On the importance of Kpi scattering for Phenomenology

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Outline

1. Introduction and Motivation
2. Test of ChPT
3. Hadron spectroscopy
4. Test of the SM and new physics
5. Conclusion and outlook

1. Introduction and Motivation

1.1 Why $K\pi$ scattering is important?

- Hadron spectroscopy: determine resonances and their nature
  - P-wave: K*(892), K*(1410), K*(1680), …
  - S-wave: “κ(~800)”, …
  - Exotics,…

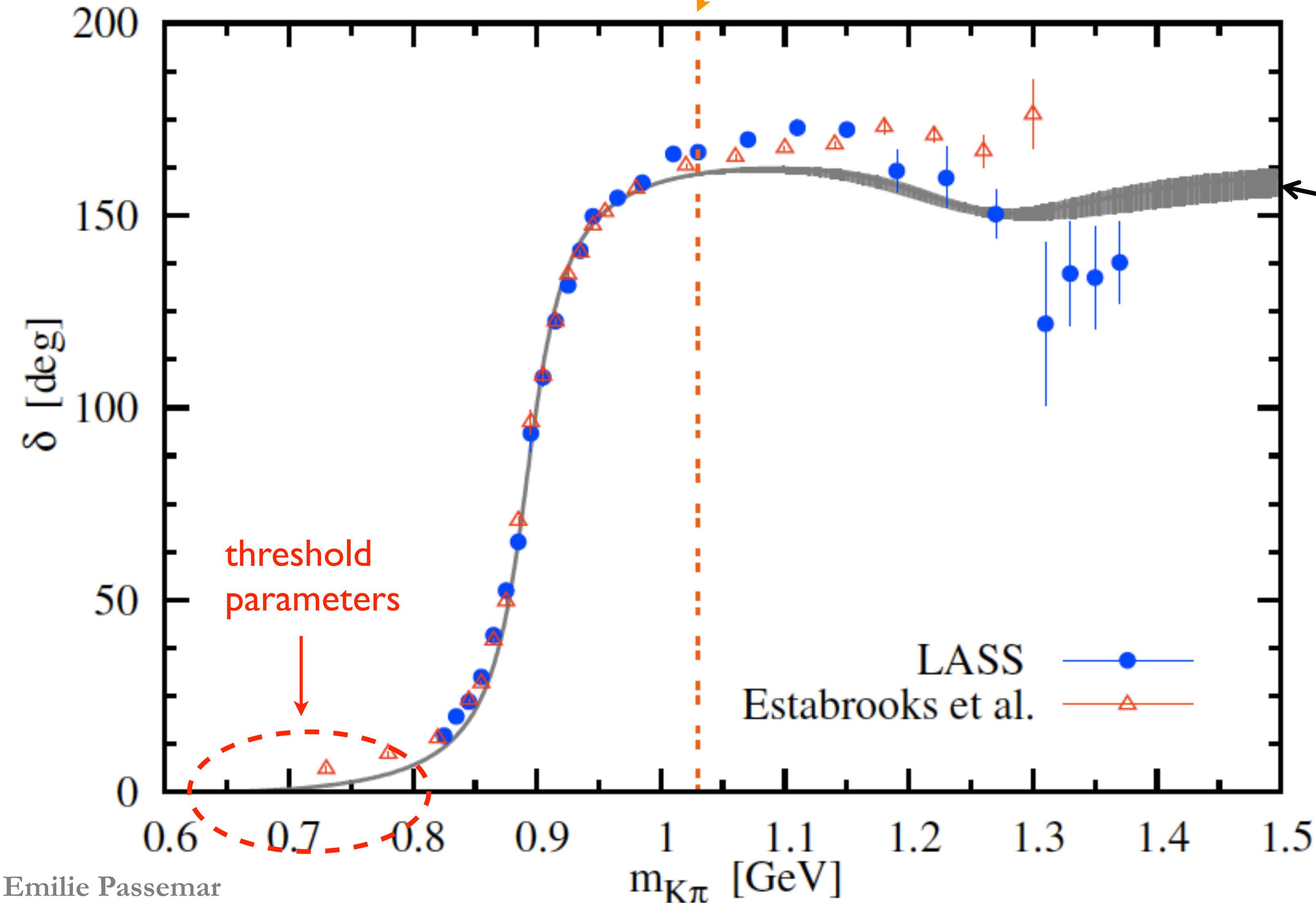
- $\pi\pi$ and $K\pi$ building blocks for hadronic physics:
  - Test of Chiral Dynamics
  - Extraction of fundamental parameters of the Standard Model
  - Look for physics beyond the Standard Model: High precision at low energy as a key to new physics?

➡ Very important when *Final State Interactions* at play!

1.2 Ex: Kπ scattering, P-wave

K*π threshold

Boito, Escribano & Jamin'10

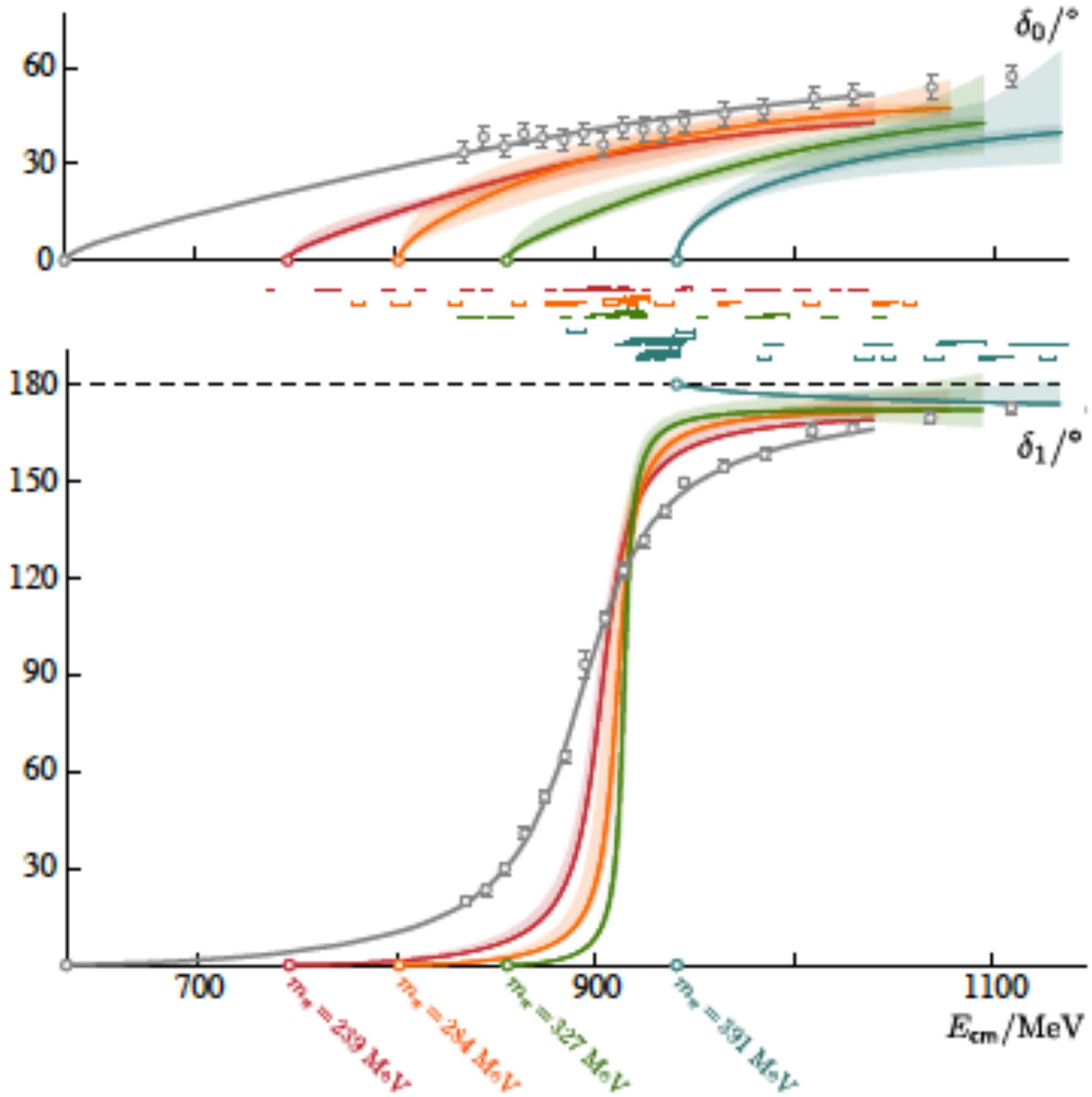


threshold parameters

Tau data

$\tau \to K\pi\nu_\tau$

1.2 Kπ scattering from lattice QCD



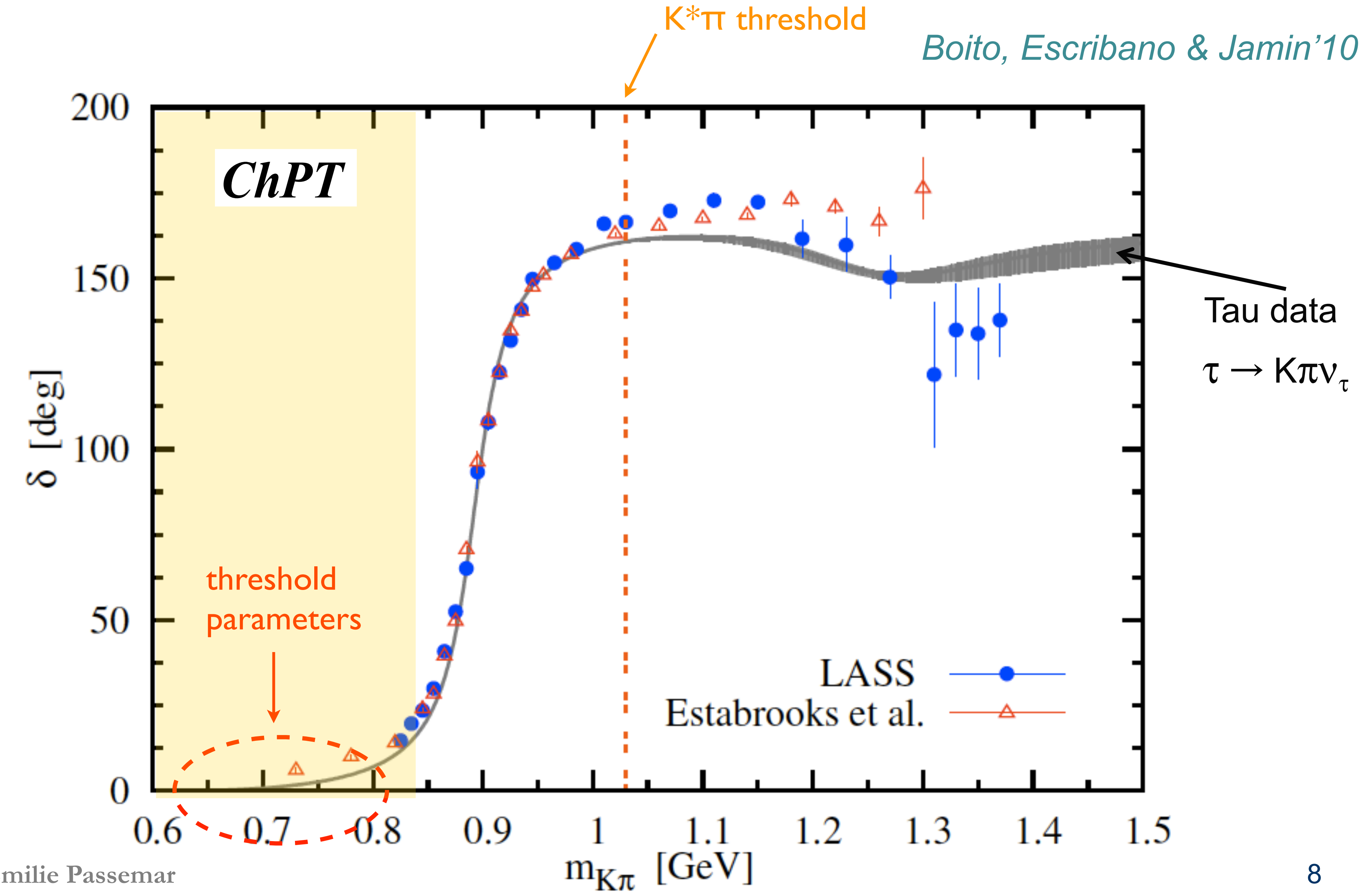
Wilson, Briceno, Dudek, Edwards, Thomas'19

Adapted by

Pelaez & Rodas'22

2. Using Kπ scattering to test ChPT

Ex: Kπ scattering, P-wave



2.1 Chiral Symmetry

- Limit $m_k \to 0$

$$\mathcal{L}_{QCD} \to \boxed{\mathcal{L}^0_{QCD} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R}, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

with $q_{L/R} \equiv \frac{1}{2}\left(1 \mp \gamma_5\right) q$

Symmetry: $\boxed{G \equiv SU(3)_L \otimes SU(3)_R \to SU(3)_V}$

- Chiral Perturbation Theory: dynamics of the Goldstone bosons (kaons, pions, eta)

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{QCD}$
Expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\boxed{\mathcal{L}_{eff} = \sum_{d \geq 2} \mathcal{L}_d \ , \mathcal{L}_d = \mathcal{O}\left(p^d\right) \ , p \equiv \left\{q, m_q\right\}}$$

$$\boxed{\mathrm{p} << \Lambda_H = 4\pi F_\pi \sim 1\ \mathrm{GeV}}$$

2.2 Chiral expansion

- $$\mathcal{L}_{ChPT} = \underbrace{\mathcal{L}_2}_{\text{LO}:\ \mathcal{O}(p^2)} + \underbrace{\mathcal{L}_4}_{\text{NLO}:\ \mathcal{O}(p^4)} + \underbrace{\mathcal{L}_6}_{\text{NNLO}:\ \mathcal{O}(p^6)} + \dots.$$

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants → LECs appearing at each order

- The method has been rigorously established and can be formulated as a set of calculational rules:

LO : tree level diagrams with $\mathcal{L}_2$

$\mathcal{L}_2$: F_0, B_0

NLO: tree level diagrams with $\mathcal{L}_4$
1-loop diagrams with $\mathcal{L}_2$

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i\, O_4^i,$$

NNLO: tree level diagrams with $\mathcal{L}_6$
2-loop diagrams with $\mathcal{L}_2$
1-loop diagrams with one vertex from $\mathcal{L}_4$

$$\mathcal{L}_6 = \sum_{i=1}^{90} C_i\, O_6^i$$

- Renormalizable and unitary order by order in the expansion

2.3 ChPT in the meson sector: precision calculations

- Today’s standard in the meson sector: 2-loop calculations

- Main obstacle to reaching high precision: determination of the LECs: $\mathcal{O}(p^6)$ LECs proliferation makes the program to pin down/ estimate all of them prohibitive

- In a specific process, only a limited number of LECs appear

- The LECs calculable if QCD solvable, instead
  - Determined from experimental measurement
  - Estimated with models: Resonances, large N_C
  - Computed on the lattice

2.4 Test of SU(3) ChPT

- Interesting framework to test ChPT is offered by the kaons: K_{l3}, K_{l4}, K → 3π, etc

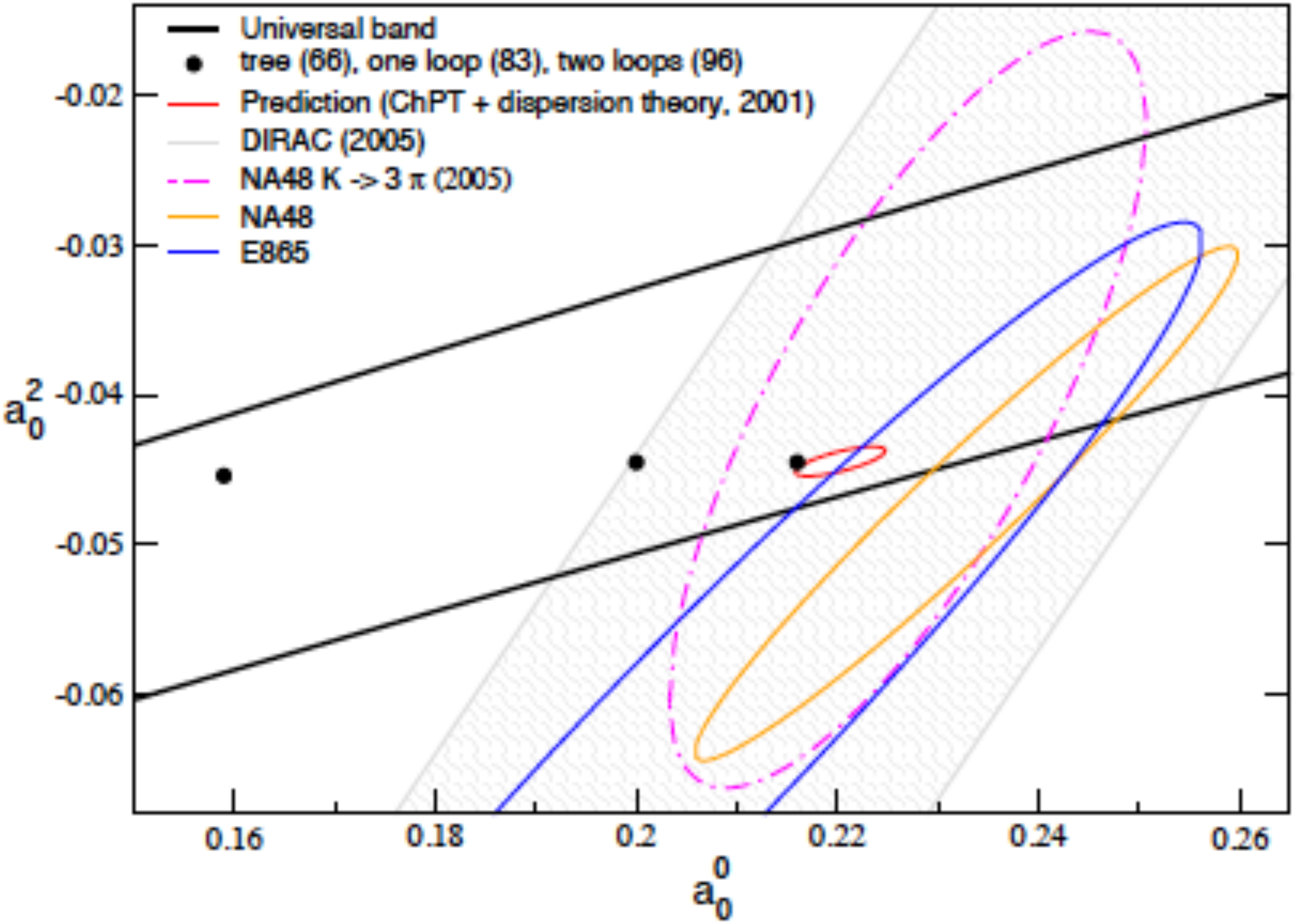
- A very interesting quantity is the scattering length: first term in the expansion:

$$\boxed{\frac{2}{\sqrt{s}}\mathrm{Re}\, t_l^I(s) = \frac{1}{2q}\sin 2\delta_l^I(q) = q^{2l}\left[\underline{a_l^I} + b_l^I q^2 + c_l^I q^4 + \mathcal{O}(q^6)\right]}$$

- For ππ: SU(2) ChPT very successful!

ππ scattering lengths

H. Leutwyler



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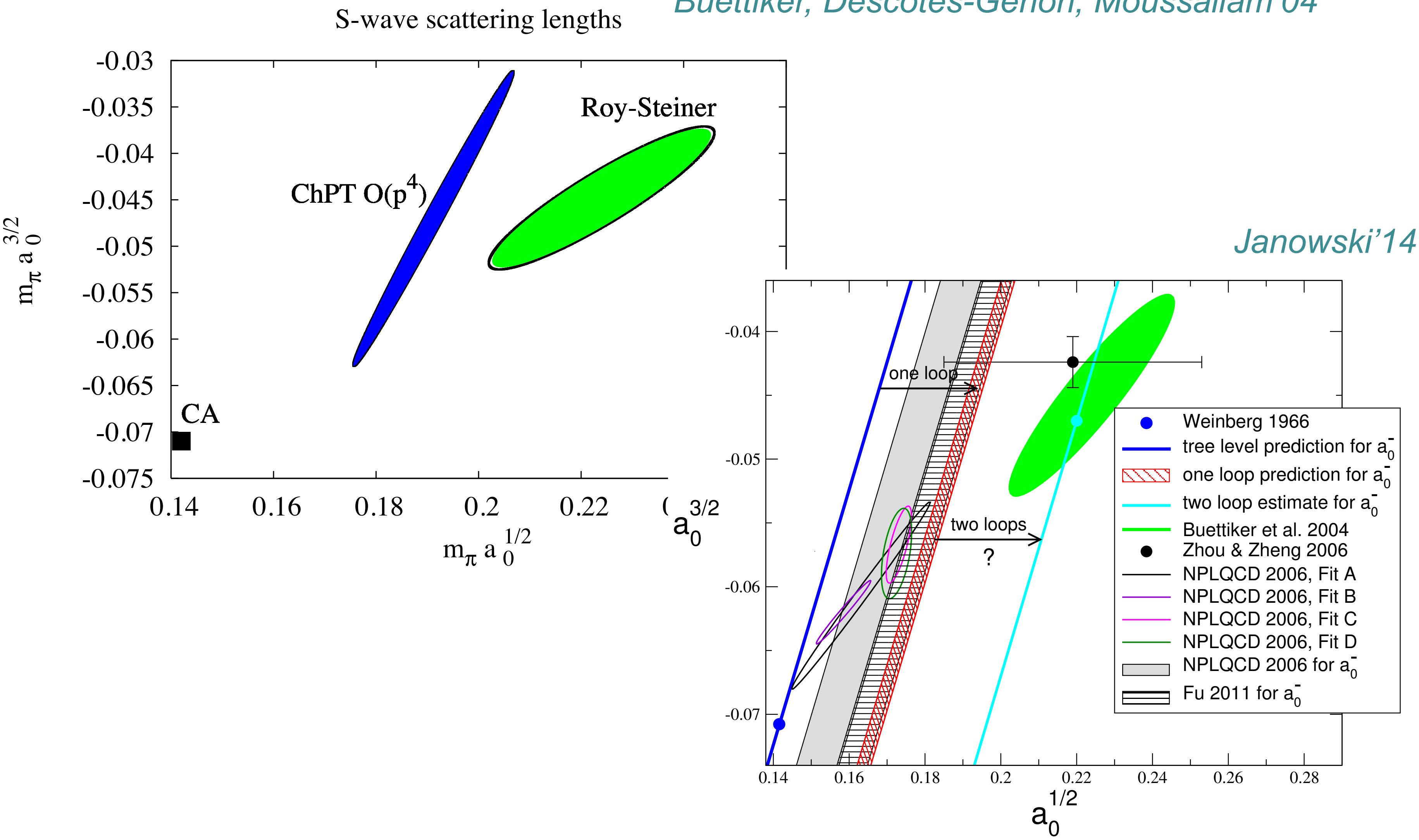
- For ππ: SU(2) ChPT very successful!

- What about SU(3) ChPT?
  In principle slower convergence if convergence at all!

Kπ scattering lengths: S-wave



Buettiker, Descotes-Genon, Moussallam'04

Janowski'14

Roy-Steiner equations for Kπ

- Unitarity effects can be calculated *exactly* using dispersive methods

- Unitarity, analyticity and crossing symmetry ≡ *Roy-Steiner equations*

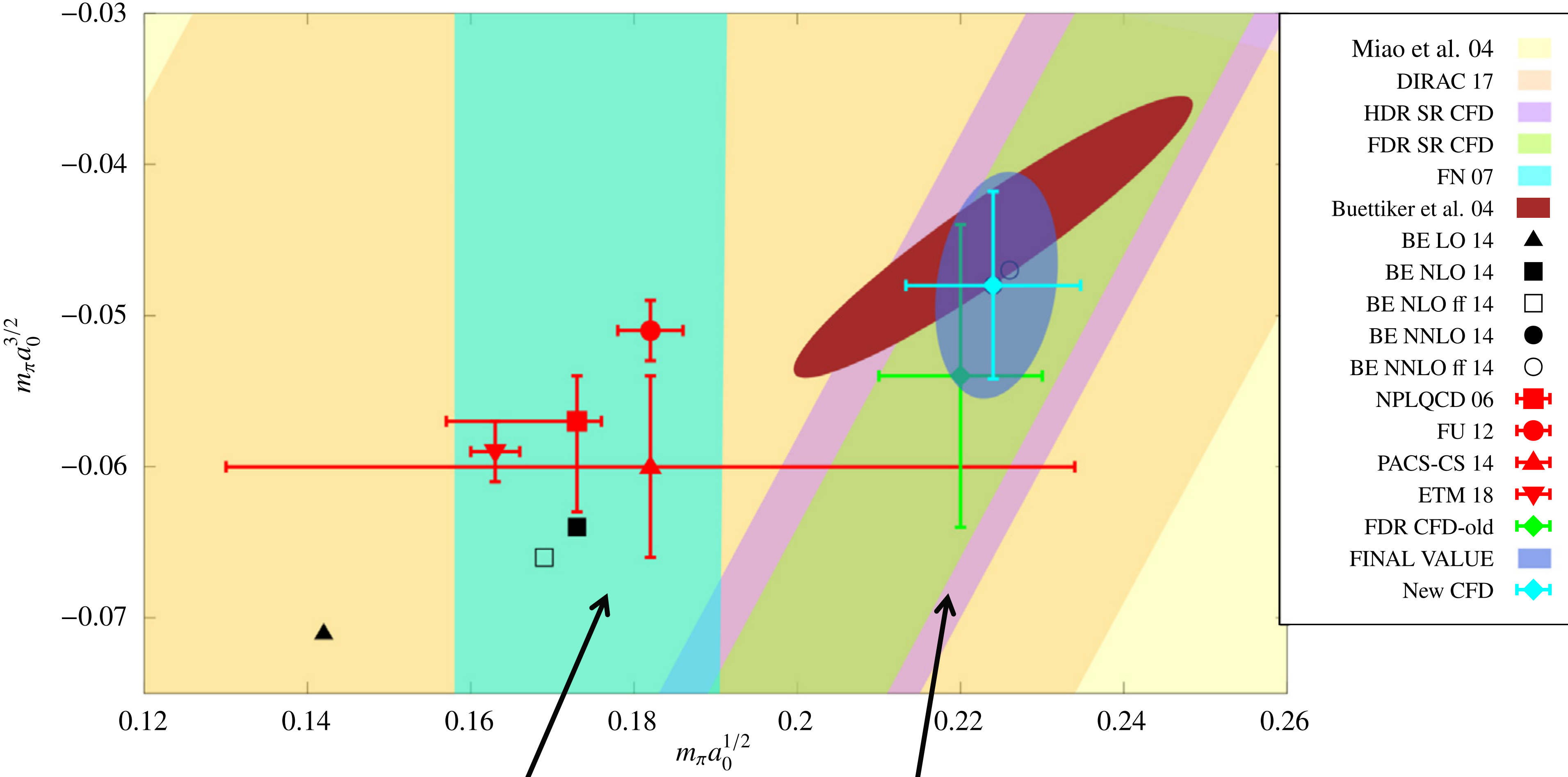
- Input: Data on Kπ → Kπ and ππ → KK for E ≥ 1 GeV
  two subtraction constants, e.g. a_0^0 and a_2^0

- Output: the full Kπ scattering amplitude below 1 GeV
  ⇒ In *poor* agreement with the experimental data

- Numerical solutions of the Roy-(Steiner) equations:
  - ππ: *Pennington-Protopopescu, Basdevant-Froggatt-Petersen (70s)*
    Bern group: Ananthanarayan et al.'00, Caprini et al.'11
    Orsay group: Descotes-Genon, Fuchs, Girlanda and Stern'01
    Madrid-Cracow group: Garcia-Martin,et al.'11
  - Kπ: *Buettiker, Descotes-Genon, Moussallam'04 , Pelaez & Rodas'16 '22*
  - KN: *Ruiz de Elvira et al'15*

Kπ scattering lengths: S-wave

Pelaez & Rodas'22



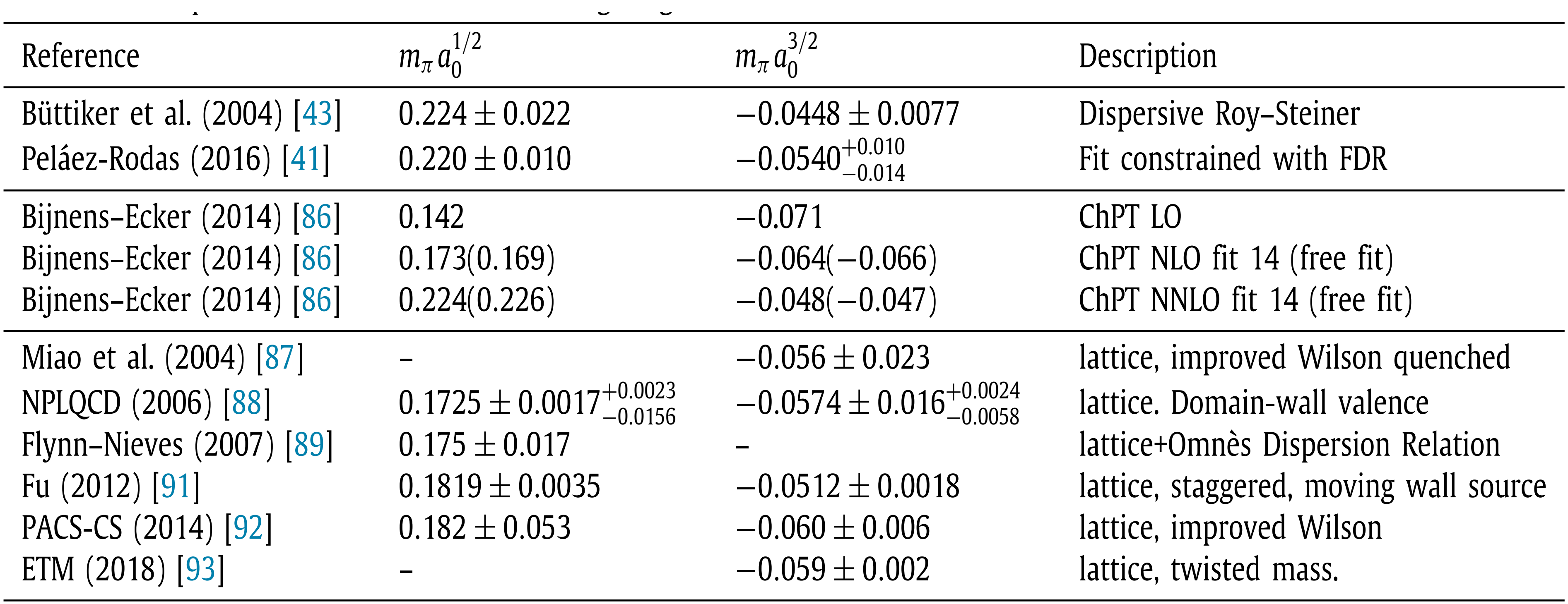
Lattice

Dispersive

BE: Bijnens & Ecker'14

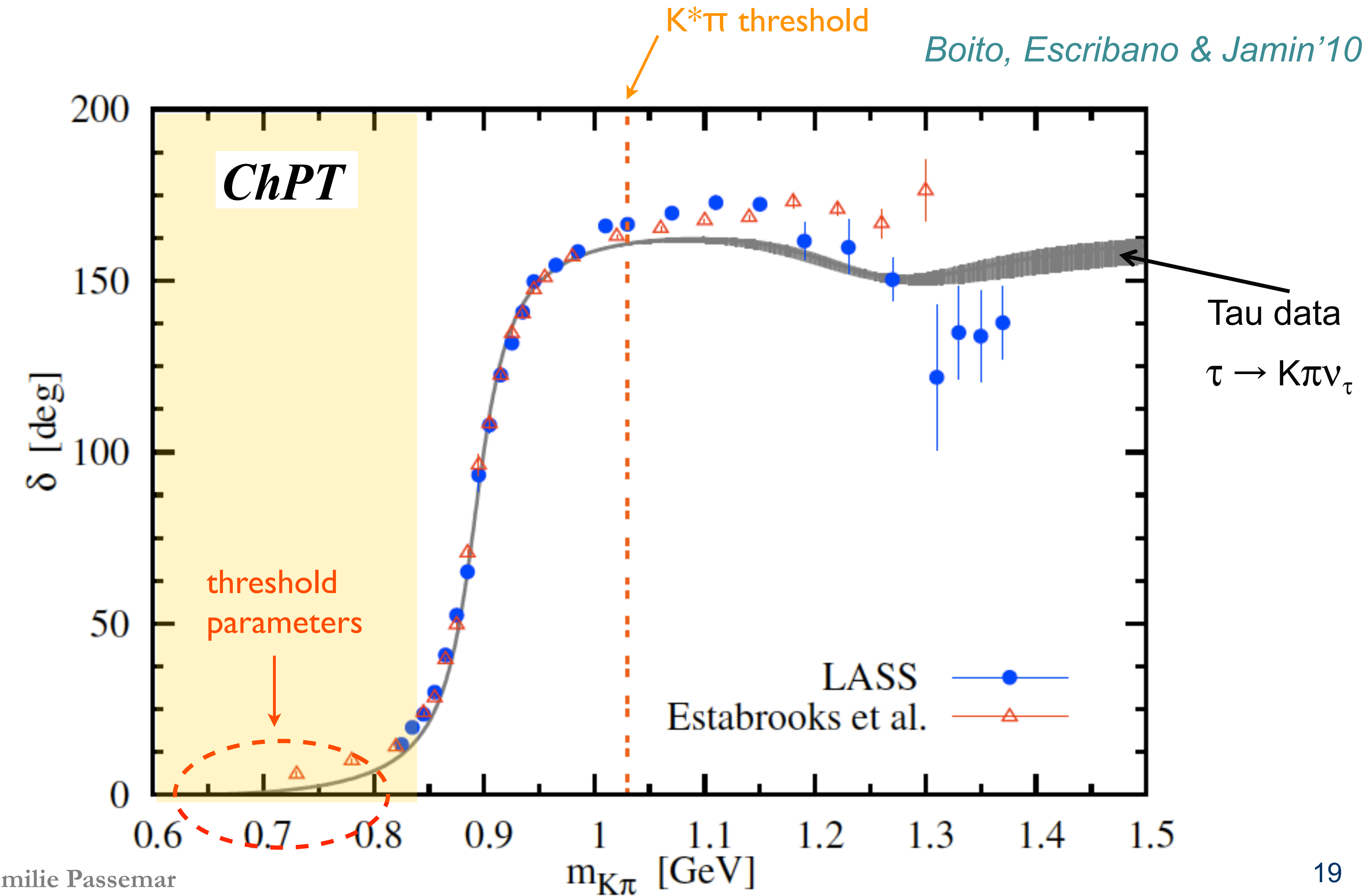
Kπ scattering lengths: S-wave

Pelaez & Rodas'22



Reference	$m_\pi a_0^{1/2}$	$m_\pi a_0^{3/2}$	Description
Büttiker et al. (2004) [43]	0.224 ± 0.022	-0.0448 ± 0.0077	Dispersive Roy–Steiner
Peláez-Rodas (2016) [41]	0.220 ± 0.010	$-0.0540^{+0.010}_{-0.014}$	Fit constrained with FDR
Bijnens–Ecker (2014) [86]	0.142	-0.071	ChPT LO
Bijnens–Ecker (2014) [86]	0.173(0.169)	$-0.064(-0.066)$	ChPT NLO fit 14 (free fit)
Bijnens–Ecker (2014) [86]	0.224(0.226)	$-0.048(-0.047)$	ChPT NNLO fit 14 (free fit)
Miao et al. (2004) [87]	–	-0.056 ± 0.023	lattice, improved Wilson quenched
NPLQCD (2006) [88]	$0.1725 \pm 0.0017^{+0.0023}_{-0.0156}$	$-0.0574 \pm 0.016^{+0.0024}_{-0.0058}$	lattice. Domain-wall valence
Flynn–Nieves (2007) [89]	0.175 ± 0.017	–	lattice+Omnès Dispersion Relation
Fu (2012) [91]	0.1819 ± 0.0035	-0.0512 ± 0.0018	lattice, staggered, moving wall source
PACS-CS (2014) [92]	0.182 ± 0.053	-0.060 ± 0.006	lattice, improved Wilson
ETM (2018) [93]	–	-0.059 ± 0.002	lattice, twisted mass.

Kπ scattering lengths: P-wave



Kπ scattering lengths: P-wave

Boito, Escribano & Jamin'10

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	Tau data	ChPT $\mathcal{O}(p^4)$ 🟥	RChPT $\mathcal{O}(p^4)$ 🟪	ChPT $\mathcal{O}(p^6)$ 🟩	Roy-Steiner 🟦
$m_\pi^3\, a_1^{1/2} \times 10$	$0.166(4)$	$0.16(3)$	$0.18(3)$	0.18	$0.19(1)$
$m_\pi^5\, b_1^{1/2} \times 10^2$	$0.258(9)$	-	-		$0.18(2)$
$m_\pi^7\, c_1^{1/2} \times 10^3$	$0.90(3)$	-	-		$0.71(11)$

⇨ Recent analysis combining K_{l3}, tau and D data : 0.249 ± 0.011 *Bernard'14*

🟥 *Bernard, Kaiser, Meissner'91*

🟪 *Bernard, Kaiser, Meissner'91*

🟩 *Bijnens, Dhonte, Talavera'04*

🟦 *Buettiker, Descotes-Genon, Moussallam'04*

- Poor agreement ⇨ need *more data*

3. Hadron spectroscopy

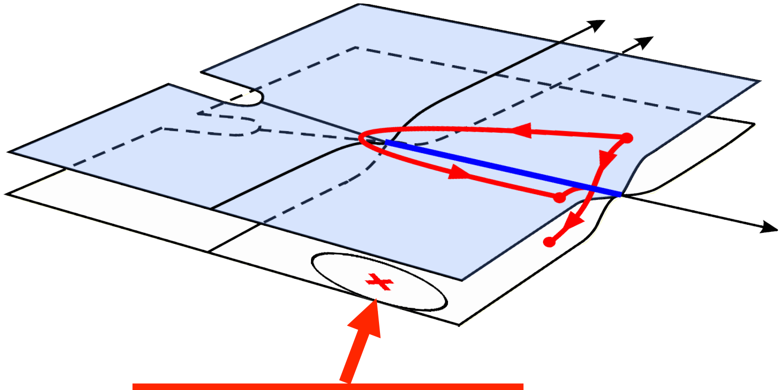
3.1 Determining of pole and width

- Once one gets Kπ scattering amplitude

  ⇨ analytical continuation into the complex plane

Poles on the second sheet correspond to zeros on the first sheet!

Plot from M. Pennington

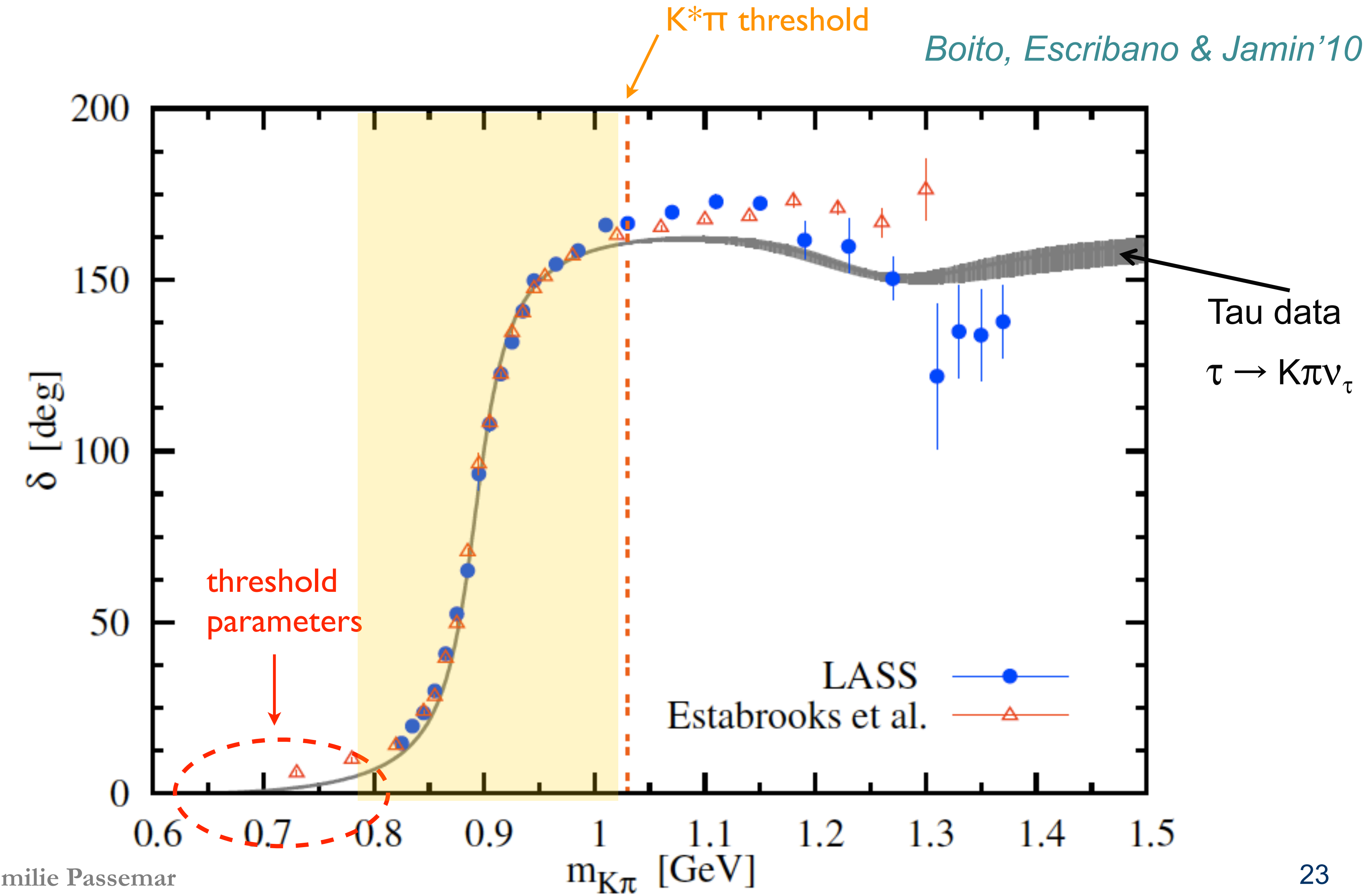


resonance pole

Dispersive analytic continuation

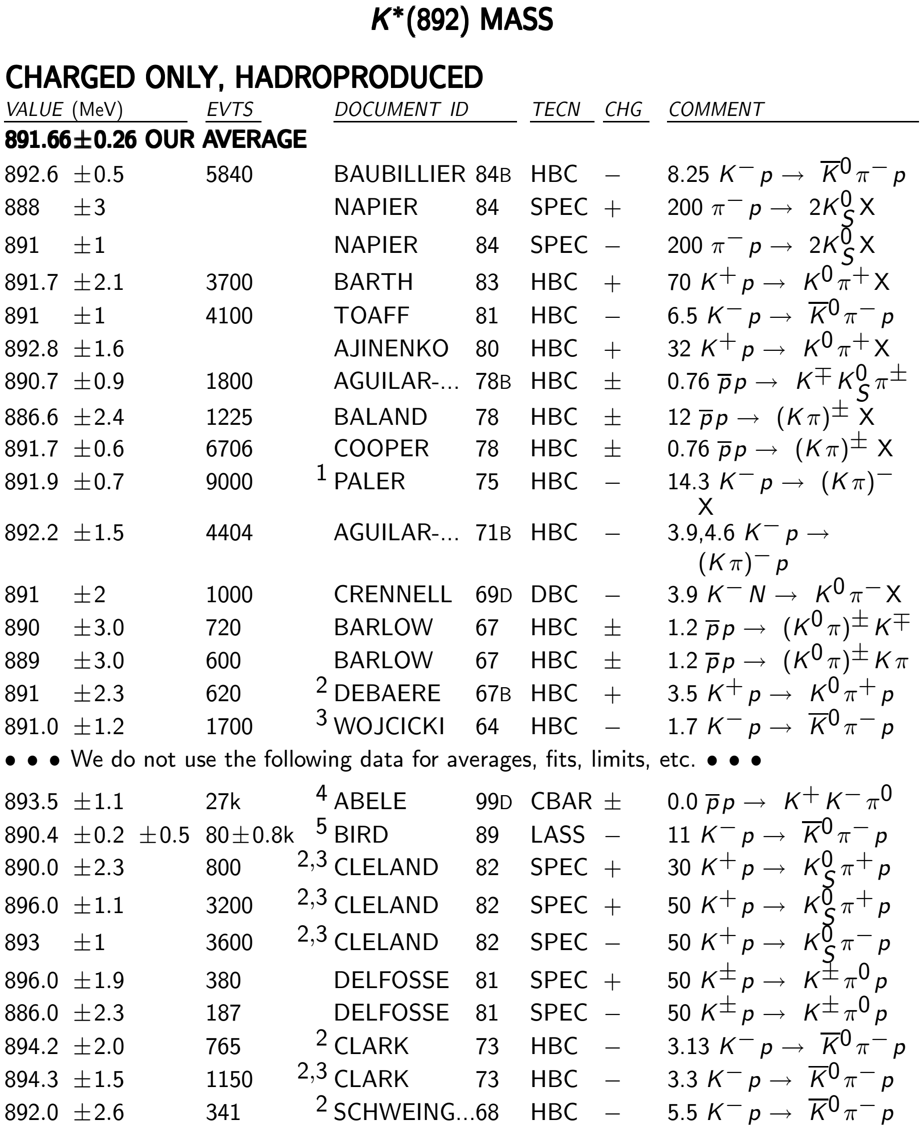
Kπ scattering, P-wave

Boito, Escribano & Jamin'10



3.2 K*(892) mass and width

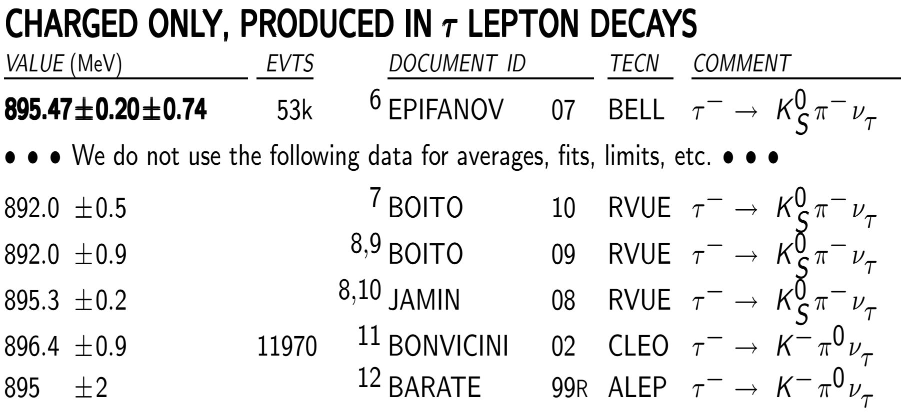
PDG 22



$K^*(892)$ MASS

CHARGED ONLY, HADROPRODUCED

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
891.66±0.26 OUR AVERAGE						
892.6 ±0.5	5840	BAUBILLIER	84B	HBC	−	8.25 $K^- p \to \overline{K}^0 \pi^- p$
888 ±3		NAPIER	84	SPEC	+	200 $\pi^- p \to 2K^0_S X$
891 ±1		NAPIER	84	SPEC	−	200 $\pi^- p \to 2K^0_S X$
891.7 ±2.1	3700	BARTH	83	HBC	+	70 $K^+ p \to K^0 \pi^+ X$
891 ±1	4100	TOAFF	81	HBC	−	6.5 $K^- p \to \overline{K}^0 \pi^- p$
892.8 ±1.6		AJINENKO	80	HBC	+	32 $K^+ p \to K^0 \pi^+ X$
890.7 ±0.9	1800	AGUILAR-...	78B	HBC	±	0.76 $\overline{p} p \to K^\mp K^0_S \pi^\pm$
886.6 ±2.4	1225	BALAND	78	HBC	±	12 $\overline{p} p \to (K\pi)^\pm X$
891.7 ±0.6	6706	COOPER	78	HBC	±	0.76 $\overline{p} p \to (K\pi)^\pm X$
891.9 ±0.7	9000	[1] PALER	75	HBC	−	14.3 $K^- p \to (K\pi)^- X$
892.2 ±1.5	4404	AGUILAR-...	71B	HBC	−	3.9,4.6 $K^- p \to (K\pi)^- p$
891 ±2	1000	CRENNELL	69D	DBC	−	3.9 $K^- N \to K^0 \pi^- X$
890 ±3.0	720	BARLOW	67	HBC	±	1.2 $\overline{p} p \to (K^0 \pi)^\pm K^\mp$
889 ±3.0	600	BARLOW	67	HBC	±	1.2 $\overline{p} p \to (K^0 \pi)^\pm K \pi$
891 ±2.3	620	[2] DEBAERE	67B	HBC	+	3.5 $K^+ p \to K^0 \pi^+ p$
891.0 ±1.2	1700	[3] WOJCICKI	64	HBC	−	1.7 $K^- p \to \overline{K}^0 \pi^- p$
• • • We do not use the following data for averages, fits, limits, etc. • • •						
893.5 ±1.1	27k	[4] ABELE	99D	CBAR	±	0.0 $\overline{p} p \to K^+ K^- \pi^0$
890.4 ±0.2 ±0.5	80±0.8k	[5] BIRD	89	LASS	−	11 $K^- p \to \overline{K}^0 \pi^- p$
890.0 ±2.3	800	[2,3] CLELAND	82	SPEC	+	30 $K^+ p \to K^0_S \pi^+ p$
896.0 ±1.1	3200	[2,3] CLELAND	82	SPEC	+	50 $K^+ p \to K^0_S \pi^+ p$
893 ±1	3600	[2,3] CLELAND	82	SPEC	−	50 $K^+ p \to K^0_S \pi^- p$
896.0 ±1.9	380	DELFOSSE	81	SPEC	+	50 $K^\pm p \to K^\pm \pi^0 p$
886.0 ±2.3	187	DELFOSSE	81	SPEC	−	50 $K^\pm p \to K^\pm \pi^0 p$
894.2 ±2.0	765	[2] CLARK	73	HBC	−	3.13 $K^- p \to \overline{K}^0 \pi^- p$
894.3 ±1.5	1150	[2,3] CLARK	73	HBC	−	3.3 $K^- p \to \overline{K}^0 \pi^- p$
892.0 ±2.6	341	[2] SCHWEING...	68	HBC	−	5.5 $K^- p \to \overline{K}^0 \pi^- p$



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VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
895.47±0.20±0.74	53k	[6] EPIFANOV	07	BELL	$\tau^- \to K^0_S \pi^- \nu_\tau$
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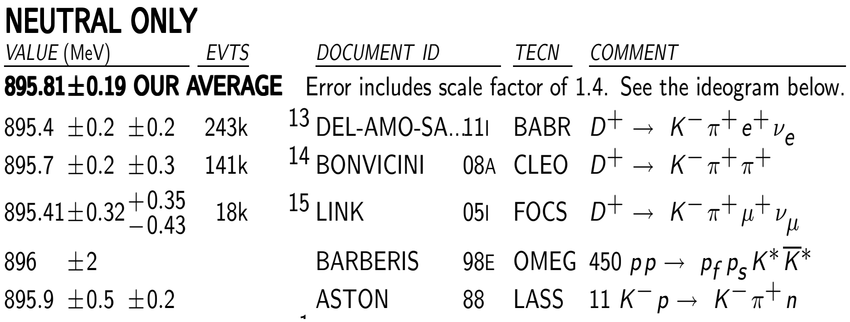
892.0 ±0.5		[7] BOITO	10	RVUE	$\tau^- \to K^0_S \pi^- \nu_\tau$
892.0 ±0.9		[8,9] BOITO	09	RVUE	$\tau^- \to K^0_S \pi^- \nu_\tau$
895.3 ±0.2		[8,10] JAMIN	08	RVUE	$\tau^- \to K^0_S \pi^- \nu_\tau$
896.4 ±0.9	11970	[11] BONVICINI	02	CLEO	$\tau^- \to K^- \pi^0 \nu_\tau$
895 ±2		[12] BARATE	99R	ALEP	$\tau^- \to K^- \pi^0 \nu_\tau$



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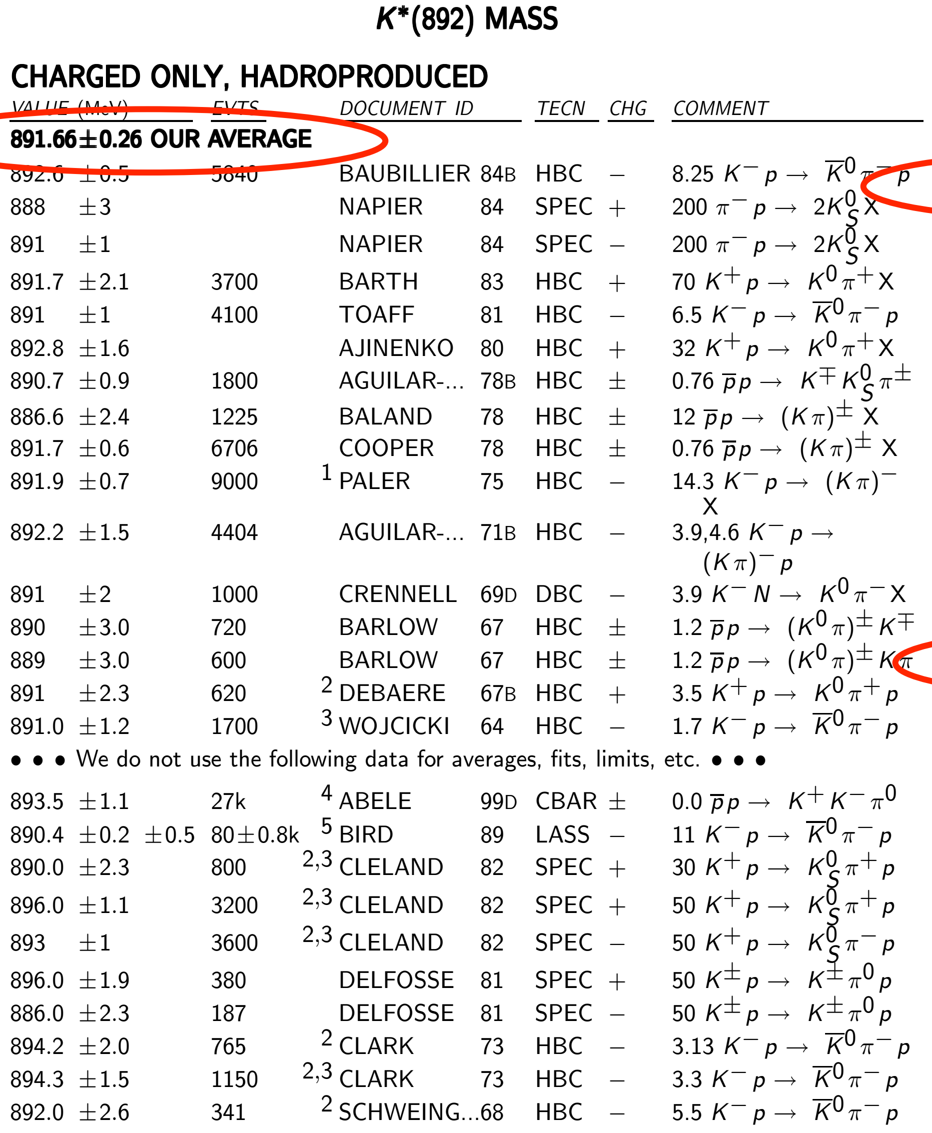
895.4 ±0.2 ±0.2	243k	[13] DEL-AMO-SA...	11I	BABR	$D^+ \to K^- \pi^+ e^+ \nu_e$
895.7 ±0.2 ±0.3	141k	[14] BONVICINI	08A	CLEO	$D^+ \to K^- \pi^+ \pi^+$
$895.41 \pm 0.32 ^{+0.35}_{-0.43}$	18k	[15] LINK	05I	FOCS	$D^+ \to K^- \pi^+ \mu^+ \nu_\mu$
896 ±2		BARBERIS	98E	OMEG	450 $p p \to p_f p_s K^* \overline{K}^*$
895.9 ±0.5 ±0.2		ASTON	88	LASS	11 $K^- p \to K^- \pi^+ n$

3.2 K*(892) mass and width

PDG 22



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886.6 ±2.4	1225	BALAND	78	HBC	±	$12\ \overline{p} p \to (K\pi)^\pm X$
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889 ±3.0	600	BARLOW	67	HBC	±	$1.2\ \overline{p} p \to (K^0 \pi)^\pm K\pi$
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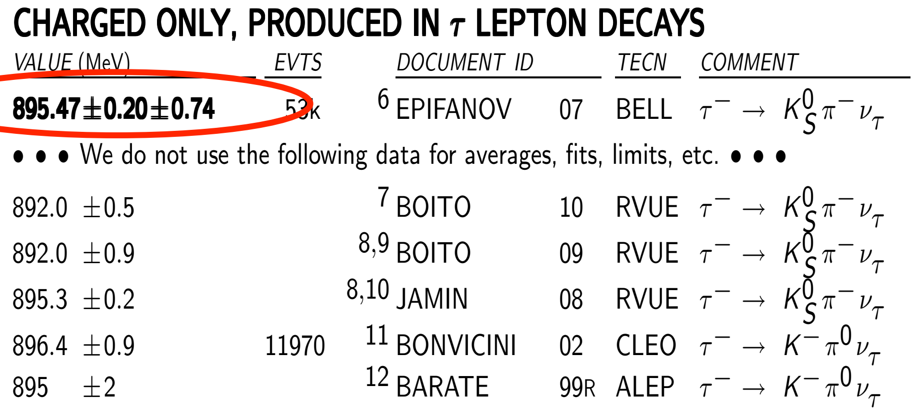

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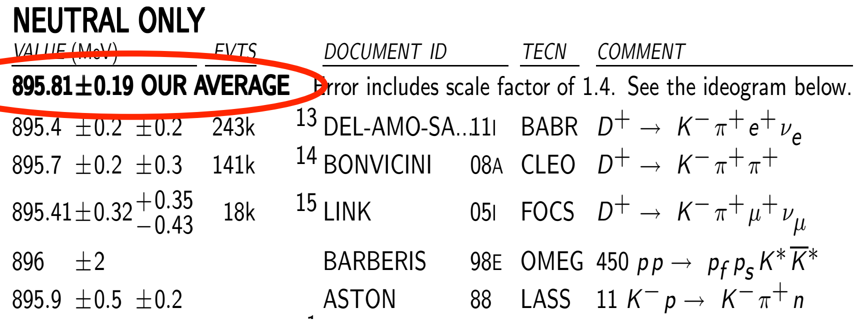

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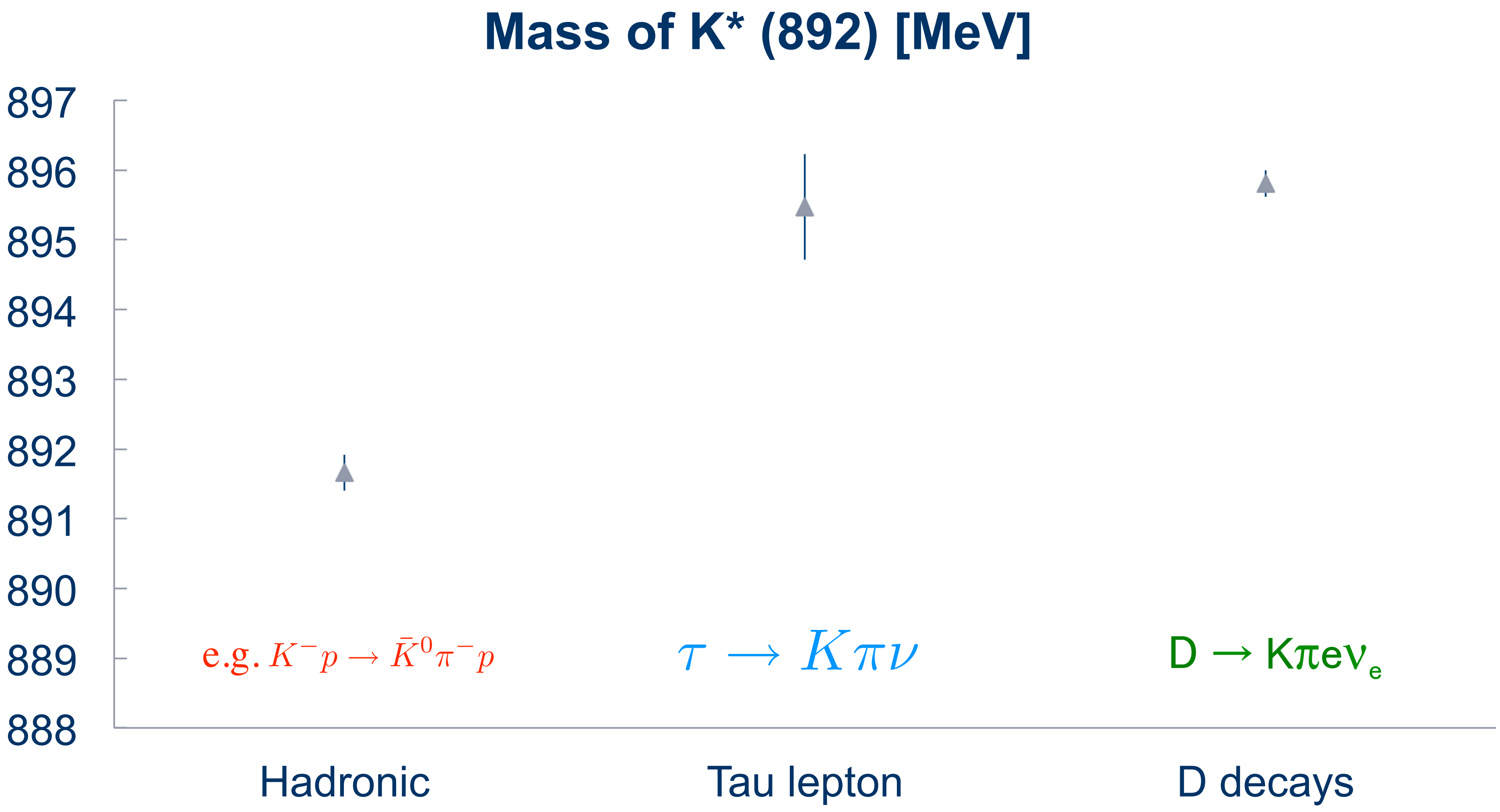
VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
895.81±0.19 OUR AVERAGE		Error includes scale factor of 1.4. See the ideogram below.			
895.4 ±0.2 ±0.2	243k	[13] DEL-AMO-SA...	11I	BABR	$D^+ \to K^- \pi^+ e^+ \nu_e$
895.7 ±0.2 ±0.3	141k	[14] BONVICINI	08A	CLEO	$D^+ \to K^- \pi^+ \pi^+$
$895.41 \pm 0.32^{+0.35}_{-0.43}$	18k	[15] LINK	05I	FOCS	$D^+ \to K^- \pi^+ \mu^+ \nu_\mu$
896 ±2		BARBERIS	98E	OMEG	$450\ pp \to p_f p_s K^* \overline{K}^*$
895.9 ±0.5 ±0.2		ASTON	88	LASS	$11\ K^- p \to K^- \pi^+ n$

3.2 $K^*(892)$ mass and width

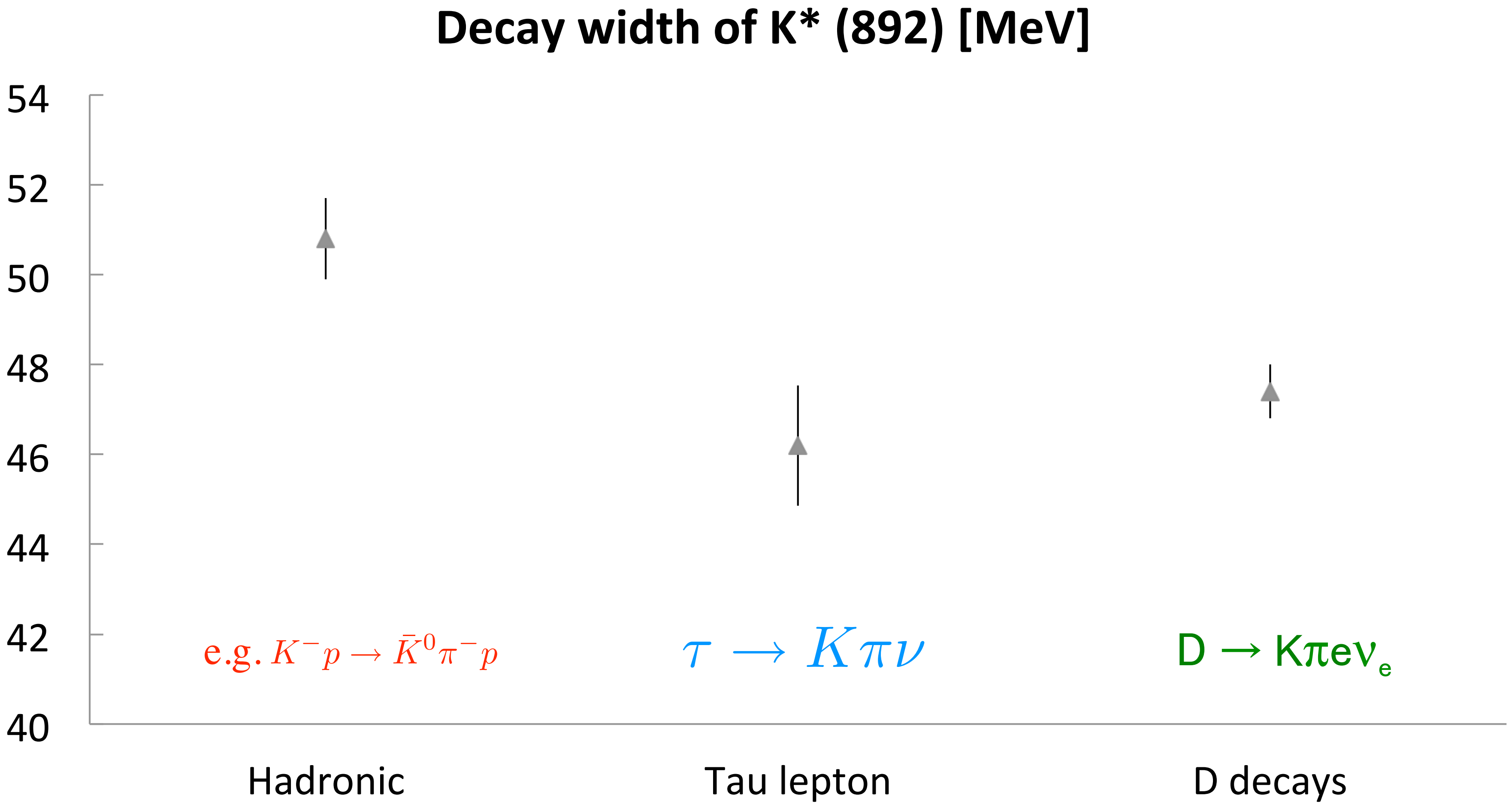
PDG 22



Emilie Passemar

3.2 $K^*(892)$ mass and width

PDG 22

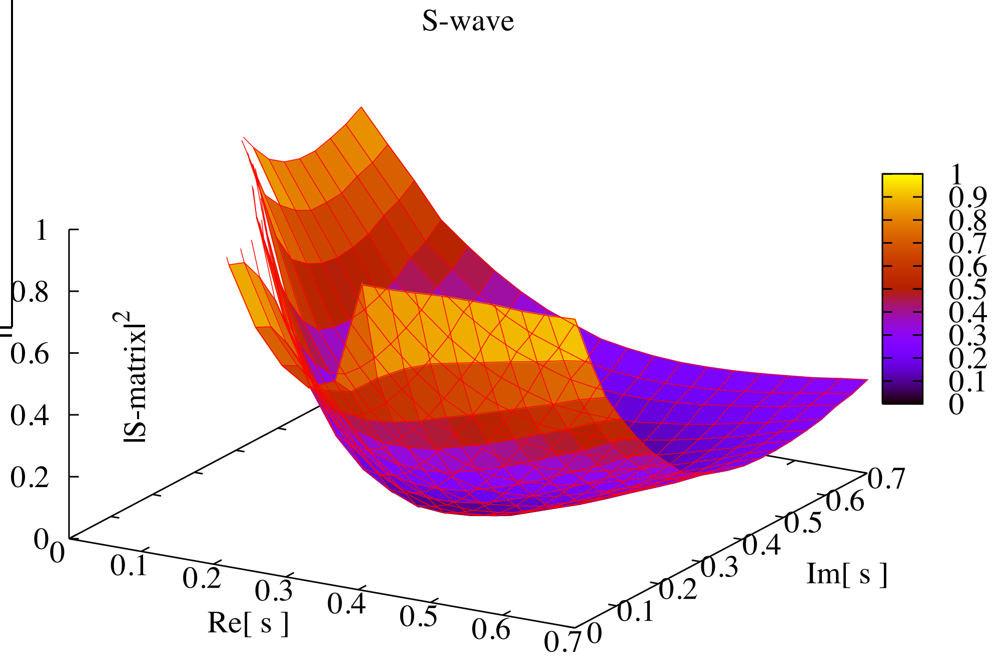


3.3 Kappa(800)

- The results coming from Roy-Steiner and data at higher energy not in agreement with low energy experimental data ➡ need improvement!
  Problem: no other precise data

Descotes-Genon, Moussallam'06

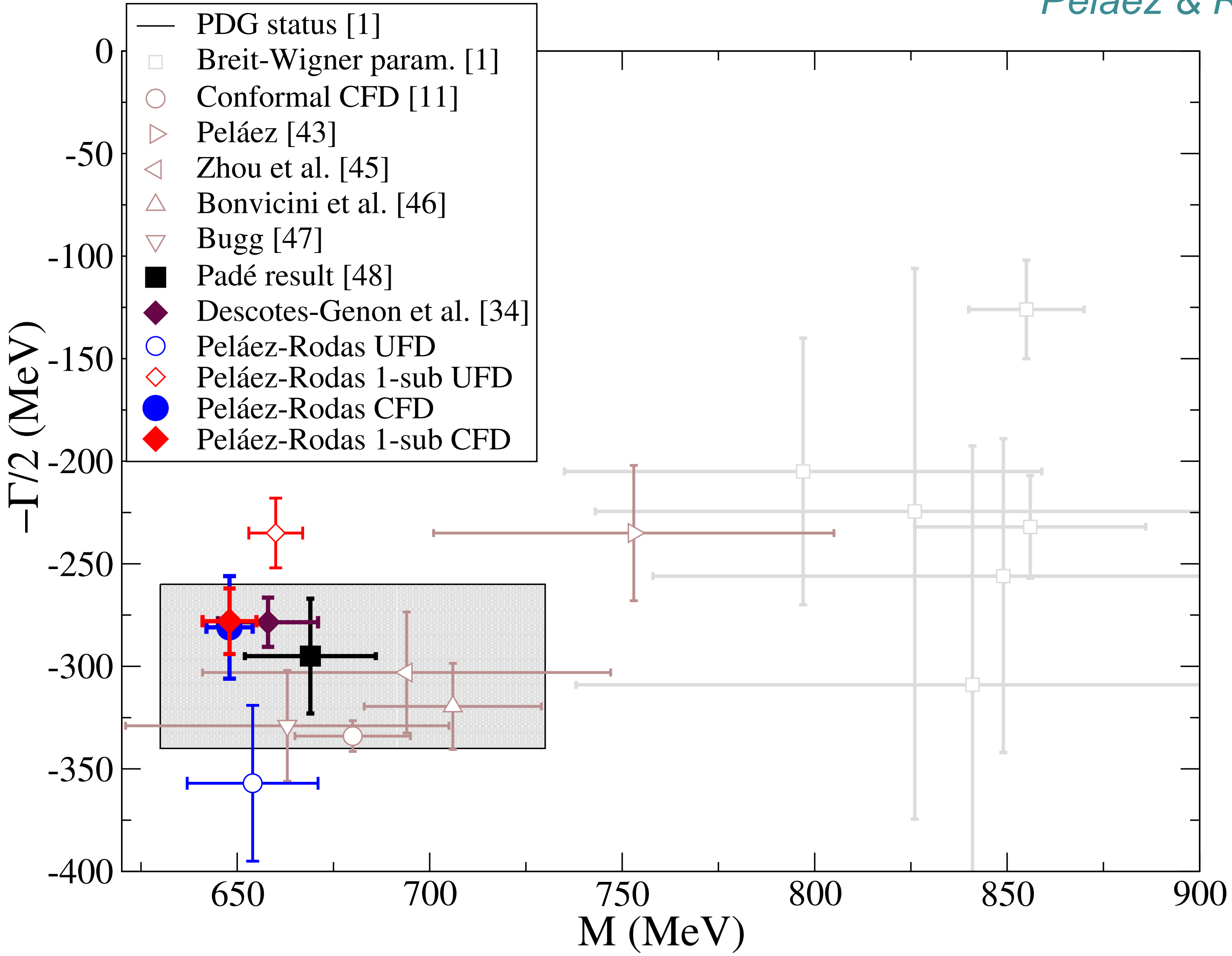
	M_κ (MeV)	Γ_κ (MeV)
This work	658 ± 13	557 ± 24
Zhou, Zheng [16]	694 ± 53	606 ± 89
Jamin et al. [18]	708	610
Aitala et al. [7]	$721 \pm 19 \pm 43$	$584 \pm 43 \pm 87$
Pelaez [19]	750 ± 18	452 ± 22
Bugg [9]	750^{+30}_{-55}	684 ± 120
Ablikim et al. [20]	$841 \pm 23^{+64}_{-55}$	$618 \pm 52^{+55}_{-87}$
Ishida et al. [14]	877^{+65}_{-30}	668^{+235}_{-110}



- Existence would suggest
  σ not a glueball

3.3 Kappa(800)

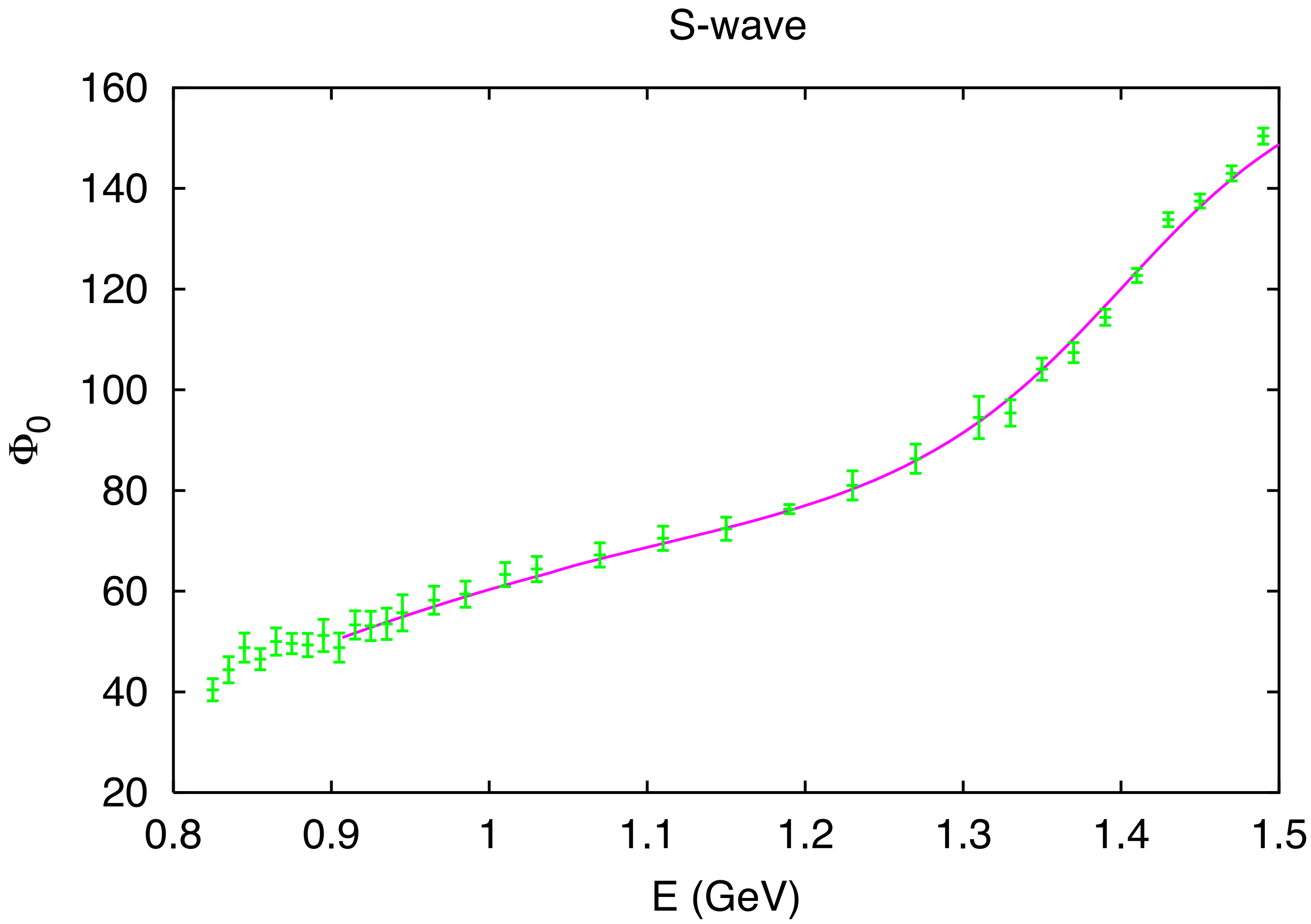
Pelaez & Rodas'22



3.3 Kappa(800)

Buettiker, Descotes and Moussallam'04

- Inputs for S wave in Roy-Steiner analysis from LASS



3.3 Kappa(800)

Buettiker, Descotes and Moussallam'04

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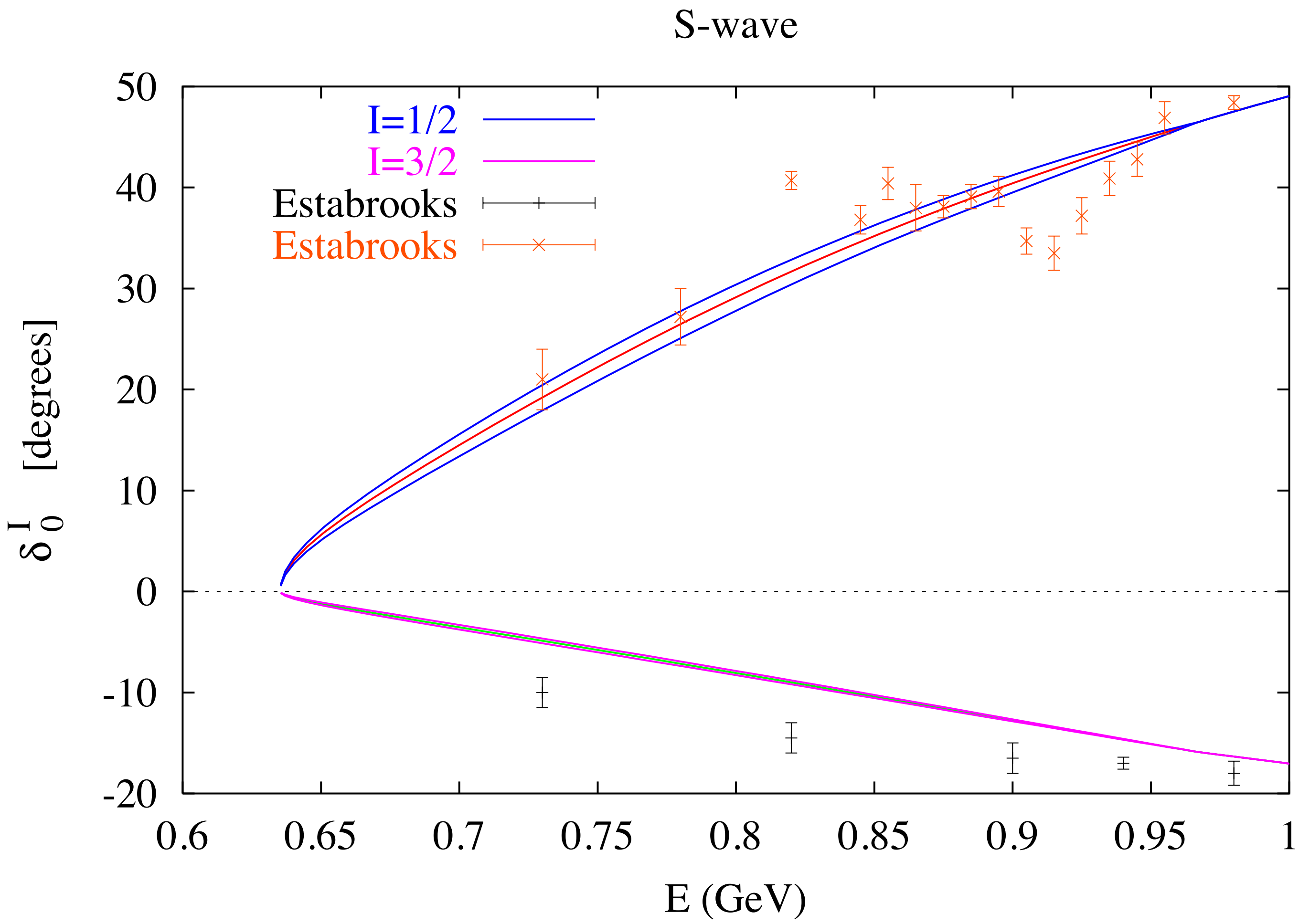
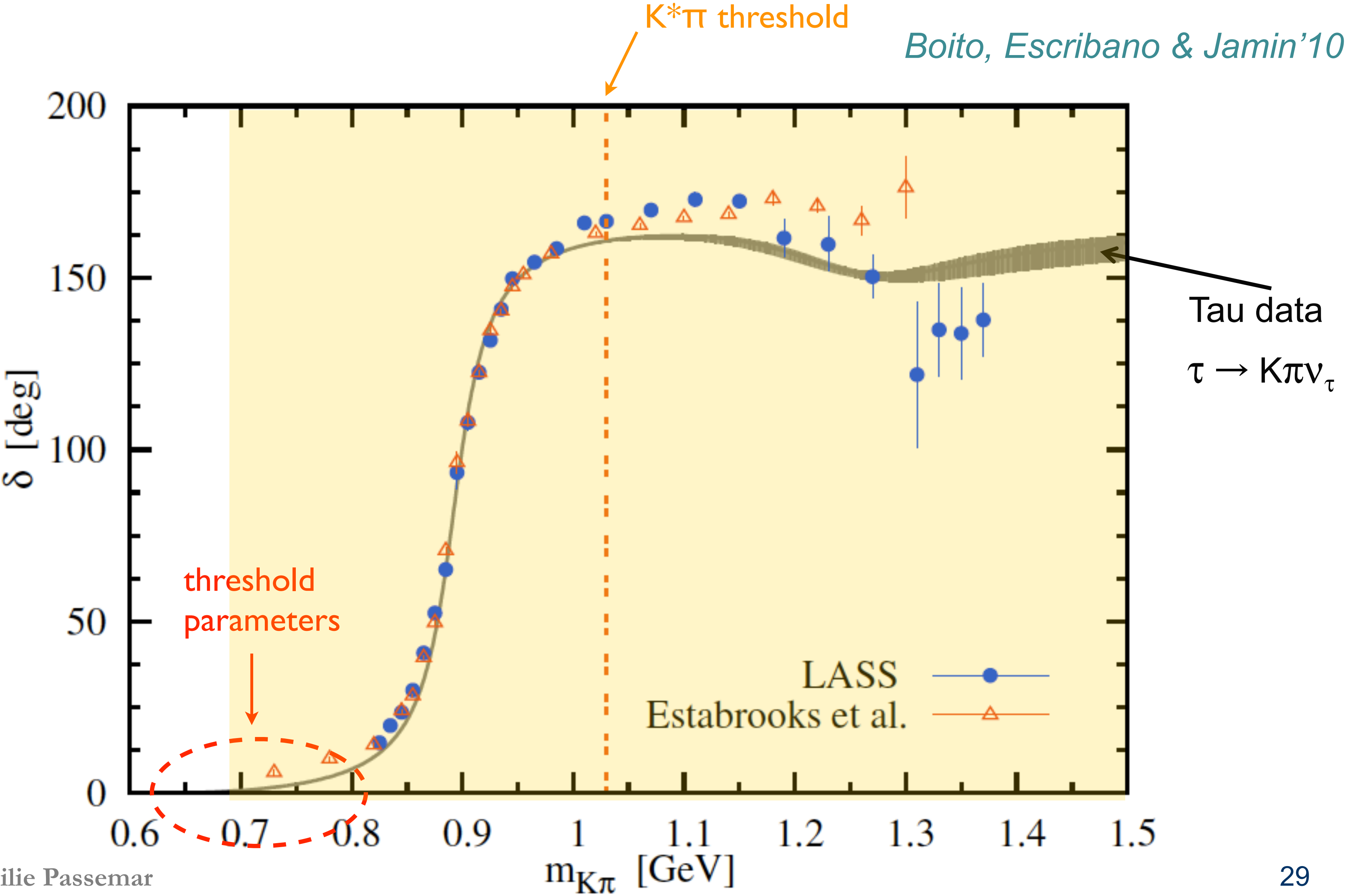


4. Tests of the SM and new physics

Kπ scattering, P-wave

Boito, Escribano & Jamin'10

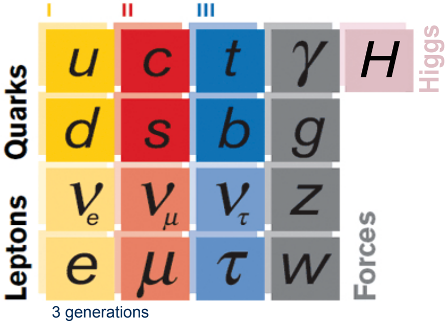


4.1 Determination of fundamental parameters: V_{us}

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

Description of the weak interactions:

$$\boxed{\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}}$$

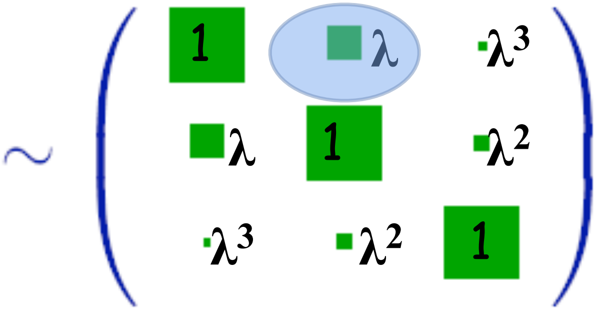


4.1 Determination of fundamental parameters: V_{us}

- The CKM Mechanism source of *Charge Parity Violation* in SM
- Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

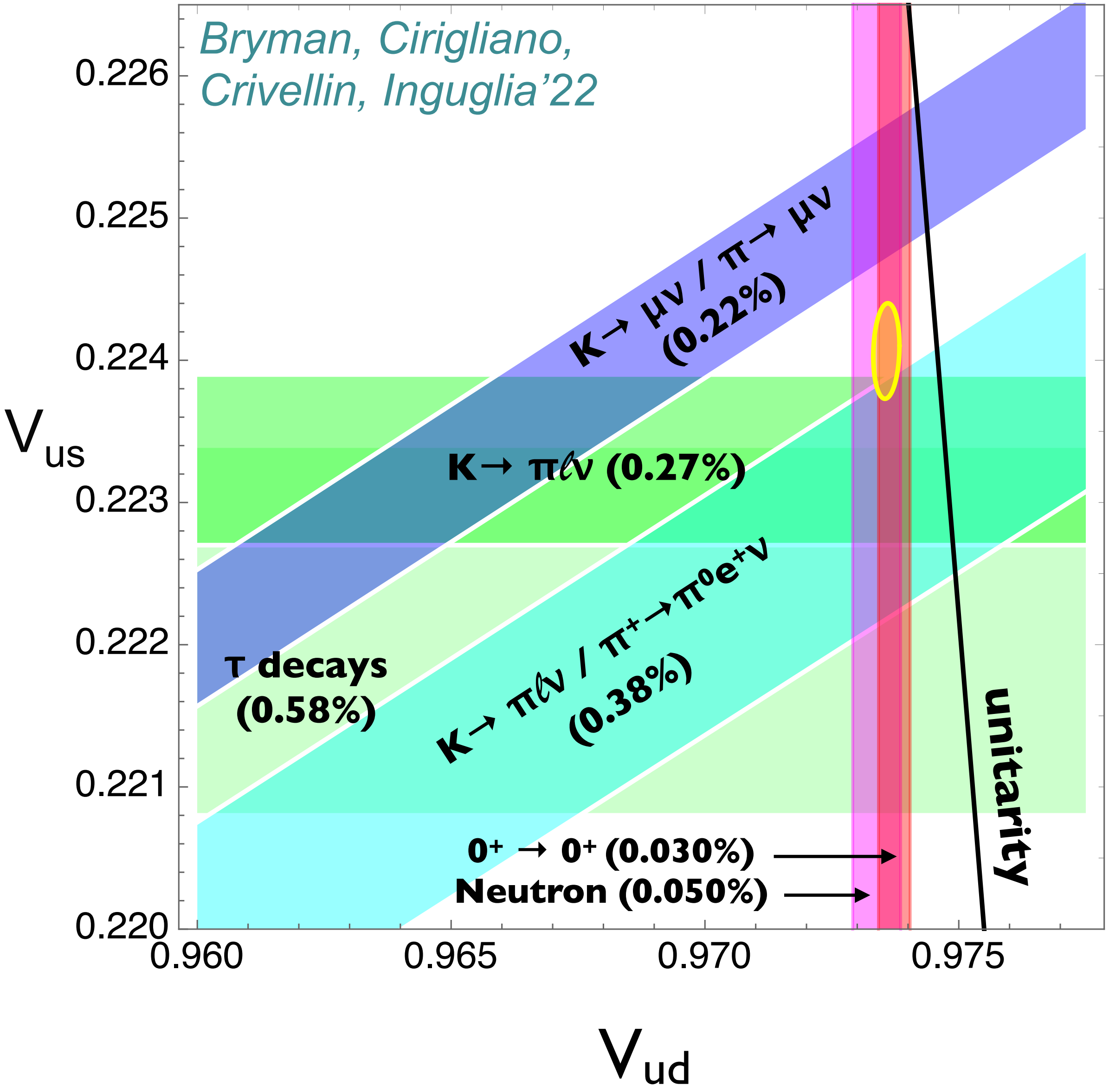
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates CKM Matrix Mass Eigenstates



$$\sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Status on V_{us} and V_{ud} Cabibbo angle anomaly



Bryman, Cirigliano, Crivellin, Inguglia'22

Moulson, E.P. @CKM2021

$|V_{ud}| = 0.97373(31)$

$|V_{us}| = 0.2231(6)$

$|V_{us}|/|V_{ud}| = 0.2311(5)$

Fit results, no constraint

$V_{ud} = 0.97365(30)$

$V_{us} = 0.22414(37)$

$\chi^2/\text{ndf} = 6.6/1\ (1.0\%)$

$\Delta_{\text{CKM}} = -0.0018(6)$

-2.7σ

$$\boxed{|V_{ud}|^2+|V_{us}|^2+|\cancel{V_{ub}}|^2=1+\Delta_{CKM}}$$

Negligible ~$2\text{x}10^{-5}$
(B decays)

Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \rightarrow 0^+$ $\pi^\pm \rightarrow \pi^0 e \nu_e$	$n \rightarrow p e \nu_e$	$\pi \rightarrow l \nu_l$
V_{us}	$K \rightarrow \pi l \nu_l$	$\Lambda \rightarrow p e \nu_e$	$K \rightarrow l \nu_l$

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{RC}) \times F_{\text{kin}}$$

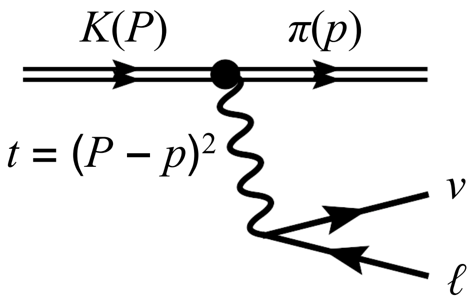
Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

The most precise determination of Vus comes from K_{l3}

4.1 Determination of fundamental parameters: V_{us}



- Master formula for $K \rightarrow \pi l \nu_l$: $K = \{K^+, K^0\}$, $l=\{e,\mu\}$

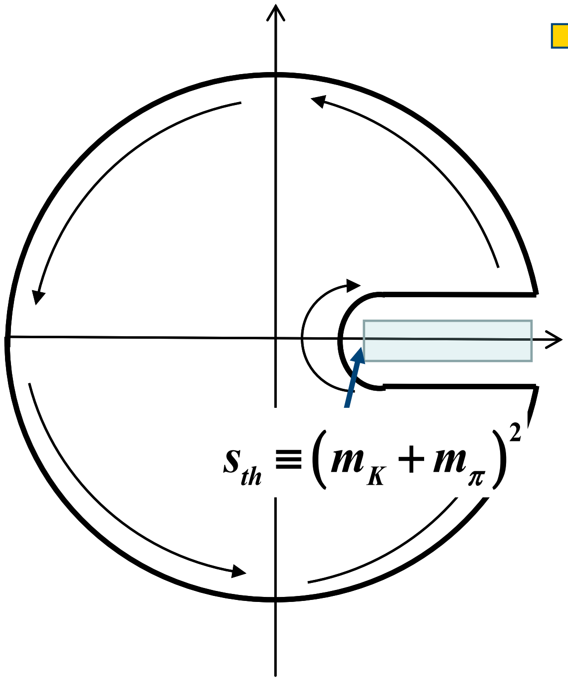
$$\Gamma\left(K \rightarrow \pi l \nu[\gamma]\right) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K \boxed{\left|V_{us}\right|^2} \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^l \left(1+\delta_{\mathrm{EM}}^{Kl}+\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2$$

$$\left\langle \pi(p_\pi)\right| \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \left|\mathrm{K}(\mathrm{p_K})\right\rangle = \left[\left(p_K+p_\pi\right)_\mu - \frac{\Delta_{K\pi}}{t}\left(p_K-p_\pi\right)_\mu\right] \underset{\substack{\uparrow \\ \text{vector}}}{f_+(t)} + \frac{\Delta_{K\pi}}{t}\left(p_K-p_\pi\right)_\mu \underset{\substack{\uparrow \\ \text{scalar}}}{f_0(t)}$$

Dispersive representation for the form factors

Bernard, Oertel, E.P., Stern'06, '09

- Omnès representation:



$$s_{th} \equiv \left(m_K + m_\pi\right)^2$$

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$

$\phi_{+,0}$ (s): phase of the form factor

$$s < s_{in} \;:\; \phi_{+,0}(s) = \delta_{K\pi}(s)$$

Kπ scattering phase

$$s \geq s_{in} \;:\; \phi_{+,0}(s)$$

$$\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \;\left(\bar{f}_{+,0}(s) \to 1/s\right)$$

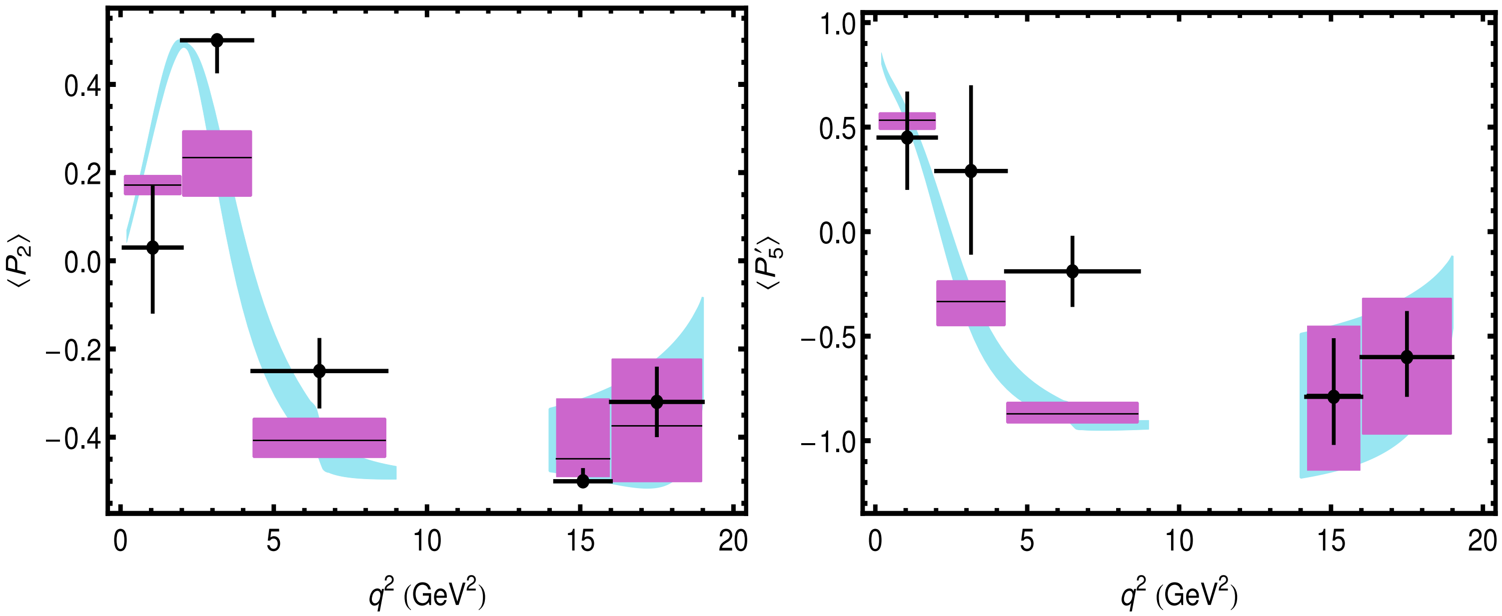
[Brodsky&Lepage]

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied.

4.2 FSI in the quest for New Physics

- Ex: CP violating asymmetries: B → K^* ll

Matthias et al'12
Camalich&Jaeger'11
Doering, Meissner, Wang'13 etc..



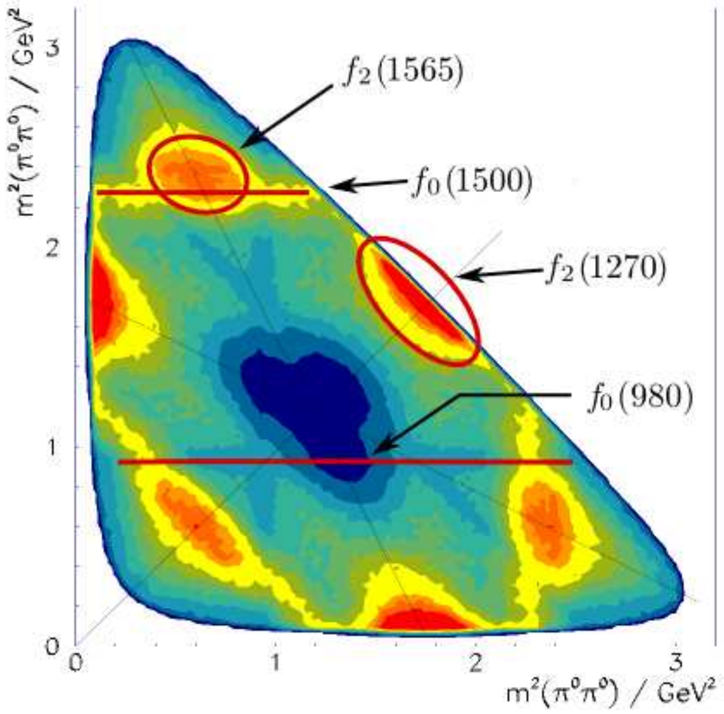
LHCb'17: 2.9 σ discrepancy in P_2, 4.0 σ in P'_5 !

[blue: SM unbinned, purple: SM binned, crosses: LHCb]

4.2 FSI in the quest for New Physics

- Ex: CP violation in D → Kππ

Ex: Dalitz plot



4.2 FSI in the quest for New Physics

- Ex: CP violation in D → Kππ

Niecknig & Kubis'15

Full set of equations

$$S^2_{\pi\pi}(u) = \Omega^2_0(u)\left\{u^2\int_{4M_\pi^2}^{\infty}\frac{\hat{S}^2_{\pi\pi}(u')}{u'^2(u'-u)}d\mu^2_0\right\}$$

$$P^1_{\pi\pi}(u) = \Omega^1_1(u)\left\{c_0 + c_1 u + u^2\int_{4M_\pi^2}^{\infty}\frac{\hat{P}^1_{\pi\pi}(u')}{u'^2(u'-u)}d\mu^1_1\right\}$$

$$S^{1/2}_{\pi K}(s) = \Omega^{1/2}_0(s)\left\{c_2 + c_3 s + c_4 s^2 + c_5 s^3 + s^4\int_{(M_K+M_\pi)^2}^{\infty}\frac{\hat{S}^{1/2}_{\pi K}(s')}{s'^4(s'-s)}d\mu^{1/2}_0\right\}$$

$$S^{3/2}_{\pi K}(s) = \Omega^{3/2}_0(s)\left\{s^2\int_{(M_K+M_\pi)^2}^{\infty}\frac{\hat{S}^{3/2}_{\pi K}(s')}{s'^2(s'-s)}d\mu^{3/2}_0\right\}$$

$$P^{1/2}_{\pi K}(s) = \Omega^{1/2}_1(s)\left\{c_6 + s\int_{(M_K+M_\pi)^2}^{\infty}\frac{\hat{P}^{1/2}_{\pi K}(s')}{s'(s'-s)}d\mu^{1/2}_1\right\}$$

$$D^{1/2}_{\pi K}(s) = \Omega^{1/2}_2(s)\left\{\int_{(M_K+M_\pi)^2}^{\infty}\frac{\hat{D}^{1/2}_{\pi K}(s')}{(s'-s)}d\mu^{1/2}_2\right\}$$

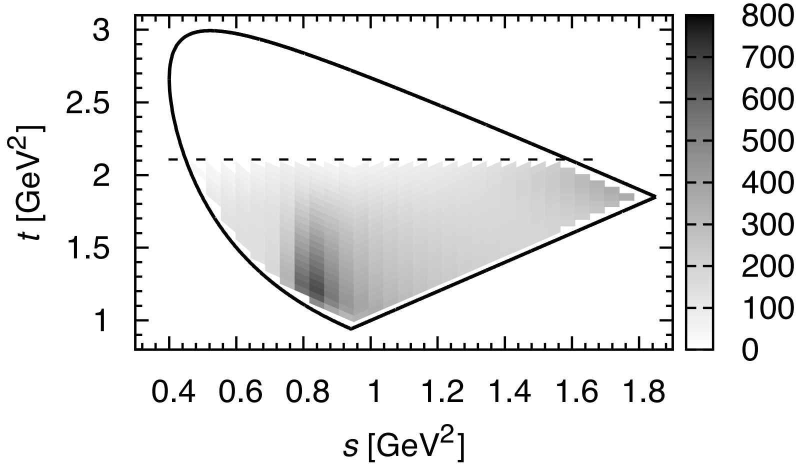
4.2 FSI in the quest for New Physics

- Ex: CP violation in D → Kππ

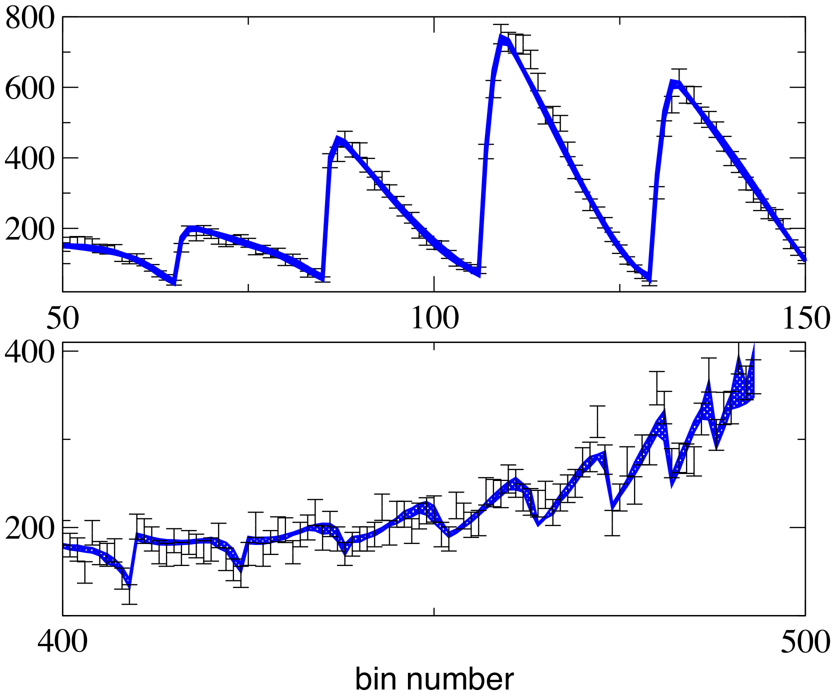
Niecknig & Kubis'15

Dalitz plot

CLEO'08



slices



- full fit: $\chi^2/\mathrm{ndof} \approx 1.1$

4.2 FSI in the quest for New Physics

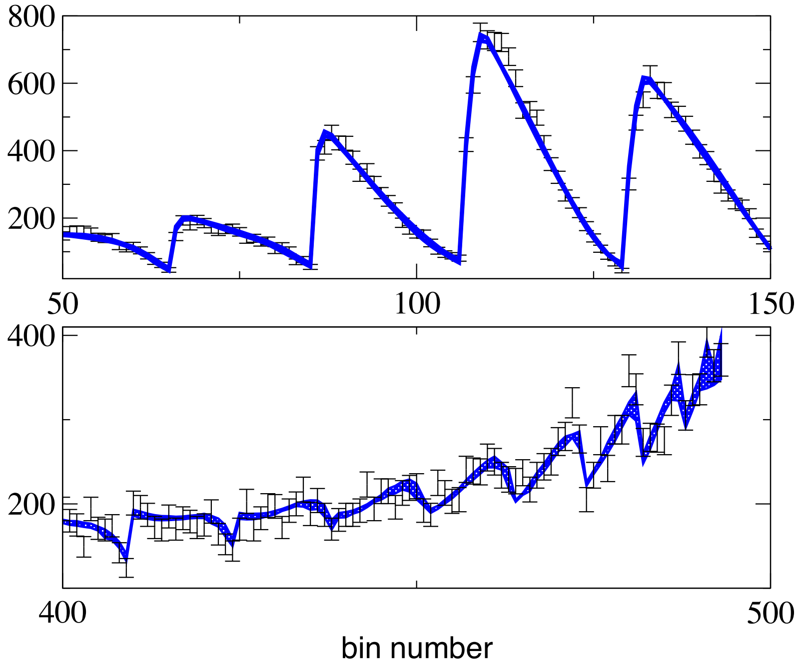
- Ex: CP violation in D → Kππ

Niecknig & Kubis'15

fit fractions

	Full fit
$S^2_{\pi\pi}$	$(8 \pm 3)\%$
$S^{1/2}_{\pi K}$	$(72 \pm 12)\%$
$P^{1/2}_{\pi K}$	$(10 \pm 2)\%$
$S^{3/2}_{\pi K}$	$(16 \pm 3)\%$
$D^{1/2}_{\pi K}$	$(0.15 \pm 0.1)\%$
Σ	$(106 \pm 20)\%$

slices



- full fit: $\chi^2/\text{ndof} \approx 1.1$
- fit fractions: hierachy of partial-wave amplitudes compare to previous analyses

5. Conclusion and outlook

5.1 Conclusion

- Determining Kπ scattering reliably very important:
  - Low energy: test of Chiral Dynamics
  - Intermediate energy: Determination of Resonance parameters
  - Very important to help taking into account final state interactions and hunting for new physics
    ⇨ CP violation in heavy meson decays

- Hadronic data on which most of the analyses rely not in good agreement with more recent data coming mainly from tau decays
  ⇨ worth remeasuring it.

- Possibility at Jlab with K_L?
  Major advantage: pure I=1/2 measurement

5.2 Outlook

- Possibility at Jlab with K_L?
  Major advantage: pure I=1/2 measurement

- Challenges: Extracting the Kpi phase shift from KN
  ⇨ Reliable interpolation at the pion pole

- Require a collaboration between theorists and experimentalists