



Departamento de Física Teórica. Universidad Complutense de Madrid

# pion-kaon scattering at KLF

J. R. Peláez  
For KLF Collaboration

*PR-12-18-002*  
*PAC46, July 17, 2018*

## Motivation to study $\pi K$ scattering

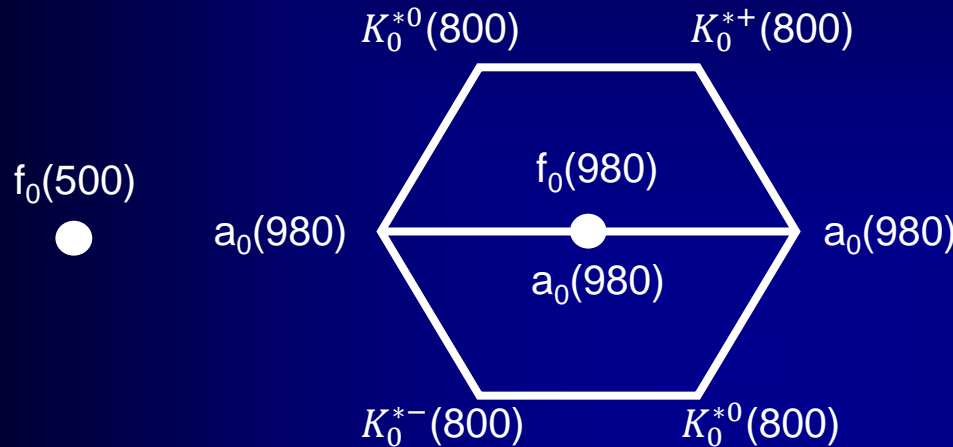
- $\pi, K$  appear as final products of almost all hadronic strange processes:  
Examples: B and D decays, CP violation studies, etc...
- $\pi, K$  are Goldstone Bosons of QCD  $\rightarrow$  Test Chiral Symmetry Breaking
- Many light resonances appear  $\rightarrow$  Strange SPECTROSCOPY

### Particularly interesting at KLF:

- $\kappa/K_0^*(800)$  light scalar meson. “Needs confirmation”@PDG.  
Light scalar mesons longstanding candidates for non-ordinary mesons.  
Settle multiplet classification?
- Scattering lengths for SU(3) Chiral Perturbation Theory (ChPT)
- $K_1^*(892)$  interesting for CP violation studies
- $K_1^*(1410)$  and  $K_0^*(1430)$  smaller discussion on parameters and nature

# The light scalar controversy. The theory side... classification

- Scalar lightest SU(3) nonet controversial for decades  
... but a picture is emerging



Non-strange heavier!!  
**Inverted hierarchy problem**  
For quark-antiquark

$f_0(500)$  and  $f_0(980)$  are really octet/singlet mixtures

Oldest candidates for non-ordinary  $q\bar{q}$  mesons (Jaffe 76)

**Only the  $\kappa(800)$  or  $K_0^*(800)$  still “Needs Confirmation” @ PDG**

# The resonance is NO LONGER the $\kappa$ nor the $K_0^*(800)$ ,

Citation: C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C*, **40**, 100001 (2016)

**$K_0^*(800)$**   
or  $\kappa$

$$I(J^P) = \frac{1}{2}(0^+)$$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under  $f_0(500)$  (see the index for the page number)

## $K_0^*(800)$ M

VALUE (MeV)	EVTS	DOCUMENT ID
<b>682 ± 29</b>	<b>OUR AVERAGE</b>	Error includes scale
826 ± 49	+49 -34	1338
849 ± 77	+18 -14	1421
841 ± 30	+81 -73	25k
658 ± 13		6 DESCOTES-G.
797 ± 19	± 43	15k 7,8 AITALA

Best analysis:  
Roy-Steiner  
dispersion relations

Plenty of room  
for improvement  
on parameters

Pade sequences

Still "Needs Confirmation" !

Citation: M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018)

**$K_0^*(700)$**

$$I(J^P) = \frac{1}{2}(0^+)$$

also known as  $\kappa$ ; was  $K_0^*(800)$

Needs confirmation. See the mini-review on scalar mesons under  $f_0(500)$  (see the index for the page number).

## $K_0^*(700)$ T-Matrix Pole $\sqrt{s}$

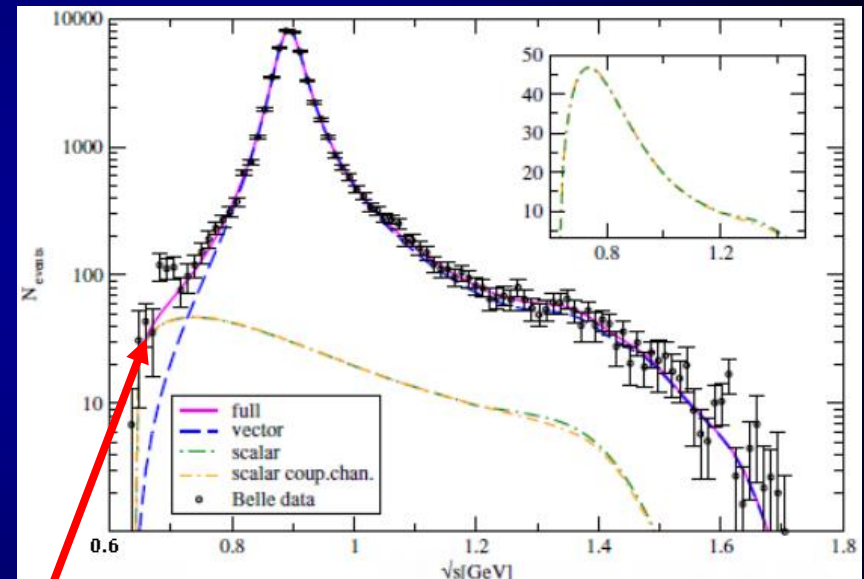
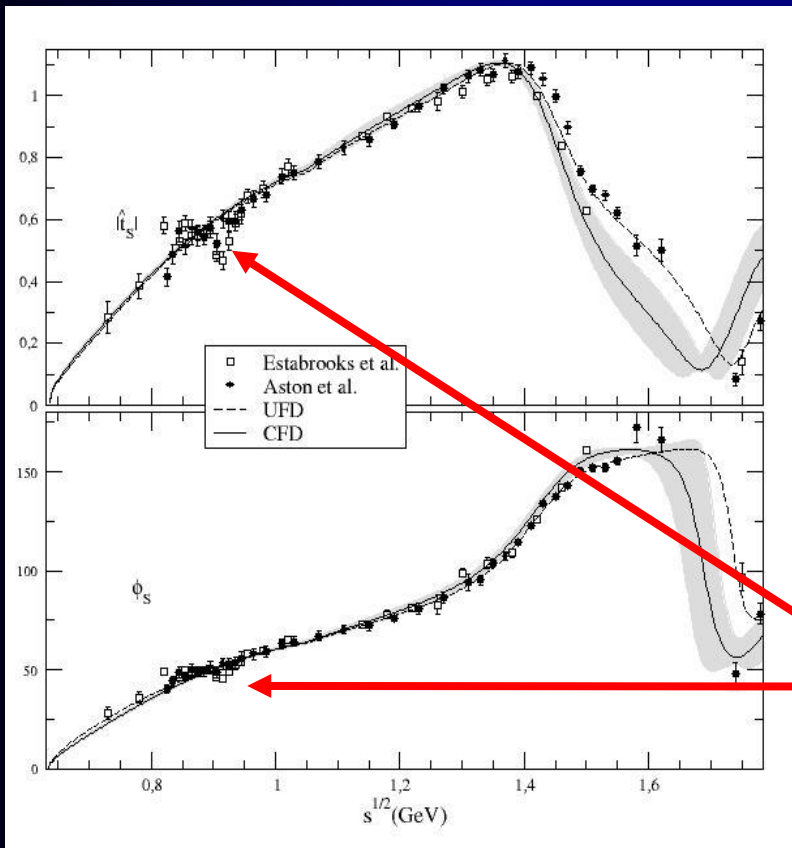
VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
<b>(630–730) – i (260–340) OUR EVALUATION</b>			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(670 \pm 18) - i (295 \pm 28)$	1 PELAEZ	17	RVUE
$(764 \pm 63^{+71}_{-54}) - i (306 \pm 149^{+143}_{-85})$	2 ABLIKIM	11B	BES2 1.3k $J/\psi \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$
$(665 \pm 9) - i (268^{+21}_{-6})$	3 GUO	11B	RVUE
$(849 \pm 77^{+18}_{-14}) - i (256 \pm 40^{+46}_{-22})$	2 ABLIKIM	10E	BES2 1.4k $J/\psi \rightarrow K^\pm K_S^0 \pi^\mp \pi^0$
$(663 \pm 8 \pm 34) - i (329 \pm 5 \pm 22)$	4 BUGG	10	RVUE S-matrix pole

# The problem with the kappa

It is a very wide resonance close to threshold

Either from scattering (SLAC)

Or produced in decays (Belle)



No clear “peak” or phase movement for  $\kappa/K_0^*(800)$  resonance, less so  $K_0^*(700)$

Definitely **NO BREIT-WIGNER** shape

Mathematically correct to use **POLES**

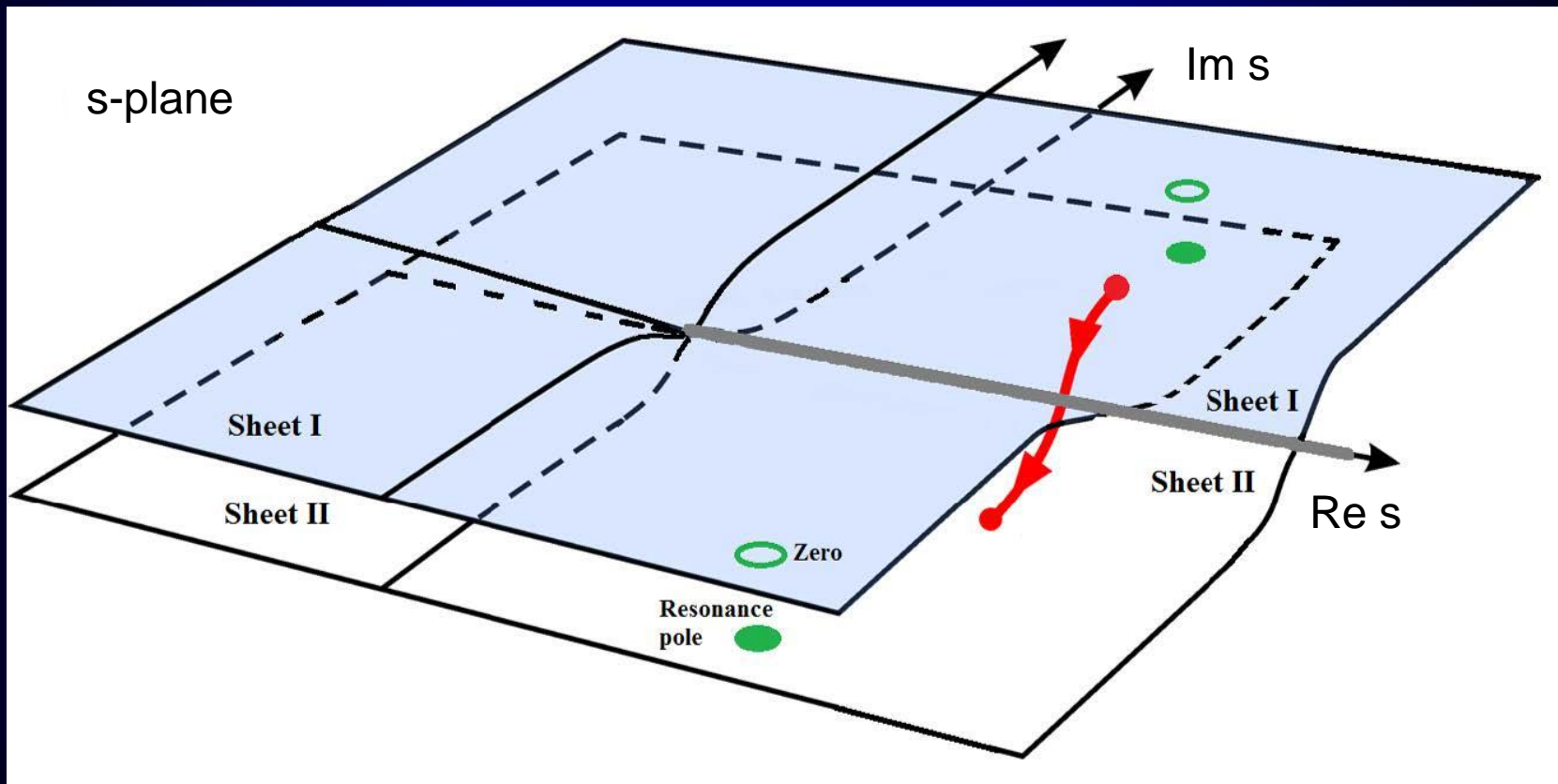
## Resonances as poles

The Breit-Wigner shape is just an approximation for narrow and isolated resonances

The universal features of resonances are their pole positions and residues \*

$$\sqrt{s_{pole}} \approx M - i \Gamma/2$$

\*in the Riemann sheet obtained from an analytic continuation through the physical cut



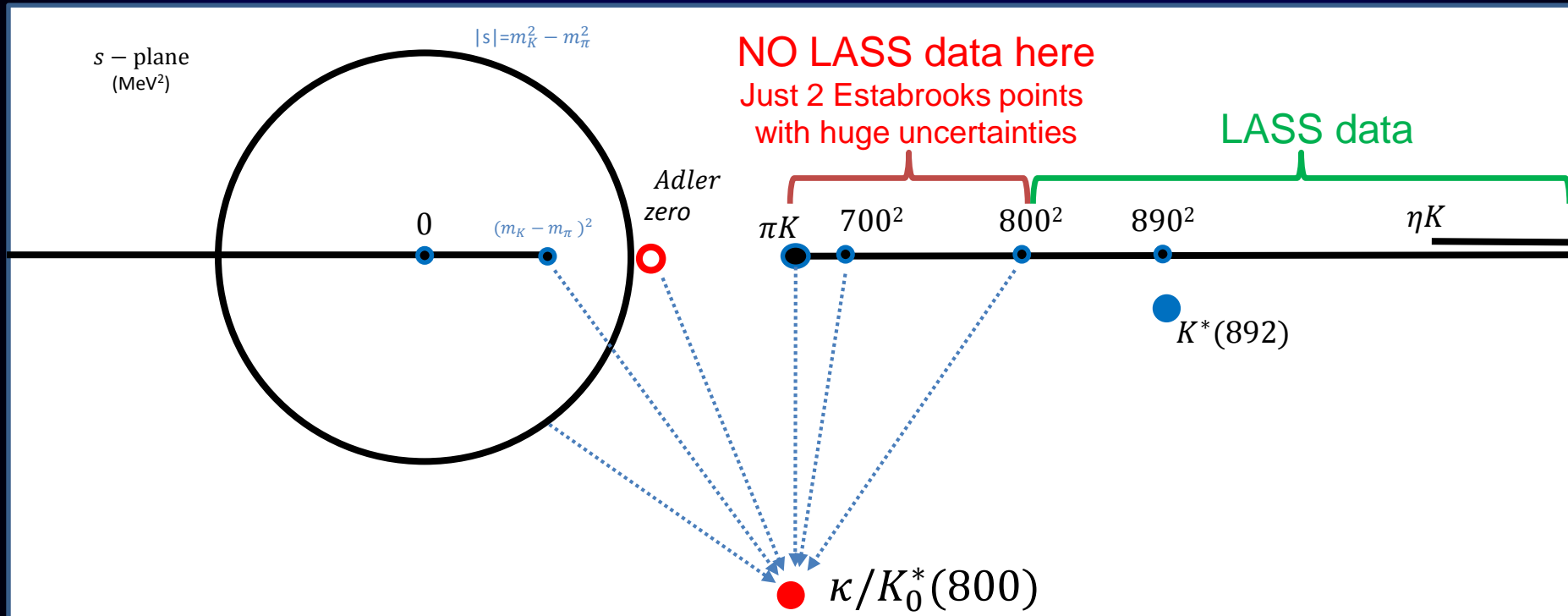
Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

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Analyticity is expressed in the  $s$ -variable, not in  $\sqrt{s}$

# Why so much worries LOW ENERGY and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the  $s$ -variable, not in  $\sqrt{s}$



Important for  
the  $\kappa/K_0^*(800)$

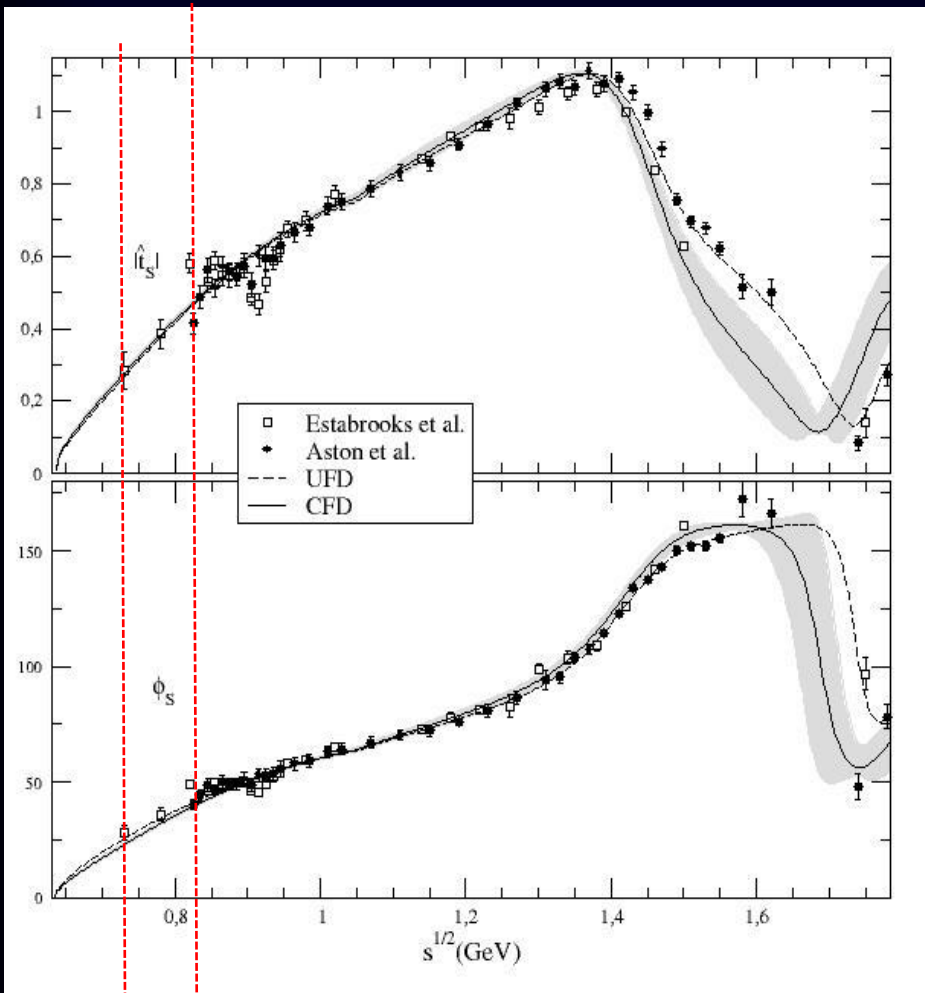
- Threshold behavior (Theory: chiral symmetry)
- Subthreshold behavior (Theory: chiral symmetry  $\rightarrow$  Adler zeros)
- Other cuts (Theory: Left & circular)
- **LOW ENERGY REGION data very relevant for the kappa**



# The problem with data on S-WAVE

Most reliable sets:

- Estabrooks et al. 78 (SLAC)
- Aston et al. 88 (SLAC-LASS) Largest statistics. But measures  $t_{1/2} + t_{3/2}/2$

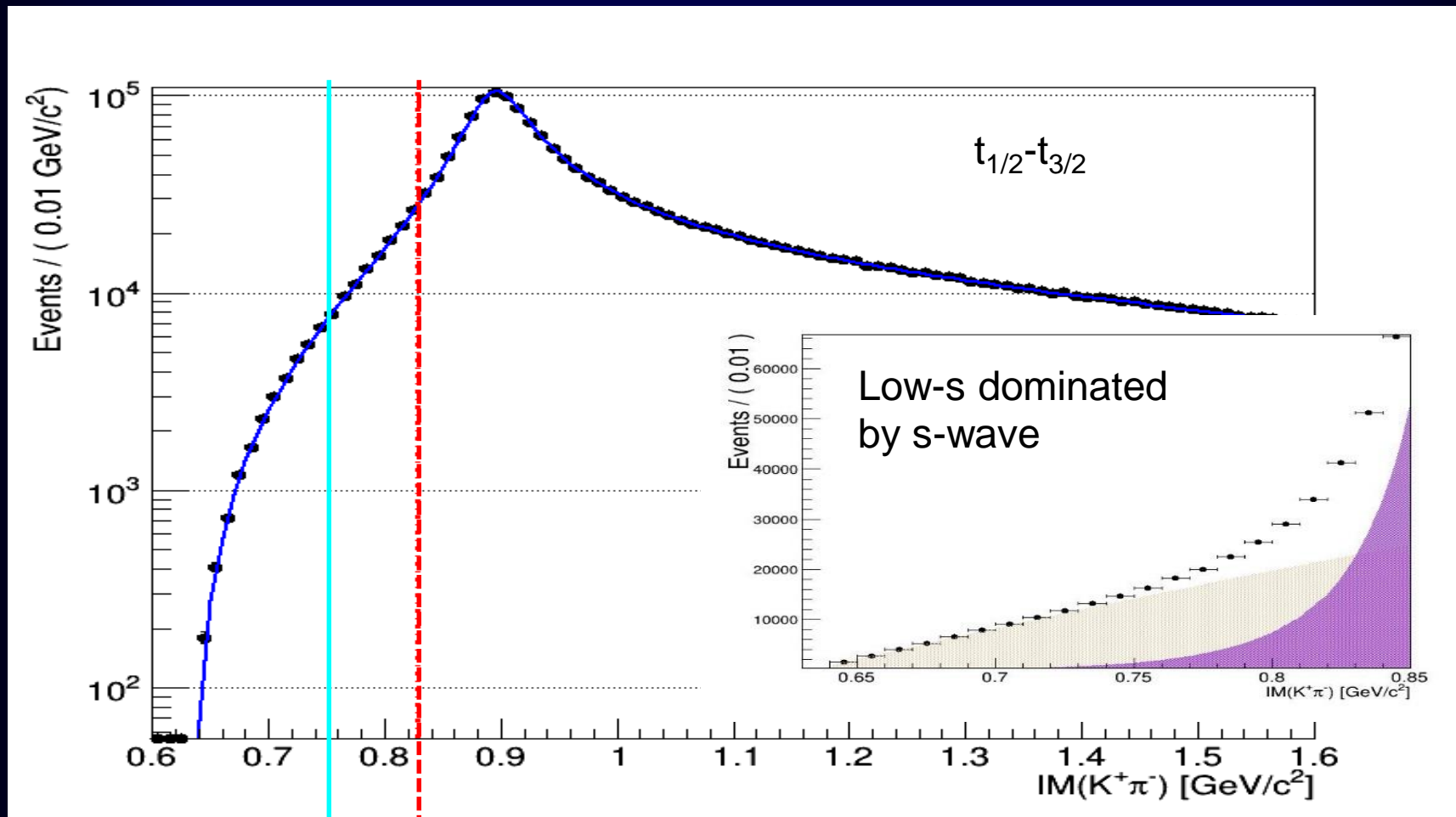


**No LASS Data below 825 MeV.** Only 2 points with huge uncertainties from Estabrooks et al. 78 below 800 MeV

**No data below 725 MeV**

**KLF will improve this**

1) FOUR ORDERS of MAGNITUDE larger tan previous data set



2) 18 NEW energy bins below 825 MeV (there were 2)

3) 11 NEW energy bins below 725 MeV (there were NONE)

- KLF will measure

$$K_L p \rightarrow (K^{*0})p \rightarrow K^+ \pi^- p$$

$$K_L p \rightarrow (\bar{K}^{*0})p \rightarrow K^- \pi^+ p$$

which are sensitive to  $t_{1/2} - t_{3/2}$ .

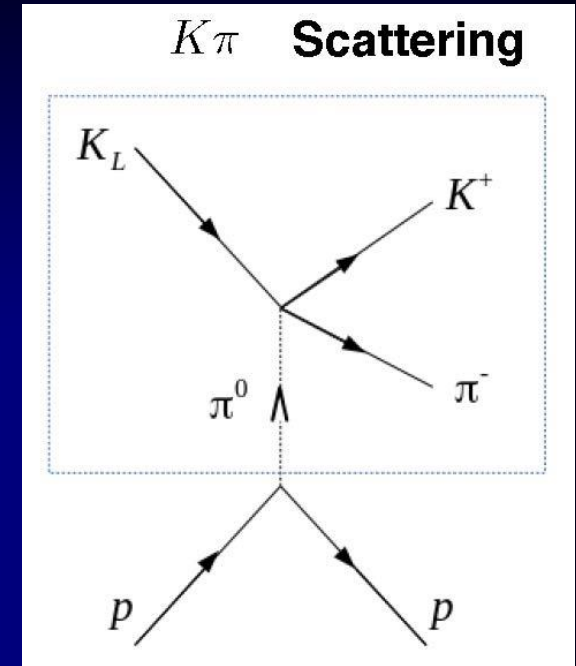
But also

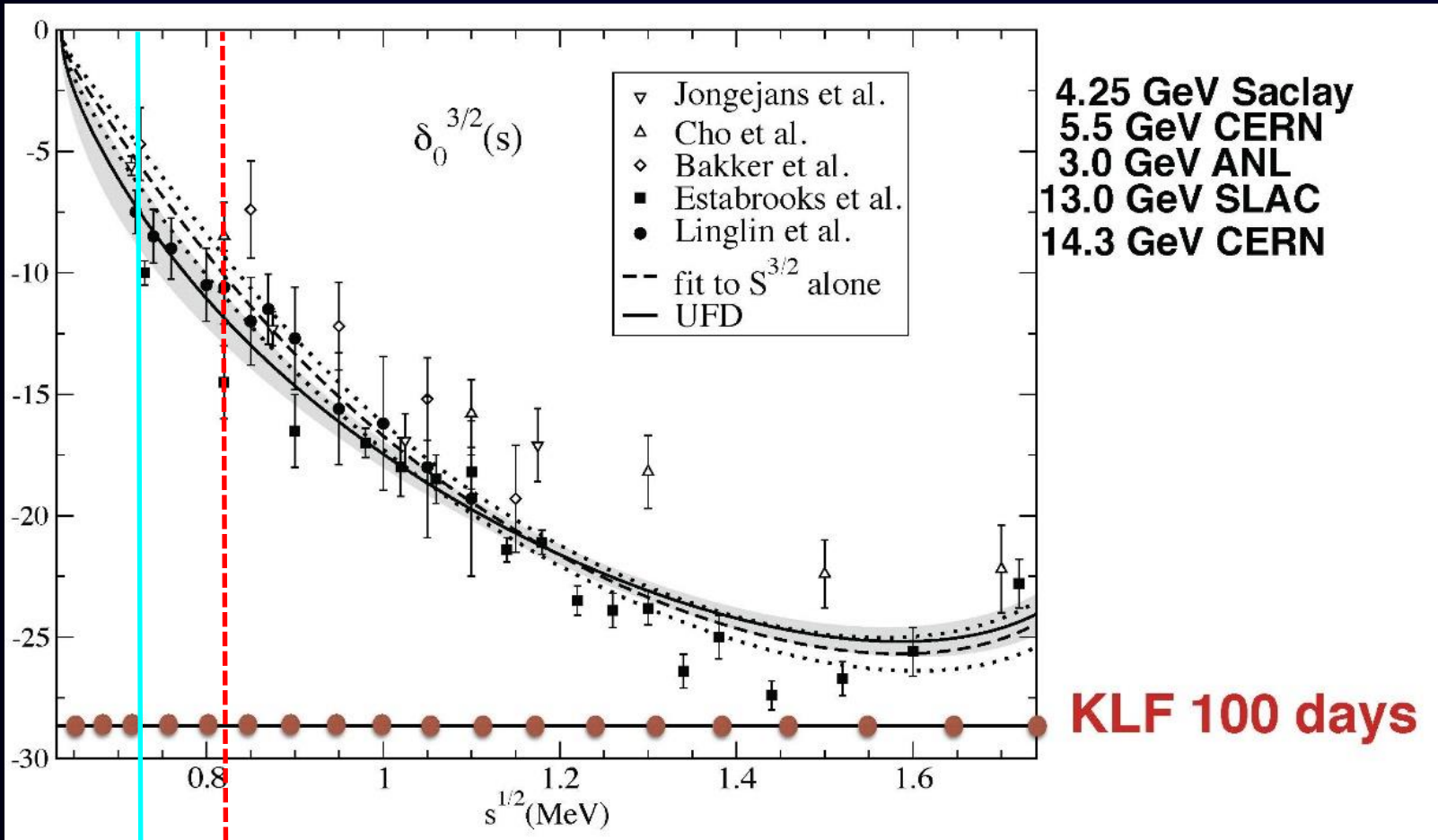
$$K_L p \rightarrow (K^{*0})p \rightarrow K_L \pi^0 p$$

which is sensitive to  $t_{1/2} + 2 t_{3/2}$

In this way the two isospin states can be separated.

For the latter the  $K_L$  will be reconstructed from the missing mass of the proton and the  $\pi^0$  and the invariant mass of the  $K_L \pi^0$  in the missing mass of the proton.



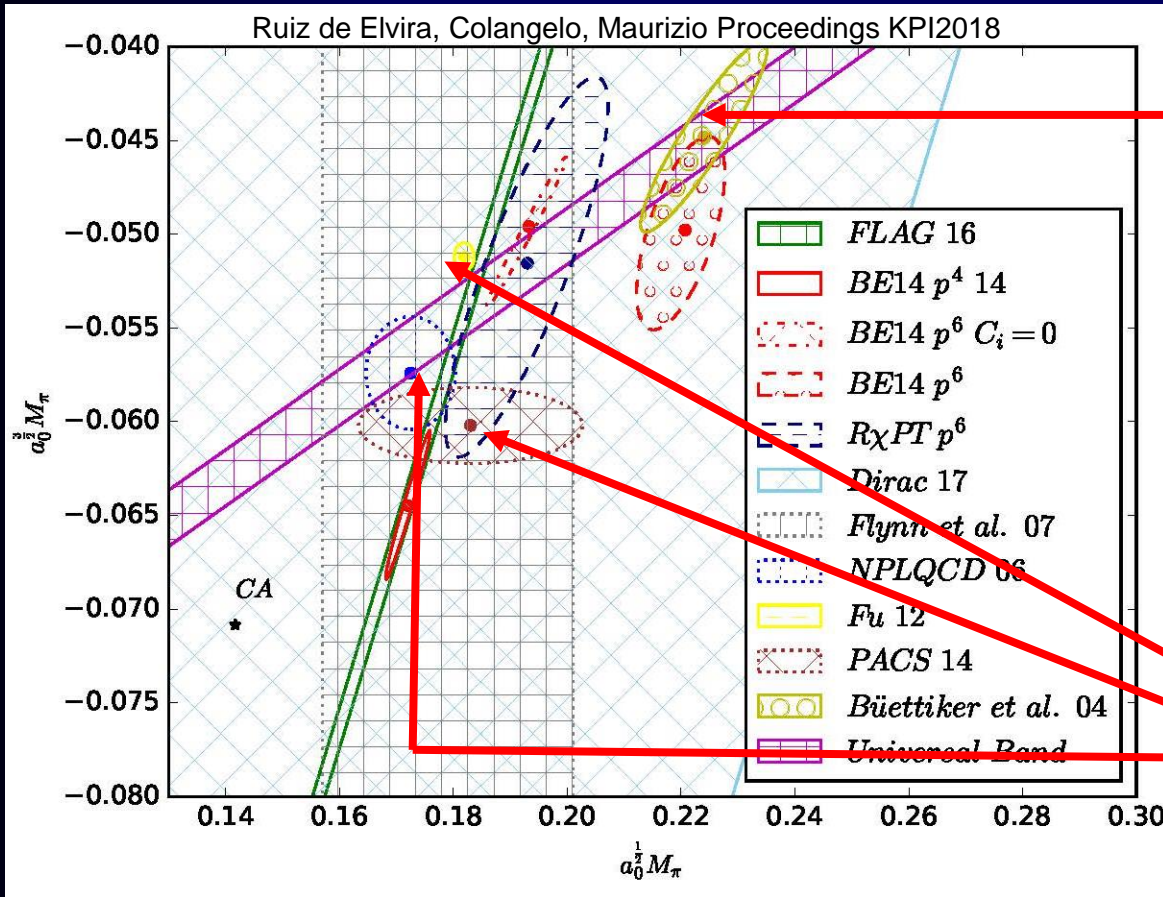


- With the missing mass reconstruction one expects 5 points below 825MeV
- 3 points below the existing data.
- With some timing improvement KLF could double the points.
- 100 x statistics than Estabrooks et al. **Stat. Error bars invisible with KLF.**
- Systematic uncertainties expected at  $\sim 5\%$  level

# Relevance for threshold parameters and ChPT

Present tension between exp+dispersion theory vs. lattice.

Important to understand applicability of SU(3) ChPT & size of strange condensate.



Dispersive solution  
or with dispersive  
constrained fit to data

Lattice

No reliable extraction from DATA. All rely on extrapolations from 750MeV to threshold

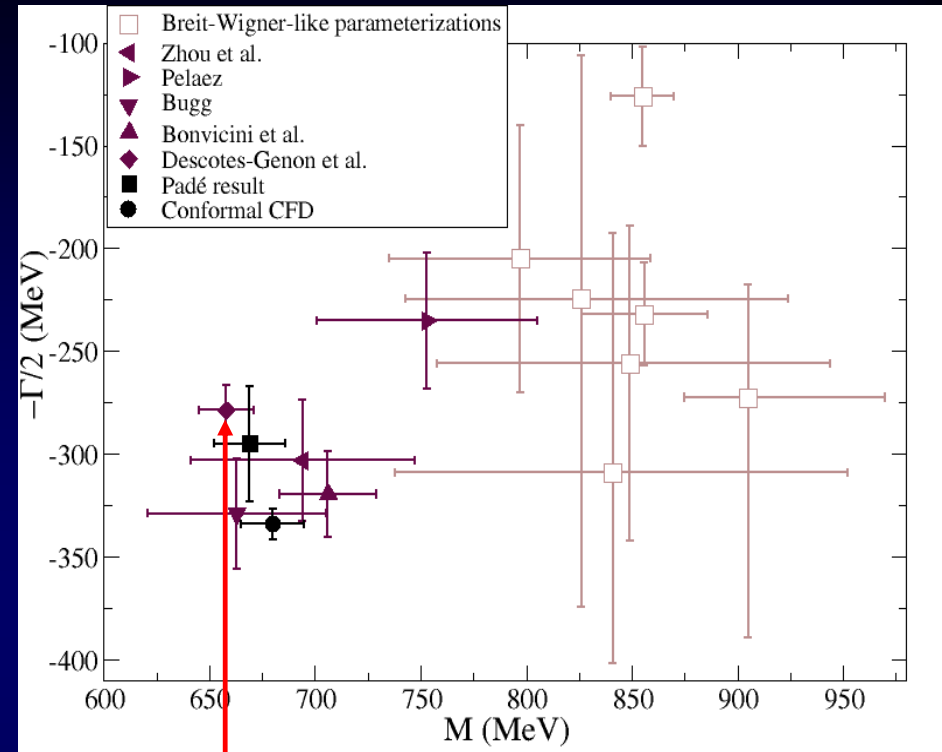
Low-energy and isospin separation @KLF crucial

# The theory problem for the kappa

Much confusion from too simple theoretical models:

- Breit Wigners !!
- No Adler zeros
- No left cuts, no circular cuts, etc...

The rigorous way to extract the pole is with partial wave dispersion relations (Roy-Steiner eqs.)



The only Roy-Steiner calculation so far DOES NOT USE DATA on S-wave below 900 MeV (In a sense is a prediction)

# Even Lattice + K-matrix gets a kappa

## Resonances in coupled $\pi K, \eta K$ scattering from quantum chromodynamics

Jozef J. Dudek,<sup>1,2,\*</sup> Robert G. Edwards,<sup>1</sup> Christopher E. Thomas,<sup>3</sup> and David J. Wilson<sup>2</sup>

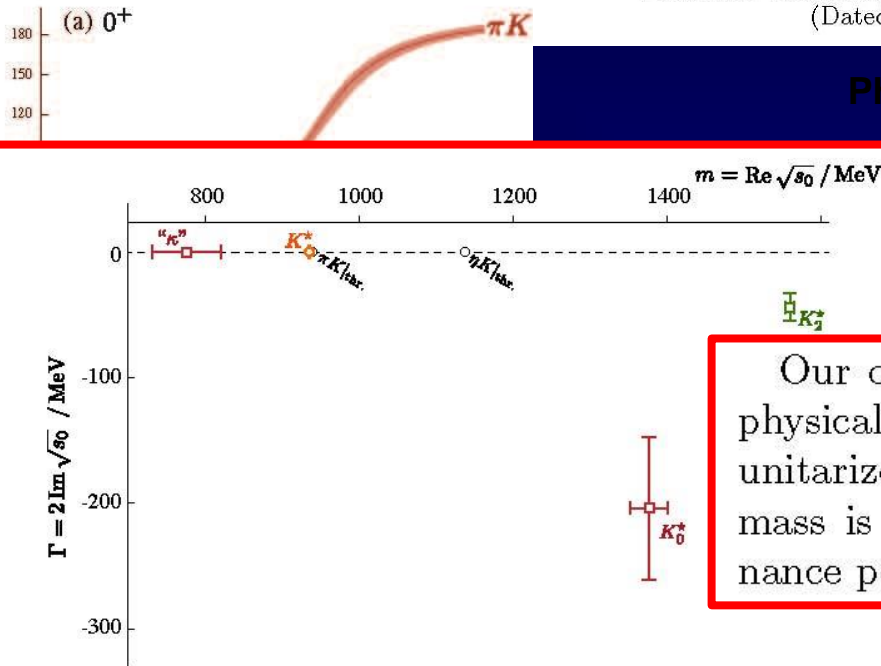
(for the Hadron Spectrum Collaboration)

<sup>1</sup>Theory Center, Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

<sup>2</sup>Department of Physics, Old Dominion University, Norfolk, VA 23529, USA

<sup>3</sup>DAMTP, University of Cambridge, Cambridge, UK

(Dated: June 18, 2014)



Phys.Rev.Lett. 113 (2014) 18, 182001

$M_\pi = 391$  MeV

$M_K = 549$  MeV

Our observation of a  $0^+$  pole below threshold on unphysical sheets agrees with the qualitative prediction of unitarized chiral perturbation theory that as the pion mass is increased above its physical value, the  $\kappa$  resonance pole becomes a virtual bound state [13].

FIG. 3. Pole singularities of partial-wave  $t$ -matrices in the complex plane for  $J^P = 0^+$  (red),  $1^-$  (orange) and  $2^+$  (green). Squares correspond to poles found on unphysical sheets, circle is a physical sheet bound-state.

around the  $\pi K$  threshold. Points determined times:  $16^3$  (boxes),  $20^3$  (circles) and  $24^3$  (triangles).  $\cot \delta_1 = (m_R^2 - s) \frac{6\pi\sqrt{s}}{g_R^2}$ . (c)  $J^P = 2^+$  amplitudes -

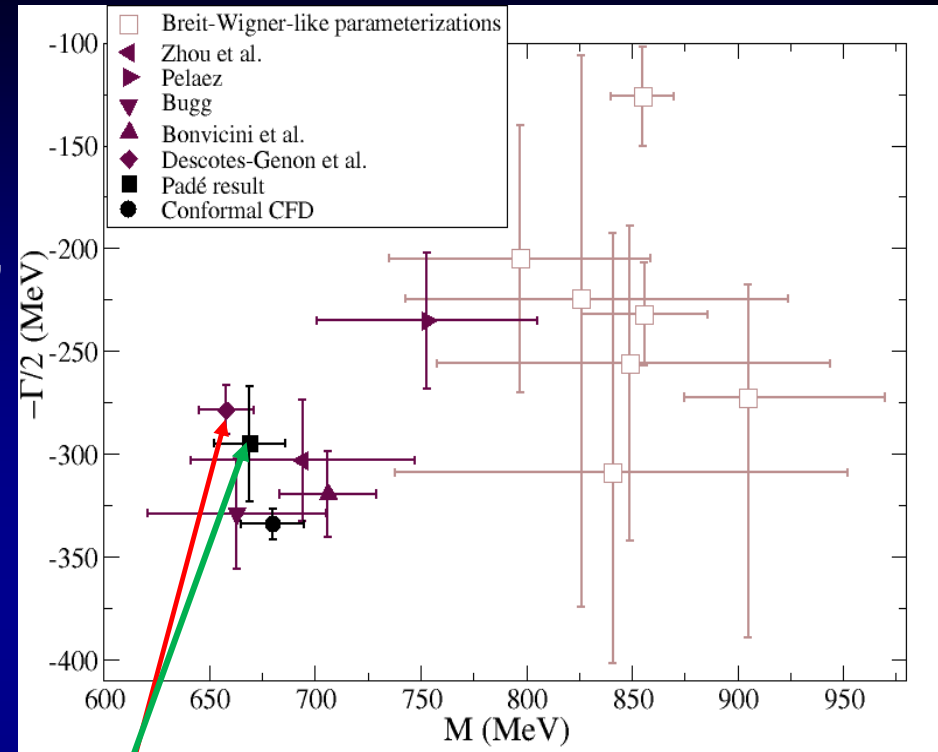
- So, there seems to be a virtual kappa on the lattice...

**but at high masses!!**

# The theory problem for the kappa

There are some sound extractions, with good chiral and analyticity properties, but still with some model dependence and without low-energy data...

**NOT GOOD FOR PRECISION**



Reference	Pole (MeV) $\sqrt{s_\kappa} \equiv M - i\Gamma/2$	Comment
Bonvicini [221]	$706.0 \pm 24.6 - i 319.4 \pm 22.4$ MeV	<i>T</i> -matrix pole model from CLEO
Bugg [222]	$663 \pm 42 - i 342 \pm 60$ MeV	Model with LO Chiral symmetry
Pelaez [139]	$753 \pm 52 - i 235 \pm 33$ MeV	Unitarized ChPT up to NLO
Conformal CFD [125]	$680 \pm 15 - i 334 \pm 8$ MeV	Conformal parameterization from dispersive fit
Padé [143]	$670 \pm 18 - i 295 \pm 28$ MeV	Analytic local extraction from dispersive fit
Zhou <i>et al.</i> [207]	$694 \pm 53 - i 303 \pm 30$ MeV	partial-wave dispersion relation. Cutoff on left cut.
Descotes-Genon <i>et al.</i> [22]	$658 \pm 13 - i 279 \pm 12$ MeV	Roy-Steiner prediction. No S-wave data used below 1 GeV.



- Impose Forward Dispersion Relations on fits to data.

(García-Martín, Kaminski,JRP, Ruiz de Elvira, Ynduráin, Rodas)

Use any parameterization to **fit DATA** imposing FDR within uncertainties.

(But you can use physical inspiration for clever choices of parameterizations)

Also needs input on other waves and high energy.

**USE ROY-STEINER EQUATIONS TO DETERMINE THE POLE**

(but not the parameterizations)

# Dispersive analysis of $\pi K$ scattering DATA up to 1.6 GeV

(not a solution of dispersion relations, but a constrained fit)

A.Rodas & JRP, PRD93,074025 (2016)

First observation:

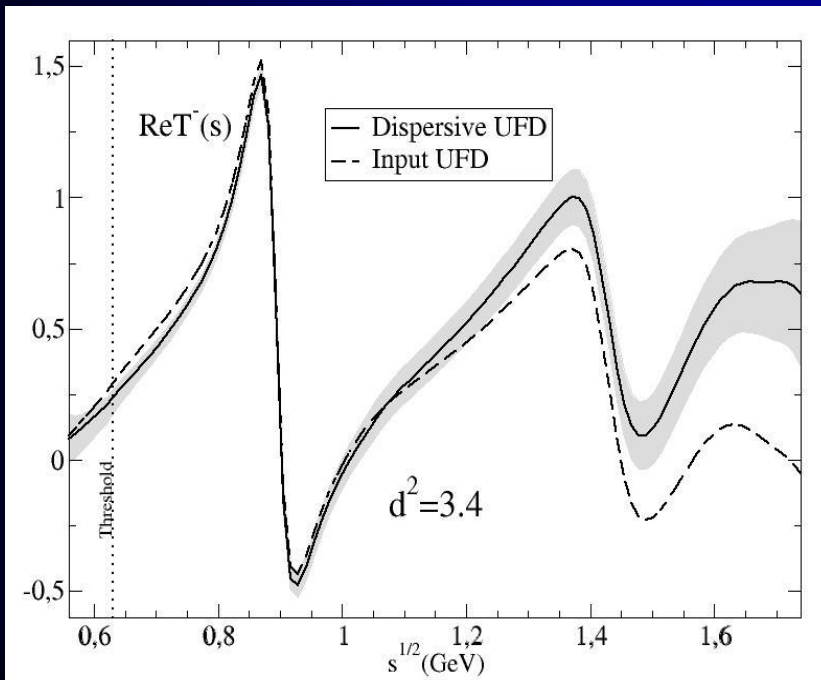
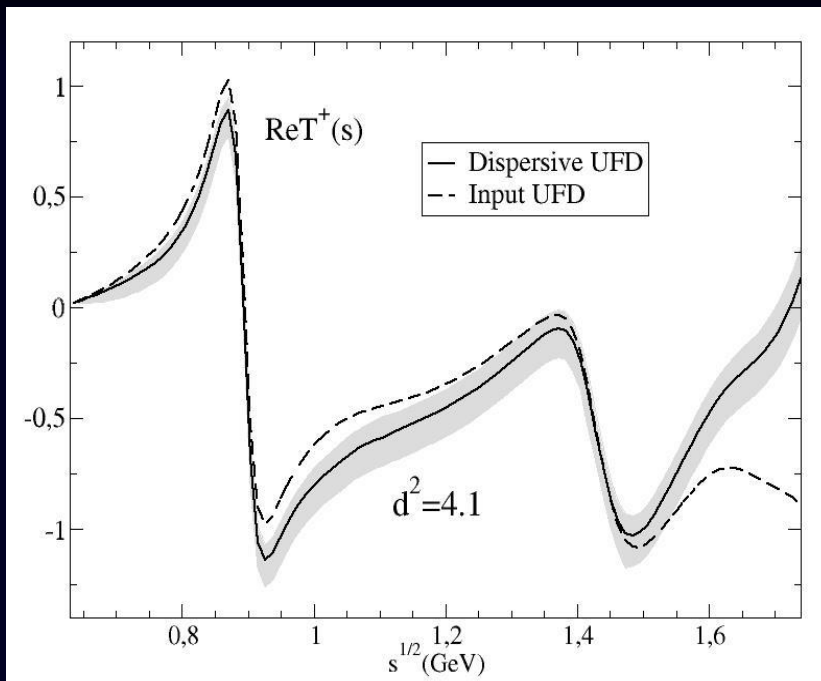
Forward Dispersion relations

Not well satisfied by data

Particularly at high energies

So we use

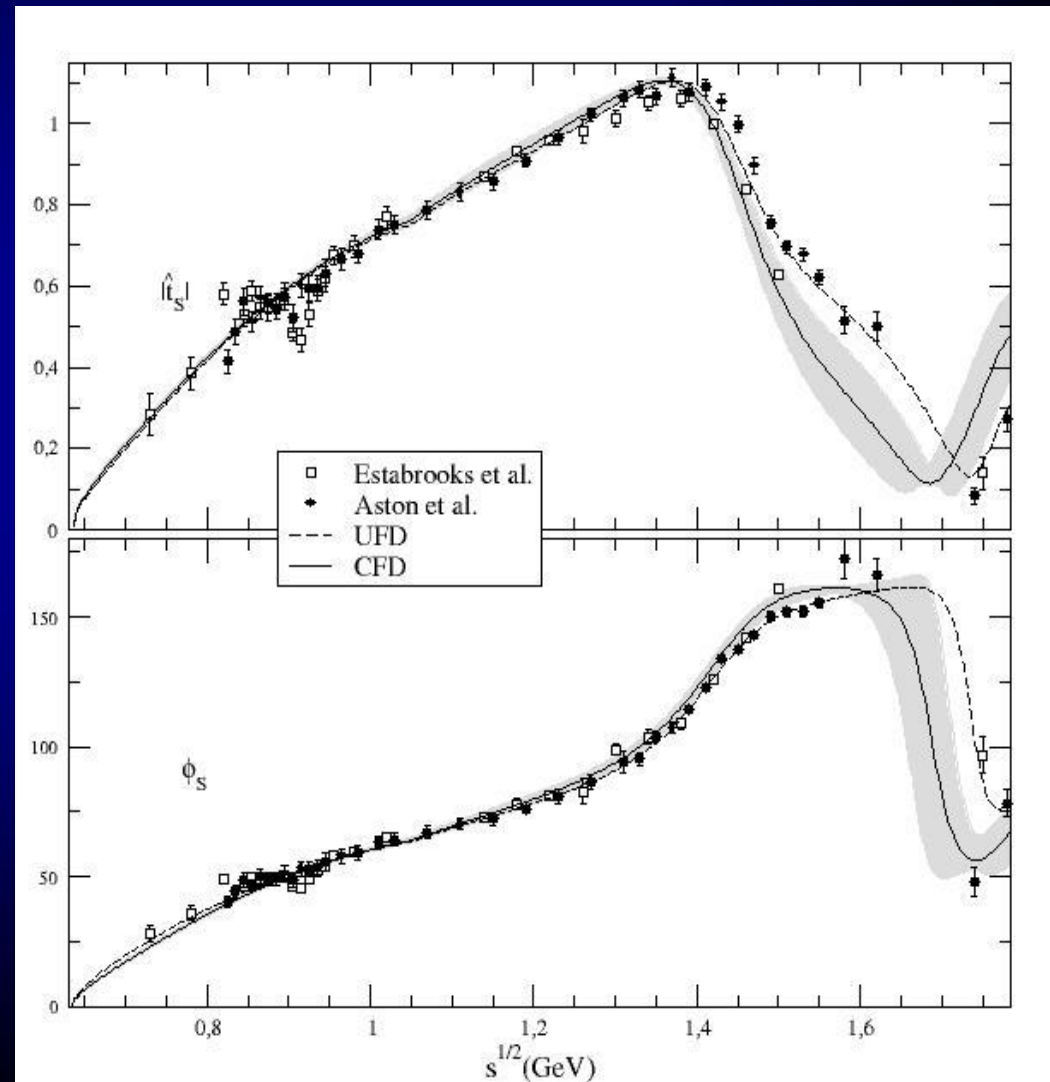
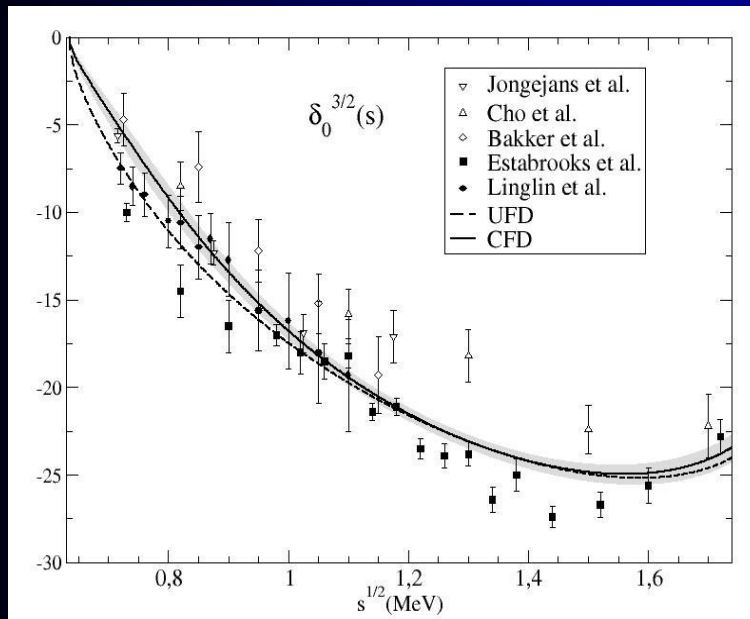
Forward Dispersion Relations as CONSTRAINTS on fits

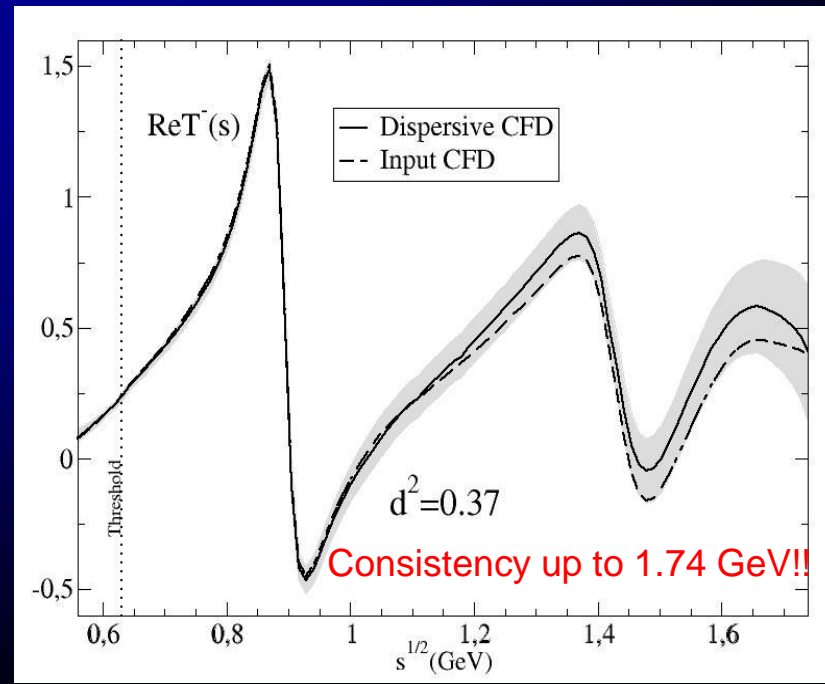
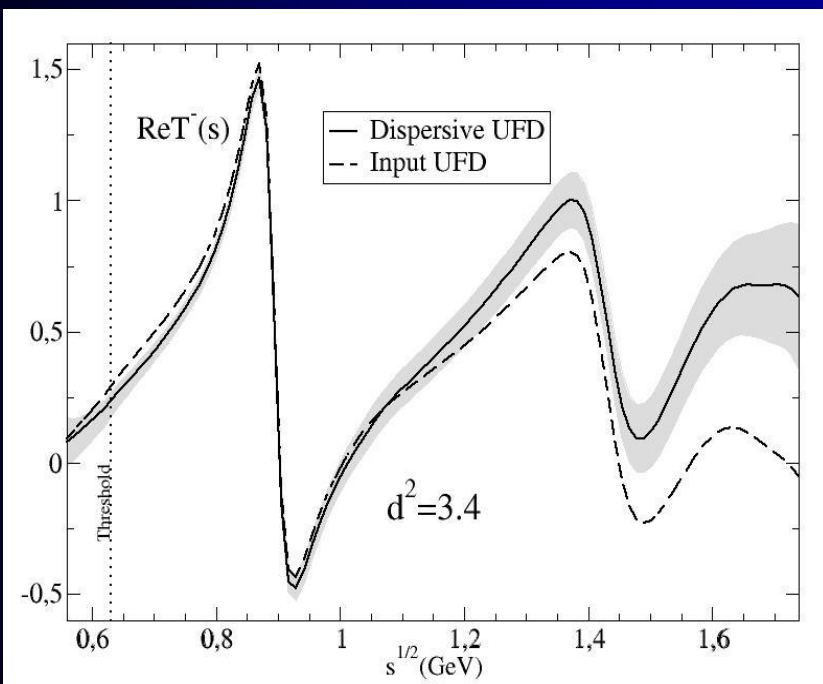
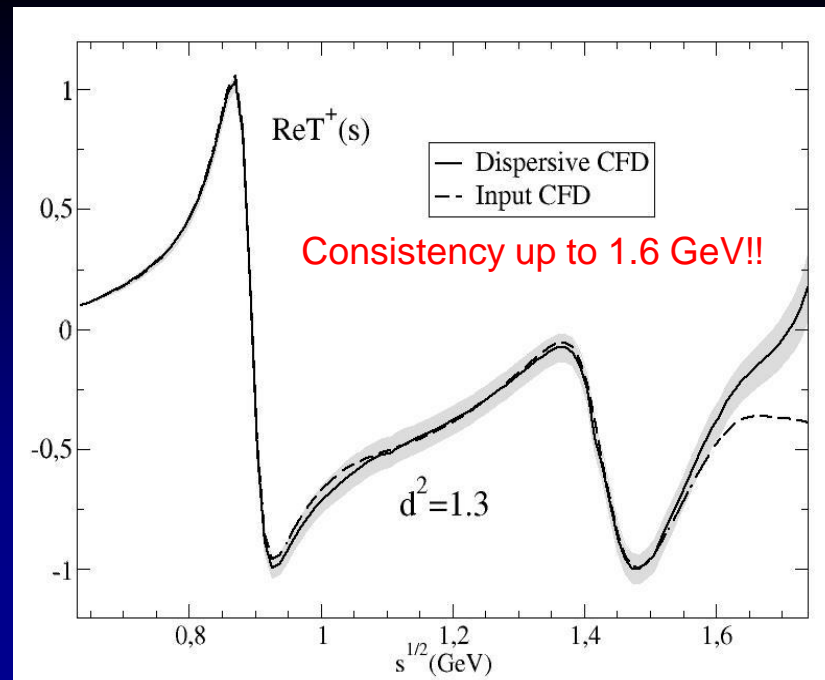
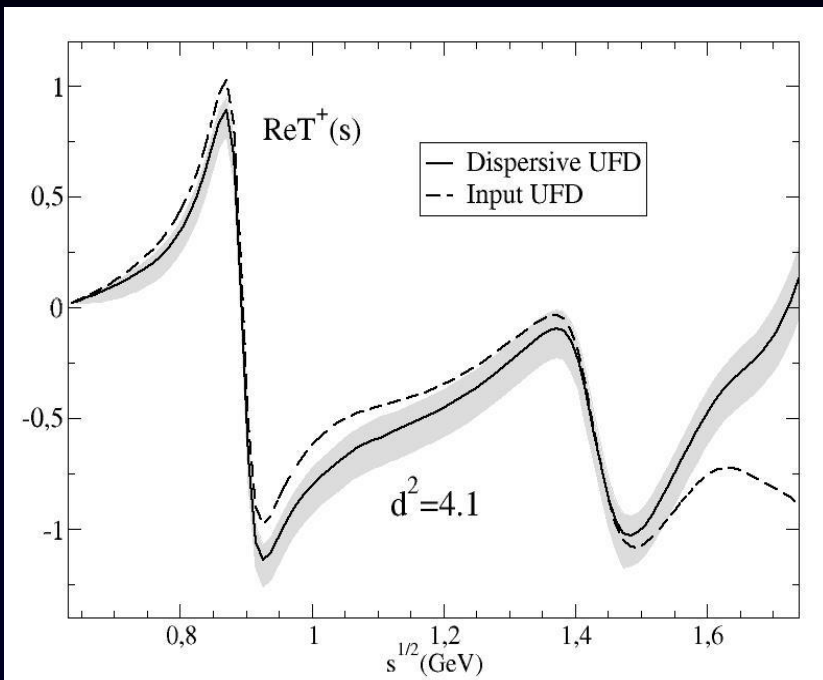


# From Unconstrained (UFD) to Constrained Fits to data (CFD)

S-waves. The most interesting for the  $K_0^*$  resonances

Largest changes from UFD to CFD  
at higher energies





# Model independent analysis with existing data

## Roy-Steiner SOLUTION from Paris group

Descotes-Genon-Moussallam 2006

## Our Roy-Steiner analysis of FIT to data

JRP, A. Rodas, in preparation

**NEW!!**

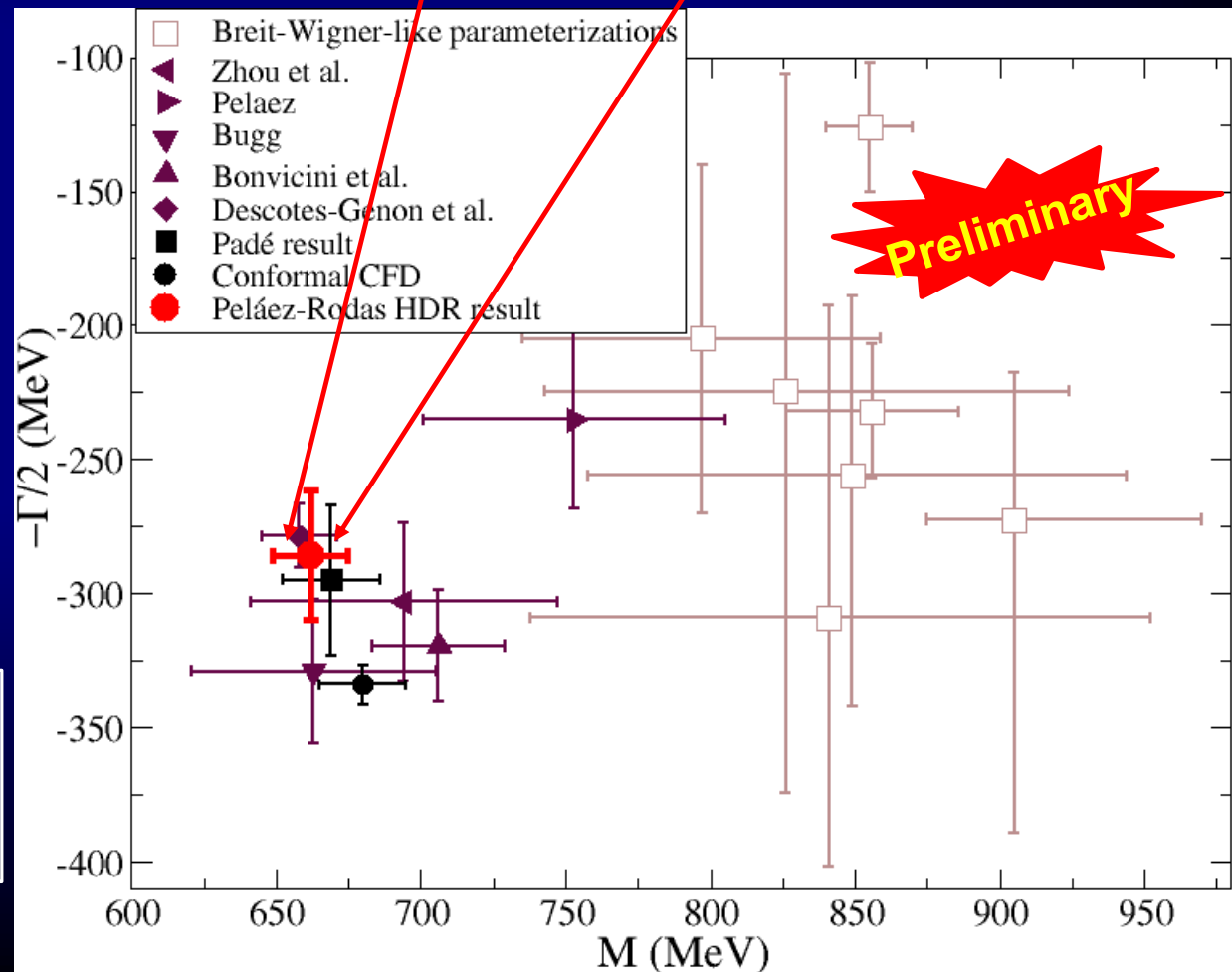
We have:

- Constrained Fit to data (not solved)
- Improved P-wave (data OK)
- Used Hyperbolic DR both in real axis and complex plane.
- Improved Pomeron
- Constrained  $\pi\pi \rightarrow KK$  input with DR
- Other technicalities

Independent dispersive  
 $K_0^*(800)$  determination  
**USING EXISTING DATA**

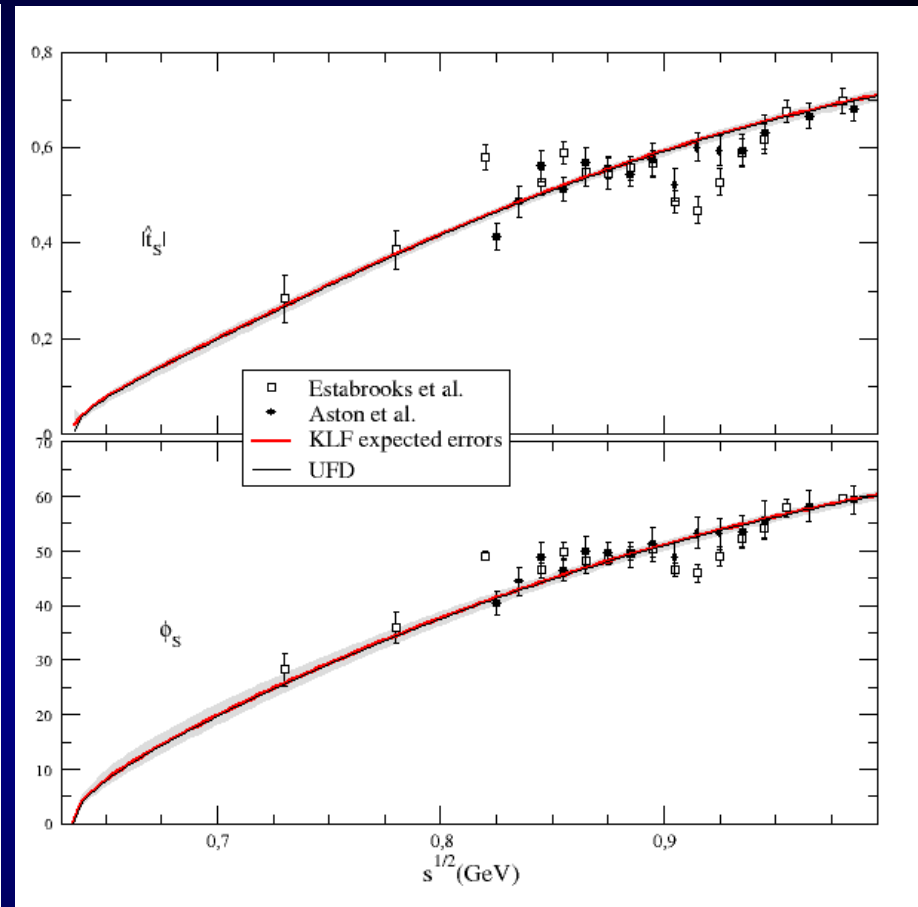
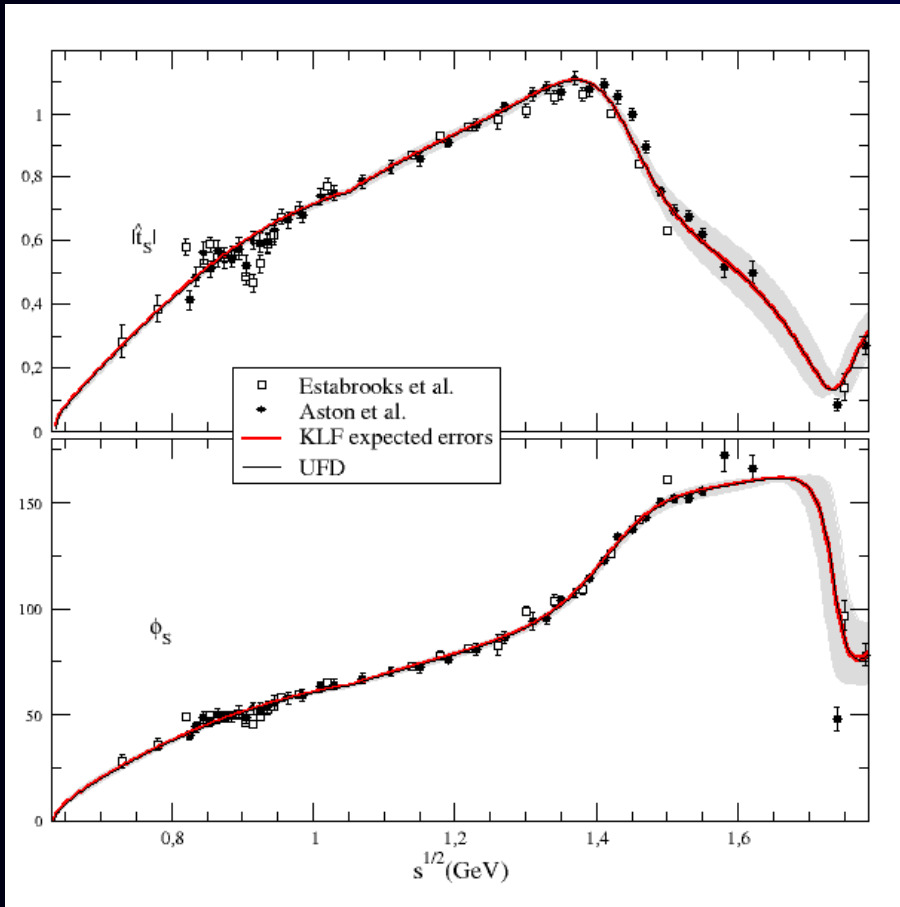
$(658 \pm 13) - i(278.5 \pm 12)$  MeV

$(663 \pm 14) - i(288 \pm 27)$  MeV



# Model independent analysis WITH EXPECTED KLF DATA

We have run the same procedure but with 50x statistics of KLF on LASS data



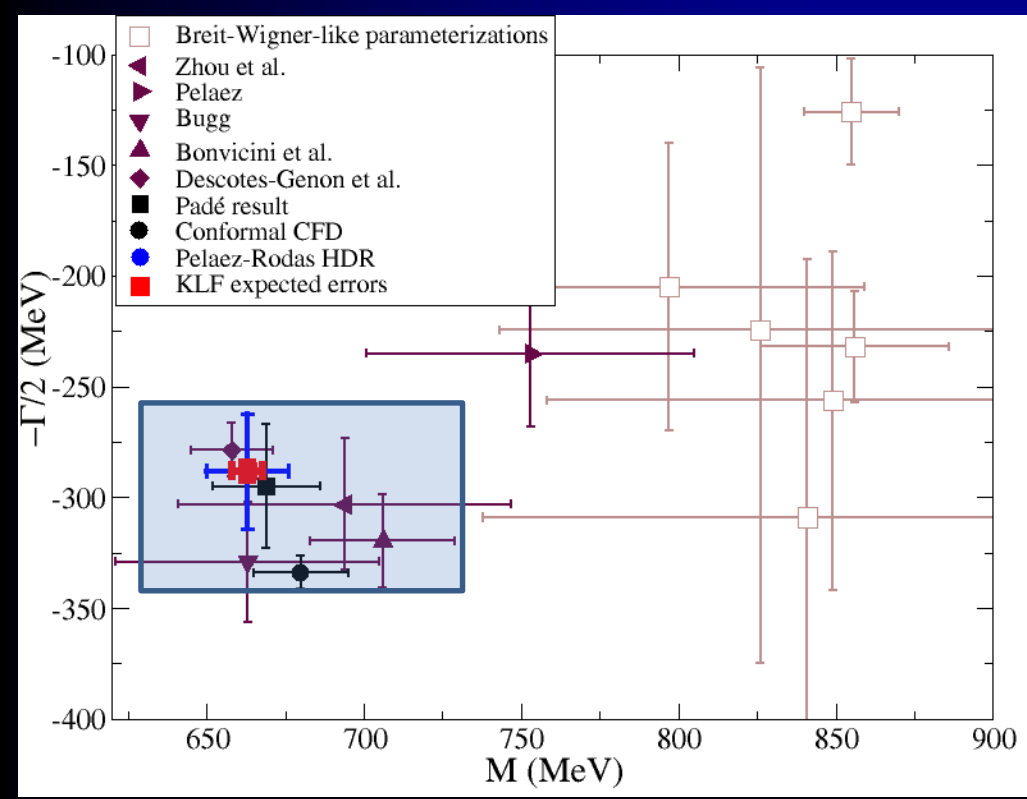
Uncertainty bands of the size of red line (not vanishing due to systematics)

Note we are NOT CONSIDERING NEW POINTS AT LOW ENERGIES

(that will decrease systematics)

# Model independent analysis WITH EXPECTED KLF DATA

Reference	Pole (MeV) $\sqrt{s_p} \equiv M - i\Gamma/2$	Comment
Bonvicini [221]	$706.0 \pm 24.6 - i 319.4 \pm 22.4$ MeV	$T$ -matrix pole model from CLEO
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Pelaez-Rodas HDR [20,126]	$663 \pm 14 - i 288 \pm 27$ MeV	Roy-Steiner analysis of scattering data
<b>KLF expected errors</b>	<b><math>663 \pm 6 - i 288 \pm 5</math> MeV</b>	<b>As previous line but with KLF expected errors</b>



with EXISTING DATA  
 **$(663 \pm 14) - i(288 \pm 27)$  MeV**

**WITH KLF**  
 **$(663 \pm 6) - i(288 \pm 5)$  MeV**

**HUGE IMPROVEMENT!!**

Systematic errors should be reduced with further points near threshold

## Remarks: Alternative/simpler methods

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Dispersion relations needed to extract pole, but also to complete the region where there is no data.

They need data from other partial waves, high energies, etc... which induce systematic uncertainties

However, with additional points below 800 or 750 MeV one might not even need to use Dispersion Relations in the real axis or to continue to the complex plane.

Other analytic methods, like sequences of Padé approximants, which need dense grids of data points, could be used.

In that case, **kappa determinations based on KLF data alone**, will become competitive with dispersion relations.



## Summary I

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- KLF plans to measure  $K\pi$  with very high statistics (estimated 50x existing data)
- Will provide isospin  $1/2$  and  $3/2$  separation. This will require  $K_L$  reconstruction
- Will provide: 18 data points below 825 MeV (LASS reach) for charged final state.  
11 of them below 725 where data does not exist
- Will provide: 5 data points below 825 MeV (LASS reach) for neutral final state.  
3 of them below 725 where data does not exist
  
- The points near threshold and the large statistics will determine precisely the value of the scattering lengths and determine the convergence radius and applicability of SU(3) ChPT, size of the strange condensate, etc...

## Summary II

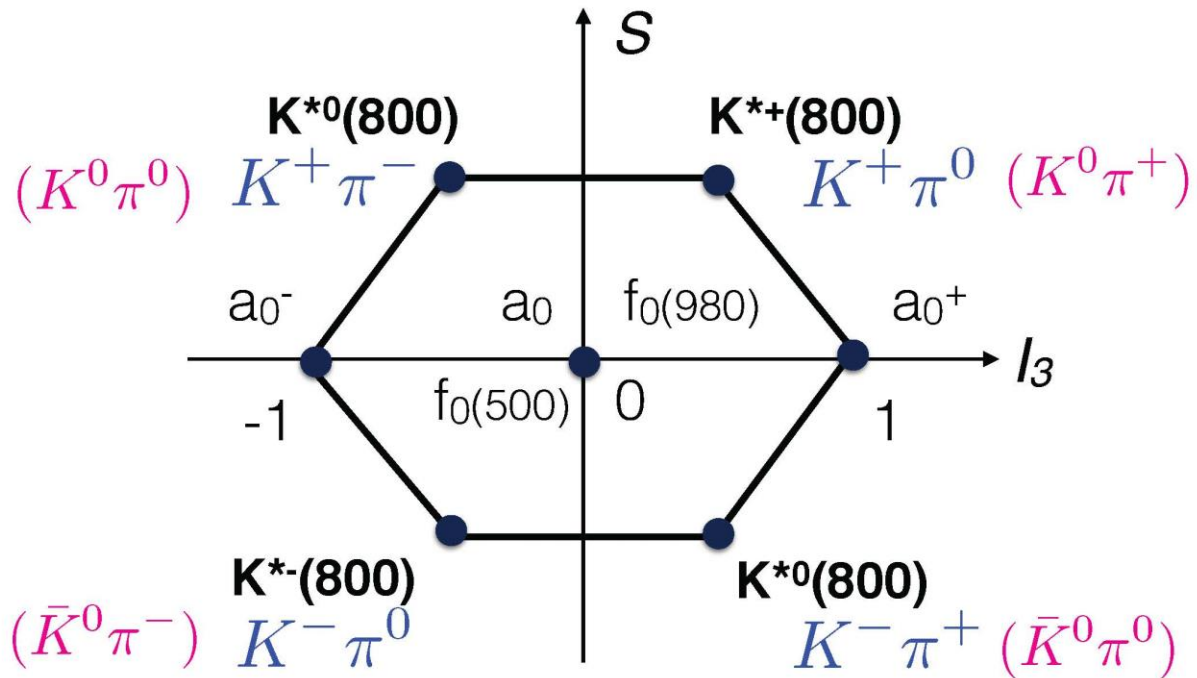
- Just with the statistical improvement + dispersion theory+ other input, a model independent analysis of the much debated  $\kappa/K_0^*(800)$  will reduce the uncertainties in its mass by more than 2 and the width (coupling) by more than 5
- Systematic effects will be reduced with the new data points close to threshold.
- This may allow for a precise determination with KLF data only with other rigorous methods based on analyticity.

This should finally settle the controversy confirming the existence of the  $\kappa/K_0^*(800)$  and determining precisely the value of its parameters. It will also fix the controversy about the existence of a light scalar nonet and its non-ordinary nature.

- KLF will also improve the precision of the  $K_1^*(892)$ ,  $K_1^*(1410)$  and  $K_0^*(1430)$

SPARE SLIDES

## Scalar Meson Nonet

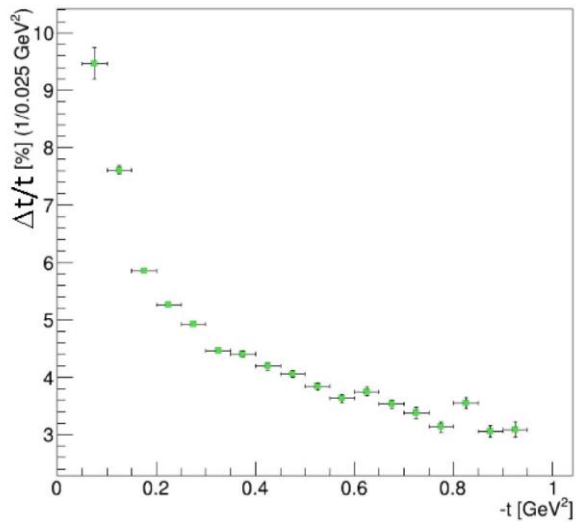


Four states called  $\mathcal{K}$

still need further confirmation(PDG)

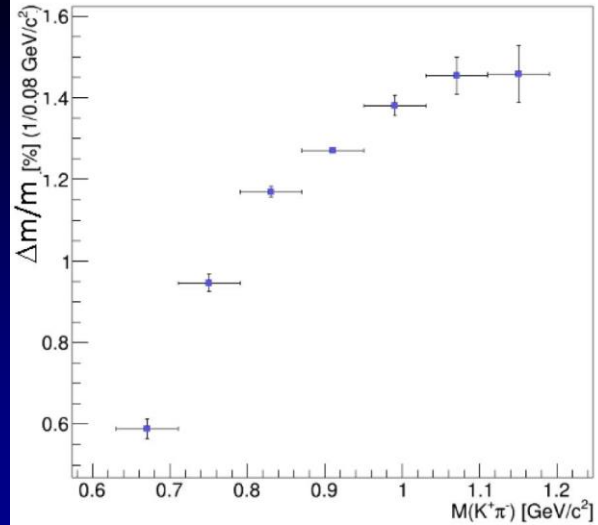
We can measure all of them

Four Momentum Resolution for  $K_L p \rightarrow K^+ \pi^- p$



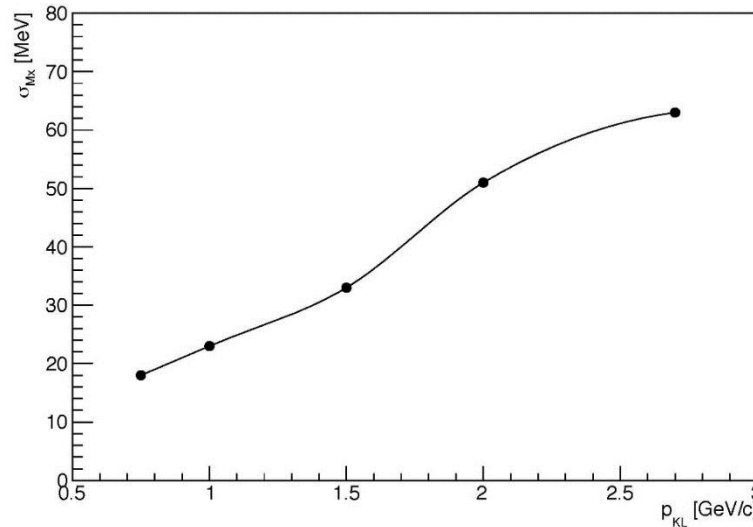
*-Good resolution at low- $t$  is needed to be on pion pole*

$K^+ \pi^-$  Invariant Mass Resolution for  $K_L p \rightarrow K^+ \pi^- p$



*-Binning in  $\sim 10 \text{ MeV}$  will cover almost entire elastic  $K$ - $\pi$  scattering range*

Missing mass resolution off proton



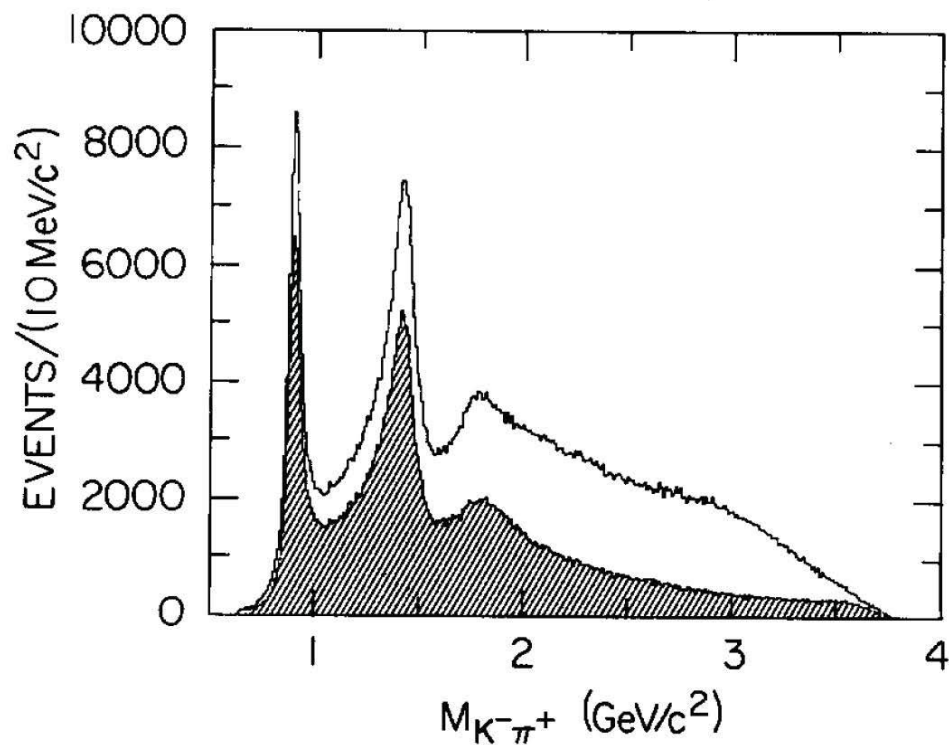


FIG. 1. The  $K^-\pi^+$  invariant mass distribution for events with  $|t'| \leq 1.0$   $(\text{GeV}/c)^2$ . The unshaded curve contains all events while the cross-hatched curve contains events with  $n\pi^+$  mass greater than  $1.7$   $(\text{GeV}/c)^2$ .

We use that

$$I(\pi) = 1, I_3(\pi^0) = 0, \quad (1)$$

$$I(K) = 1/2, I_3(K^0) = -1/2, I_3(\bar{K}^0) = 1/2, \quad (2)$$

and that

$$\langle K_L | = \frac{\langle K^0 | + \langle \bar{K}^0 |}{\sqrt{2}}, \quad (3)$$

$$\langle K_S | = \frac{\langle K^0 | - \langle \bar{K}^0 |}{\sqrt{2}}, \quad (4)$$

now by construction

$$\langle K_L \pi^0 | = \frac{\langle K^0 \pi^0 | + \langle \bar{K}^0 \pi^0 |}{\sqrt{2}}, \quad (5)$$

$$\langle K_S \pi^0 | = \frac{\langle K^0 \pi^0 | - \langle \bar{K}^0 \pi^0 |}{\sqrt{2}}, \quad (6)$$

so that

$$\langle K_L \pi^0 | T | K_S \pi^0 \rangle = \frac{1}{2} (\langle K^0 \pi^0 | T | K^0 \pi^0 \rangle - \langle \bar{K}^0 \pi^0 | T | \bar{K}^0 \pi^0 \rangle). \quad (7)$$

**The minus sign was a plus in the previous calculation**

Now one can use the Clebsch-Gordan coefficients for the states with defined  $I_3$

$$\langle K^0 \pi^0 | = \frac{1}{\sqrt{3}} \langle 1/2, -1/2 | + \frac{\sqrt{2}}{\sqrt{3}} \langle 3/2, -1/2 |, \quad (8)$$

$$\langle \bar{K}^0 \pi^0 | = -\frac{1}{\sqrt{3}} \langle 1/2, 1/2 | + \frac{\sqrt{2}}{\sqrt{3}} \langle 3/2, 1/2 |. \quad (9)$$

Finally by introducing this coefficients in Eq. (7) we get

$$\langle K_L \pi^0 | T | K_S \pi^0 \rangle = \frac{1}{2} (T^{1/2}/3 + 2T^{3/2}/3 - T^{1/2}/3 - 2T^{3/2}/3) = 0 \quad (10)$$

$$\langle K_L \pi^0 | T | K_L \pi^0 \rangle = \frac{1}{2} (T^{1/2}/3 + 2T^{3/2}/3 + T^{1/2}/3 + 2T^{3/2}/3), \quad (11)$$

$$= T^{1/2}/3 + 2T^{3/2}/3 \quad (12)$$