and a Δ Regge trajectories exist up to states with orbital angular momentum L = 6. Our knowledge on the hyperon spectrum of Λ and Σ resonances is much poorer: even the first excitation band is not complete, and some states in the first band are known with little evidence. Only few states in the second excitation band have been reported so far. No Regge trajectory can be drawn with more than two states. Our knowledge on the cascade resonances is even worse: Apart from the ground states (in the SU(3) octet and decuplet), only one resonance has been reported with spin and parity.

In particular the Λ states are sensitive to details of quark models. Consider the two spin doublets with $J^P = 3/2^+ / 5/2^+$: $\Lambda(1890) / \Lambda(1820)$ and $\Lambda(2070) / \Lambda(2110)$. The low-mass doublet is interpreted in all quark models as SU(3) partner of N(1720) / N(1680). The high-mass doublet, however, is interpreted as partner of N(1880) / N(1900) in the celebrated Isgur-Karl model [9] that uses

an effective gluon exchange for the quark-quark interaction, in contrast to the Bonn model [10] that is based on instanton-induced interactions. This model predicts the A(2070) / A(2110) doublet as SU(3) singlet states. The latter two states have been reported in a recent Bonn-Gatchina analysis [8], with fair evidence only; the decay modes seem to favor their SU(3) singlet nature. The example demonstrates the sensitivity of the hyperon spectrum to the interaction between quarks in the confinement region.

Not only quark models are on the test bench. Modern approaches to hadron spectroscopy generate resonances from their decay products. A famous example is the $\Lambda(1405)$ region which is supposed to house two Λ and one or two Σ resonances with spin-parity $J^P = 1/2^-$. The well-known $\Lambda(1405)$ is seen with a large SU(3)-octet contribution, a new wider $\Lambda(1380)$ as largely SU(3) singlet. Quark models predict only one state in this mass region, the $\Lambda(1405)$ as SU(3) singlet state. Alternative approaches will be discussed below. In any case, sensitive experiments with high precison are required to resolve this conflict.

After a period of great enthusism, the interest in searches for pentaquarks had degraded considerably even though the discussion continued (see, e.g. [11]). The discovery of three $J/\psi p$ structures observed in 246.000 $\Lambda_b^0 \rightarrow J/\psi p K^$ events by the LHCb collaboration, the pentaquark candidates $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ [12], has reinitiated the interest. A search the pentaquark candidates in the photoproduction reaction $\gamma p \rightarrow J/\psi p$ with 469 J/ψ events did not find evidence for a pentaquark enhancement [13]. In the light-quark sector, three pentaquarks are predicted with quantum numbers that are not accessible to three-quark systems. These states are, of course, of particular interest and should be searched for with high statistics and, in the case of narrow states, with good resolution.

These topics ask for a new facility allowing for the study of Kaon-nucleon interactions in the mass range from threshold to 3 GeV. Such a facility is offered by the proposal PAC47 at JLab [14]. A beam of K_L -mesons will be provided with an intensity of 10⁴ particles per second. This facility will provide for new insights into the spectroscopy of hyperons and cascade baryons. Also the COMPASS experiment [15], J-PARC [16], and the forthcoming PANDA experiment [17] may provide substantial contributions to hyperon spectroscopy.

The paper is organized as follows. After this Introduction, the physics of hyperons in the first and third excitation band – the negative-parity states – will be discussed in Section 2, including a proposal how to determine the SU(3) structure of the $\Lambda(1405)$ resonance. In Section 3, the positive-parity states in the second excitation band will be discussed as well as the possibility to search for two of the three light-quark pentaguarks that have flavor quantum-numbers that are incompatible with three-quark states. Three-quark baryons have two oscillators that can be excited. In most excitations, only one of the two oscillators is excited (with rapid changes from one oscillator to the other one), or there is at least a component in the baryon wave function in which only one oscillator is excited. In quark models there is, however, also a class of resonances that always carry excitation in both oscillators. Yet, no member of this class has ever been observed. A scenario in which such a state should be observable is presented at the end of Section 3. A short discussion of the physics of Regge trajectories and of cascade baryons follows in Section 4 and 5.

2 The negative-parity states

2.1 The first and third excitation band

Baryon resonances are often classified by the leading representation in the flavor-spin SU(6) basis by $J^P(D, L_N^P)S$ where J^P is the spin and parity of a resonance, D the dimensionality of the SU(6) representation, L the intrinsic orbital angular momentum, N the excitation band, and Sthe total quark spin.

The first excitation band has an intrinsic orbital angular momentum L = 1; the resonances in the first band belong to a $(D, L_N^P) = (70, 1_1^-)$ representation. The 70-plet can be expanded into a spin-doublet SU(3) decuplet, a spin-doublet and a spin-quartet SU(3) octet, and a spin-doublet SU(3) singlet:

$$70 = {}^{2}10 \oplus {}^{2}8 \oplus {}^{4}8 \oplus {}^{2}1.$$
 (1)

The N and Δ resonances belonging to the first excitation band are all well established. Resonances with a leading configuration with spin and isospin 1/2 have a mass around 1530 MeV; states with a leading spin-1/2 and isospin-3/2 or spin-3/2 and isospin-1/2 configuration

Table 1. Star rating of nucleon and Δ resonances in the first excitation band. J^P is the spin and parity of a resonance, S the total internal quark spin. The table gives the leading configuration; mixing between states with identical external quantum numbers can mix.

Π		Octet		Decuplet		
	$J^P =$	$1/2^{-}$	$3/2^{-}$	$5/2^{-}$	$1/2^{-}$	$3/2^{-}$
Π	$70 [^210]$				**** N(1620)	**** N(1700)
	$70 \ [^48]$	**** N(1650)	**** N(1700)	**** N(1675)		
	$70 [^28]$	**** N(1535)	$****_{N(1520)}$			

Table 2. Star rating of Λ resonances in the first excitation band. See caption of Table 1.

		Singlet				
.	$J^P =$	$1/2^{-}$	$3/2^{-}$	$5/2^{-}$	$1/2^{-}$	$3/2^{-}$
70	$[^{4}8]$			**** A(1800)	-	$****_{\Lambda(1830)}$
70	$[^{2}8]$			$**** \\ \Lambda(1670)$	**** $\Lambda(1690)$	
70	$[^{2}1]$	**** $\Lambda(1405)$	$**** \\ \Lambda(1520)$			

Table 3. Star rating of Σ resonances in the first excitation band. See caption of Table 1.

	Octet		Decuplet		
$J^P =$	$1/2^{-}$	$3/2^{-}$	$5/2^{-}$	$1/2^{-}$	$3/2^{-}$
$70 [^210]$				* $\Sigma(1950)$	$* \\ \Sigma(1920)$
$70 [^48]$	*** $\Sigma(1750)$	-	**** $\Sigma(1775)$		
$70 [^28]$	$** \\ \Sigma(1620)$	$**** \Sigma(1670)$			

have masses that fall into a (1660 ± 40) MeV window (see Table 1).

The Λ excitations in the first band are mostly also well established except for the state $J^P = 3/2^-$ state that has dominantly internal quark spin S = 3/2, see Table 2. This state was shown to have a very small K^-N decay width [18,19], it is hence difficult to find it in K^-p scattering. It may be more advantageous to search for the state in the decay sequence Σ^+ (high mass) $\rightarrow \Lambda(xxx)3/2^-\pi^+ \rightarrow$ $(\Sigma\pi)\pi^+$. Unfortunately, $N(1700)3/2^-$ was not yet seen as intermediate state in a cascade process, hence no predictions can be made concerning the best suited cascade.

In the third excitation band, a large number of resonances can be expected. In quark models they fall into one of the following eight representations:

$$(56, 1_3^-), (70, 3_3^-), (56, 3_3^-), (20, 3_3^-), (70, 2_3^-), (70, 1_3^-), (70, 1_3^-), (20, 1_3^-).$$
(2)

In the N plus Δ sector, 30 nucleon and 15 Δ resonances are expected. All known states fit into the first two respresentations, see Table 4.

 $\Delta(1930)5/2^{-1}$

 $\Delta(1940)3/2^{-1}$

 $N(1875)3/2^{-1}$

Table 4. N and Δ resonances and their SU(6) multiplet assignments in the third excitation band. All known resonances in this mass range can be assigned to two multiplets. The other six multiplets are empty (from Ref. [20].

Particularly interesting are the resonances that can be assigned to the $(56, 1^-_3)$ representation. The 56-plet can be expanded into

56 $[^28]$

 $\Delta(1900)1/2$

 $N(1895)1/2^{-1}$

$$56 = {}^{4}10 \oplus {}^{2}8,$$
 (3)

where the three resonances $\Delta(1900)1/2^{-}$, $\Delta(1940)3/2^{-}$ $\Delta(1930)5/2^{-}$ form a (degenerated) spin-quartet and $N(1895)1/2^-$ and $N(1875)3/2^-$ a spin-doublet. In quark models, these states belong to the third excitation band but their masses are rather compatible with resonances falling into the second excitation band. In these states, one oscillator is excited to carry one unit of orbital angular momentum, one oscillator carries one unit of radial excitation. Note that the Roper resonance – carrying one unit of radial excitation and belonging to the second excitation shell – has a smaller mass than $N(1520)3/2^{-}$ carrying one unit of orbital angular momentum. Considering the masses of these five resonances above, we should expect a spindoublet of negative-parity Λ resonances, and five negativeparity Σ states, all only slightly above 2 GeV. Three onestar candidates are known, $\Lambda(2000)1/2^{-}$, $\Lambda(2050)3/2^{-}$, and $\Sigma(2010)3/2^-$. They could be members of the $(56, 1_3^-)$ representation. We emphasized again that states with identical quantum numbers can mix. The mixing angles are, however, predicted to be often small, and experimentally, an assignment to multiplets seems to be possible.

Some N and Δ states can be assigned to the $(70, 3_3^-)$ representation. Expected are resonances with a total orbital angular momentum of three units. Nucleons in a 70-plet can carry spin 1/2 or 3/2; Δ 's only spin 1/2. The two pairs of nucleon resonances with $J^P = 5/2^-$ and $7/2^-$ can have intrinsic quark spin S = 1/2 or 3/2, respectively, and can be expected to be separated in mass by about 110 MeV. So far, they have not been identified separately. In the remaining multiplets, many more states predicted but no candidates known.

Totally 45 Σ resonances are expected in the third excitation band, in a comparatively narrow mass interval from 2000 to 2400 MeV. It seems hopeless to identify them all. The aim in a study of $K_L N$ interactions should be to verify that the N and Δ resonances at about 1900 MeV with negative parity belong to a 56-plet and have Λ and Σ partners. Two Λ and five Σ states with negative-parity falling into the 2000 to 2100 MeV mass region are to be expected. These are states with one unit of orbital and one unit of radial excitation as dominant configuration. Also the *leading* resonances with L = 3 and S = 1/2 coupling to $J^P = 7/2^-$, and L = 3 and S = 3/2 coupling to $9/2^-$ should be identified. In the Λ sector, a 4* $\Lambda(2100)7/2^-$ is known that likely belongs to the SU(3) singlet series (see Fig. 1). The recently suggested 1* $\Lambda(2080)5/2^-$ [8] could be its spin partner. The 1* $\Sigma(2100)7/2^-$ is a bit low in mass; it could be the strange partner of $N(2190)7/2^-$ or $\Delta(2200)7/2^-$.

2.2 The $\Lambda(1405)$

The $\Lambda(1405)1/2^-$ resonance was discovered in 1961 [21]. Its spin and parity were first taken from the quark model in which the $\Lambda(1405)$ and $\Lambda(1520)$ hyperons are interpreted as qqq resonances with a dominant SU(3)-singlet structure [22]. The SU(3) assignments of $\Lambda(1405)$ and $\Lambda(1520)$ as mainly SU(3) singlet states were confirmed by Tripp *et al.* [23] by a comparison of the phases at the resonance position of the $K^-p \to \Lambda(1405) \to KN$ and $K^-p \to \Lambda(1405) \to \pi\Sigma$ transition amplitudes. The preference for the SU(3)-singlet nature of $\Lambda(1405)$ was statistically significant even though the data base was meagre. The spin and parity of $\Lambda(1405)$ were only established in 2014 [24].

The SU(3) singlet assignment was challenged by coupled-channels calculations based on chiral SU(3) effective field theories. Kaiser, Waas and Weise constructed an effective potential from a chiral Lagrangian, and the $\Lambda(1405)$ emerged as quasi-bound state in the $\bar{K}N$ and $\pi\Sigma$ coupledchannel system [25]. Oller and Meissner [26] derived the interaction of the SU(3) octet of pseudoscalar mesons and the SU(3) octet of stable baryons and studied the S-wave $\bar{K}N$ interactions in a relativistic chiral unitary approach. Two isoscalar resonances at 1379.2 MeV and at 1433.7 MeV and one isovector resonance at 1444.0 MeV governed the interaction. The two Λ^* poles as well as a third state at $1680 \,\mathrm{MeV}$ were interpreted as combinations of the $\mathrm{SU}(3)$ singlet state and the two octet states expected in the expansion $8 \otimes 8$ into $1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \overline{10} \oplus 27$. The first wider state (at 1390 MeV in their analysis) was interpreted as a



Fig. 1. The signs of the imaginary parts of resonating amplitudes in the $\overline{KN} \to \Lambda \pi$ and $\Sigma \pi$ channels. The signs of the $\Sigma(1385)$ and $\Lambda(1405)$, marked with a \bullet , are set by convention, and then the others are determined relative to them. The signs required by the SU(3) assignments of the resonances are shown with an arrow, and the experimentally determined signs are shown with an \times . (From Ref. [1])

mainly SU(3)-singlet state, a second and a third state at 1426 MeV and 1680 MeV were interpreted as mainly octet states. The two expected isovector states were found to be much more sensitive to the details of the coupled channel approach [27]. These results were confirmed in a series of further studies [28,29,30,31,32,33,34,35,36,37,38,39]. A survey of the literature and a discussion of the different approaches can be found in Ref. [40].

The SU(3) structure of a baryon can be deduced from its decays, in particular from the sign of transition amplitudes at the resonance position. The amplitude for $K^-p \rightarrow \overline{K}^0 n$ scattering can be decomposed into isospin-0 and -1 elastic scattering amplitudes A_0 and A_1 and written as $\pm (A_1 - A_0)/2$, where the sign depends on conventions. It is custom to chose the overall phase so that the amplitude of any Σ at resonance will point "up" and any Λ at resonance will point "down" (along the negative imaginary axis): The phase at resonance determines the isospin.

The separation of Λ SU(3) singlet and octet states requires a second decay mode, here $K^-p \rightarrow \Lambda(1405) \rightarrow \pi \Sigma$. Again, a convention has to be adopted for some overall phases. We use the convention of Levi-Setti [41] that is shown in Fig. 1. The figure compares experimental results with theoretical predictions for the signs of several resonances. Since this approach is not very well known, we first derive the amplitude relations shown graphically in Fig. 1.

The decay amplitude of hyperons into a baryon and a meson are governed by two SU(3) structure constants, the symmetric (d_{ijk}) and the antisymmetric (f_{ijk}) . These are tabulated, e.g., in the RPP. Their relative contribution is governed by the parameter α that depends on the SU(6) classification of the baryon (see Table 5). The corresponding SU(6) coupling constants can be found in Refs. [18,19]) and are listed in Table 5. The production cancels in the comparison, and the relative sign of the amplitudes can be

Table 5. SU(3) coupling constants for hyperon decays and the SU(6) predictions for the coefficient α in decays of octet hyperons.

Decay mo	de $8 \rightarrow$	8 + 8	$1 \rightarrow 8 +$	8
$\Lambda \to N \dot{F}$	$\bar{X} = \sqrt{\frac{2}{3}}(2\epsilon)$	$(\alpha + 1)A_8$	$\frac{1}{2}A_{1}$	
$\Lambda \to \varSigma \tau$	$\tau = 2(\alpha - $	$(-1)A_8$	$\sqrt{\frac{3}{2}}A_1$	
	² 8	8[56]	$^{2}8[70]$	$^{4}8[70]$
α		$\frac{2}{5}$	$\frac{5}{8}$	$-\frac{1}{2}$
	$^{2}1[70]$	$^{2}8[56]$	$^{2}8[70]$	$^{4}8[70]$
$\frac{A(\Lambda \to \Lambda)}{A(\Lambda \to \Sigma)}$	$\frac{V\bar{K})}{Z\pi}$ $\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{2}}$	$-\sqrt{6}$	0
Sign	n +	_	_	

used to determine the SU(3) structure of a hyperon. The relative signs are listed in the last line in Table 5. Mixing of the ²8[70] component into the ²1[70] wave function could reverse the sign from +1 to -1, which would make $\Lambda(1405)$ appear as "mainly" octet state.

The BnGa collaboration analyzed the CLAS data on the three charge states in $\gamma p \to K^+(\Sigma \pi)$ [42], combined with data on the total cross sections for K^-p induced reactions: $K^-p \to K^-p$, $K^-p \to \bar{K}^0n$, $K^-p \to \pi^0\Lambda$, $K^-p \to \pi^+\Sigma^-$, $K^-p \to \pi^0\Sigma^0$, $K^-p \to \pi^-\Sigma^+$ [43,44, 45,46], elastic and inelastic K^-p scattering [47], the lowenergy BNL data on $K^-p \to \pi^0\pi^0\Lambda(\Sigma^0)$ [48,49], bubble chamber data on $K^-p \to \pi^-\pi^+\pi^\pm\Sigma^\mp$ [50], K^-p annihilation frequencies at rest [51,52], and the recent experimental results on the energy shift and width of kaonic hydrogen atoms which constrain the K^-p S-wave scattering length [53,54]. Very important are the data on $K^-p \to \pi\Sigma$ [55] which constrain the SU(3) structure of $\Lambda(1405)$. In the preferred solution, the BnGa partial-wave analy-

Table 6. The signs of the SU(6) amplitudes for $\Sigma^+(1670)3/2^- \rightarrow \pi^+\Lambda(1405); \Lambda(1405) \rightarrow \Sigma^{\pm}\pi^{\mp}$ and $\Sigma^+(1670)3/2^- \rightarrow \pi^+\Lambda(1405); \Lambda(1405) \rightarrow \Sigma^{\pm}\pi^{\mp}$

$\Lambda(1405)$ SU(3) structure:	1	8
$\Sigma^+(1670)3/2^- \rightarrow \Lambda(1405)\pi^+$	+	+
$\hookrightarrow \Sigma^{\pm} \pi^{\mp}$	+	-
Sign of transition amplitude at pole:	+	-
$\Sigma^+(1670)3/2^- \rightarrow \Sigma^0(1385)\pi^+$	+	+
$\hookrightarrow \varSigma^{\pm} \pi^{\mp}$	+	+
Sign of transition amplitude at pole:	+	+

sis [56] required only one isoscalar resonance with a pole at $[(1421\pm3)-i((23\pm3)]$ MeV. The pole can be identified with the $\Lambda(1405)$ at a slightly higher mass compared to the nominal mass. The isovector interactions were described by two resonances, one below, one above the considered mass range $(1300 - 1500 \,\mathrm{MeV})$. The SU(3) structure was determined to be consistent with a singlet but not with an octet state. There was, however, a second solution with a description of the data with similar quality. This second solution was compatible with a second broader isoscalar resonance with a fixed mass at 1380 MeV. In this solution, the $\Lambda(1405)$ changed its SU(3) structure from being dominant SU(3) singlet to dominant SU(3) octet. Obviously, the $\Lambda(1405)$ SU(3) structure cannot be determined in a model-independent way from existing K^-p scattering alone, even when the CLAS data on photo-induced data on $\Lambda(1405)$ production are included in the analysis.

The K^-p threshold is at 1432 MeV, considerably above the nominal $\Lambda(1405)$ mass. At present, data on differential cross sections for $K^-p \rightarrow \Lambda(1405) \rightarrow KN$ exist only above 1470 MeV, those for $K^-p \rightarrow \Lambda(1405) \rightarrow \pi\Sigma$ only above 1530 MeV. It will be important to repeat the BnGa analysis with data on K^-p scattering covering a mass range starting from close to the threshold to about 1540 MeV.

In the reaction $K^-p \to \pi^-\pi^+ \pi^\pm \Sigma^\mp$ studied in [50], the full $\Lambda(1405)$ line shape can be investigated. In this reaction, the SU(3) assignment follows from the correlation in the production and decay dynamics. The derivation relies on approximate SU(6) symmetry in baryon decays. We consider the two decay sequences

$$\Sigma^{+}(1670)3/2^{-} \to \pi^{+}\Lambda(1405); \ \Lambda(1405) \to \Sigma^{\pm}\pi^{\mp}$$
 (4a)

$$\Sigma^{+}(1670)3/2^{-} \to \pi^{+}\Sigma(1385); \ \Sigma(1385) \to \Sigma^{\pm}\pi^{\mp}$$
 (4b)

that are shown to contribute to this reaction [56].

The SU(6) amplitude for reaction (4a) depends on the SU(3) structure of $\Lambda(1405)$ and on the primary $\Sigma^+(1670)$ $3/2^-$ (see Table 6). The sign of this amplitude is given by the product of the signs for $\Sigma^+(1670)3/2^- \to \Sigma\pi$ and $\Lambda^+(1405)1/2^- \to \Sigma\pi$. The $\Sigma^+(1670)3/2^-$ belongs dominantly to a spin-1/2 SU(3) octet in the SU(6) 70-plet; $\alpha =$ 5/8. The sign of the SU(6) amplitude for $\Sigma^+(1670)3/2^ \to \Sigma\pi$ is given by $2\sqrt{2} \cdot \alpha$, hence +1; the sign for the $\Lambda^+(1405) \to \Sigma\pi$ transition depends on the SU(3) structure of $\Lambda^+(1405)$: if it is an octet with spin-1/2 in the SU(6) 70-plet, it is given by $2(\alpha - 1)$ with $\alpha = 5/8$, hence negative. If it is a singlet, it is $\sqrt{6/4}$ and positive. The sign of the transition amplitudes for reactions (4a) and (4b) are the same when $\Lambda(1405)$ is an octet, they are different when $\Lambda(1405)$ is an octet.

3 The positive-parity states in the second excitation band

3.1 Missing resonances

The second excitation band contains a number of representations:

$$(56, 0_0^+); (70, 0_2^+); (56, 0_2^+); (70, 0_2^+); (20, 1_2^+).$$
 (5)

In total, there are 8 Δ and 8 Ω resonances expected in the 2nd excitation shell, 13 nucleon resonances, 19 Λ resonances, and 21 Σ and 21 Ξ resonances. The Particle Data Group classifies baryon resonances with a star rating; 3* and 4* resonances are considered to be established, 1* and 2* resonances not. Table 7 gives the number of predicted states and compares this number with the number of established and the number of 1* or 2* states.

Table 7. Number of expected and observed resonances that can be assigned to the 2^{nd} excitation shell for $J^P = 1/2^+, ..., 7/2^+$. The first number gives the expected number of resonances, followed by the number of observed resonances with 3^{*} and 4^{*}, 1^{*} and 2^{*} (in parentheses).

		$1/2^{+}$	$3/2^{+}$	$5/2^{+}$	$7/2^{+}$	Sum
seen	N	4(4,0)	5(3,1)	3(1,2)	1(1,0)	13 (9,3)
seen	Δ	2(1,1)	3(2,0)	2(1,0)	1(1,0)	8(5,1)
seen	Λ	6(2,1)	7(1,1)	5(2,0)	1(0,1)	19(5,3)
seen	$\boldsymbol{\Sigma}$	6(1,1)	8(0,4)	5(1,1)	2(1,0)	21(3,6)
seen	Ξ	6(0,0)	8(0,0)	5(0,0)	2(0,0)	21 (0,0)
seen	Ω	2(0,0)	3(0,0)	2(0,0)	1(0,0)	8(0,0)

In the nucleon spectrum, thirteen states are expected in the second excitation level. Nine states are established, three states need further confirmation, one state is missing. The number of $J^P = 1/2^+$ states seems complete; yet the state with highest mass, $N(2100)1/2^+$, may already belong to the fourth excitation shell. (It could be low in mass like the Roper resonance in the second excitation shell, see Ref. [10].) Then, one state would be missing. For $J^P = 3/2^+$, one state is missing. Below we will discuss the reasons why we might expect not to observe the two nucleon states (with $J^P = 1/2^+$ and $3/2^+$) in the 20-plet. In the Δ spectrum, one state with $J^P = 3/2^+$, one with $J^P = 5/2^+$ are missing, one further states with $J^P = 1/2^+$ is seen with little evidence only. The situation is much worse in for Λ and Σ hyperons: only 17 of 42 states are seen, only 8 of them are established. No Ξ

resonance or Ω with known spin-parity that might belong to the second excitation shell is listed in the RPP.

In $K_L p$ scattering experiments, Σ resonances can be searched for in formation. Λ resonances are formed only by scattering off neutrons. The reactions

$$K_L p \to \Sigma^{+*} \to K_S p$$
 (6a)

$$K_L p \to \Sigma^{+*} \to \pi^+ \Lambda; \ \pi^+ \Sigma^0; \ \pi^0 \Sigma^+$$
 (6b)

and there analysis, reconstruction and partial wave analysis, were described in detail in Ref. [14]. Here, we refrain from further discussions.

3.2 On the sideline: Δ^{++} excitations

A K_L be amline can also be used to study \varDelta excitations in the reaction

$$K_L p \to K^- \Delta^{++}$$
 (7)

The advantage is similar to $\pi^+ p$ scattering: only Δ and no N excitations can be produced. Admittedly, $\pi^+ p$ scattering as formation experiment is superior.

3.3 Search for the states in the SU(3) 20-plet

The baryonic spatial wave can be constructed from the two degrees of freedom of a three-particle system (neglecting the cms motion). In the three-particle system, two quarks can oscillate (the ρ oscillator) or two quarks can oscillate against the third quark (the λ oscillator). With these oscillators spatial wave functions can be formed that are symmetric with respect to (w.r.t.) the exchange of any pair of quarks

$$S = \frac{1}{\sqrt{2}} \left\{ \left[\phi_{0s}(\boldsymbol{\rho}) \times \phi_{0d}(\boldsymbol{\lambda}) \right] + \left[\phi_{0d}(\boldsymbol{\rho}) \times \phi_{0s}(\boldsymbol{\lambda}) \right] \right\}^{(L=2)}, \quad (8)$$

or they can have mixed symmetry

$$\mathcal{M}_{\mathcal{S}} = \frac{1}{\sqrt{2}} \left\{ \left[\phi_{0s}(\boldsymbol{\rho}) \times \phi_{0d}(\boldsymbol{\lambda}) \right] - \left[\phi_{0d}(\boldsymbol{\rho}) \times \phi_{0s}(\boldsymbol{\lambda}) \right] \right\}^{(L=2)}$$
(9a)
$$\mathcal{M}_{\mathcal{A}} = \left[\phi_{0p}(\boldsymbol{\rho}) \times \phi_{0p}(\boldsymbol{\lambda}) \right]^{(L=2)} ,$$
(9b)

with one part that is symmetric in the ρ and antisymmetric in the λ oscillator and one part antisymmetric in the ρ and symmetric in the λ oscillator. Both parts are required in the full wave function. The part $\mathcal{M}_{\mathcal{A}}$ describes a component in which the ρ and the λ oscillator are both excited simultaneously.

Finally, the spatial wave function can be antisymmetric w.r.t. the exchange of any quark pair:

$$\mathcal{A} = [\phi_{0p}(\boldsymbol{\rho}) \times \phi_{0p}(\boldsymbol{\lambda})]^{(L=1)} .$$
 (10)

The multiplets 70, 56, and 20 arise from the combination of the three light quarks u, d, s having spin 1/2:

$$6 \otimes 6 \otimes 6 = 56_S \oplus 70_M + 20_A \tag{11}$$



Fig. 2. (Color online) Classical orbits of nucleon excitations with L = 2 (upper row) and L = 1 (lower row). The first two pictures in both rows show excitations of the ρ and λ oscillators, in the third picture in the first row both, ρ and λ are excited [58].

The spin-flavor wave functions can be symmetric (S) or antisymmetric (A) or can be of mixed symmetry. In the (M). Since the total spin-flavor-spatial wave function needs to be symmetric, the spatial wave function has to carry the same symmetry. Hence the antisymmetric wave function (10) is combined with the spin-flavor wave function of the 20-plet.

In an analysis of data on the reaction $\gamma p \to \pi^0 \pi^0 p$ [57] it was shown that the dominant decay modes of resonances like the quartet of Δ resonances, $\Delta(1910)1/2^+$, $\Delta(1920)3/2^+, \ \Delta(1905)5/2^+, \ \Delta(1950)7/2^+, \ having sym$ metric wave functions of type (8), are decays into two ground-state hadrons like $N\pi$ or $\Delta\pi$. Nucleon resonances in the 70-plet with a mixed symmetry, (9a) and (9b), have sizable branching ratios into final states in which one of the decay product carries orbital excitation like $N(1520)\pi$ or $Nf_{\pi\pi_{\rm S-wave}}$. This observation was interpreted as evidence for the three-body nature of nucleon excitations [58]. The situation is depicted in Fig. 2. When the λ or ρ oscillator is excited, it can de-excite into the ground state. When both oscillators are excited, then first one oscillator de-excites into an intermediate excited states while the other oscillator remains in an excited state. A second step, a cascade, is required to reach the final state.

This scenario forbids (or suppresses) a direct excitation of resonances that have no symmetric component. Reversing the argument, it forbids excitations from the ground state into resonances having a wave function of type (10). The antisymmetry of the orbital wave function needs to be combined with an antisymmetric spin-flavor wave function. These belong to the 20-plet representations. The 20plet can be expanded into

$$20 = {}^{2}8 \oplus {}^{4}1. \tag{12}$$

So far, no member of a 20-plet has ever been identified. In the case of nucleon resonances, this is rather difficult: there are several possibilities to realize nucleon excitations with internal total quark spin 1/2. But the discovery of a Λ state that decays mostly via a cascade process would provide strong evidence that a member of a 20-plet has been identified. The observation of a series of states

 $\mathbf{6}$

with $J^P = 1/2^+, 3/2^+, 5/2^+$ decaying via cascades would strengthen the conjecture.

In Ref. [10] the three states are predicted to have masses of 2099 MeV; 2176 MeV; 2150 MeV. We suggest to search first for the member of the 20-plet with $J^P = 3/2^+$ in the reaction

$$K_L p \to \pi^+ \Lambda_{20}, \quad \Lambda_{20} \to \Lambda(1520)\eta \text{ or } \Lambda(1670)\eta.$$
(13)

This is an S wave decay to an intermediate state with orbital angular momentum excitation. The first decay mode has the disadvantage that $\Lambda(1520)$ is dominantly a SU(3) singlet, η dominantly SU(3) octet but the mixing angles deviate significantly from pure SU(3) eigenstates. The second mode might be forbidden kinematically if the mass of the expected resonance is low. With L = 2 between η and excited hyperon, also the states with $J^P = 1/2^+$ and $5/2^+$ could be observed. Note that Λ excitations with a total quark spin S = 3/2 exist only in the SU(6) 20-plet.

3.4 Pentaquark search

The concept of a nucleon composed of three constituent quarks is certainly oversimplified, and the hadronic properties of nucleons cannot be understood or, at least, are not understood in terms of quarks and their interactions. Skyrme studied the pion field and discovered that by adding a non–linear " σ term" to the pion field equation, stable solutions can result [59]. These solutions have half integer spin and a winding number identified by Witten [60] as the baryon number. These stable solutions of the pion field equation are called soliton solutions.

The chiral soliton model predicts the existence of a full antidecuplet of states [61, 62] with quantum numbers $J^P = 1/2^+$. The antidecuplet is shown in Fig. 3; the states are called pentaquarks [63]. Note that the three corner states have quantum numbers which cannot be constructed out of three quarks. In the minimum quark model configuration, the flavor wave function of the state with positive strangeness is given by $\Theta^+ = uudd\bar{s}$. The strange quark fraction increases from 1 to 2 units in steps of 1/3additional s quark. The masses of the pentaquark states were predicted in Ref. [63]. The increase in mass per unit of strangeness is is 540 MeV, instead of the 120 MeV that are derived when the ρ or ω mass is compared to the K^{*} mass. The splitting is related to the so-called $\sigma_{\pi N}$ term in low-energy π N scattering. Its precise value is difficult to determine and has undergone a major revision [64].

Pentaquarks were highly discussed when the so-called Θ^+ was observed in different experiments [65, 66, 67, 68]. It has positive strangeness S = +1, its flavor wave function has a minimal quark content $uudd\bar{s}$. However, in a series of precision experiments, the evidence for pentaquarks has faded away (see, e.g., Ref. [69, 70, 71]) even though some evidence remains that a narrow state with $J^P = 1/2^+$ at 1720 MeV might exist [72, 73, 74]. High-precision experiments are mandatory to settle this important issue. Particularly convincing would be, of course, the discovery or confirmation of one o the states having quantum numbers that are incompatible with a qqq interpretation. Attractive and easily accessible is the Θ^+ . It is best searched for in the reaction

$$K_L p \to K^+ n$$
. (14)

The reaction does not receive contributions from Σ resonances, nor from Pomeron exchange nor from the exchange of f_0/f_2 mesons. In this paper, we concentrate on inelastic scattering processes and do not expand on reaction (14).

Particularly interesting is the search for a member of the quartet of Ξ pentaquarks. The minimal quark content of the $\Xi^+(2070)$ is $uuss\bar{d}$. It can be produced in the K_Lp induced reaction

$$K_L p \to K_S \Xi^+(2070) \tag{15}$$

At the first moment, the reaction looks like an elastic scattering process. However, the reaction (15) is more complicated. The minimal quark flow is depicted in Fig. 4. The process can be described as formation of a Σ^+ state belonging to the antidecuplet.

Evidence for an isospin partner of $\Xi^+(2070)$ with S = -2, Q = -2 was reported [75] studying proton proton collisions at the CERN SPS. Its mass of (1862 ± 2) MeV was a bit low when compared to the prediction [63]. The state was not confirmed in later experiments [69].

The $\Xi^+(2070)$ is best searched for in its decay into $\Xi^0\pi^+$, predicted with 30% branching ratio, followed by the decay $\Xi^0 \to A\pi^0$ (~ 100%). Thus the reaction

$$K_L p \to K_S \pi^+ \pi^0 \Lambda \qquad \Lambda \to p \pi^-; K_S \to \pi^+ \pi^-$$
(16)

needs to be studied. The K^0 mass and momentum can be reconstructed from the $\pi^+\pi^-$ pair. With a known K_L momentum, the $\Xi^+(2070)$ mass and momentum can be determined. Then, using the π^+ four-vector, the Ξ^0 mass and momentum can be deduced. The Λ mass and momentum can be deduced from its decay particles; the crossing of the Ξ^0 and Λ trajectories defines the decay point of the Ξ^0 . The Ξ^0 has a mean free path $c\tau = 8.71$ cm. Thus, the reaction chain will be reconstructed with very little background. An alternative attractive decay mode is given by $\Xi(2070) \to K^{*+}\Sigma^0$. The threshold for this decay mode is 2084 MeV.

The non-strange and strange partners in the anti-decuplet suffer from the difficulty that their identification as members of the anti-decuplet is model-dependent. Evidence for the possible existence of two narrow states at 1685 and 1720 MeV has been reported [72,73,74]. The peak at 1685 MeV is discussed extensively in the literature, see, e.g.,Refs. [76,77,78,79,80,81,82]. It seems to belong to the $J^P = 1/2^-$ partial wave and to be unrelated to pentaquark spectroscopy. The structure at 1720 MeV certainly requires further investigations but we do not see a particular advantage to use a K_L beam.

There is a triplet of Σ states in the antidecuplet. It is predicted to mix with its *nns*-partners. In Ref. [83] the mass of the additional mainly- $\overline{10}$ state is calculated to fall into the range $1795 < M_{\overline{10}} < 1830 \,\text{MeV}$; its main decay modes with estimated branching ratios of nearly



Fig. 3. The antidecuplet and its quark model decomposition. The antidecuplet predicted by the chiral soliton model describes nucleons in terms of the pion field and not by the number of quarks [63]. The three corner-states are incompatible with a qqq assignment.



Fig. 4. (Color online)Left: Quark flow diagram for the reaction $K_L p \rightarrow K_S \Xi^+(2070)$. *s*-quarks in red, \bar{s} in orange, *d*-quarks in blue, \bar{d} in green, *u*-quarks in black. Right: Hadron representation of the scattering process.

60% (16%) are $\bar{K}N$ ($\pi \Lambda$). The Σ^+ decuplet state can be searched for in a formation experiment. The main difficulty is to identify it against the expected *nns* states. Quark models, e.g. the Isgur quark model, predicts six $J^P = 1/2^+$ states in the second excitation band at 1720, 1915, 1970, 2005, 2030, 2105 MeV. Given the uncertainties with the calculation of Roper-like states in the quark model and the uncertainty of the predictions using the chiral soliton model, there is certainly a significant modeldependence in any attempt to assign a specific state with non-exotic quantum numbers to the antidecuplet.

4 The Regge trajectories

The masses of light-quark baryons fall onto Regge trajectories. Figure 5 shows the Regge trajectory of Δ baryons; plotted is the squared baryon mass M^2 versus the total angular momentum J. The four states $\Delta(1232)3/2^+$, $\Delta(1950)7/2^+$, $\Delta(2420)11/2^+$, and $\Delta(2950)15/2^+$ – all having J = L + S with L = 0, ..., 4 and S = 3/2 – are compatible with a linear trajectory. This trajectory is compared with the mesonic trajectory, again for mesons with



Fig. 5. The Regge trajectories M^2 versus J for mesons and Δ baryons have the same slope. This observation suggests for stretched states with J = L + S a string excitation between a quark and a diquark in baryons (from Ref. [20]).

J = L + S but S = 1 and for even and odd angular momenta. (Note that the negative parity $\Delta(1700)3/2^-$, $\Delta(2200)7/2^-$ and likely $\Delta(2750)11/2^-$ have spin S = 1/2. Nevertheless, they fall onto the trajectory shown in Fig. 5 when the orbital angular momentum L instead of J is considered.)

For Σ resonances, there are only two states that can be considered at the moment: $\Sigma(1385)3/2^+$ and $\Sigma(2030)7/2^+$. Their squared-mass difference suggests an identical slope as the one for Δ states. Nevertheless, it would be important to increase our knowledge on high-mass Σ resonances.

The Λ Regge trajectory could be extracted from an analysis of $K_L n$ interactions. Here, Λ resonances in SU(3) singlet and octet and Σ resonances in SU(3) octet and

decuplet contribute. Compared to $K_L p$ interactions, this is certainly a more complicated task.

5 Cascade baryons

There is only one single Ξ resonance, $\Xi(1820)3/2^-$, with known spin-parity. It is 290 MeV heavier than the SU(3) decuplet ground state, $\Xi(1530)3/2^+$. The difference corresponds to the mass gap between $N(1520)3/2^-$ and $\Delta(1232)$ $3/2^+$. Hence, very likely, $\Xi(1820)3/2^-$ is a SU(3) octet state and the first orbital excitation of the Ξ . Otherwise, the Ξ resonances are an uncharted territory.



Fig. 6. Left: Quark flow diagram for the production of a Ξ^{0*} resonance via $K_L p \to K^+ \Xi^{0*}$. *s*-quarks in red, \bar{s} in orange, *d*-quarks in blue, \bar{d} in green, *u*-quarks in black. Right: Hadron representation of the scattering process.

Figure 6 shows the flux diagram for the production of Ξ^{0*} resonances. The intermediate state is a Σ^+ excitation. In the first searches for states, the decay of Ξ resonances into $\pi\Xi$ with a subsequent weak Ξ decay should greatly reduce the background.

6 Summary

The planned K_L beamline at JLab in connection with the GlueX experiment provides a powerful tool to study KN and $\bar{K}N$ interactions. In formation experiments, Σ resonances can be studied using a proton target; a deuteron target makes Λ resonances accessible. In this paper, we emphasize the highlights of production experiments with several particles in the final state. We propose to determine the disputed SU(3) structure of $\Lambda(1405)$ and propose a method how to search for members of the missing 20-plet, in particular for Λ states in the ⁴1 multiplet, in a cascade process. Further, we suggest to search for light-quark pentaquarks with quantum numbers that are incompatible with a qqq interpretation. The study can be extended to identify resonances of hyperons with two units of strangeness.

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