Departamento de Física Teórica
Institute of Particle & Cosmos Physics (IPARCOS)
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Strange light-scalar mesons: Spectroscopy & scattering
Where do we stand?

J. R. Peláez

JRP. Physics Reports 658-(2016)-1
JRP, A.Rodas, J. Ruiz de Elvira, Eur. Phys. J. Spec. Top. (2021) 230:1539
JRP & A.Rodas, Physics Reports 969-(2022)-1

Jlab Physics Division Seminar
Associated with the 7th KLF Collaboration Meeting
September 19th 2023

Supported by:

Motivation to study the scattering of pions & kaons

- π,K,η are Goldstone Bosons of QCD → Test Chiral Symmetry Breaking

- π,K appear as final products of almost all hadronic processes: B,D, decays, CP violation…

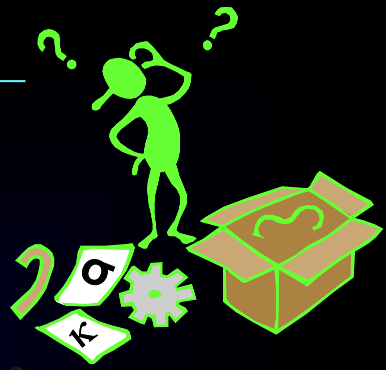
- **SPECTROSCOPY**:
  Scattering main or relevant source for PDG parameters of most light resonances.

  - Relevant for glueball identification

  - **CRYPTOEXOTICS**: The controversial **light scalar resonances** appear here: **σ/f_0(500)**, f_0(980), a_0(980) and strange κ/**K^*_0(700)**.
    Strong indications for predominant non quark-antiquark nature of light scalars

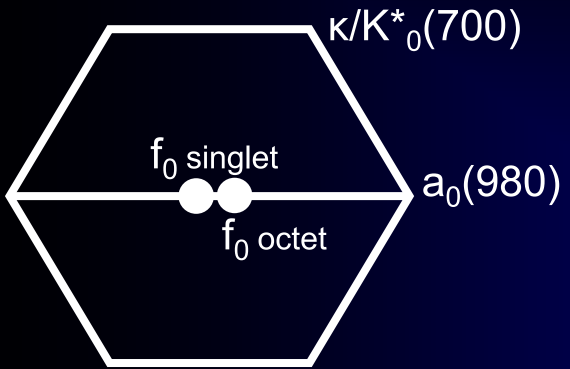
  - K π scattering particularly relevant for PDG mass and width values of **strange scalars** below 2 GeV

Light-scalars: spectroscopic classification



Lightest scalar SU(3) multiplets <2 GeV. Accepted picture at PDG

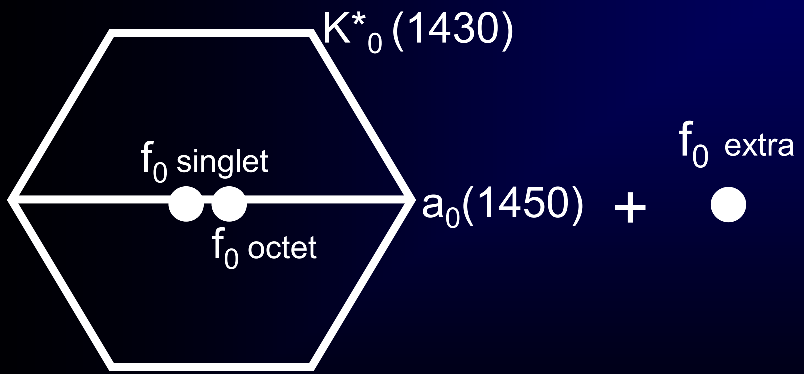
Light scalar nonet <1 GeV:



Non-strange heavier!!
Hugely Inverted $q\bar{q}$ hierarchy.
Cryptoexotics? (Tetraquarks? R.Jaffe 1976)

σ/f$_0$(500) and f$_0$(980) octet/singlet mixtures
Lightest strange: κ/K*$_0$(700) “well established” @PDG only in 2021

Scalar nonet + extra state >1 GeV:



One extra state f$_0$(1370), f$_0$(1500), f$_0$(1710)
A glueball?
Also, not quite a $q\bar{q}$ hierarchy

Non-strange, complicated mixtures
f$_0$(1370) worst determined and still contested

Identifying number of **Strange Resonances** = Identifying number of nonets

The“kappa” controversy… very very briefly

- Dalitz 1965: “Quite apart from the model discussed here,…such K* states are expected to exist simply on the basis of SU(3)” Procs. Oxford Int. Conf. on Elementary Particles 1965
- Many claims at different masses, narrow, wide… claims of absence. Confusion
- 1967 attitude

REVIEWS OF

MODERN PHYSIC

VOLUME 39, NUMBER 1

Data on Particles and Resonant States*

ARTHUR H. ROSENFELD, ANGELA BARBARO-GALTIERI, WILLIAM J. PODOLSKY, L
PAUL SODING, CHARLES G. WOHL
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Dept. of Physics, Yale University, New Haven, Connecticut

Data on the properties of leptons, mesons, and baryons are listed, referenced, averaged, and summarized in tables and wallet cards. This is an updating of the *Reviews of Modern Physics* article of October 1965.

1. The κ(725) (Lynch, Rittenberg, Rosenfeld, Söding, Dec. 1966)

We are beginning to think that κ should be classified along with flying saucers, the Loch Ness Monster, and the Abominable Snowman. We have heard of several experiments which were supposed to confirm it, and each one has either failed completely or failed to find it in the sought-for channel, but found instead a small Kπ peak near 725 MeV in some other channel.

- Removed from Review of Particle Physics in 1976 (with the σ)
- Back to PDG in 2004 as “**controversial**” $K_0^*(800)$. Omitted from summary tables
- Finally considered “Well established” in 2021 mainly due to dispersive analyses

Two longstanding sources of trouble
In meson-meson scattering

DATA PROBLEM

MODEL-DEPENDENCE PROBLEM

THIS TALK

Overview of effort to discard inconsistent data and **eliminate or reduce model dependence:**
by using

DISPERSIVE/ANALYTICITY APPROACHES

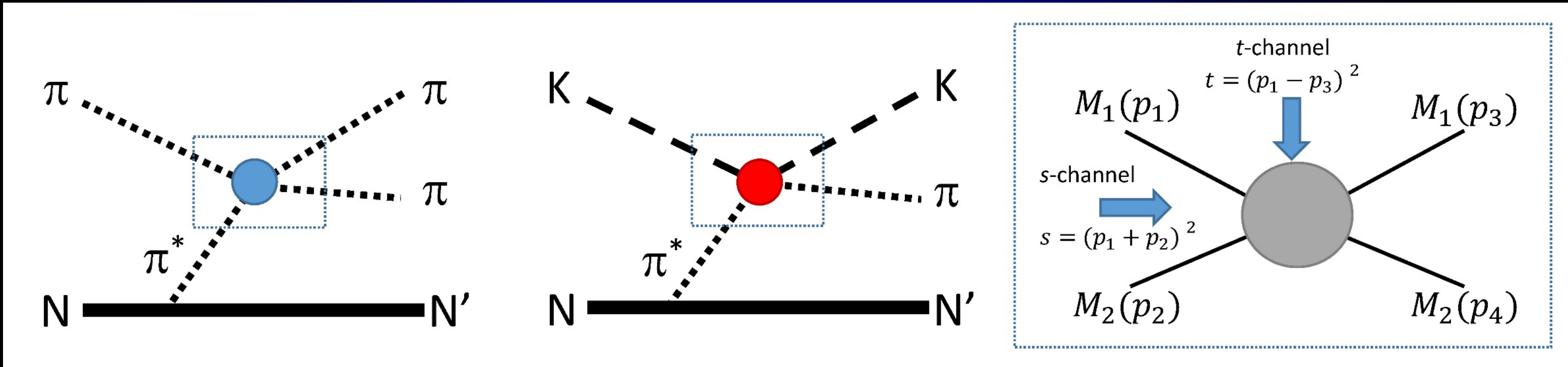
DATA PROBLEM: **Meson-meson SCATTERING data are poor**

π and K unstable. Beams NOT luminous enough for **ππ and πK** collisions:

Indirect measurements

1) Very few good data from $K\to\pi\pi e\nu$. But $E<M_K$. Geneva-Saclay (77), E865 (01), **NA48/2 (2010)**

2) ALMOST ALL DATA from Meson-Nucleon scattering (In the 70's and 80's)



CAVEATS: One-Pion-Exchange (OPE) Approximation

In initial state virtual pion not well defined, Chew-Low off-shell extrapolation

More contributions: absorption, A_2 exchange...

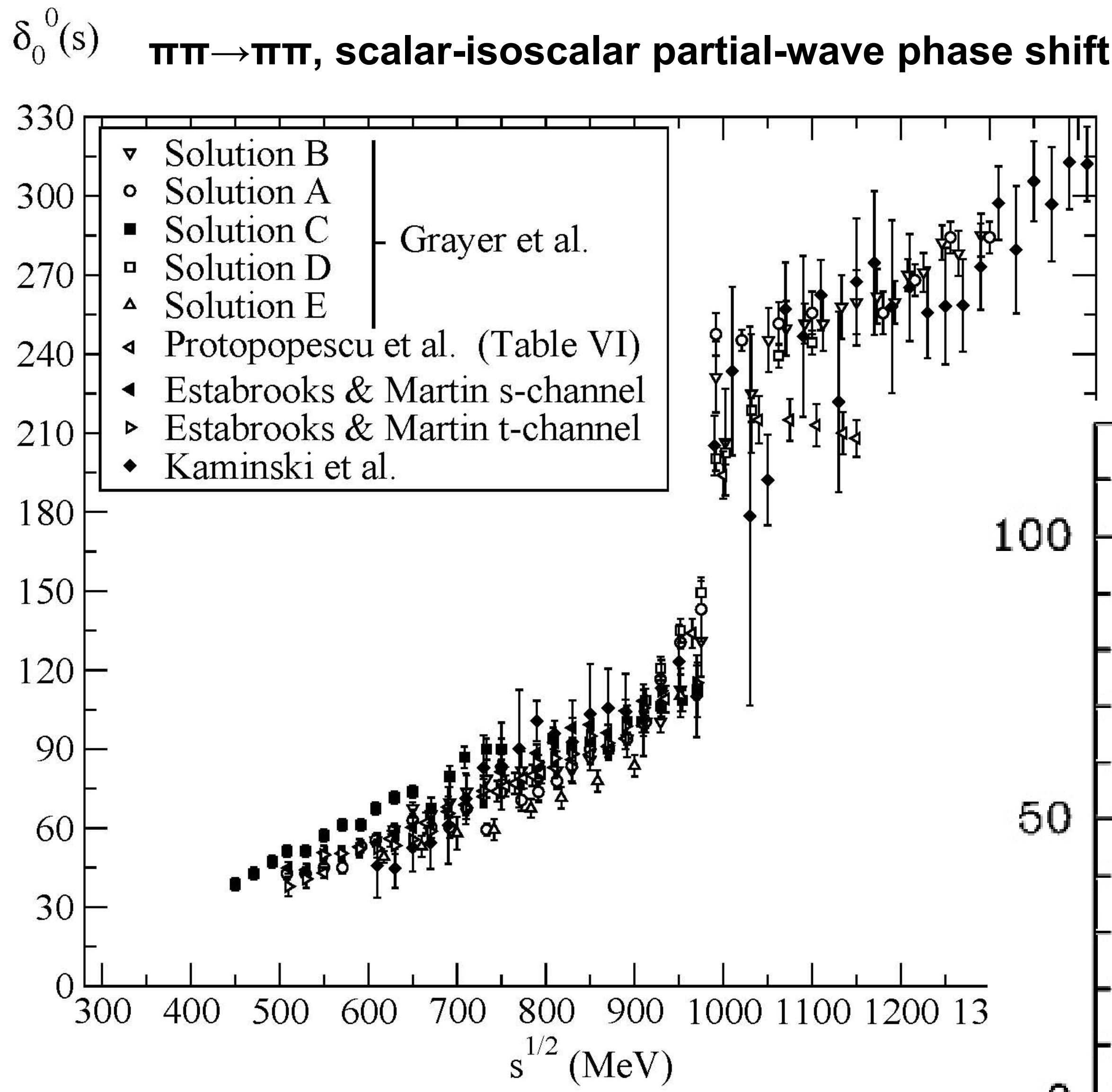
Needs Meson-N partial-wave extraction. Problems with phase shift ambiguities, etc...

As a consequence... VERY LARGE SYSTEMATIC UNCERTAINTIES

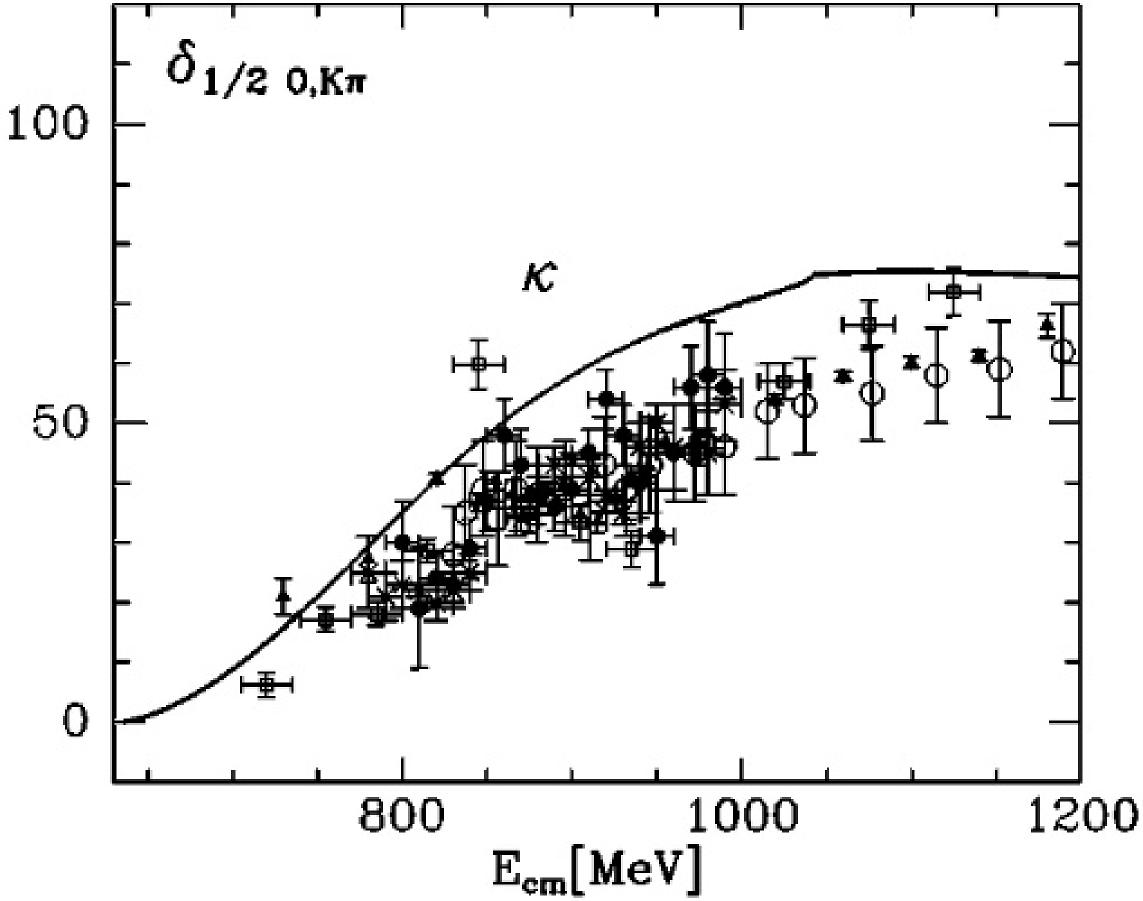
DATA PROBLEM **ππ and πK SCATTERING data are often in conflict**

CONFLICTING DATA SETS & SYSTEMATIC uncertainties larger than STATISTICAL

ππ→ππ, scalar-isoscalar partial-wave phase shift

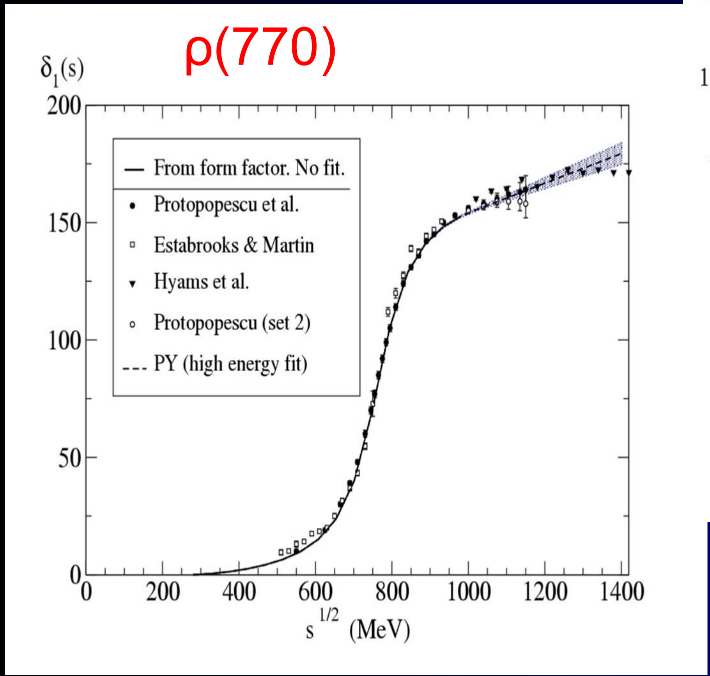


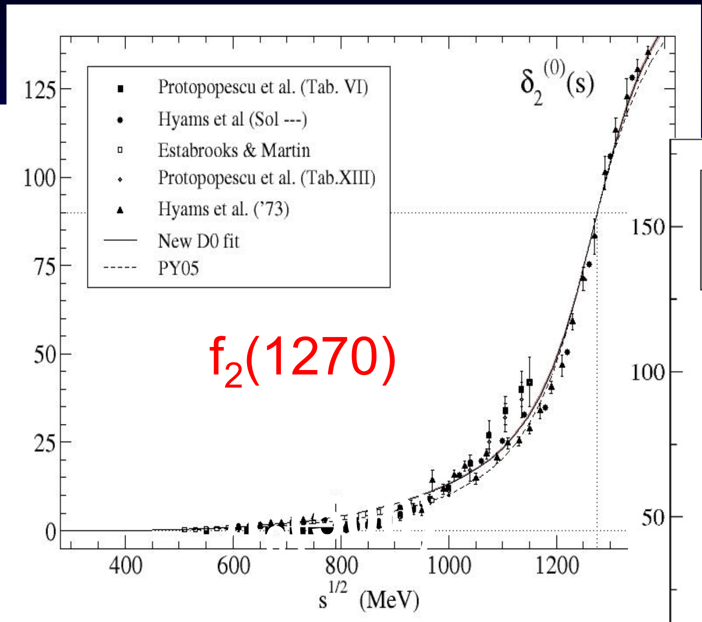
Kπ→Kπ, scalar-isospin ½ partial-wave phase shift

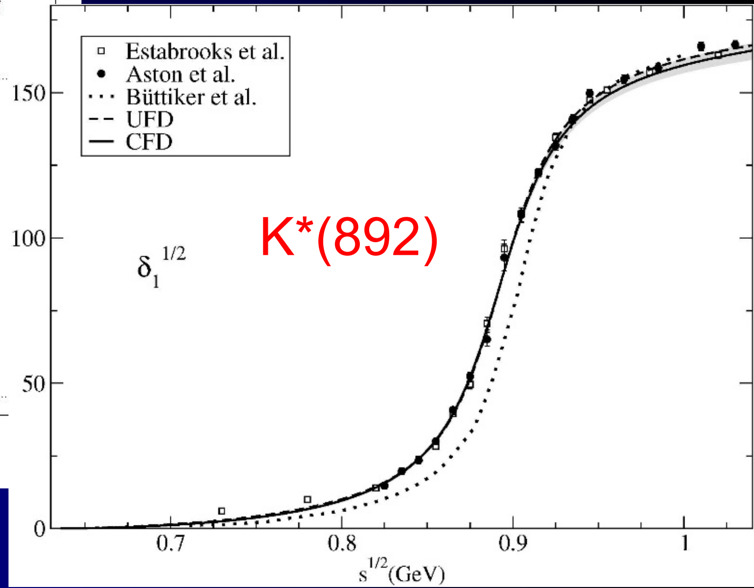


MODEL PROBLEM: many models used to fit data and extract resonances…

Narrow resonances when far from other resonances or singularities (thresholds, cuts, etc…) produce typical peaks and rapid 180º phase motions

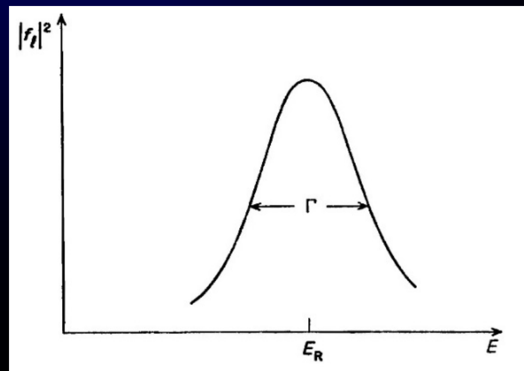






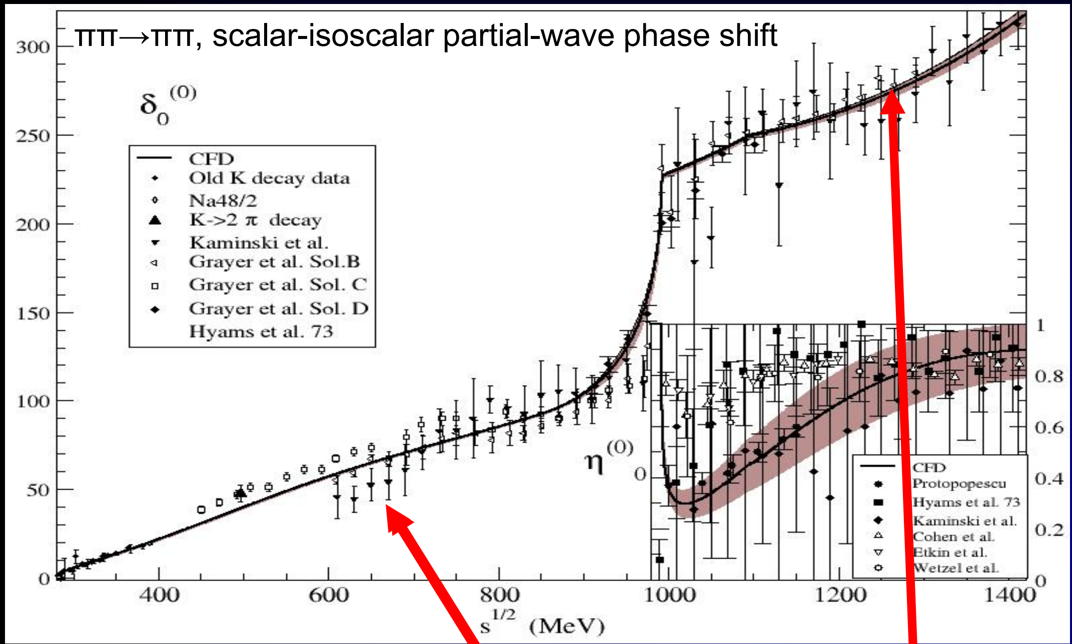
These were the “easy ones” and models are usually fine.
For instance, they may be reasonably well approximated by Breit-Wigner shapes

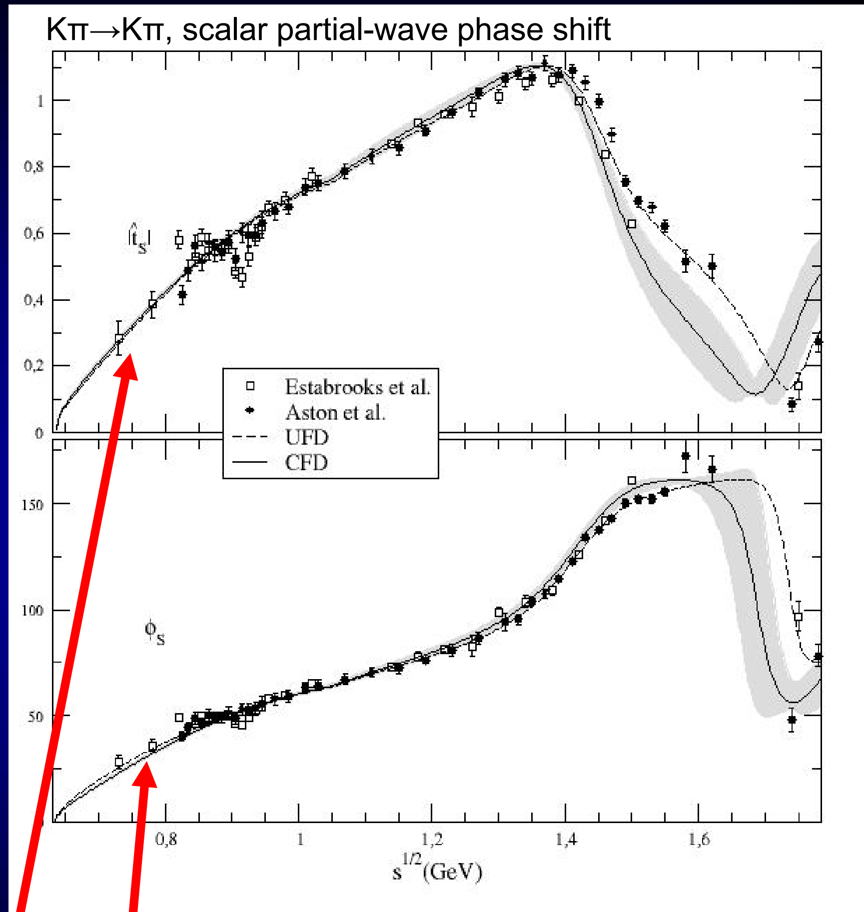
$$\sim \frac{M\,\Gamma(s)}{M^2 - s - iM\,\Gamma(s)}$$



MODEL PROBLEM: Resonances in meson-meson scattering

"Breit-Wigner" shapes are easily recognizable… but life is not that easy





Do you see resonances there?

Nevertheless there are resonances (poles) in these regions: the $\sigma/f_0(500)$, $f_0(1370)$ and $\kappa/K_0^*(700)$ light scalars

MODEL PROBLEM: Resonance shapes process dependent

The not so easy ones…

- Light scalars are wide, or even extremely wide and frequently overlap with one another or with thresholds like KK.

- Very often they do not produce clear peaks, nor rapid phase motions, and “peak searching” not valid anymore

- Moreover since they are not clear-cut peaks, **their shape**, apparent position, width… **can be different depending on the process where they are observed**.

- Model fits to different process or partial data can yield different resonances

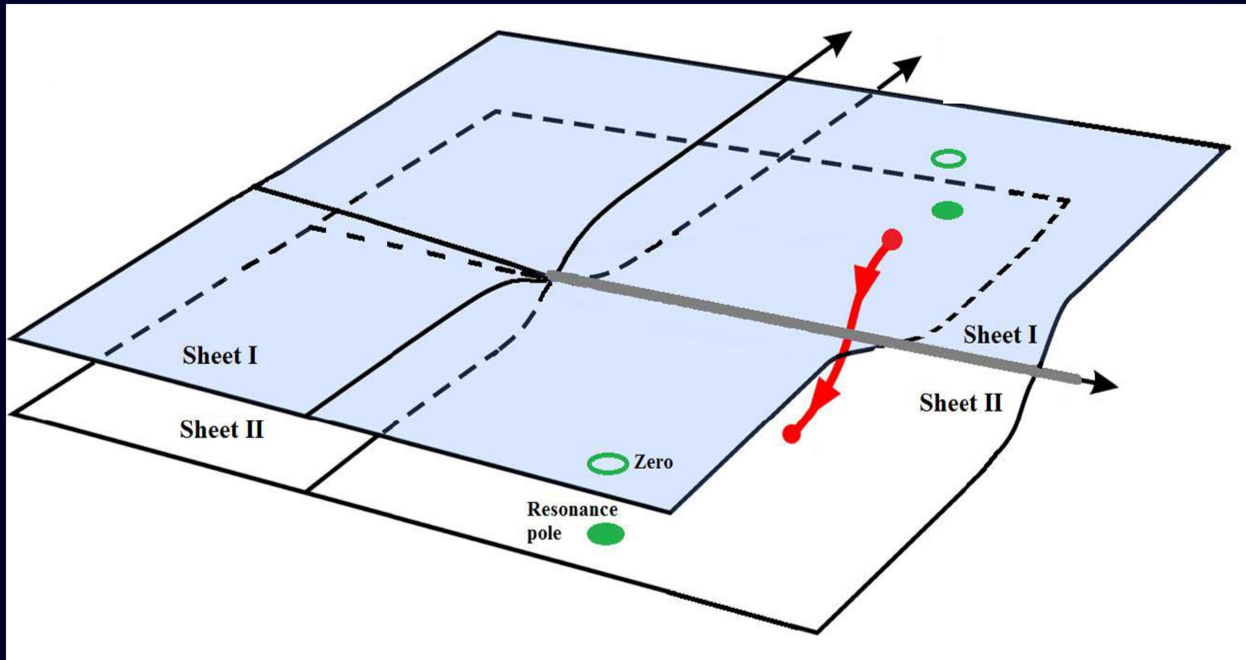
- Meson-meson scattering has the strongest theory constraints and is the most reliable theoretically to go to the complex plane. (non-linear unitarity condition)

Process dependent peaks vs. Process independent poles

The universal features of resonances are their pole positions and residues *

$\sqrt{s_{pole}} \approx M\text{-}i\,\Gamma/2$

*in the Riemann sheet obtained from an analytic continuation through the physical cut



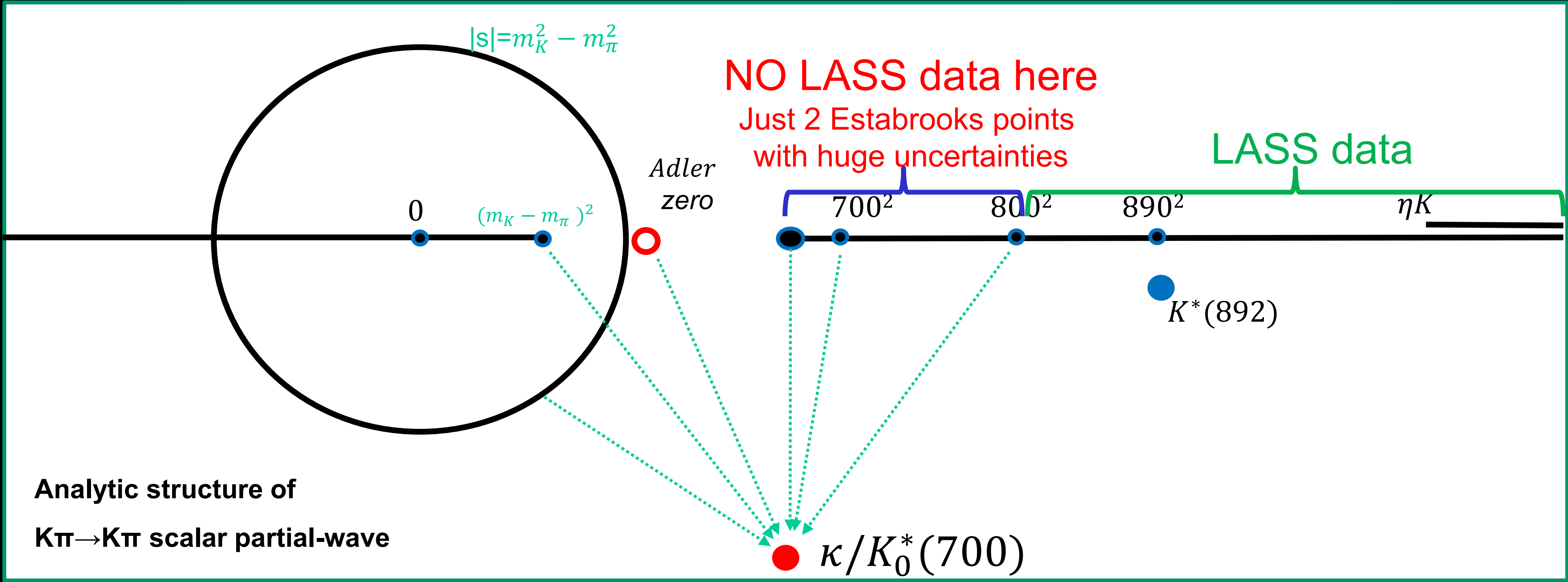
The Review of Particle Physics has been adding pole determinations for more and more resonances

unfortunately keeping also Breit-Wigner parameters even when not applicable

However, analytic continuations are a delicate mathematical problem and a good control of the analytic structure is needed. Many models fail at this.

Why worry about LOW ENERGY and CORRECT ANALYTIC STRUCTURE? Kπ case

Analyticity is expressed in the s-variable, not in $\sqrt{s}$



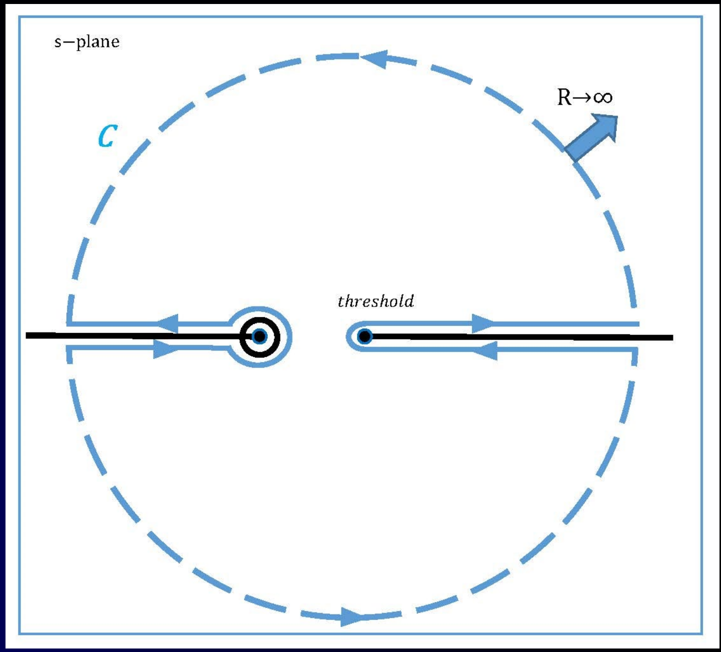
Important for the $\kappa/K_0^*(700)$

- Threshold behavior (Theory: chiral symmetry)
- Subthreshold behavior (Theory: chiral symmetry →Adler zeros)
- Other cuts (Theory: Left & circular)

Thus, LOW ENERGY behavior and ANALYTICITY crucial for the $\kappa/K_0^*(700)$

What is a dispersion relation.? (Very Briefly)

- CAUSALITY ⇒ Amplitudes t(s) are ANALYTIC in complex s plane with cuts due to thresholds (also in crossed channels)



- Cauchy Theorem:

If t(s)→0 fast enough at high s, curved part vanishes

$$t(s) = \frac{1}{\pi}\int_{th}^{\infty} \frac{Im\, t(s')}{s-s'} ds' + LC + CC$$

Otherwise, determined up to polynomial (subtractions)

Good for:

1) Calculating t(s) as an integral where there is not data
2) Constraining data analysis: Input =output
3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane

<u>Last decades</u>
Effort to **eliminate or reduce model dependence** by using **dispersive approaches** often combined with **Chiral Perturbation Theory (ChPT).**

We need to get rid of one variable to write CAUCHY THEOREM for the other

1) Fix one variable in terms of the other (fixed-t, hyperbolic relations…)

Most popular: $t_0=0$, Forward Dispersion Relations (FDRs).
(Kaminski, Pelaez , Yndurain, Garcia Martin, Ruiz de Elvira, Rodas)

PROS: One eq. per amplitude. Simple. High energy reliable. Applicable to all energies
Precision
CONS: No direct access to poles… until recently (see below)

2) Integrate one variable: Partial wave dispersion relations

- "Roy-like" equations. GKPY eqs, Roy Steiner Equation-

Crossing to rewrite Left/circular cuts with. crossing in terms of physical region.
CONS: Different partial waves or channels coupled. In practice, limited to a finite energy
PROS: Directly partial waves. Better to look for poles. Precision

Precision Dispersive Studies

(The ones with highest impact on the PDG)

- Unitarized Amplitudes (IAM, N/D, Chew-Mandelstam…)

2-body unitarity exact on dispersion relation for inverse amplitude (single or coupled channels)
Ideally combined with ChPT for these approximations, but additional bare/preexisting resonances could be added, simple models for real part, use of Lagrangians, effective theories etc…
CONS: Unphysical cuts, higher energies, multibody, approximated
PROS: Directly partial waves. Better to look for poles. Connection with QCD through ChPT in UChPT

Precision studies. Two strategies on real axis:

- **SOLVE equations:** (Ananthanarayan, Colangelo, Gasser, Leutwyler, Caprini, Moussallam, Stern…)

  S and P wave solution for Roy or GKPY equations unique at low energy

  Needs input on other waves and high energy.

  NO scattering DATA used at low energies ($\sqrt{s} \leq 0.8 \sim 1\ GeV$)

  Good if interested in low energy scattering and do not trust data.

  Uses ChPT input for threshold parameters

- **Impose Dispersion Relations on fits to data.** (García-Martín, Kaminski,JRP, Ruiz de Elvira, Ynduráin)

  Also known as "**DATA driven dispersive analyses**"

  Also needs input on other waves and high energy.

DATA driven dispersive analyses: How do they work?

1) Obtain set of **Unconstrained Fits to Data (UFD).** Realistic statistical+systematic uncertainties

2) TEST dispersion relations with UFD set. Discard some data if inconsistent

3) IMPOSE dispersion relations to fits. Uniformly, as penalty functions.

↓

Constrained fits to Data (CFD)

↓

CFD Applications

- Inside dispersion relations to find resonance poles
  Model Independent. Most rigorous but only feasible for elastic region
- With analytic continuation methods to look for poles.
  Very reduced model dependence.
- Inside sum rules to obtain threshold parameters
  Relevant for effective theories and QCD

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  Very reduced model dependence.
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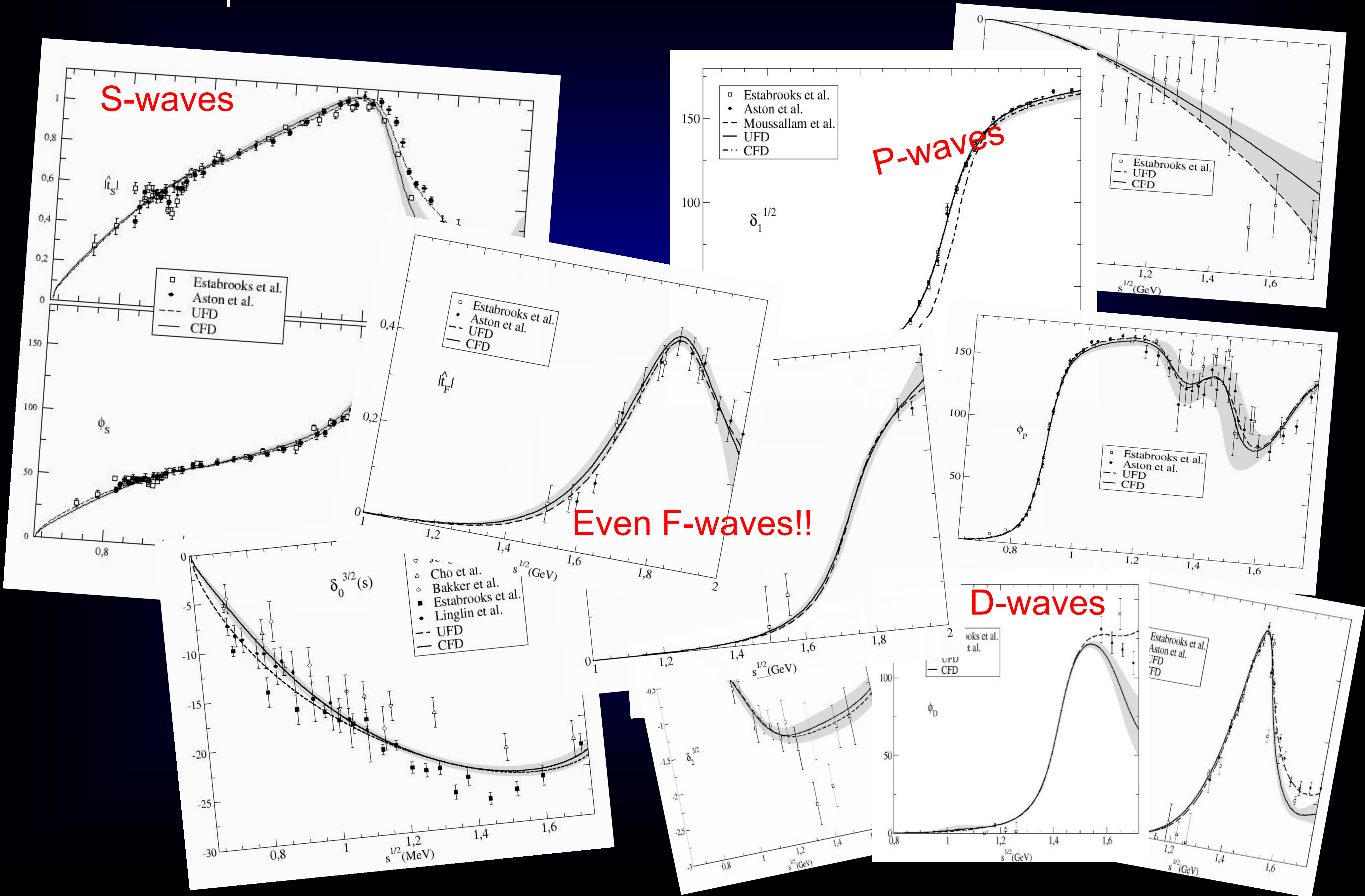
FIRST STEP: Simple Unconstrained Fits (UFD) to data

Estimation of statistical and SYSTEMATIC errors

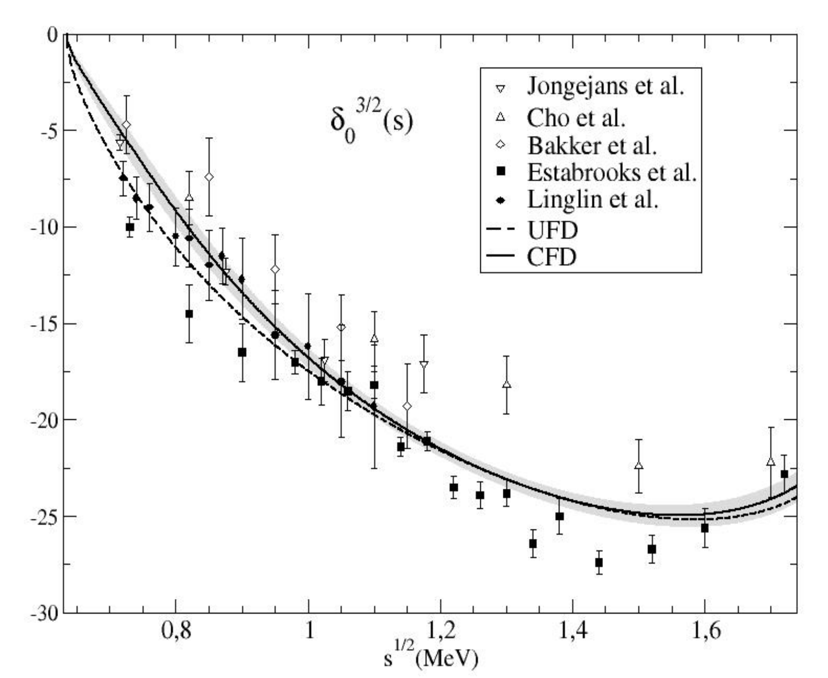
πK and ππ→KK partial-wave Data

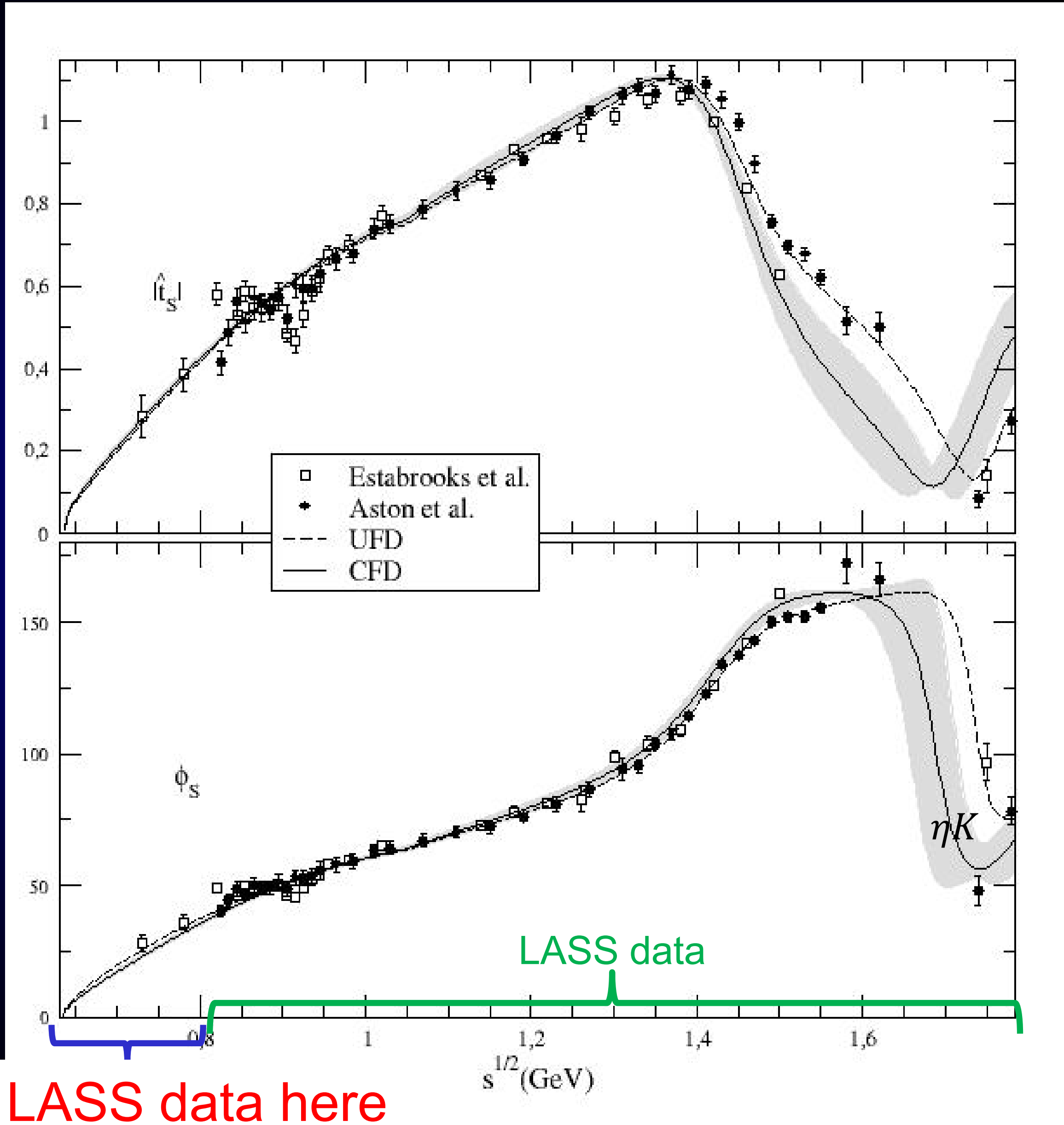


From Unconstrained (UFD) to Constrained Fits to data (CFD): Detail for S-waves

JRP, A.Rodas-PhysRevLett.124.172001-2020

The most interesting for the $K_0{}^*$ resonances and the $K_0^*(700)$ in particular





LASS does
NOT SEPARATE ISOSPIN
I=1/2 and 3/2 together

KLF@Jlab
Is expected to help in these two issues
(see A. Rodas talk)

NO LASS data here
Just 2 Estabrooks points
with huge uncertainties

DATA driven dispersive analyses: How do they work?

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2) TEST dispersion relations with UFD set. Discard some data if inconsistent

g^I_J = ππ → KK partial waves. We study (I,J)=(0,0),(1,1),(0,2)
f^I_J = Kπ → Kπ partial waves. Taken from previous dispersive study

JRP, A. Rodas PRD 2018

$$g_0^0(t) = \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_0^0(t')}{t'(t'-t)}dt' + \frac{t}{\pi}\sum_{\ell\geq2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G^0_{0,2\ell-2}(t,t')\mathrm{Im}\, g^0_{2\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G^+_{0,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'),$$

$$g_1^1(t) = \frac{1}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_1^1(t')}{t'-t}dt' + \sum_{\ell\geq2}\int_{4m_\pi^2}^\infty dt' G^1_{1,2\ell-1}(t,t')\mathrm{Im}\, g^1_{2\ell-1}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G^-_{1,\ell}(t,s')\mathrm{Im}\, f^-_\ell(s'),$$

$$g_2^0(t) = \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_2^0(t')}{t'(t'-t)}dt' + \sum_{\ell\geq2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G'^0_{2,4\ell-2}(t,t')\mathrm{Im}\, g^0_{4\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G'^+_{2,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'). \quad (39)$$

$G^I_{J,J'}$(t,t') =integral kernels, depend on a parameter
Lowest # of subtractions. Odd pw decouple from even pw.

$$g_\ell^0(t) = \Delta_\ell^0(t) + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{dt'}{t'}\frac{\mathrm{Im}\, g_\ell^0(t)}{t'-t}, \qquad \ell = 0,2,$$

$$g_1^1(t) = \Delta_1^1(t) + \frac{1}{\pi}\int_{4m_\pi^2}^\infty dt' \frac{\mathrm{Im}\, g_1^1(t)}{t'-t}, \quad (40)$$

Δ(t) depends on higher waves or on Kπ→Kπ.

Integrals from 2π threshold ! "Unphysical region"

Solve in descending J order

We have used models for higher waves, but give very small contributions

ππ→KK Hyperbolic Dispersion Relations (HDR)

For unphysical region below KK threshold, we used Omnés function

$$\Omega_\ell^I(t) = \exp\left(\frac{t}{\pi}\int_{4m_\pi^2}^{t_m}\frac{\phi_\ell^I(t')dt'}{t'(t'-t)}\right),$$

$$\Omega_\ell^I(t) \equiv \Omega_{l,R}^I(t) e^{i\phi_\ell^I(t)\theta(t-4m_\pi^2)\theta(t_m-t)},$$

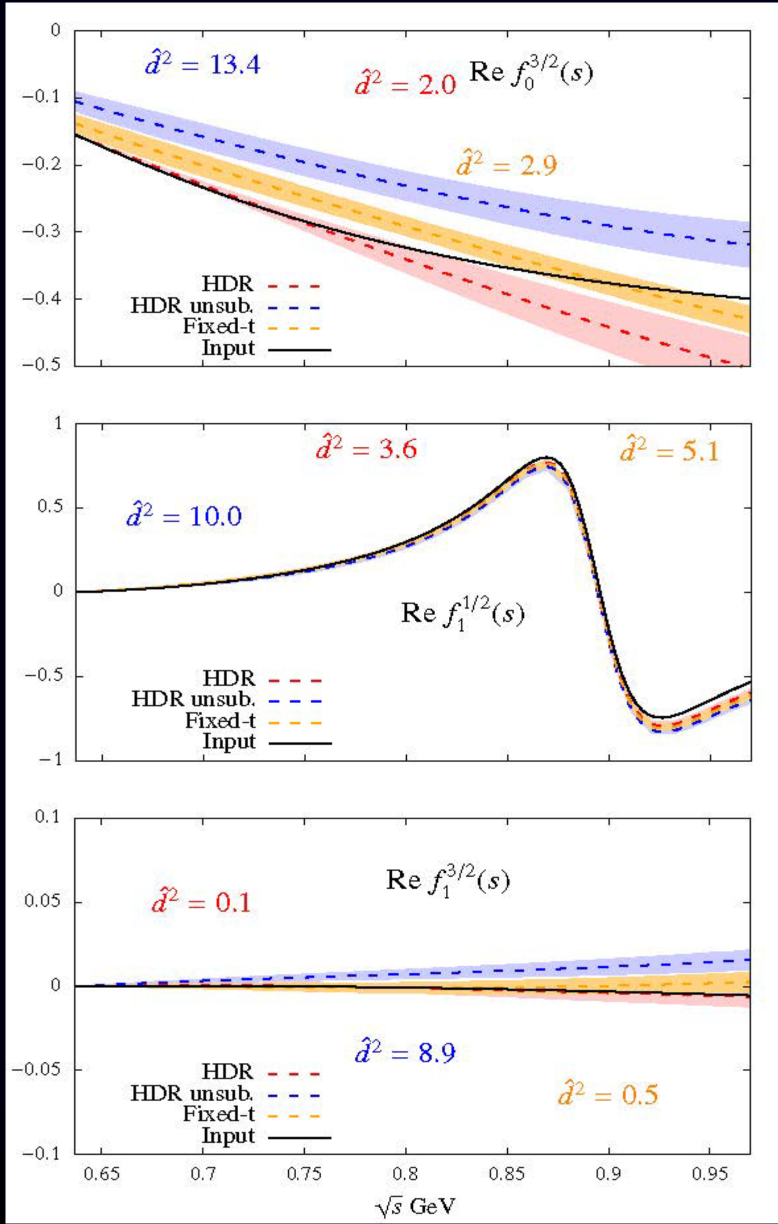
This is the form of our HDR: Roy-Steiner+Omnés formalism

$$\begin{aligned}
g_0^0(t) &= \Delta_0^0(t) + \frac{t\Omega_0^0(t)}{t_m - t}\left[\alpha + \frac{t}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{(t_m - t')\Delta_0^0(t')\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)} + \frac{t}{\pi}\int_{t_m}^{\infty} dt' \frac{(t_m - t')|g_0^0(t')|\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)}\right] \\
g_1^1(t) &= \Delta_1^1(t) + \Omega_1^1(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_1^1(t')\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_1^1(t')|\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)}\right], \\
g_2^0(t) &= \Delta_2^0(t) + t\Omega_2^0(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_2^0(t')\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_2^0(t')|\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)}\right].
\end{aligned}$$

We can now check how well these HDR are satisfied

LARGE inconsistencies IF UNCONSTRAINED

Unconstrained Fit to Data

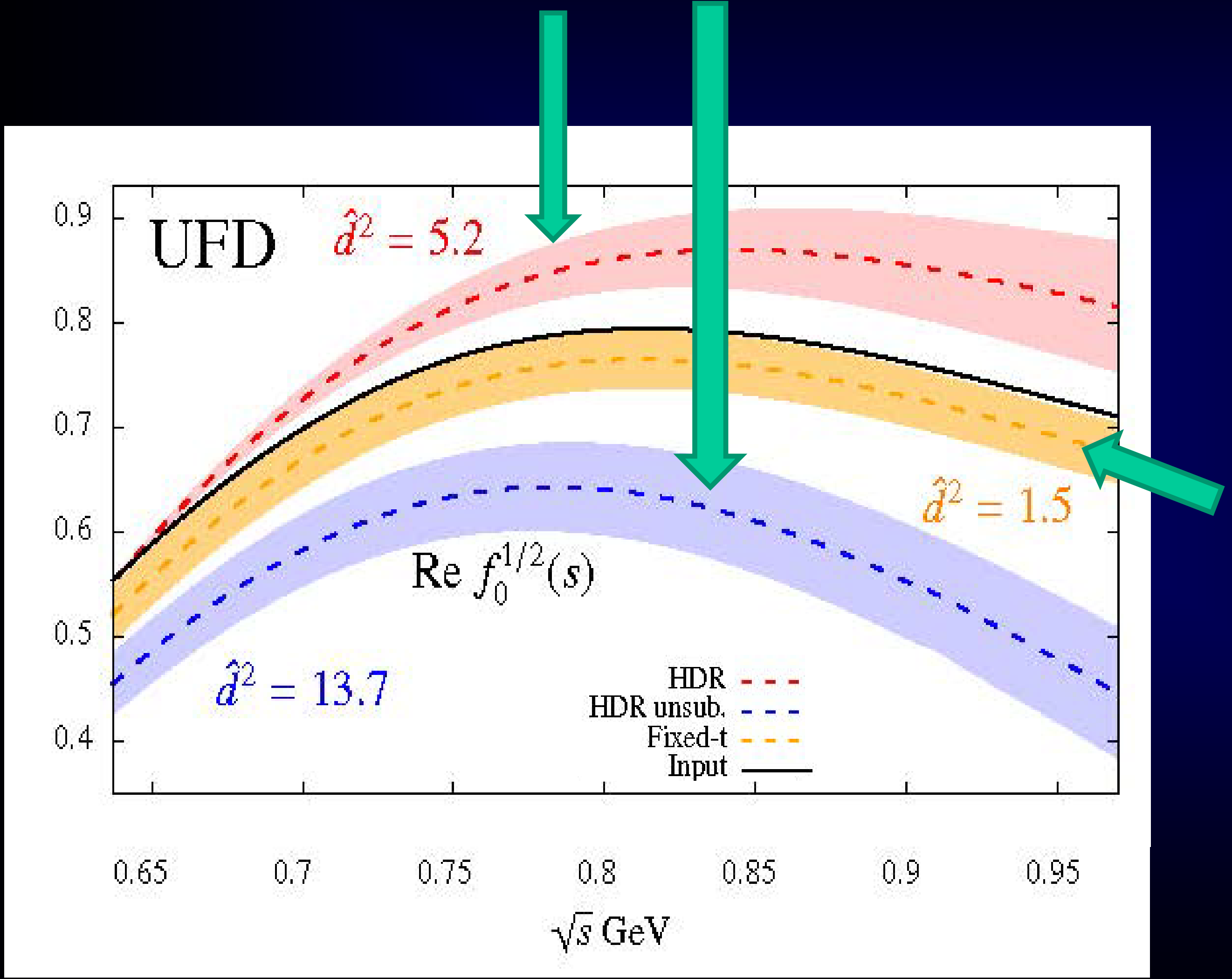


πK Hiperbolic Dispersion Relations I=1/2, J=0

The most relevant wave for the kappa resonance.

LARGE inconsistencies with HDR Roy-Steiner from unconstrained fits (UFD)

One or no subtraction for F^- lie on opposite sides of input



Fixed-t Roy-Steiner is fair but kappa pole outside their applicability region

We have chosen the hyperbolae family so that the kappa pole and its uncertainties lie within their applicability region

DATA driven dispersive analyses: How do they work?

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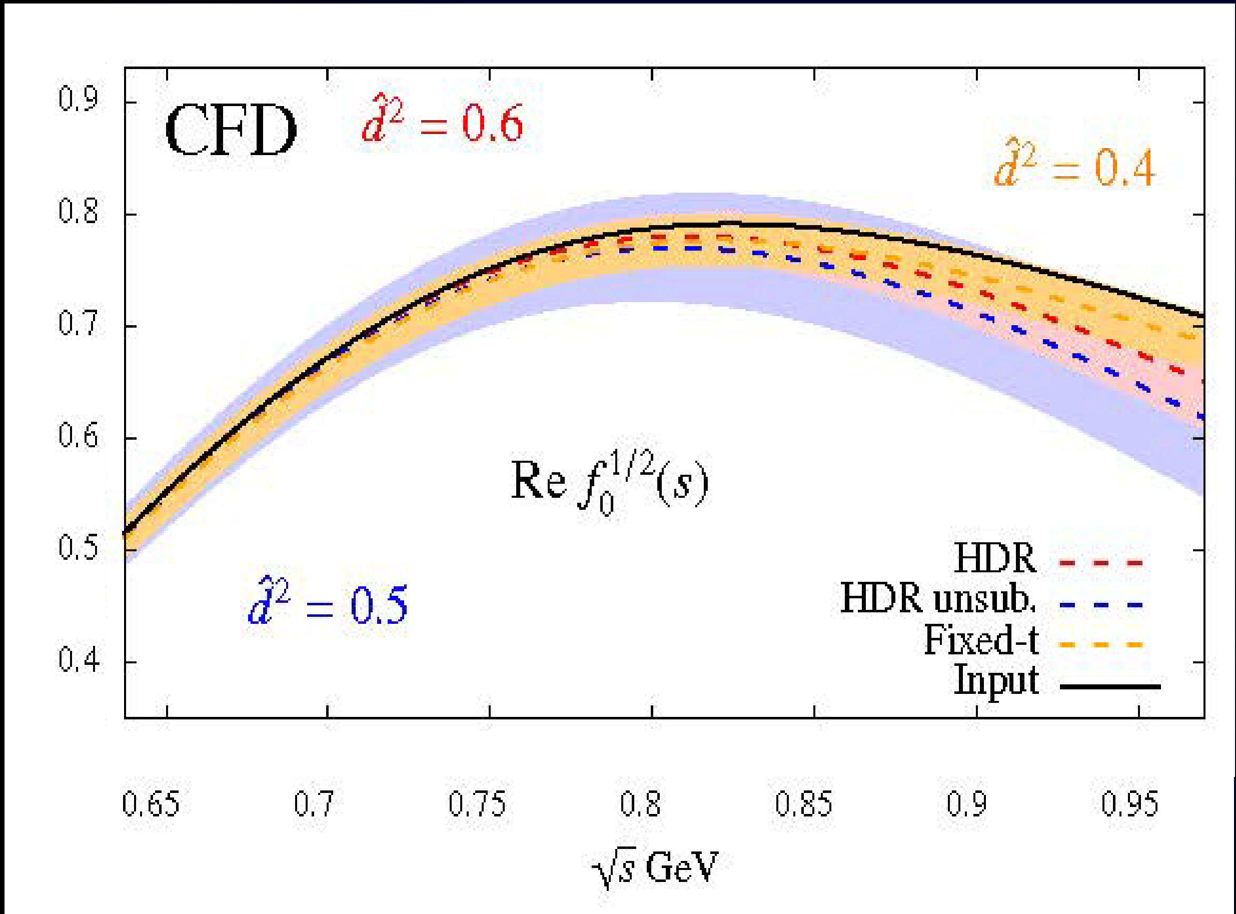
πK Partial Wave Hiperbolic Dispersion Relations. Roy-Steiner Equarions

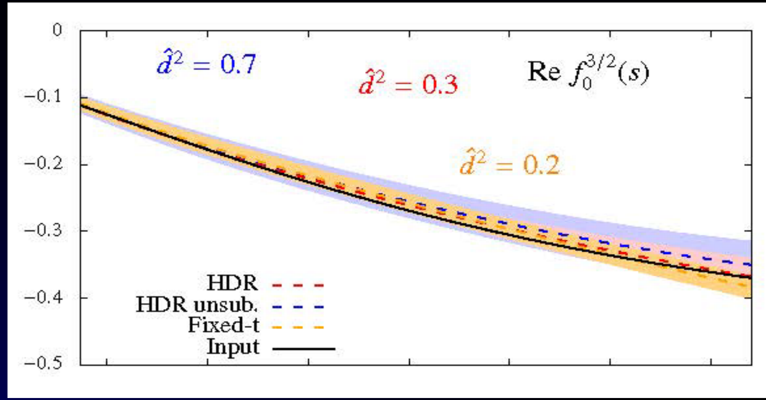
JRP, A.Rodas-PhysRevLett.124.172001-2020

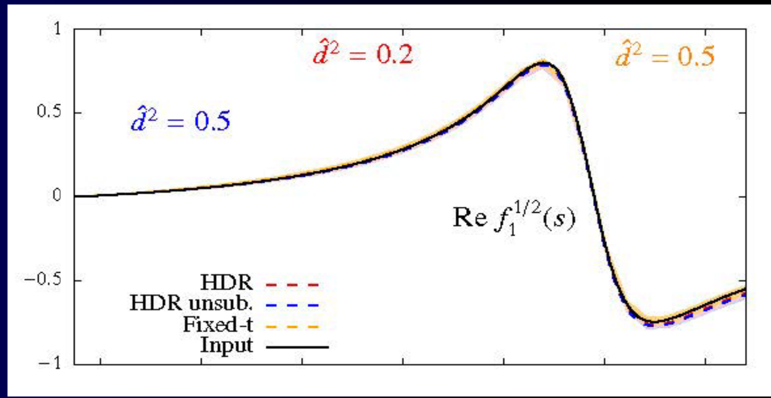
Unconstrained fits (UFD):
LARGE inconsistencies with 3 Roy-Steiner Eqs.

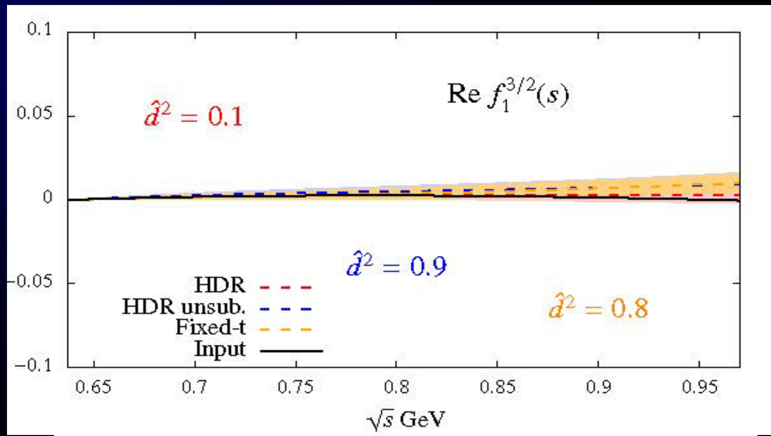
One or no subtraction for F- lie on opposite sides of input

The most relevant wave for the kappa resonance.









Constrained fits (CFD):
Consistent with dispersive constraints within uncertainties

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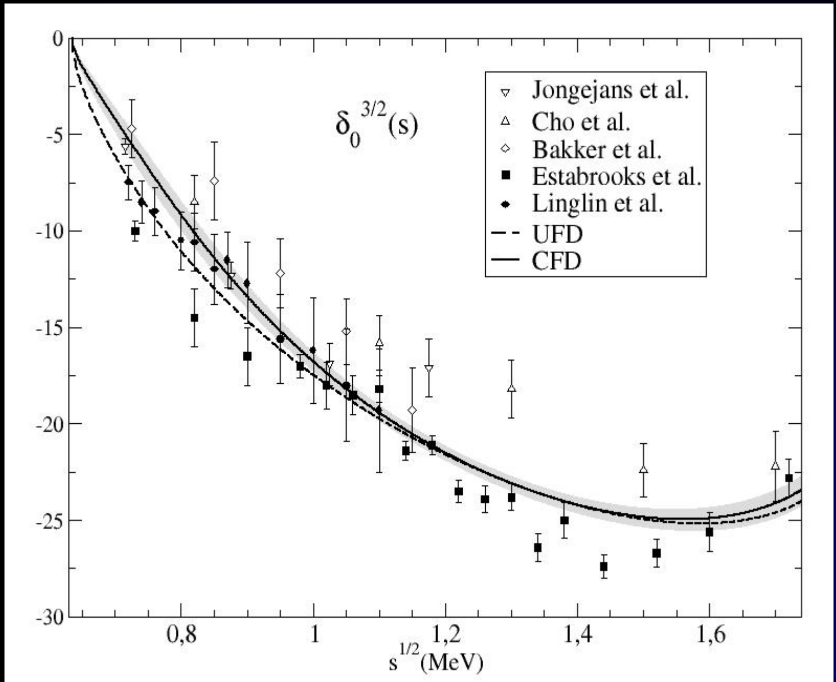
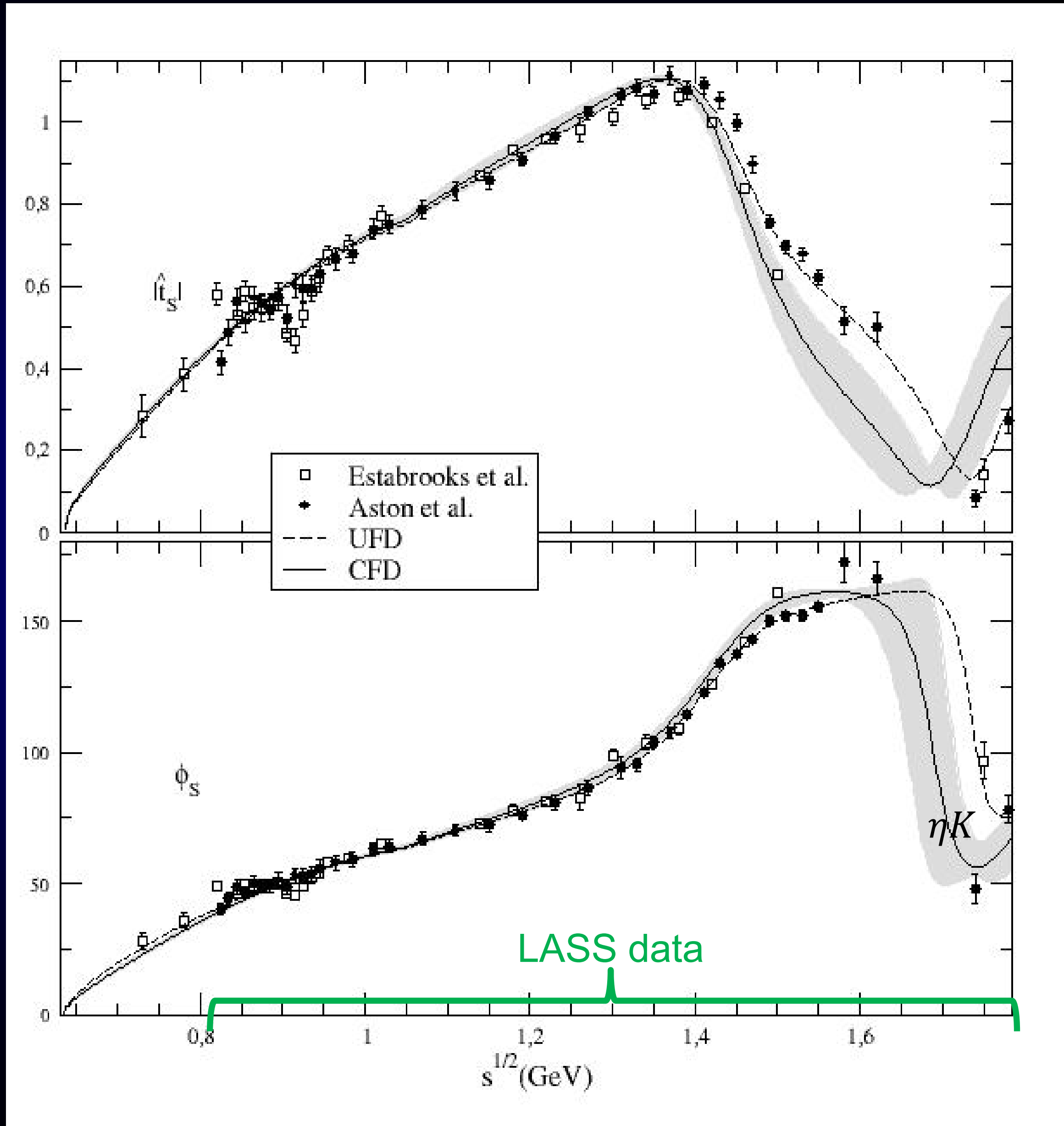
Constrained fits to Data (CFD)

JRP, A.Rodas-PhysRevLett.124.172001-2020

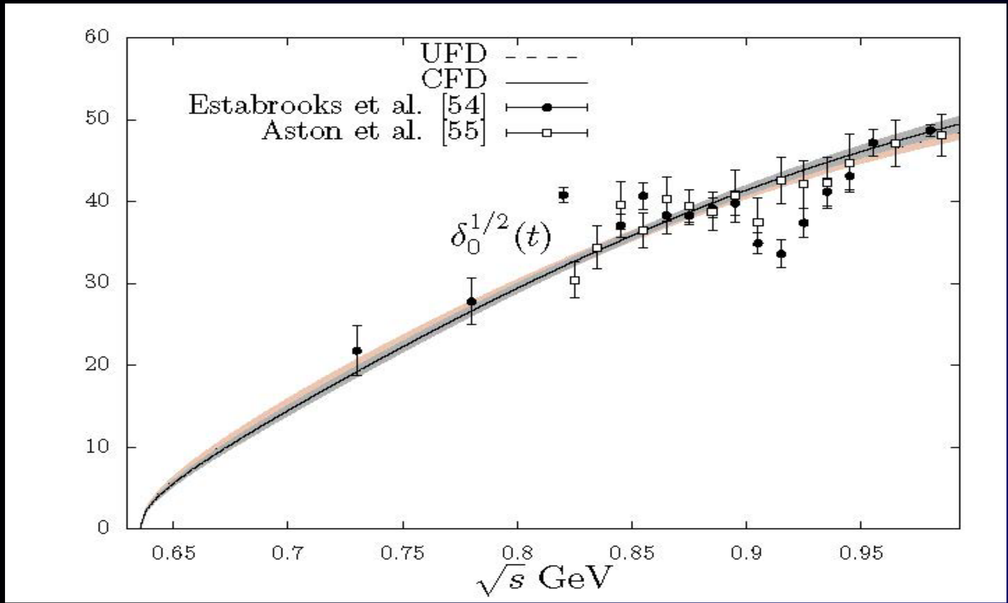
The most interesting for the K_0^* resonances and the $K_0^*(700)$ in particular

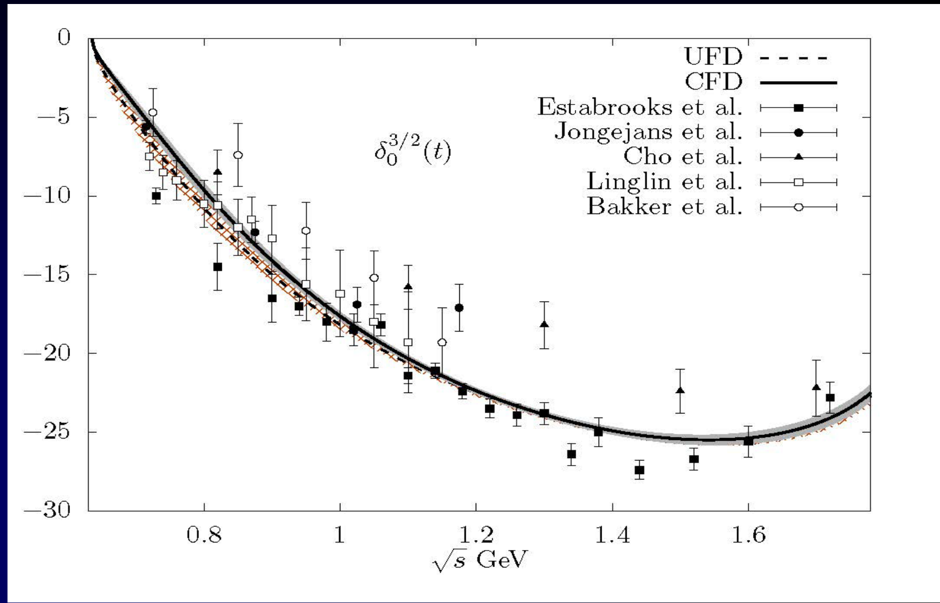


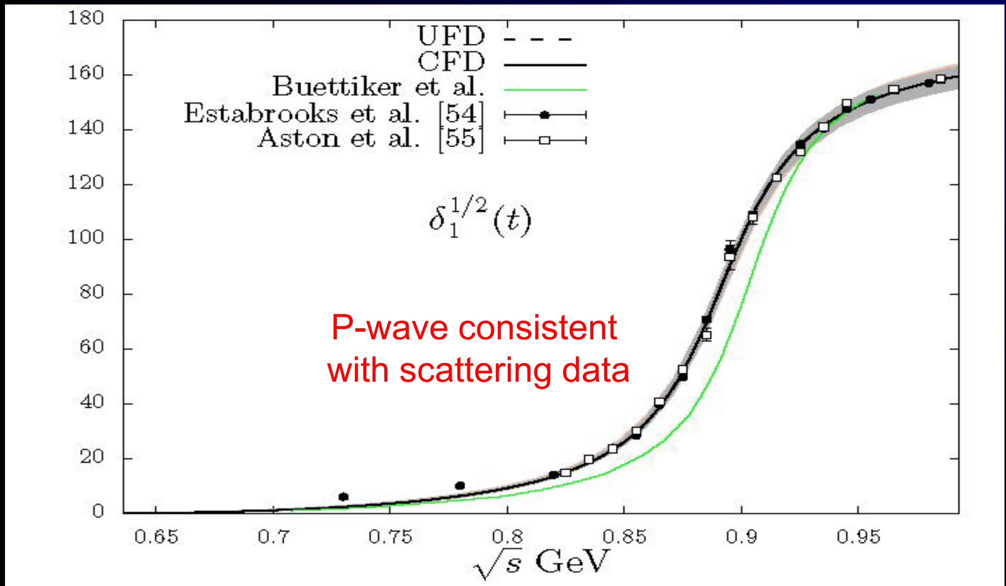


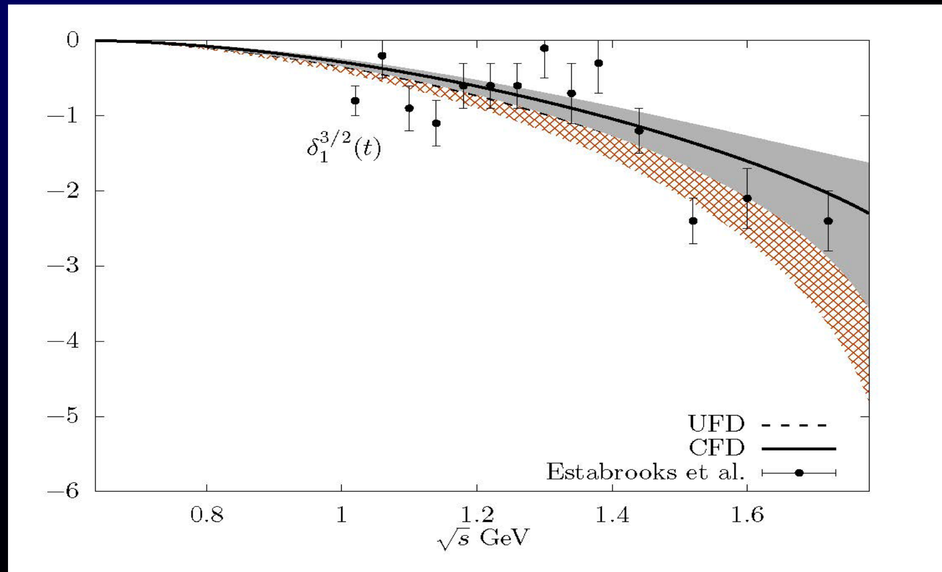
CFD still describes data,
but changes wrt UFD
at high energies
and near threshold

Constrained parameterizations suffer minor changes but still describe πK data fairly well. Here we compare the unconstrained fits (UFD) versus the constrained ones (CFD)









DATA driven dispersive analyses: How do they work?

1) Obtain set of **Unconstrained Fits to Data (UFD).** Realistic statistical+systematic uncertainties

2) TEST dispersion relations with UFD set. Discard some data if inconsistent

3) IMPOSE dispersion relations to fits. Uniformly, as penalty functions.

Constrained fits to Data (CFD)

CFD Applications

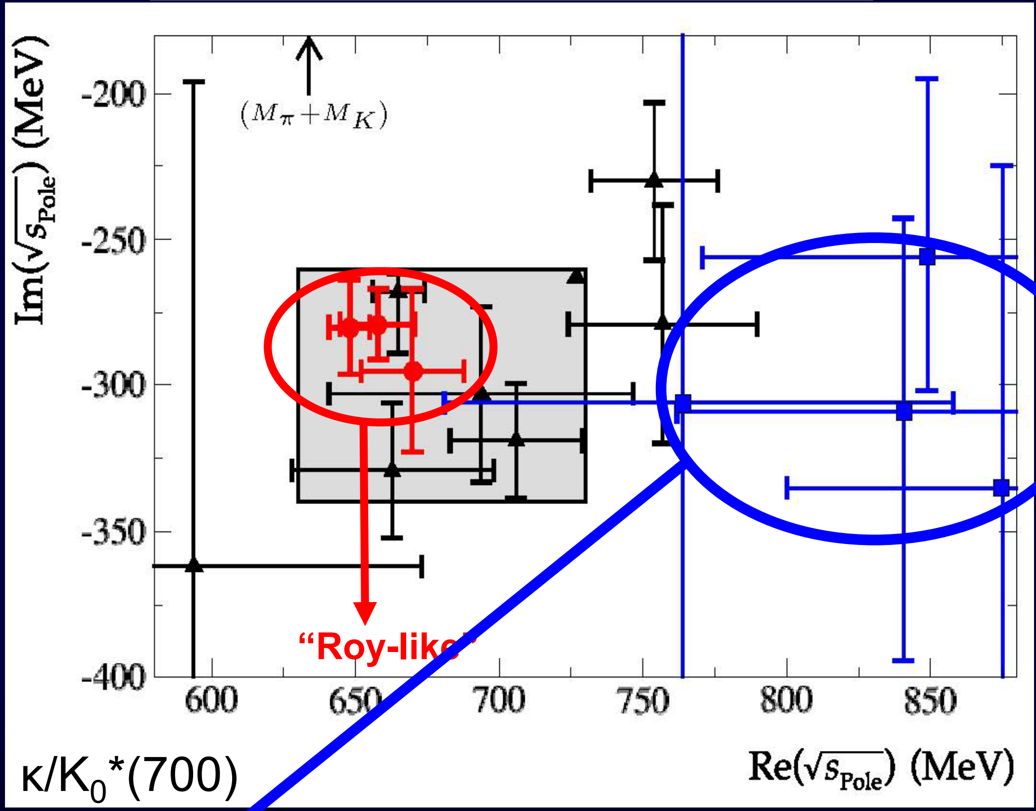
- Inside dispersion relations to find resonance poles

  Model Independent. Most rigorous but only feasible for elastic region

"Roy-like" and "Breit-Wigner" poles identified separately from the rest
Not all from meson-meson scattering

$\kappa/K_0^*(700)$ estimate
2021 No longer "Needs Confirmation"

(400-550)-i(200-350) MeV



From our data driven Roy-Steiner analysis:

No sub: (648±6)-i(283±26) MeV
1 sub: (648±7)-i(280±16) MeV

JRP, A.Rodas-PhysRevLett.124.172001-2020

☹ **But still Breit-Wigners @PDG!!**

DATA driven dispersive analyses: How do they work?

1) Obtain set of **Unconstrained Fits to Data (UFD).** Realistic statistical+systematic uncertainties

2) TEST dispersion relations with UFD set. Discard some data if inconsistent

3) IMPOSE dispersion relations to fits. Uniformly, as penalty functions.

↓

Constrained fits to Data (CFD)

↓

CFD Applications

- Inside dispersion relations to find resonance poles
  Model Independent. Most rigorous but only feasible for elastic region
- With analytic continuation methods to look for poles.
  Very reduced model dependence.

Dispersion relations provide model-independent analytic continuation to **first** Riemann sheet, but the most relevant resonance poles live in the CONTIGUOUS sheet

- For elastic resonances (only second sheet), $S^{II}=1/S^{I}$

σ/f_0(500), κ/K_0*(700), f_0(980),
Purely Dispersive Determination
from meson-meson scattering

- To reach the contiguous sheet in the inelastic case, we need an analytic continuation to the second sheet by means of general analytic functions reproducing the Dispersion Relation in the real axis or the upper-half complex plane.

Several methods in the literature
- **Sequences of Padés**
- **Continued Fractions**
- Laurent-Pietarinen functions
- Conformal expansions…

These methods avoid specific parameterizations, reducing drastically the model-dependence
Tested then with the σ/f_0(500) and κ/K_0*(700). Compatible results.

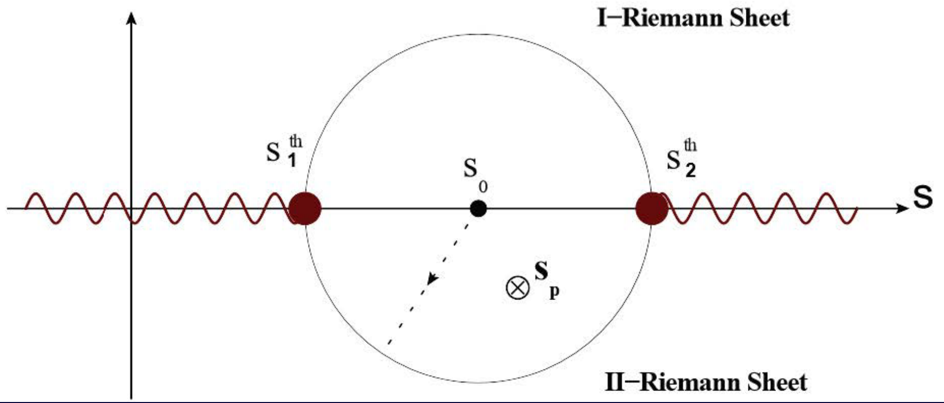
Almost model independent: Does not assume any particular functional form

But requires a few derivatives. There are powerful convergence theorems

If many derivatives needed, poor convergence

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de Elvira, JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

- The method is suitable for the calculation of both elastic and inelastic resonances.
- The Padé sequence gives us the continuation to the continuous Riemann Sheet.
- We take care of the calculation of the errors. Apart from the experimental and systematic errors of each parameterization we also include different fits.



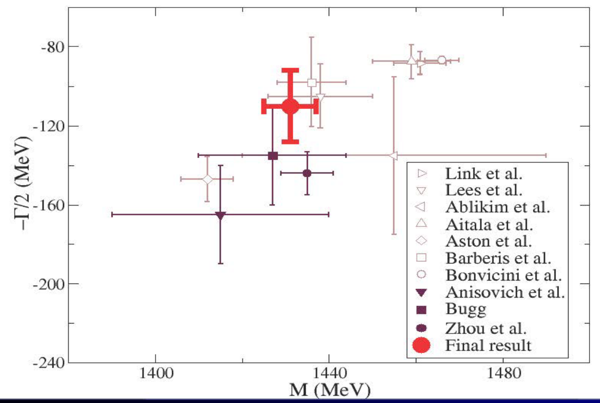
CAVEAT: Requires higher order derivatives of the function to be continued

Still succesfully applied to determine strange resonances from πK scattering up to 1.8 GeV

This DATA DRIVEN method can be used for inelastic resonances too. Provides STRANGE-resonance parameters WITHOUT ASSUMING SPECIFIC FUNCTIONAL FORM

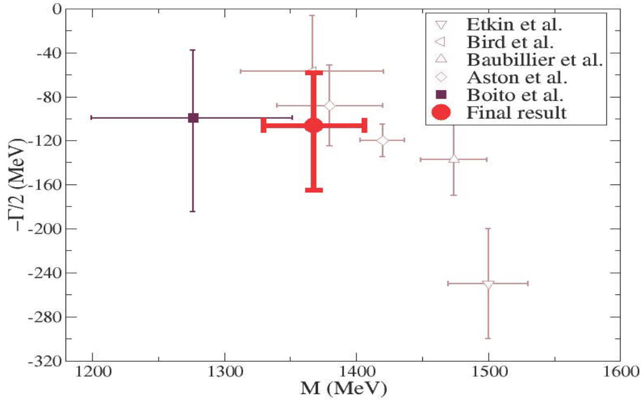
- For the $K_0^*(1430)$ we find

$$\sqrt{s_p} = (1431 \pm 6) - i(110 \pm 19)MeV$$
$$\sqrt{s_p} = (1425 \pm 50) - i(135 \pm 40)MeV(PDG)$$



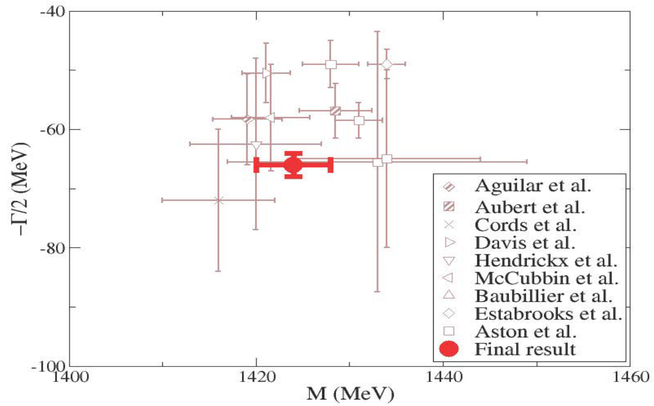
- For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59})MeV$$
$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10)MeV(PDG)$$



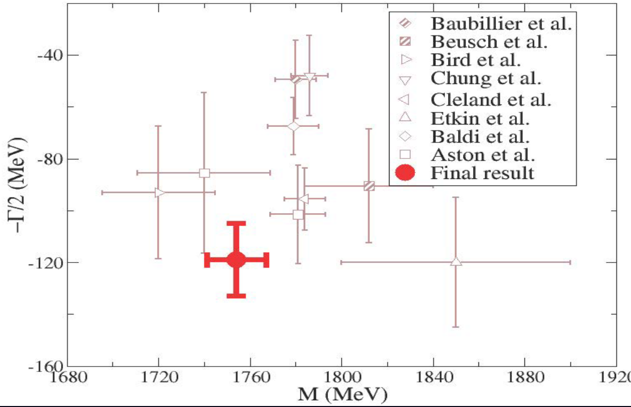
- For the $K_2^*(1430)$ we find

$$\sqrt{s_p} = (1424 \pm 4) - i(66 \pm 2)MeV$$
$$\sqrt{s_p} = (1432.4 \pm 1.3) - i(55 \pm 3)MeV(PDG)$$



- For the $K_3^*(1780)$ we find

$$\sqrt{s_p} = (1754 \pm 13) - i(119 \pm 14)MeV$$
$$\sqrt{s_p} = (1776 \pm 7) - i(80 \pm 11)MeV(PDG)$$



Using Padé Sequences, the kappa: (670±18)-i(295± 28) MeV Consistent with dispersive value

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

DATA driven dispersive analyses: How do they work?

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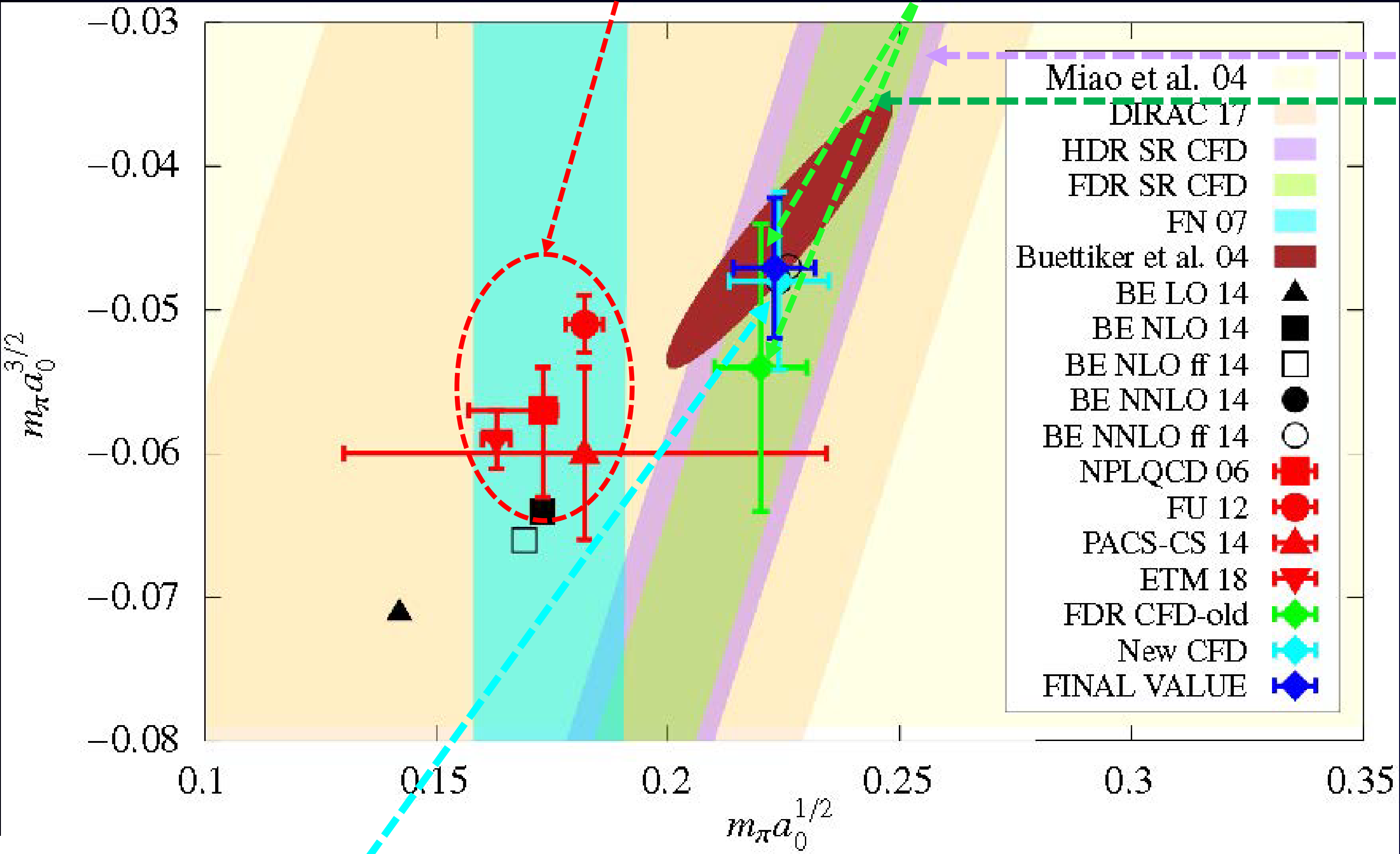
Constrained fits to Data (CFD)

CFD Applications

- Inside dispersion relations to find resonance poles
  Model Independent. Most rigorous but only feasible for elastic region
- With analytic continuation methods to look for poles.
  Very reduced model dependence.
- Inside sum rules to obtained threshold parameters
  Relevant for effective theories and QCD

- Threshold parameters relevant to test ChPT (NNLO at present).
- Present tension between lattice and dispersive results



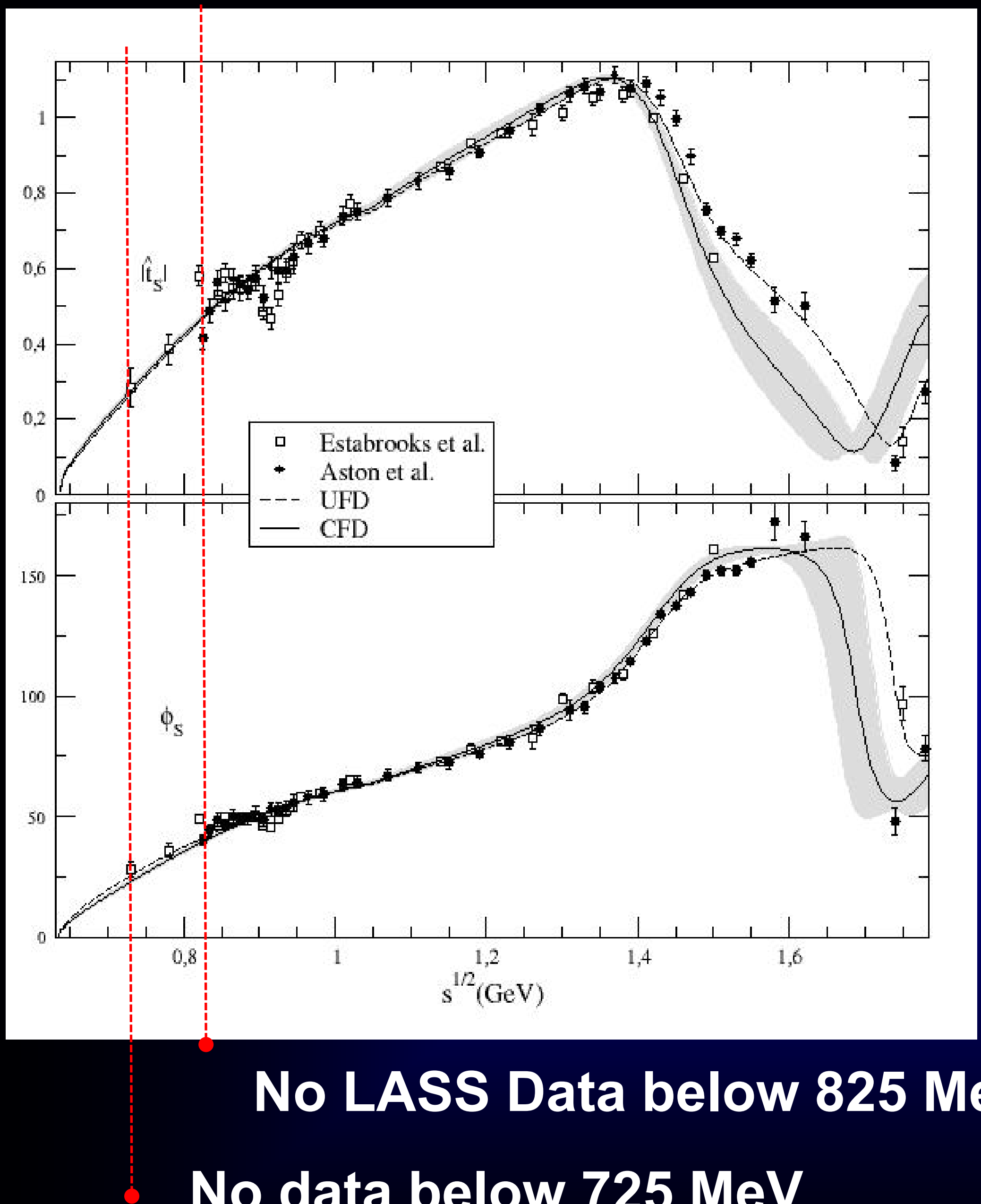
Our Dispersive SUM RULES for a_0^-

Our dispersive CFD

	UFD	CFD	Ref. [43]
$a_0^{1/2}$	0.241±0.012	**0.224±0.011**	0.224±0.022
$a_0^{3/2}$	-0.067±0.012	**-0.048±0.006**	-0.0448±0.0077

But remember DATA GAP below 750MeV and no isospin separation in LASS

KLF may be of relevance here !!



Most reliable sets:

- Estabrooks et al. 78 (SLAC)

- **Aston et al.88 (SLAC-LASS)**
- Largest statistics.
- But measures $t_{1/2}+t_{3/2}/2$. No isospín separation

No LASS Data below 825 MeV. Only 2 points with huge uncertainties from Estabrooks et al. 78 below 800 MeV

No data below 725 MeV

KLF will improve this

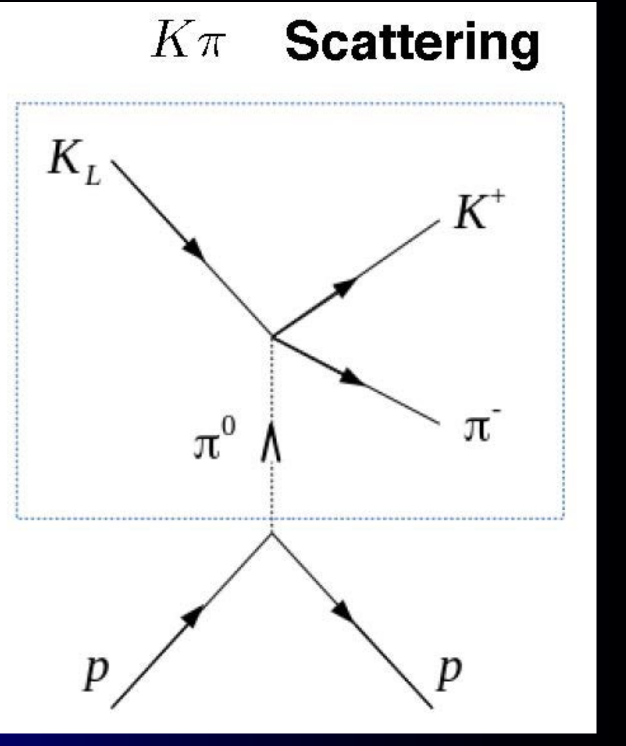
Isospin separation @KLF

- KLF will measure

$$K_L p \to (K^{*0})p \to K^+\pi^- p$$

$$K_L p \to (\overline{K}^{*0})p \to K^-\pi^+ p$$

which are sensitive to $t_{1/2}$-$t_{3/2}$.



But also

$$K_L p \to (K^{*0})p \to K_L\pi^0 p$$

$$K_L p \to (K^{*0})p \to K_L\pi^-\Delta^{++}$$

which are sensitive to $t_{1/2}$+2 $t_{3/2}$

In this way the two isospin states can be separated.

For the latter the K_L will be reconstructed from the missing mass of the proton and the π^0 and the invariant mass of the $K_L\pi^0$ in the missing mass of the proton.

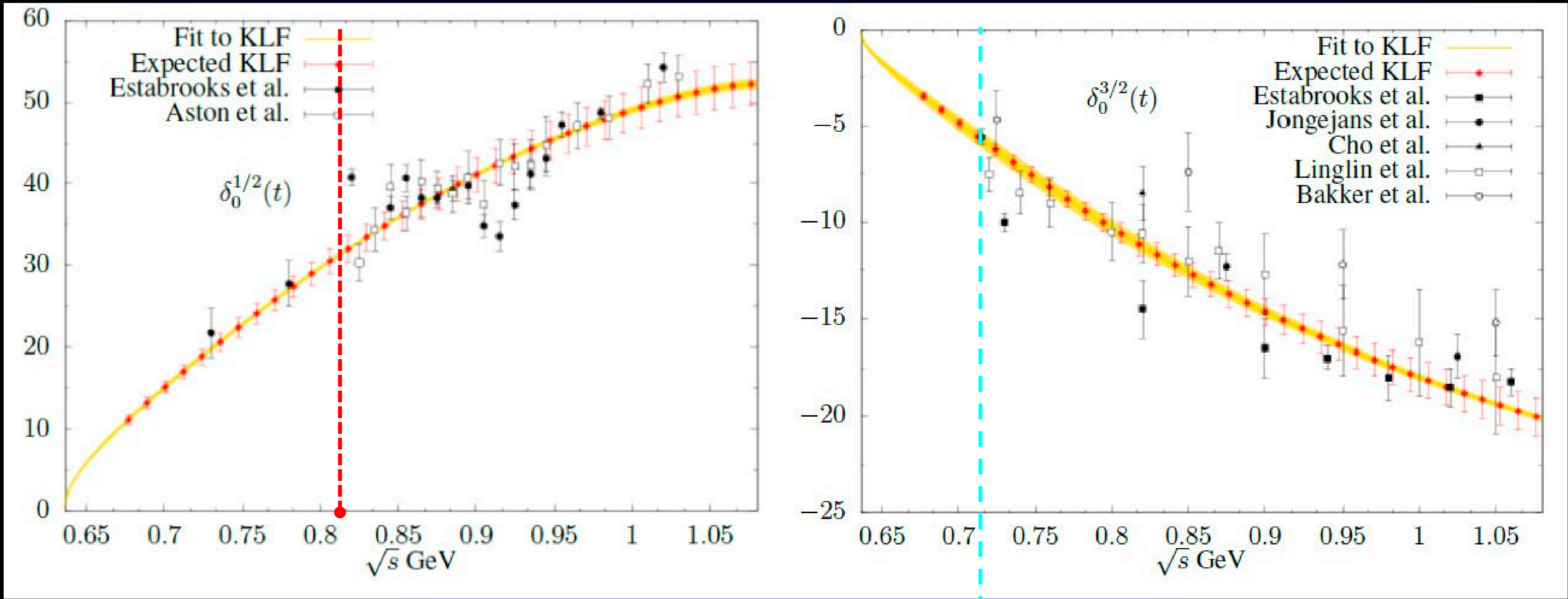
Proposal for JLab PAC48

Strange Hadron Spectroscopy with Secondary K_L Beam in Hall D

arXiv:2008.08215v1 [nucl-ex] 19 Aug 2020

KLF@Jlab EXPECTATIONS:

50 times the LASS data set (was K^- there)



For I=1/2:

- Many energy bins below 825 MeV (there were 2)
- Of which several below 725 MeV (there were)

For I=3/2,

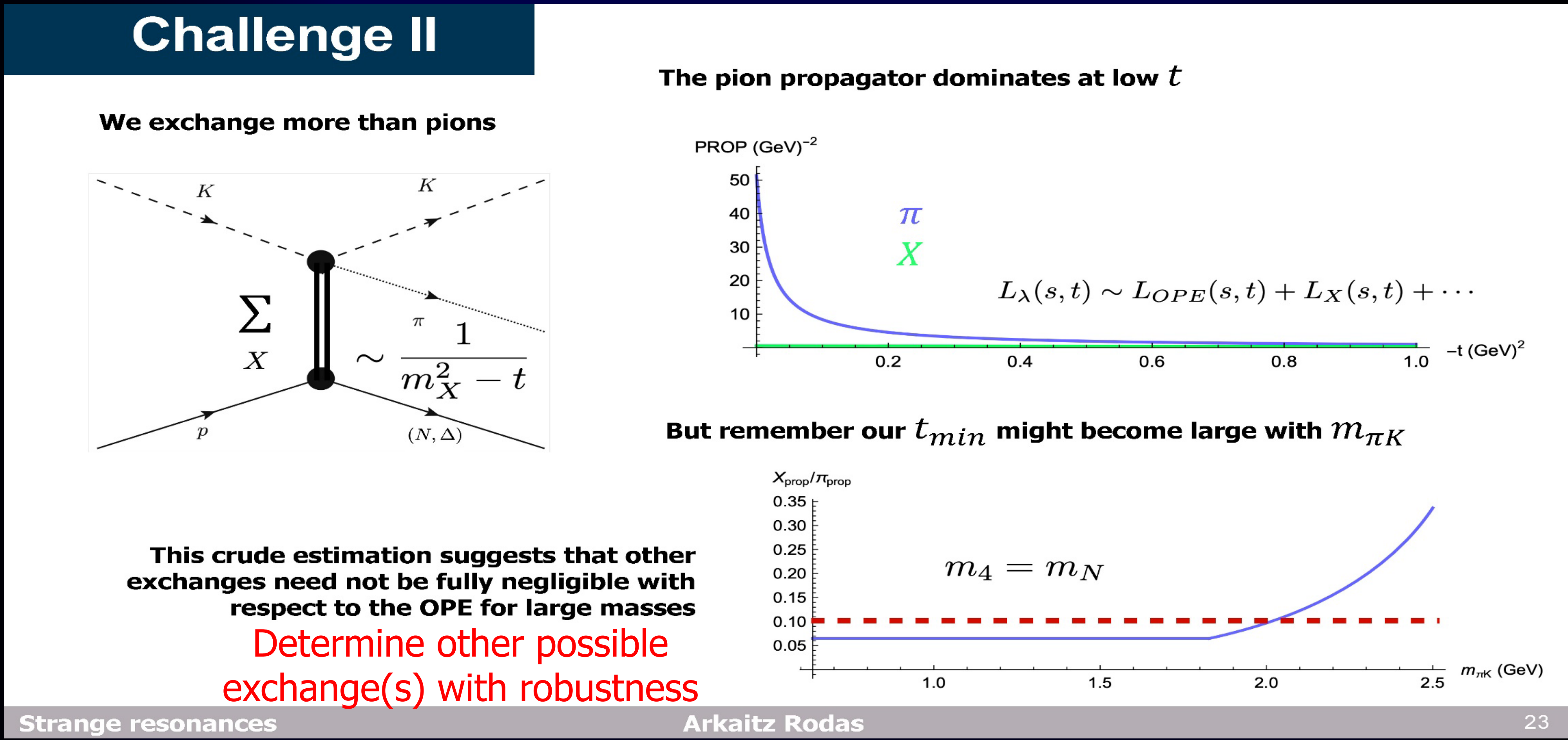
- 3 points below the existing data, possibly more.
- 100 x statistics than Estabrooks et al.
- **Stat. Error bars invisible with KLF.**

κ/$K_0^*(700)$ pole @KLF
Expected uncertainty reduced to ~50%
and similarly for scattering lengths

Pelaez-Rodas HDR [23,80,81]	648±7-i 280±16
KLF expected errors	**648±4-i 280±8**

Caveat

- The previous expectations do NOT include uncertainties due to pion pole dominance model and other contributions to the *t*-dependence (nor did the LASS or other previous data)
- Given the accuracy goal, these will be extremely relevant @KLF (see A. Rodas talk) and will require a delicate treatment.



Attract theoretical talent/experts on reaction theory/exchanges
Polarized target?
Increased K-beam energy? This will help with higher resonances and may allow for KK->ππ,KK studies

SUMMARY

- Over the last years, and as late as 2021, analyticity and dispersion theory applied to meson-meson scattering have settled the longstanding controversy about the existence of two light scalar nonets below 2 GeV.

-The last piece of the lightest scalar puzzle was the strange resonance $\kappa/K^*_0(700)$.
However, its data driven determination has a low-energy data gap, closest to the resonance pole.

-The $K\pi$ scalar scattering lengths show a sizable tension with lattice.
SU(3) Chiral Perturbation Theory convergence?
Dispersive analyses require large extrapolations to threshold

- All strange resonances below 2 GeV have large room for improvement @PDG.
Often due to conflicting data or use of naive models (BW)

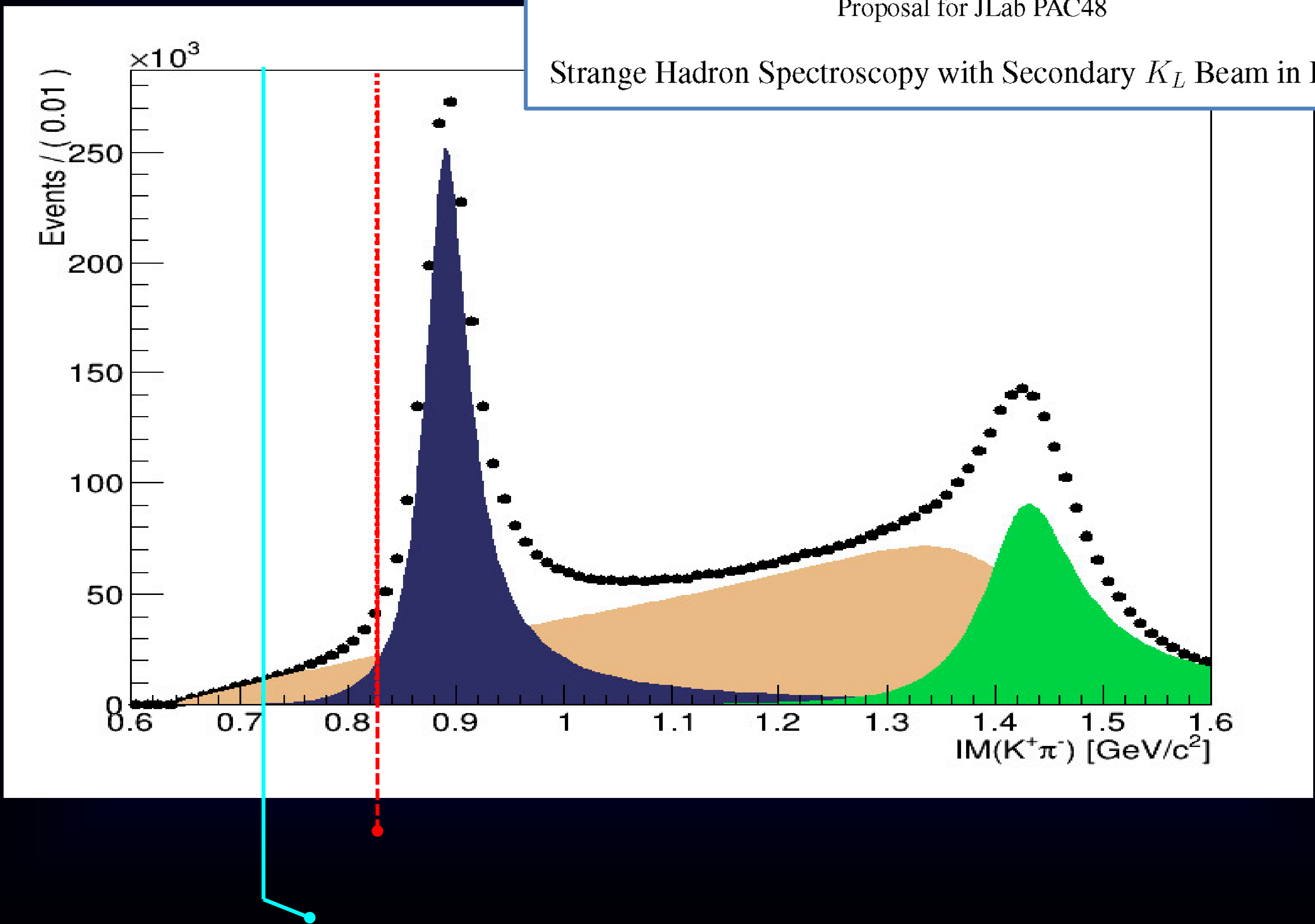
- One of the KLF@Jlab proposal goals is to obtain a huge statistical sample of $K\pi$ scattering data, covering the low-energy gap and providing isospin separation. Systematic t-dependence effects will be relevant, possibly dominant in the uncertainties. Further future upgrades could help taming these effects.

Expected number of events after 100-day run @ KLF

arXiv:2008.08215v1 [nucl-ex] 19 Aug 2020

Proposal for JLab PAC48

Strange Hadron Spectroscopy with Secondary K_L Beam in Hall D



We use that

$$I(\pi) = 1, I_3(\pi^0) = 0, \tag{1}$$
$$I(K) = 1/2, I_3(K^0) = -1/2, I_3(\bar{K^0}) = 1/2, \tag{2}$$

and that

$$\langle K_L| = \frac{\langle K^0| + \langle \bar{K}^0|}{\sqrt{2}}, \tag{3}$$
$$\langle K_S| = \frac{\langle K^0| - \langle \bar{K}^0|}{\sqrt{2}}, \tag{4}$$

now by construction

$$\langle K_L\pi^0| = \frac{\langle K^0\pi^0| + \langle \bar{K}^0\pi^0|}{\sqrt{2}}, \tag{5}$$
$$\langle K_S\pi^0| = \frac{\langle K^0\pi^0| - \langle \bar{K}^0\pi^0|}{\sqrt{2}}, \tag{6}$$

so that

$$\langle K_L\pi^0| T |K_S\pi^0\rangle = \frac{1}{2}\left(\langle K^0\pi^0| T |K^0\pi^0\rangle - \langle \bar{K}^0\pi^0| T |\bar{K}^0\pi^0\rangle\right). \tag{7}$$

The minus sign was a plus in the previous calculation

Now one can use the Clebsch-Gordan coefficients for the states with defined I_3

$$\langle K^0\pi^0| = \frac{1}{\sqrt{3}}\langle 1/2, -1/2| + \frac{\sqrt{2}}{\sqrt{3}}\langle 3/2, -1/2|, \tag{8}$$
$$\langle \bar{K}^0\pi^0| = -\frac{1}{\sqrt{3}}\langle 1/2, 1/2| + \frac{\sqrt{2}}{\sqrt{3}}\langle 3/2, 1/2|. \tag{9}$$

Finally by introducing this coefficients in Eq. (7) we get

$$\langle K_L\pi^0| T |K_S\pi^0\rangle = \frac{1}{2}\left(T^{1/2}/3 + 2T^{3/2}/3 - T^{1/2}/3 - 2T^{3/2}/3\right) = 0 \tag{10}$$

$$\langle K_L\pi^0| T |K_L\pi^0\rangle = \frac{1}{2}\left(T^{1/2}/3 + 2T^{3/2}/3 + T^{1/2}/3 + 2T^{3/2}/3\right), \tag{11}$$
$$= T^{1/2}/3 + 2T^{3/2}/3 \tag{12}$$