Lattice QCD for Hyperon Spectroscopy

David Richards
Jefferson Lab

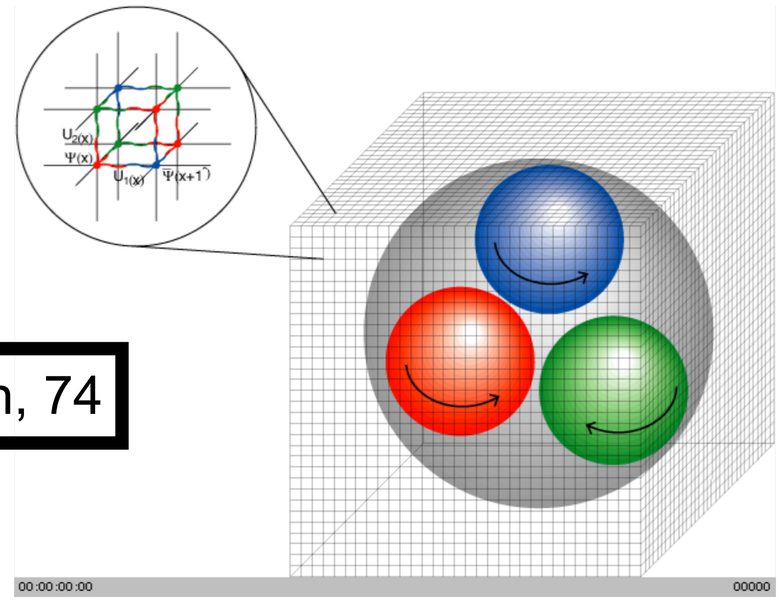
KL Collaboration Meeting

Outline

- Lattice QCD - the basics…..
- Baryon spectroscopy
  – What's been done….
  – Why the hyperons?
- What are the challenges….
- What are we doing to overcome them…

Lattice QCD

We'll see why later…



• Continuum Euclidean space time replaced by four-dimensional lattice, or grid, of "spacing" a

• Gauge fields are represented at SU(3) matrices on the links of the lattice - work with the elements rather than algebra

$$U_\mu(n) = e^{iaT^a A^a_\mu(n)}$$

Wilson, 74

Quarks ψ, ψ are Grassmann Variables, associated with the sites of the lattice

Work in a finite 4D space-time volume

– Volume V sufficiently big to contain, e.g. proton
– Spacing a sufficiently fine to resolve its structure

Gattringer and Lang, *Lattice Methods for Quantum Chromodynamics*, Springer

DeGrand and DeTar, *Quantum Chromodynamics on the Lattice*, WSPC

Lattice QCD - Summary

Lattice QCD is QCD formulated on a Euclidean 4D spacetime lattice. It is systematically improvable. For *precision calculations:*:

- Extrapolation in lattice spacing (cut-off) $a \to 0$: $a \leq 0.1$ fm
- Extrapolation in the Spatial Volume $V \to \infty$: $m_\pi L \geq 4$
- Sufficiently large temporal size T: $m_\pi T \geq 10$
- Quark masses at physical value $m_\pi \to 140$ MeV: $m_\pi \geq 140$ MeV
- Isolate ground-state hadrons

Ground-state masses

Hadron form factors, structure functions, GPDs

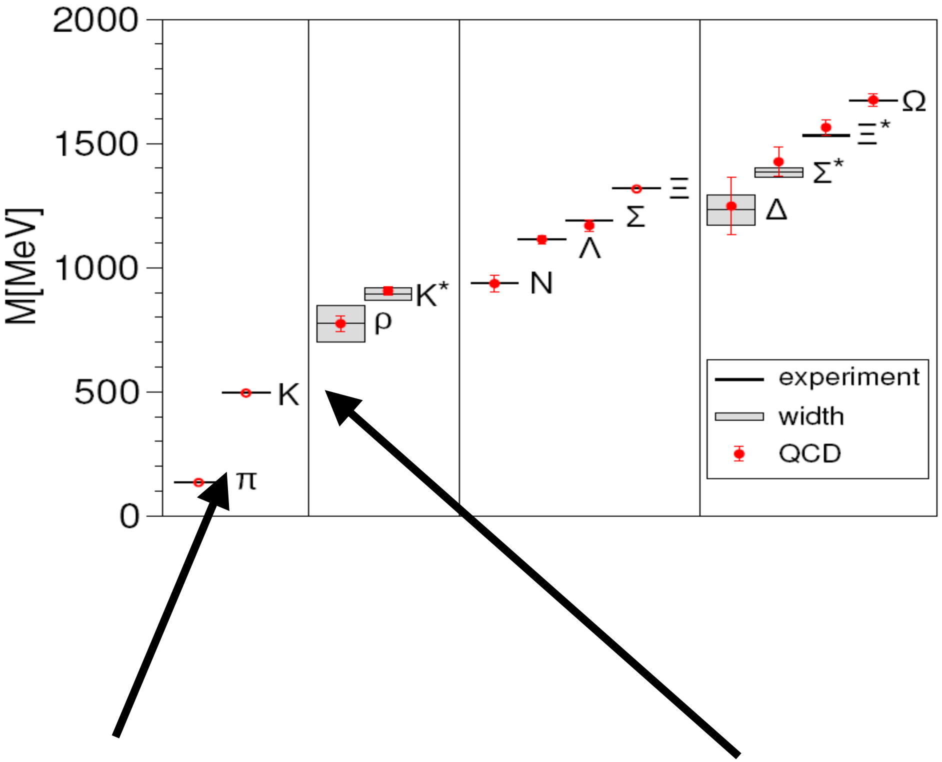
Nucleon and precision matrix elements

Low-lying Spectrum

$$C(t) = \sum_{\vec{x}} \langle 0 \mid \Phi(\vec{x},t)\Phi^\dagger(0) \mid 0\rangle$$

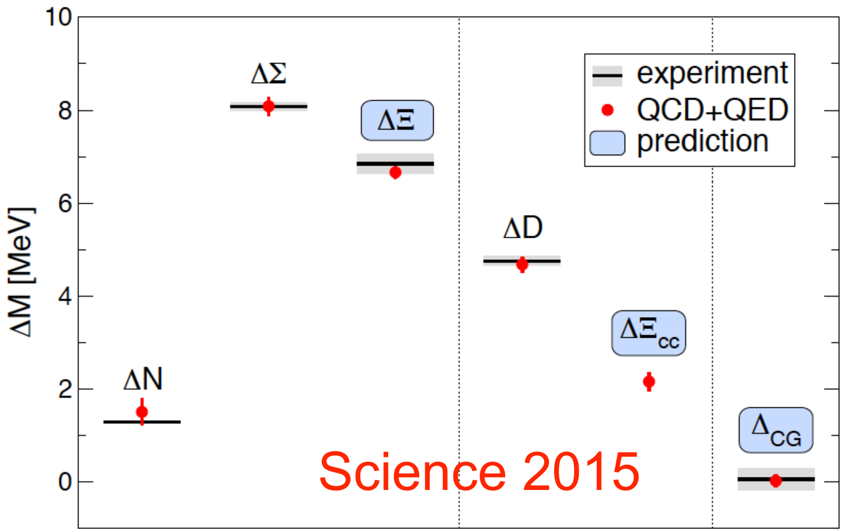
$$C(t) = \sum_{\vec{x},n} \langle 0 \mid e^{ip\cdot x}\Phi(0)e^{-ip\cdot x} \mid n\rangle\langle n \mid \Phi^\dagger(0) \mid 0\rangle$$

$$= \sum_{n} |\langle 0|\Phi(0)|n\rangle|^2 e^{-E_n t}$$



Science 2008 Durr et al., BMW Collaboration

Now with electro-magnetic splittings included



Need physical “ratios” to fit: $m_{u/d}$, m_s

Variational Method

Subleading terms → *Excited states*

Construct matrix of correlators with *judicious choice of operators*

$$C_{ij}(t,0) = \frac{1}{V_3}\sum_{\vec{x},\vec{y}} \langle \mathcal{O}_i(\vec{x},t)\mathcal{O}_j^\dagger(\vec{y},0)\rangle = \sum_N \frac{Z_i^{N*} Z_j^N}{2E_N} e^{-E_N t}$$

Delineate contributions using *variational method*: solve

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$

$$\lambda_N(t,t_0) \rightarrow e^{-E_N(t-t_0)}(1+\mathcal{O}(e^{-\Delta E(t-t_0)}))$$

Can pull out excited-state energies - but pion and nucleon only states stable under strong interactions!

Baryon Operators

Aim: interpolating operators of *definite* (continuum) JM: O^{JM}

Starting point

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

$$B = \left(\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}\right)\{\psi_1 \psi_2 \psi_3\}$$

Flavor   Spin   Orbital

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x - i\overleftrightarrow{D}_y\right)$$

$$\overleftrightarrow{D}_{m=0} = i\overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x + i\overleftrightarrow{D}_y\right).$$

Chromomagnetic

$$[D_i, D_j] \equiv F_{ij}$$

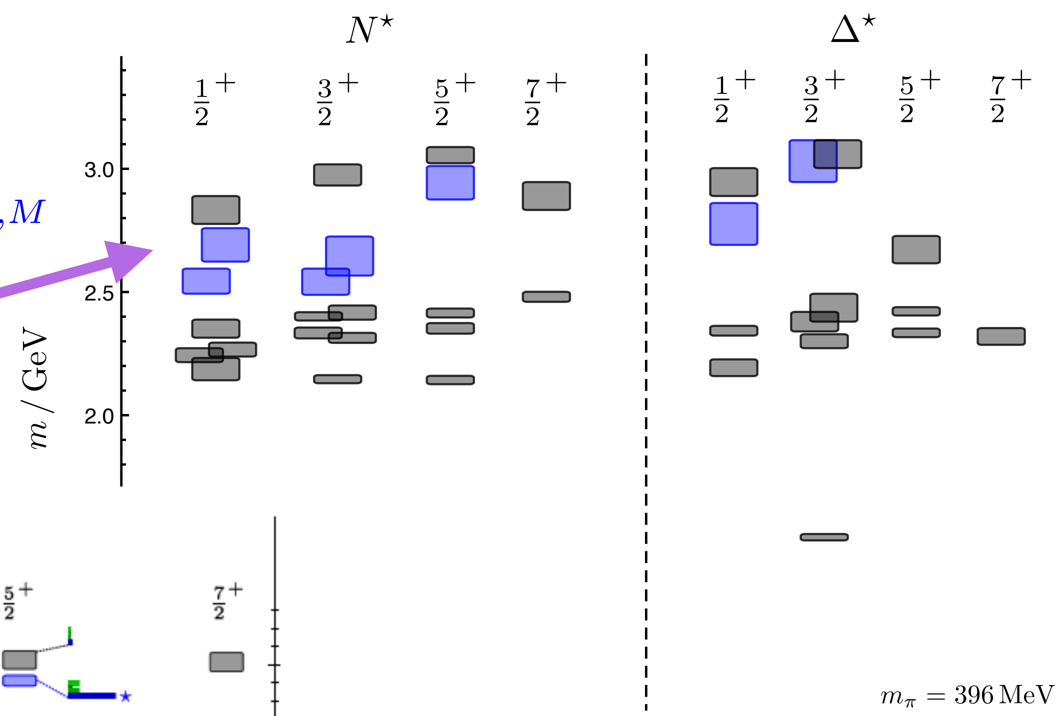
Straighforward to project to definite spin: $J = 1/2, 3/2, 5/2$

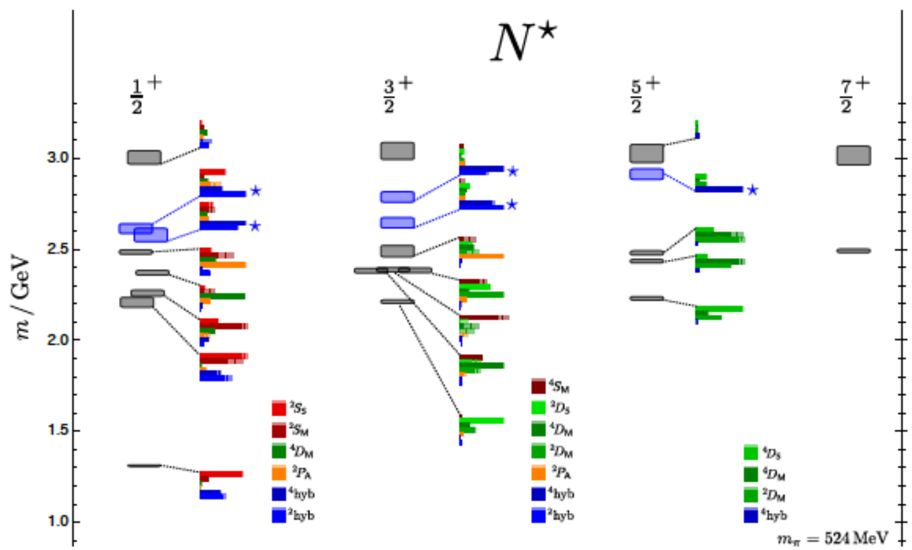
$$\left|[J, M]\right\rangle = \sum_{m_1, m_2} \left|[J_1, m_1]\right\rangle \otimes \left|[J_2, m_2]\right\rangle \langle J_1 m_1; J_2 m_2 | JM \rangle$$

Positive-parity Baryon Spectrum

$D_{l=1,M}^{[2]}$

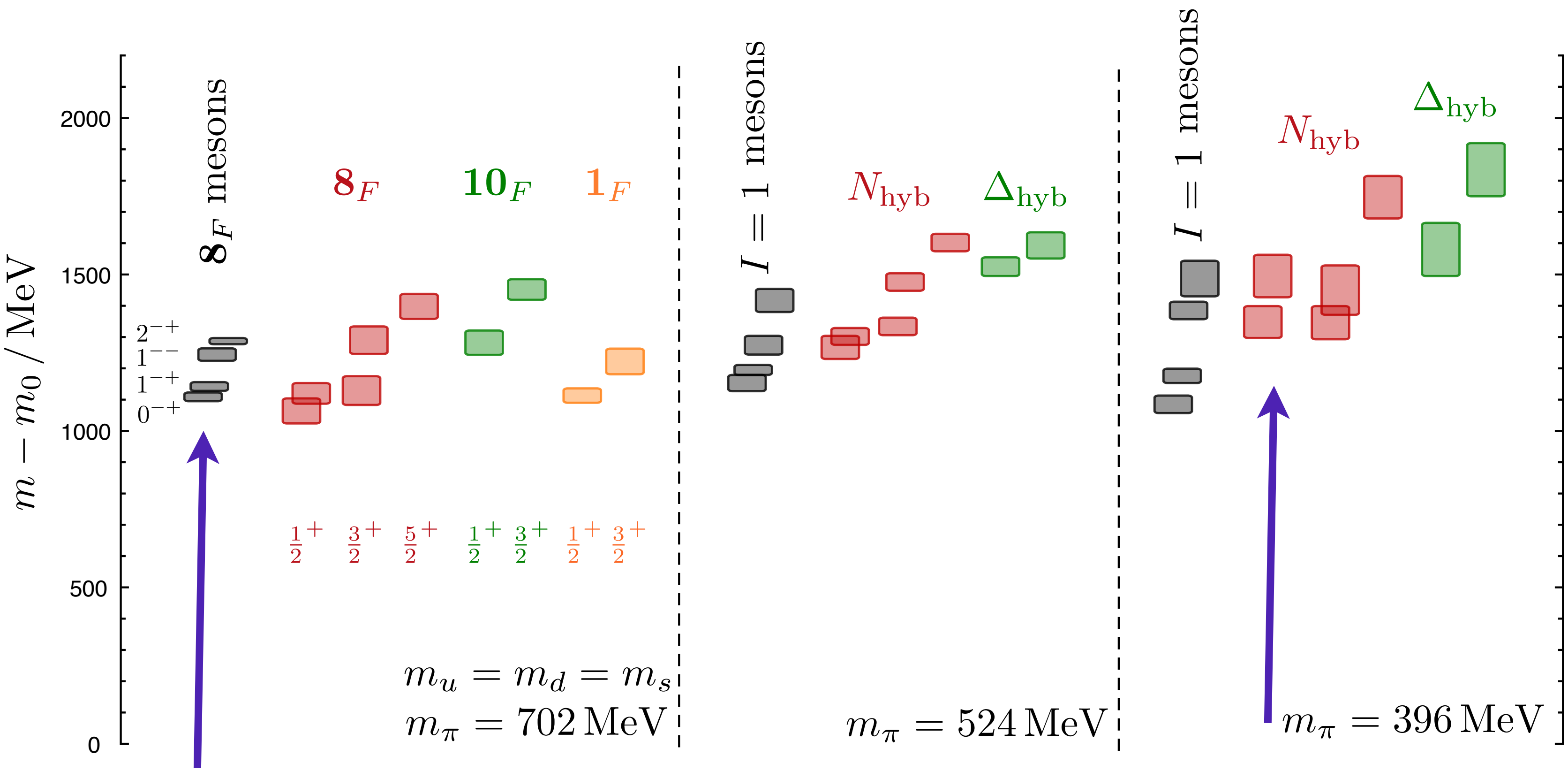
Hybrid: gluons structural





Dudek, Edwards, arXiv:1201.2349

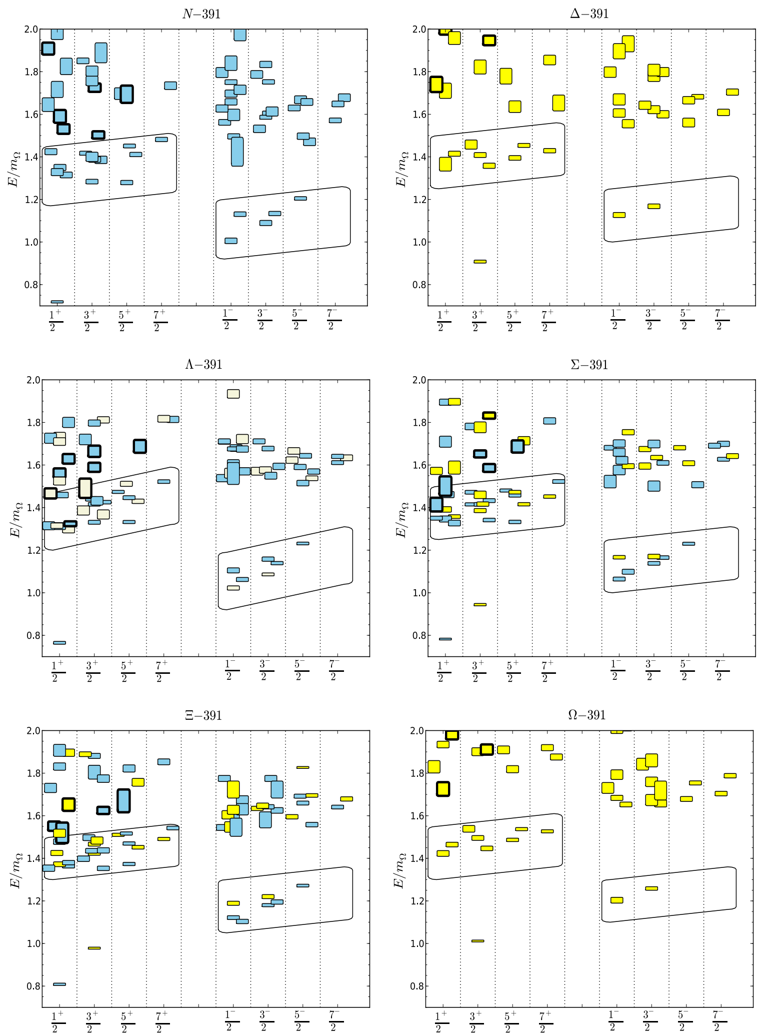
Putting it Together



Subtract ρ

Subtract N

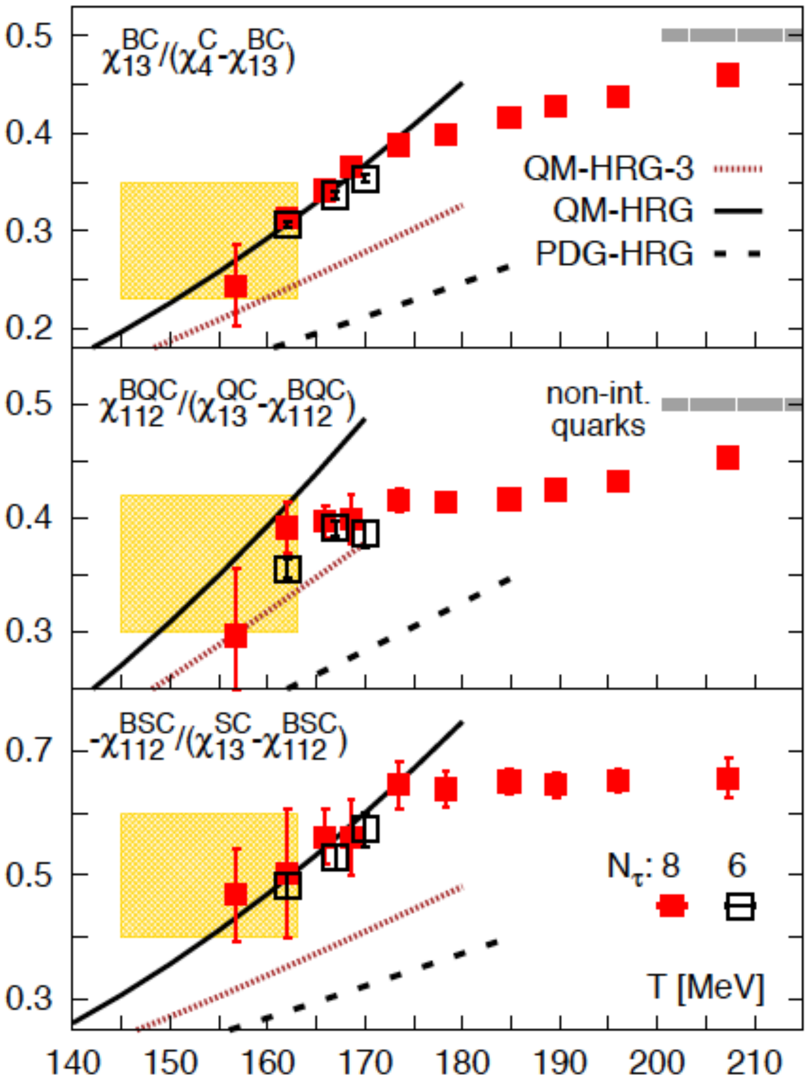
Common mechanism in meson and baryon hybrids: chromomagnetic field with *$E_g \sim$ 1.2 - 1.3 GeV*



Spectrum is *at least* as rich as quark model - *plus hybrid states*

R. Edwards et al., Phys. Rev. D87 (2013) 054506

Evidence for many charmed Baryons



Bazavov et al, PLB 737, 210 (2014)

All charmed mesons/baryons

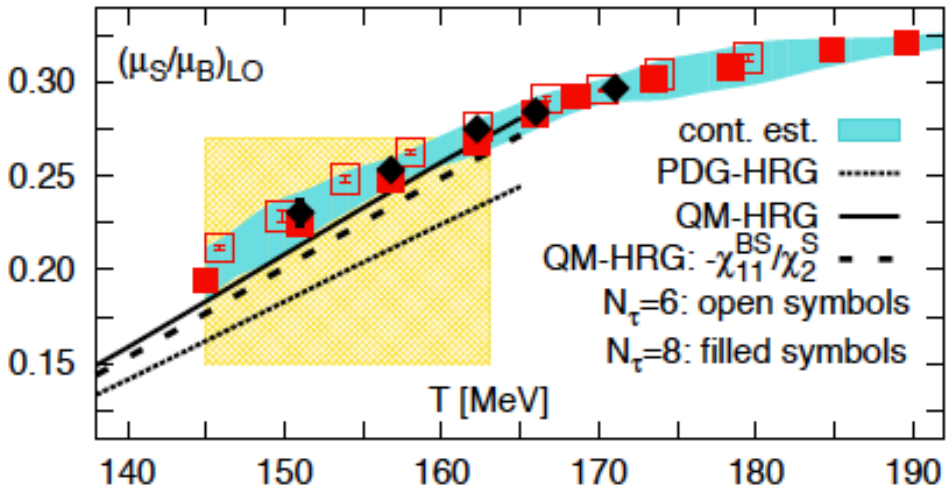
Charged charmed mesons/baryons

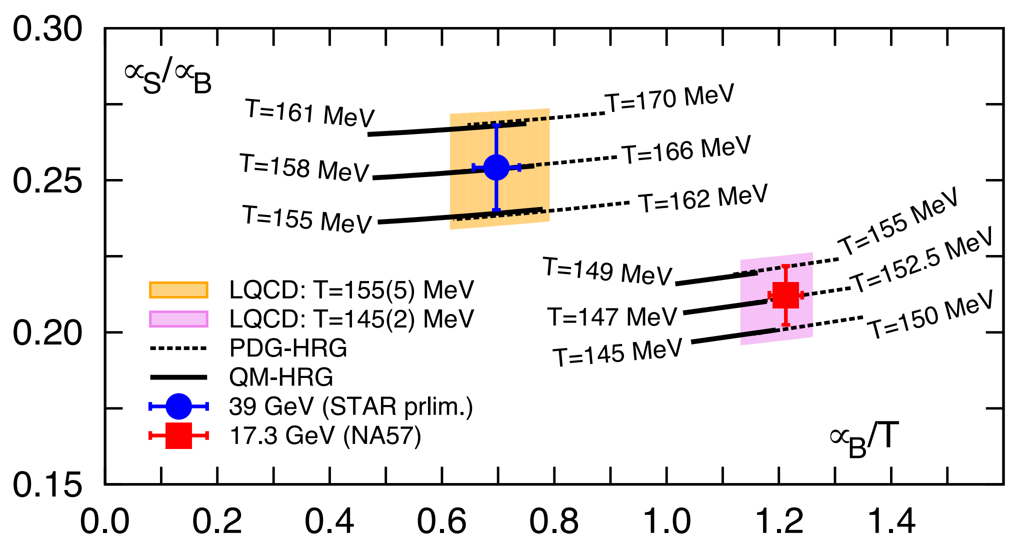
Strange charmed mesons/baryons

HRG with richer spectrum of states than PDG to describe lattice calculations

Thermal Conditions at Freeze-out

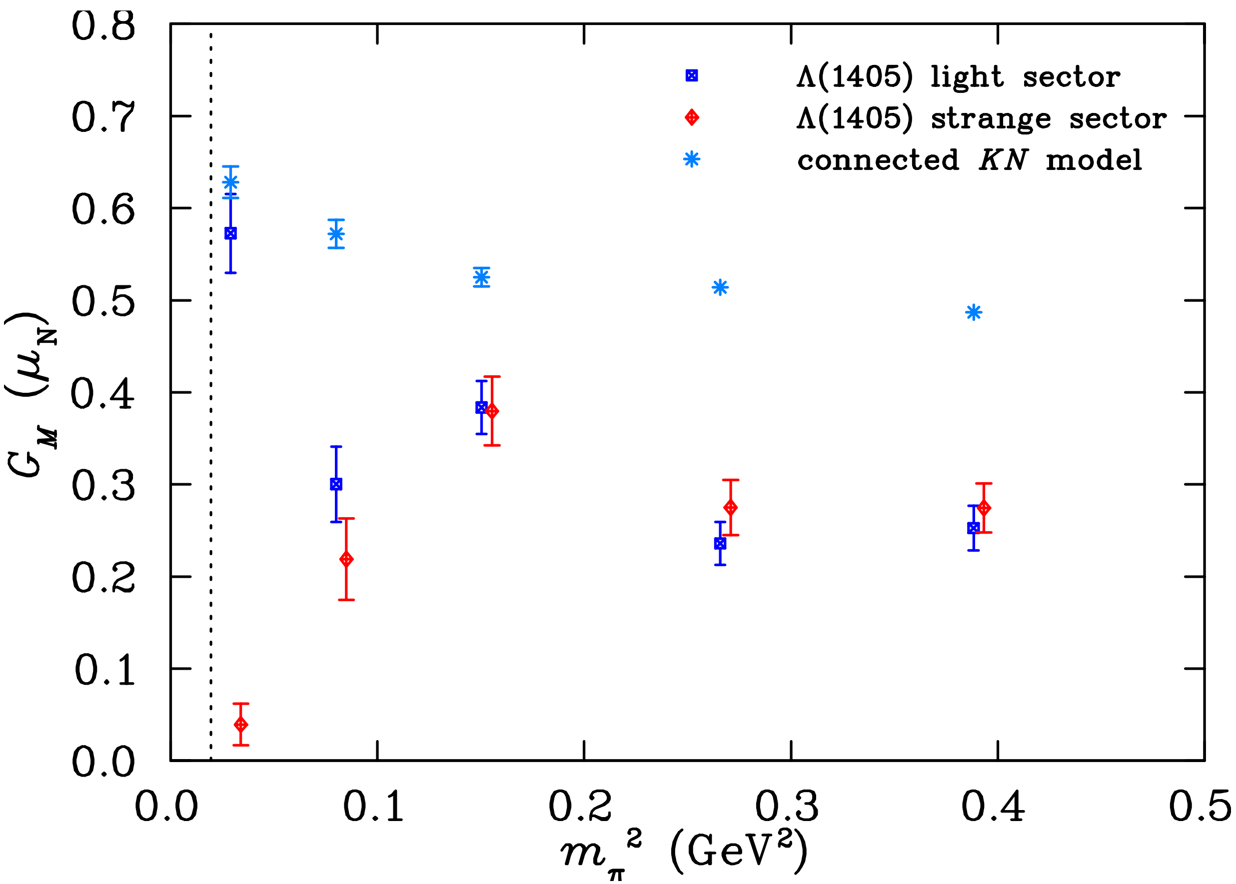
Bazavov et al, PRL 113, 072001 (2014)





Including additional strange states → lower freeze out temperature

Hints at structure of $\Lambda(1405)$?



Hall *et al, Phys. Rev. D 95, 054510*

Spectrum is rich - and strange-quark states essential component

↓

Caveat Emptor! - states are resonances, unstable under strong interactions

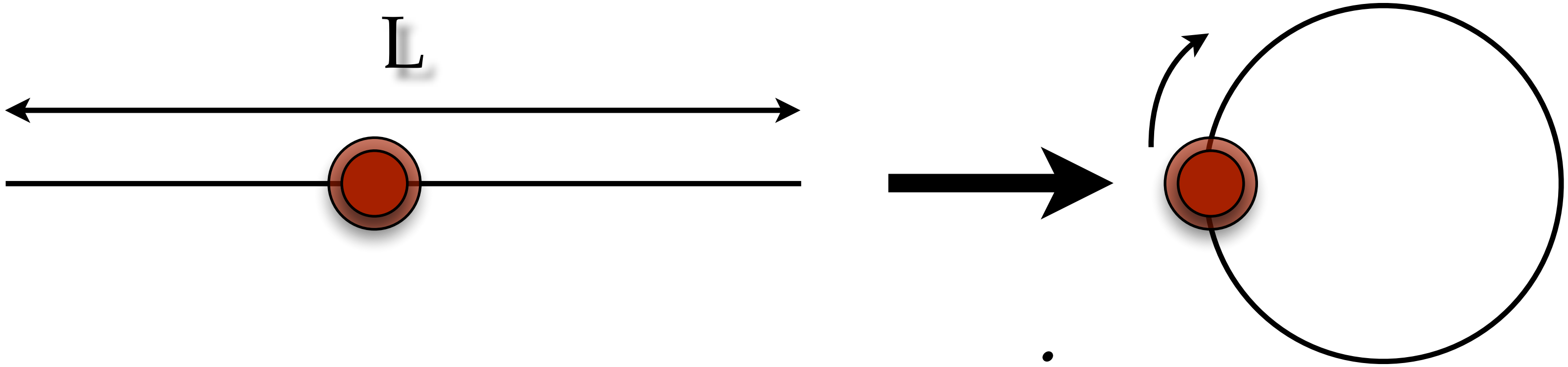
↓

"Luscher method" - relate energies shifts at finite volume to infinite-volume scattering amplitudes

R.Briceno,J.Dudek,R.Young, Rev.Mod.Phys. 90 (2018), 025001

Reinventing the *quantum-mechanical* wheel

Thanks to Raul Briceno (in 1+1 dimensions)

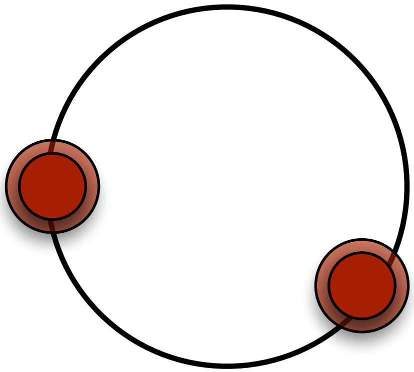


$$\phi(x) \sim e^{ipx}$$

Periodicity: $$L\, p_n = 2\pi n$$

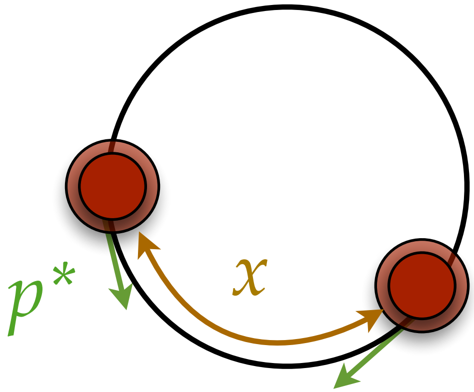
Reinventing the *quantum-mechanical* wheel

Two particles:



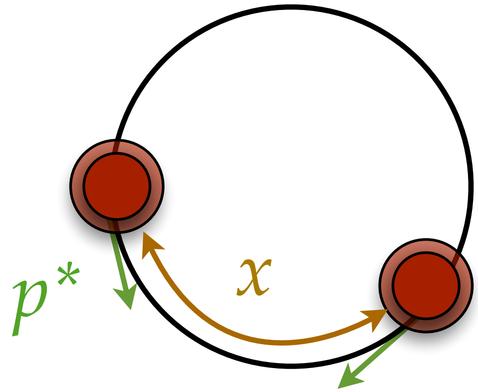
Reinventing the *quantum-mechanical* wheel

Two particles:



Reinventing the *quantum-mechanical* wheel

Two particles:



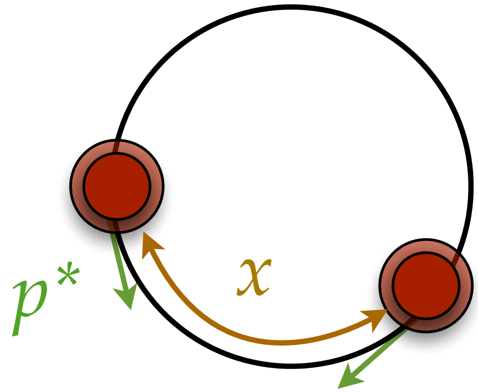
$$\psi(x) \sim e^{ip^*|x| + i2\delta(p^*)}$$

Asymptotic wavefunction

infinite volume scattering phase shift

Reinventing the *quantum-mechanical* wheel

Two particles:



infinite volume scattering phase shift

$$\psi(x) \sim e^{ip^*|x| + i2\delta(p^*)}$$

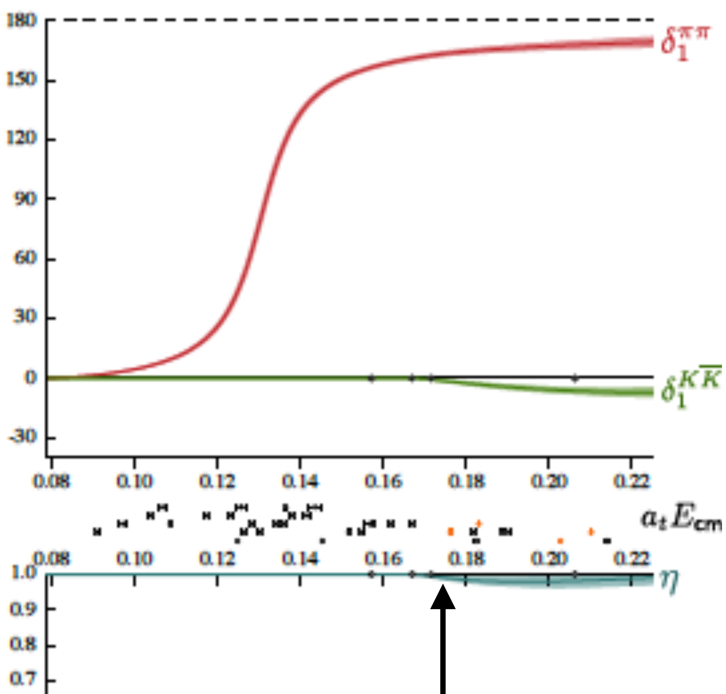
Asymptotic wavefunction

Periodicity: $$L\, p_n^* + 2\delta(p_n^*) = 2\pi n$$

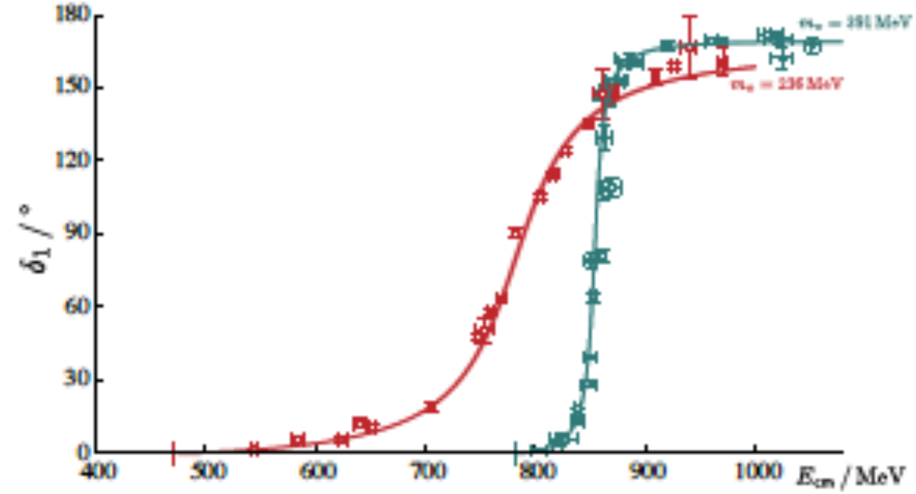
See Colin Morningstar's seminar….

Resonant Phase Shift

We have treated excitations as stable states - *resonances under strong interaction*
Luscher: finite-volume energy levels to infinite-volume scattering phase shift



Hadspec collaboration



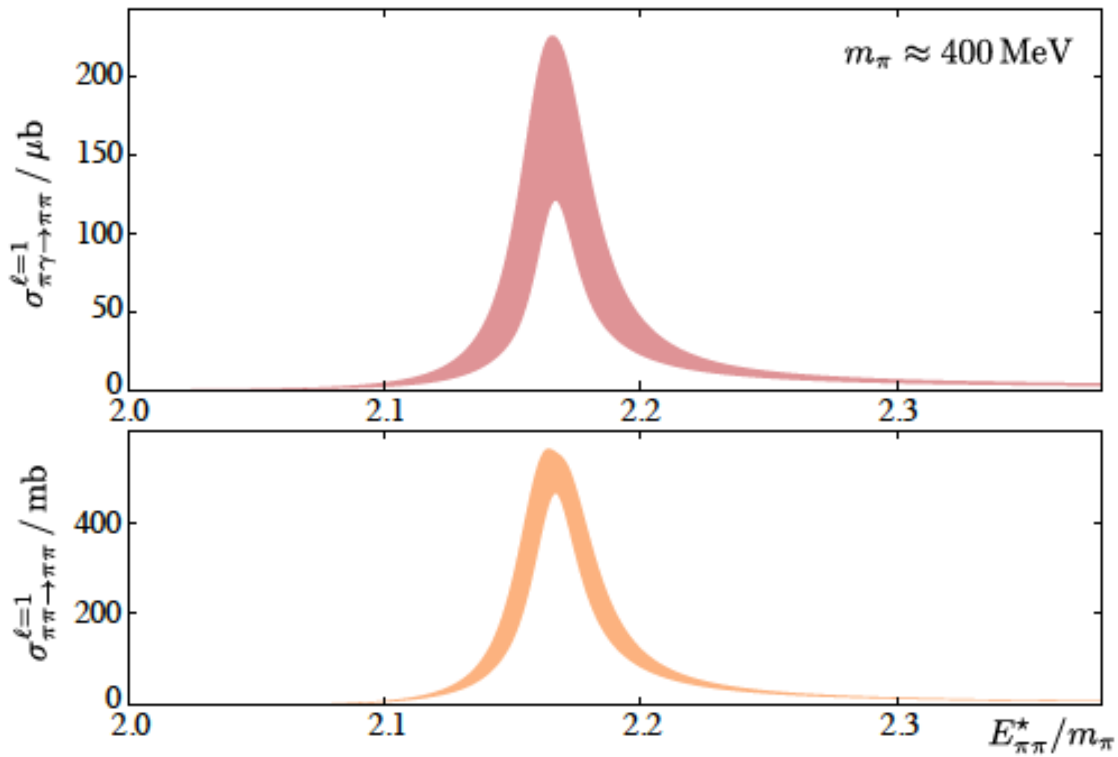
Decreasing Pion Mass

Inelastic Threshold

$$\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_m \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^m_\ell(\hat{p})\, \mathcal{O}_\pi(\vec{p})\mathcal{O}_\pi(-\vec{p})$$

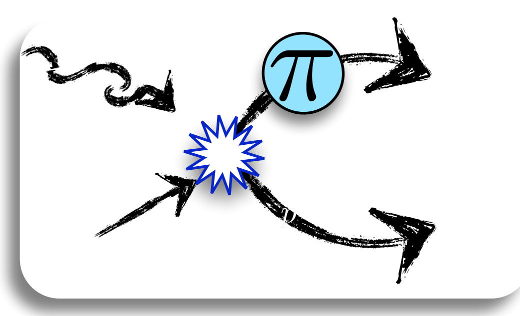
Wilson, Briceno, Dudek, Edwards, Thomas, arXiv:1507.02599

Transition form factor of ρ



ρ resonance

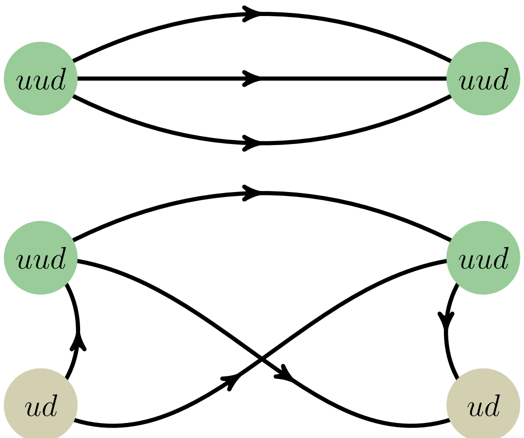
Briceno, Hansen and Walker-Loud, PRD 91, 034501 (2015)



Briceno et al., Phys. Rev. D 93, 114508 (2016)

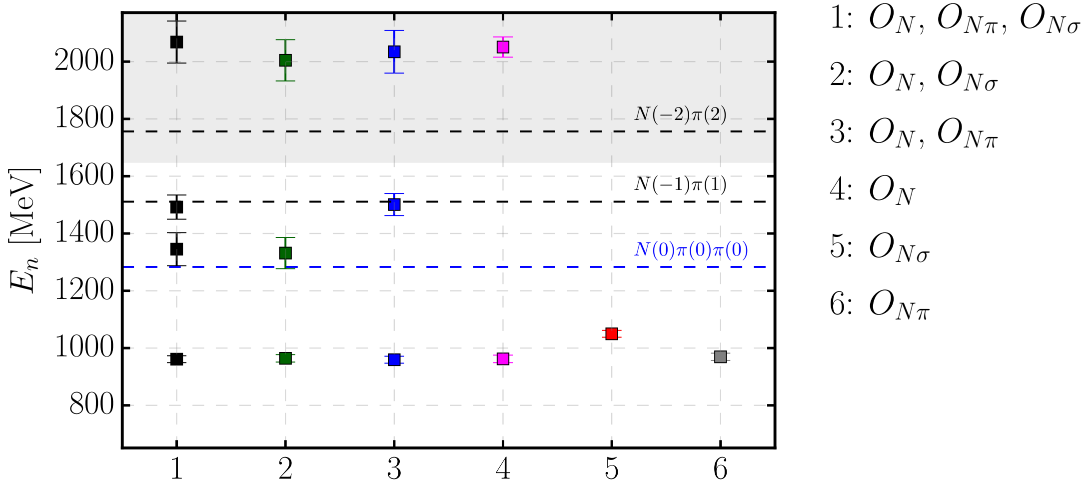
What about Baryons - and hyperons?

The theoretical elements are in place……



Combinatorics of Wick contractions much more demanding….

Luka Leskovec *et al., arXiv:1806.02363*



Combinatorics are limiting factor

Hierarchy of Computations

Capability Computing -
Gauge Generation

e.g. Summit at ORNL

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

Several *V, a, T, m_π*

~ 10% Leadership-
Class Resources

Capacity Computing -
Observable Calculation

e.g. GPU/KNL cluster at
JLab, BNL, FNAL

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

$$\text{e.g. } C(t) = \sum_{\vec{x}} \langle N(\vec{x}, t) \bar{N}(0) \rangle$$

Computationally Dominant

"Desktop" Computing -
Physical Parameters

e.g. Mac at your desk

$$C(t) = \sum_n A_n e^{-E_n t}$$

$$M_N(a, m_\pi, V)$$

hadspec

Hadron Spectrum Collaboration

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ECP

EXASCALE COMPUTING PROJECT

Centered at JLab (not me!)

Major effort at JLab - led by Robert Edwards

Important element is speeding up the contractions!

Distillation

Measure matrix of correlation functions:
$$C_{ij}(t) \equiv \sum_{\vec{x},\vec{y}} \langle N_i(\vec{x},t)\bar{N}_j(\vec{y},0)\rangle$$

M. Peardon *et al.*, PRD80,054506 (2009)

Can we evaluate such a matrix efficiently, for reasonable basis of operators?

Introduce $\tilde{\psi}(\vec{x},t) = L(\vec{x},\vec{y})\psi(\vec{y},t)$ **where L is 3D Laplacian**

Write $L \equiv (1-\kappa\nabla/n)^n = \sum_i f(\lambda_i)\xi^i \times \xi^{*i}$ **where λ_i and ξ_i are eigenvalues and eigenvectors of the Laplacian.**

We now truncate the expansion at i = N_{eigen} where N_{eigen} is sufficient to capture the low-energy physics.

Insert between each quark field in our correlation function.

Perambulators $\tau^{ij}_{\alpha\beta}(t,0) = \xi^{*i}(t)M^{-1}(t,0)_{\alpha\beta}\xi^j$

$$C_{ij}(t) = \phi^{i,(pqr)}_{\alpha\beta\gamma)}(t)\phi^{j,(\bar{p}\bar{q}\bar{r})}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}(0) \times \left[\tau^{p\bar{p}}_{\alpha\bar{\alpha}}(t,0)\tau^{q\bar{q}}_{\beta\bar{\beta}}(t,0)\tau^{r\bar{r}}_{\gamma\bar{\gamma}}(t,0) + \ldots\right]$$

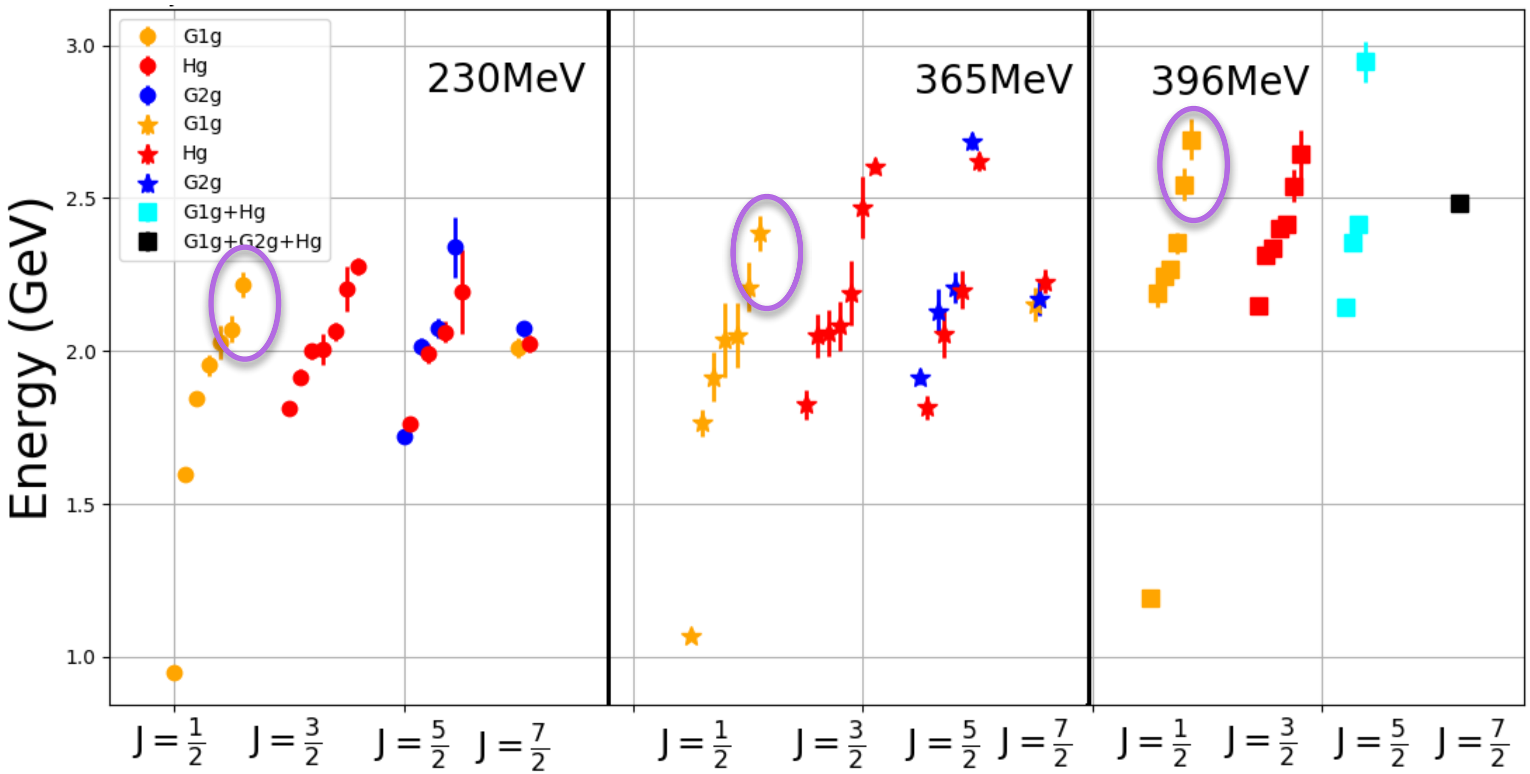
- Meson correlation functions N^3
- Baryon correlation functions N^4 ← **Severely constrains baryon lattice sizes**
- Stochastic sampling of eigenvectors - *stochastic LaPH*

– Lower pion mass
– Finer isotropic lattice
– Different action

Tanjib Khan, *preliminary*

Convergence between Spectroscopy Structure

Hybrid-like states



Summary

- Lattice QCD enables the solution of QCD - it is not modeling QCD!
- Lattice calculations have already demonstrated that the importance of a hyperon program:
  - Spectrum is rich
  - New states needed to describe phase structure of QCD
- Theoretical framework is in place:
  - "Luscher" approach and its extension to multi-channel and inelastic processes
  - External currents - *transition form factors*
- Alignment of theoretical advances, exascale computers, and the software to exploit them!
- Convergence of hadron structure and spectroscopy efforts.