Outline

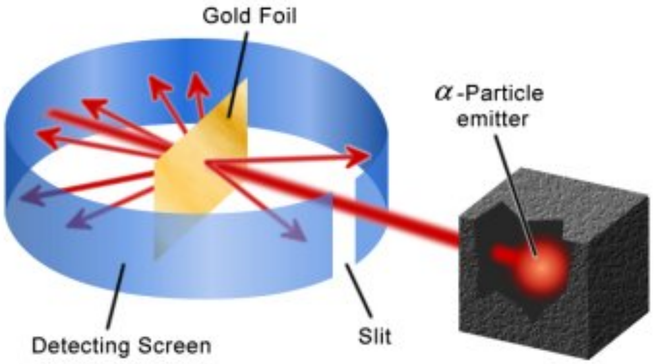
- The proton
- Proton Radius Puzzle: current status
- Our approach for a new ep scattering experiment: Prad
- Final ~~Preliminary~~ results
- Summary and outlook

Randolph Pohl et al.

Story of the Proton

Proton is the most studied sub-atomic particle

- It has been over hundred years since Rutherford postulated the existence of the nucleus



- In 1933 Stern measured the anomalous magnetic moment of the proton to show that proton is NOT an elementary point like particle.

Story of the Proton

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From Wikipedia

In 1917 (in experiments reported in 1919 and 1925), Rutherford proved that the hydrogen nucleus is present in other nuclei, a result usually described as the discovery of protons.[14] These experiments began after Rutherford had noticed that, when alpha particles were shot into air (mostly nitrogen), his scintillation detectors showed the signatures of typical hydrogen nuclei as a product. After experimentation Rutherford traced the reaction to the nitrogen in air and found that when alpha particles were introduced into pure nitrogen gas, the effect was larger. In 1919 Rutherford assumed that the alpha particle knocked a proton out of nitrogen, turning it into carbon. After observing Blackett's cloud chamber images in 1925, Rutherford realized that the opposite was the case: after capture of the alpha particle, a proton is ejected, so that heavy oxygen, not carbon, is the end result i.e. Z is not decremented but incremented. This was the first reported nuclear reaction, $14N + \alpha \rightarrow 17O + p$. **Depending on one's perspective, either 1919 or 1925 may be regarded as the moment when the proton was 'discovered'.**

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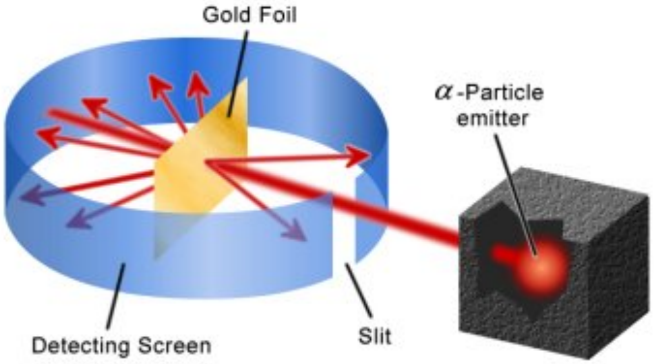

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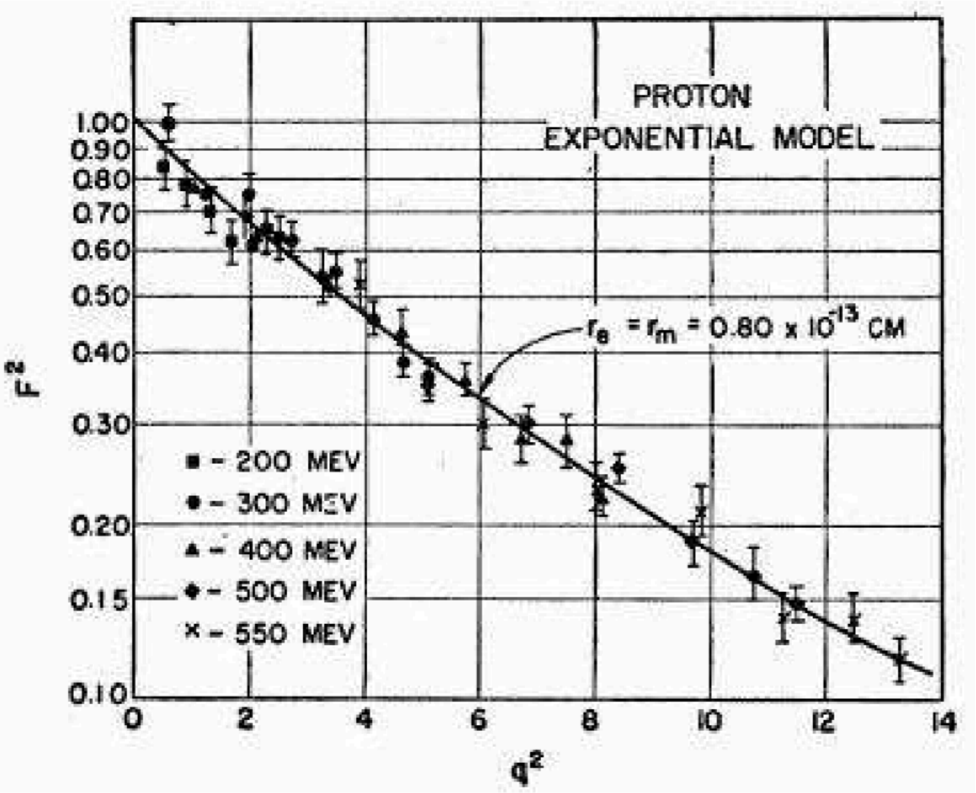
Electron Scattering to Probe the Proton

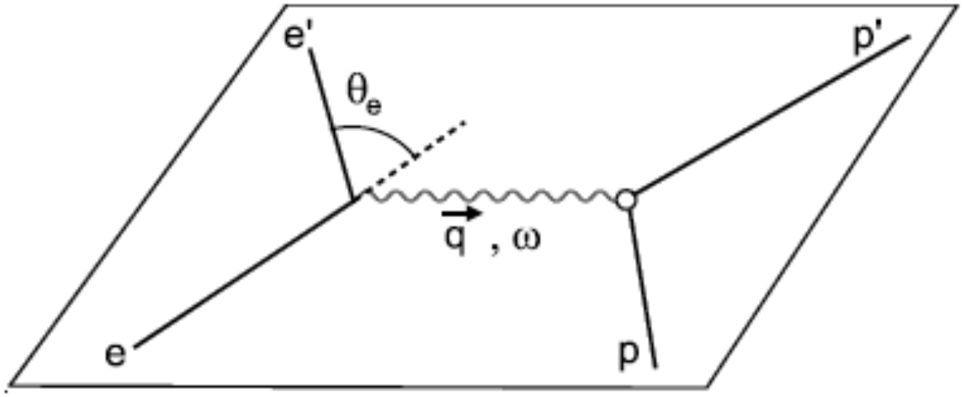
Hofstadter 1958: electron scattering to measure proton radius ~ 0.8 fm.



Electron Scattering to Probe the Proton

Hofstadter used the charge form factor to describe the charge distribution of the proton:

$$F(q) = \int_{volume} \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3 r$$



F(q) is the probability amplitude for the proton to absorb the exchanged photon

So the probability of elastically scattering off the proton:

$$\sigma(\theta_e) = \sigma_{Mott} |F(\mathrm{q})|^2 \Rightarrow [F(\mathrm{q})]^2 = \frac{\sigma(\mathrm{q})}{\sigma_{Mott}(\mathrm{q})}$$

Story of the Proton, continued….

MIT-SLAC experiments 1967: Deep Inelastic electron Scattering off protons to confirm the quarks inside the proton.

Kendall, Friedman and Taylor et al.

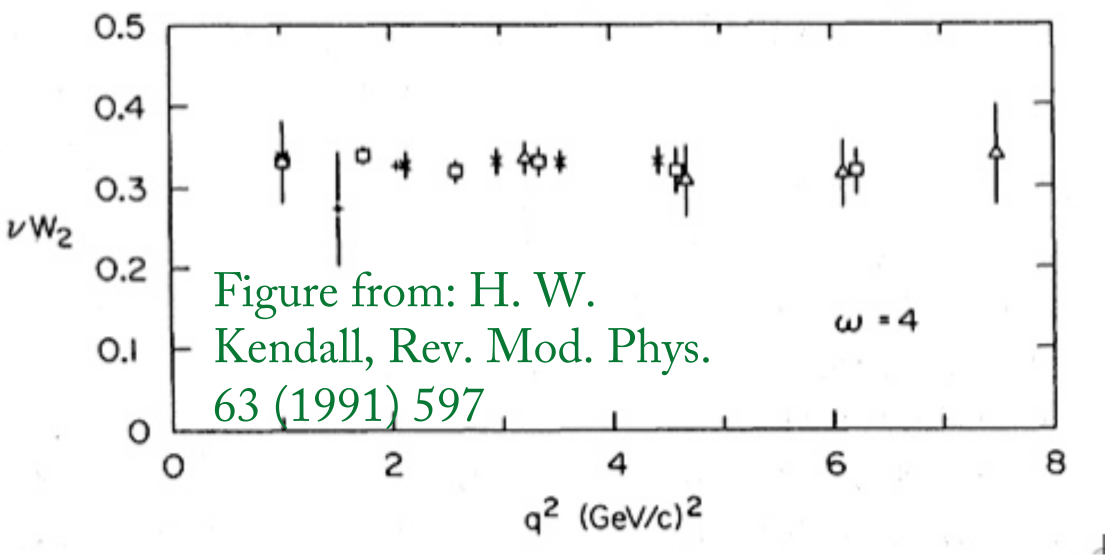
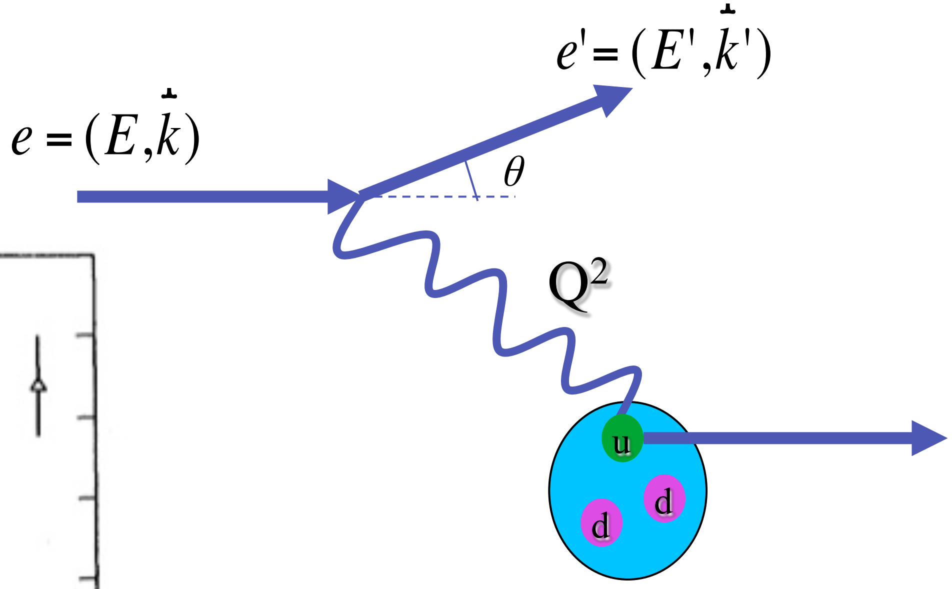
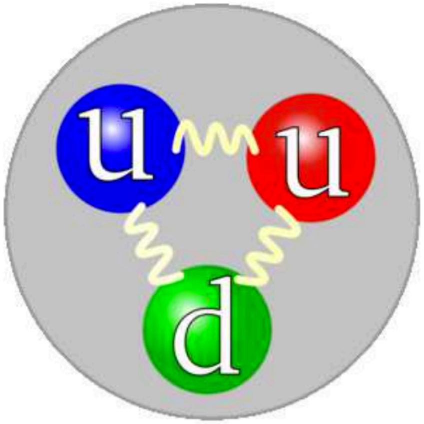


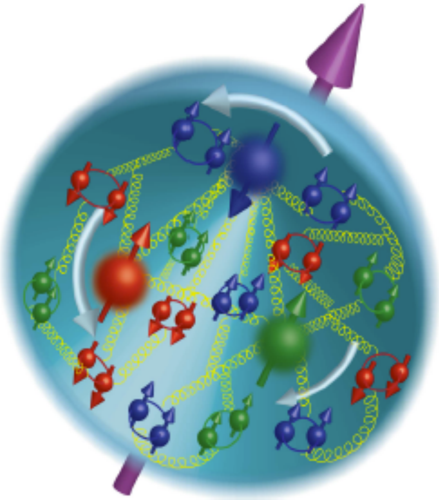
Figure from: H. W. Kendall, Rev. Mod. Phys. 63 (1991) 597



Story of the Proton, continued….

- 1970's: Quantum Chromo Dynamics (QCD): theoretical framework for strong interaction between quarks medicated by gluons.
- 1980's – Today: Looking deep inside the proton





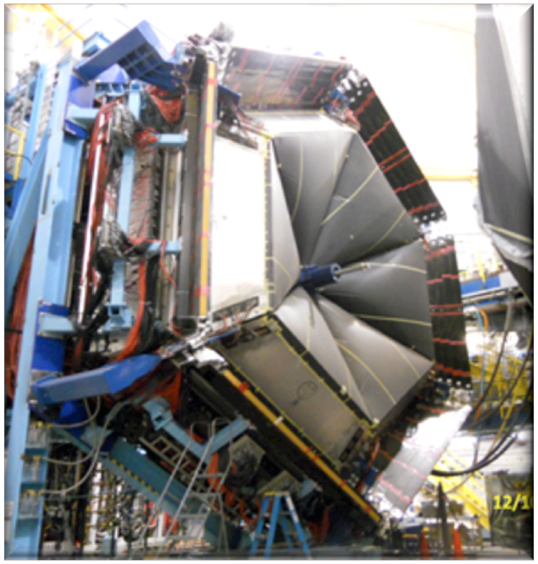
Proton: an ideal laboratory to understand strong interaction

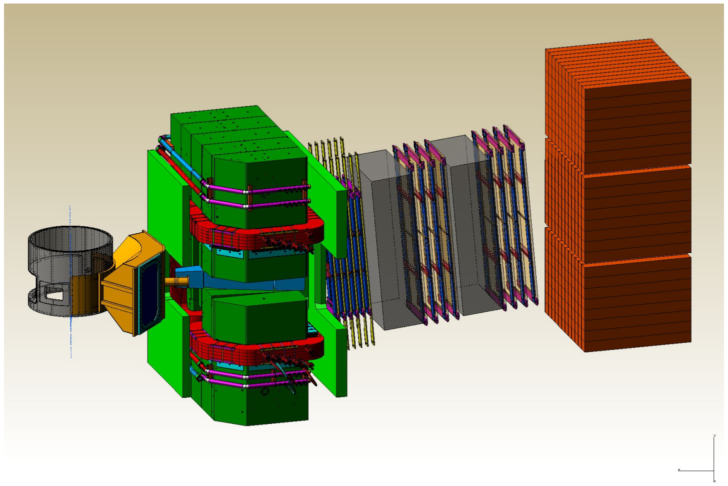
Many deep questions to answer

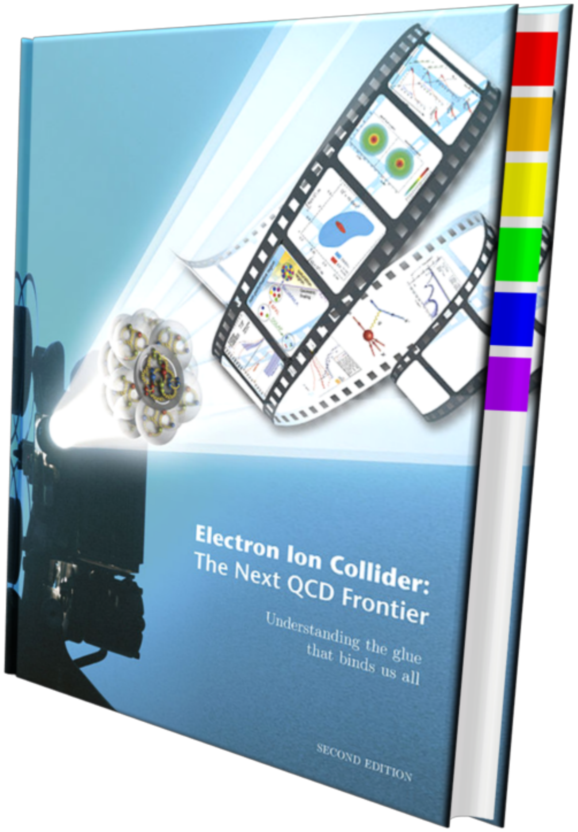
- How does proton acquire its mass: only ~1% of proton mass comes from Higgs.
- What are the different contributions to nucleon spin ?
- How does the confinement come about ?
- What role does the gluon play in all these ??

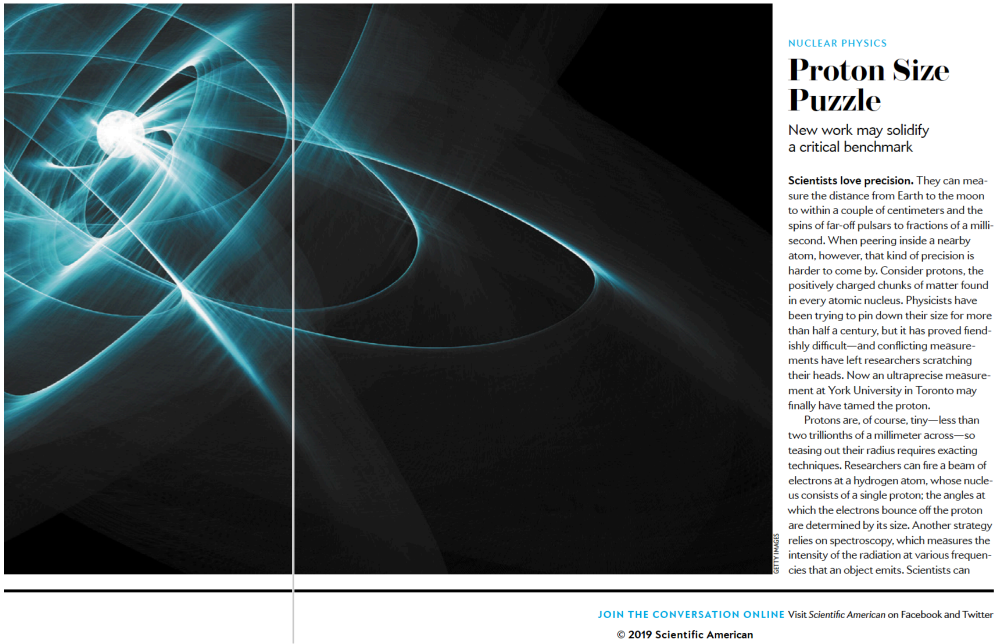
Exciting times ahead for the proton

- Jefferson Lab 12 GeV
    - 3D structure of the proton: GPDs
    - Ground stated properties with high resolution: high Q^2 FF.
- Electron Ion Collider
    - Understand the role of gluon









NUCLEAR PHYSICS

Proton Size Puzzle

New work may solidify
a critical benchmark

Scientists love precision. They can measure the distance from Earth to the moon to within a couple of centimeters and the spins of far-off pulsars to fractions of a millisecond. When peering inside a nearby atom, however, that kind of precision is harder to come by. Consider protons, the positively charged chunks of matter found in every atomic nucleus. Physicists have been trying to pin down their size for more than half a century, but it has proved fiendishly difficult—and conflicting measurements have left researchers scratching their heads. Now an ultraprecise measurement at York University in Toronto may finally have tamed the proton.

Protons are, of course, tiny—less than two trillionths of a millimeter across—so teasing out their radius requires exacting techniques. Researchers can fire a beam of electrons at a hydrogen atom, whose nucleus consists of a single proton; the angles at which the electrons bounce off the proton are determined by its size. Another strategy relies on spectroscopy, which measures the intensity of the radiation at various frequencies that an object emits. Scientists can

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Meanwhile there are other secrets the proton has yet to give up. For starters, we know protons and neutrons both consist of three quarks bound by the strong nuclear force—but the exact nature of that binding is poorly under- [redacted]

“Protons are the stuff we’re made of,” says [redacted] who has tackled the proton radius puzzle through electron-scattering experiments at the Jefferson Lab in Virginia. And “99.9 percent of our mass—of ourselves, of everything in the universe—comes from protons and neutrons.” The proton radius is a critical benchmark quantity, he adds: “It’s a very important particle, and we need to understand it.” —*Dan Falk*

Directions
in
Hadron Physics (th)

Barbara Pasquini

Università di Pavia & INFN Pavia

3DSPIN
MAPPING
THE PROTON IN 3D

PI: A. Bacchetta

erc
European Research Council

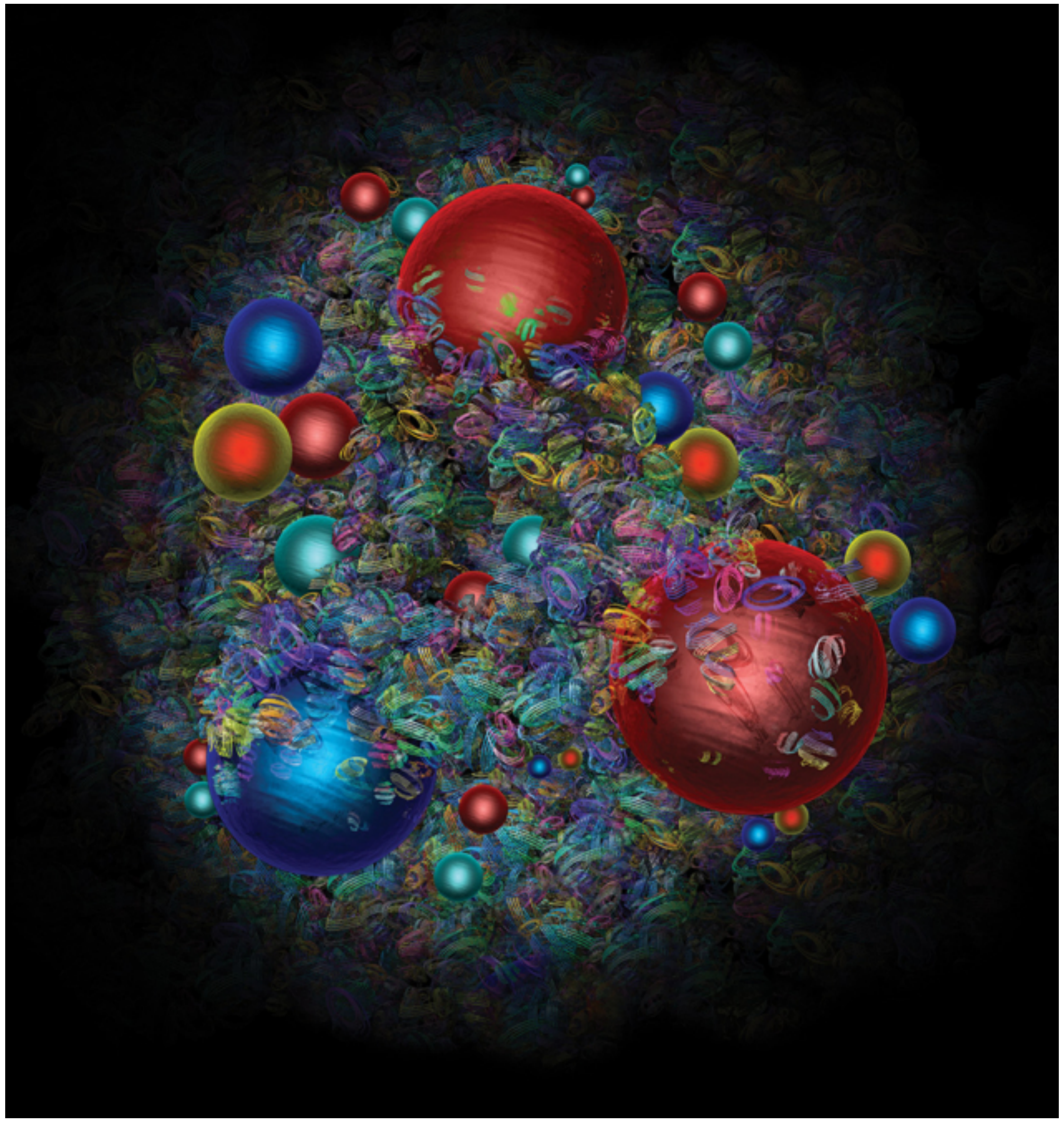
INFN
Istituto Nazionale
di Fisica Nucleare

UNIVERSITÀ
DI PAVIA

100 years of the discovery of the proton

``What proton is depends on how you look at it, or rather on how hard you hit it''

A. Cooper-Sarkar, CERN Courier, June, 2019



Q^2

hadronic d.o.f. → nucleon resonances → partonic d.o.f.

How can we explain the evolving picture of hadrons
from low to high Q^2?

The existence of hadrons, their properties and their binding into nuclei,
do not appear in the Lagrangian of QCD

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\left(i\gamma_\mu D^\mu - m\right)\psi - \frac{1}{2}\mathrm{Tr}\left\{G_{\mu\nu}G^{\mu\nu}\right\}$$

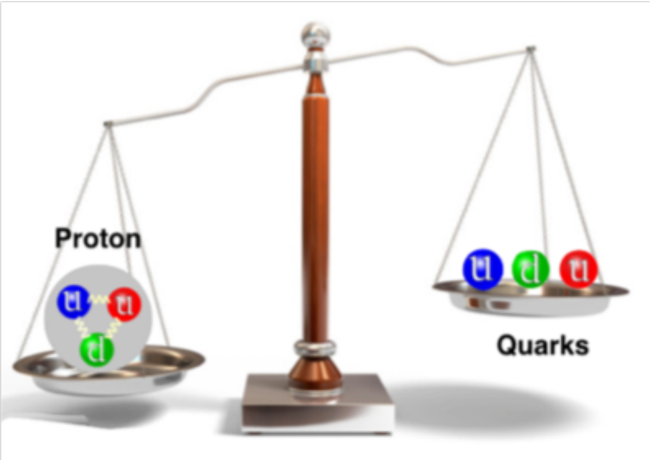
The Science questions:
The 2015 Long Range Plan for Nuclear Science

- How did visible matter come into being and how does it evolve?
- How does subatomic matter organise itself and what phenomena emerge?
- Are the fundamental interactions that are basic to the structure of matter fully understood?

Size

Spin

Mass



The Proton Charge Radius

Charge distribution
(up to relativistic corrections)

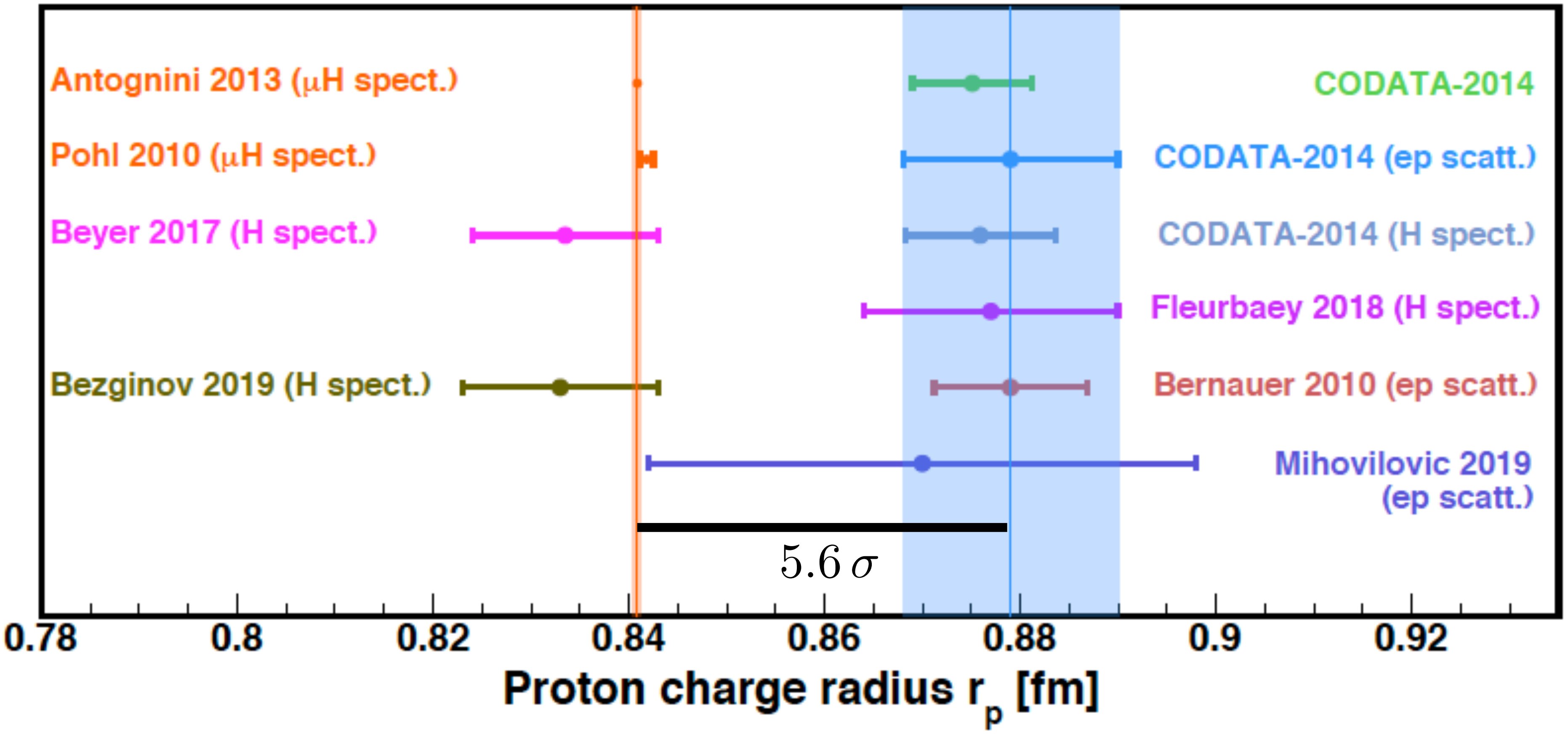
$$\rho_E(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} G_E(-\vec{q}^2)$$

Mean square radius:

$$\langle r^2 \rangle_E = \int d^3r\, r^2 \rho_E(\vec{r}) = 6\, \frac{\mathrm{d}G_E(0)}{\mathrm{d}Q^2}$$

Spectroscopy measurements: $V_C = -\frac{\alpha_{em}}{q^2} - 4\pi\alpha_{em} \frac{\mathrm{d}G_E(0)}{\mathrm{d}Q^2}$

Elastic electron scattering: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_0}{\epsilon(1+\tau)} \left[\epsilon\, G_E(Q^2) + \tau\, G_M(Q^2)\right]$



Final PRad (JLab) result from ep scattering supports the smaller radius from spectroscopy

CODATA 2018 (all available data through 31 Dec. 2018): 0.8414 ± 0.019 fm

⟶ *Talks of N. Liyanage, R. Pohl*

The missing piece

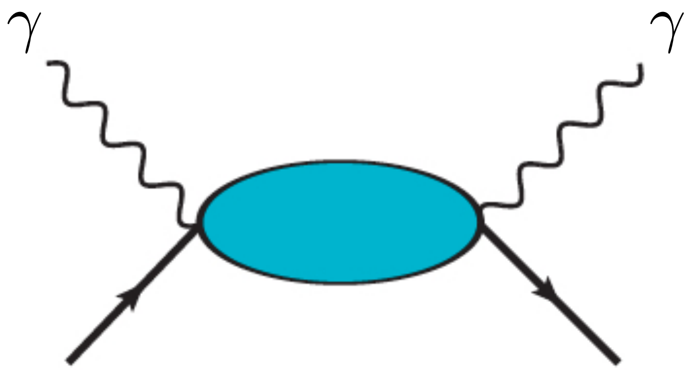
r_E (fm)	ep	μp
Spectroscopy	0.8758 ± 0.077	0.84087 ± 0.00039
Scattering	0.8770 ± 0.060	**???**

Measure radius with proton-muon scattering!

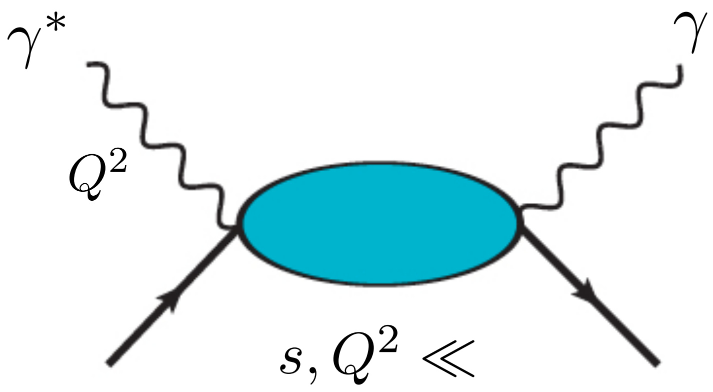
- MUSE at PSI
- ProRad at PRAE, Paris/Orsay
- ELPH, Tohoku U., Japan
- MAMI at Mainz
- COMPASS++ at CERN

Two-photon Physics

RCS polarizabilities



VCS generalized pol.



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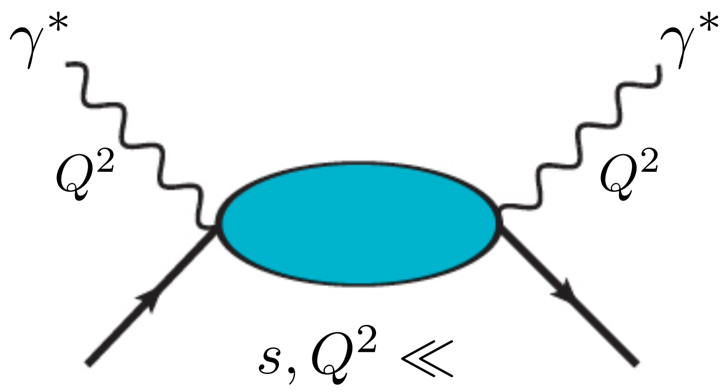
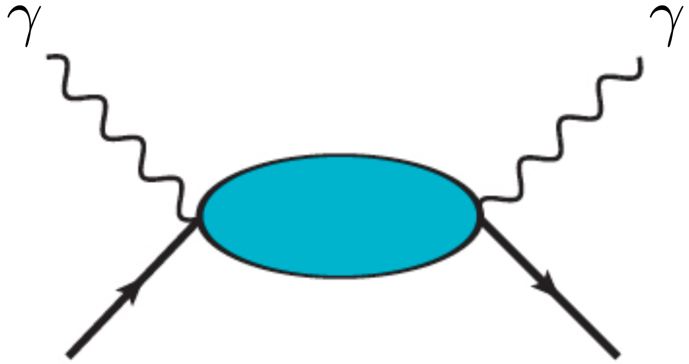


Compton scattering at threshold can be interpreted as electron scattering by a target which is in constant electric and magnetic fields

Two-photon Physics

RCS polarizabilities



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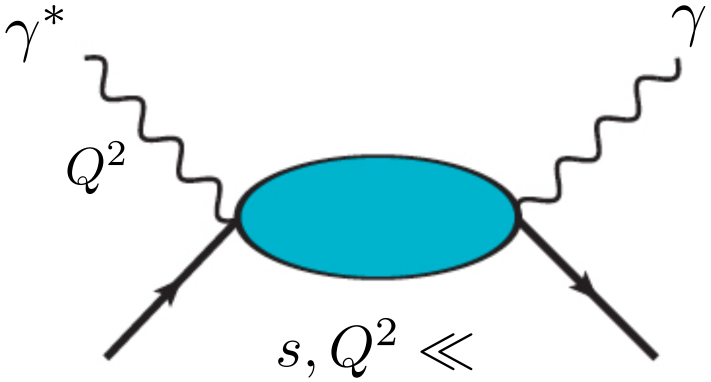
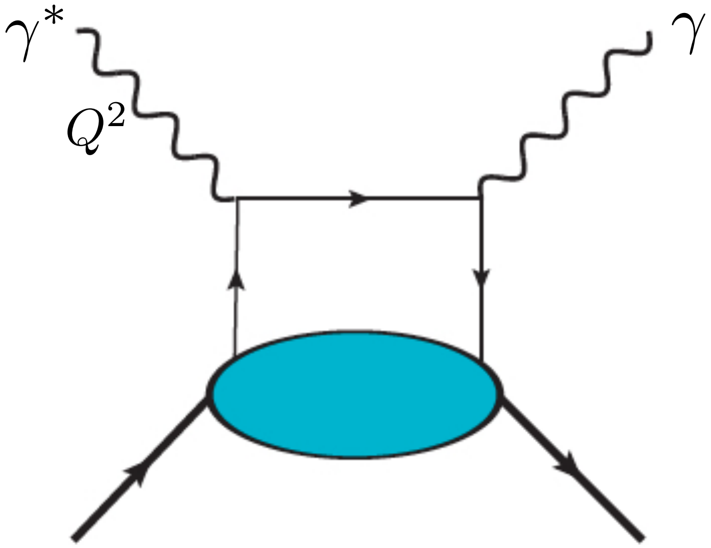


$s, Q^2 \ll$

$s, Q^2 \gg$

DVCS
generalized parton distributions



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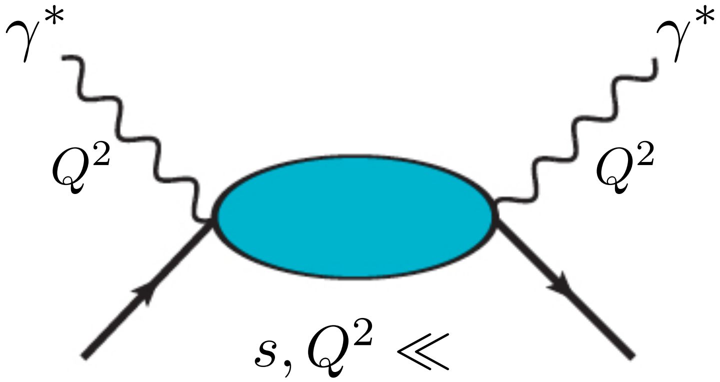

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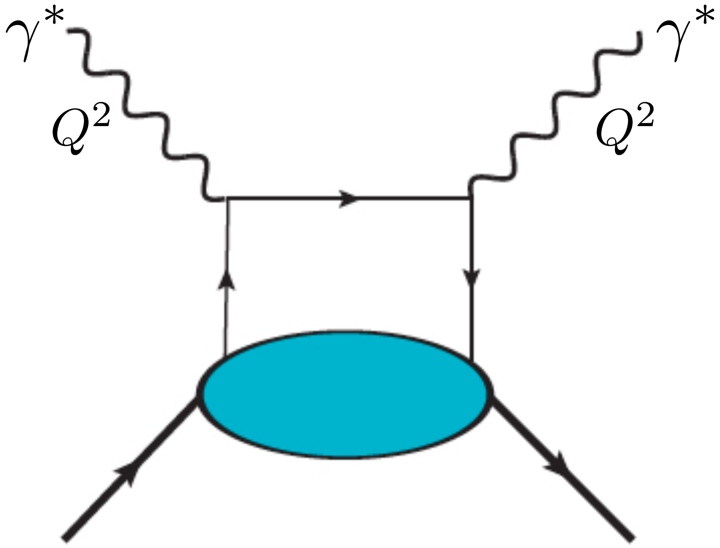
DIS
parton distributions



Real Compton Scattering at low energies

Measure of the strength of induced polarizations: 2 scalar polarizabilities + 4 spin polarizabilities

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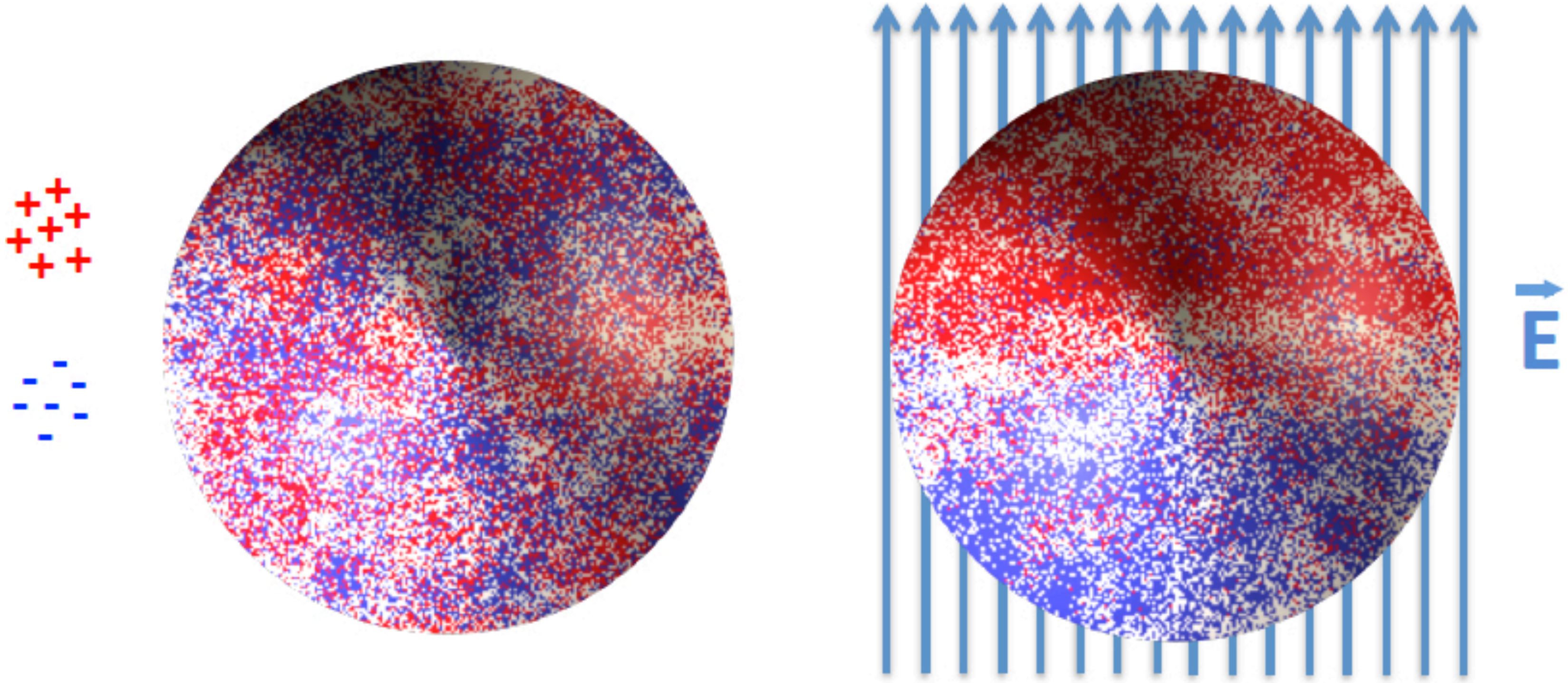

$$\vec{D}_E \sim \alpha_{\mathrm{E1}}\,\vec{E}$$

Unlike atoms,
it is not proportional to volume

$$V \sim \langle r_p \rangle^3 \approx 0.6\,\mathrm{fm}^3$$

$$\alpha_{\mathrm{E1}} \approx 10^{-4}\,V_p$$

much ``stiffer'' than hydrogen!

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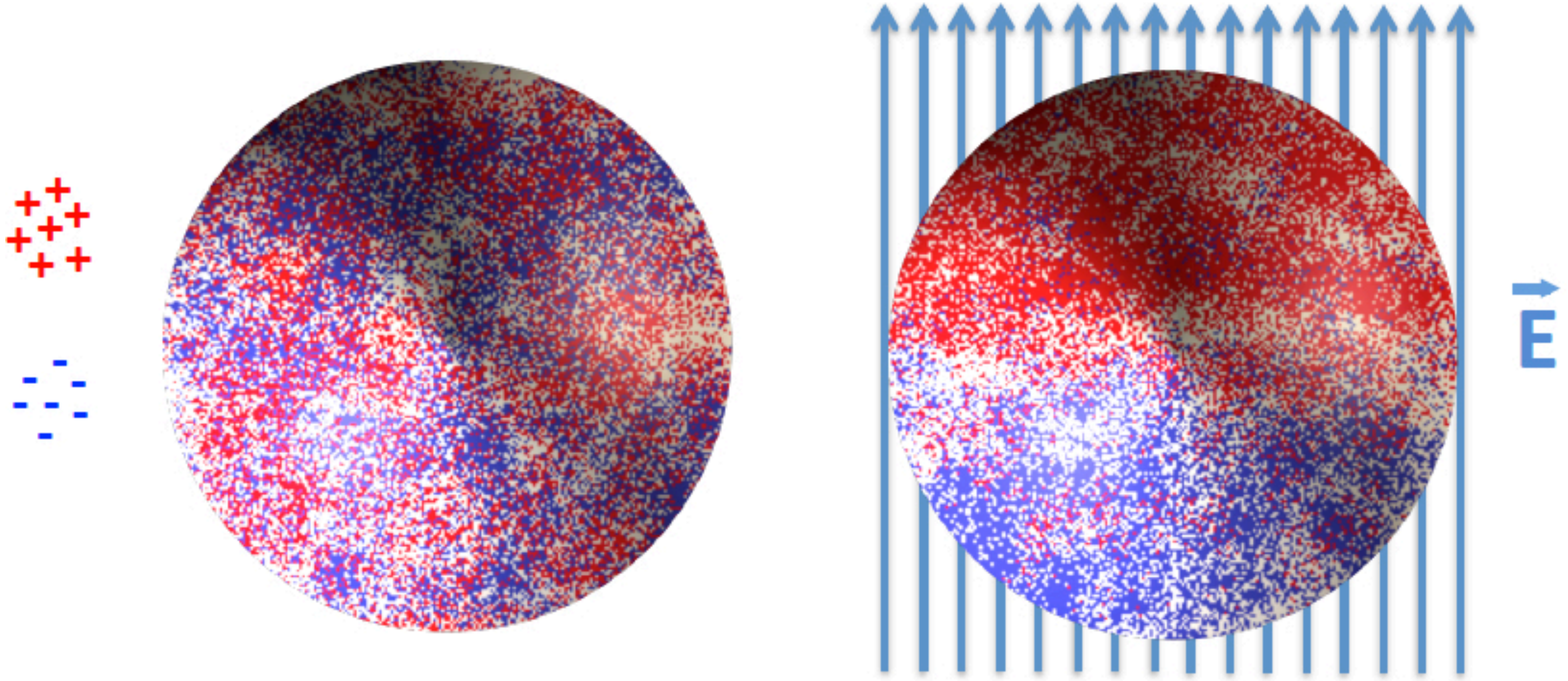
Measure of the strength of induced polarizations: 2 scalar polarizabilities + 4 spin polarizabilities



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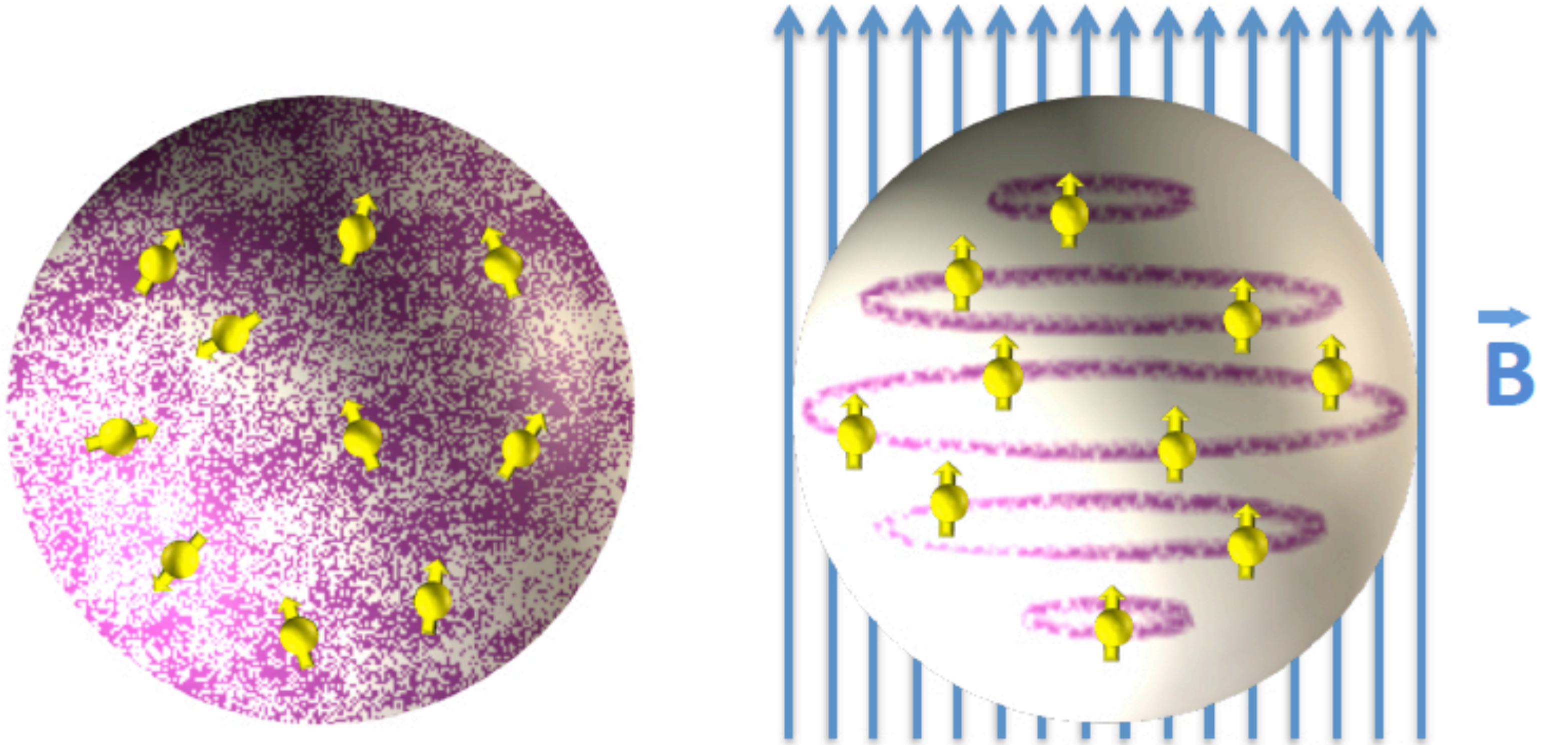

$$\vec{D}_M \sim \beta_{\rm M1}\, \vec{B}$$

$\beta_{\rm M1}^{\rm para} > 0$ proton spin aligns with external field

$\beta_{\rm M1}^{\rm dia} < 0$ induced current of pion cloud generates field opposite to the external one

Status of RCS scalar polarizabilities

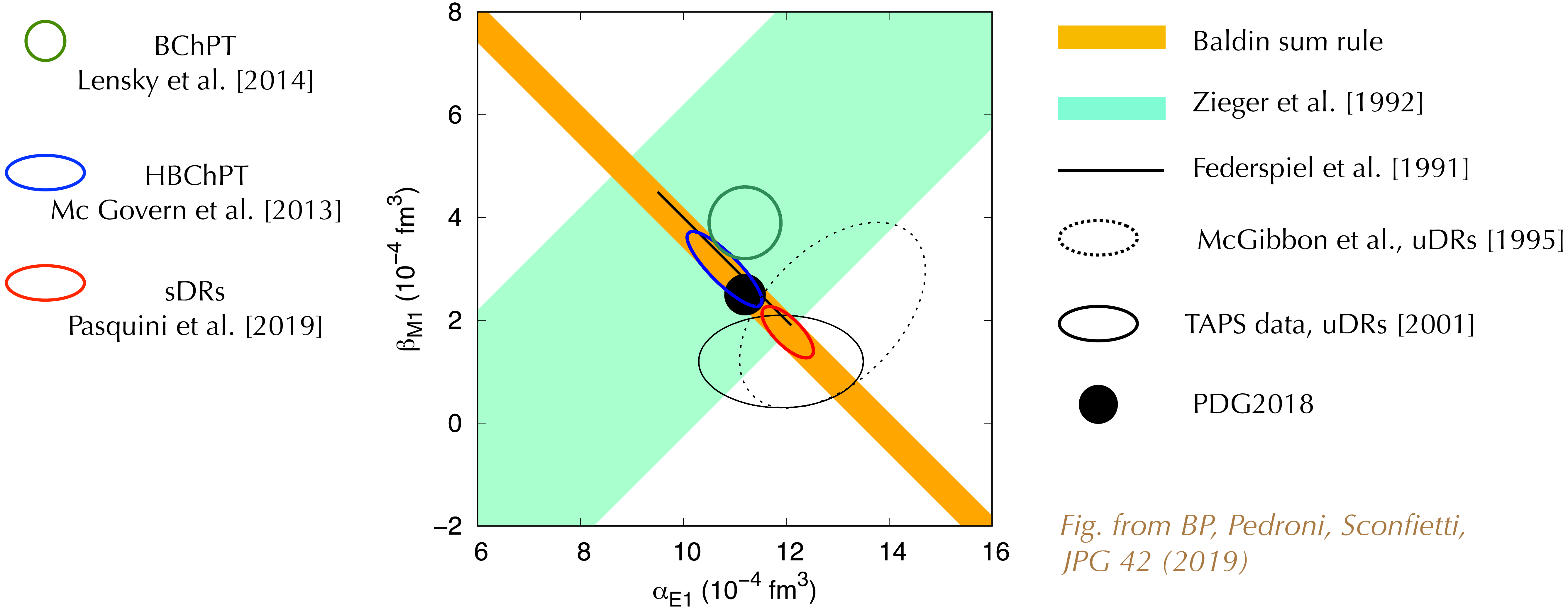

Fig. from BP, Pedroni, Sconfietti, JPG 42 (2019)

PDG2018: $\alpha_{\rm E1} = 11.2 \pm 0.4 \quad \beta_{\rm M1} = 2.5 \pm 0.4$

Baldin sum rule: $\alpha_{\rm E1} + \beta_{\rm M1} = 13.8 \pm 0.4$

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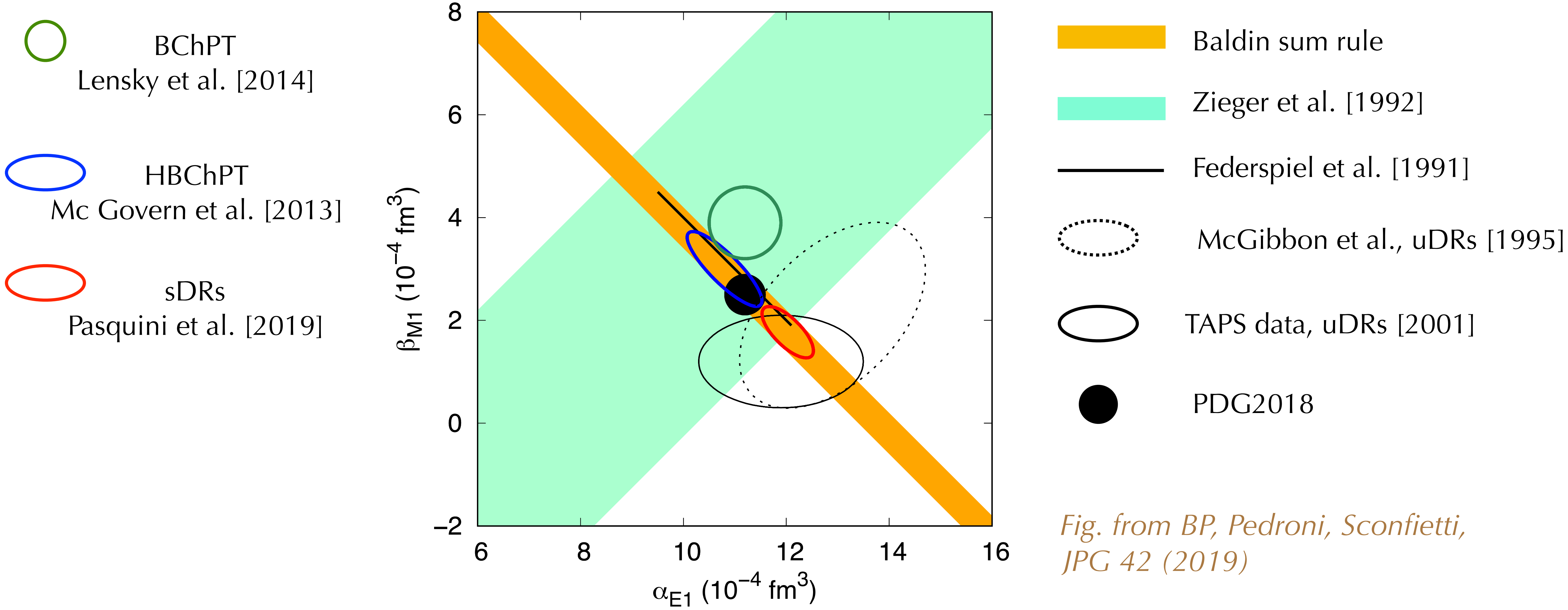

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Extractions obtained using different data sets and different theoretical models:

HBChPT

$$\alpha_{\rm E1} = 10.65 \pm 0.35\,({\rm stat.}) \pm 0.2\,({\rm Baldin}) \pm 0.3\,({\rm th.})$$

$$\beta_{\rm M1} = 3.15 \pm 0.35\,({\rm stat.}) \pm 0.2\,({\rm Baldin}) \pm 0.3\,({\rm th.})$$

Subtracted Dispersion Relations

$$\alpha_{\rm E1} = 12.03^{+0.48}_{-0.54}$$

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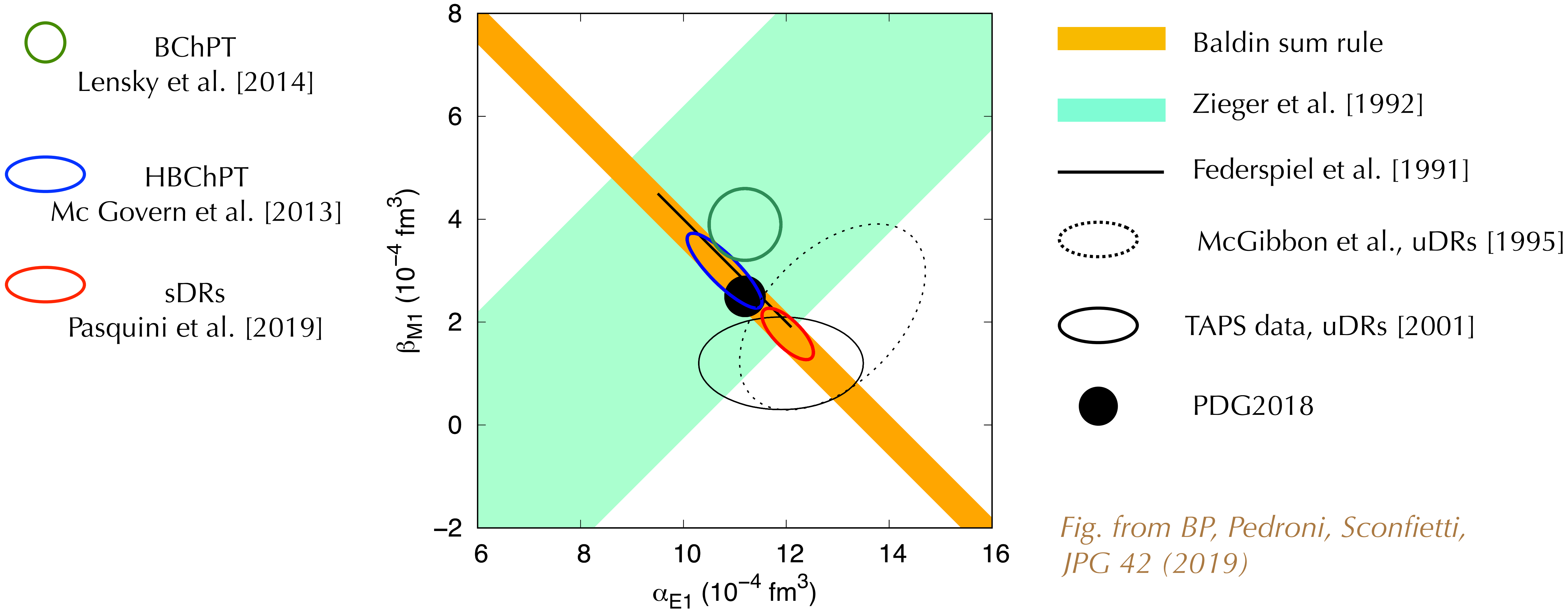
Status of RCS scalar polarizabilities



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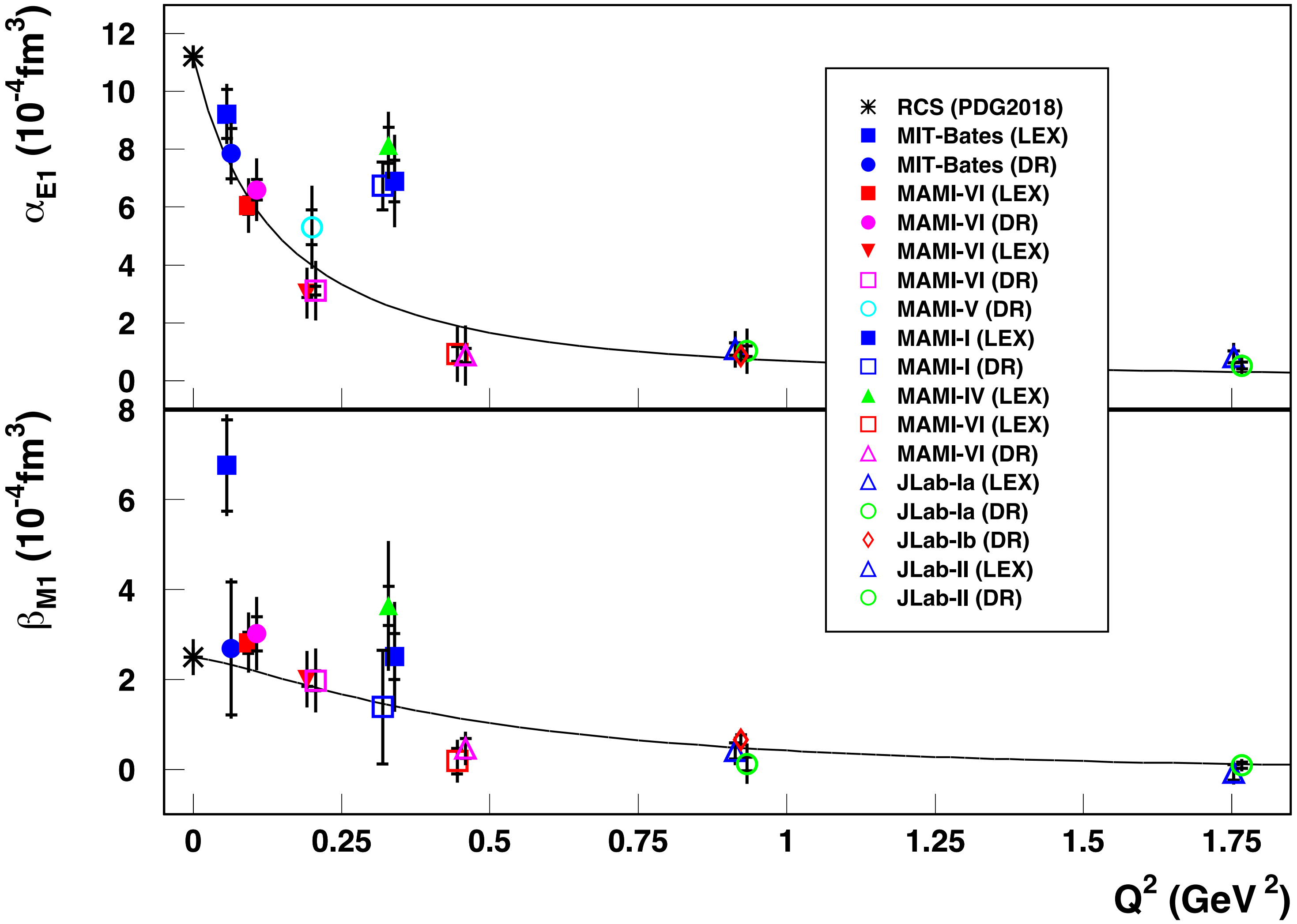
First extraction of spin pol. and very accurate data for scalar pol. from MAMI:

talks of P. Martel and E. Mornacchi

Status of VCS scalar polarizabilities

DR
fitted to data



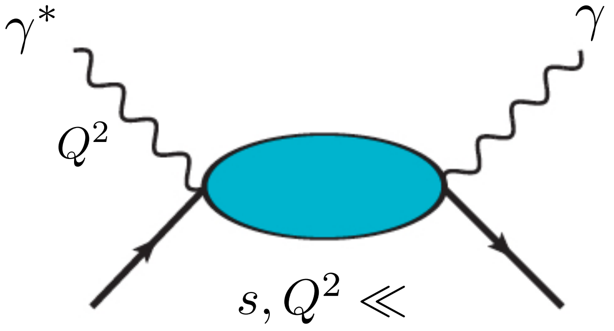


Fig. from Fonvieille, BP, Sparveris, arXiv:1910.11071

Two analysis methods: Low-Energy Expansion (LEX)
Dispersion Relations (DRs)

⇨

Model dependence:
spin GPs are taken from DR theory

New JLAB data under analysis: $0.3\,\mathrm{GeV}^2 \leq Q^2 \leq 0.75\,\mathrm{GeV}^2$

Plans to extract spin GPs directly from data under study
and
Efforts to reduce theoretical model dependence

Mean square polarizabilities radius

radius of induced electric and magnetic polarizations

(up to relativistic corrections)

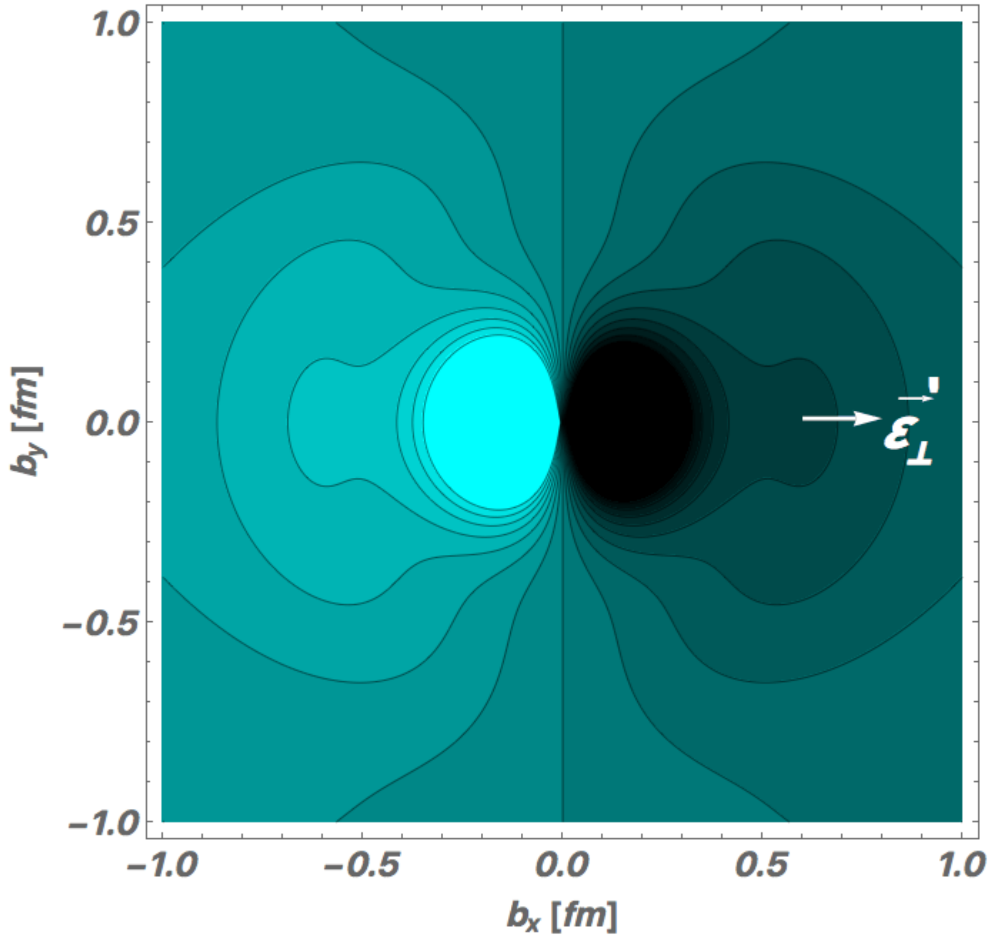
$$\langle r^2 \rangle_{\mathrm{GP}} = -\frac{6}{\mathrm{GP}(0)} \frac{d}{dQ^2} \mathrm{GP}(Q^2)\Big|_{Q^2=0}$$

$<r^2>_{\mathrm{GP}}$ (fm²)	resonance excitation	pion cloud	Total
α_{E1}	$0.60\ ^{+0.32}_{-0.26}$	$1.10\ ^{+0.04}_{-0.04}$	$1.70\ ^{+0.33}_{-0.24}$
β_{M1}	$2.67\ ^{+0.51}_{-0.37}$	$-3.91\ ^{+1.47}_{-2.00}$	$-1.24\ ^{+1.38}_{-1.86}$

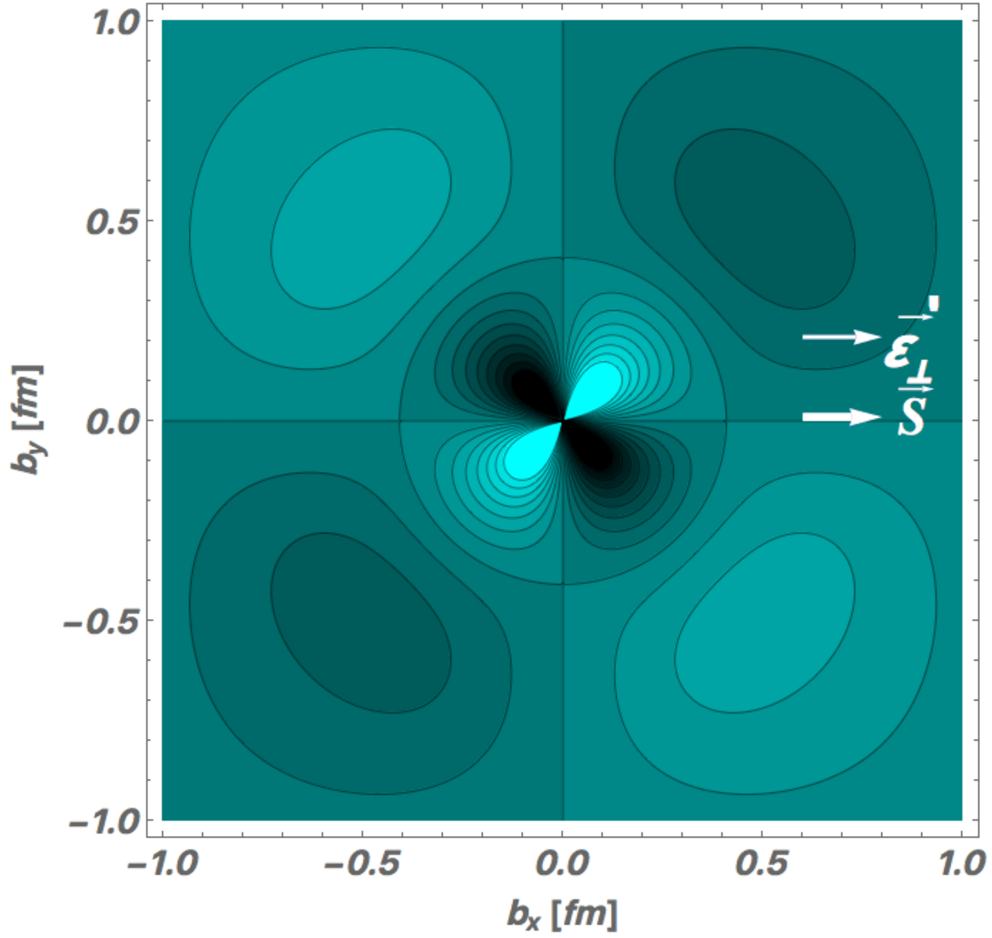
- Square radius of electric GP much larger than square radius of charge distribution
- Dominance of long range effects of pion cloud

Fonvieille, BP, Sparveris, arXiv:1910.11071

Spatial density of induced polarizations



unpolarized proton target



transversely pol. proton target

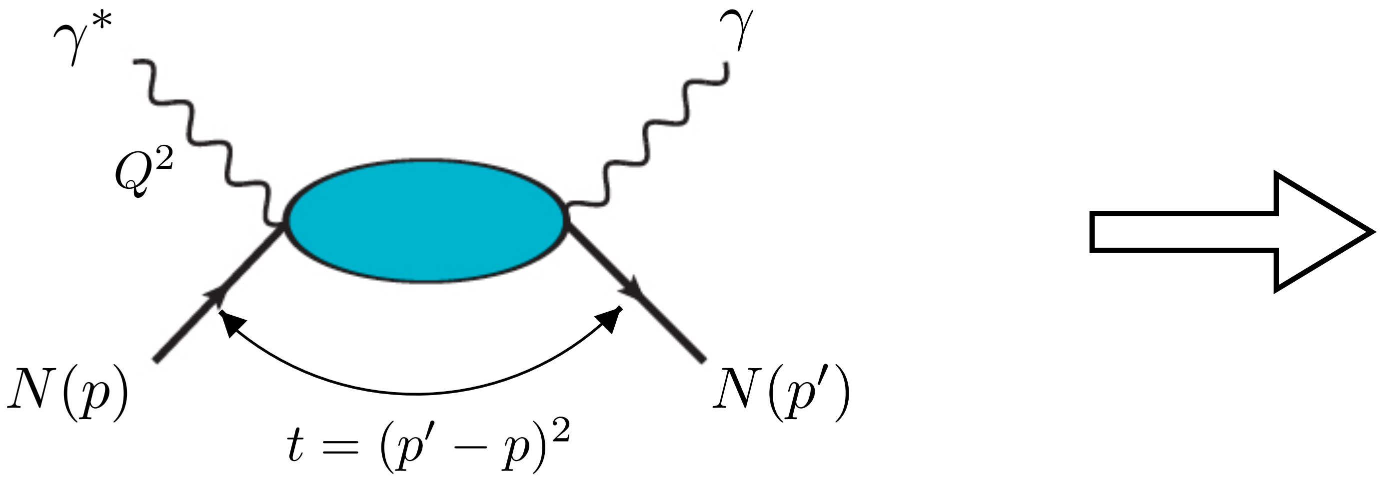
light (dark) regions ⟶ larger (smaller) values

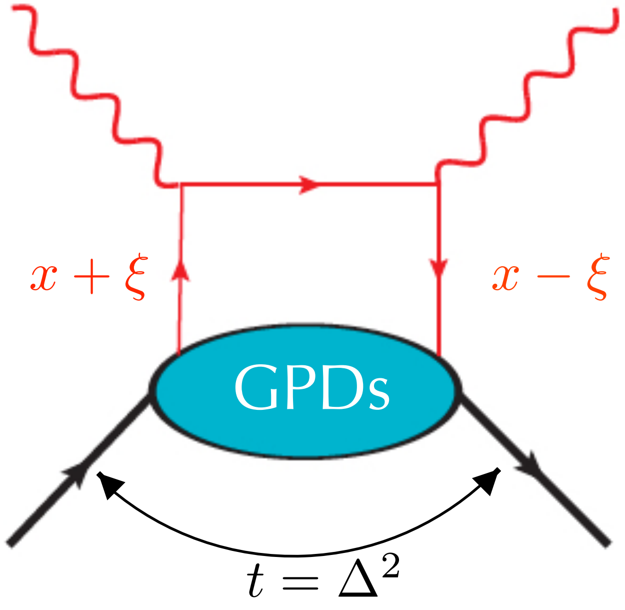
Light-front frame with fast moving proton in the longitudinal direction and $Q^2 = q_\perp^2$ $\quad \vec{q}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$ true probabilistic interpretation!

$\vec{E} \sim i q'^0 \vec{\epsilon}\,'_\perp$ quasi-static electric field $\longrightarrow$ $\vec{P}$ induced polarization depending on scalar and spin GPs

Gorchtein, Lorcé, BP, Vanderhaeghen, PRL104 (2010) 112001

Partonic description: Deeply Virtual Compton Scattering





factorization for large Q^2, $|t| << Q^2$, s

$\mathcal{M}$ =[parton Ampl.] $\otimes$ [GPDs]

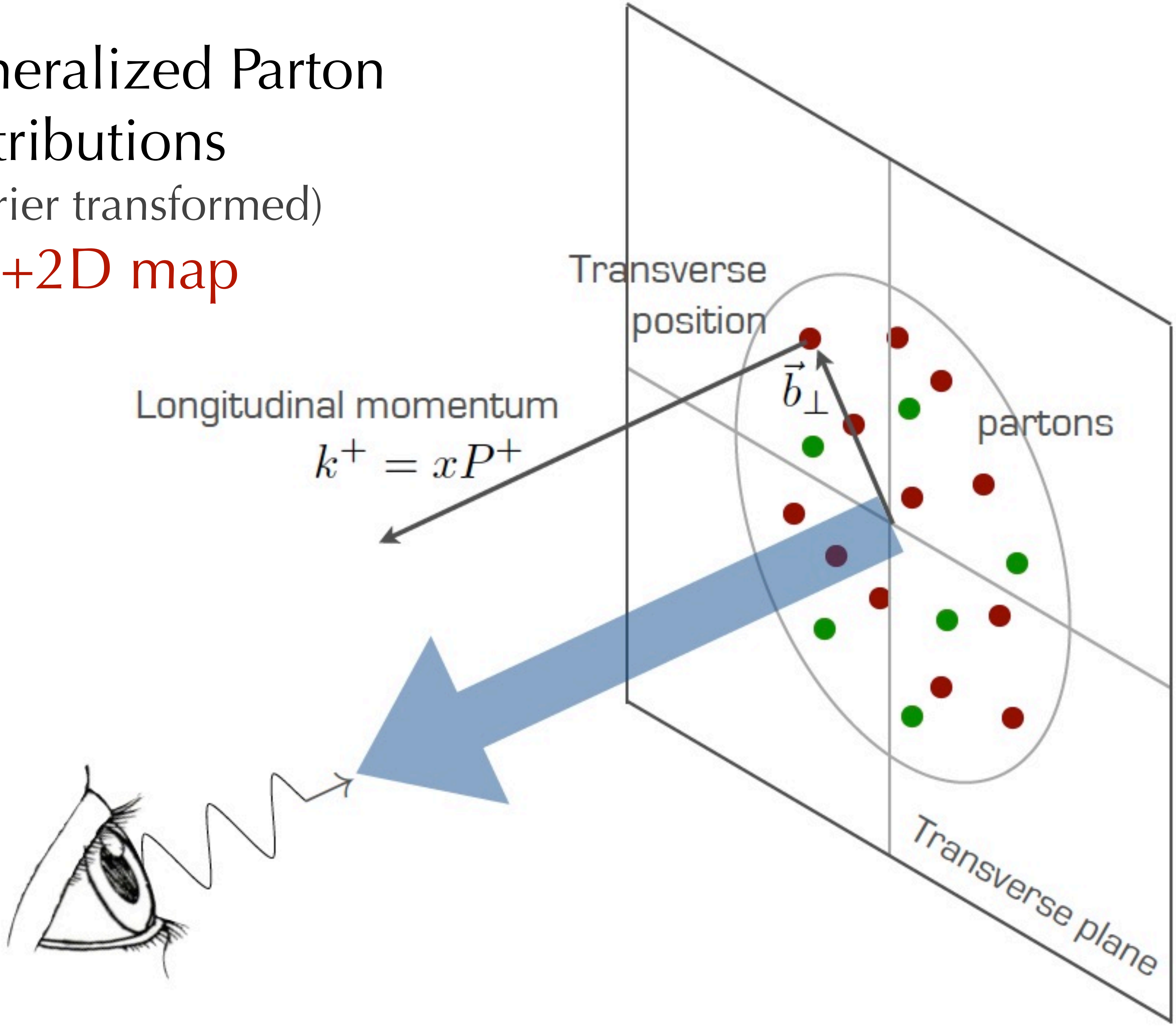
GPDs=GPDs (x, ξ, t)

- Transverse position size as function of x (2D+1D map)
- Form Factors of Energy Momentum Tensor ⟶ ``mechanical'' properties of the nucleon

Generalized Parton Distributions

(Fourier transformed)

1D+2D map



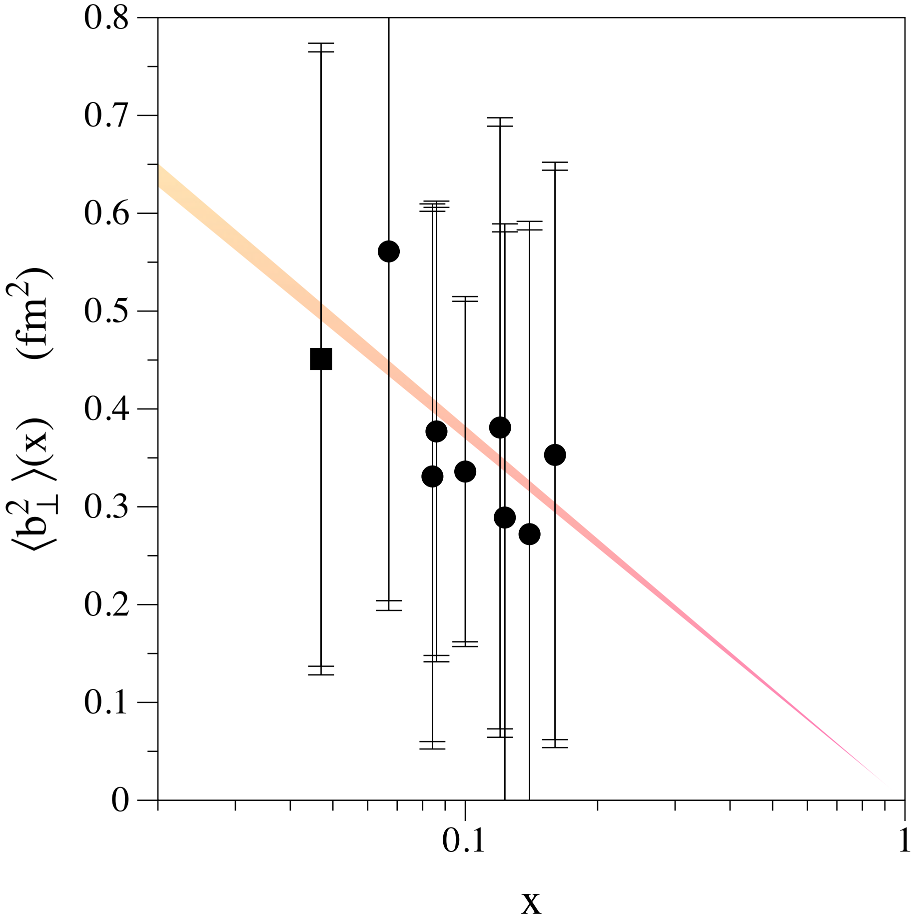
x-dependent transverse squared charge radius

$$H(x,0,\vec{b}_\perp) = \int_{-\infty}^{+\infty} \mathrm{d}^2\vec{\Delta}_\perp \, \underset{\downarrow}{H(x,0,t)} \, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \longrightarrow \langle \vec{b}_\perp^2(x) \rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp \, \vec{b}_\perp^2 \, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp H(x,0,b_\perp)}$$

$(t = -\vec{\Delta}_\perp^2)$ $\xi = 0$ extrapolation from data

x-dependent transverse squared radius

CLAS and HERMES data



The errors are large,
but slowly we are getting some 3D information

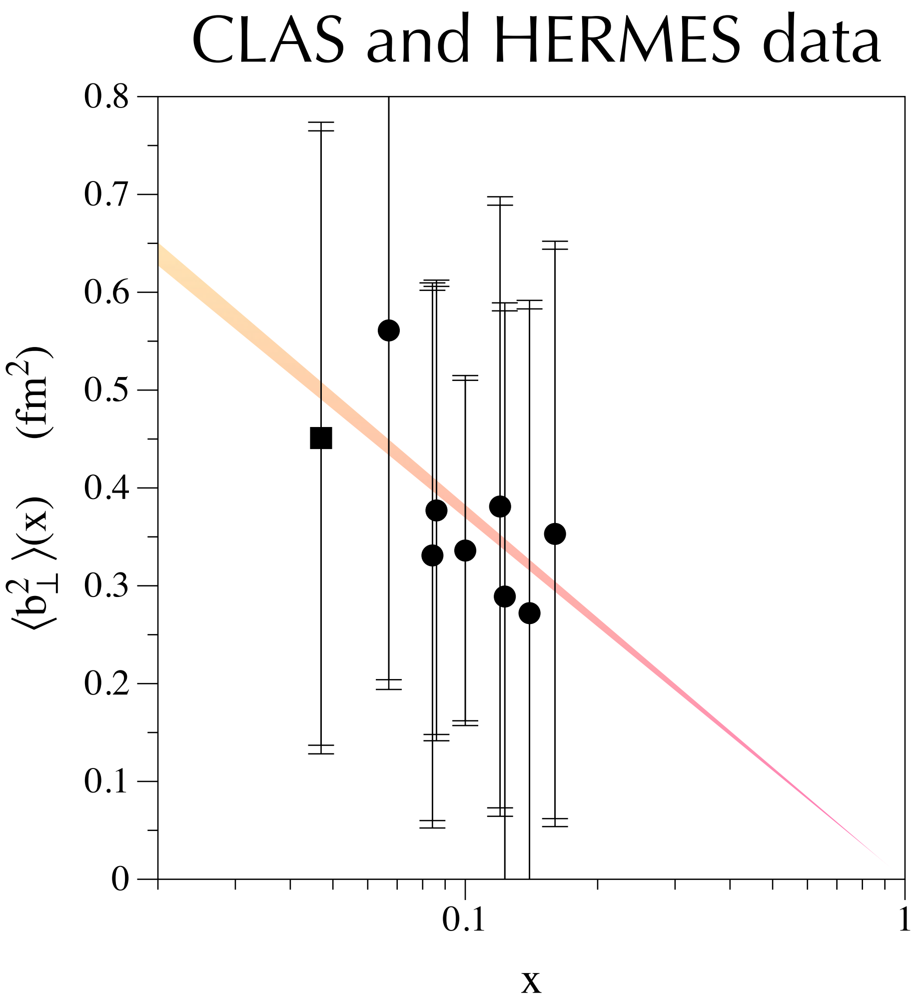
Dupré et al., PRD95(2017)011501

x-dependent transverse squared charge radius

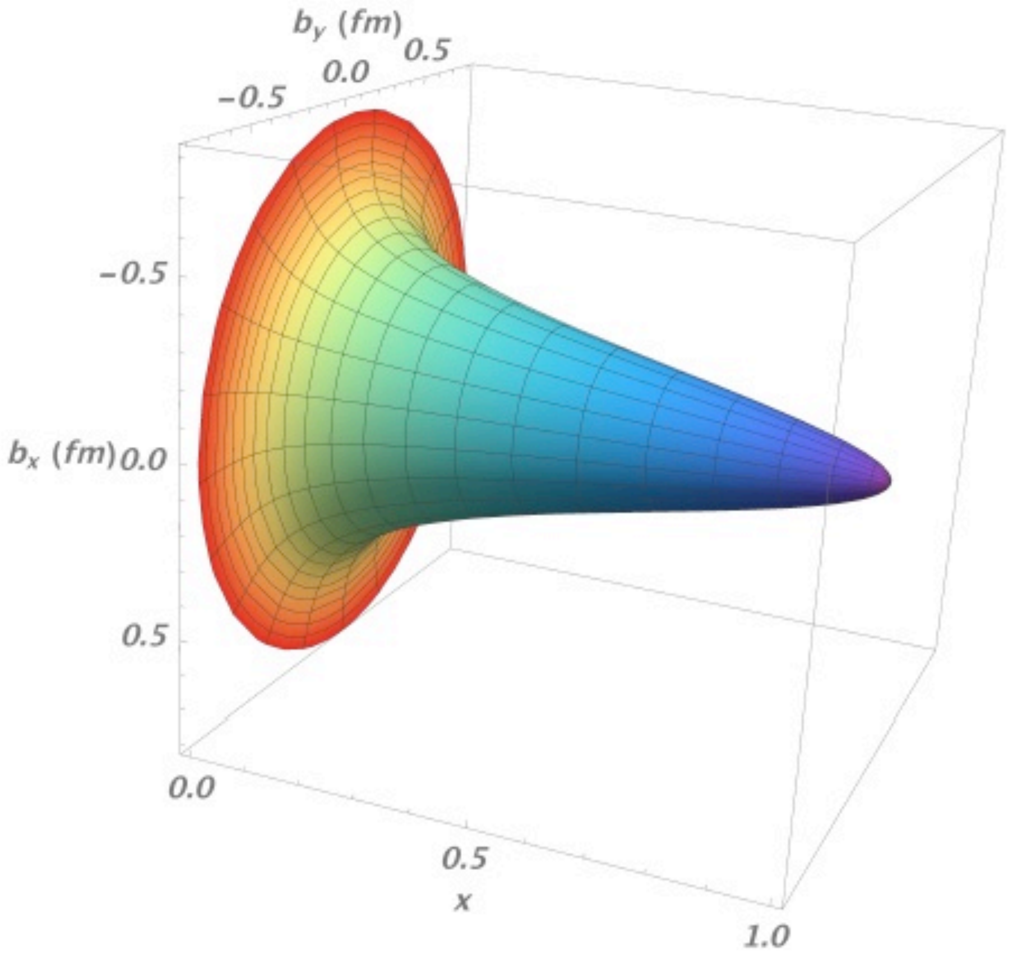
$$H(x,0,\vec{b}_\perp) = \int_{-\infty}^{+\infty} \mathrm{d}^2\vec{\Delta}_\perp \, \underset{\downarrow}{H(x,0,t)} \, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \longrightarrow \langle \vec{b}_\perp^{\,2}(x)\rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp \, \vec{b}_\perp^{\,2} \, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp H(x,0,b_\perp)}$$

$(t = -\vec{\Delta}_\perp^2)$ $\xi = 0$ extrapolation from data

x-dependent transverse squared radius



extrapolating
in the unmeasured
x-range

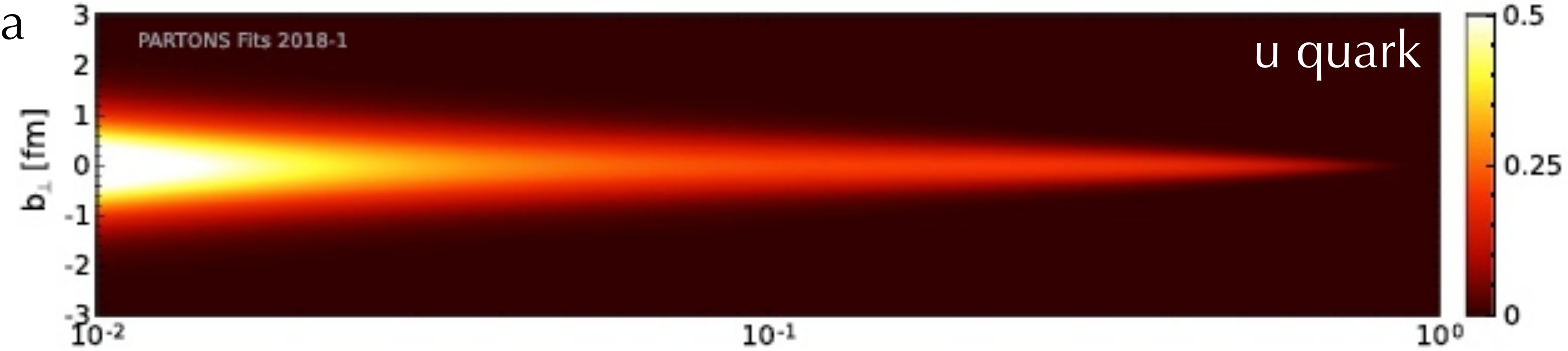


As x ⟶ 1, the active parton carries all the momentum
and represents the centre of momentum

Dupré et al., PRD95(2017)011501

New parametrization based on DRs: reduce problems related to the extrapolation to $\xi = 0$

CLAS and HERMES data

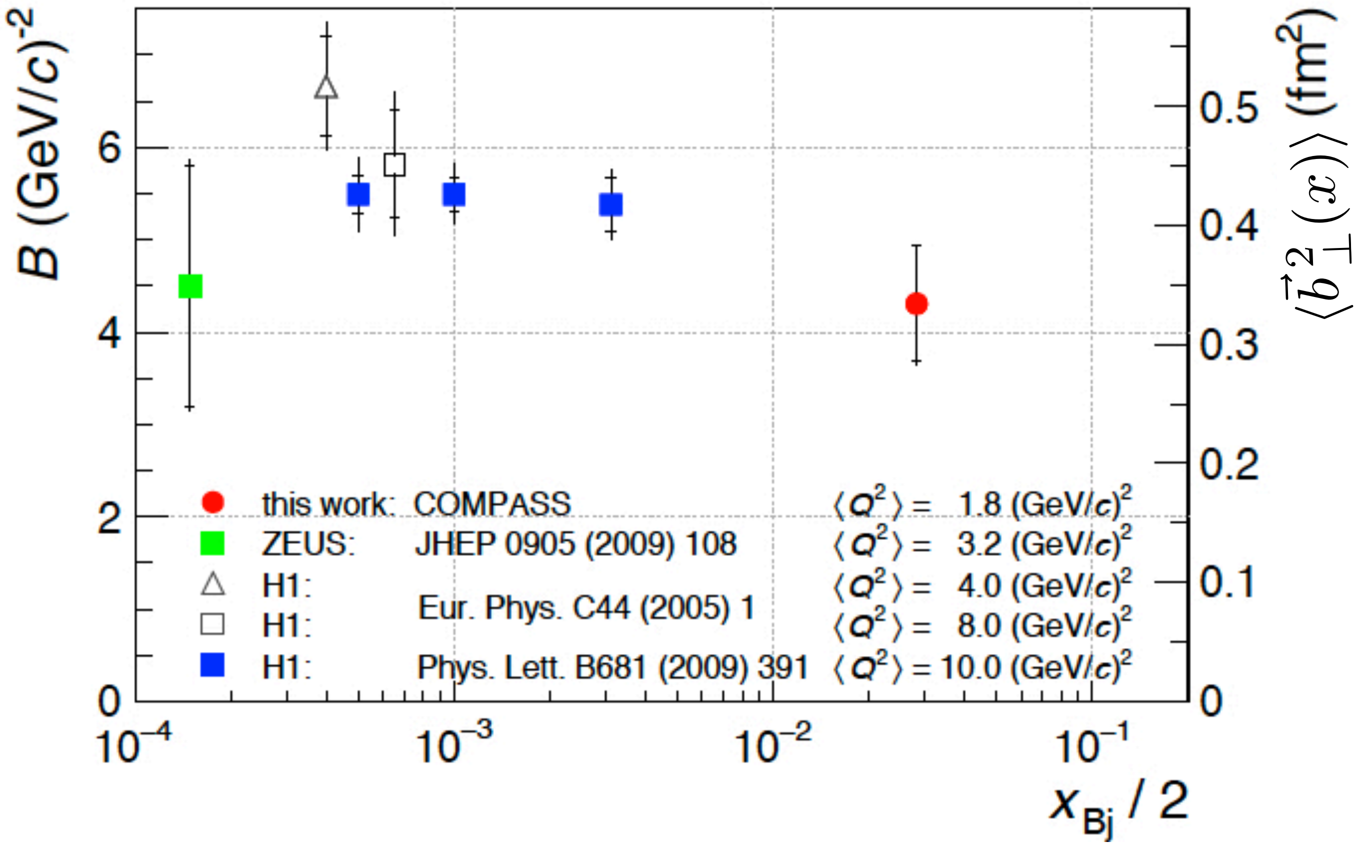


Moutarde et al., EPJC (2018)78

New results from COMPASS Coll.: arXiv:1802.02739

$$\frac{d\sigma}{dt} \approx e^{-B(x)|t|}$$

$$\langle \vec{b}_\perp^{\,2}(x) \rangle = 2\langle B(x) \rangle$$



Model dependence can not be avoided, but different fit methods and parametrizations can help to constraint the theoretical uncertainties

⟶ *Talk of D. Sokhan*

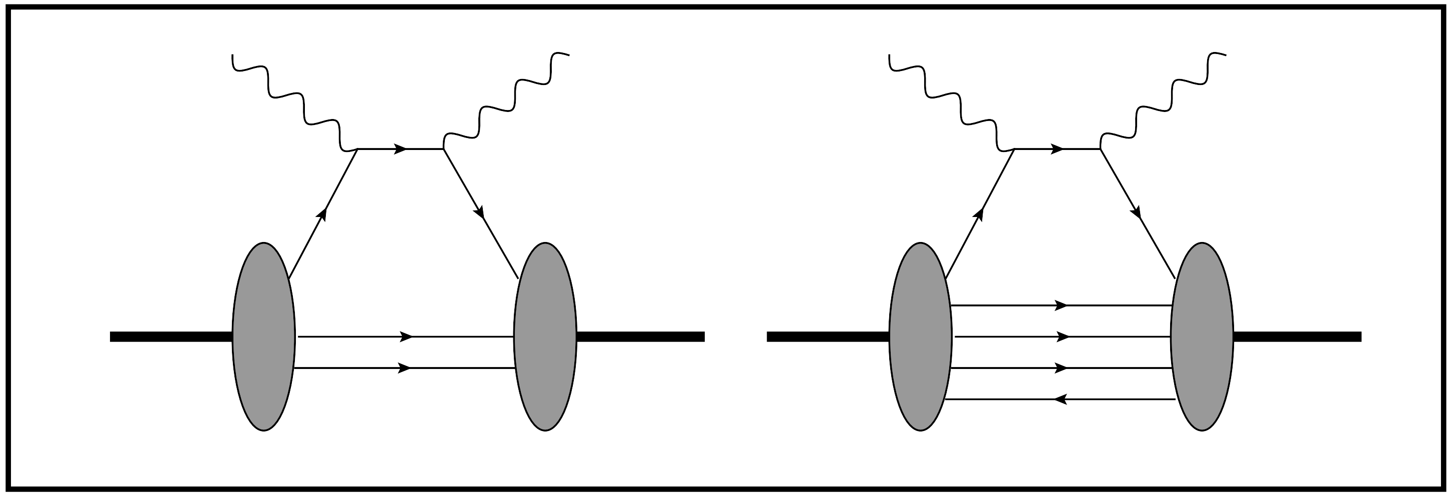
Probabilistic interpretation

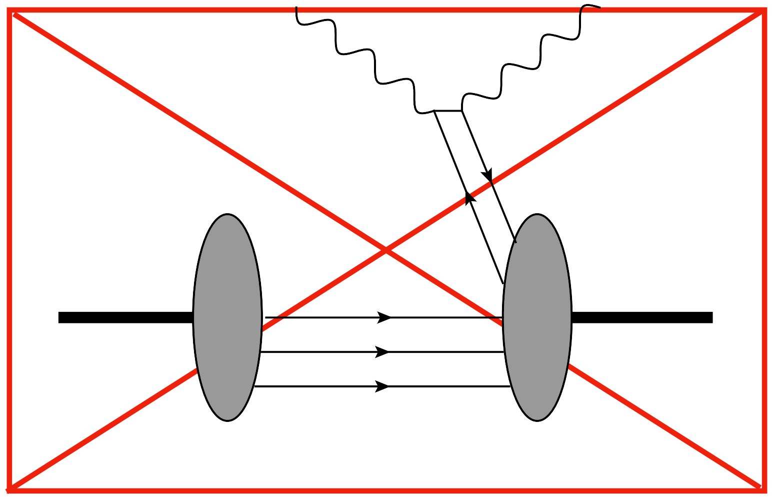
Drell-Yan frame: $\Delta^+ = 0 \quad \vec{\Delta}_\perp \neq 0$

✓ $\Delta^+ = 0 \longrightarrow$ no sensitivity to longitudinal Lorentz contraction

✓ $\vec{\Delta}_\perp \neq 0$: Transverse boosts $\longrightarrow$ no transverse Lorentz contraction

✓ Particle number is conserved in Drell-Yan frame $\Delta^+ = 0$





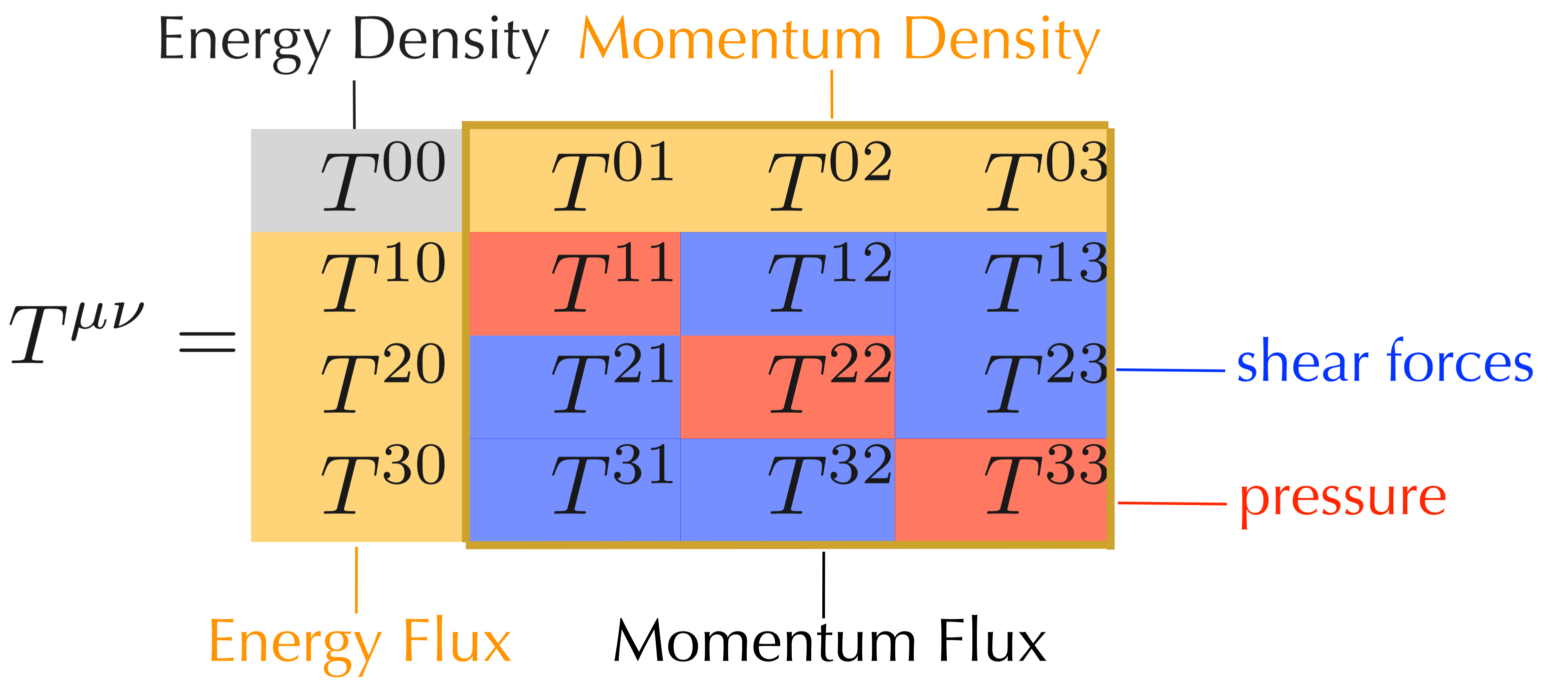
Relation with means square radius measured extracted from G_E

$$\langle b_\perp^2\rangle_{\rm NR} = \int d^2\, b_\perp^2\; b_\perp^2\; \rho_{\rm NR}(b) = -4G'_E(0) = \tfrac{2}{3}\langle r^2\rangle_{\rm NR}$$

$$\langle b_\perp^2\rangle_{\rm NR} = \langle b_\perp^2\rangle + \tfrac{\kappa_N}{4M_N^2} = \langle b_\perp^2\rangle + 0.02\ \ \mathrm{fm}^2$$

G.A. Miller, PRC99 (2019)

Form Factors of Energy Momentum Tensor



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{matrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{matrix}$$

shear forces

pressure

Energy Flux

Momentum Flux

$$\langle p| T^{Q,G}_{\mu\nu} |p'\rangle = \bar{u}(p') \left[M_2^{Q,G}(t) \frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu} \right] u(p)$$

Relation with second-moments of GPDs:

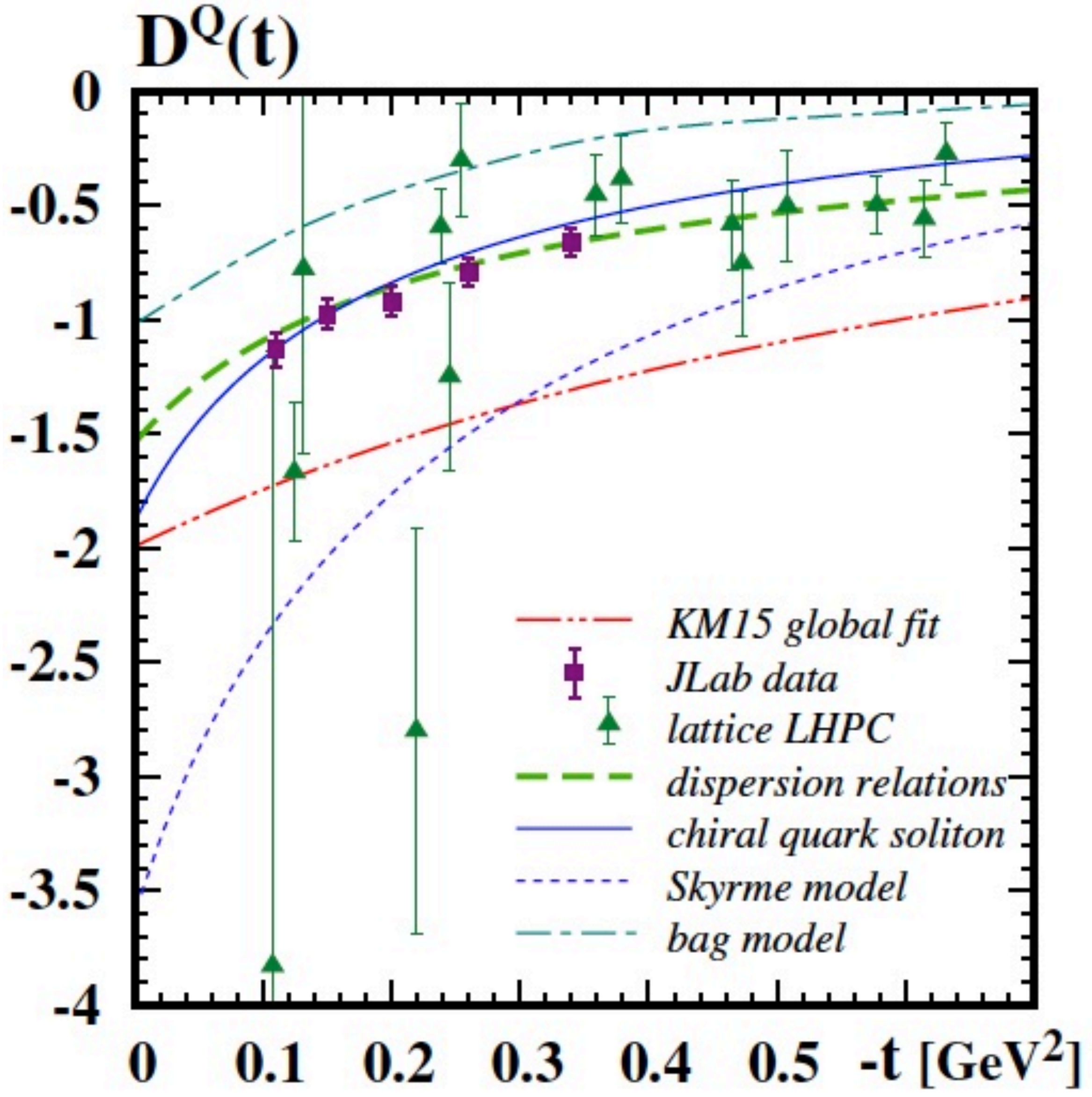
$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = M_2^Q(t) + \frac{4}{5} d_1^Q(t)\xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5} d_1^Q(t)\xi^2$$

"Charges" of the EMT Form Factors at t=0

$M_2(0)$	nucleon momentum carried by parton
$J(0)$	angular momentum of partons
$d_1(0)$	D-term ("stability" of the nucleon)

D-term form factor



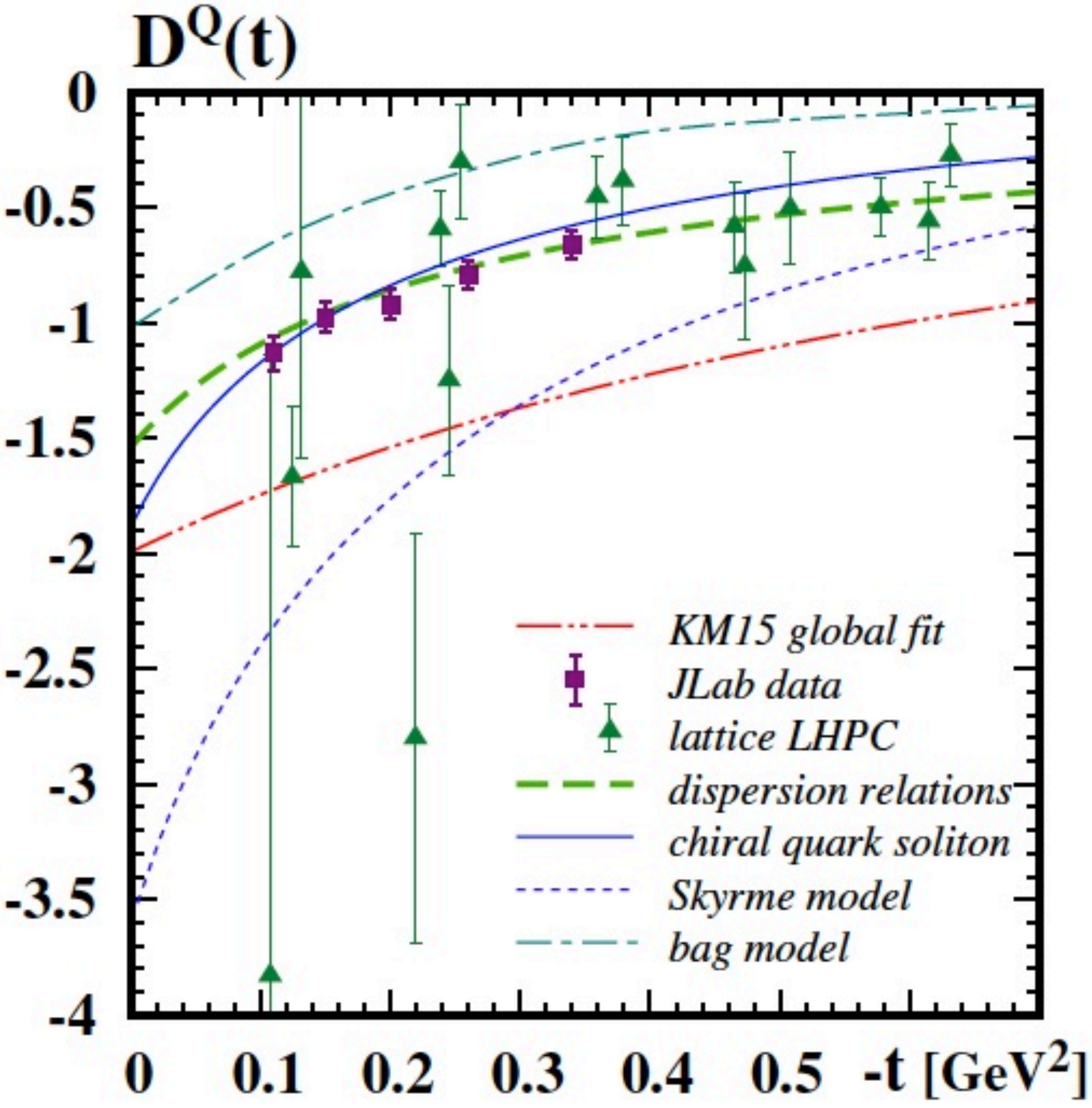
Polyakov and Schweitzer,
Int. J. Mod. Phys. A33 (2018) 1830025

Normal force distribution in the system:

$$\langle r^2\rangle_{\text{mech}} = \frac{\int d^3r\, r^2[\frac{2}{3}s(r)+p(r)]}{\int d^3r\,[\frac{2}{3}s(r)+p(r)]} = \frac{6\,D(0)}{\int_{-\infty}^{0} dt D(t)}$$

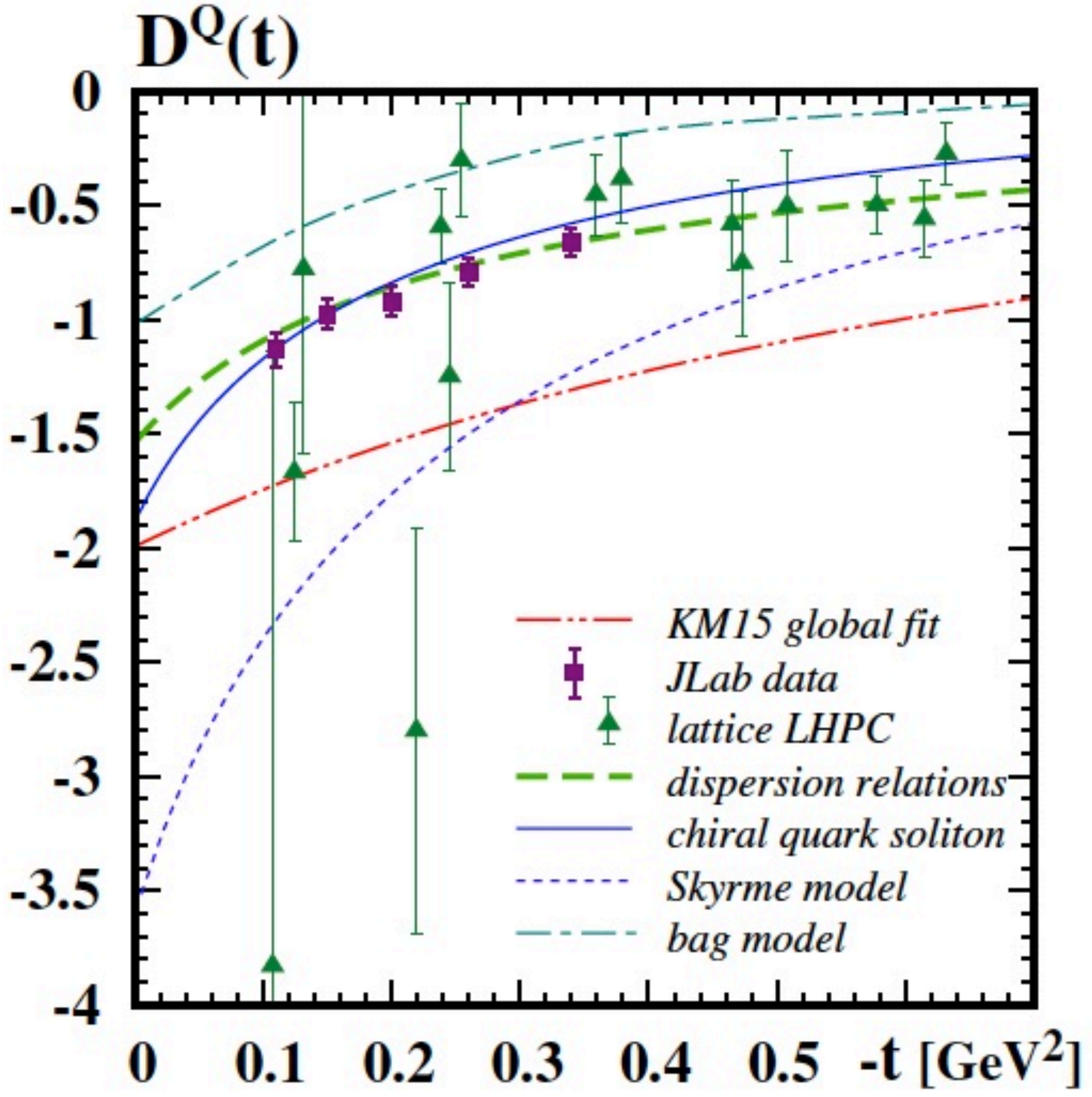
$\langle r^2\rangle_{\text{mech}} \approx 0.75\,\langle r^2\rangle_{\text{charge}}$ Chiral quark soliton model

D-term form factor



Polyakov and Schweitzer,
Int. J. Mod. Phys. A33 (2018) 1830025

D-term form factor

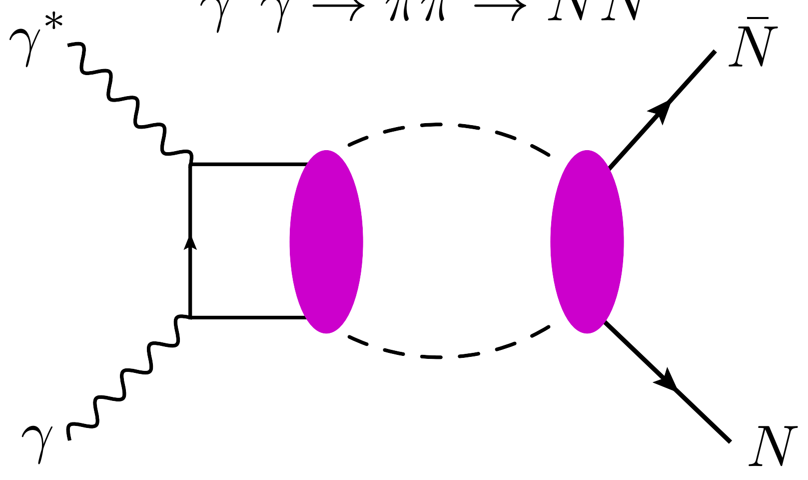


Polyakov and Schweitzer,
Int. J. Mod. Phys. A33 (2018) 1830025

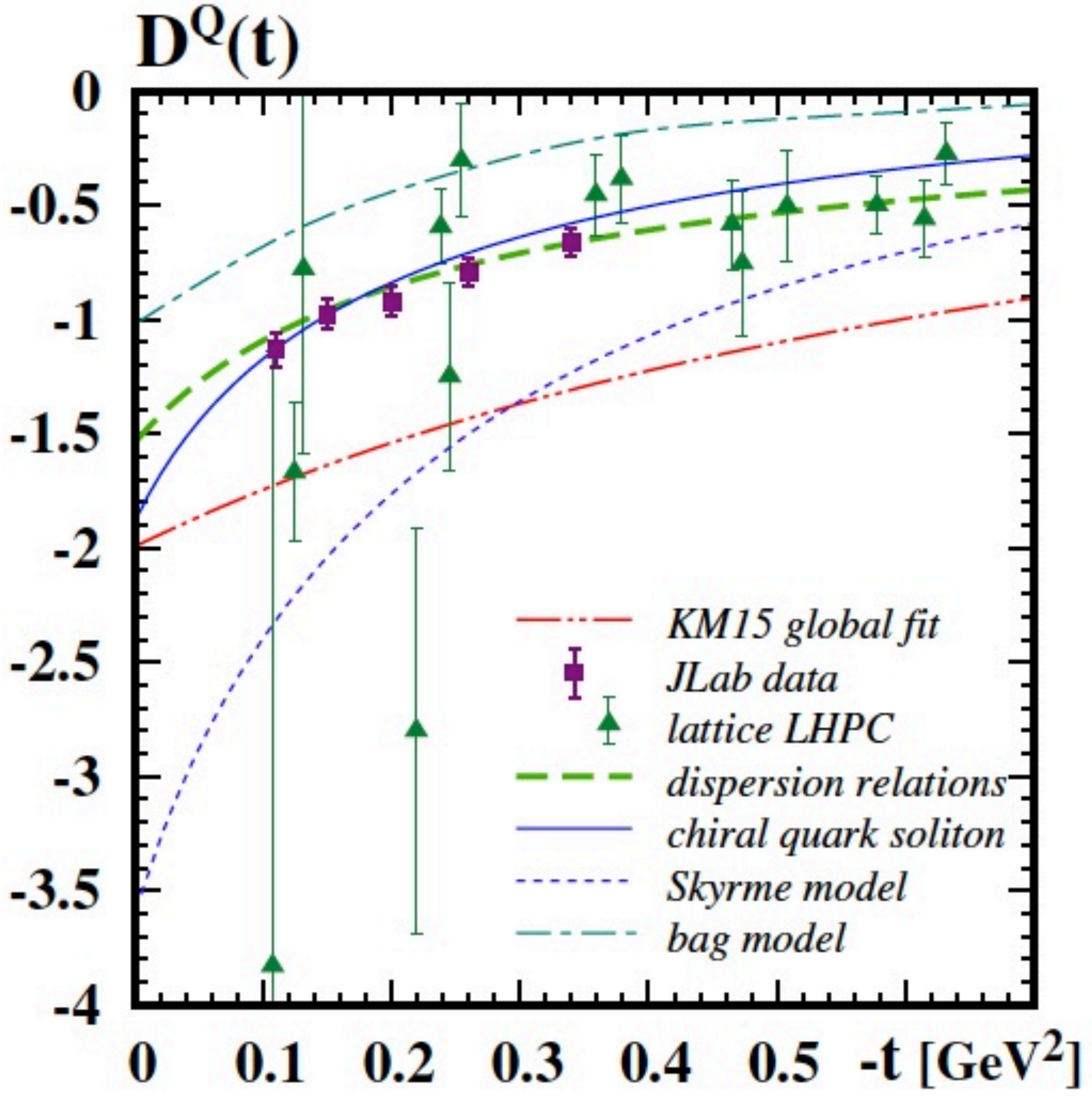
Dispersion Relations:
BP, Polyakov, Vanderhaeghen, PLB739(2014)133

D-term from t-channel dispersion relations

$\gamma^*\gamma \to \pi\pi \to N\bar{N}$



D-term form factor



Polyakov and Schweitzer,
Int. J. Mod. Phys. A33 (2018) 1830025

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BP, Polyakov, Vanderhaeghen, PLB739(2014)133

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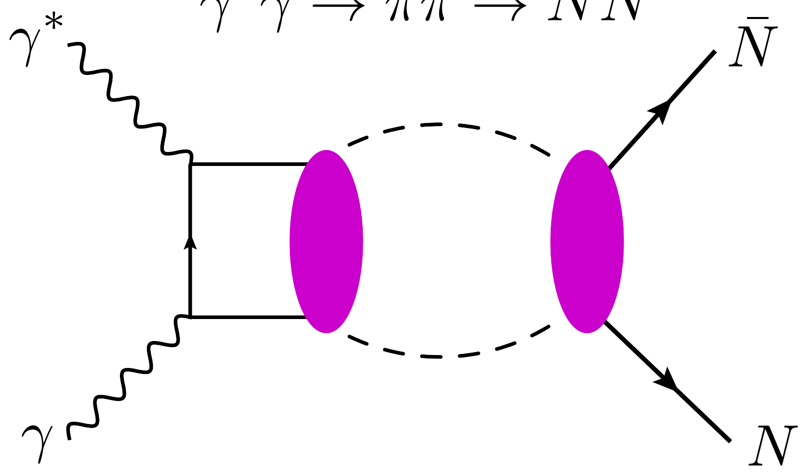
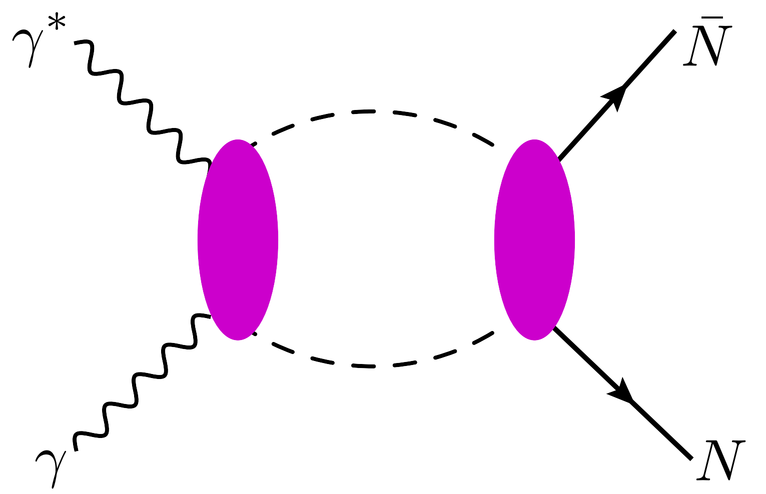


the same two-pion correlated state enters

the diamagnetic contribution to $\beta_{\mathrm{M1}}(Q^2)$ from DRs

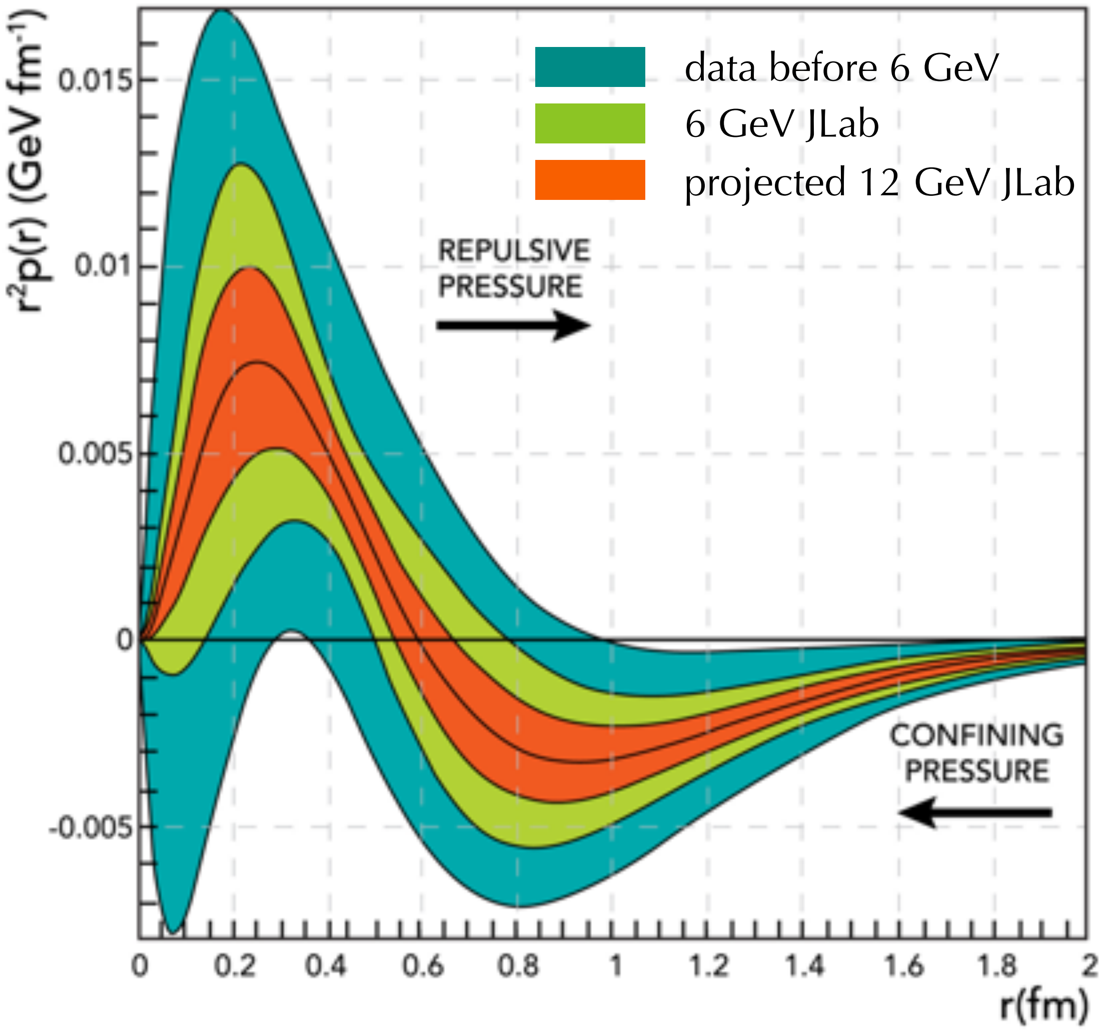


Efforts to develop unified framework connecting low and high Q² regimes

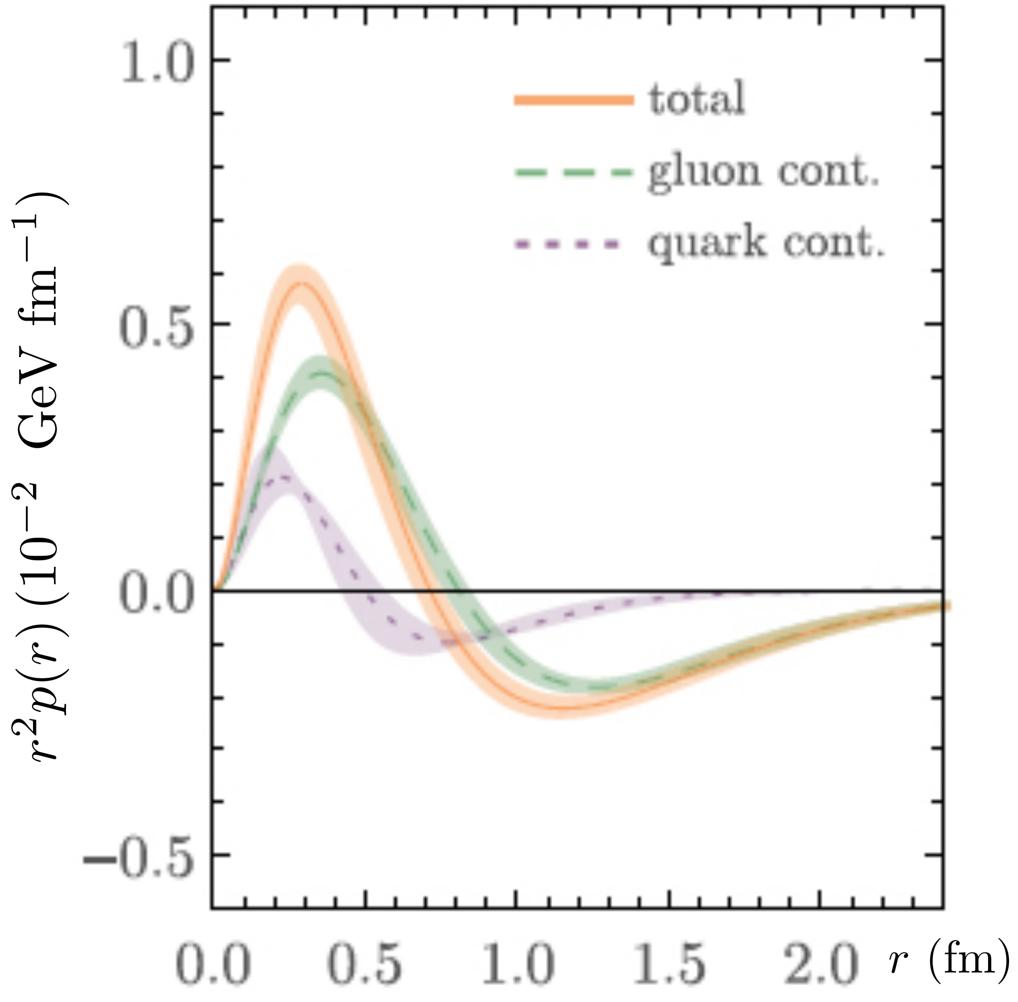
Belitsky, Mueller, Yao Ji, NPB878(2014)214; Eichmann, Fischer, PRD87 (2013)

Radial pressure distribution

$$\tilde{D}(r) \overset{\text{FT}}{\longleftrightarrow} D(t) \qquad r^2 p(r) = \frac{1}{3}\frac{d}{dr} r^2 \frac{d}{dr}\tilde{D}(r)$$



Girod, Elouadrhiri,Burkert, Nature 557 (2018) 7705

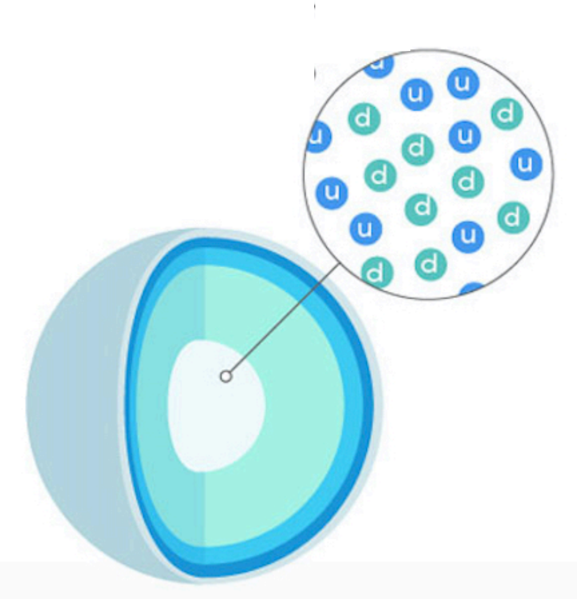


Shanahan, Detmold, PRL122 (2019) 072003

Necessary to verify model assumptions in the exp extraction with more data coming from JLab, COMPASS and the future EIC

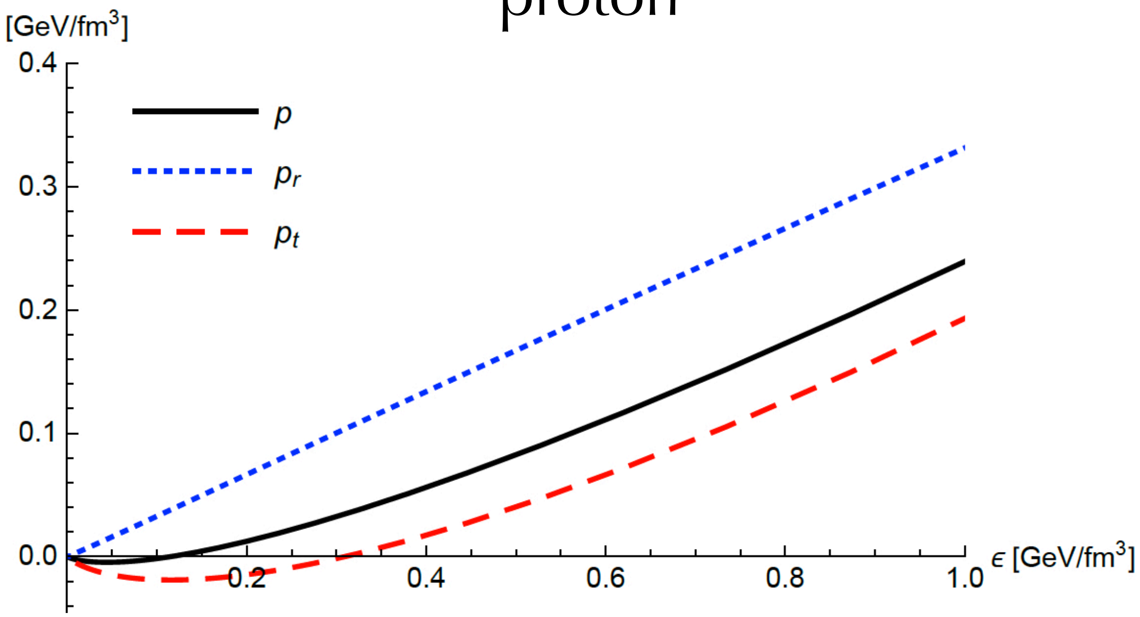
Kumericki, Nature 570 (2019) 7759

⟶ *Talks of Elouadrhiri, Shanahan, Trawinski*



The knowledge of pressure in hadronic matter can in principle allows us to make predictions on the behaviour of neutron stars

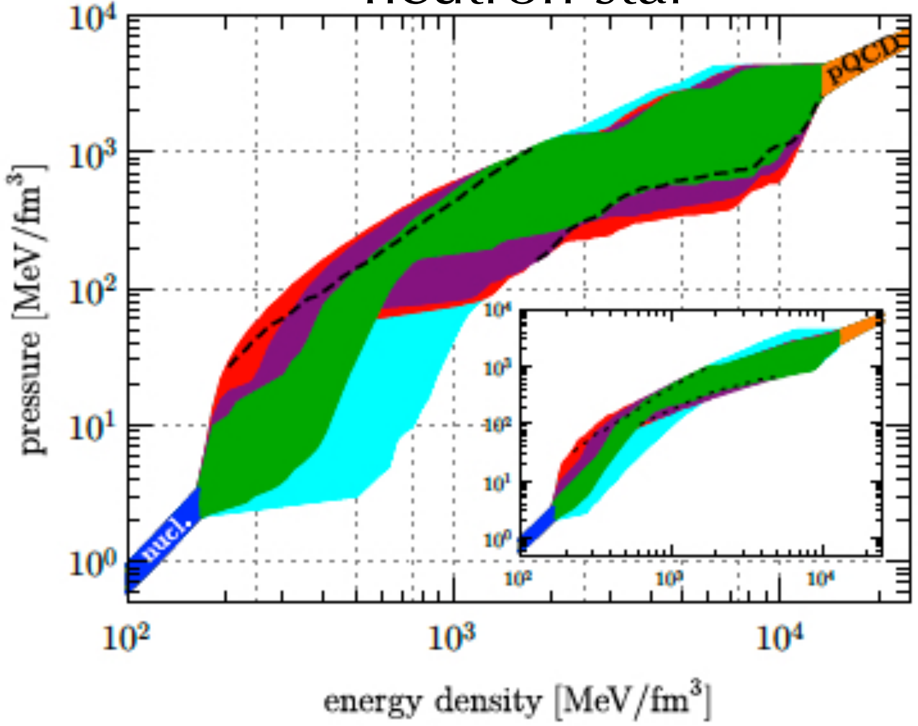
proton



Lorcè, Moutarde,Trawinski, EPJ C79 (2019) 89

Rajan, Liuti, Yagi, arXiv:1812.01479

neutron star



Annala et al., PRL120 (2018) 172703

Exciting results but need more solid underpinnings!

⟶ *Talk of J. Van den Brand*

Angular Momentum Relation (Ji's Sum Rule)

X. Ji, PRL 78 (1997) 610

quark and gluon contribution to the nucleon spin

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x\, \big(\underset{\downarrow}{H^{q,g}(x,\xi,0)} + \underset{\downarrow}{E^{q,g}(x,\xi,0)}\big)$$

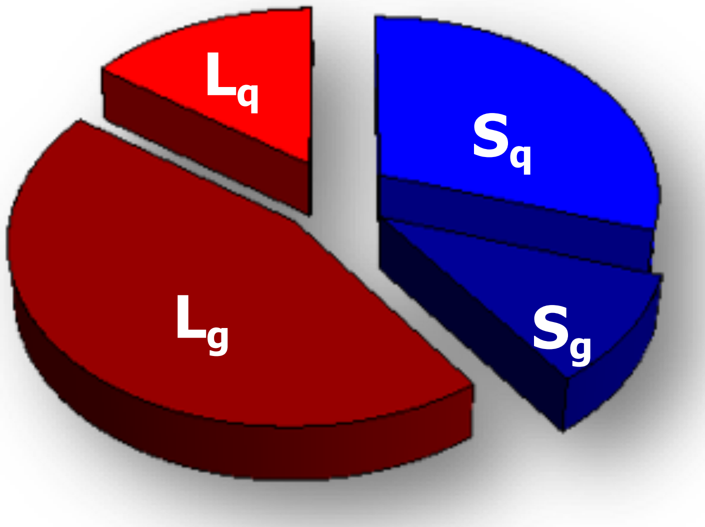
at $\xi = 0$ unpolarized PDF

not directly accessible

$\frac{1}{2}\Delta\Sigma$ from DIS

$\uparrow$

$J^q = L^q + S^q$



$\frac{1}{2}\Delta g$ from DIS

$\uparrow$

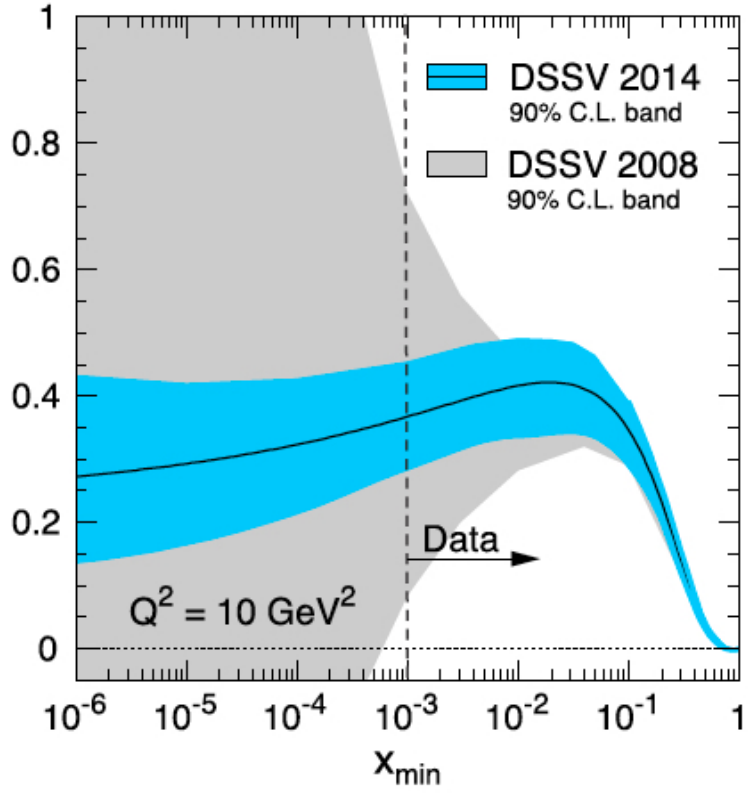
$J^g = L^g + S^g$

- Requires extrapolation at t=0
- Requires spanning x at fixed values of ξ ($\xi = 0$ is the most convenient)
- Does not have an interpretation as angular momentum density as a function of x

Spin contributions to proton angular momentum from data

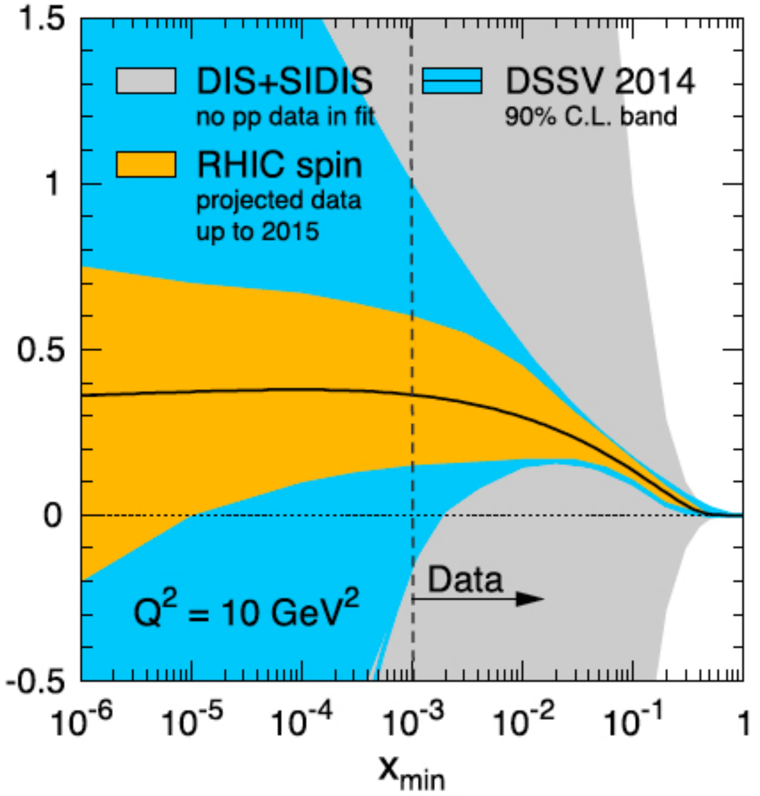
Quark spin

$$\int_{x_{min}}^{1} dx\, \Delta\Sigma(x, Q^2)$$



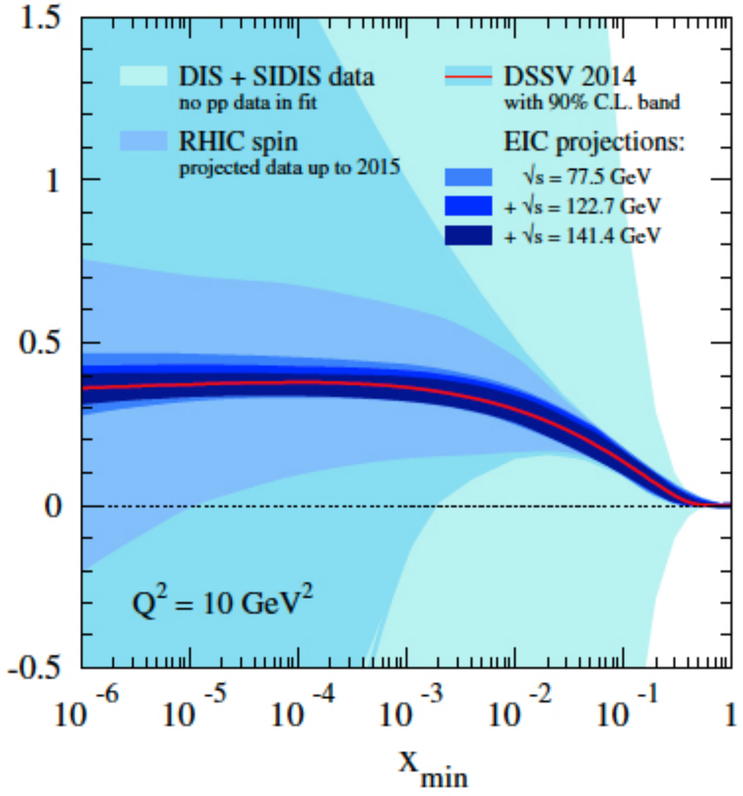
Gluon spin

$$\int_{x_{min}}^{1} dx\, \Delta g(x, Q^2)$$



Gluon spin with EIC

$$\int_{x_{min}}^{1} dx\, \Delta g(x, Q^2)$$



Aschenauer, Sassot and Stratmann, PRD92 (2015) 094030; Aschenauer et al. Rep.Prog.Phys. 82 (2019) 024301

We are constantly improving the knowledge of the contributions to the spin of the nucleon

However the details on the flavor and sea contributions are still sketchy

What about a direct measurement of orbital angular momentum?

⟶ *Talk of R. Fatemi*

Orbital angular momentum of the proton from GPDs

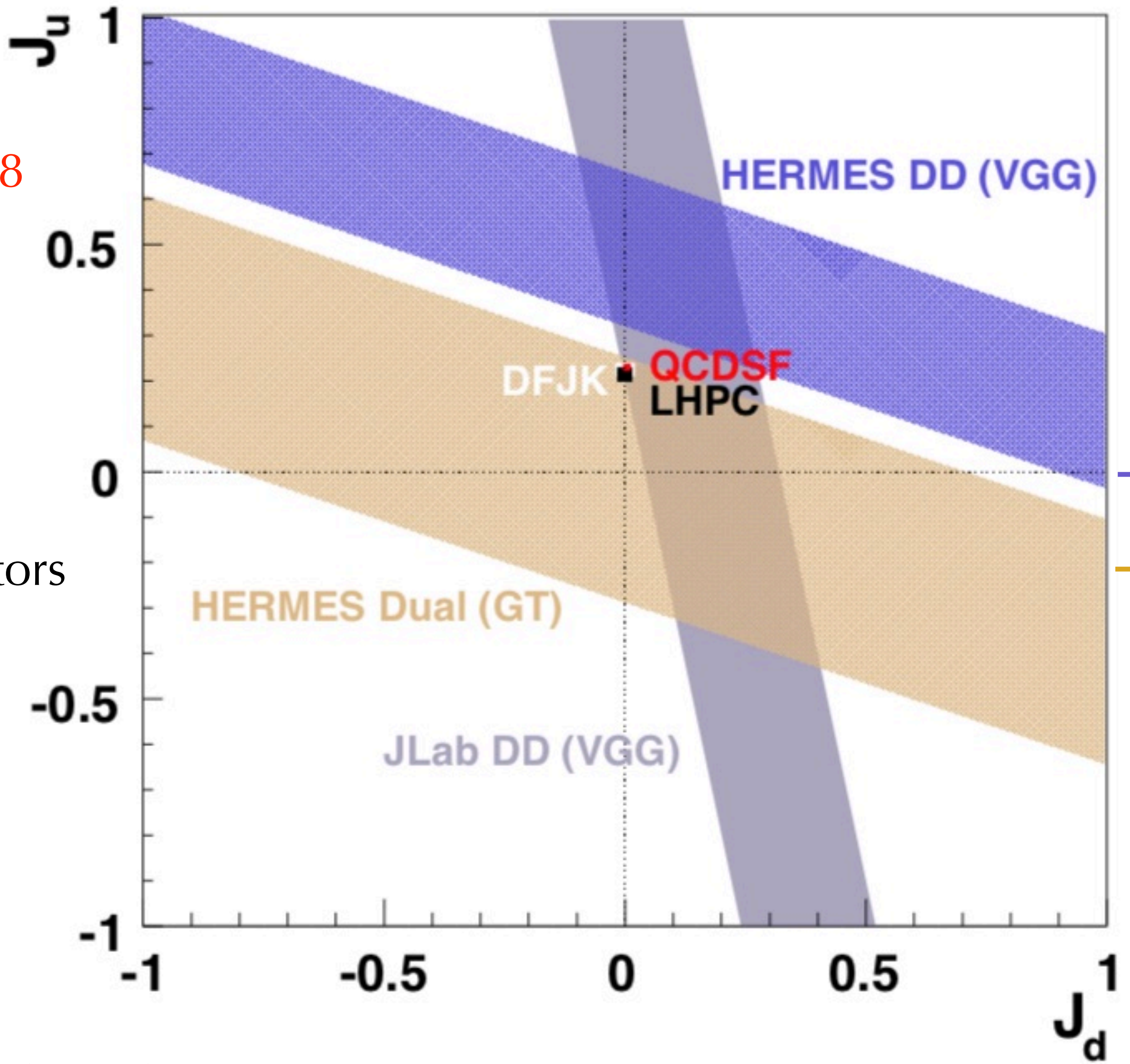
Lattice results

QCDSF: PoS (Lattice 2007) 158

LHPC: PRD77 (2008) 094502

GPDs extracted from form factors

DFJK, EPJC39 (2005) 1



extractions from HERMES data using two different models

JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

Hermes Coll., JHEP 06 (2008) 066

Problem of model dependent extractions

Orbital angular momentum of the proton from GPDs

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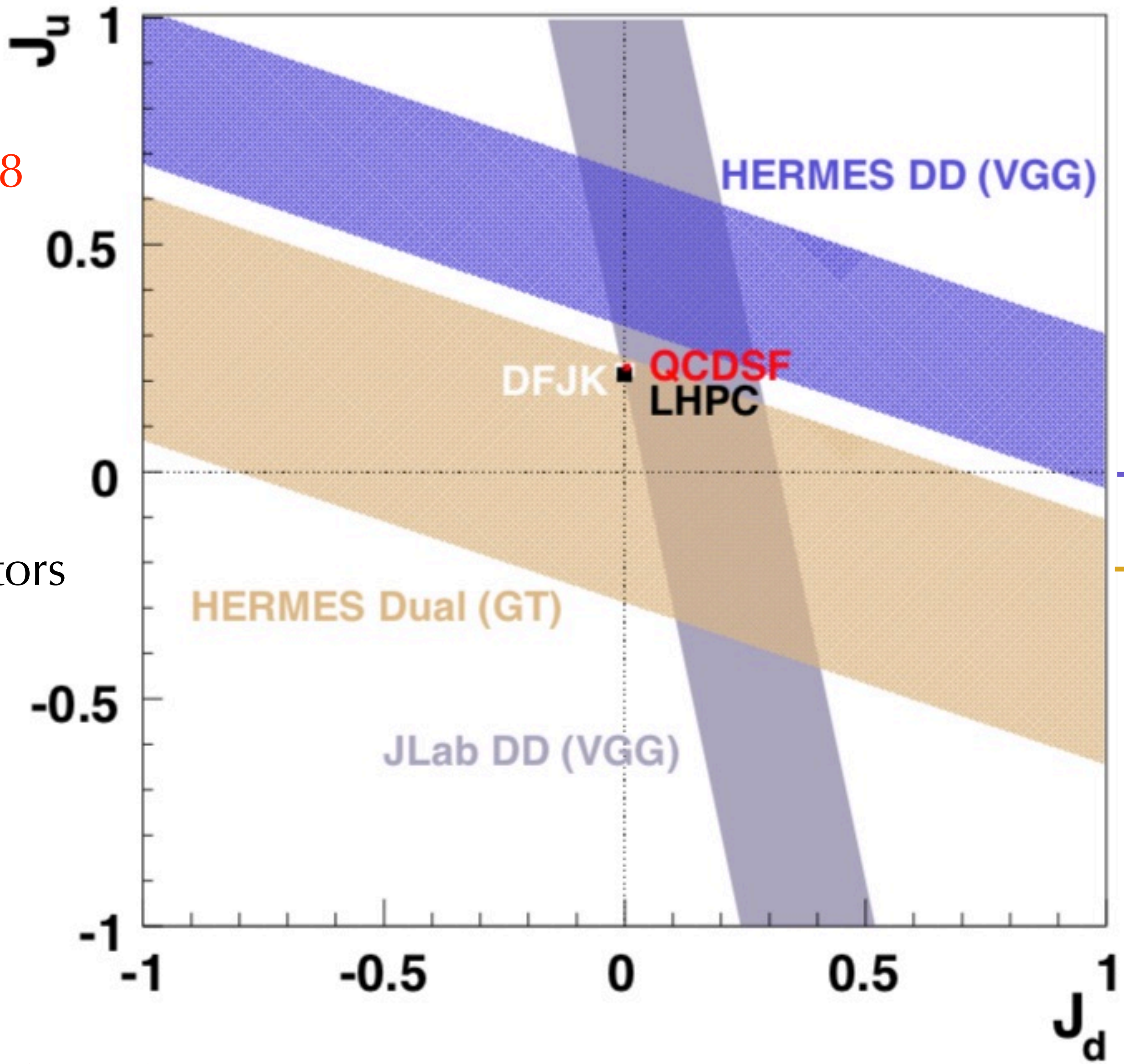
DFJK, EPJC39 (2005) 1



extractions from HERMES data using two different models

JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

Hermes Coll., JHEP 06 (2008) 066

Problem of model dependent extractions

Twist-3 GPDs?

$$L^q = -\int_{-1}^{1} \mathrm{d}x\, x\, G_2^q(x, \xi = 0, t = 0)$$

Very challenging! We can not address the individual twist-3 GPDs *[Aslan et al., PRD 98 (2018) 014038]*

Recent formalism: Kriesten et al., arXiv:1903.05742

Orbital angular momentum of the proton from Wigner functions

$$L_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp\mathrm{d}^2\vec{b}_\perp(\vec{b}_\perp \times \vec{k}_\perp)\mathcal{W}_{LU}^q(x,\vec{b}_\perp,\vec{k}_\perp)$$

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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relation to GTMD: $L_z^q = -\int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{M^2} F_{1,4}^q(x, \vec{k}_\perp^{\,2})\big|_{\Delta=0}$

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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- intuitive definition of OAM
- mutually orthogonal components of quark position and momentum

  ⟶ no conflict with uncertainty principle
- the integrand L_z^q represents the OAM density
- same equation for both Jaffe-Manohar (staple-like link) and Ji (straight link) OAM
- equation holds also for gluon OAM
- it can be calculated in LQCD *Engelhardt, PRD95 (2017) 094505*

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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$$L_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp) \rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\, \vec{k}_\perp\, \rho_{LU}^q(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorcé, BP, PRD 84 (2011) 014015

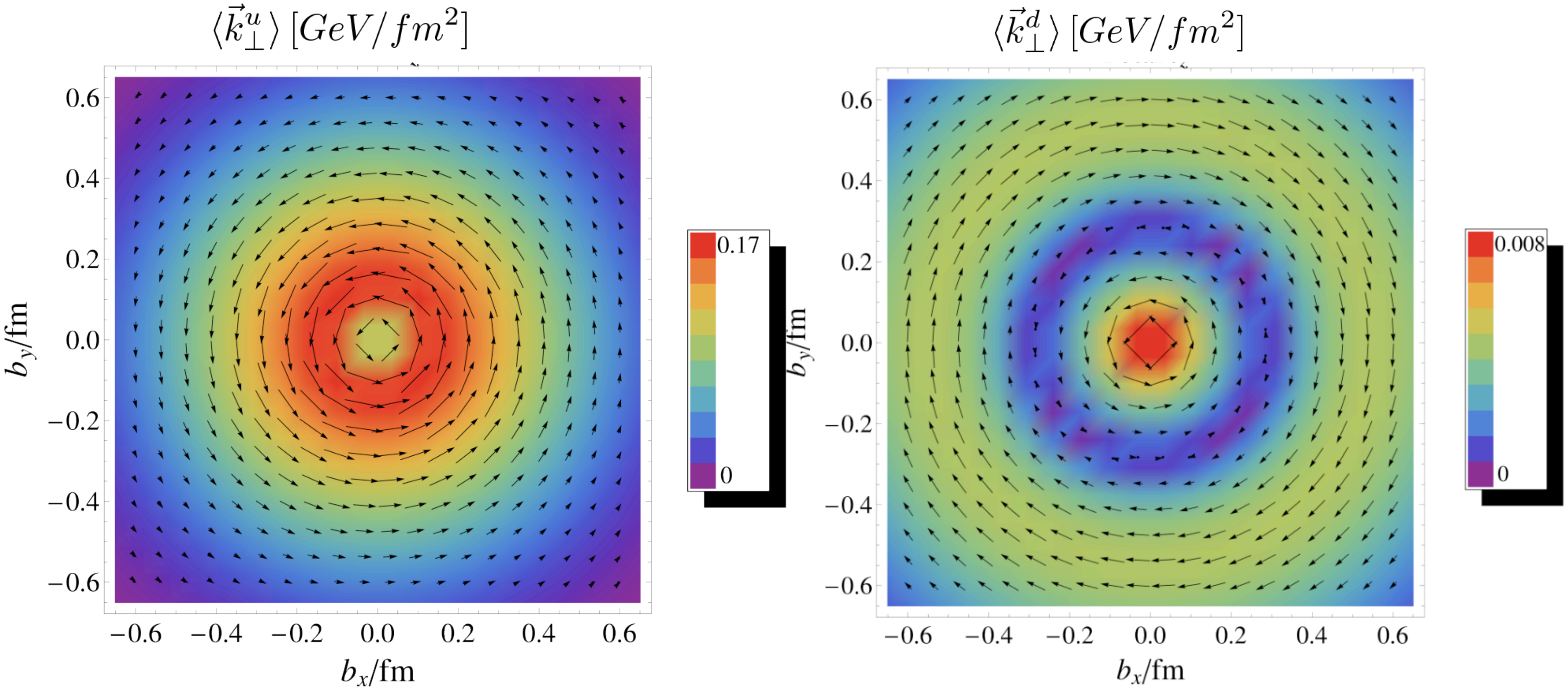
Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Orbital angular momentum of the proton from Wigner functions

$$L_z^q = \int \mathrm{d}x\,\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}^2\vec{b}_\perp\,(\vec{b}_\perp \times \vec{k}_\perp)\,\mathcal{W}^q_{LU}(x,\vec{b}_\perp,\vec{k}_\perp)$$

$$L_z^q = \int \mathrm{d}^2\vec{b}_\perp\,\vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp)\rangle = \int \mathrm{d}x\,\mathrm{d}\vec{k}_\perp\,\vec{k}_\perp\,\rho^q_{LU}(\vec{b}_\perp,\vec{k}_\perp,x)$$



Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Orbital angular momentum of the proton from Wigner functions

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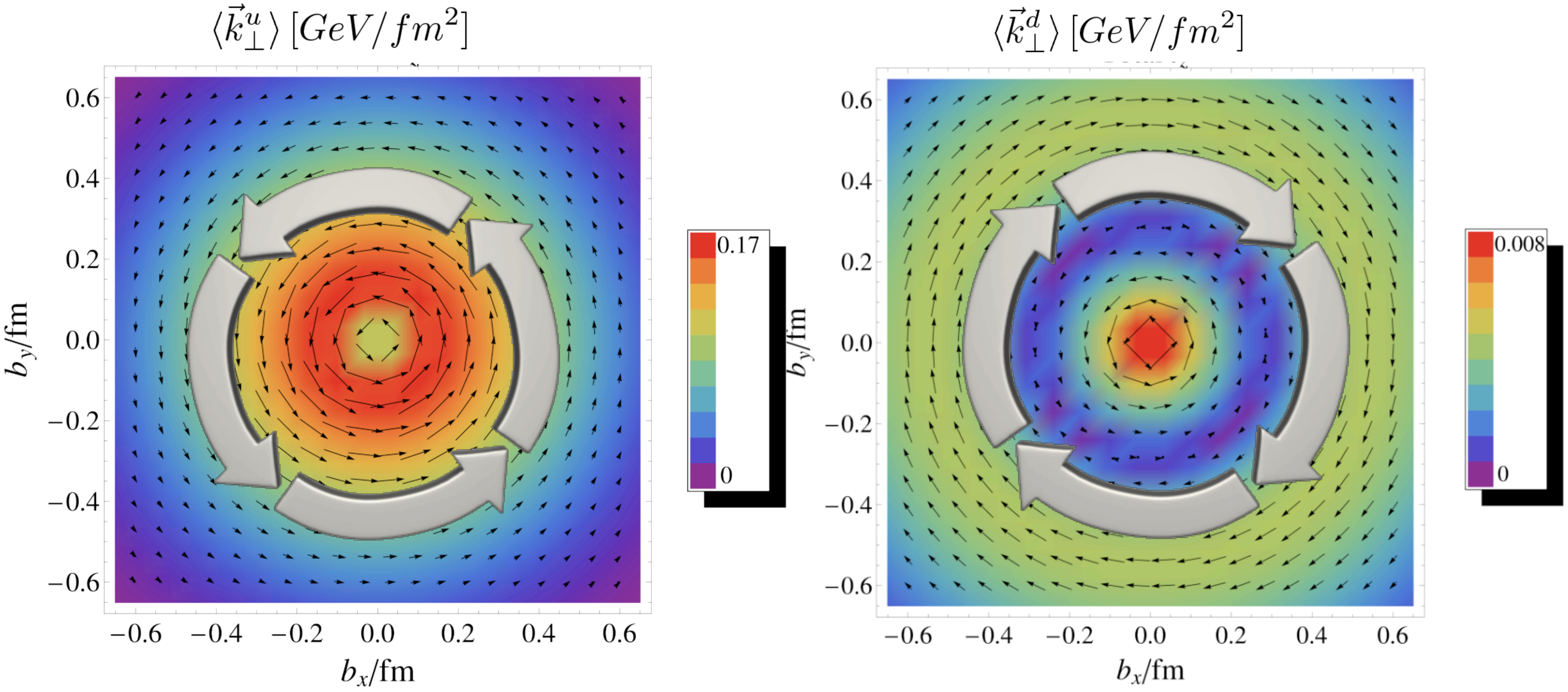

$\langle \vec{k}_\perp^u \rangle \, [GeV/fm^2]$

$\langle \vec{k}_\perp^d \rangle \, [GeV/fm^2]$

→ Proton spin

→ u-quark OAM

← d-quark OAM

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Angular correlations

quark polarization

nucleon polarization

$\mathcal{W}_X$	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

$\boxed{\xi = 0}$

$\int d^2\vec{k}_\perp$

$\int d^2\vec{b}_\perp$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T, \ \tilde{H}_T$

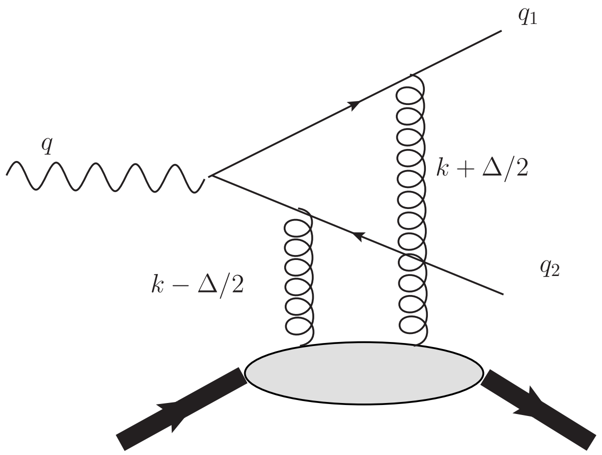
TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, \ h_{1T}^\perp$

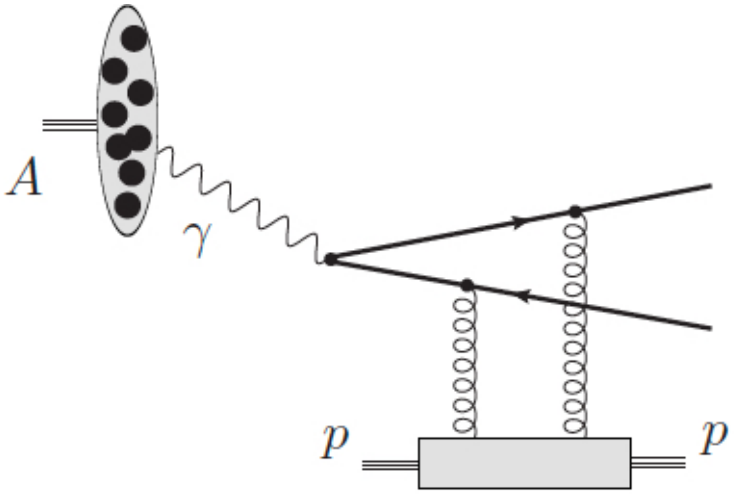
each distribution contains unique information

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

GTMDs from observables

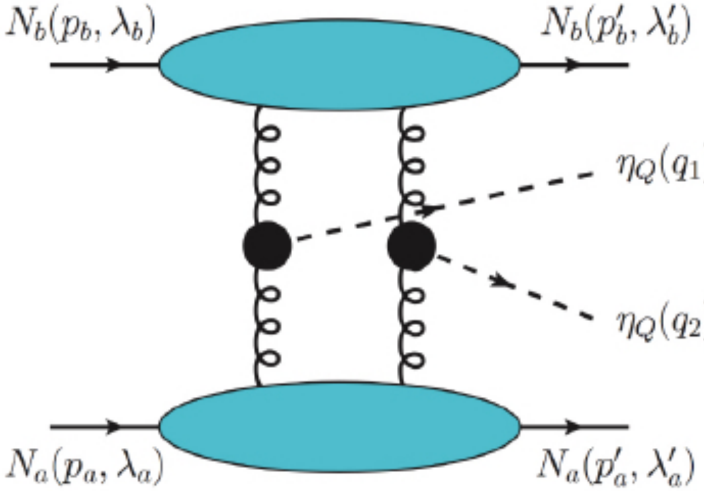




Exclusive dijet production in ep DIS and in pa UPC (gluon GTMDs)

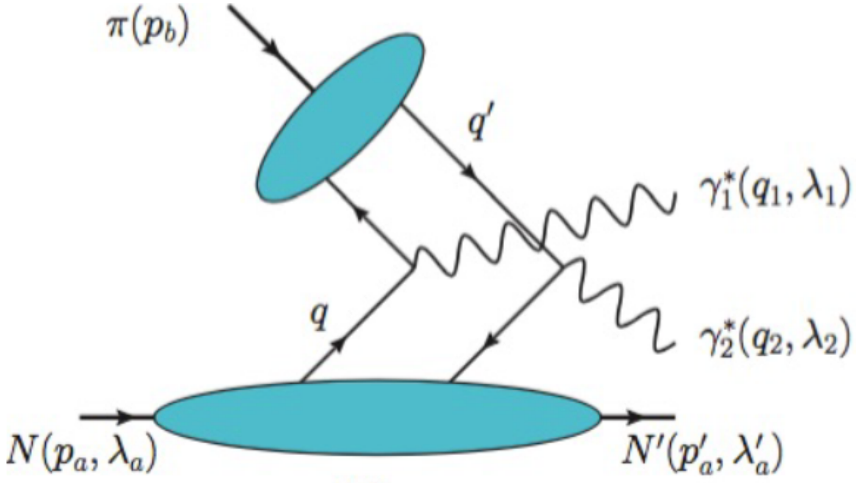
Hatta, Xiao, Yuan, PRL 116 (2016) 202301
Hatta, Nakagawa, Xiao, Yuan, Zhao, PRD 95 (2017) 114032

Ji, Yuan, Zhao, PRL 118 (2017) 192004
Hagiwara, et al., PRD 96 (2016) 034009



Exclusive double quarkonia production
in nucleon-nucleon collisions (gluon GTMDs)

Bhattacharya, Metz, Ojha, Tsai, Zhou, arXiv:1802.10550
Boussarie, Hatta, Xiao, Yuan, arXiv: 1807.08697



Exclusive pion-nucleon double Drell-Yan
(quark GTMDs)

Bhattacharya, Metz, Zhou, PLB 771 (2017) 396

Key information from Transverse Momentum Dependent PDFs

- Complete momentum spectrum of single particle
- Transverse momentum size as function of x (3D map)
- Spin-Spin and Spin-Orbit Correlations of partons
- Information on parton orbital angular momentum
  (no direct model-independent relation)
- Extractions from SIDIS require knowledge of Fragmentation Functions
- Test what we can calculate with QCD (perturbative and lattice)

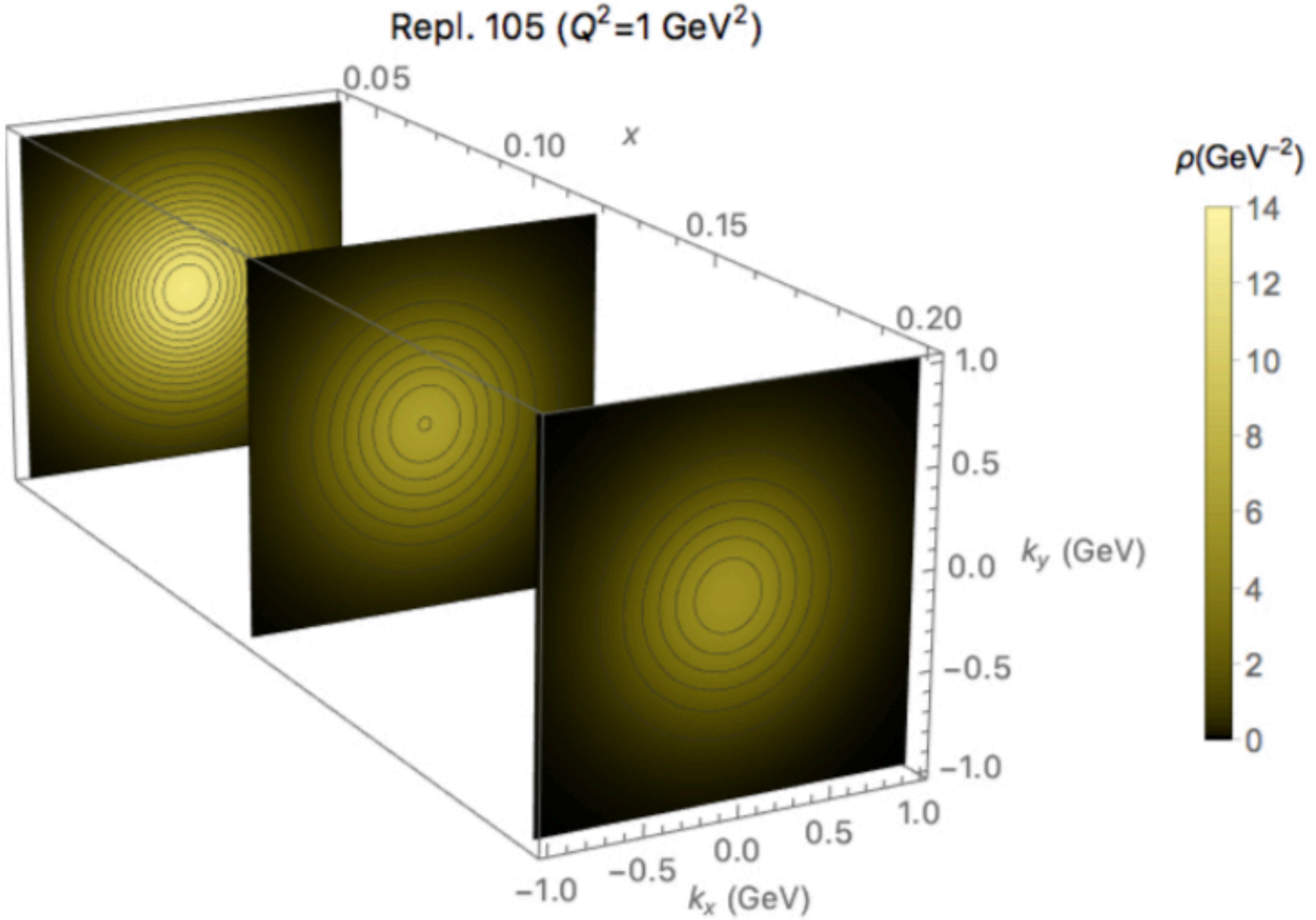
Talks of A. Prokudin, M. Chiosso, M. Engelhardt

Quark unpolarized TMD extractions

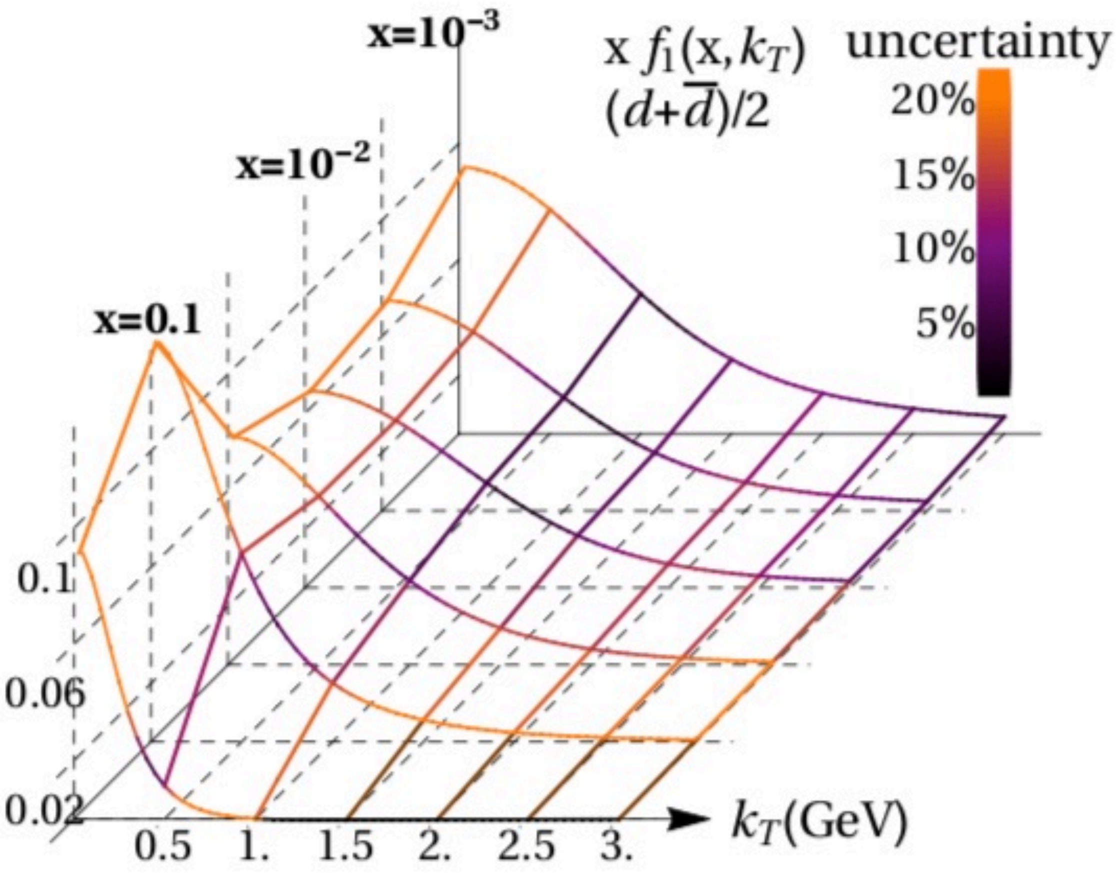
	Framework	HERMES	COMPASS	DY	Z Production	N of points
KN 2006 hep-ph/0506225	NLL/NLO	✗	✗	✓	✓	98
Pavia 2013 arXiv:1309.3507	No evo	✓	✗	✗	✗	1538
Torino 2014 arXiv:1312.6261	No evo	✓ (separately)	✓ (separately)	✗	✗	576(H) 6284(C)
DEMS 2014 arXiv:1407.3311	NNLL/NLO	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	NLL/LO	1(x,Q^2)bin	1(x,Q^2)bin	✓	✓	500
Pavia 2016 arXiv:1703.10157	NLL/LO	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLL/NNLO	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLL/NNLO	✗	✗	✓	✓	457

Quark unpolarized TMD extractions

$$f_1(x, \vec{k}_\perp)$$



Bacchetta, Delcarro, Pisano, Radici, Signori,
JHEP 1706 (2017) 081

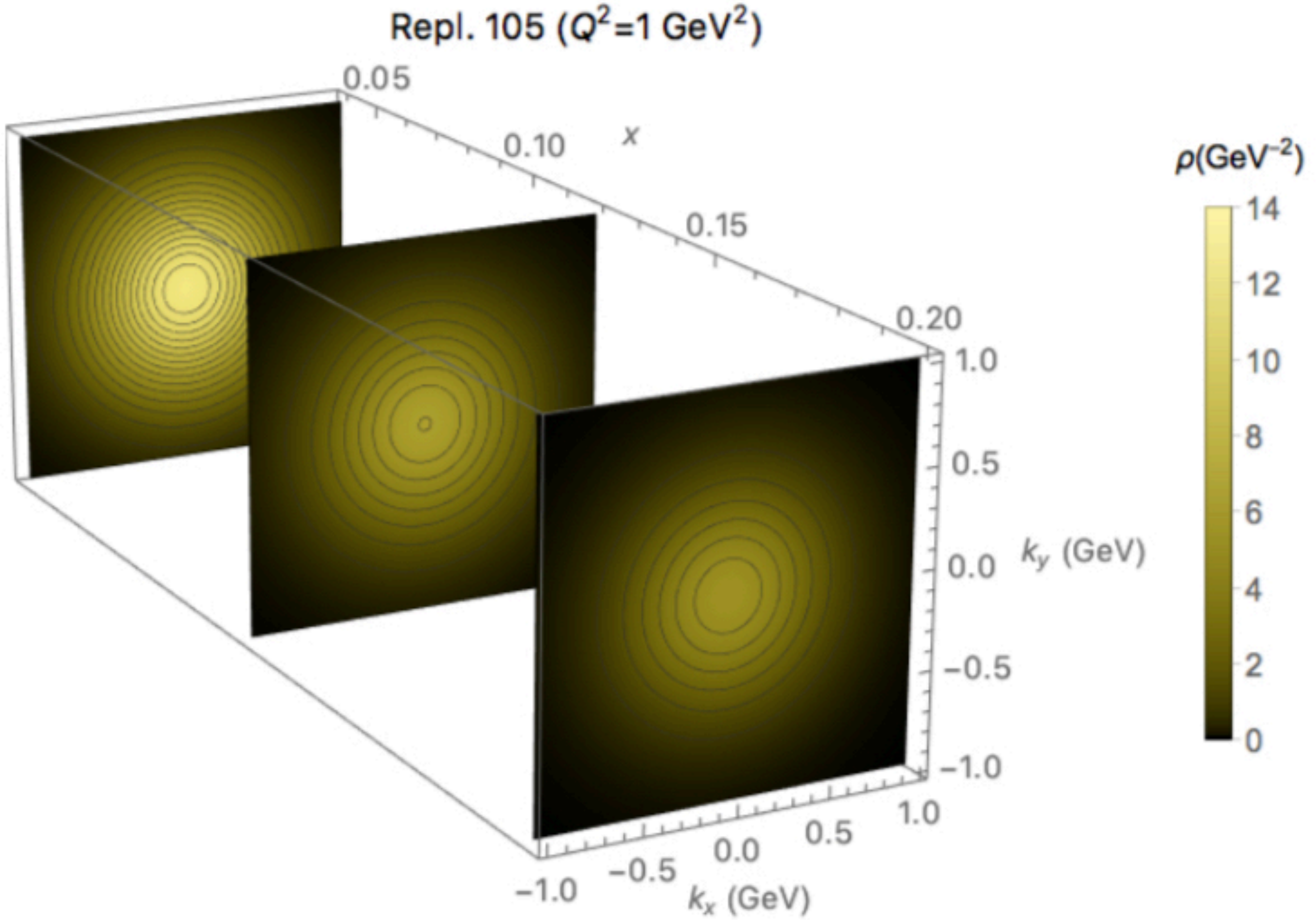


Bertone, Scimemi, Vladimirov,
JHEP 1906 (2019) 28

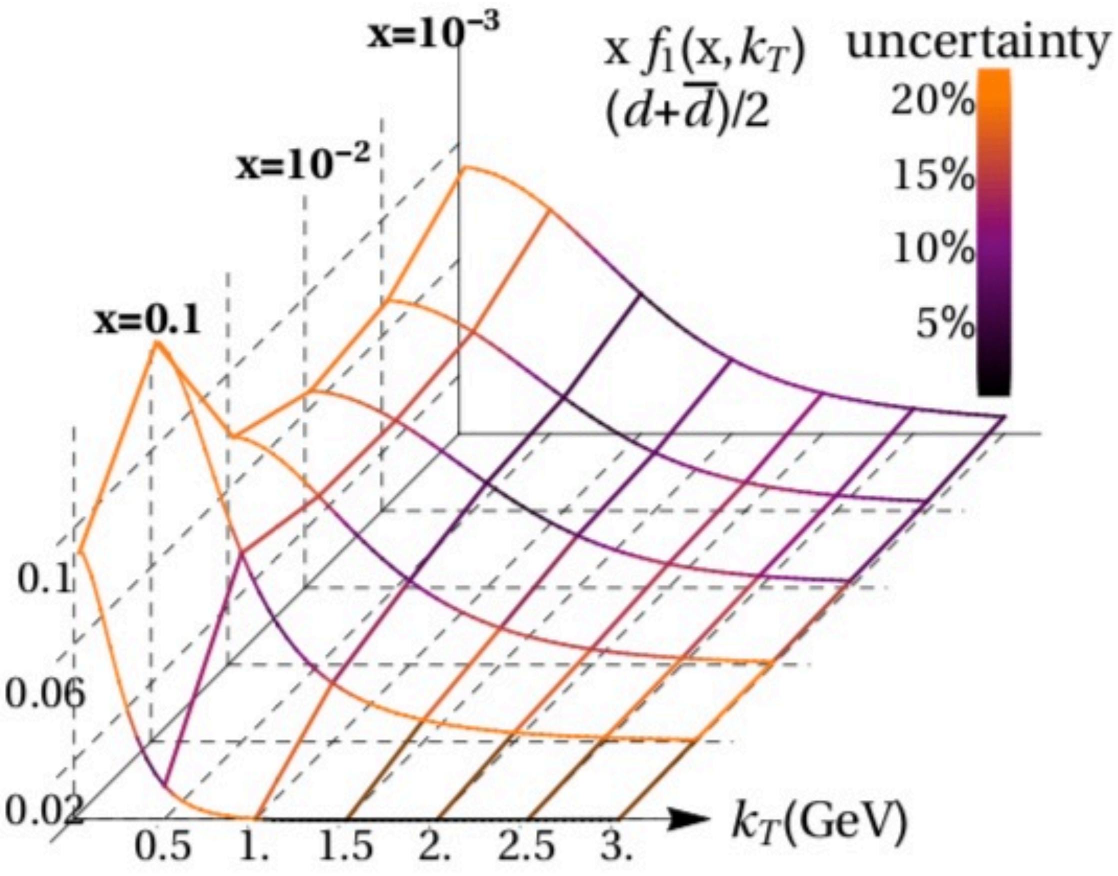
- Density in transverse-momentum space for unpolarized quark in unpolarized nucleon
  ⟶ monopole distribution, wider at smaller x_B
  ⟶ reconstructed from measured data

Quark unpolarized TMD extractions

$$f_1(x, \vec{k}_\perp)$$



Bacchetta, Delcarro, Pisano, Radici, Signori,
JHEP 1706 (2017) 081



Bertone, Scimemi, Vladimirov,
JHEP 1906 (2019) 28

Open issues:

- ☐ Flavor dependence and more flexible functional forms
- ☐ Different choices in implementation of TMD formalism
- ☐ More data needed to test the formalism
- ☐ Improvements on the knowledge of the fragmentation functions

Library and Plotting tools for collinear parton distributions

LHAPDF
lhapdf.hepforge.org

xFitter
www.xfitter.org

APFEL++
github.com/vbertone/apfelxx
apfel.mi.infn.it

Dedicated Softwares to study GPDs

PARTONS
partons.cea.fr

PARtonic
Tomography
Of
Nucleon
Software

GeParD

not yet public

Dedicated software to study and fit TMDs

arTeMiDe
teorica.fis.ucm.es/artemide

TMD lib and TMD Plotter
tmdlib.hepforge.org

NangaParbat
public soon

Efforts to combine different inputs to understand TMDs and GPDs in an unified framework

Proton mass decomposition

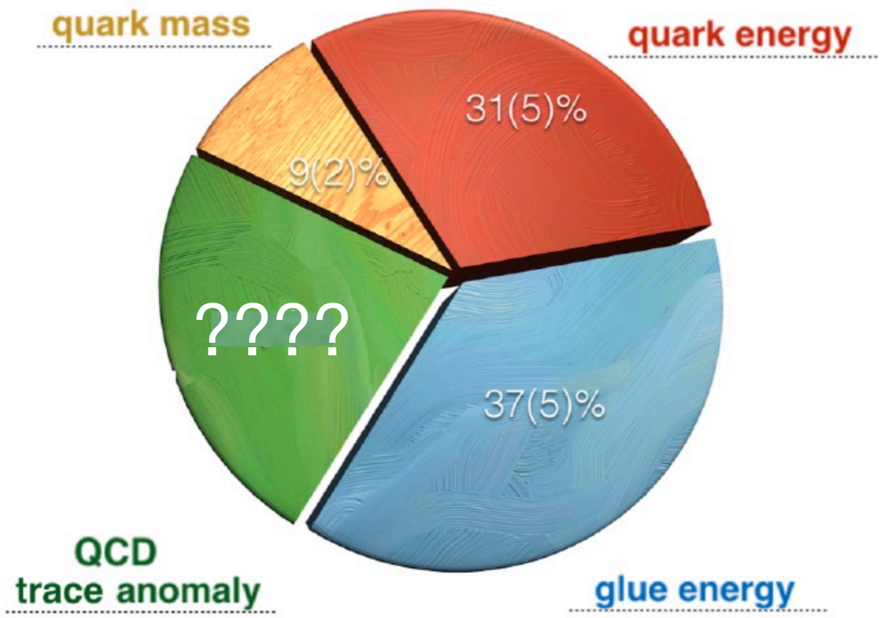
$$M = \underbrace{M_q + M_g}_{\text{quark/gluon kinetic energy}} + \overset{\text{quark mass}}{M_m} + \overset{\text{trace anomaly}}{M_a}$$

X. Ji, PRD 52 (1995) 271

$M_q + M_g$: related to $\langle x \rangle_{q,g} \longrightarrow$ from DIS

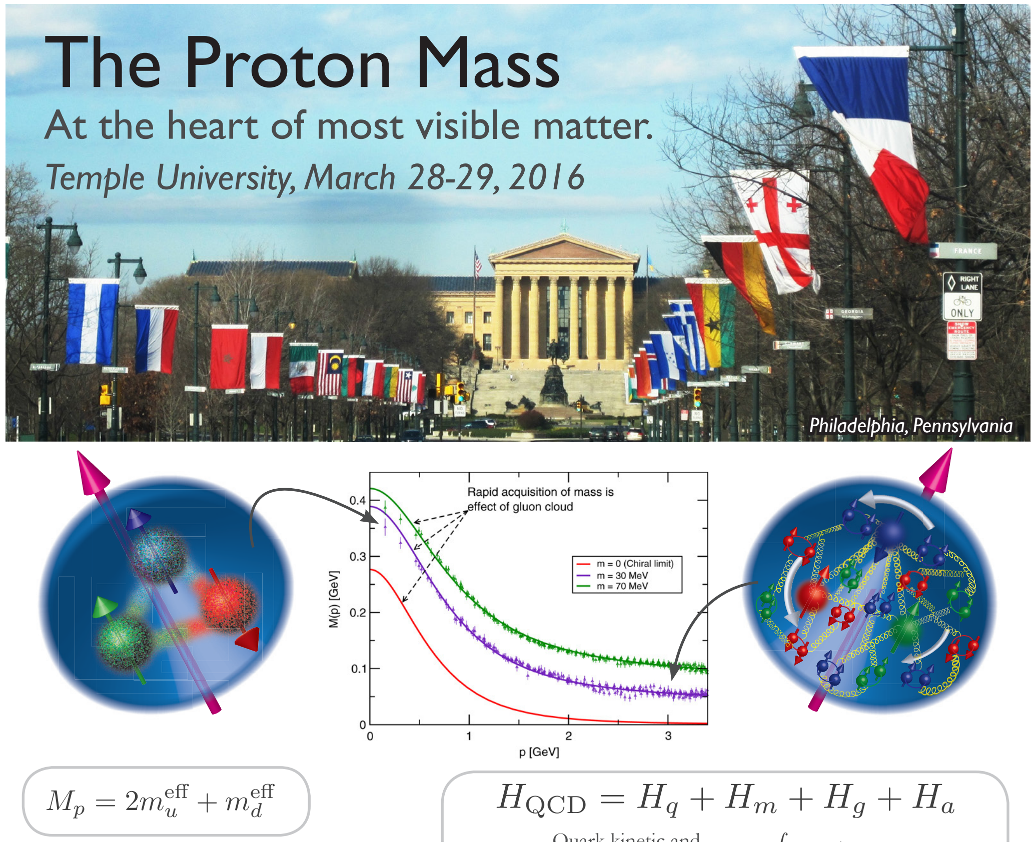
M_m : quark condensate $\longrightarrow \pi N$ sigma term

M_a : ???? possibly from exclusive production of heavy quarkonia at threshold ⟶ *talk of Y. Hatta*



Lattice QCD
Y.-B. Yang, et al., PRL 121 (2018)

- different proton mass decompositions *[C. Roberts, C. Lorcé]*
- clearly identify observables directly linked to gluon anomaly and measurable at JLab and EIC



The Proton Mass

At the heart of most visible matter.

Temple University, March 28-29, 2016

Philadelphia, Pennsylvania

$$M_p = 2m_u^{\rm eff} + m_d^{\rm eff}$$

$$H_{\rm QCD} = H_q + H_m + H_g + H_a$$

Speakers

Stan Brodsky (S
Xiandong Ji (Ma
Dima Kharzeev
Keh-Fei Liu (Ur
David Richards
Craig Roberts (A
Martin Savage (U
Stepan Stepanya
George Stermar

Moderat

Alfred Mueller (C

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Institutional Member of the European Expert Committee NUPECC

Castello di Trento ("Trint"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum,

The Proton Mass: At the Heart of Most Visible Matter

Trento, April 3 - 7, 2017

Main Topics

aly contribution, ...

ical model approaches, ...

r structure function, …

, Chen Jian-Ping (*Jefferson Lab*),
ade Abhay (*Stony Brook University*),
Lin Huey-Wen (*Michigan State University*),
Amsterdam), Papavassiliou Joannis
berts Craig (*Argonne National Lab*),
titute of Technology), Dima Kharzeev

" (Provincia Autonoma di Trento),
of the University of Trento.

le 286 - 38123 Villazzano (Trento) -Italy
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INT Workshop INT-20-77W

Origin of the Visible Universe: Unraveling the Proton Mass

May 4 - 8, 2020

Organizers:
Ian Cloët
Argonne National Laboratory
icloet@anl.gov

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zmeziani@anl.gov

Barbara Pasquini
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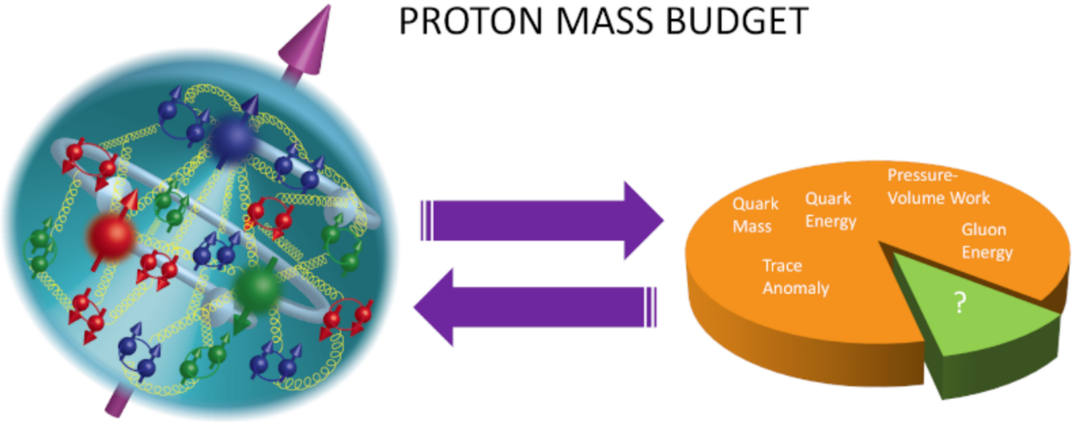
PROTON MASS BUDGET

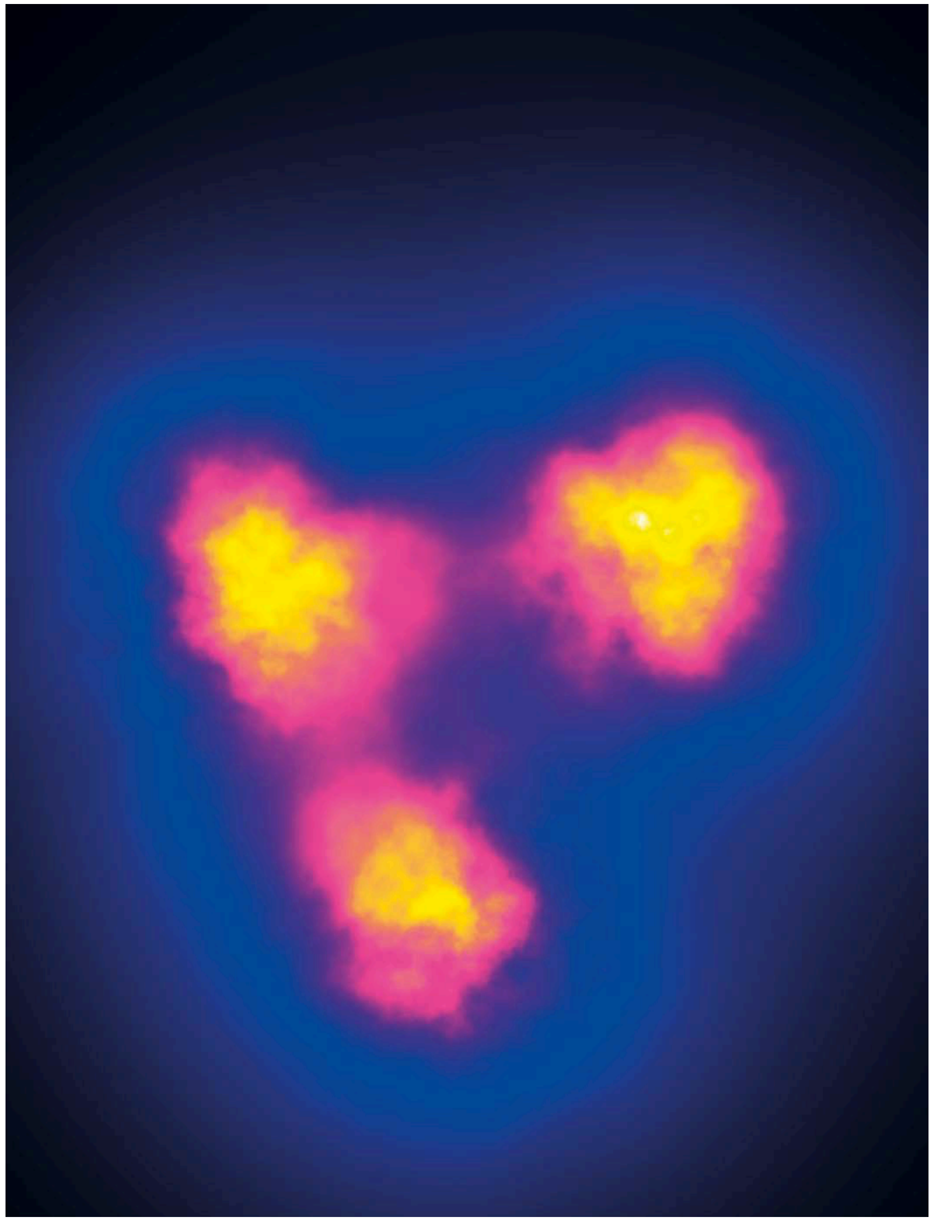


100 years from the discovery of the existence of the proton

100 years of evolving understanding of the proton

There is still much to learn about the proton…..

New challenges to interpret upcoming data from JLab12, COMPASS, MAMI, JPARC,EIC,..



CERN Courier cover, June 2019

Surely, there is a lot to learn about the proton, …

But we thought we at least understood the ground state bulk properties of the proton well, until….

8 July 2010 | www.nature.com/nature | £10

THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE

nature

OIL SPILLS
There's more to come

PLAGIARISM
It's worse than you think

CHIMPANZEES
The battle for survival

SHRINKING THE PROTON
New value from exotic atom trims radius by four per cent

NATUREJOBS
Researchers for hire

nature 466, 151–284 8 July 2010

www.nature.com/nature

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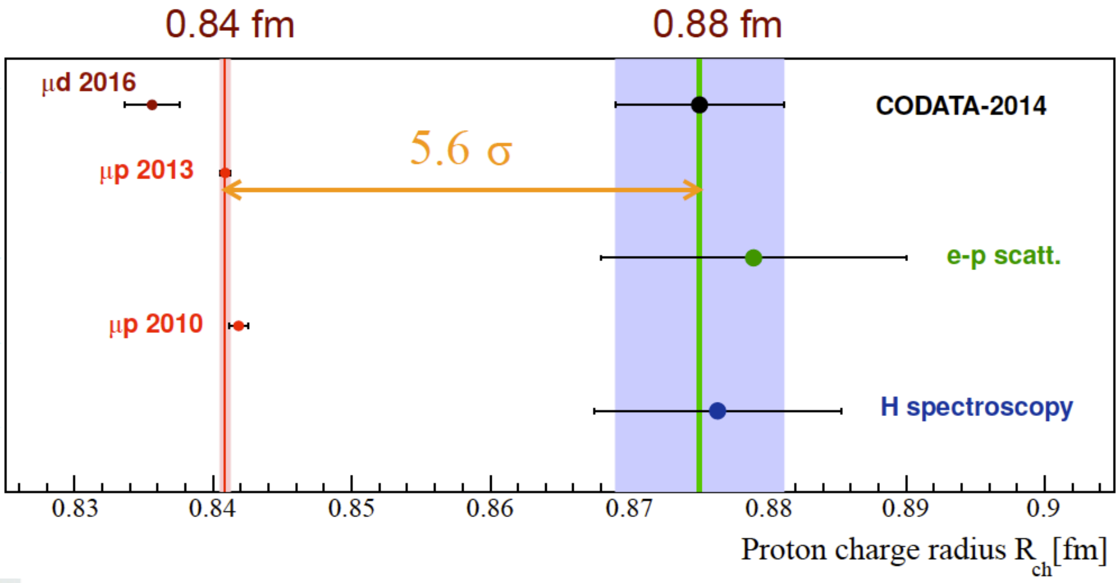
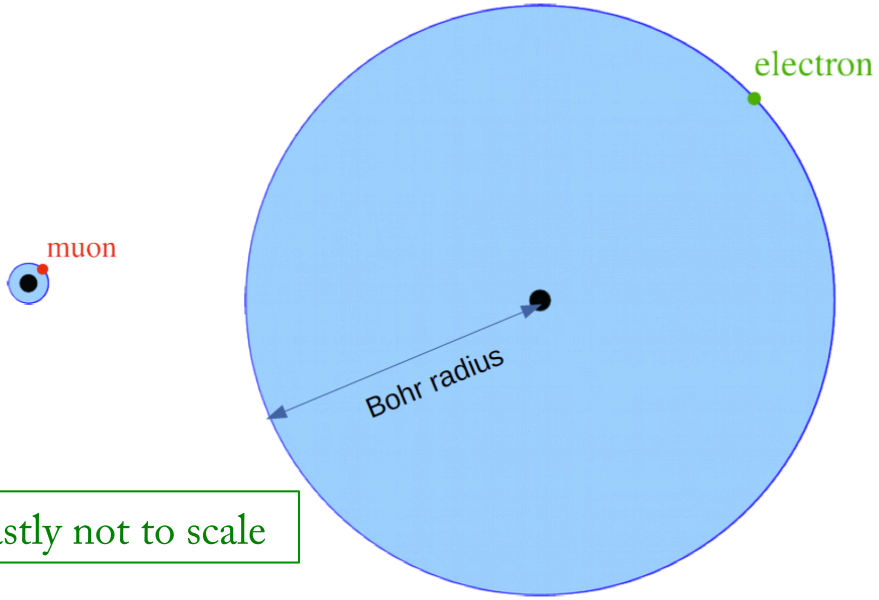
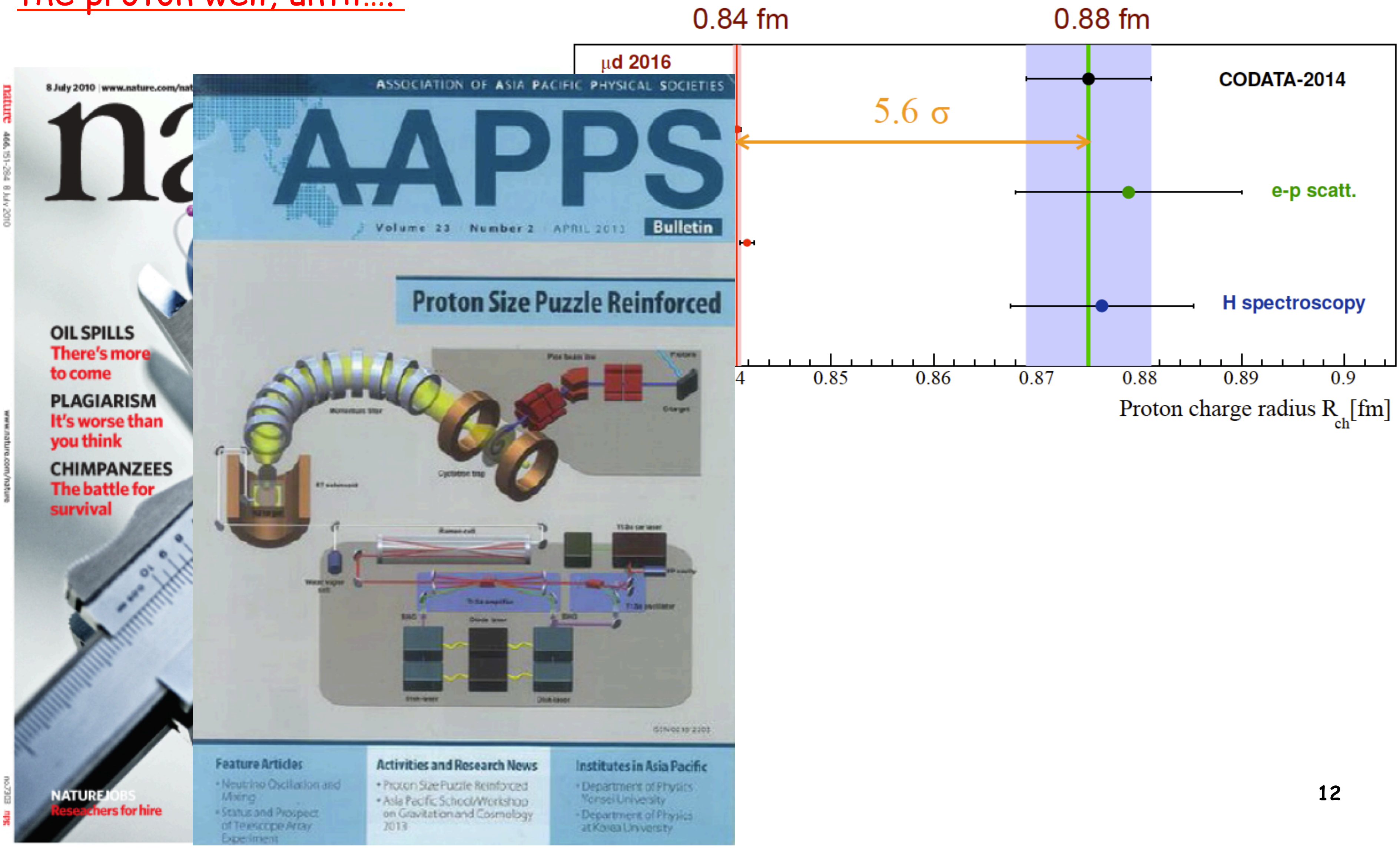




vastly not to scale

Surely, there is a lot to learn about the proton, …

But we thought we at least understood the ground state bulk properties of the proton well, until….



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TINY PARTICL BIG PROBLEM
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The Proton Problem
Could scientists be seeing signs of a whole new realm of physics?


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O enigma do
Próton


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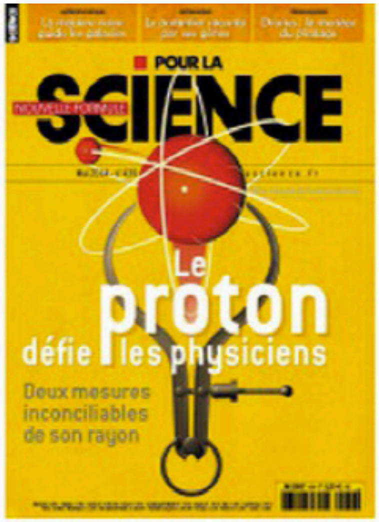
2013年度创新榜:
最具影响力研发中心
Top 10
质子半径之谜


POUR LA SCIENCE
Le proton défie les physiciens
Deux mesures inconciliables de son rayon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Le Scienze
Il dilemma del protone


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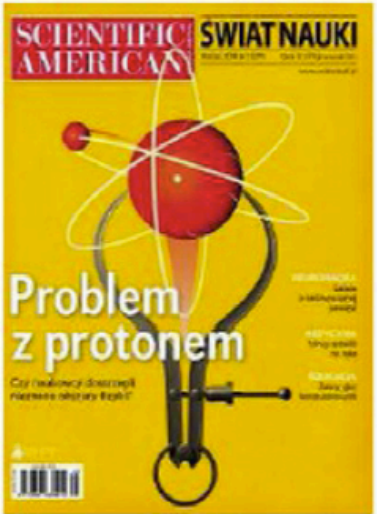
В Мире Науки
ПРОТОН:
НОВОЕ ИЗМЕРЕНИЕ ФИЗИКИ


INVESTIGACIÓN Y CIENCIA
El problema del protón
Las discrepancias sobre su tamaño podrían esconder nueva física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Problem z protonem


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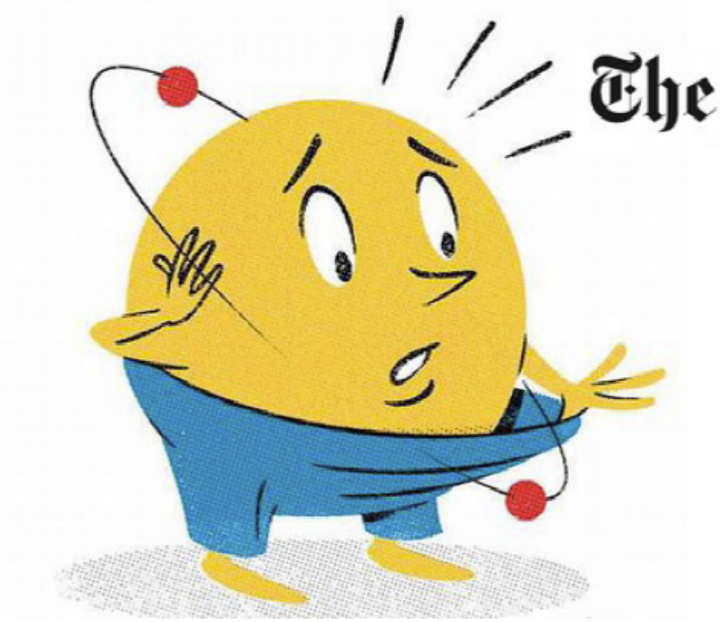
質子半徑的量子問題

The Big deal about the Proton Radius



The New York Times

OK, the proton may be ~5% smaller than we thought, so what's the big deal ?

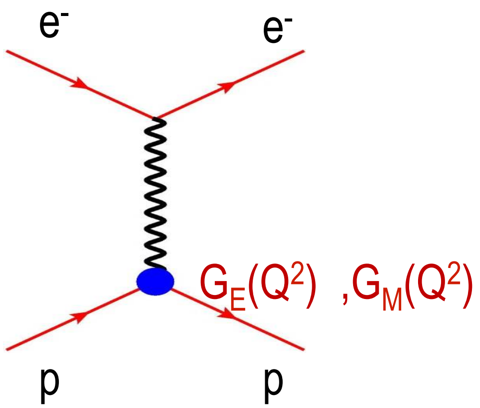
- Important bench-mark quantity for many calculations.
  - nuclear physics (QCD, Lattice, …)
  - atomic physics (QED, Lamb shifts, …)
- directly correlated to the Rydberg constant (most accurately known constant in physics)
- potential for "New Physics"
  - Lepton universality in question ??? !!
  - Coupling to unknown particles ?

Elastic electron-proton Scattering Formalism

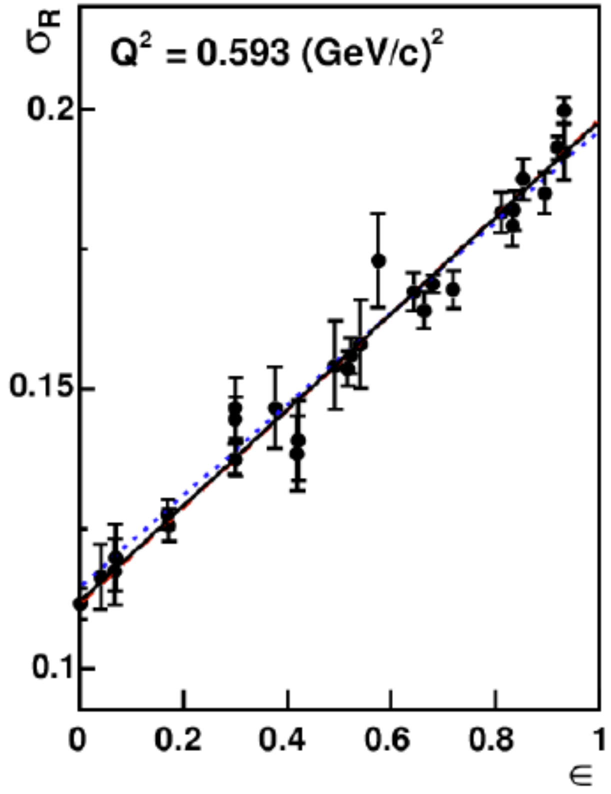
In one photon approximation the elastic *ep* scattering

$$\boxed{\sigma_R = (d\sigma/d\Omega)/(d\sigma/d\Omega)_{\text{Mott}} = \tau G_M^2 + \varepsilon G_E^2}$$

$$\boxed{Q^2 = 4EE'\sin^2\frac{\theta}{2} \qquad \tau = \frac{Q^2}{4M_p^2} \qquad \varepsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}}$$



- $G_E(Q^2)$ and $G_M(Q^2)$ extracted using Rosenbluth separation
- Measure the reduced cross section at several values of ε while keeping Q^2 fixed.
- Extract G_E from the slope
- At extremely low Q^2 the G_M contribution is small, like in the PRad experiment



Proton Mean Square Charge Radius

Classically:

$$\boxed{\left\langle r^2 \right\rangle = \int \rho(r) r^2 d^3 r}$$

Using the QED formalism: with the

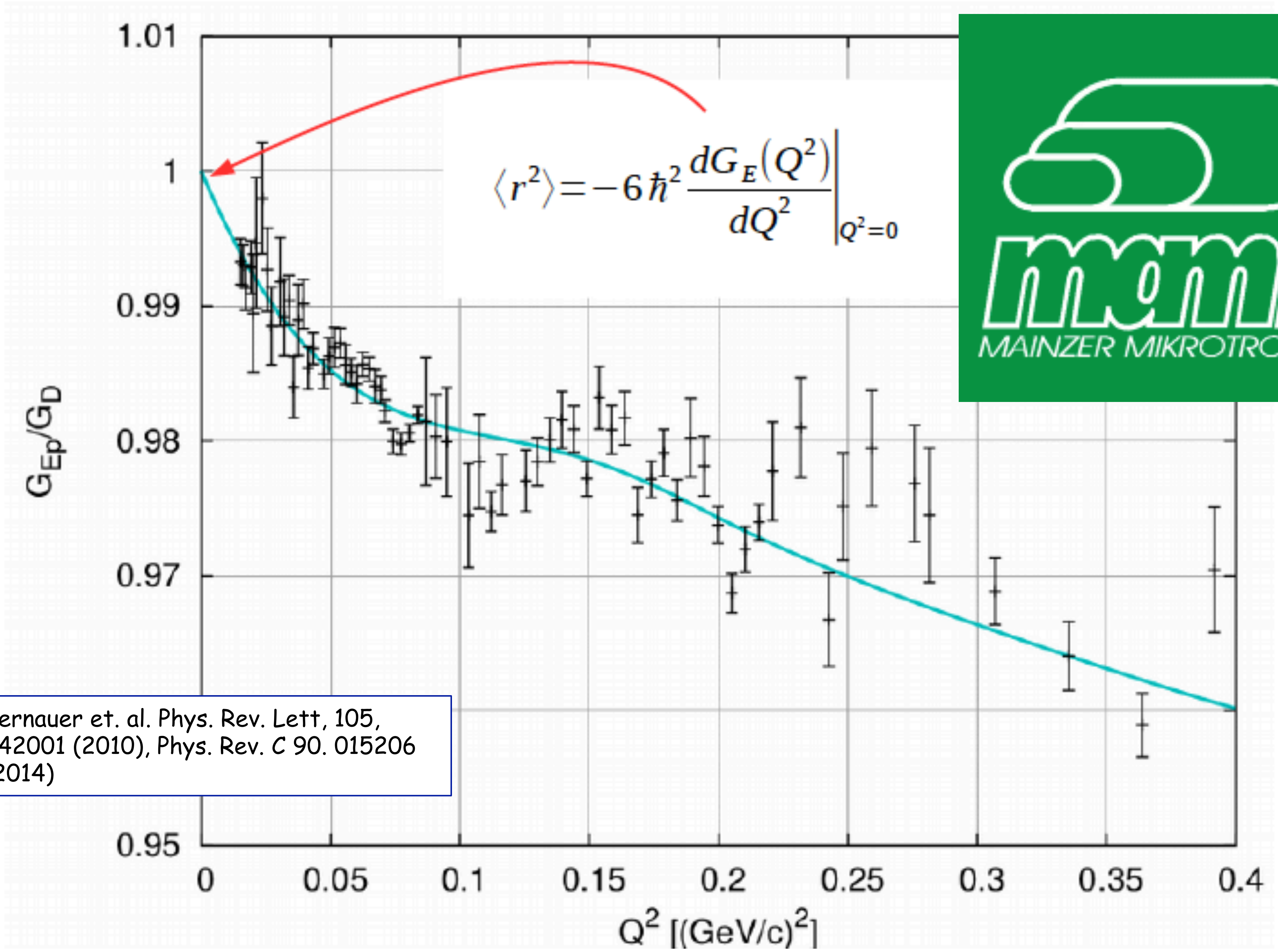
Expanding Electric FF $G_E(Q^2)$ in Taylor series:

$$\boxed{G_E^p(Q^2) = 1 - \frac{Q^2}{6}\langle r^2 \rangle + \frac{Q^4}{120}\langle r^4 \rangle + \ldots}$$

We have:

$$\boxed{\left\langle r^2 \right\rangle = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}}$$

Proton Mean Square Charge Radius

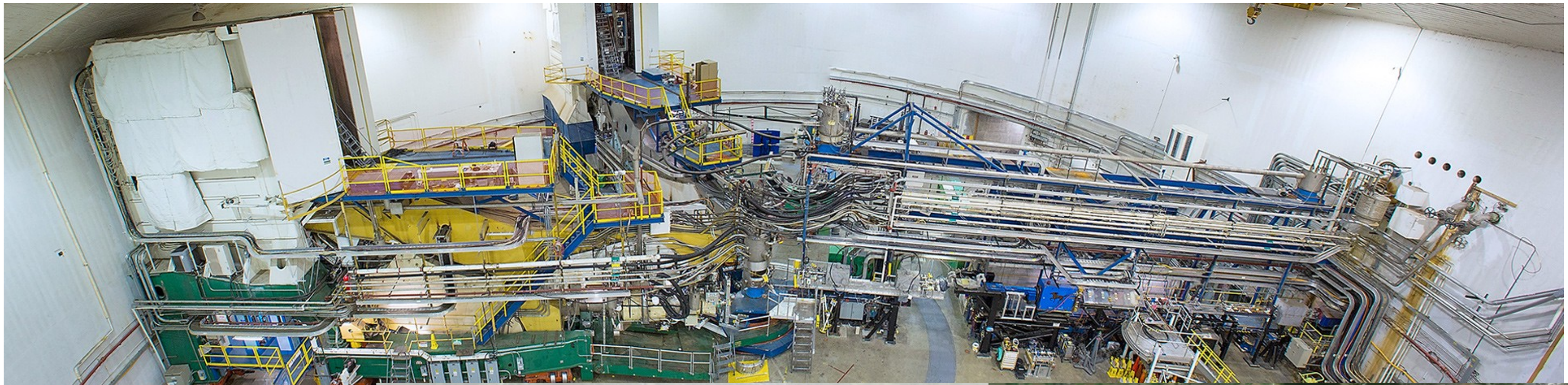


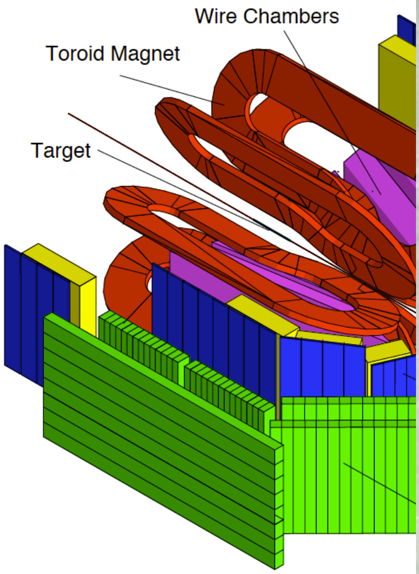
$$\langle r^2\rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

mami
MAINZER MIKROTRON

Bernauer et. al. Phys. Rev. Lett, 105, 242001 (2010), Phys. Rev. C 90. 015206 (2014)

Proton Radius from electron-proton Scattering



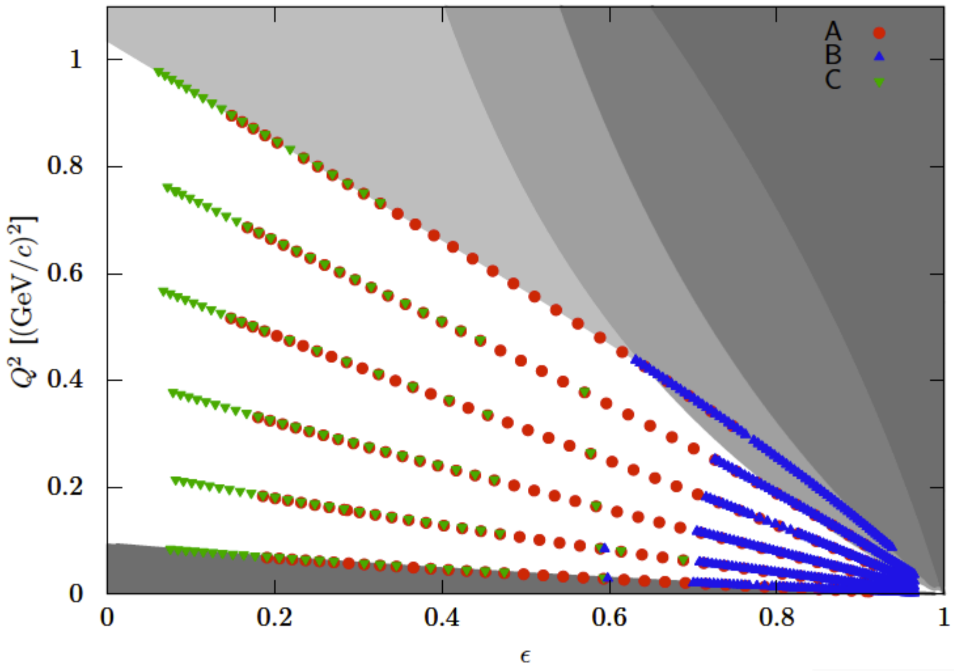


Mainz Microtron

- cw electron beam
- 10 μA polarized,
  100 μA unpolarized
- MAMI A+B: 180-855 MeV
- MAMI C: 1.6 GeV

A1 3-spectrometer facility

- 28 msr acceptance
- angle resolution: 3 mrad
- momentum res.: 10^{-4}

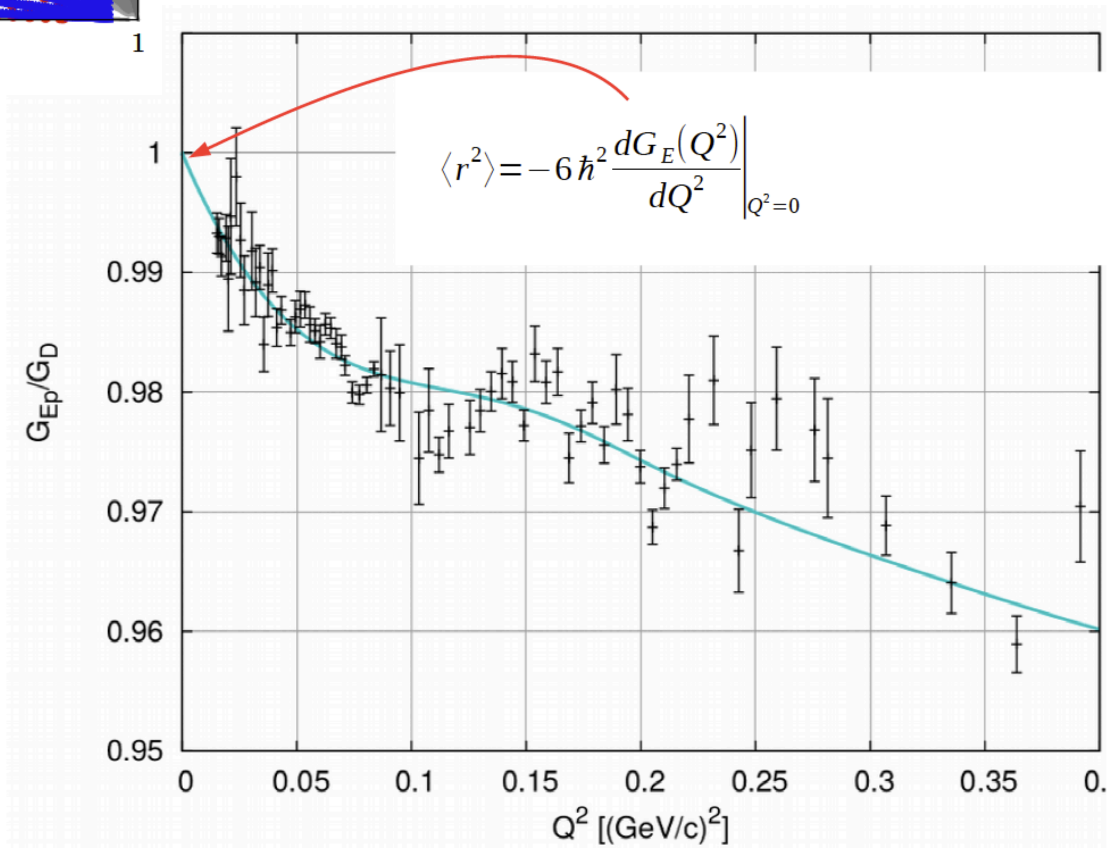


electron-proton Scattering data from Mainz

Bernauer et. al. Phys. Rev. Lett, 105, 242001 (2010), Phys. Rev. C 90. 015206 (2014)

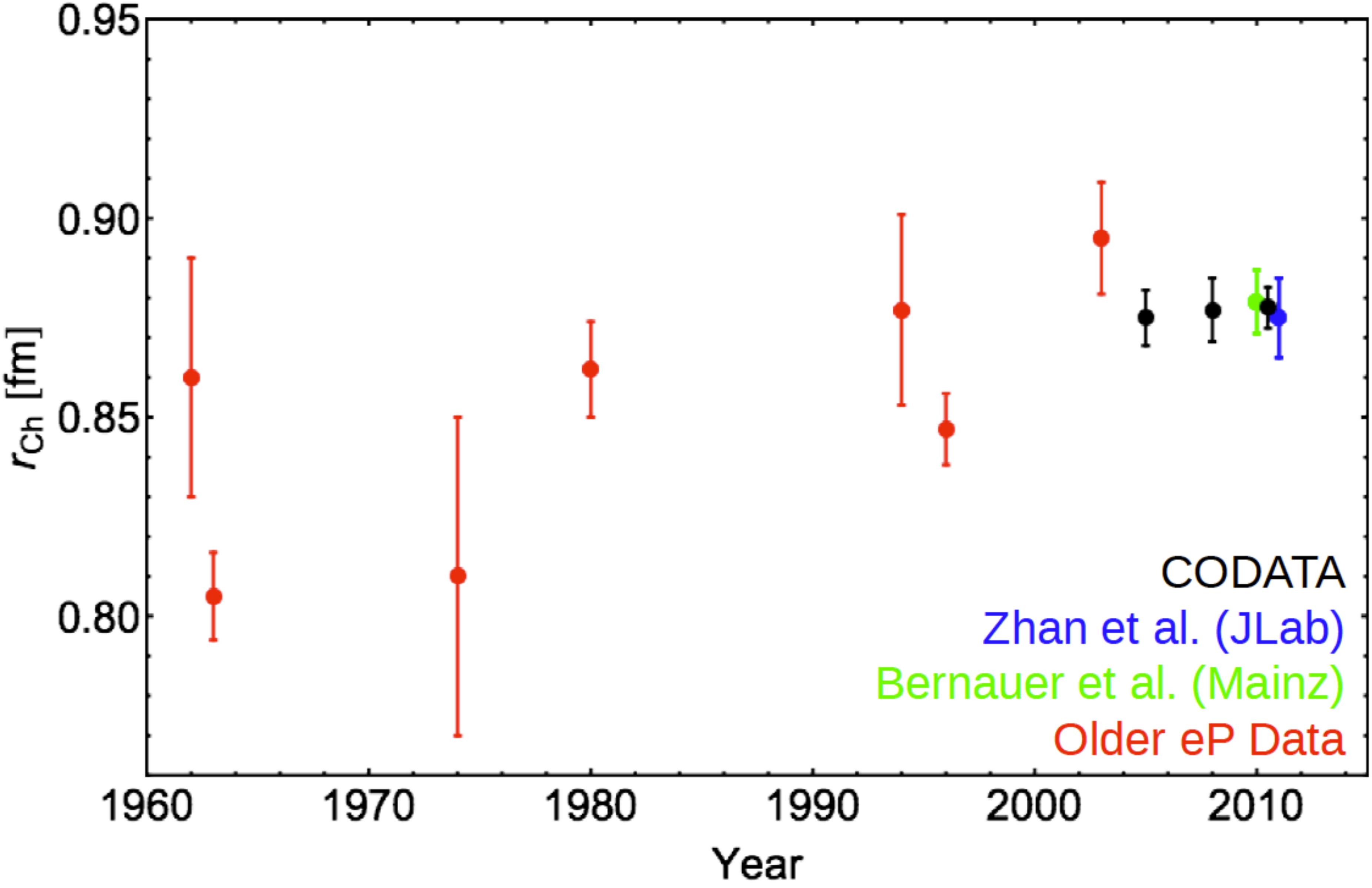
- Mainz data come from a wide range of beam energies and spectrometer angles: required separation of G_E from G_M

- Mainz G_E agrees with G_E from Jlab Hall A; but G_M disagrees

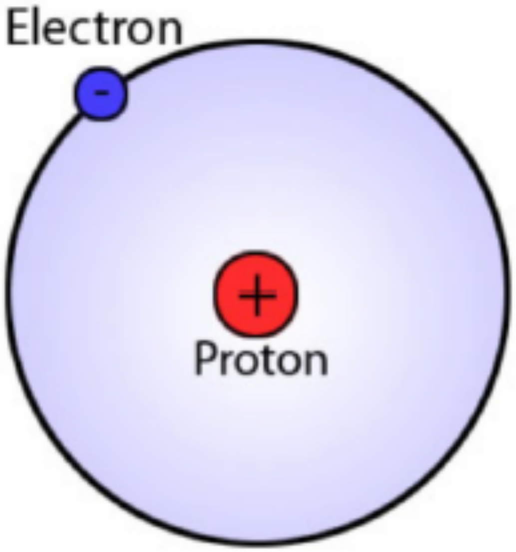


$$\langle r^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

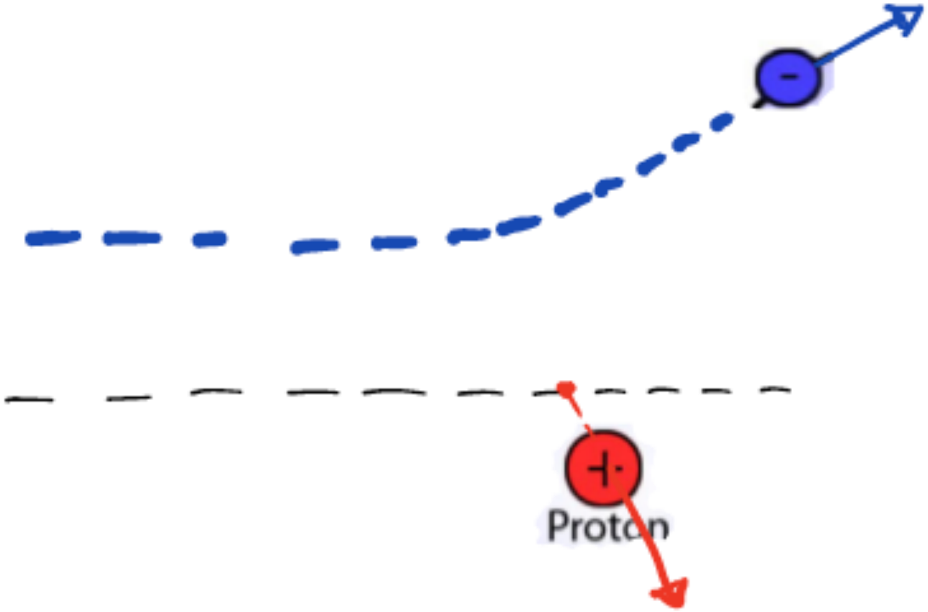
Time evolution of Proton Radius from e-p Scattering



How is the same $<r^2>$ measured in Atomic Physics ?



e-p bound state:
Atomic Physics

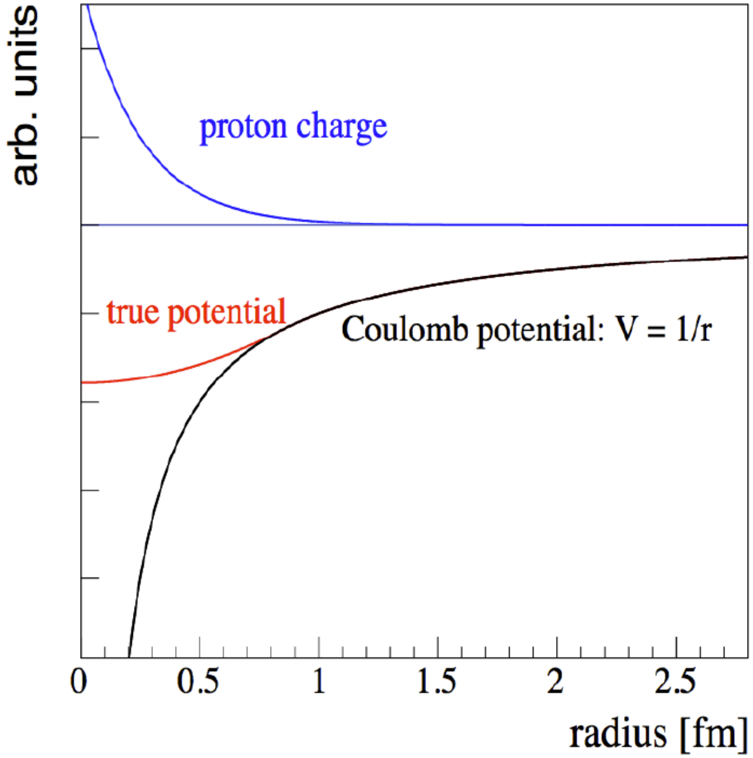


e-p scattering state:
Nuclear Physics

In either case electron interacting with proton through Coulomb interaction,

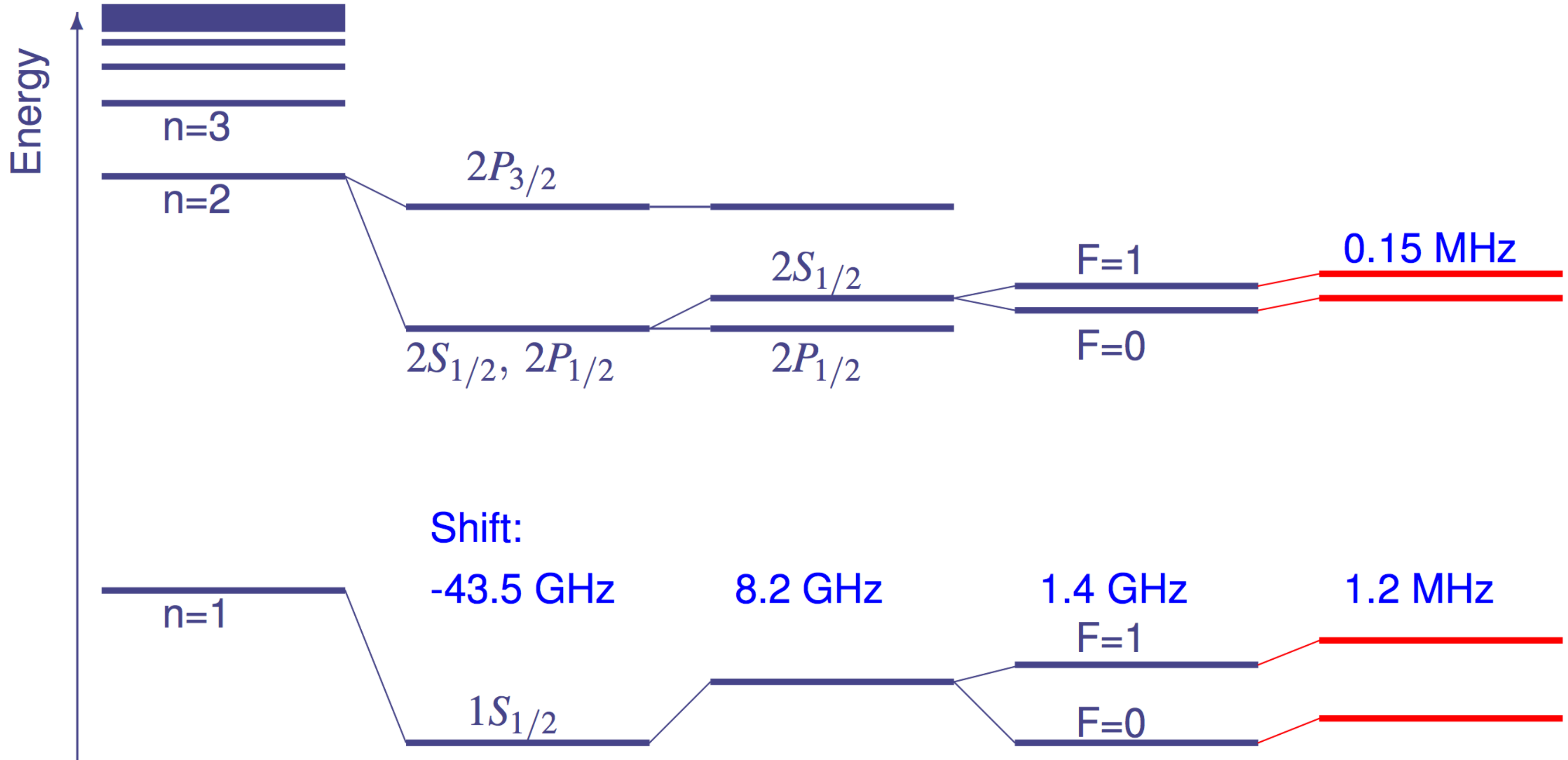
Coulomb interaction which is modified due to the extended charge distribution of the proton

The difference between true proton potential and the potential for a point like proton



$$\delta V(\mathbf{r}) \equiv V_C(\mathbf{r}) - V_C^{\mathrm{pt}}(\mathbf{r}) = -4\pi\alpha \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{[G_{\mathrm{E}}(\mathbf{q}^2) - 1]e^{-i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2}.$$

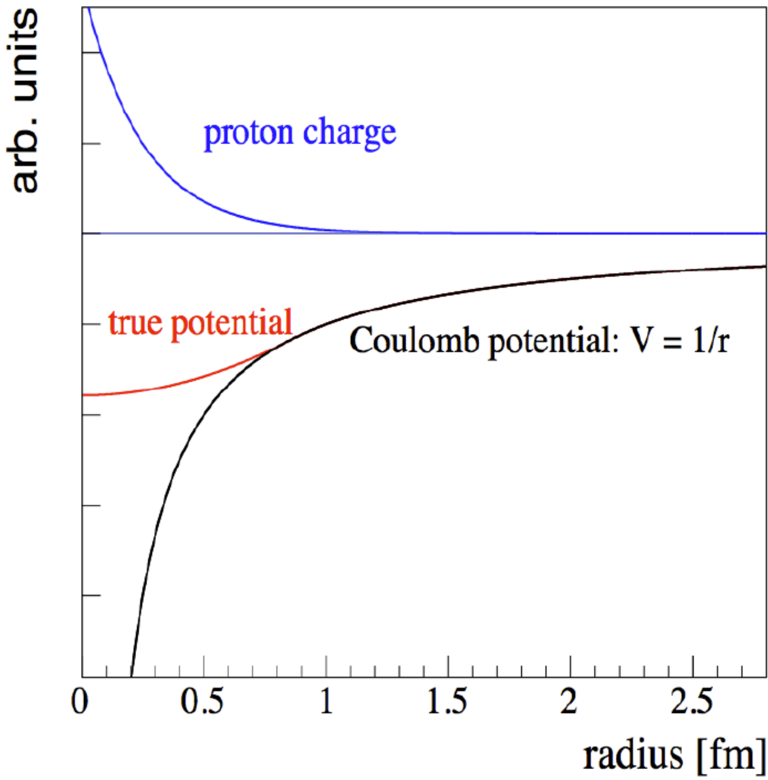
Regular Hydrogen spectroscopy

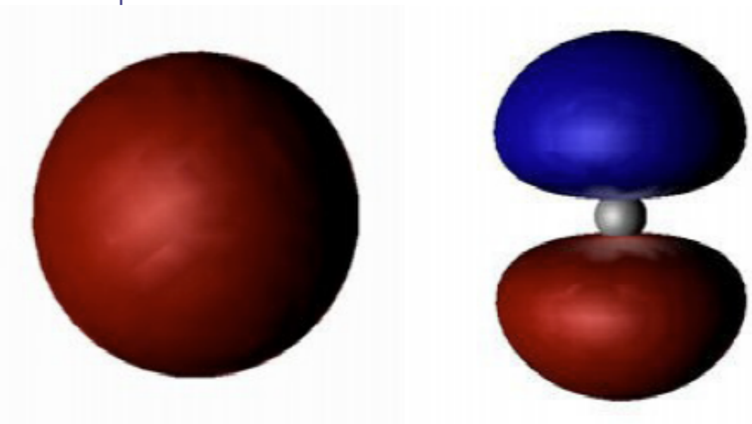


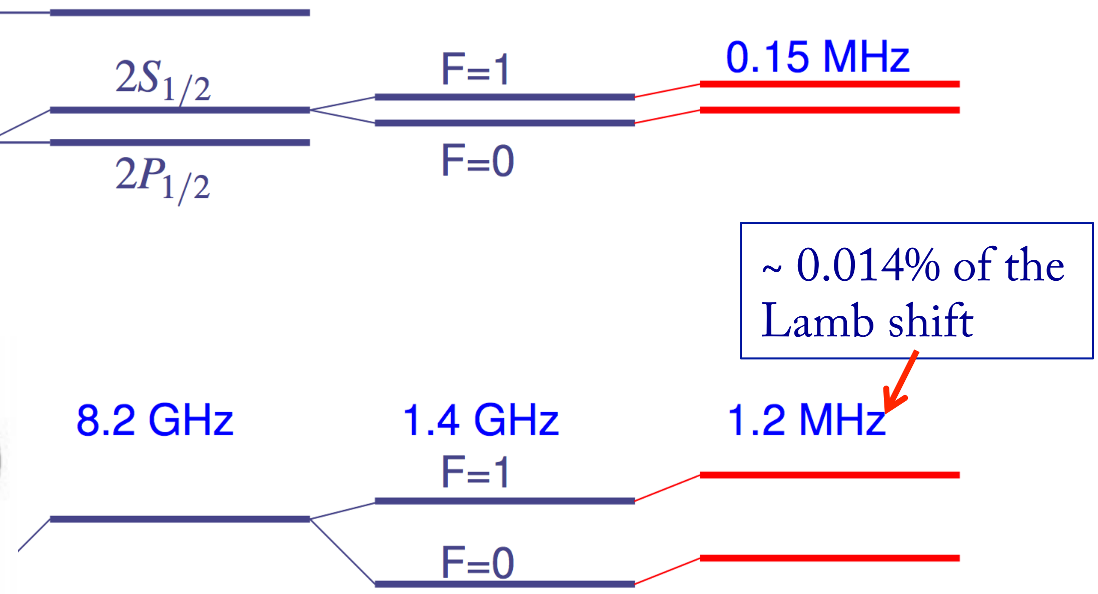
Bohr	**Dirac**	**Lamb**	**hfs-splitting**	r_p
$E = R_\infty / n^2$ $V \sim 1/r$	e^- spin relativity	QED	proton-spin $H^{\text{hfs}} \sim \vec{\mu}_p \cdot \vec{\mu}_e$	proton size $V \not\sim 1/r$

R. Pohl

- While electron is inside the proton attractive potential is lower.
- Strongly affects the S orbital, much less so the P.







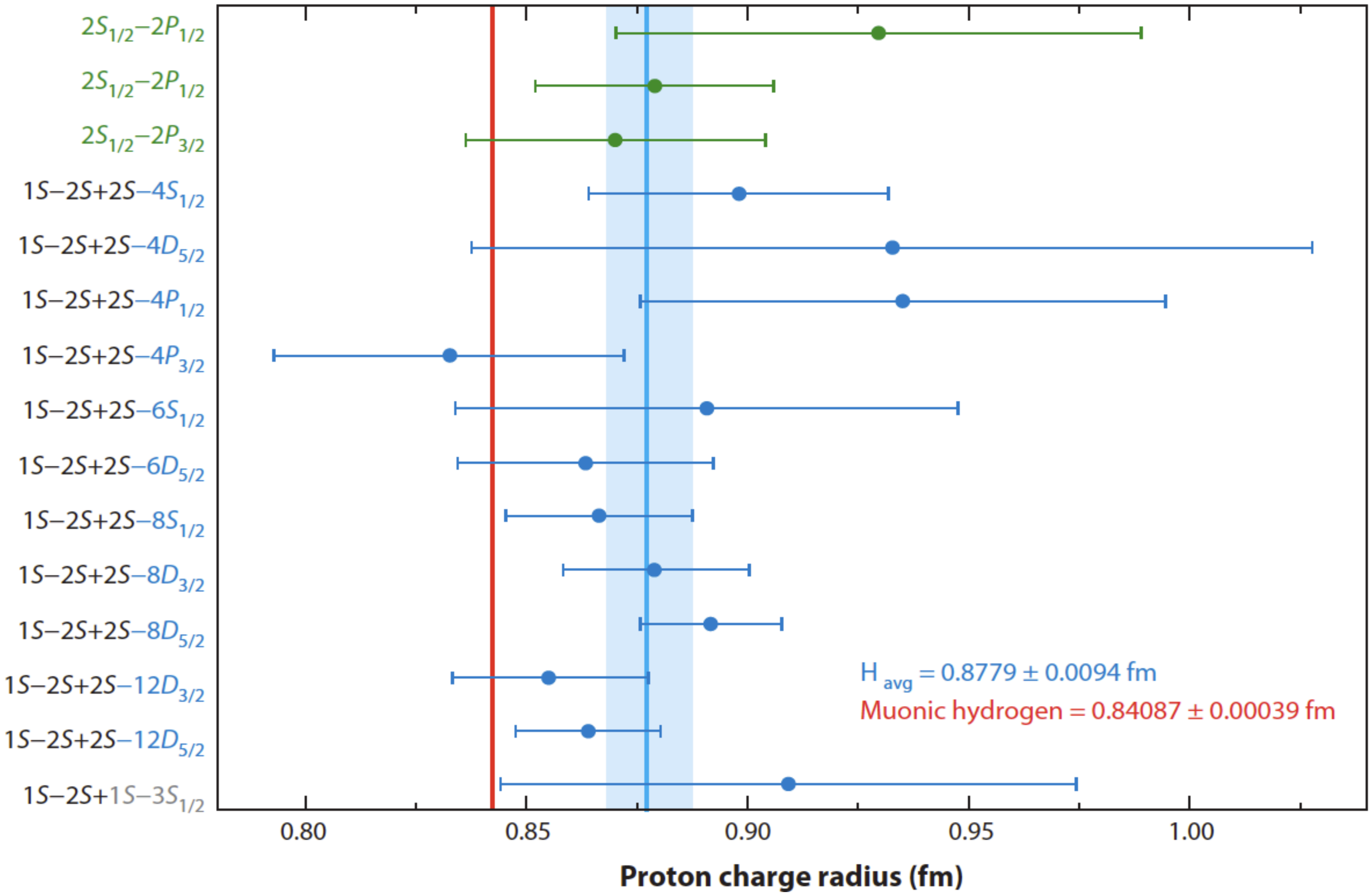
~ 0.014% of the Lamb shift

Lamb — QED

hfs-splitting — proton-spin — $H^{\text{hfs}} \sim \vec{\mu}_p \cdot \vec{\mu}_e$

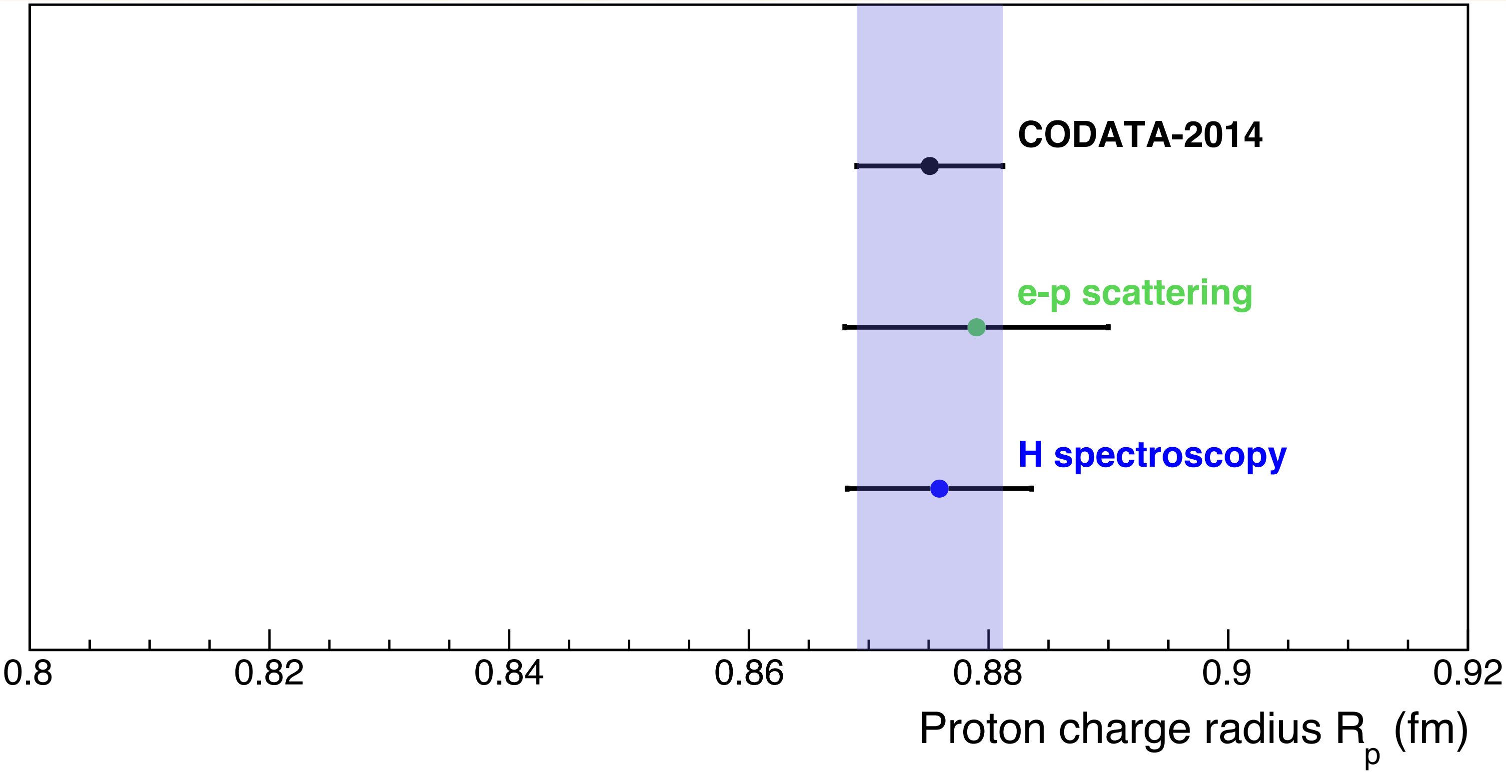
r_p — proton size — $V \not\approx 1/r$

R. Pohl



$2S_{1/2}-2P_{1/2}$
$2S_{1/2}-2P_{1/2}$
$2S_{1/2}-2P_{3/2}$
$1S-2S+2S-4S_{1/2}$
$1S-2S+2S-4D_{5/2}$
$1S-2S+2S-4P_{1/2}$
$1S-2S+2S-4P_{3/2}$
$1S-2S+2S-6S_{1/2}$
$1S-2S+2S-6D_{5/2}$
$1S-2S+2S-8S_{1/2}$
$1S-2S+2S-8D_{3/2}$
$1S-2S+2S-8D_{5/2}$
$1S-2S+2S-12D_{3/2}$
$1S-2S+2S-12D_{5/2}$
$1S-2S+1S-3S_{1/2}$
$H_{avg} = 0.8779 \pm 0.0094$ fm
Muonic hydrogen = 0.84087 ± 0.00039 fm
0.80
0.85
0.90
0.95
1.00
Proton charge radius (fm)

Proton Radius Before 2010



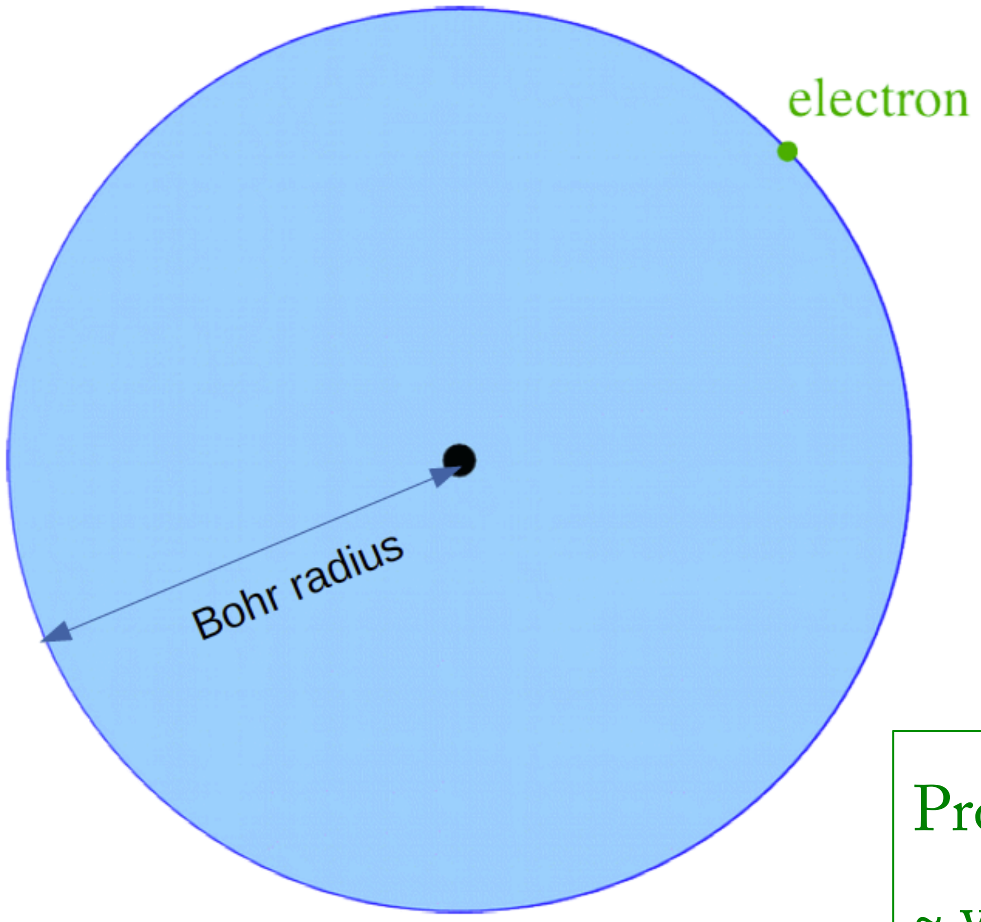
CODATA average:	0.8751 ± 0.0061 fm
ep-scattering average (CODATA):	0.879 ± 0.011 fm
Regular H-spectroscopy average (CODATA):	0.859 ± 0.0077 fm

Very good agreement between ep-scattering and H-spectroscopy results !

Electronic and Muonic Hydrogen

Regular hydrogen:

Proton + Electron



Muonic hydrogen:

Proton + Muon

Muon **mass** = **200** * electron mass

Bohr **radius** = **1/200** of H

200^3 = a **few million times** more sensitive to proton size

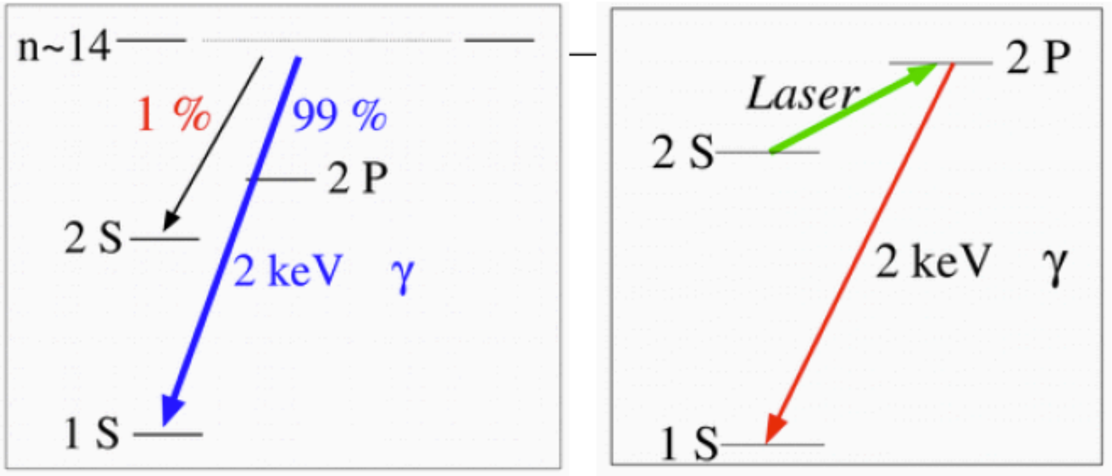
Probability for lepton inside proton ~ volume of proton / volume of atom

R. Pohl

Vastly not to scale

Muonic Hydrogen Spectroscopy Experiment

- Form μH* (n~14) by firing muon beam on 1 mbar H_2 target.
- 99% decay to 1S emitting prompt 2 keV photons.
- 1% decay to long lived 2S state.
- Excite from 2S to 2P using tuned laser: decay from 2P to 1S emitting delayed 2 keV photons.
- Vary laser frequency, find 2S-2P resonance.

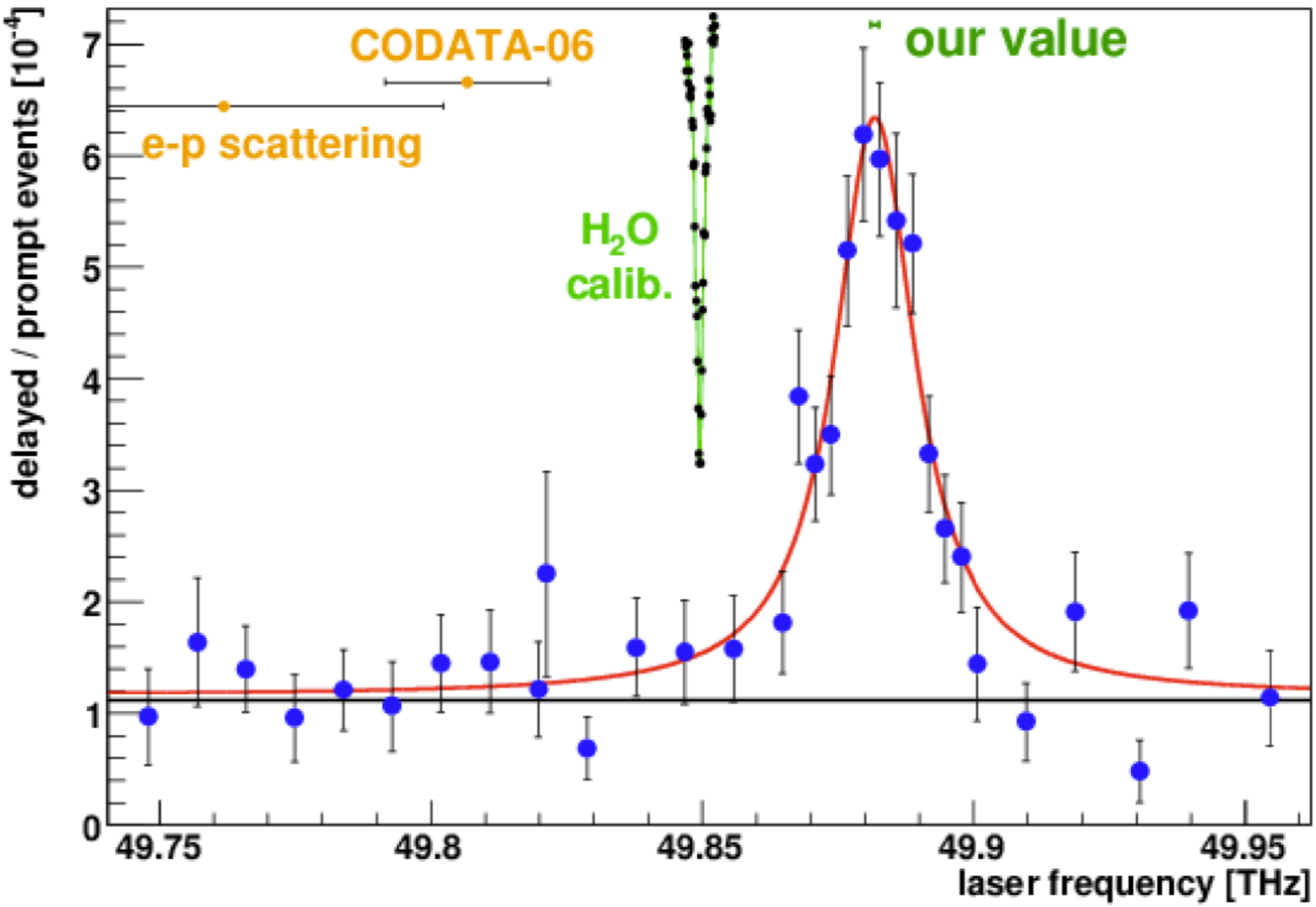


prompt (t=0) “delayed” (t = 1 μs)

Plots from R. Pohl

Muonic Hydrogen Spectroscopy Experiment

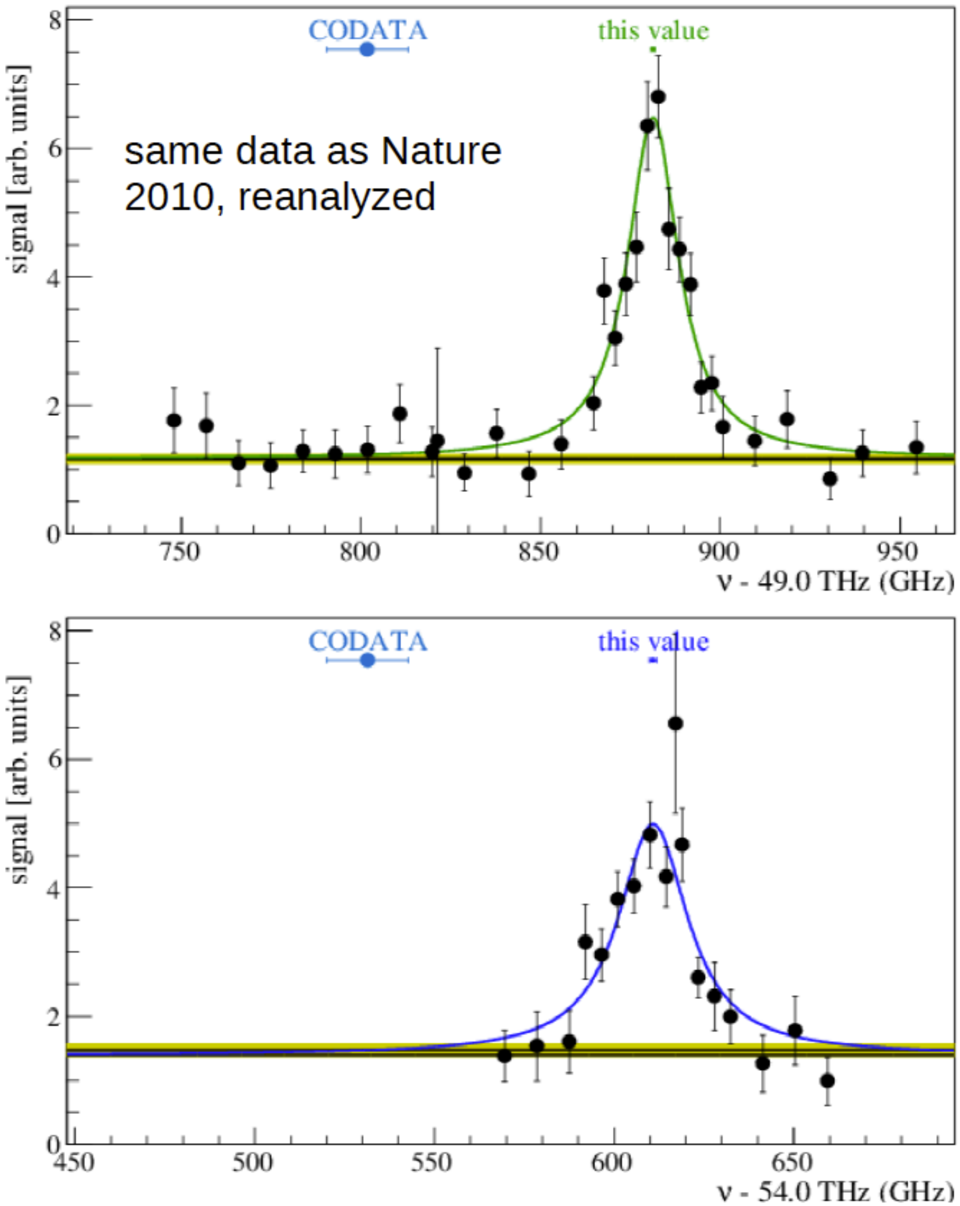
Resonance in muonic hydrogen

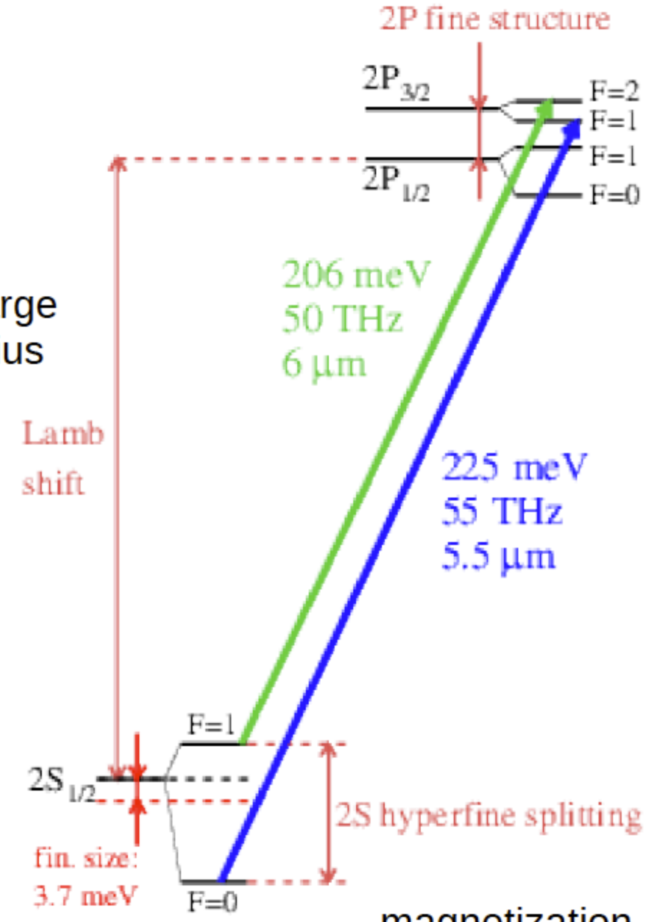


Pohl et al. (CREMA Coll.), Nature 466, 213 (2010)

Muonic Hydrogen Spectroscopy Experiment

2 transitions in muonic H


same data as Nature 2010, reanalyzed



charge radius

magnetization distribution (Zemach radius)

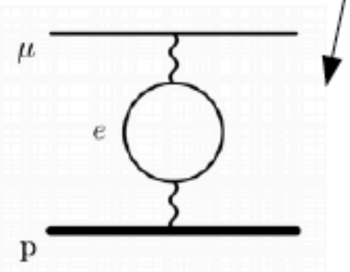
R. Pohl

Antognini et al. (CREMA Coll.), Science 339, 417 (2013)

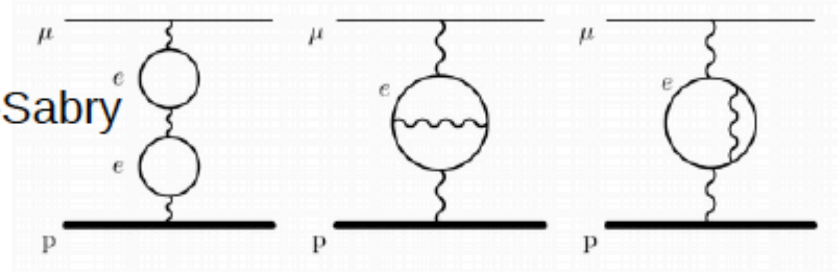
Theory in muonic H

$$\Delta E_{Lamb} = 206.0336\ (15)\ \mathrm{meV}_{QED} + 0.0332\ (20)\ \mathrm{meV}_{TPE} - 5.2275\ (10)\ \mathrm{meV/fm^2} * \mathbf{R_p}^{\mathbf{2}}$$

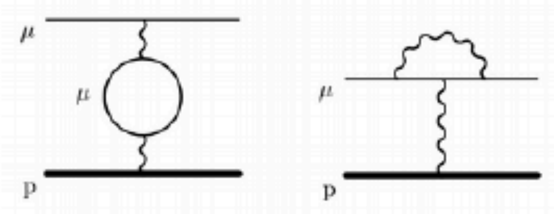
Uehling



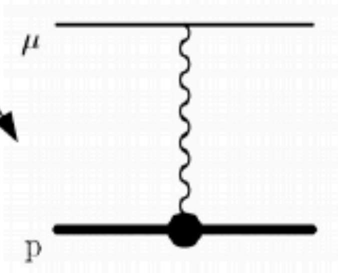
Källen-Sabry



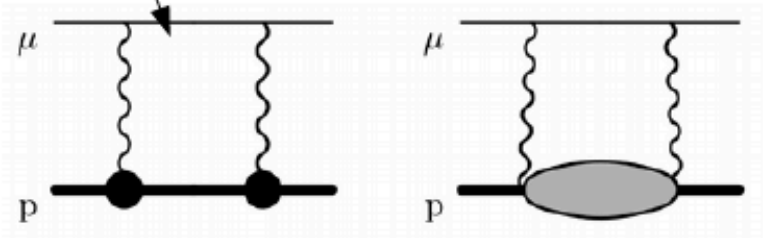
Muon SE+VP



and 20 more….



Proton form factor



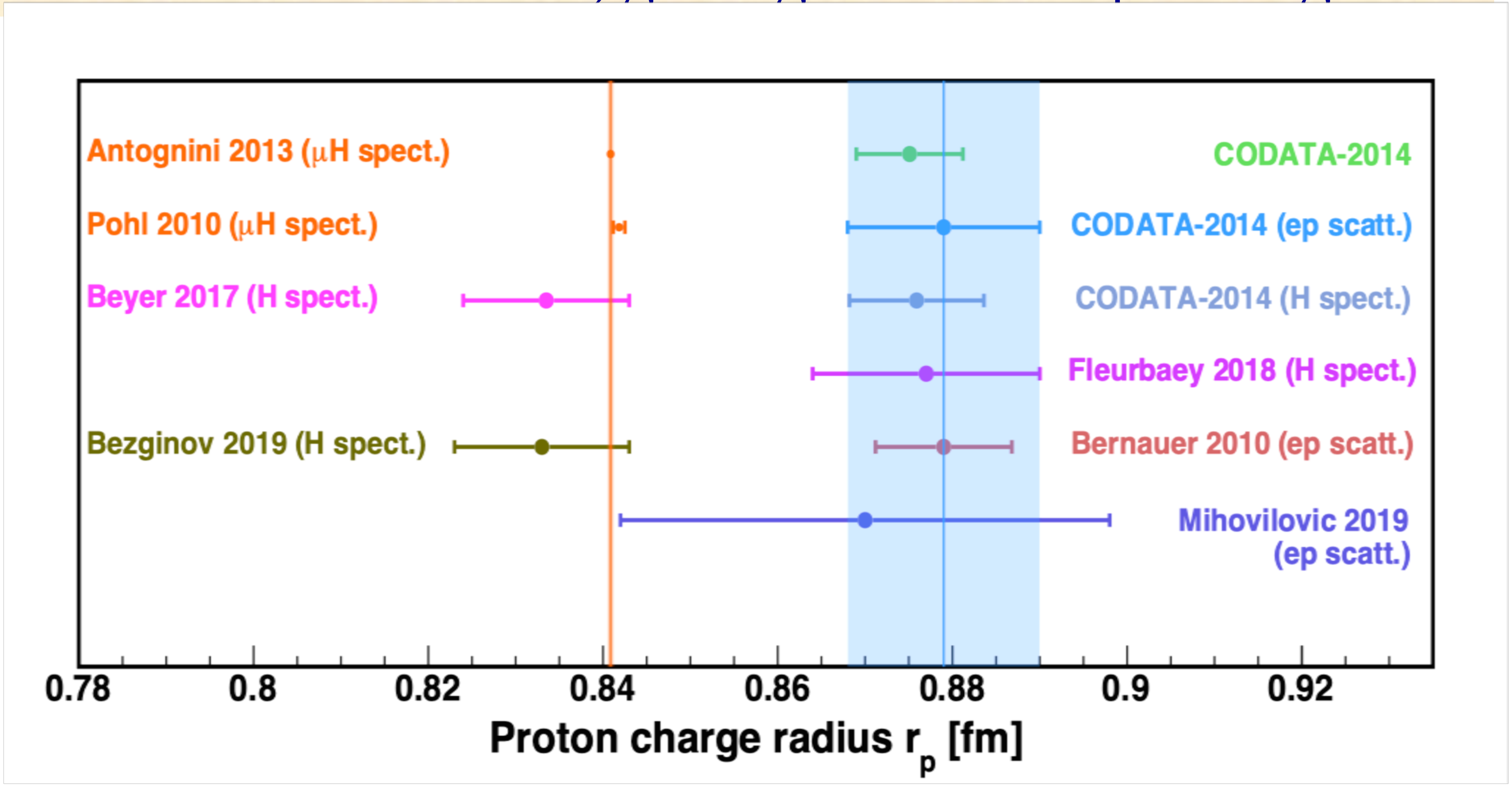
elastic and inelastic two-photon exchange
(Friar moment and polarizability)

R. Pohl

So, how do we resolve this puzzle ?

	ep	μp
Spectroscopy	New measurements with ▪ lower systematics ▪ new transitions	✓
Scattering	New measurements with ▪ lower systematics ▪ reaching lower Q^2 ProRAD, ULQ2, ISR @ MESA, PRad	No data yet. MUSE at PSI coming soon

Proton Radius Puzzle, getting even more puzzling……



Regular hydrogen average (CODATA):	0.8751 ± 0.0061 fm
Muonic hydrogen (CREMA coll.):	0.8409 ± 0.0004 fm
Regular H (2S ➔ 4P, CREMA coll.):	0.8335 ± 0.0095 fm
Regular H (1S ➔ 3S, LKB, Paris):	0.877 ± 0.013 fm

Regular H-spectr. (2S ➔ 2P, York Univ. Canada, Just published in Science)

So, how do we resolve this puzzle ?

	ep	μp
Spectroscopy	New measurements with ▪ lower systematics ▪ new transitions	✓
Scattering	New measurements with ▪ lower systematics ▪ reaching lower Q^2 ProRAD, ULQ2, ISR @ MESA, PRad	No data yet. MUSE at PSI coming soon

A New ep Scattering Experiment?

A 1% level *Rp* measurements requires

- Q^2 down to 10^{-4} GeV^2 level or lower
- Measurements over wide enough Q^2 range for a fit
- ~< 0.5% accuracy in absolute cross section
- ~< 0.2 mrad in scattering angle determination

These conditions are VERY difficult to achieve with the standard methods used for *ep* scattering experiments

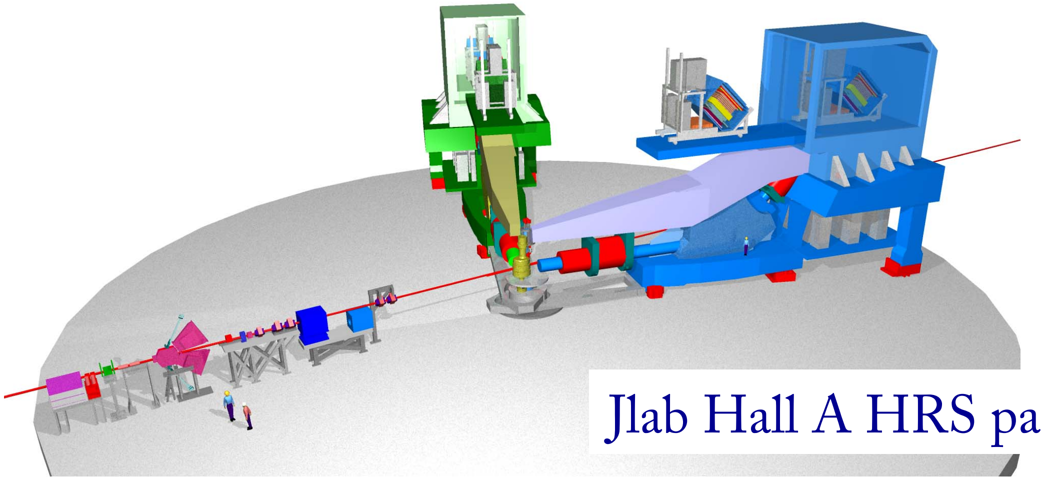
Difficulties with traditional ep Scattering Experiments

Practically all *ep*-scattering experiments have been performed with magnetic spectrometers and LH_2 targets.

- many experimental settings to cover the Q^2 range!
  - angle (Θ_e), energies (E')
- limitation on minimum Q^2: 10^{-3} GeV/C^2

Thr on:

Mainz magnetic spectrometers



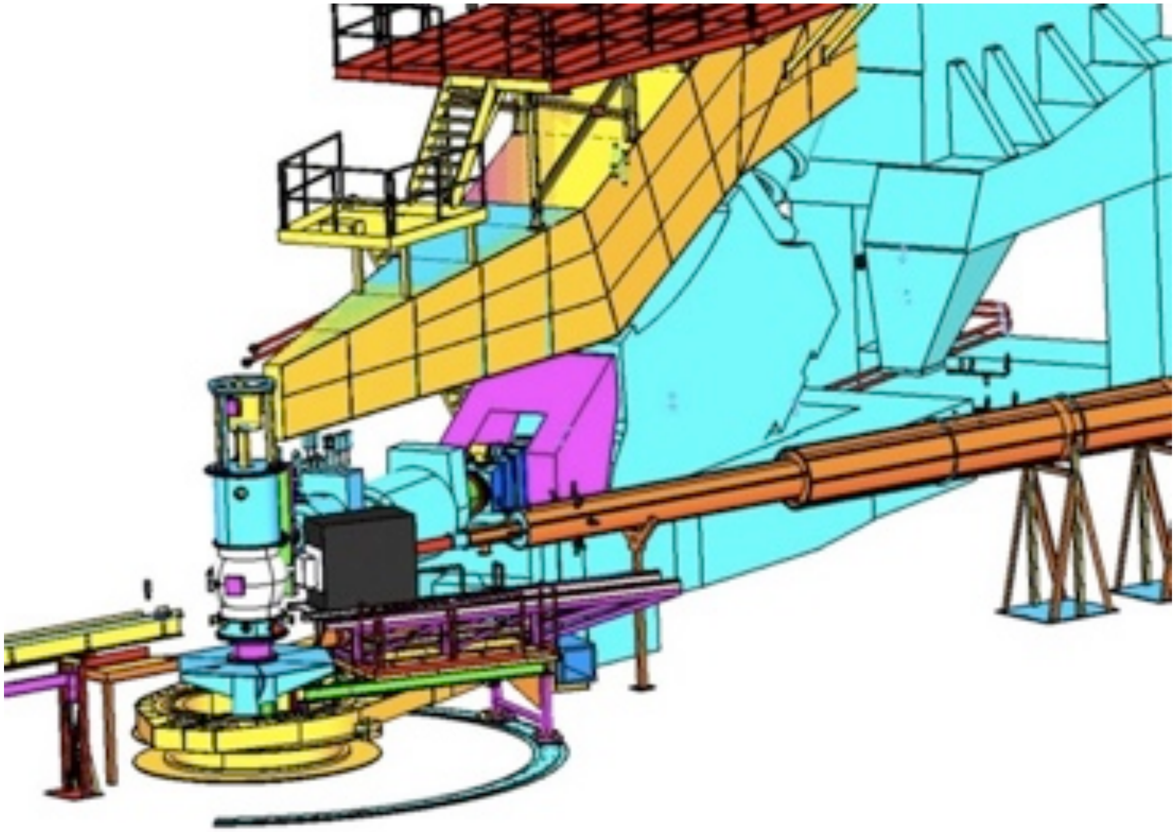
Jlab Hall A HRS pair

limitation on minimum Q^2: 10^{-3} GeV/C^2

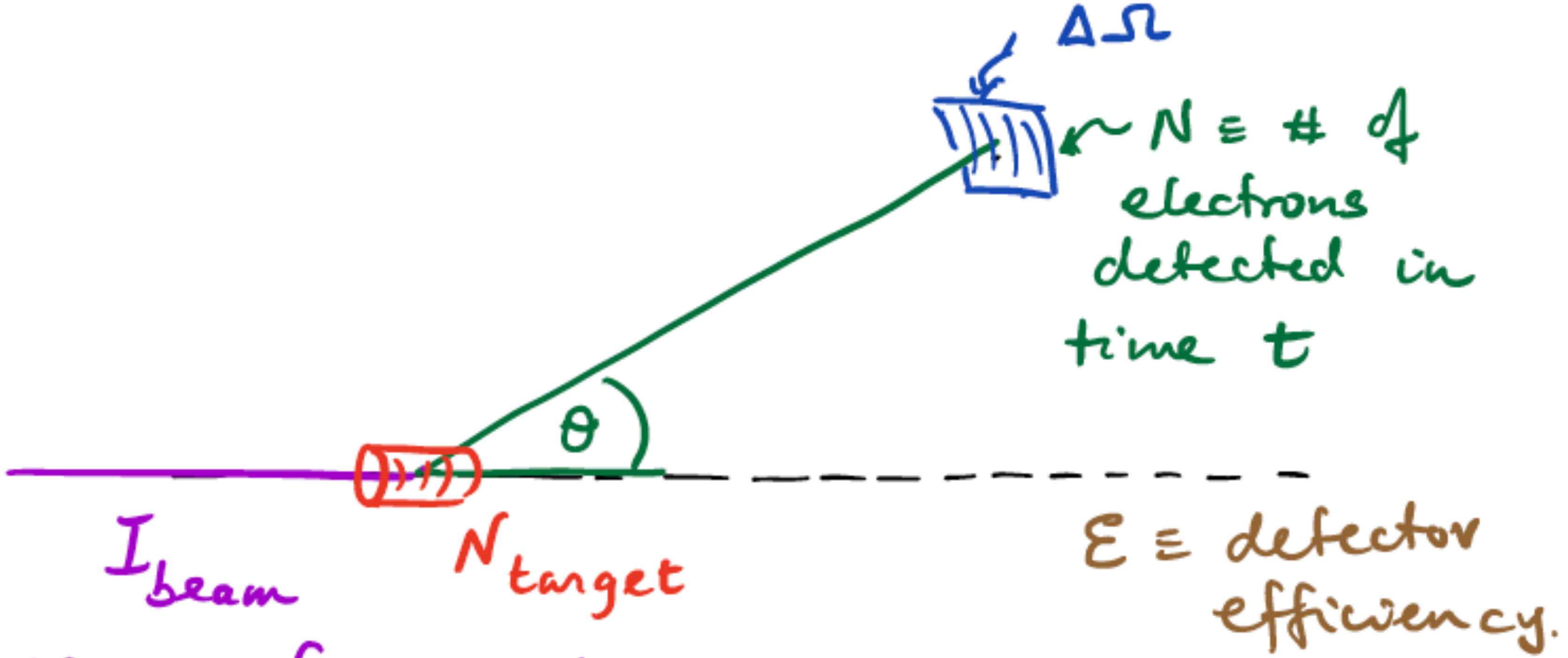
min. scattering angle: $\theta_e \approx 5^0$

beam energies: ~ 0.1 ÷1 GeV

Jlab Hall A HRS



A New ep Scattering Experiment?



$N \equiv$ # of electrons detected in time t

$\varepsilon \equiv$ detector efficiency.

$$N_{beam} = \int I_{beam}\, dt / e$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{N\,\delta}{N_{beam} \cdot N_{target}\, \Delta\Omega\, \varepsilon}$$

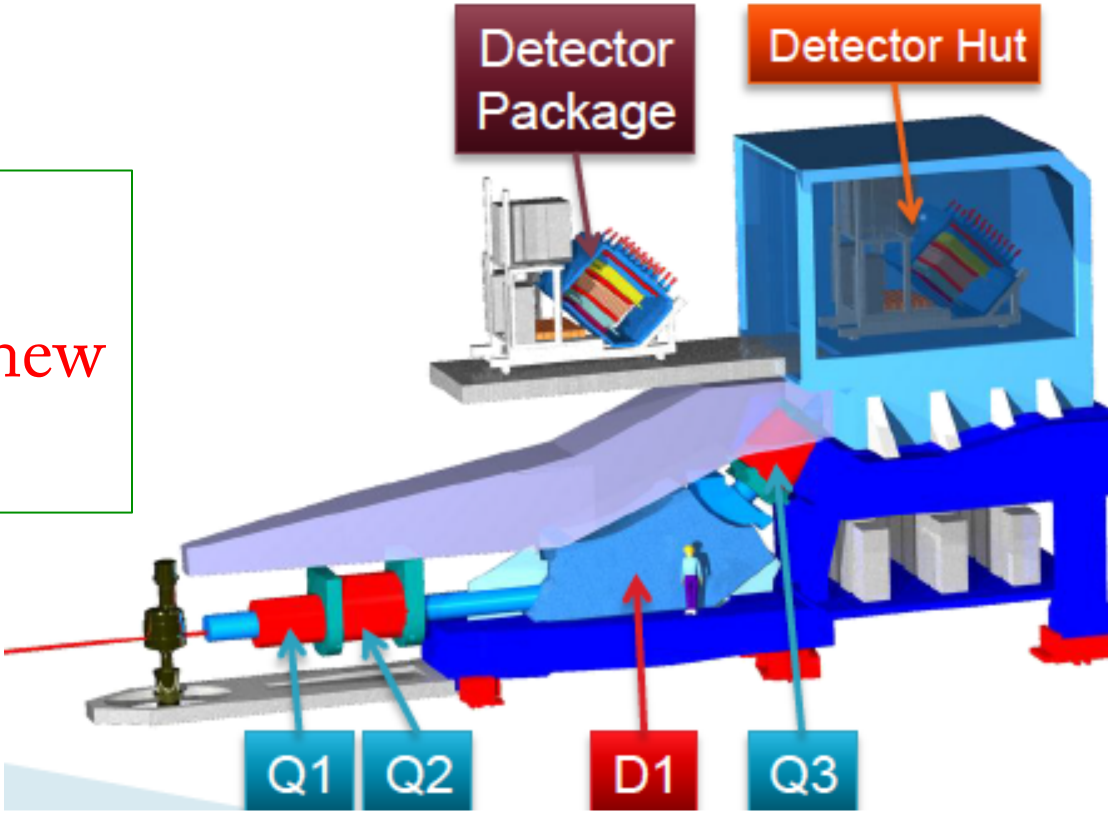
$\delta \equiv$ other corrections. (radiative etc.)

A New ep Scattering Experiment?

limitation on absolute cross sections (dσ/dΩ): ~ 2 ÷ 3%

- statistics is not a problem (<0.2%)
- control of systematic errors???
- beam flux, target thickness, windows,
- acceptances, detection efficiencies,
- …

A new high precision measurement requires a new experimental method.



The PRad Experimental Approach

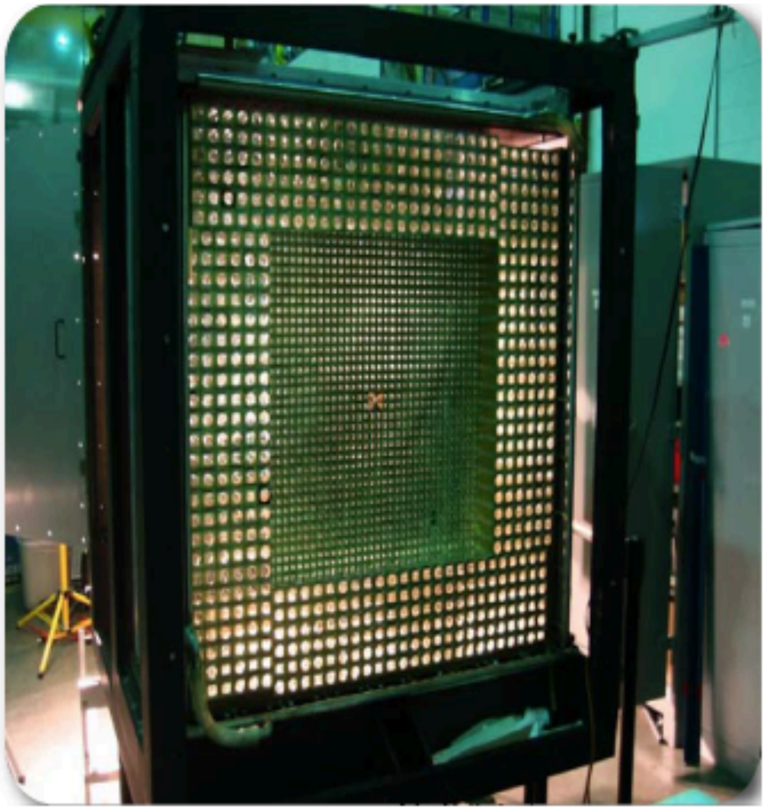
PRad initial goals:

- large Q^2 range in one experimental setting
- reach to very low Q^2 range (~ 10^{-4} GeV/C^2)
- reach to sub-percent precision in cross section

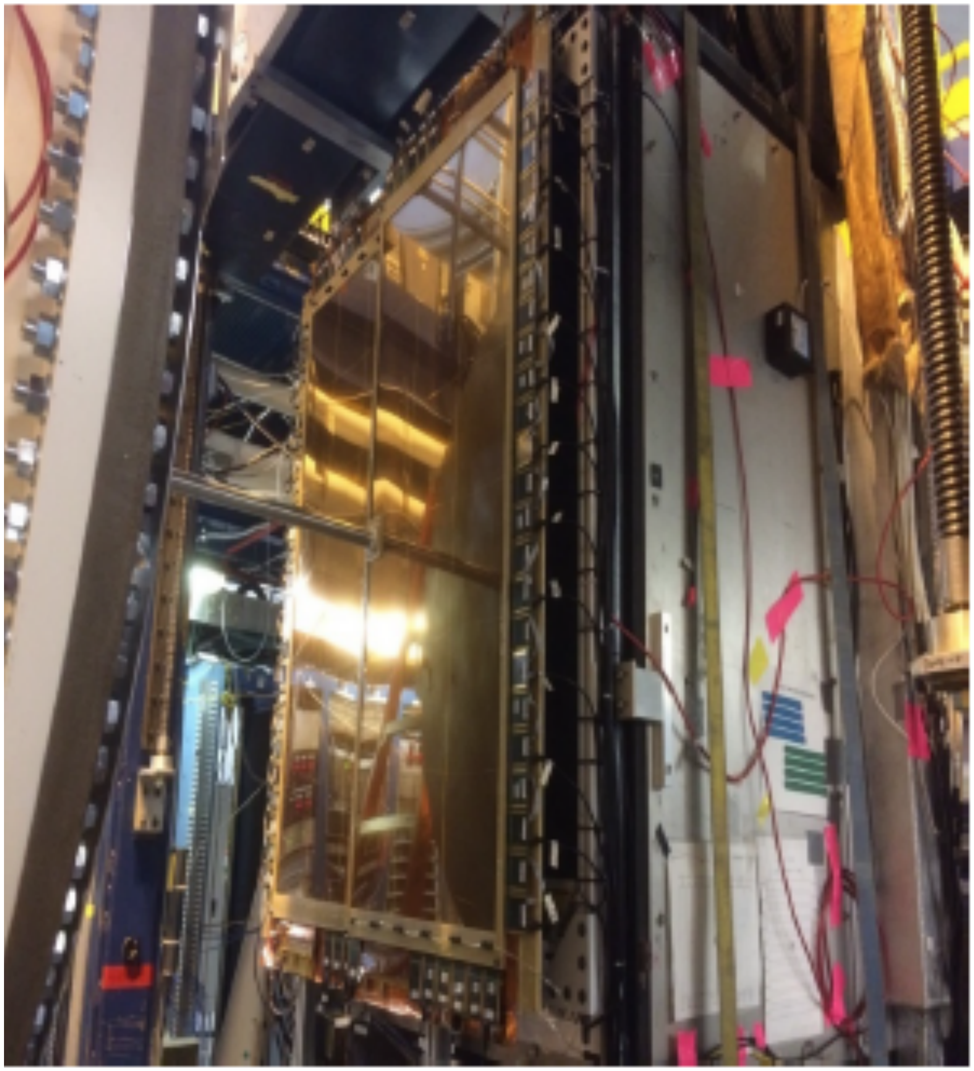
PRad suggested solutions

use high resolution high acceptance calorimeter:

- ✓ reach smaller scattering angles: ($\theta_e = 0.7^0 - 7.0^0$): ($Q^2 = 2\text{x}10^{-4} \div 6\text{x}10^{-2}$) GeV/c^2 ;
- ✓ large Q^2 range in one experimental setting!;

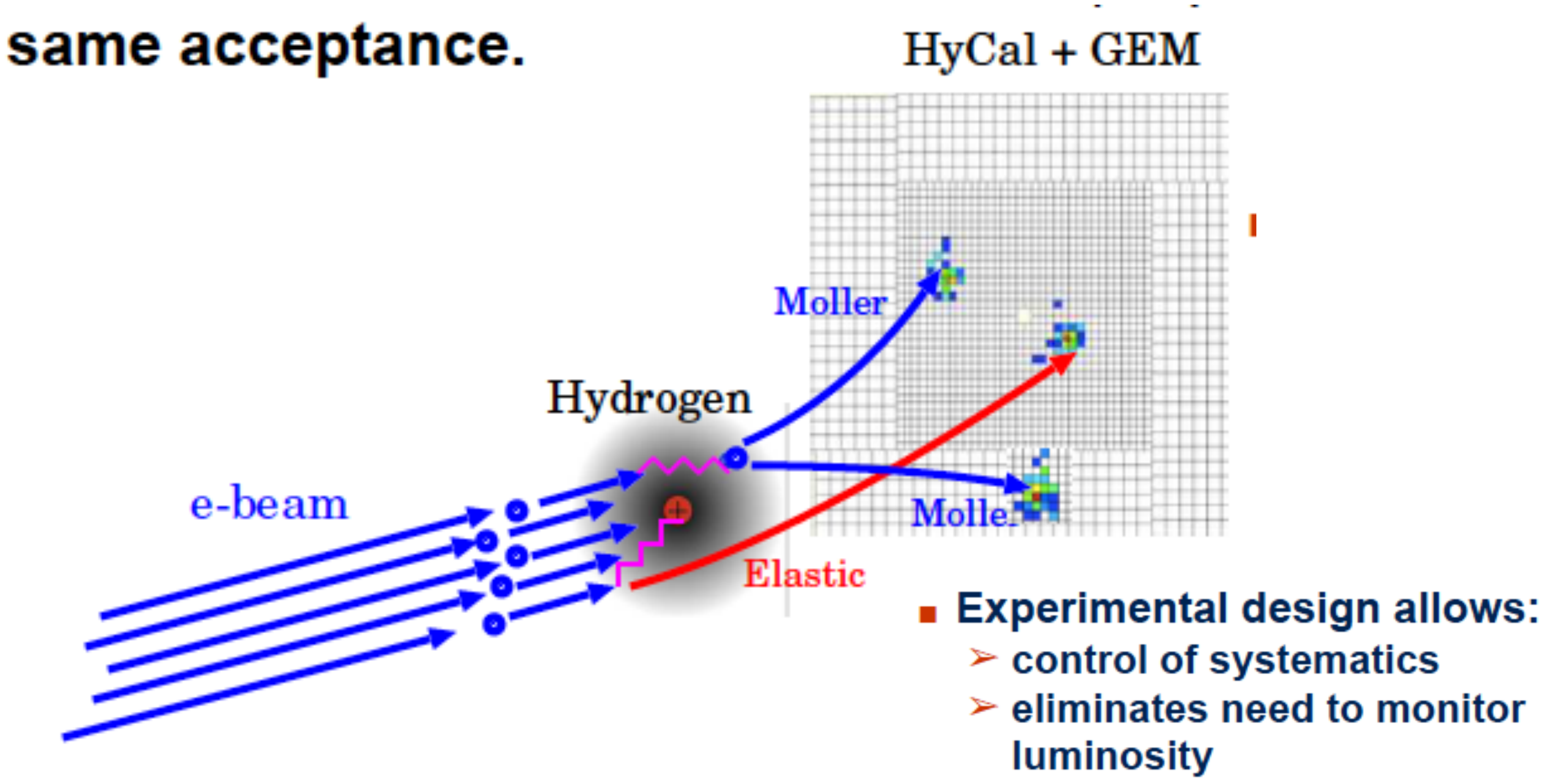


PRad Experimental Apparatus: GEM coordinate detectors



- Two large area GEM detectors (largest GEM detectors in the world at the time)
- Small overlap region in the middle
- Excellent position resolution (72 μm)
- Improve position resolution of the setup by > 20 times
- Large improvements in Q^2 determination

Our setup also allowed simultaneous detection of $ee \to ee$ Moller scattering (best known control of systematics).

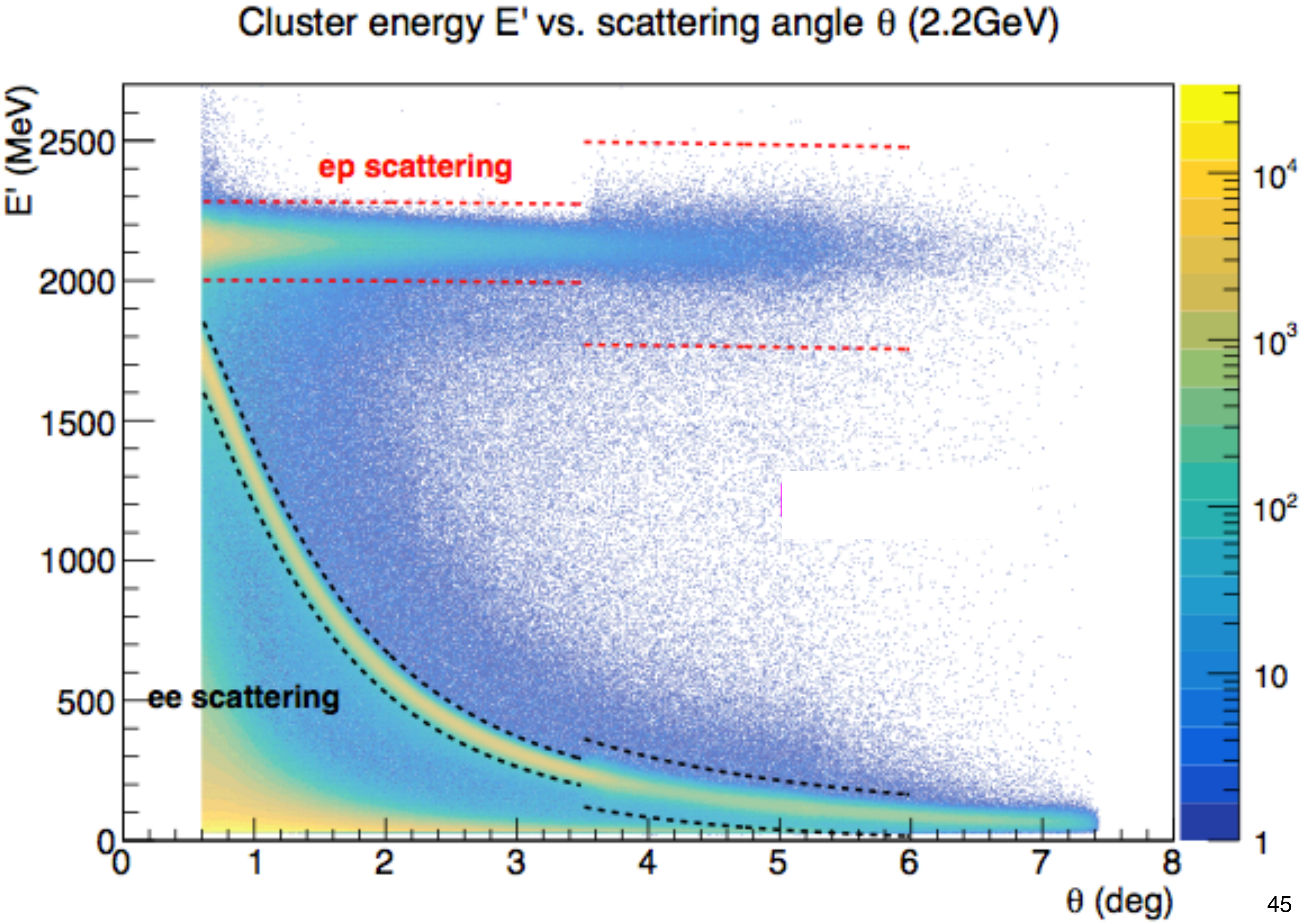


same acceptance.

- Experimental design allows:
  - control of systematics
  - eliminates need to monitor luminosity

$ee \to ee$ Moller scattering cross section is known with very high accuracy from QED

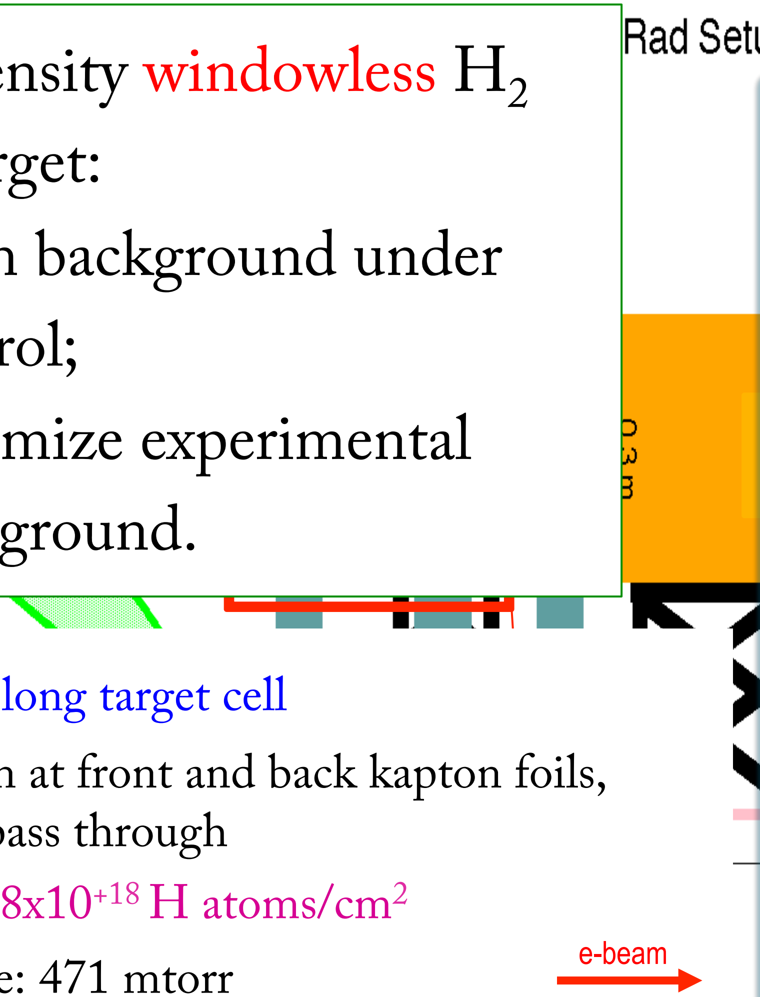
PRad data


Cluster energy E' vs. scattering angle θ (2.2GeV)
E' (MeV)
2500
2000
1500
1000
500
0
ep scattering
ee scattering
0
1
2
3
4
5
6
7
8
θ (deg)
10^4
10^3
10^2
10
1

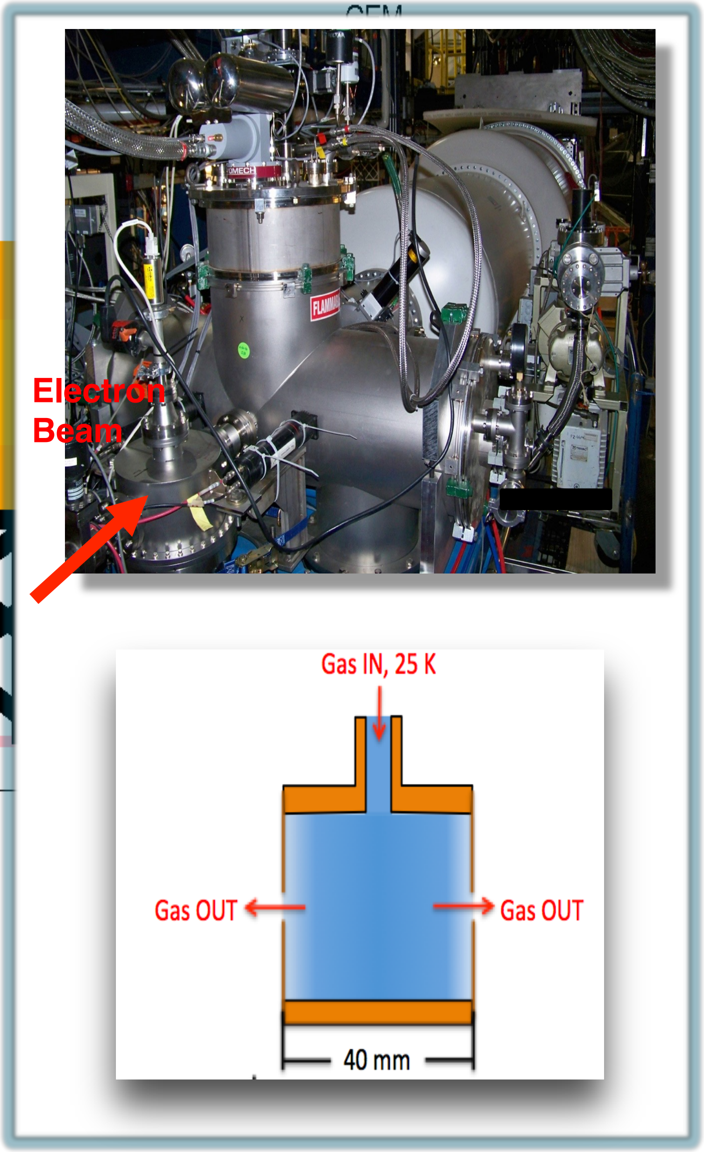
Windowless Gas Flow Target

use high density windowless H_2 gas flow target:

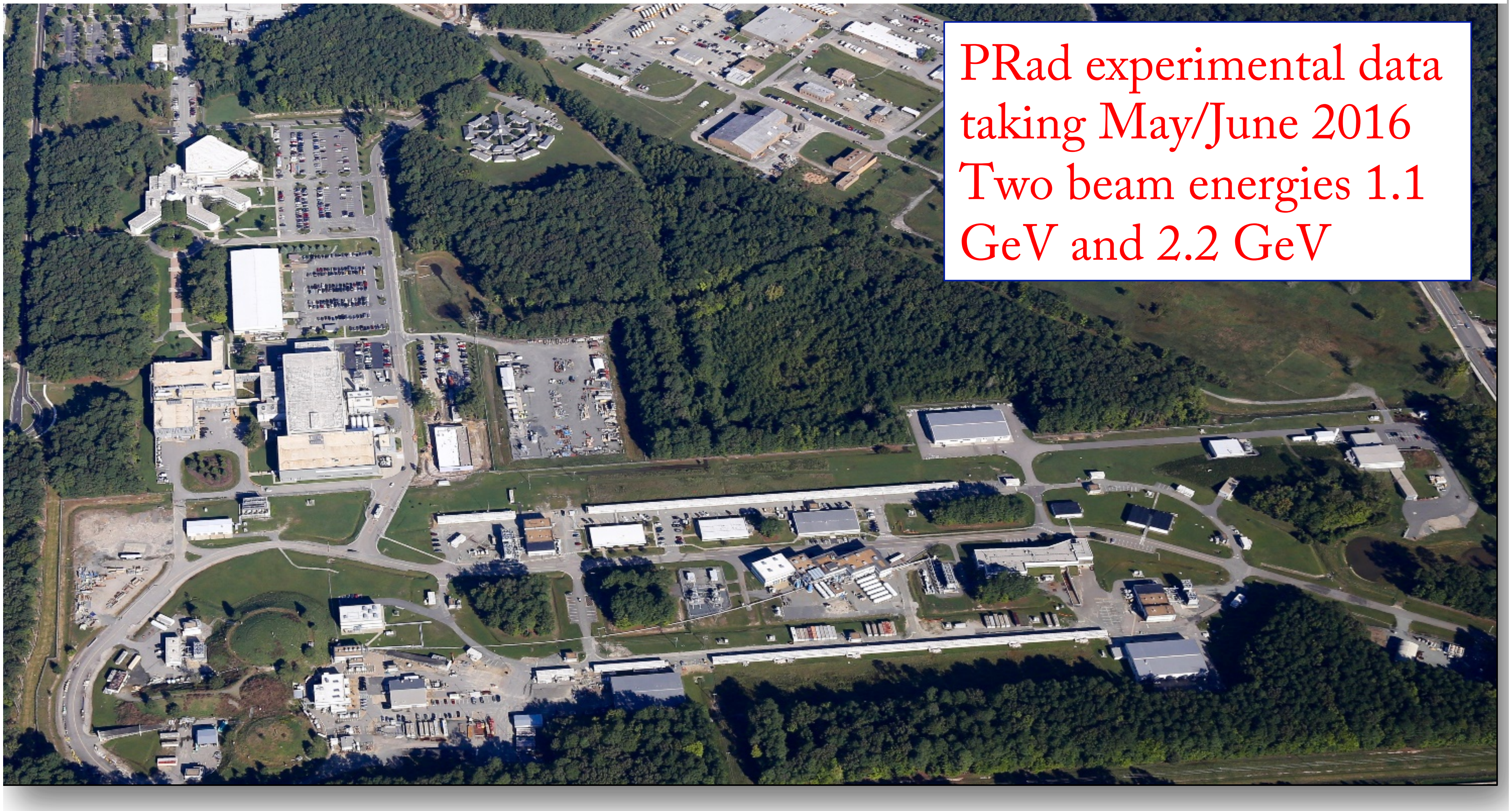
- ✓ beam background under control;
- ✓ minimize experimental background.



- 8 cm dia x 4 cm long target cell
- 2 mm holes open at front and back kapton foils, allows beam to pass through
- Areal density: $1.8\times10^{+18}$ H atoms/cm^2
  - cell pressure: 471 mtorr
  - chamber pressure: 2.34 mtorr
  - cell vs. chamber pressures: 200:1 was reached.
- Gas temperature: 19.5 K



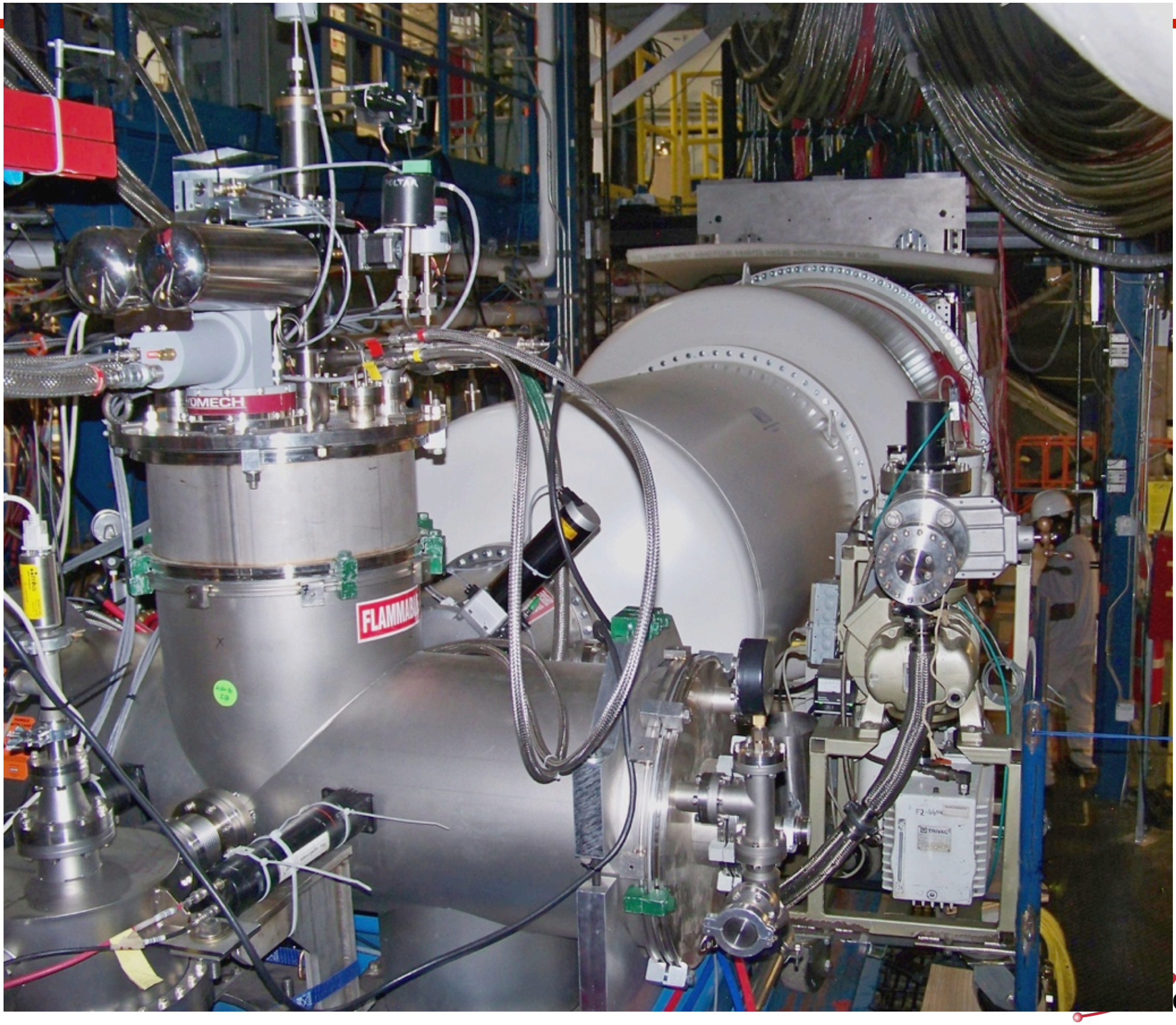
PRad experiment was carried out at Jefferson lab, located in Virginia



PRad experimental data taking May/June 2016
Two beam energies 1.1 GeV and 2.2 GeV

PRad was one of the first experiments to run at Jefferson lab after its major upgrade

PRad Experimental in Hall B

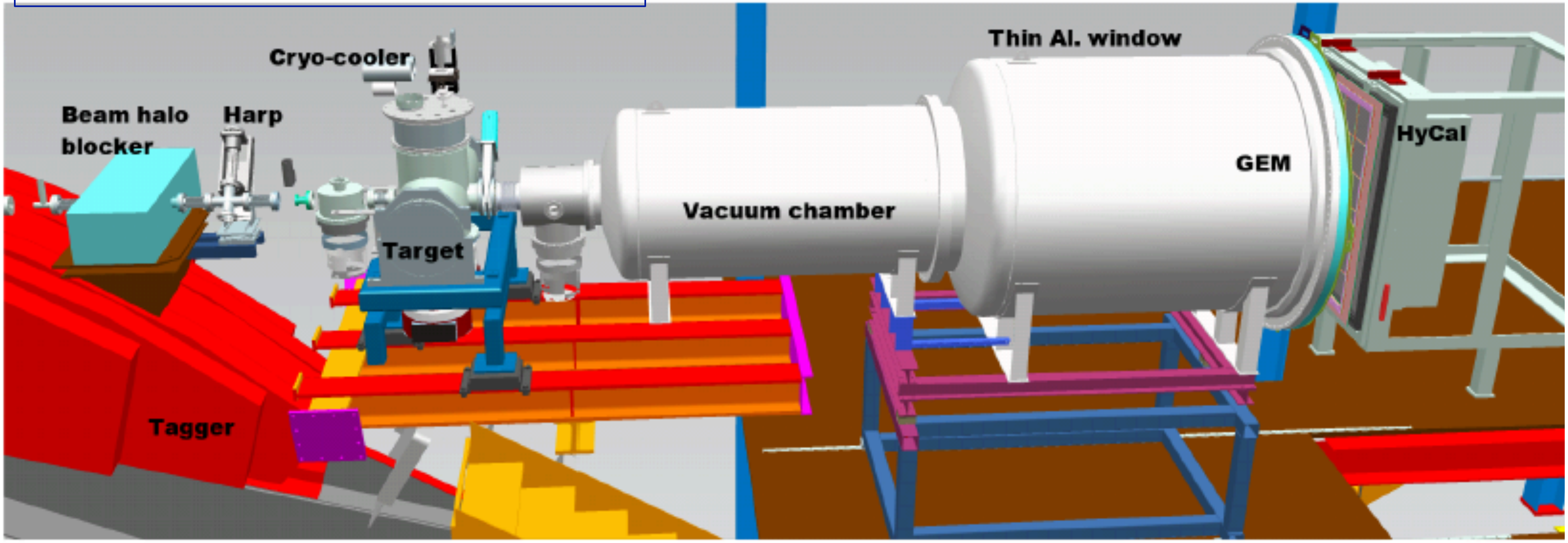


PRad Experimental Setup in Hall B at JLab (schematics)

- Main detector elements:
  - windowless H_2 gas flow target
  - PrimEx HyCal calorimeter
  - vacuum box with one thin window at HyCal end
  - X,Y – GEM detectors on front of HyCal

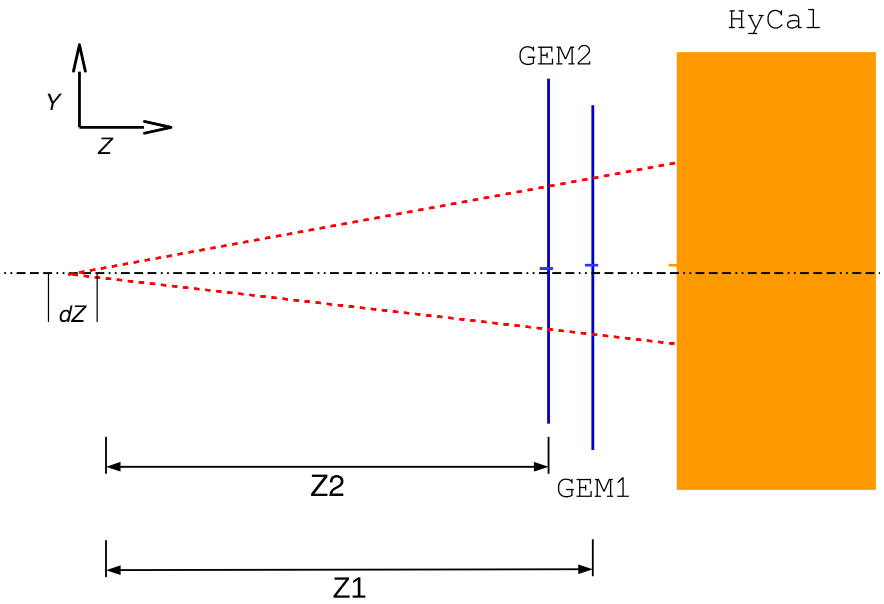
- Beam line equipment:
  - standard beam line elements (0.1 – 50 nA)
  - photon tagger for HyCal calibration
  - collimator box (6.4 mm collimator for photon beam, 12.7 mm for e^- beam halo "cleanup")
  - Harp 2H00
  - pipe connecting Vacuum Window through HyCal

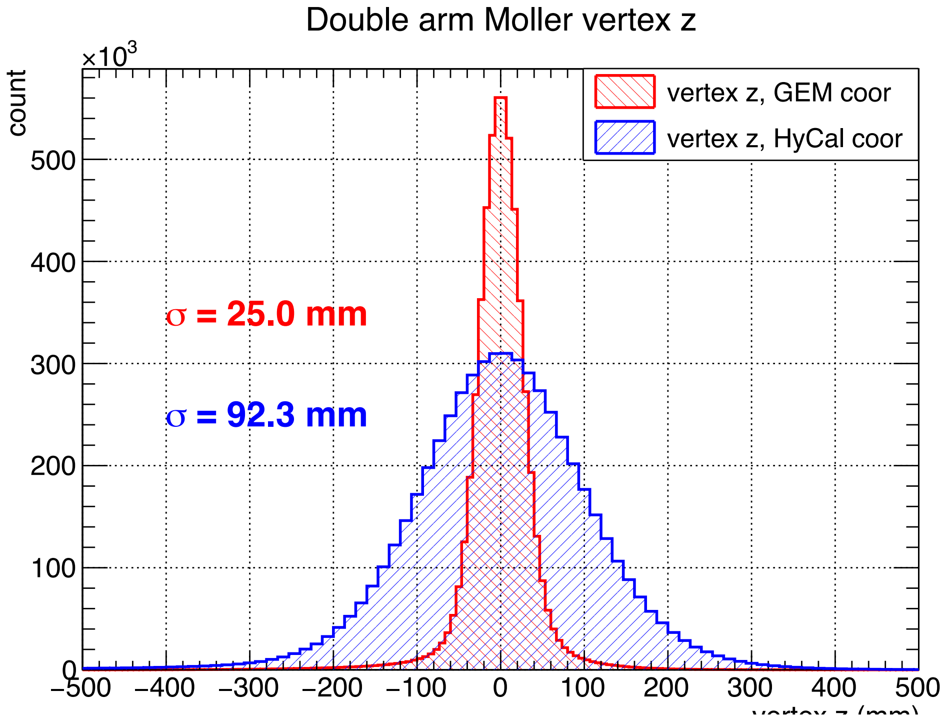
PRad experimental data taking May/June 2016: Two beam energies 1.1 GeV and 2.2 GeV



Detector Position Calibration

- Engineering survey, done before the experiment.
- Detector offsets and z position from double-arm Moller events:
  - co-planarity to determine offsets;
  - Møller kinematics to determine detector z position (cross check surveyed data);
  - offset with ~ 50 μm and z with ~ 1 mm precision;





Recent Developments in Fitting Procedures

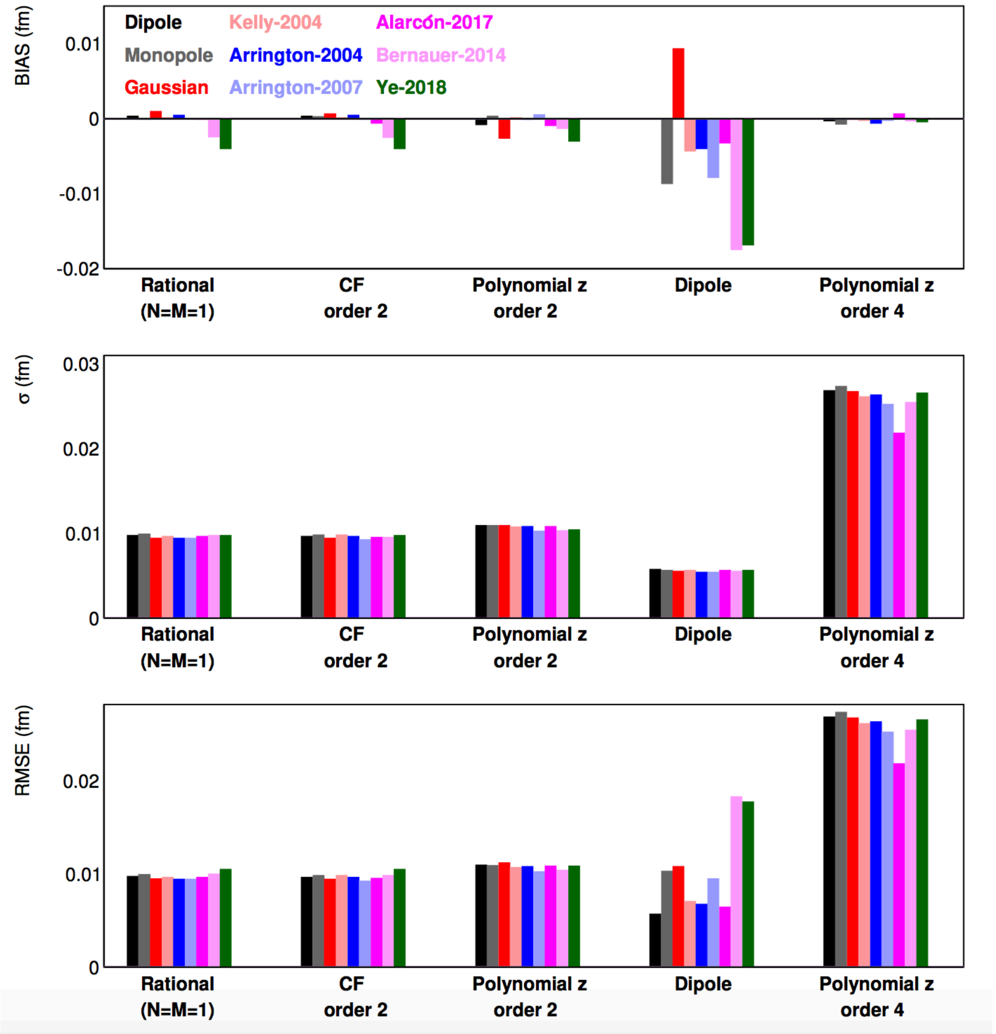
✓ X. Yan, D.W. Higinbotham, D. Dutta et al.
"*Robust extraction of the proton charge radius from electron-proton scattering data*"
Published in: *PRC 98, 2, 025204, 2018*

➢ The input form factors (with known Rp) are used to generate pseudodata with fluctuations mimicking the binning and random uncertainty of a set of real data.

➢ All combinations of input functions and fit functions can then be tested repeatedly against regenerated pseudodata.

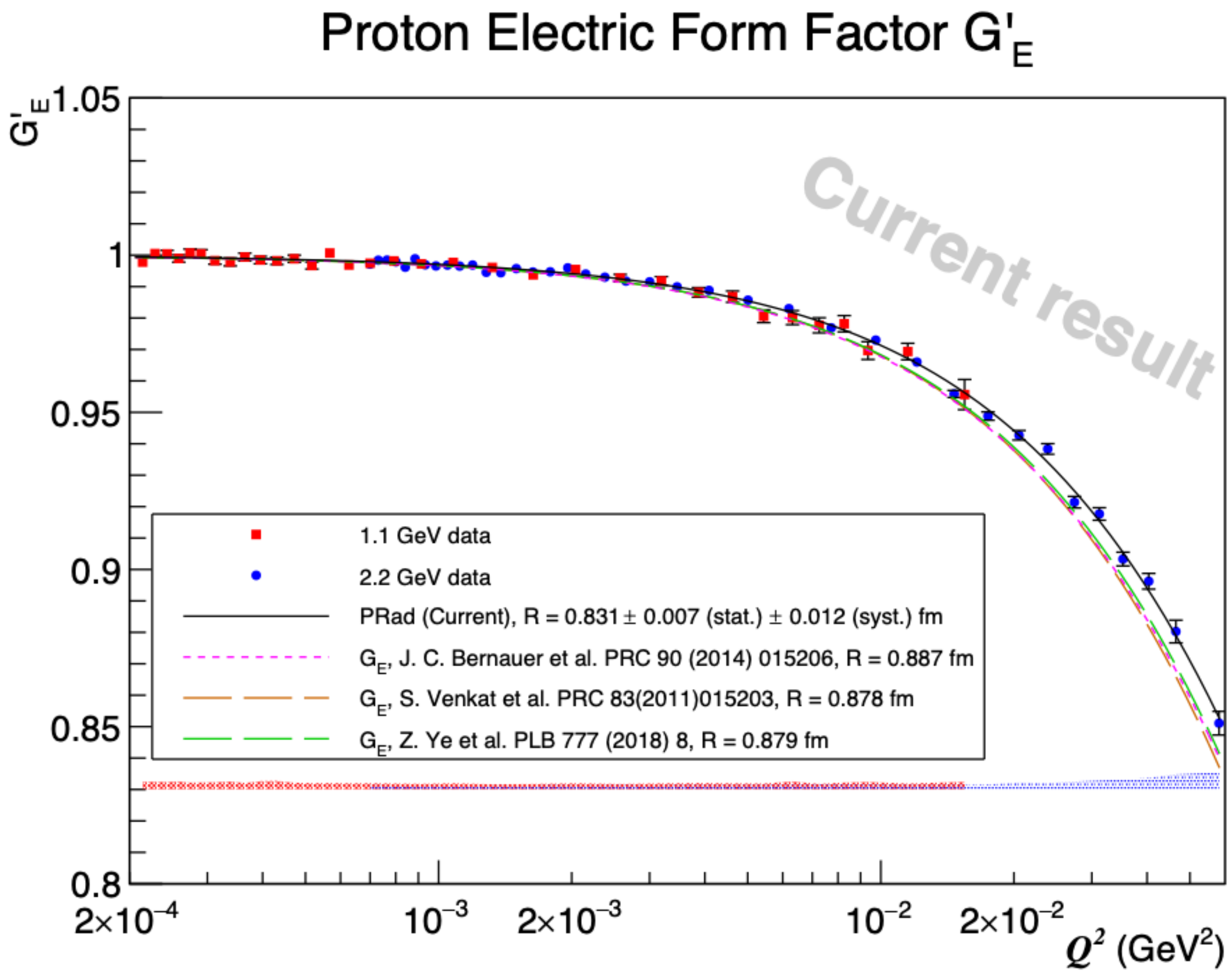
➢ Since the input radius is known, this allows us to find fitting functions that are robust for proton radius extractions in an objective fashion.

➢ … we find that a two-parameter rational function, a two-parameter continued fraction, and the second-order polynomial expansion of z can extract the input radius regardless of the input charge form factor function that is used.



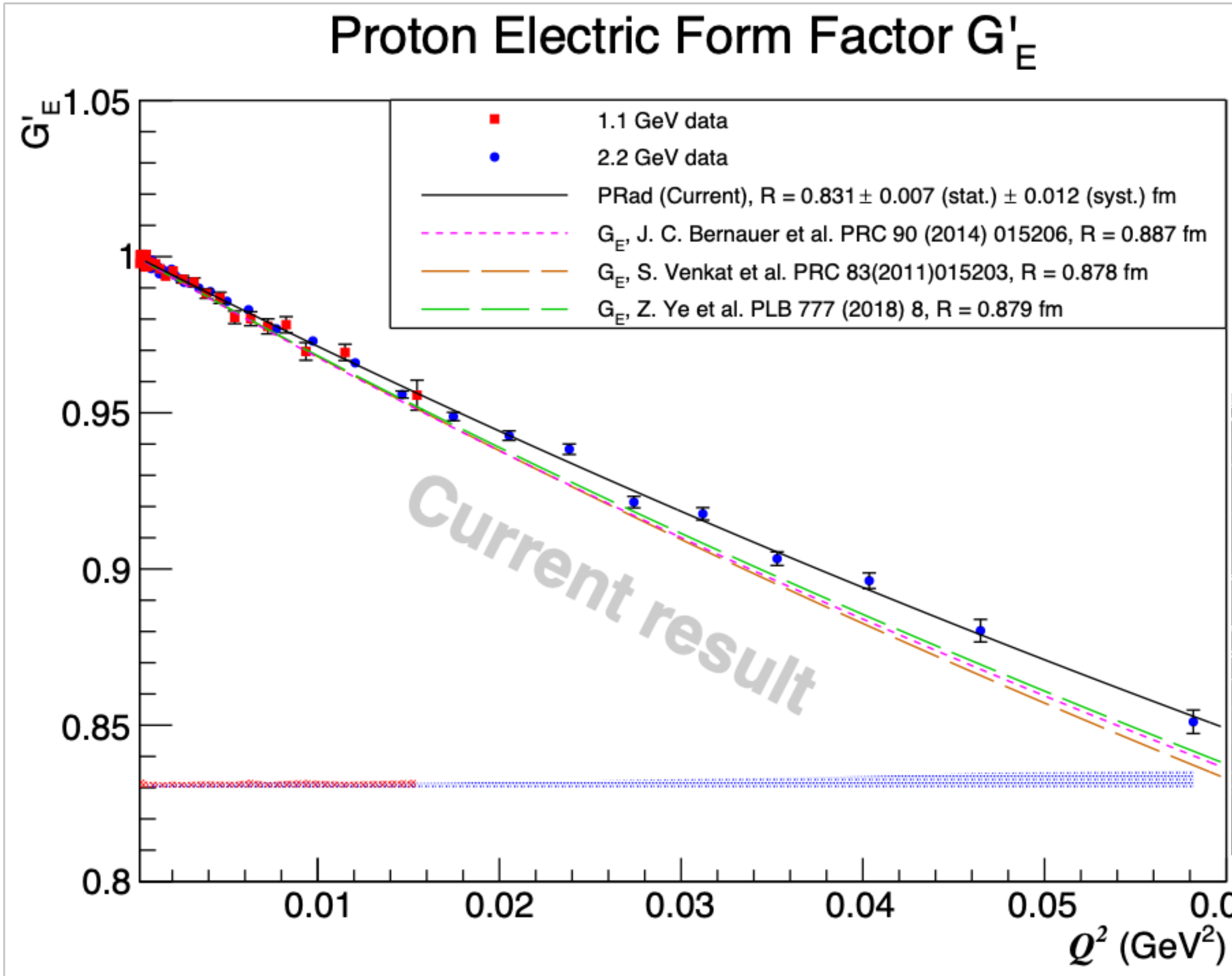
$$f_{\text{rational}}(Q^2) = p_0 G_E(Q^2) = p_0 \frac{1 + \sum_{i=1}^{N} p_i^{(a)} Q^{2i}}{1 + \sum_{j=1}^{M} p_j^{(b)} Q^{2j}},$$

Proton Electric Form Factor with Recent Models



Plots courtesy of Weizhi Xiong

PRad result from Duke analysis

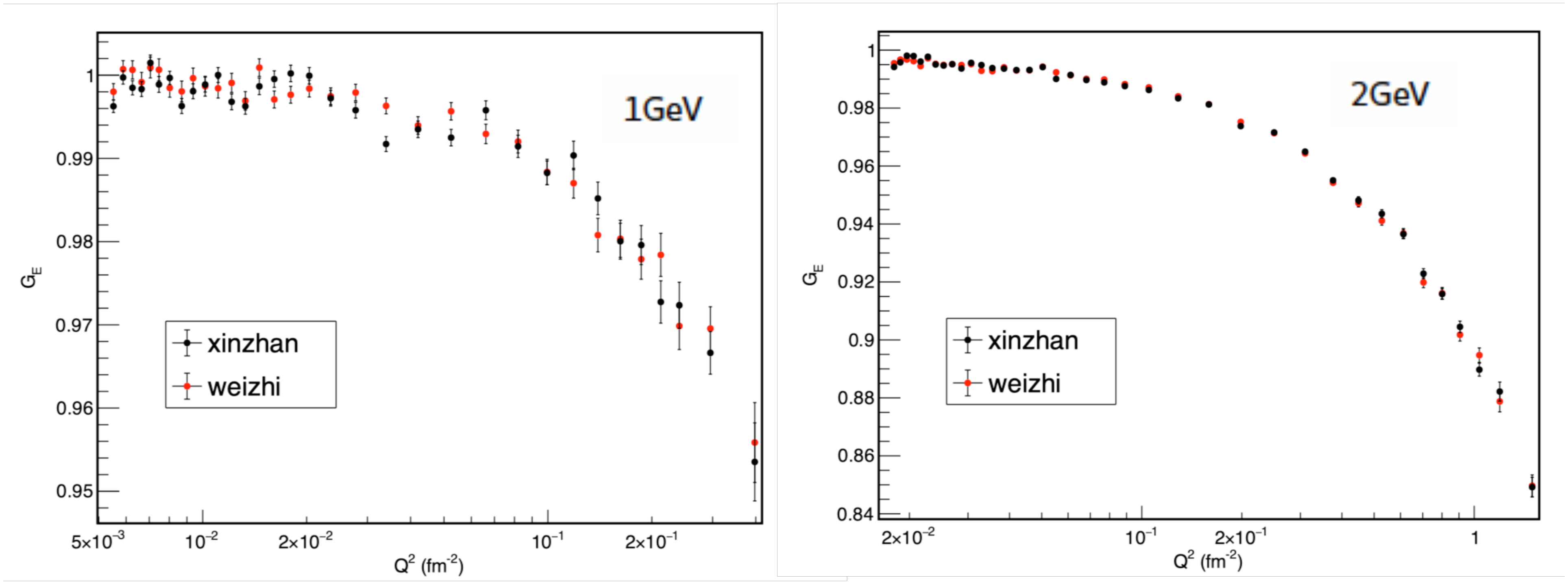


Prad result:
R_p = 0.831 ± 0.007 (stat.) ± 0.012 (syst.) fm

Plots courtesy of Weizhi Xiong

PRad result from UVa analysis

courtesy of Xinzhan Bai

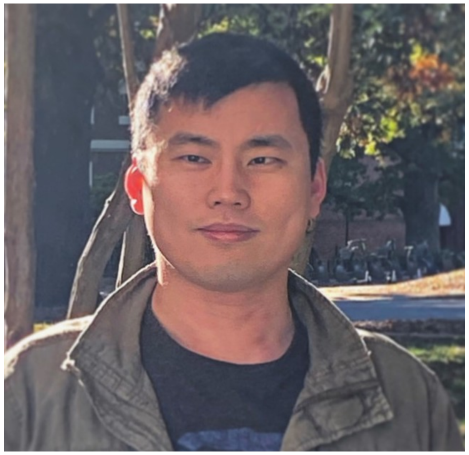


Prad result from Duke:

R_p = 0.831 ± 0.007 (stat.) ± 0.012 (syst.) fm

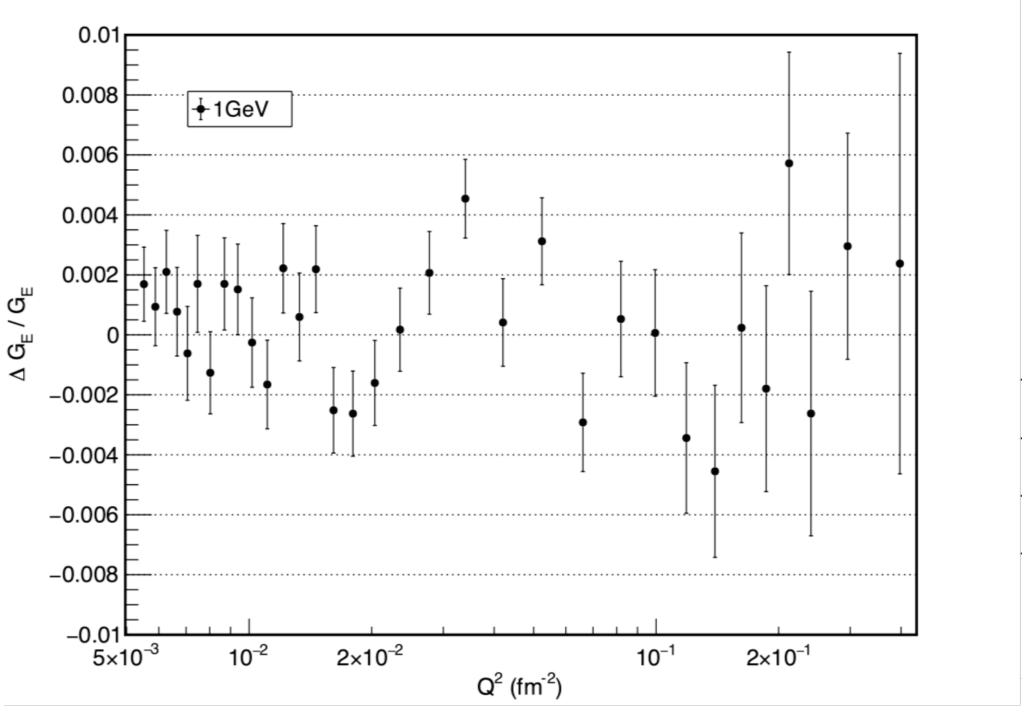
Prad result from UVa :

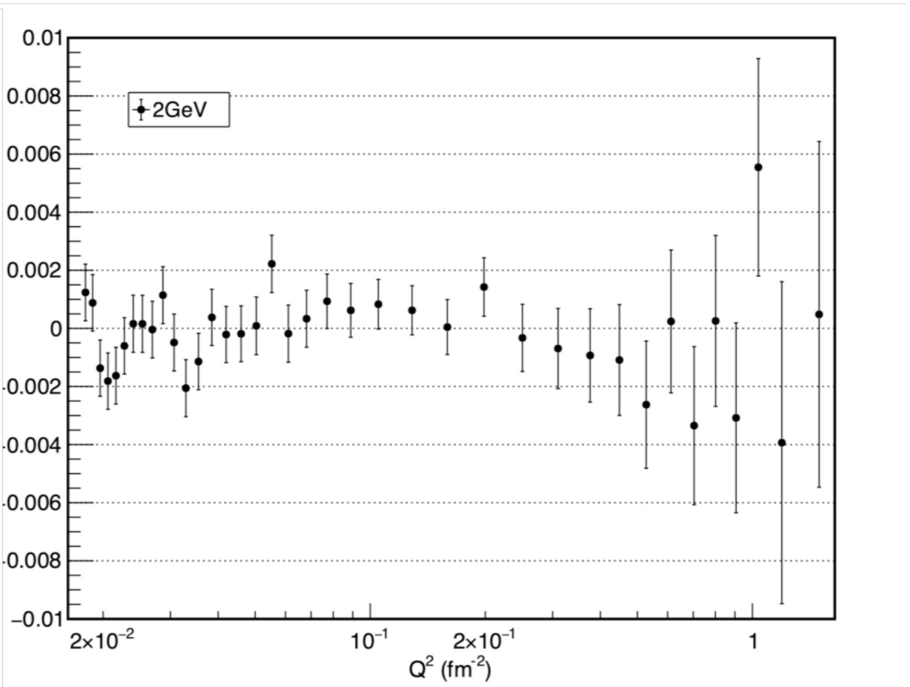
R_p = 0.833 ± 0.007 (stat.) ± 0.012 (syst.) fm



PRad result: UVa analysis compared to Duke analysis.

courtesy of Xinzhan Bai





Prad result from Duke:

R_p = 0.831 ± 0.007 (stat.) ± 0.012 (syst.) fm

Prad result from UVa :

R_p = 0.833 ± 0.007 (stat.) ± 0.012 (syst.) fm

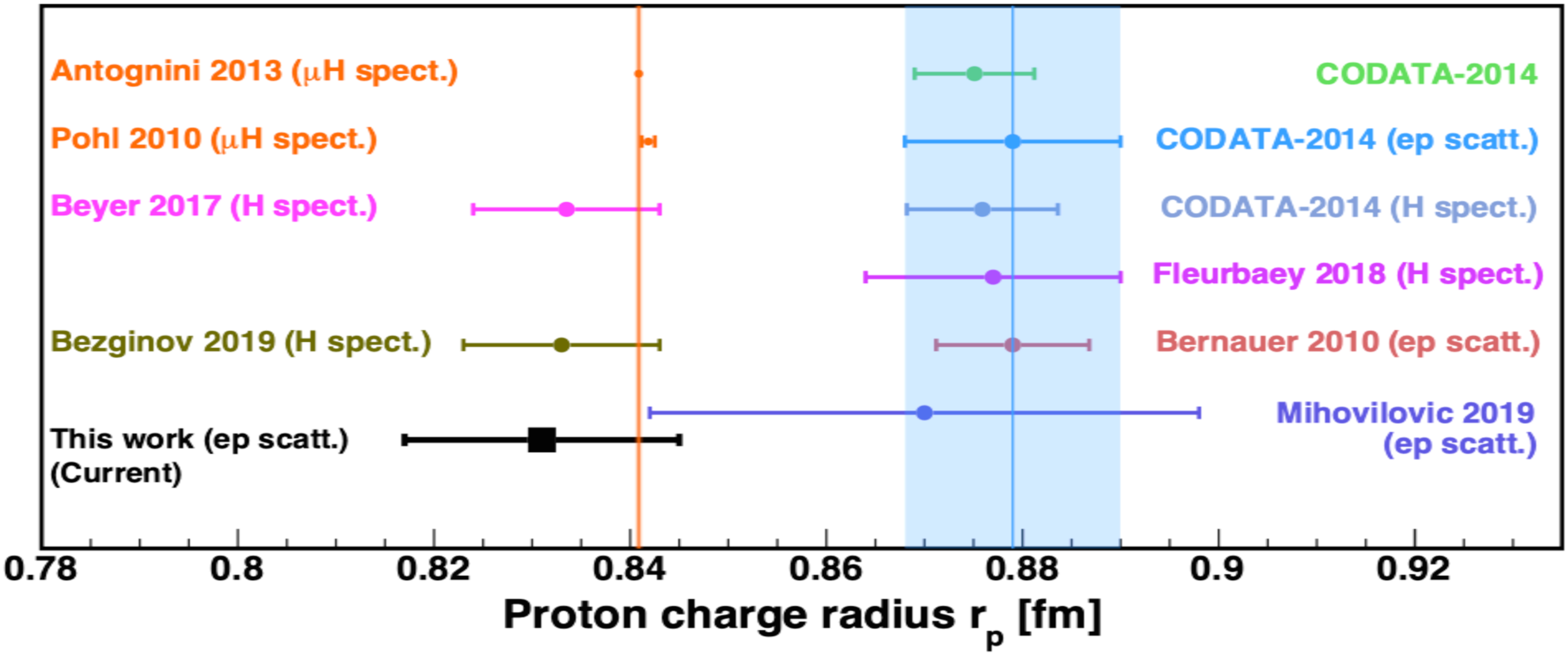
Systematic Uncertainties on Rp (Preliminary)

Showing only major items

Item	**R_p uncertainty (fm)**	**n_1 uncertainty (1GeV)**	**n_2 uncertainty (2GeV)**
Event selection	(0.0052)0.0092	(0.0002)0.0008	(0.0005)0.0011
Acceptance	(0.0024)0.0054	(0.0001)0.0001	(0.0001)0.0001
Beam background	(0.0038)0.0039	(0.0017)0.0020	(0.0003)0.0003
Detector efficiency	(0.0038)0.0045	(0.0001)0.0001	(0.0001)0.0001
Beam energy	(0.0022)0.0084	(0.0001)0.0001	(0.0002)0.0003
HyCal response	(0.0020)0.0032	(0.0000)0.0000	(0.0000)0.0001
Inelastic ep	(0.0009)0.0051	(0.0000)0.0001	(0.0000)0.0000
Radiative corrections	(0.0070)0.0070	(0.0011)0.0009	(0.0011)0.0009
Total	(0.0109)0.0175	(0.0020)0.0023	(0.0013)0.0015

(Current numbers in brackets)

Proton Radius from PRad



Prad result:
R_p = 0.831 ± 0.007 (stat.) ± 0.012 (syst.) fm

Nature paper in print: will come out on Nov 7

What's Next ?

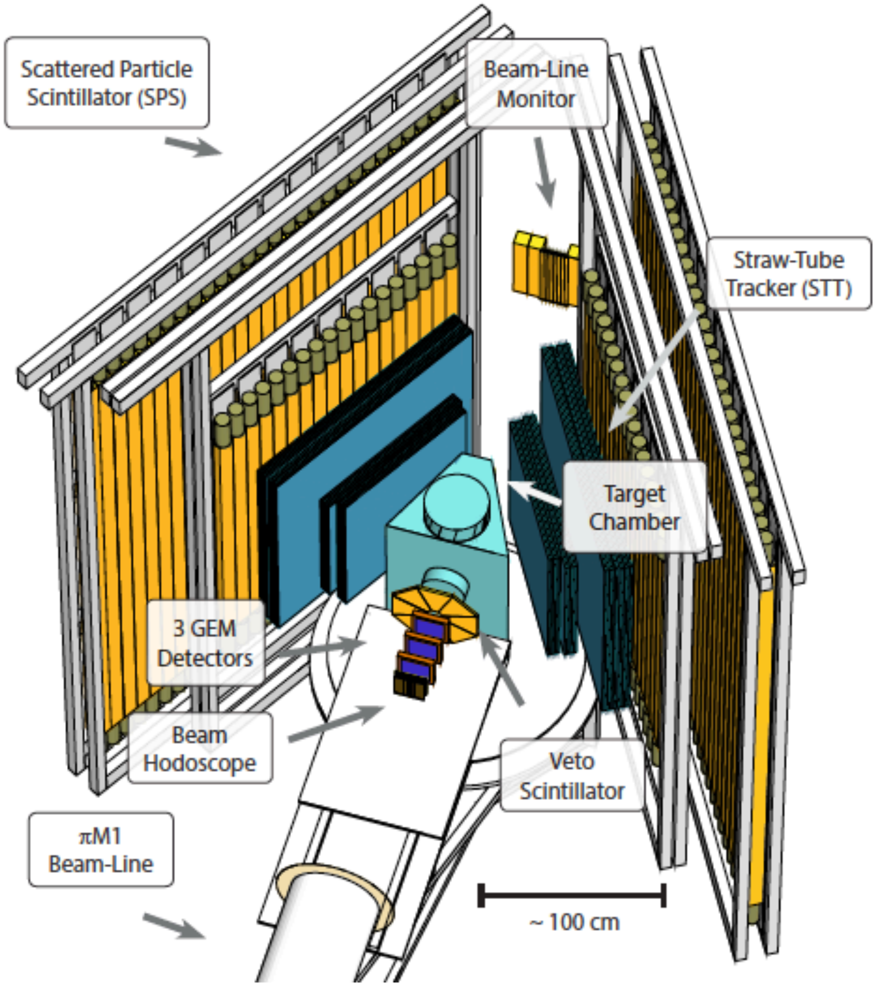
- ☐ Several other experiments around the world.
    - ☐ μP scattering: MUSE at PSI
    - ☐ ProRad at Grenoble
    - ☐ ULQ2 at Tokohu
    - ☐ ISR measurement at MESA @ Mainz
- ☐ DRad and an even more precise PRad

MUSE @ PSI

Paul Scherrer Institute πM1 Beam

- 590 MeV proton beam, 2.2mA, 1.3MW beam, 50.6MHz RF frequency
- World's most powerful proton beam
- Converted to $e^\pm$, $\mu^\pm$, $\pi^\pm$ in piM1 beamline
- Separate out particle species by timing relative to beam RF
- Cut as many pions as possible, trigger on $e^\pm$, $\mu^\pm$

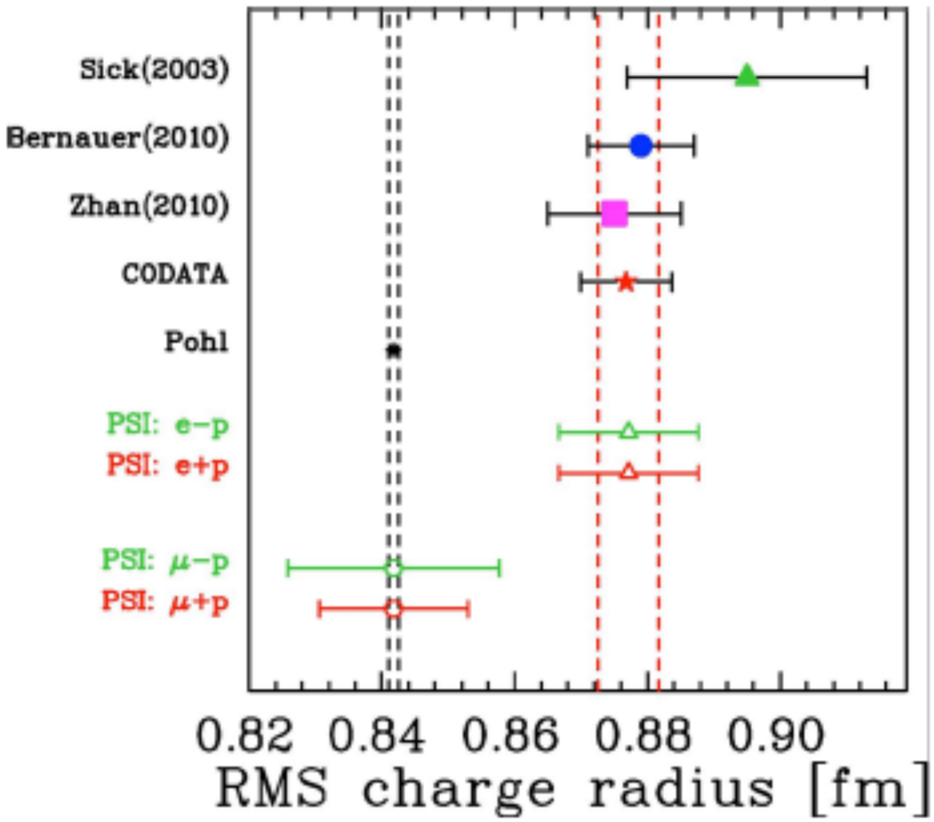
MUSE Experiment



- Low beam flux. → Large angle, non-magnetic detectors.
- Secondary beam. → Tracking of beam particles to target.
- Mixed beam. → Identification of beam particle in trigger.

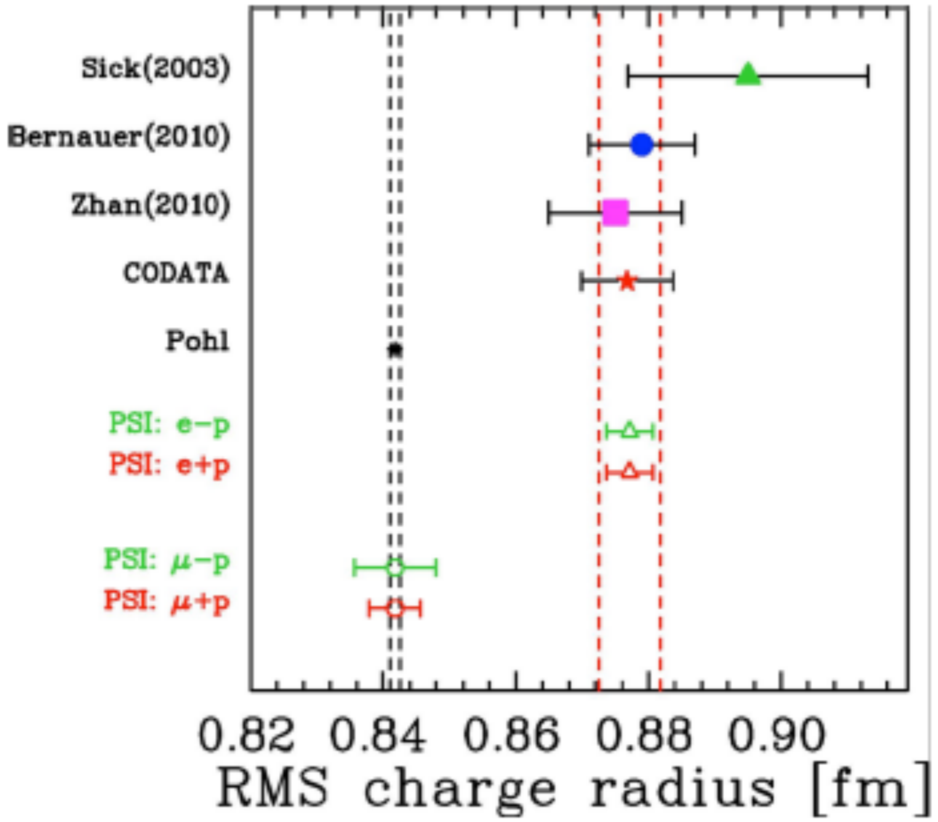
MUSE expected results

Absolute radius extraction uncertainties similar to previous experiments

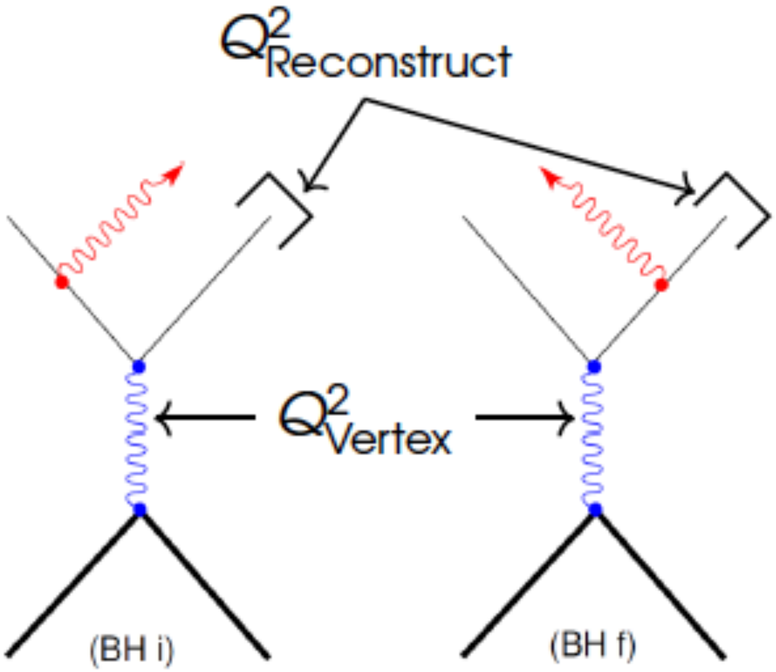


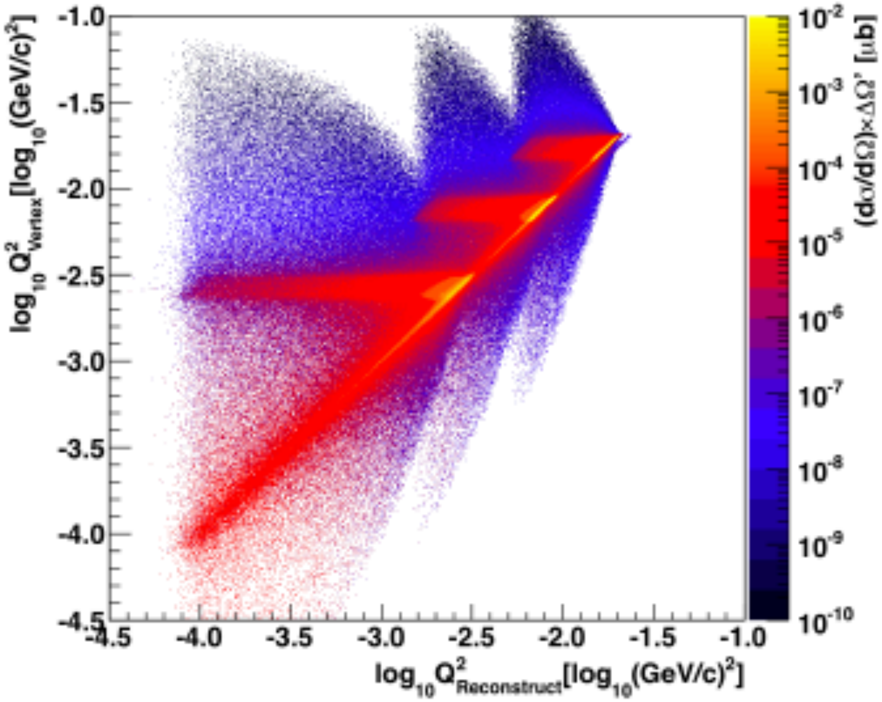
MUSE expected results

- Absolute radius extraction uncertainties similar to previous experiments
- However, common uncertainties cancel



Mainz ISR

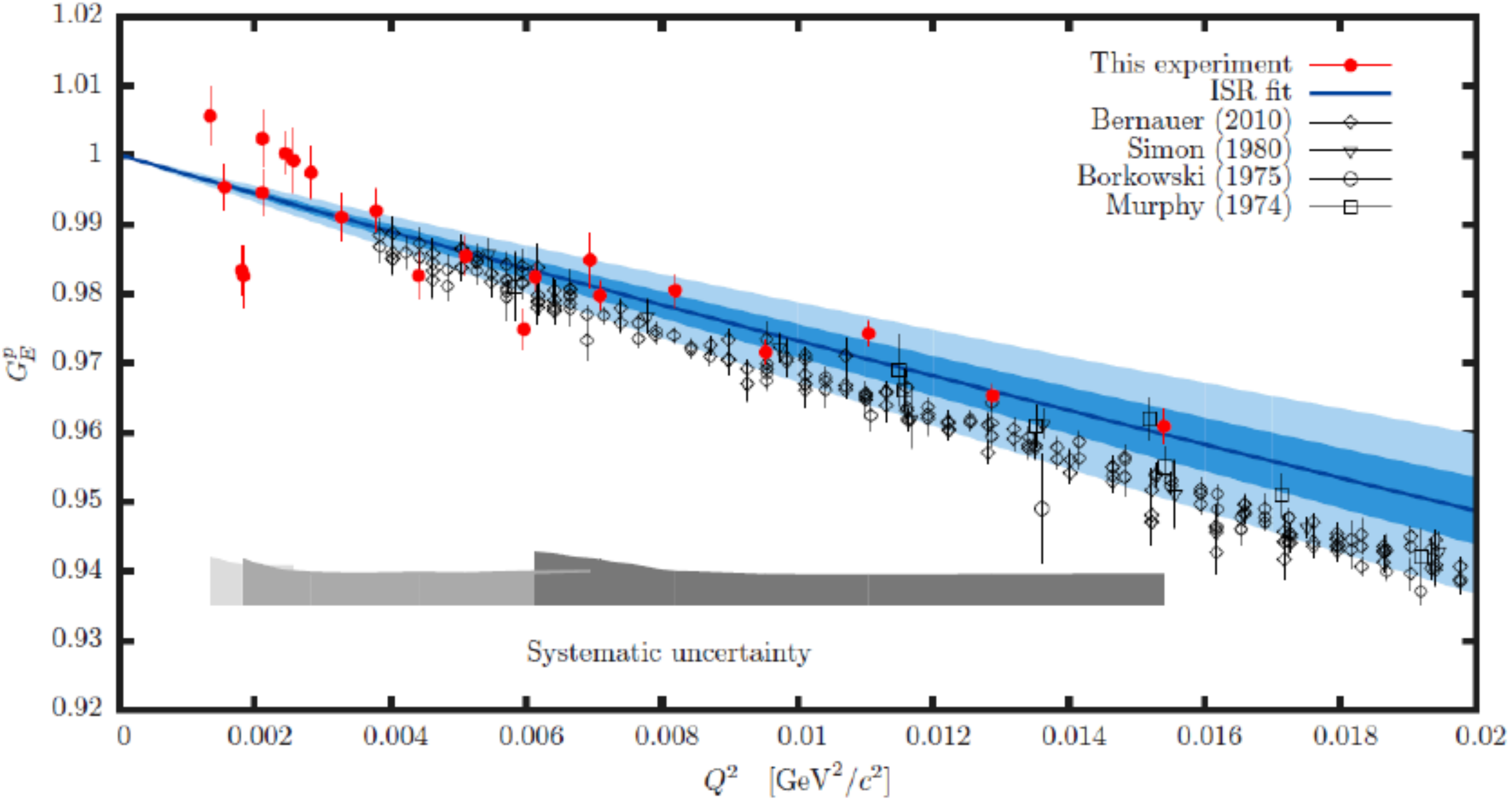




- Use initial state radiation to reduce effective beam energy
- Have to subtract FSR

Jan Bernauer

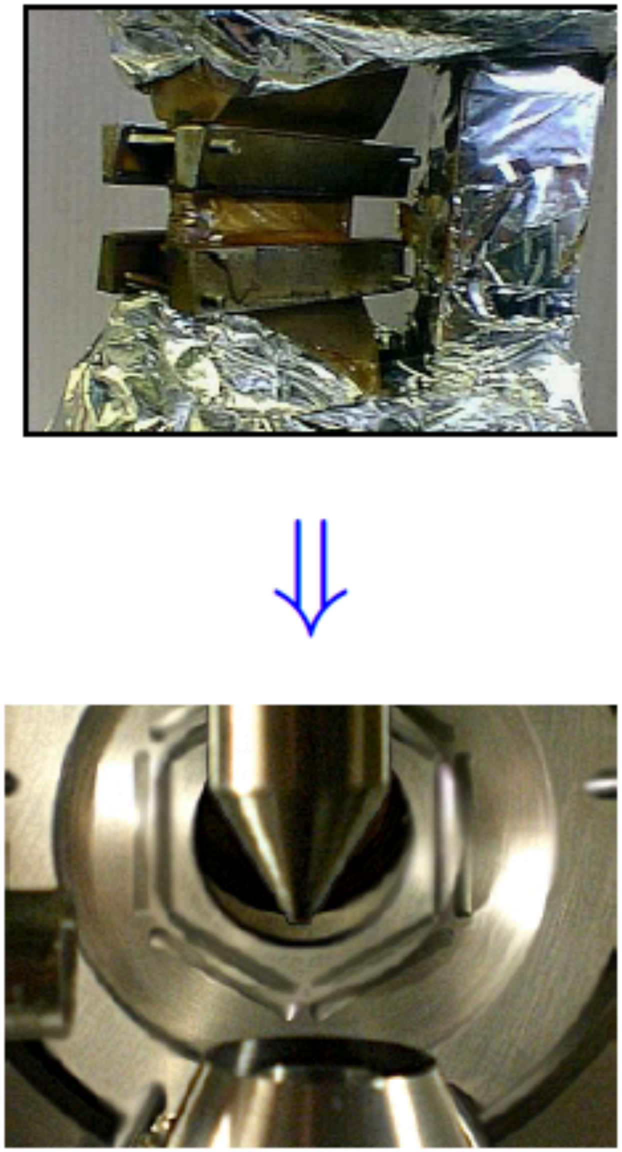
Mainz ISR



- Result from the pilot measurement: R_p = 0.810 ± 0.035 (stat.) ± 0.074 (syst.) fm
- Not competitive
- New measurement planed with MESA with the Jet target

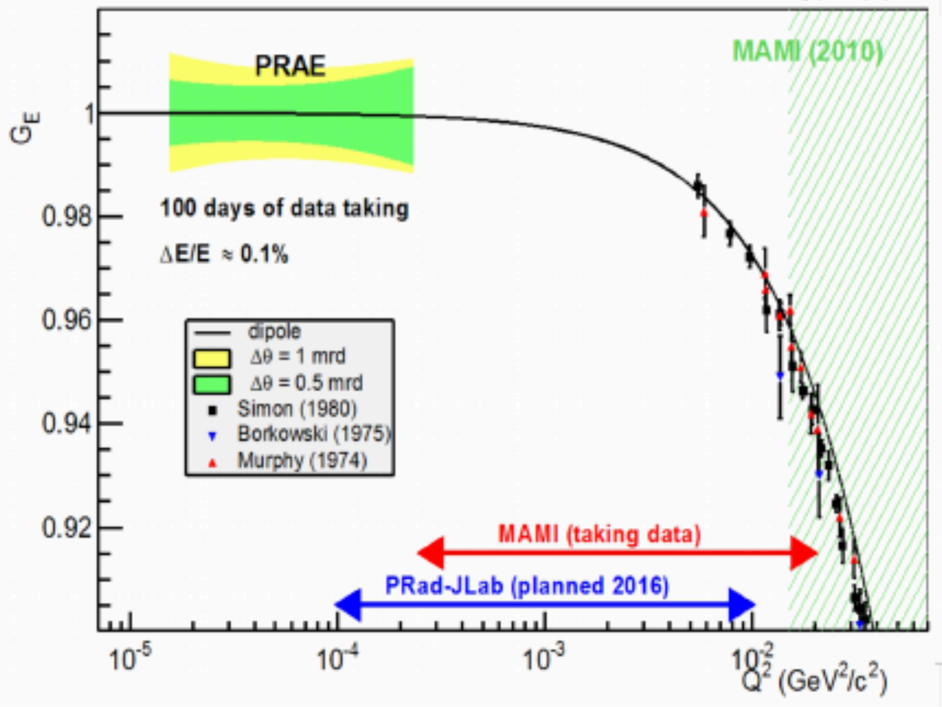
Mainz ISR

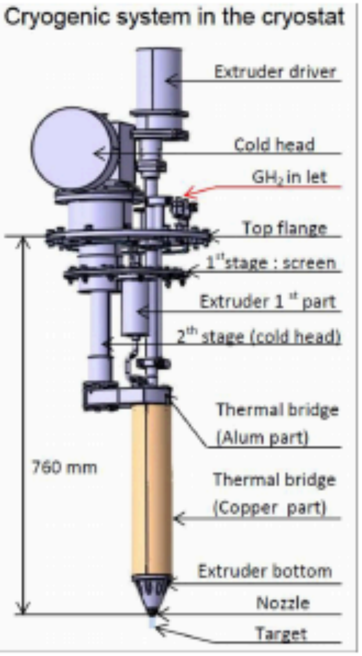
- For Mainz data, systematic errors dominate
  - Background from target walls
  - Acceptance correction for extended target
- Eliminate with jet target
  - point-like
  - no walls
  - but less density
- Rinse, repeat with D,^{3}He,^{4}He, ...



Platform for Research and Applications with Electrons: ProRad

J.-M. Gheller et al. AIP Conf. Proc. 1573 (2014) 58



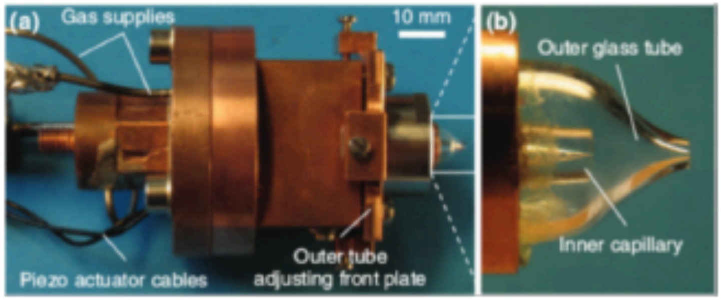


CHyMENE

Details from
Eric Voutier
LPSC, Grenoble
(France).

Bi-national ANR proposal with Francfort University submitted.

Droplet Sream



- New accelerator to be built in France,
- Beginning measurement 2020
- Measurements in unexplored Q^2-range
  - ➔ **1.5×10^{-5} – 3×10^{-4} (GeV/c^2)^2**
- Constrain Q^2-dependence of G_E and extrapolation to zero
- Non-magnetic spectrometer, frozen hydrogen wire / film target

ULQ^2 @ Sendai

ULQ^2 collaboration
(Ultra-Low Q^2)


60 MeV e-linac


1.3 GeV synchrotron

1) elastic e+p scattering at ultra-low Q^2 region

2) $G_E(Q^2)$ at $0.0003 \leq Q^2 \leq 0.008$ $(\mathrm{GeV}/c)^2$

3) G_E is extracted by Rosenbluth separation

4) Absolute cross section measurement

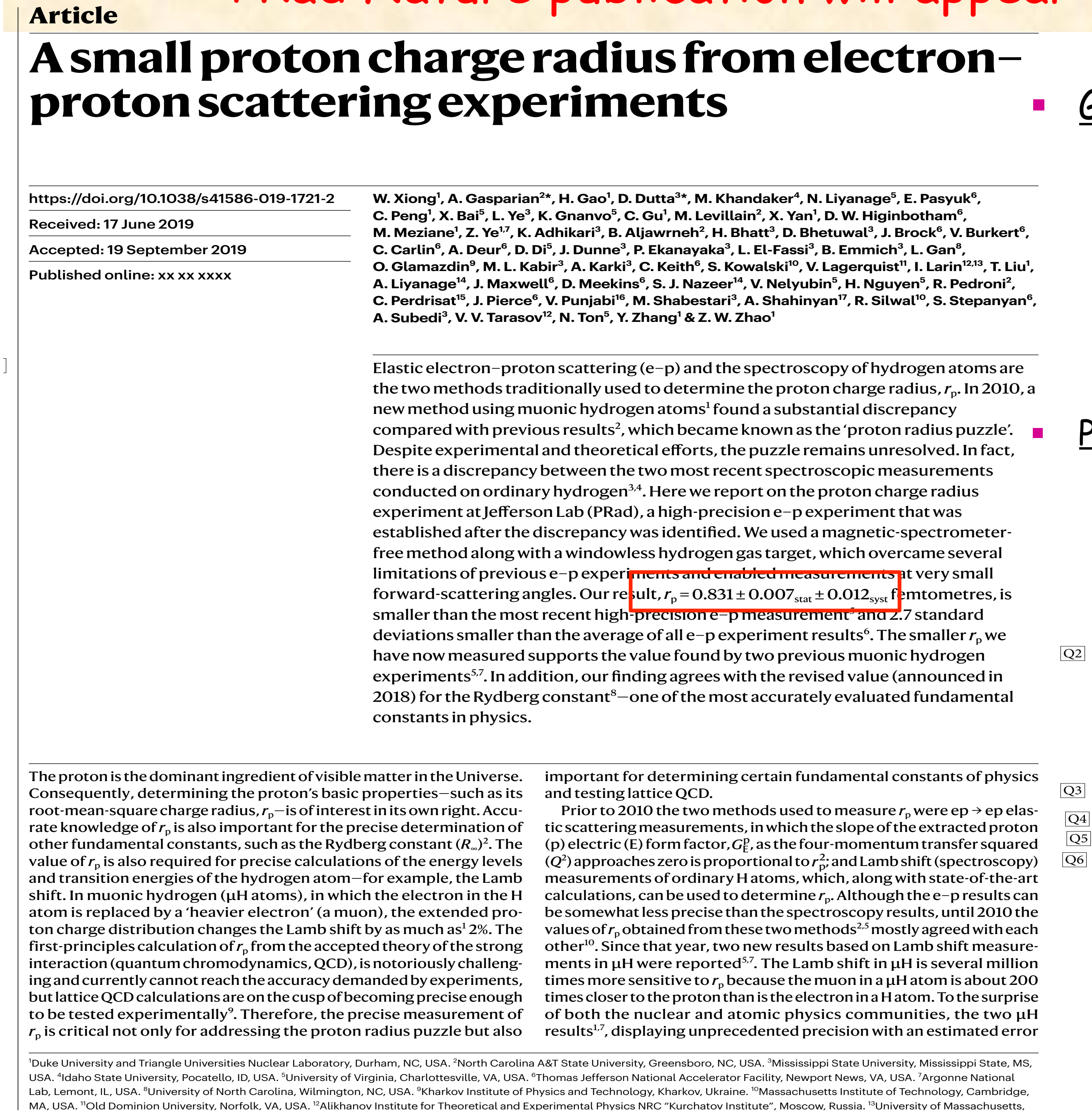
relative to $^{12}C(e,e)^{12}C$: sys. err. ~3×10^{-3}

5) Ee = 20 - 60 MeV, θ = 30 - 150°

6) the new beam line, and spectrometer are under construction

7) the experiments will start in 2019

PRad Nature publication will appear on Nov 7.



Article

A small proton charge radius from electron–proton scattering experiments

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Elastic electron–proton scattering (e–p) and the spectroscopy of hydrogen atoms are the two methods traditionally used to determine the proton charge radius, r_p. In 2010, a new method using muonic hydrogen atoms[1] found a substantial discrepancy compared with previous results[2], which became known as the 'proton radius puzzle'. Despite experimental and theoretical efforts, the puzzle remains unresolved. In fact, there is a discrepancy between the two most recent spectroscopic measurements conducted on ordinary hydrogen[3,4]. Here we report on the proton charge radius experiment at Jefferson Lab (PRad), a high-precision e–p experiment that was established after the discrepancy was identified. We used a magnetic-spectrometer-free method along with a windowless hydrogen gas target, which overcame several limitations of previous e–p experiments and enabled measurements at very small forward-scattering angles. Our result, $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ femtometres, is smaller than the most recent high-precision e–p measurement[5] and 2.7 standard deviations smaller than the average of all e–p experiment results[6]. The smaller r_p we have now measured supports the value found by two previous muonic hydrogen experiments[5,7]. In addition, our finding agrees with the revised value (announced in 2018) for the Rydberg constant[8]—one of the most accurately evaluated fundamental constants in physics.

Q2

The proton is the dominant ingredient of visible matter in the Universe. Consequently, determining the proton's basic properties—such as its root-mean-square charge radius, r_p—is of interest in its own right. Accurate knowledge of r_p is also important for the precise determination of other fundamental constants, such as the Rydberg constant $(R_\infty)^2$. The value of r_p is also required for precise calculations of the energy levels and transition energies of the hydrogen atom—for example, the Lamb shift. In muonic hydrogen (μH atoms), in which the electron in the H atom is replaced by a 'heavier electron' (a muon), the extended proton charge distribution changes the Lamb shift by as much as[1] 2%. The first-principles calculation of r_p from the accepted theory of the strong interaction (quantum chromodynamics, QCD), is notoriously challenging and currently cannot reach the accuracy demanded by experiments, but lattice QCD calculations are on the cusp of becoming precise enough to be tested experimentally[9]. Therefore, the precise measurement of r_p is critical not only for addressing the proton radius puzzle but also important for determining certain fundamental constants of physics and testing lattice QCD.

Prior to 2010 the two methods used to measure r_p were ep → ep elastic scattering measurements, in which the slope of the extracted proton (p) electric (E) form factor, G_E^p, as the four-momentum transfer squared (Q^2) approaches zero is proportional to r_p^2; and Lamb shift (spectroscopy) measurements of ordinary H atoms, which, along with state-of-the-art calculations, can be used to determine r_p. Although the e–p results can be somewhat less precise than the spectroscopy results, until 2010 the values of r_p obtained from these two methods[2,5] mostly agreed with each other[10]. Since that year, two new results based on Lamb shift measurements in μH were reported[5,7]. The Lamb shift in μH is several million times more sensitive to r_p because the muon in a μH atom is about 200 times closer to the proton than is the electron in a H atom. To the surprise of both the nuclear and atomic physics communities, the two μH results[1,7], displaying unprecedented precision with an estimated error

Q3
Q4
Q5
Q6

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Summary

- PRad was uniquely designed and performed in May/June of 2016 to address the *"Proton Radius Puzzle"*:
  - data in a large Q^2 range have been recorded with the same experimental settings, $[2\mathrm{x}10^{-4} \div 6\mathrm{x}10^{-2}]$ GeV/C^2.
  - lowest Q^2 data set ($\sim 10^{-4}$ GeV/C^2) has been collected for the first time in ep-scattering experiments;
  - simultaneous measurement of the Moller and Mott scattering processes has been demonstrated to control systematic uncertainties.

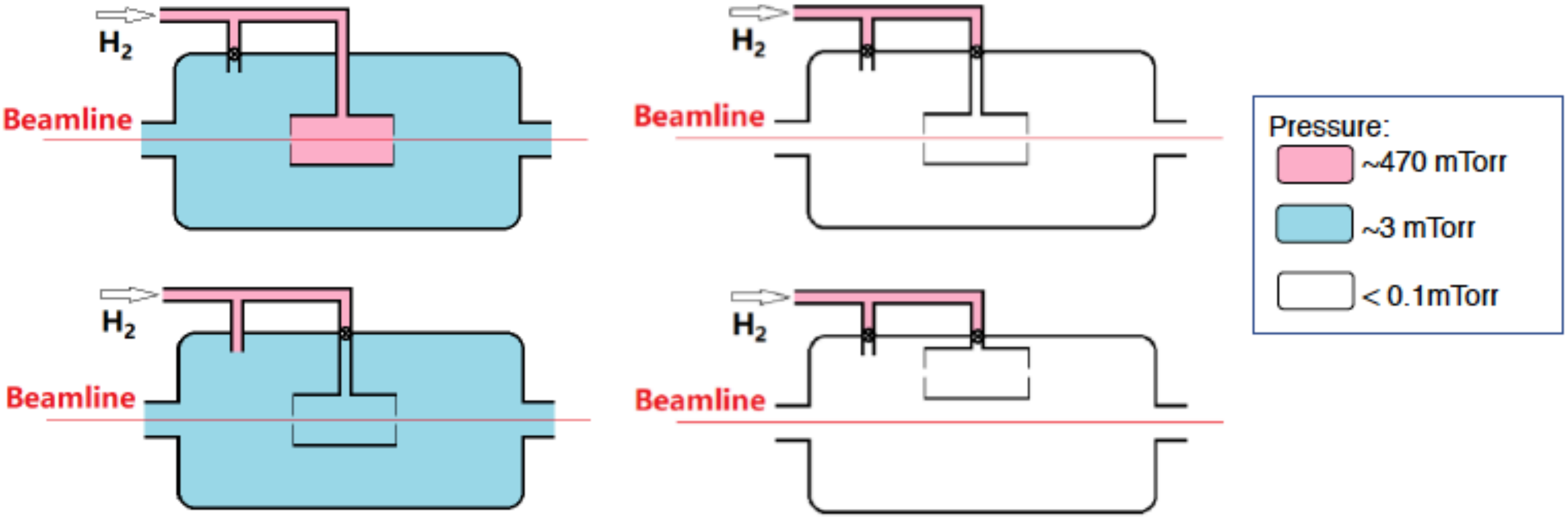
- The final result from the PRad experiment is:

$$R_p = 0.831 \pm 0.007\ (\text{stat.}) \pm 0.012\ (\text{syst.})\ \text{fm}$$

- The article with the Final result will appear online in a few days.
- Stay tuned for PRad-II and DRad

Background Subtraction

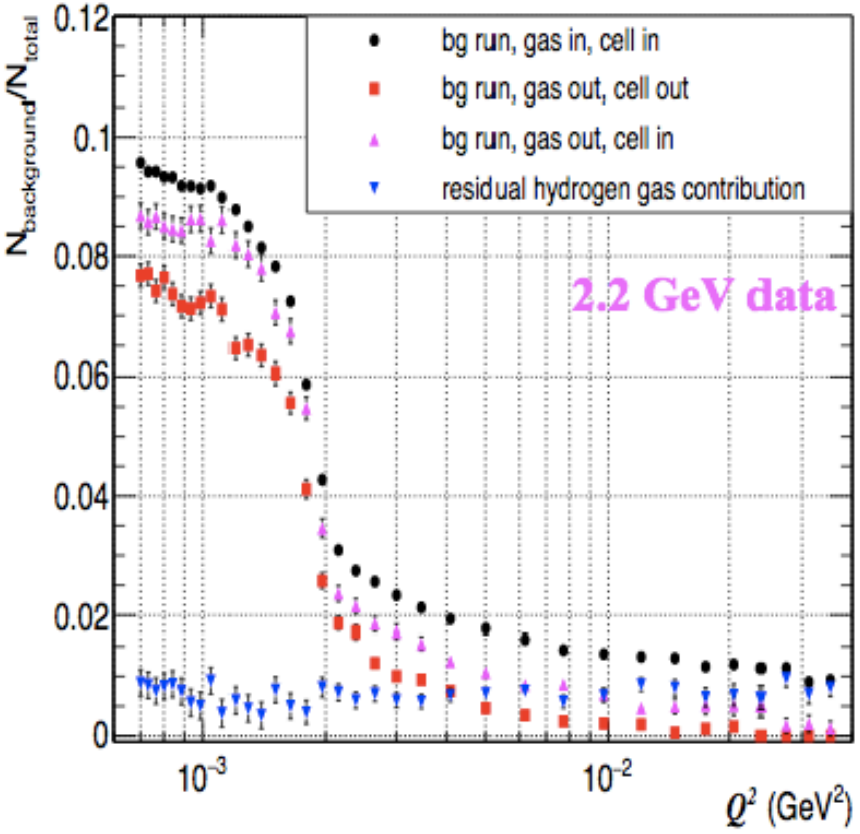
- Runs with different target condition taken for background subtraction and studies for the systematic uncertainty
- Developed simulation program for target density (COMSOL finite element analysis)



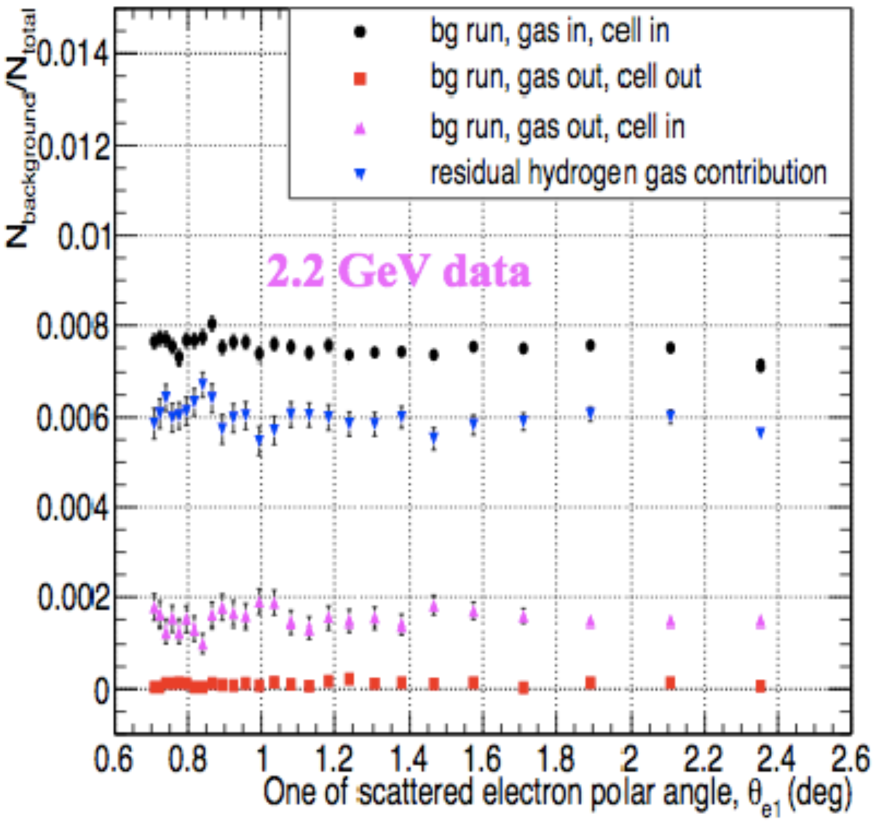
Background Subtraction

- *ep* background rate ~ 10% at forward angle (<1.3 deg, dominated by upstream collimator), less than 2% otherwise
- ee background rate ~ 0.8% at all angles

ep **Background Contribution**

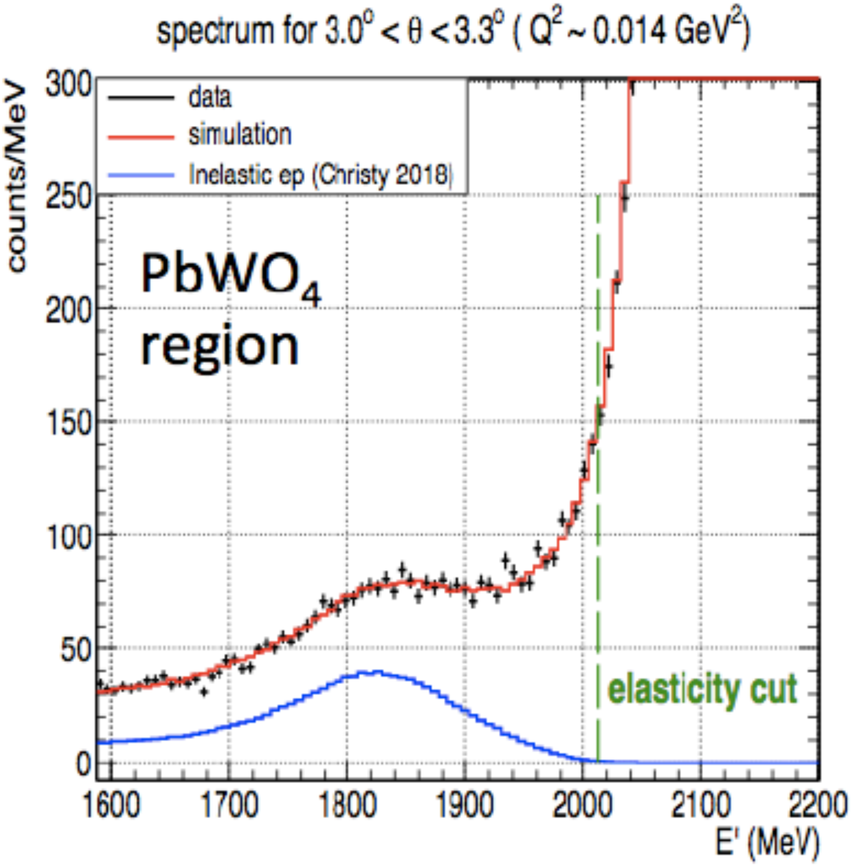


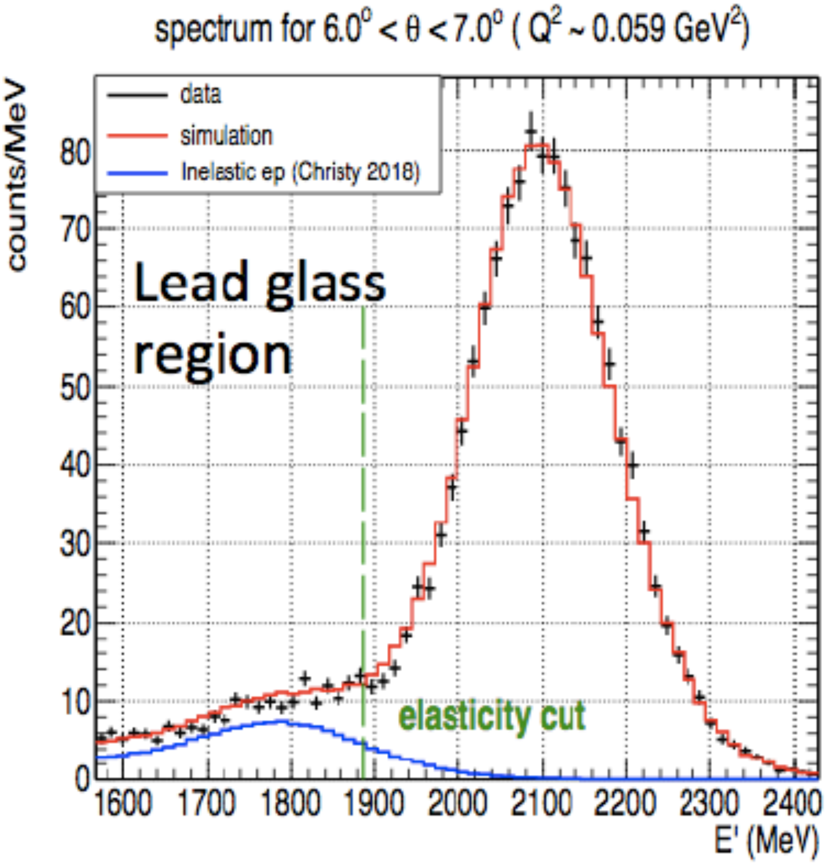
ee **Background Contribution**



Elastic cut and inelastic contribution

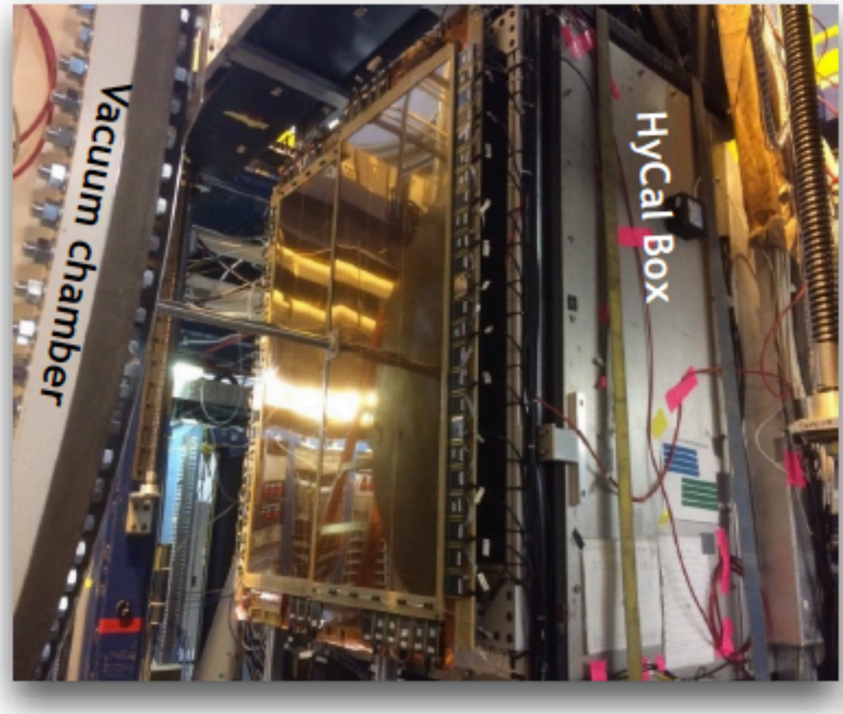
- Using Christy 2018 empirical fit to study inelastic ep contribution
- Good agreement between data and simulation
- Negligible for the $PbWO_4$ region (<3.5°), less than 0.2%(2.0%) for 1.1GeV(2.2GeV) in the Lead glass region


spectrum for $3.0^\circ < \theta < 3.3^\circ$ ($Q^2 \sim 0.014$ GeV^2)

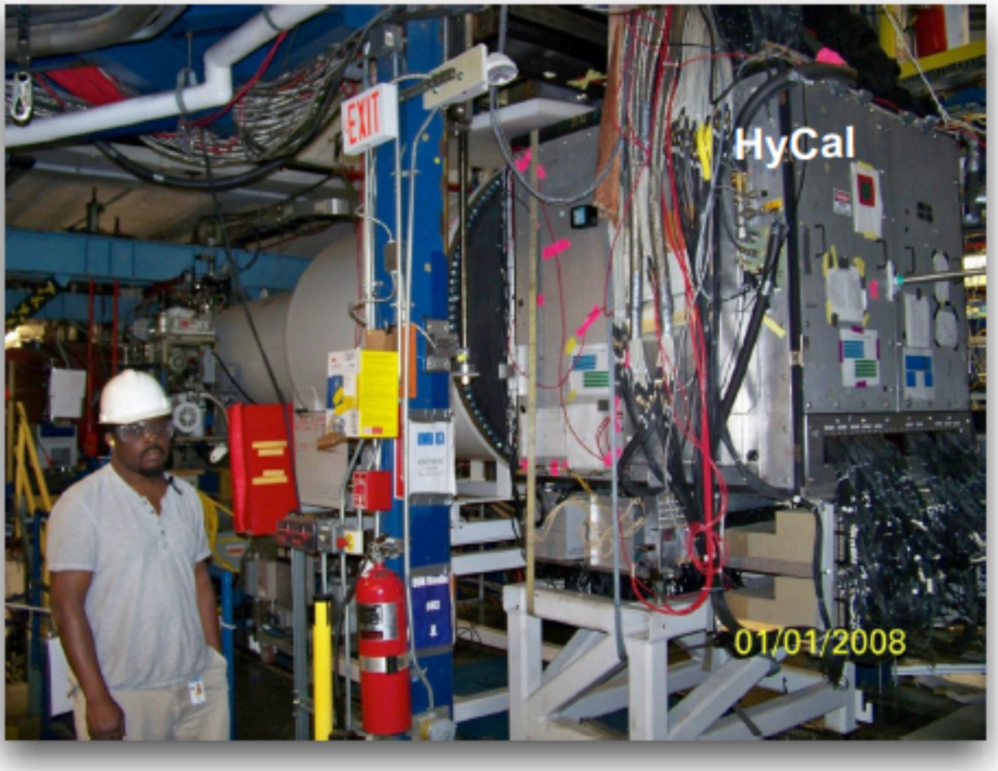

spectrum for $6.0^\circ < \theta < 7.0^\circ$ ($Q^2 \sim 0.059$ GeV^2)

HyCal and GEMs on the beamline

beam view

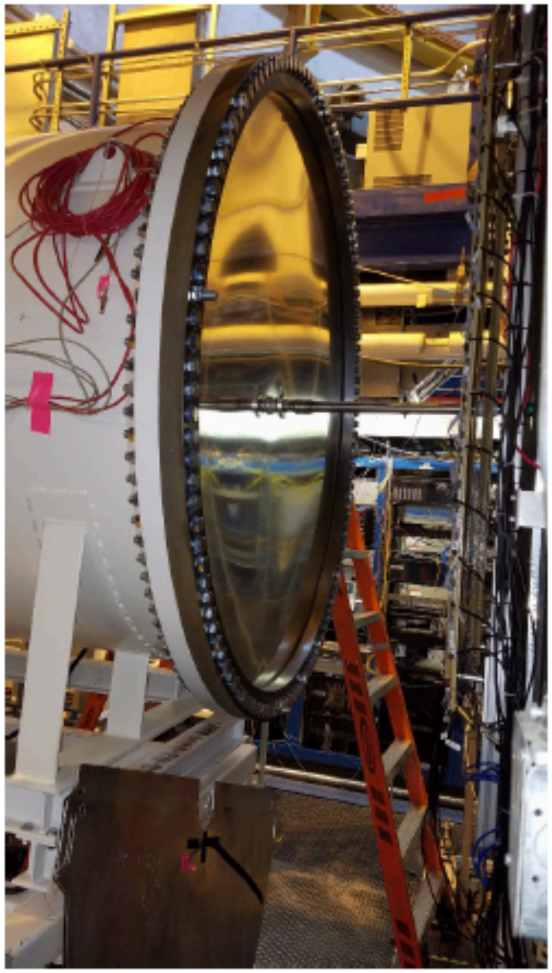


downstream view



Vacuum chamber with one thin window

two stage, 5 m long vacuum box



1.7 m dia, 2 mm thick
Al window

Extraction of *ep* Elastic Scattering Cross Section

- To reduce the systematic uncertainty, the ep cross section is normalized to the Møller cross section:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{ep} = \left[\frac{N_{\mathrm{exp}}(ep \to ep \text{ in } \theta_i \pm \Delta\theta_i)}{N_{\mathrm{exp}}(ee \to ee)} \cdot \frac{\varepsilon^{ee}_{\mathrm{geom}}}{\varepsilon^{ep}_{\mathrm{geom}}} \cdot \frac{\varepsilon^{ee}_{\mathrm{det}}}{\varepsilon^{ep}_{\mathrm{det}}}\right] \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{ee}$$

- Event generators for unpolarized elastic ep and Møller scatterings have been developed based on complete calculations of radiative corrections
  1. A. V. Gramolin et al., J. Phys. G Nucl. Part. Phys. 41(2014)115001
  2. I. Akushevich et al., Eur. Phys. J. A 51(2015)1 (fully beyond ultra relativistic approximation)

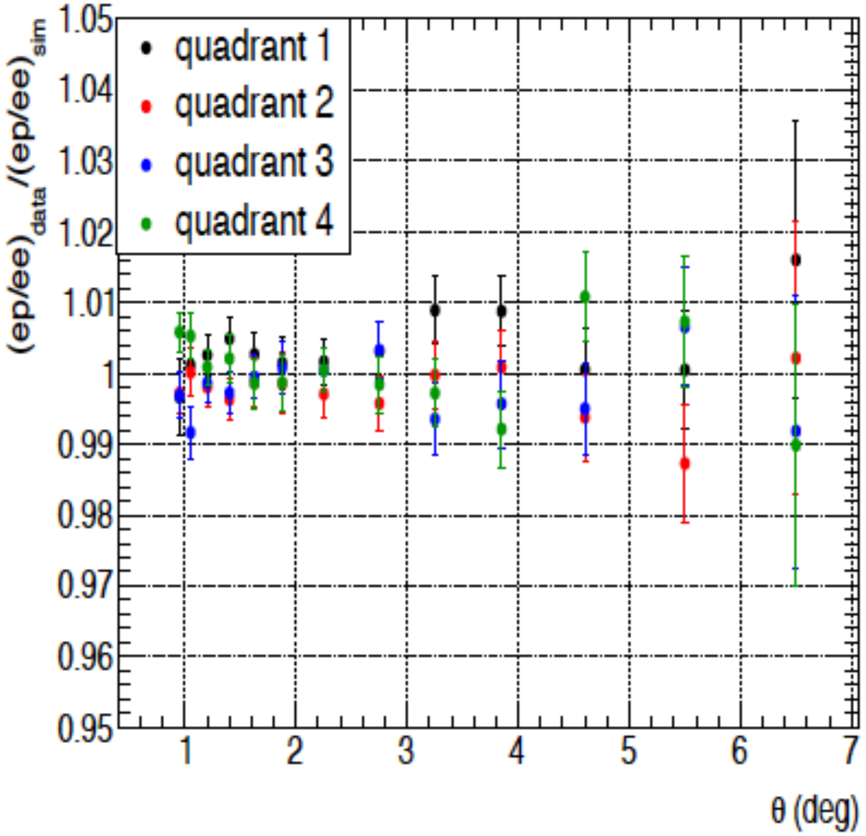
- A Geant4 simulation package is used to study the radiative effects:

$$\sigma_{ep}^{Born(exp)} = \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{exp} / \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{sim} \cdot \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{Born(model)} \cdot \sigma_{ee}^{Born(model)}$$

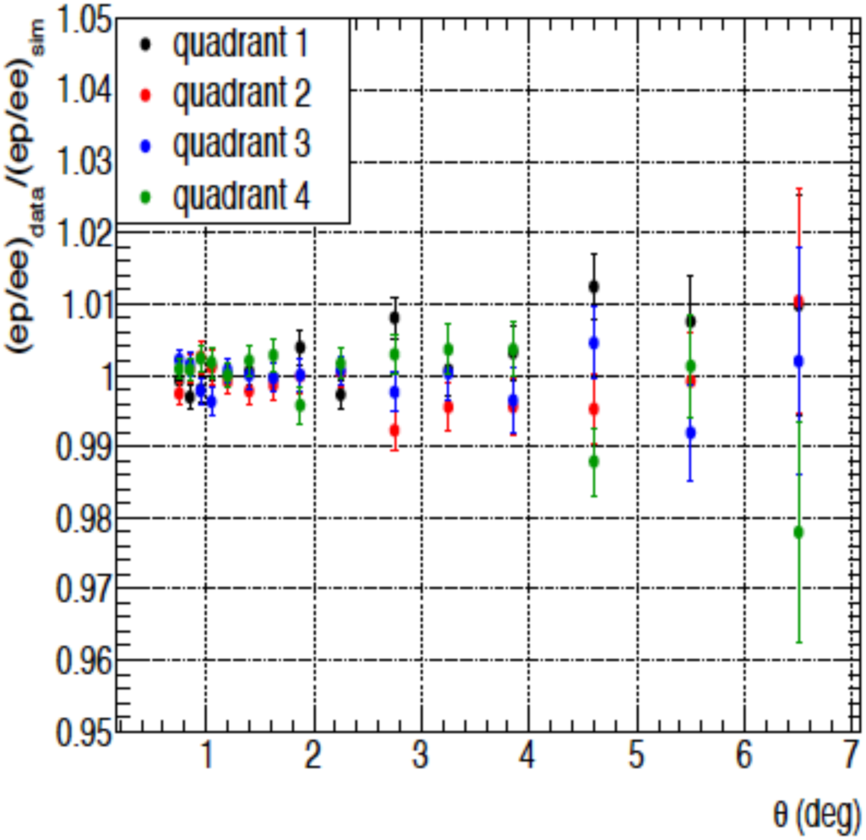
- Iterative procedure applied for radiative correction

Normalized super ratio by quadrants

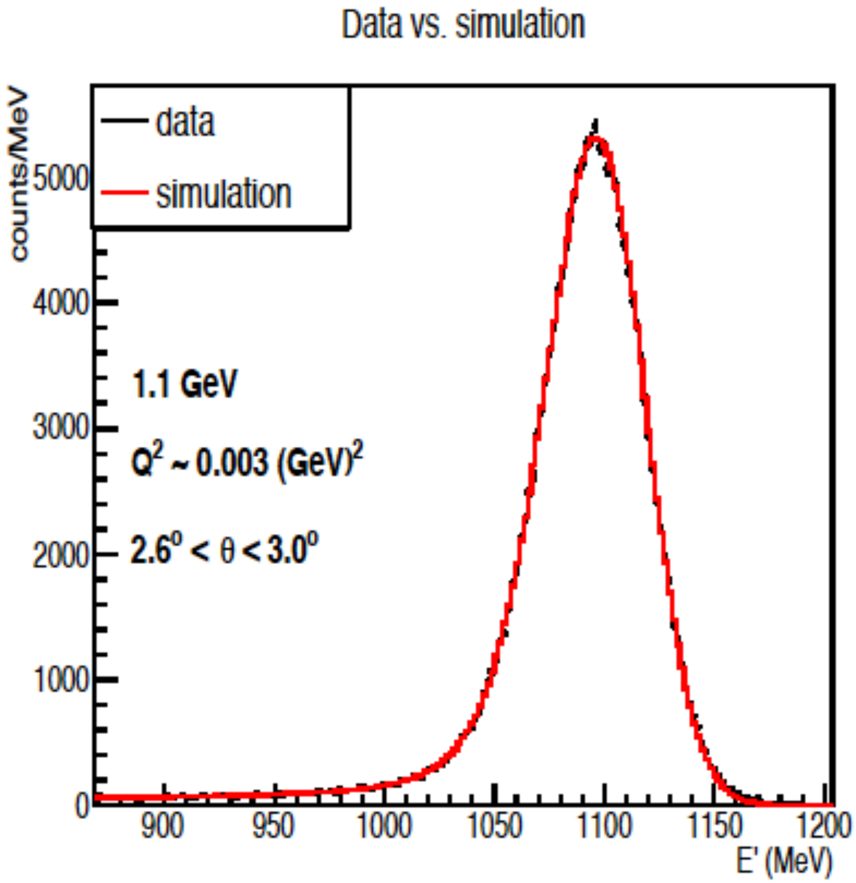
1GeV

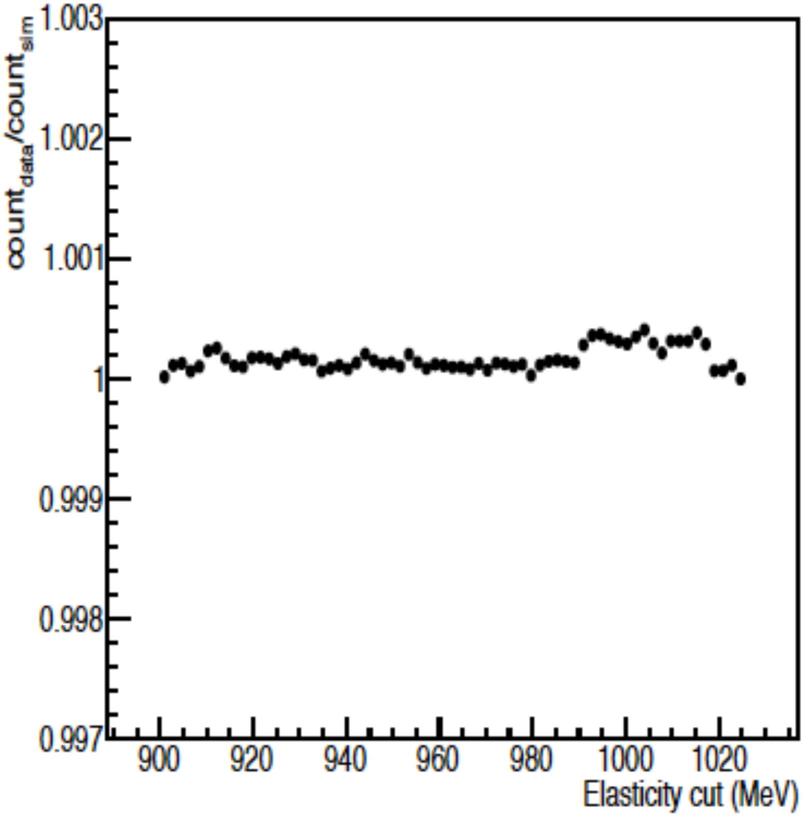


2GeV

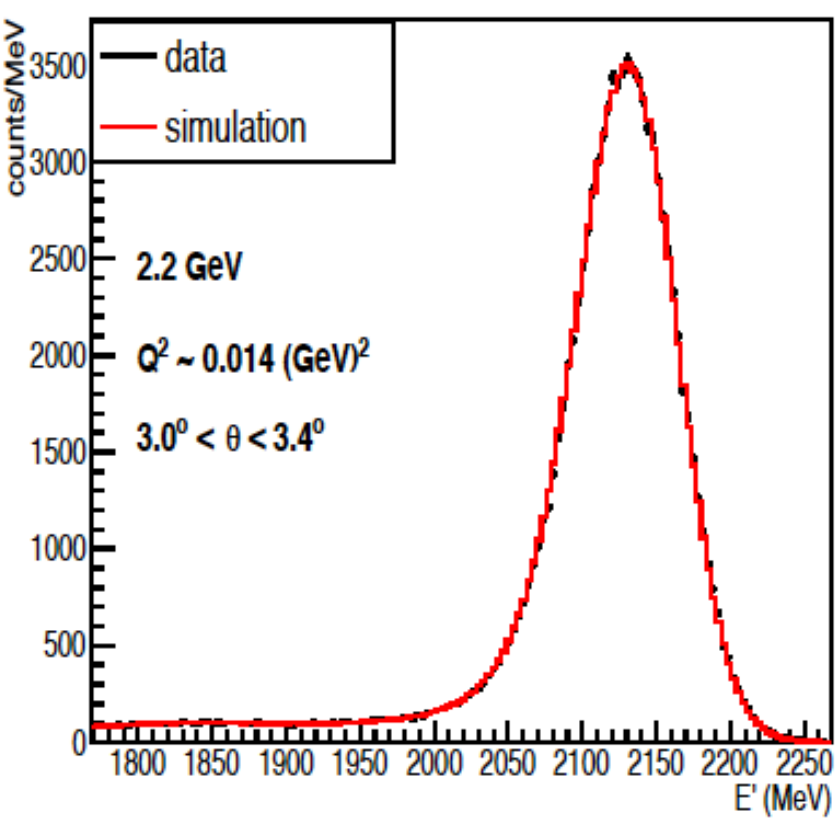


1GeV elasticity cut sensitivity

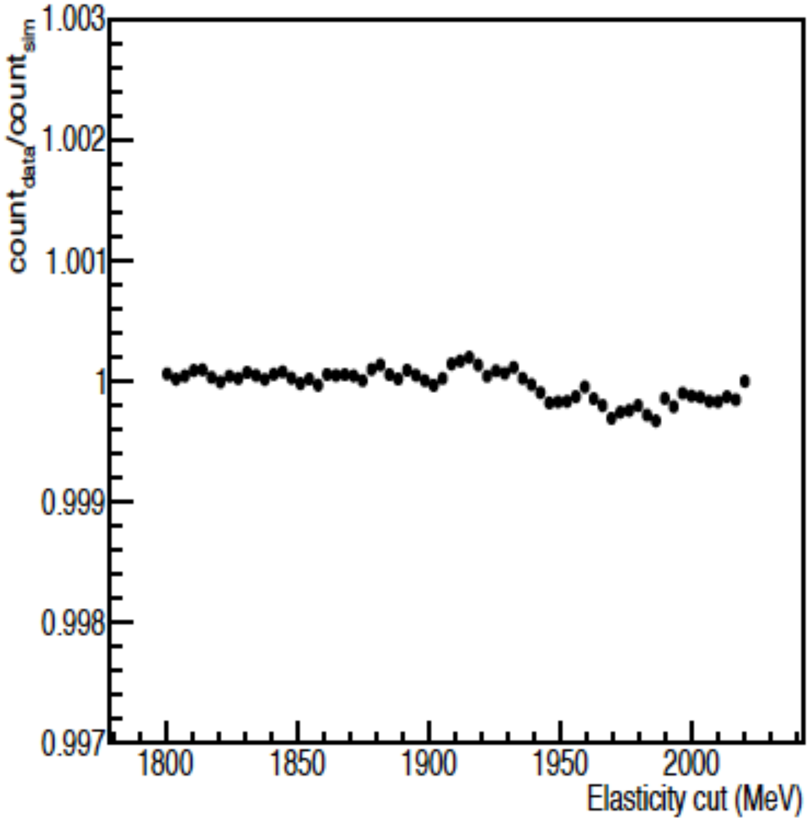

Data vs. simulation



2GeV elasticity cut sensitivity

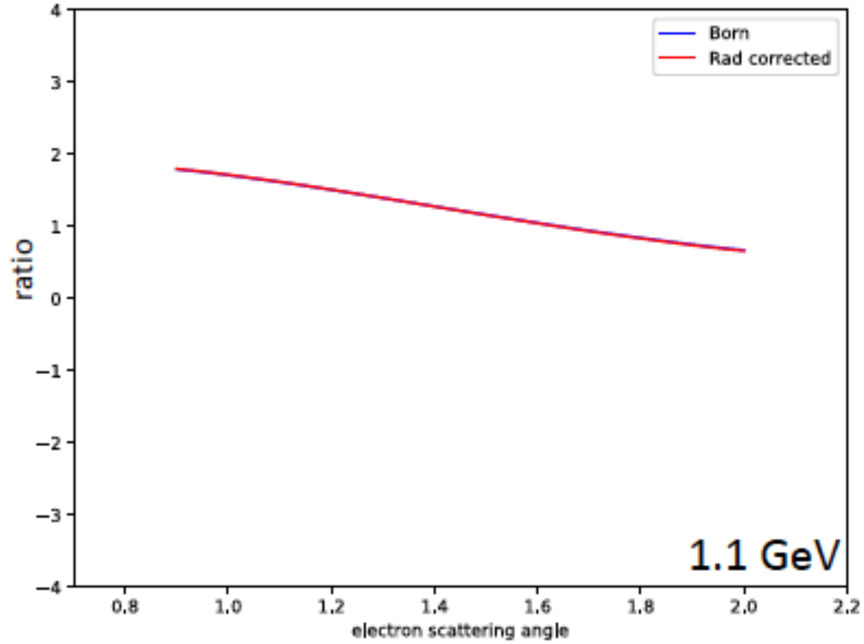


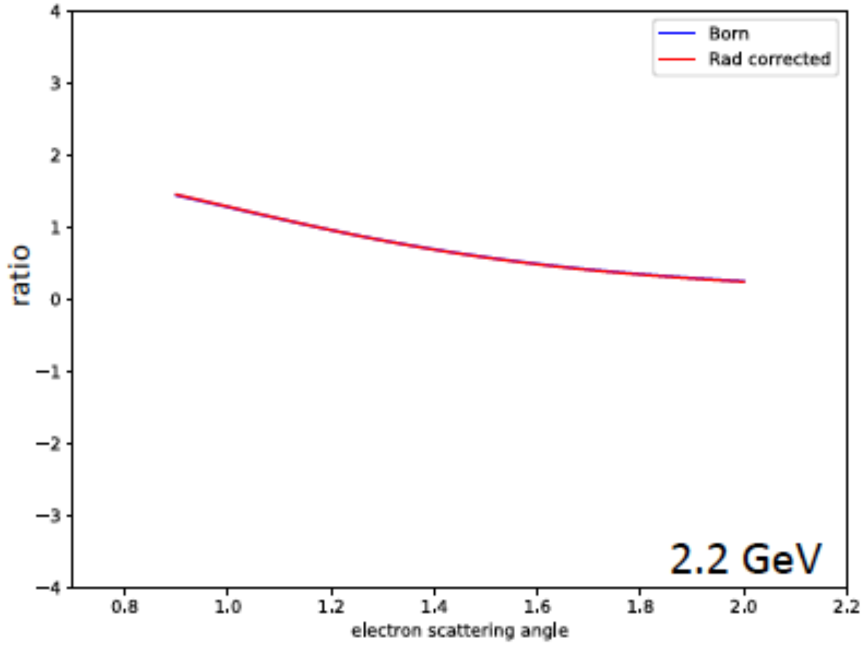
Data vs. simulation



Radiative corrections

ep cross section / *ee* cross section





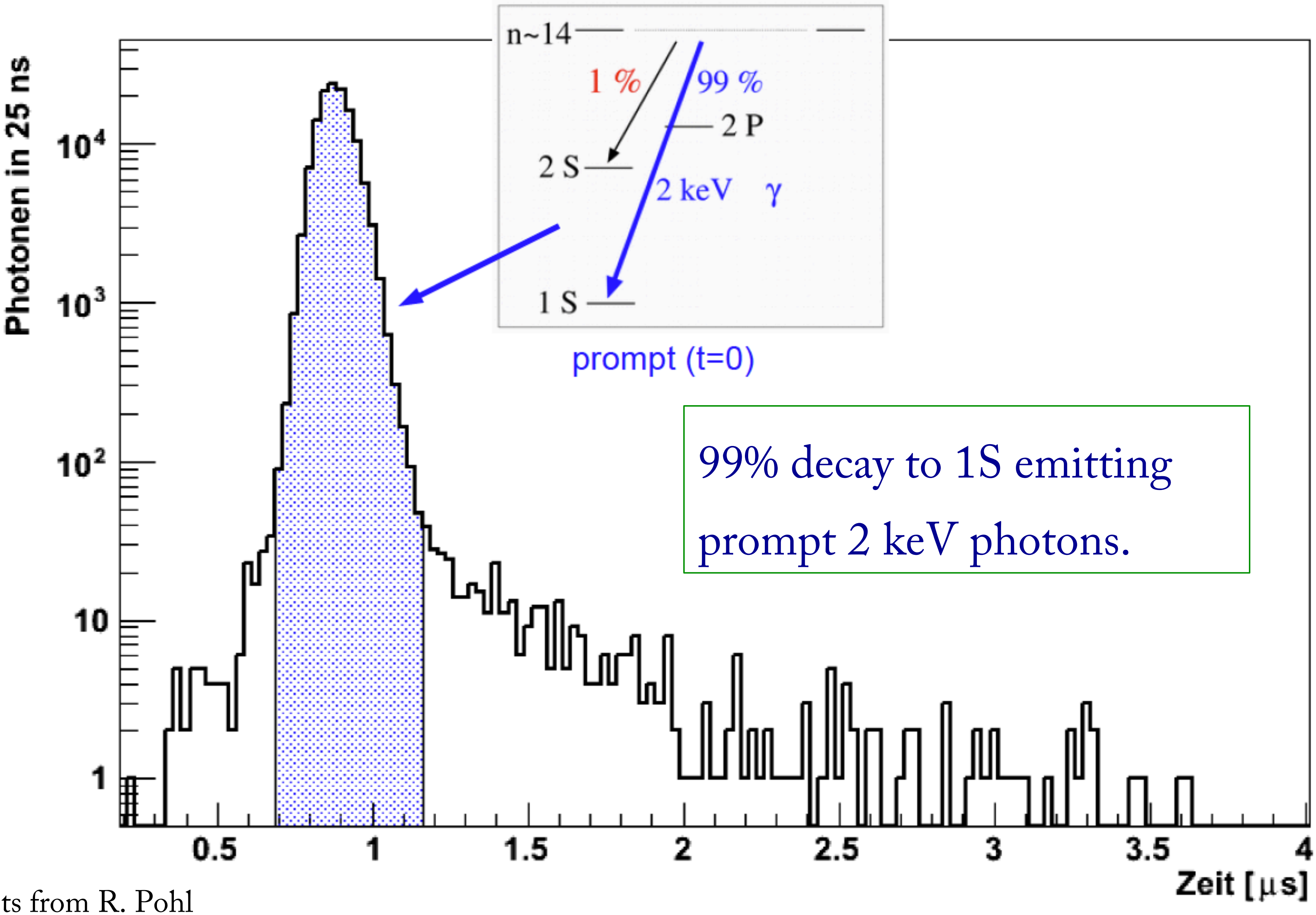
- Event generators for *ep* and *ee* elastic scattering developed based on complete calculation of radiative correction beyond URA[1]
- Cross checked with results from second generator[2]
- Include hard emission radiative photons for full correction of radiative effect with HyCal
- Include effect from two photon exchange[3]

[1] I. Akushevich et al., Eur. Phys. J. A 51(2015)1

[2] A. V. Gramolin et al., J. Phys. G Nucl. Part. Phys. 41(2014)115001

[3] O. Tomalak, Few Body Syst. **59**, no. 5, 87 (2018)

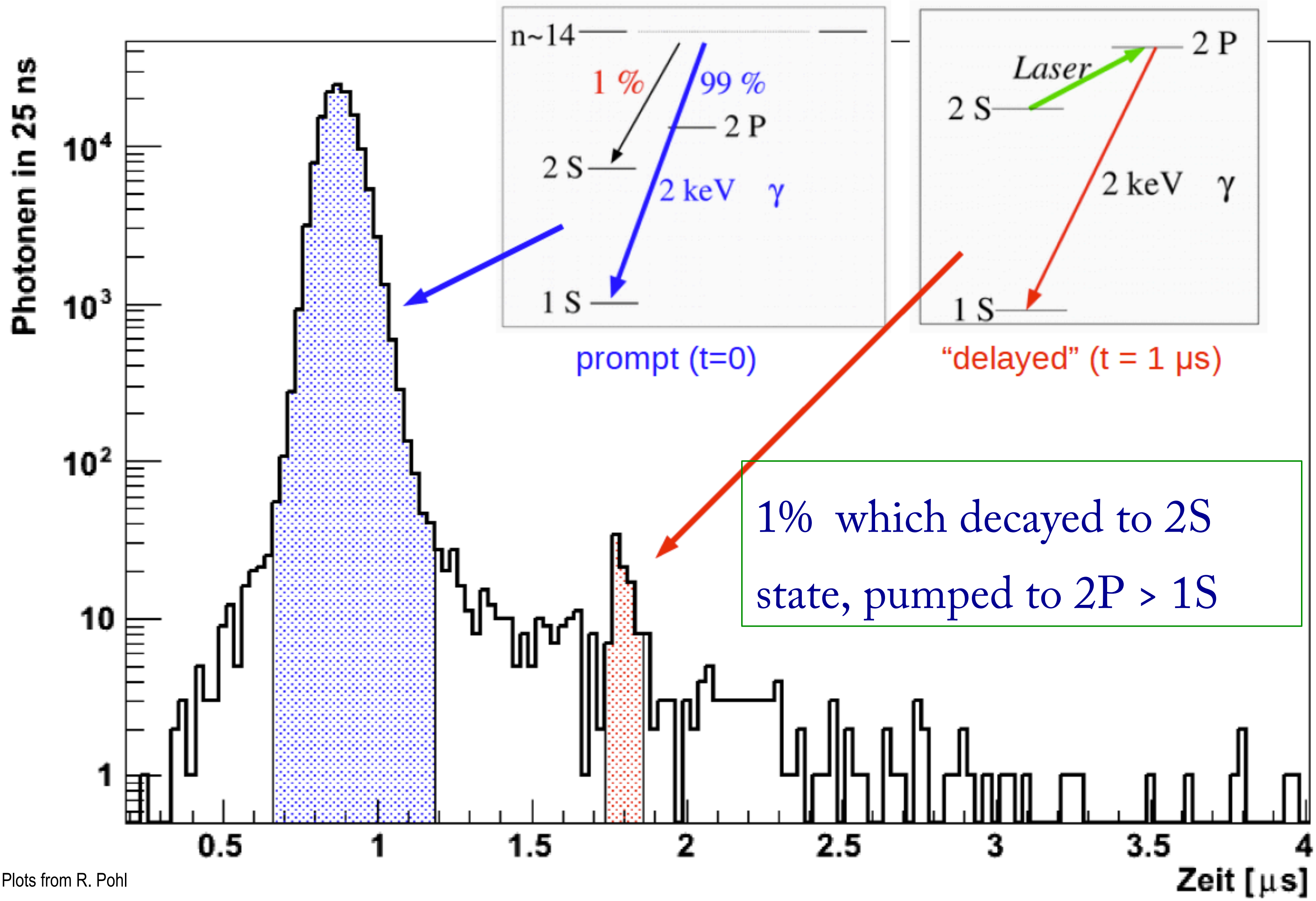
Muonic Hydrogen Spectroscopy Experiment



99% decay to 1S emitting prompt 2 keV photons.

Plots from R. Pohl

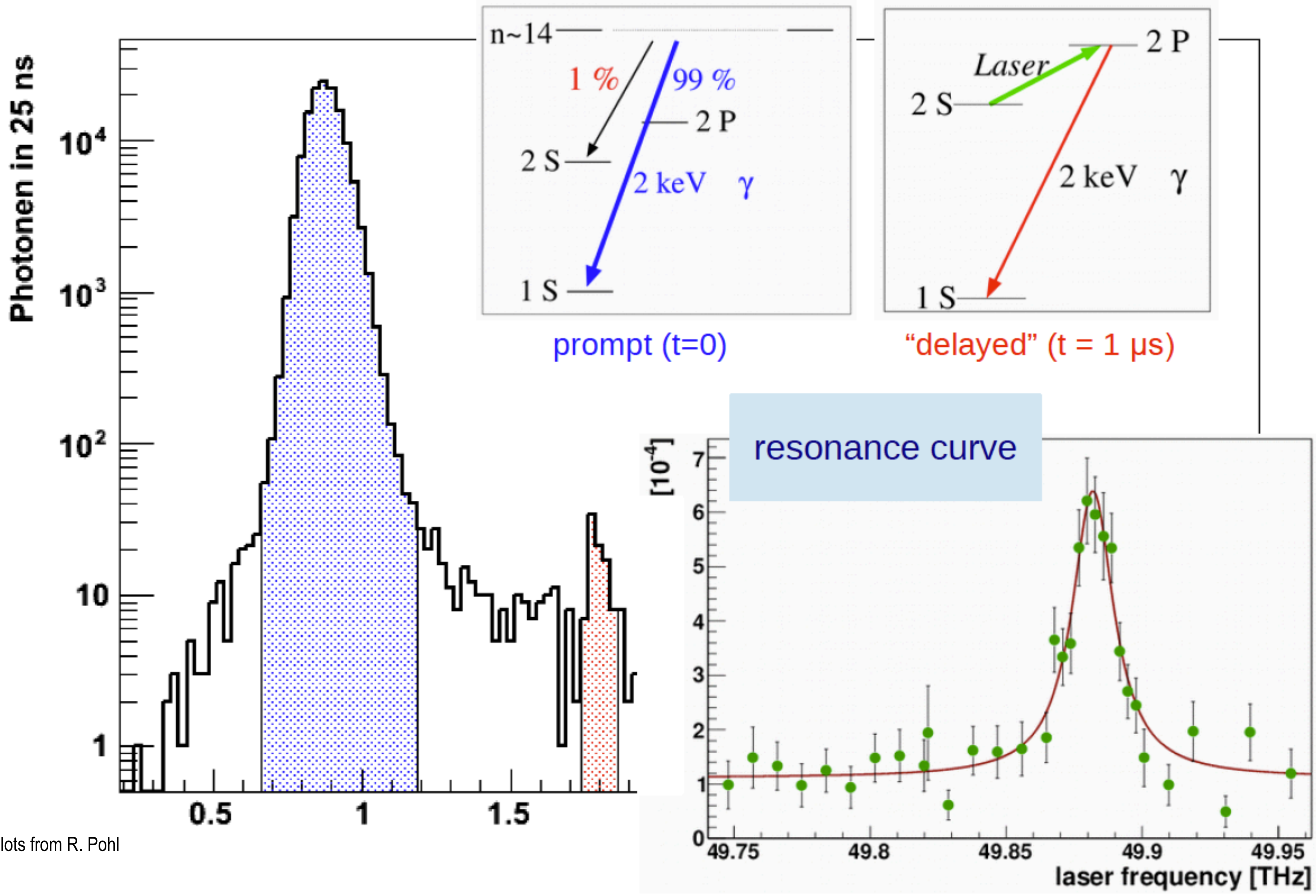
Muonic Hydrogen Spectroscopy Experiment



1% which decayed to 2S state, pumped to 2P > 1S

Plots from R. Pohl

Muonic Hydrogen Spectroscopy Experiment



prompt (t=0)

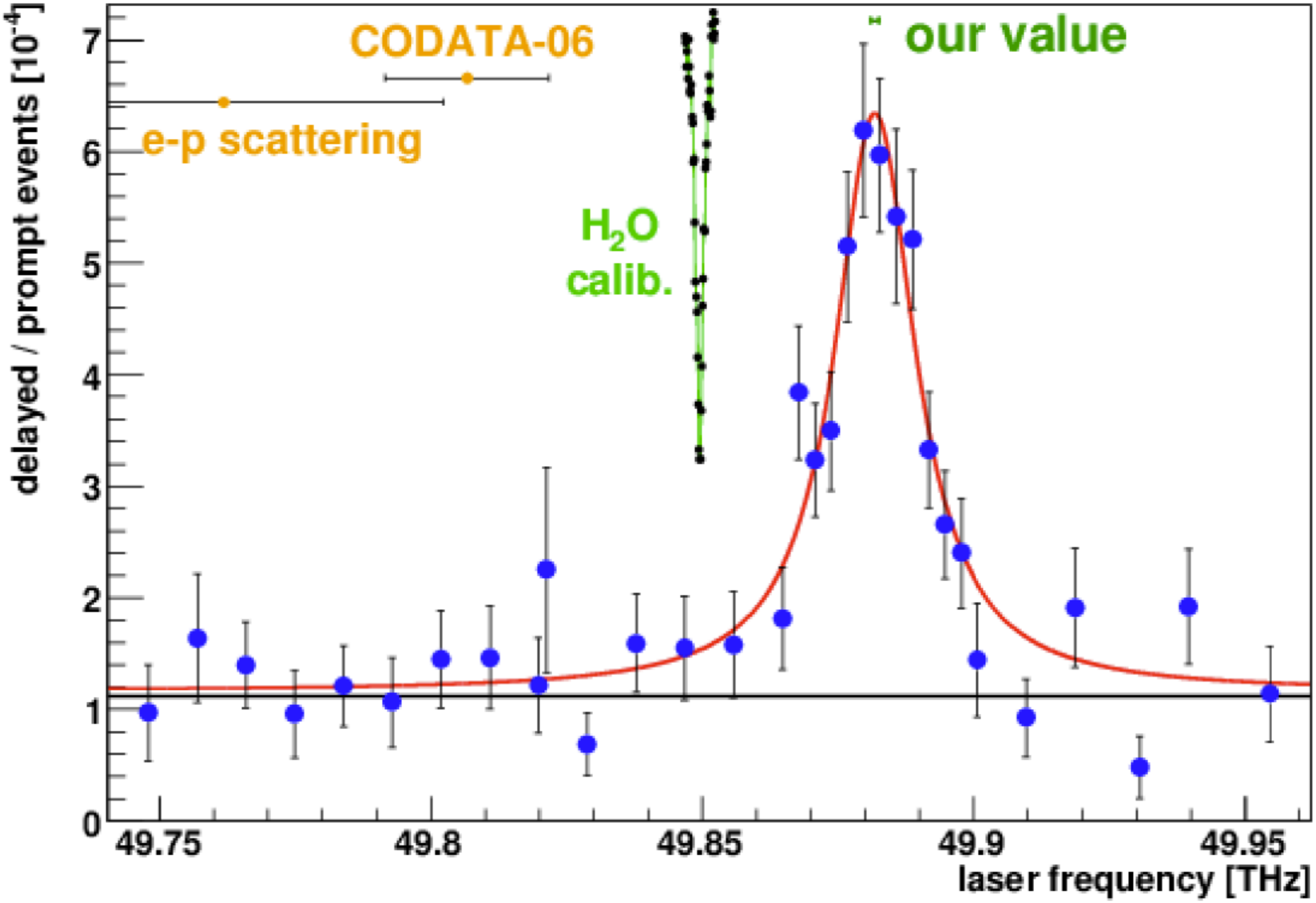
“delayed” (t = 1 μs)

resonance curve

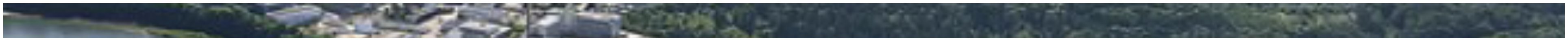
Plots from R. Pohl

Muonic Hydrogen Spectroscopy Experiment

Resonance in muonic hydrogen

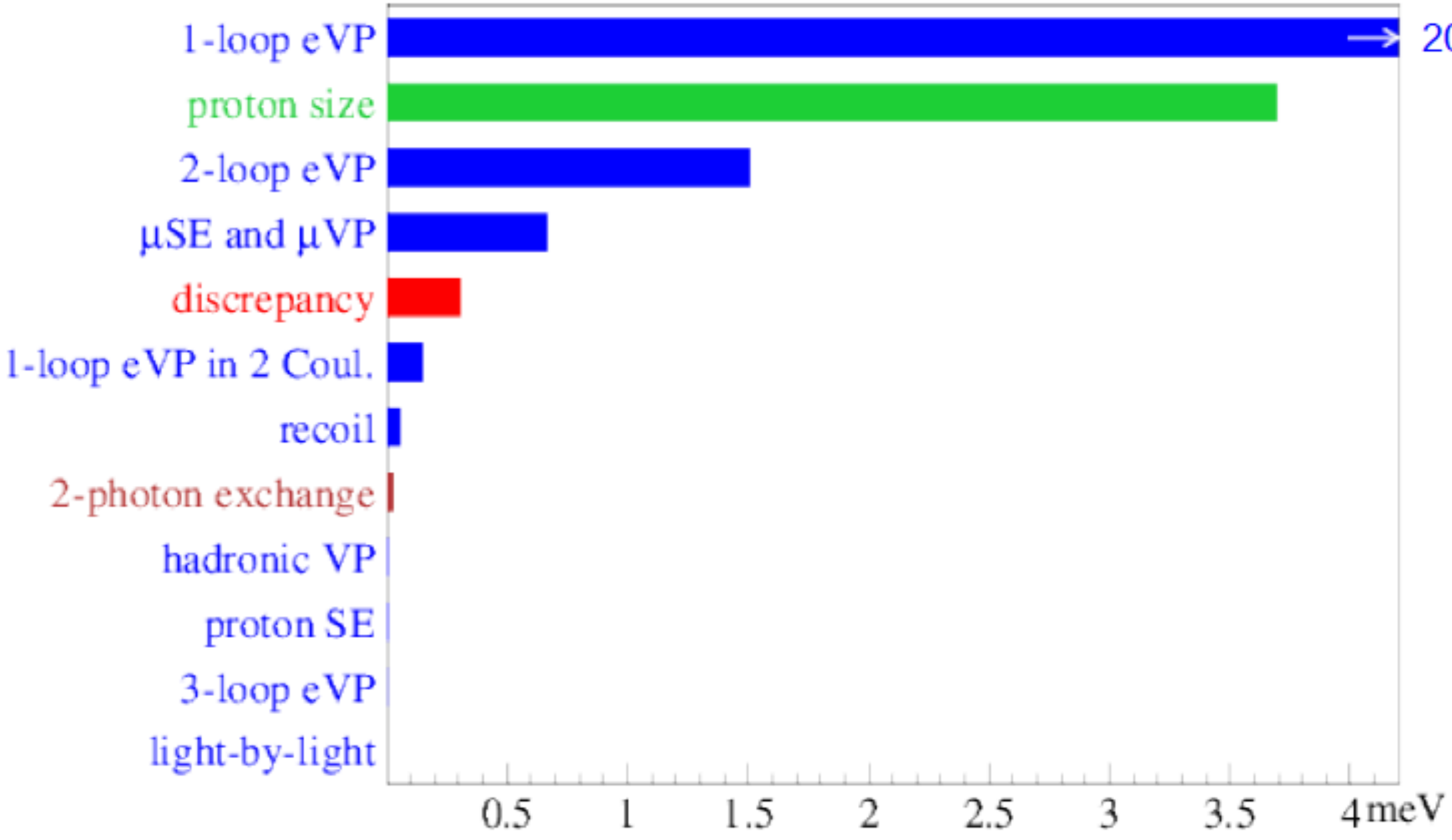


Pohl et al. (CREMA Coll.), Nature 466, 213 (2010)



Theory in muonic H

$$\Delta E_{\mathrm{Lamb}} = 206.0336\ (15)\ \mathrm{meV}_{\mathrm{QED}} + 0.0332\ (20)\ \mathrm{meV}_{\mathrm{TPE}} - 5.2275\ (10)\ \mathrm{meV/fm^2} * \mathbf{R_p}^{\mathbf{2}}$$

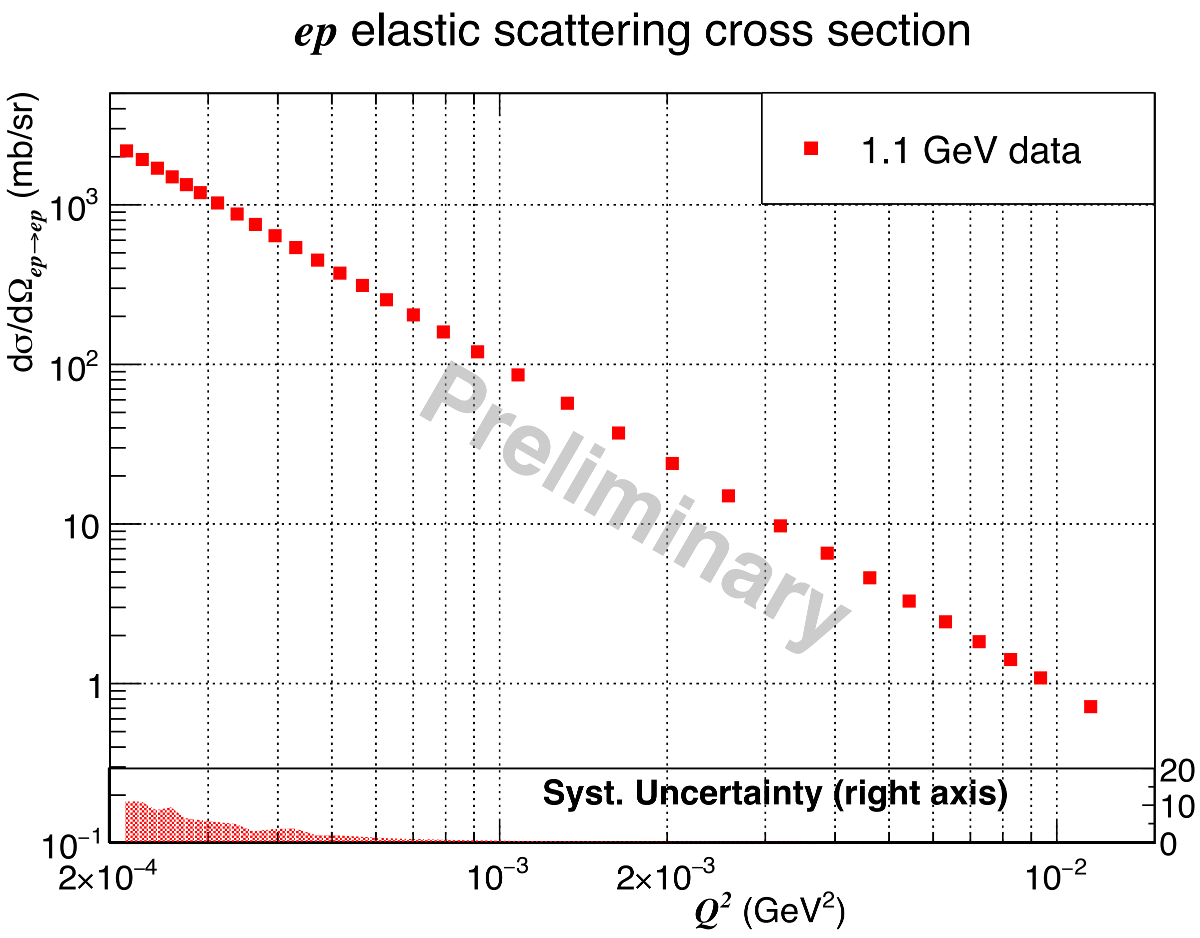


Nice hierarchy

R. Pohl

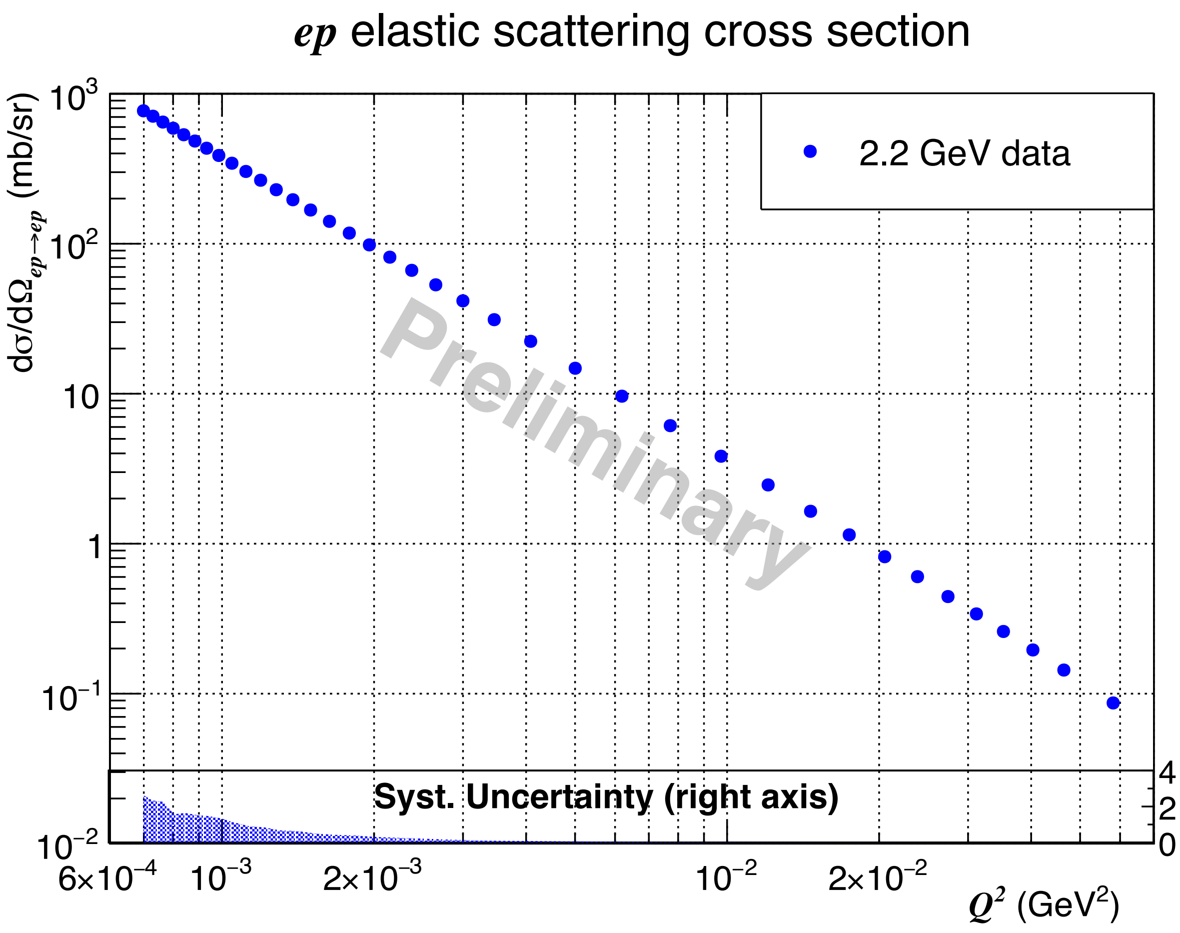
Elastic ep→ep Cross Sections, 1.1 GeV (Preliminary)

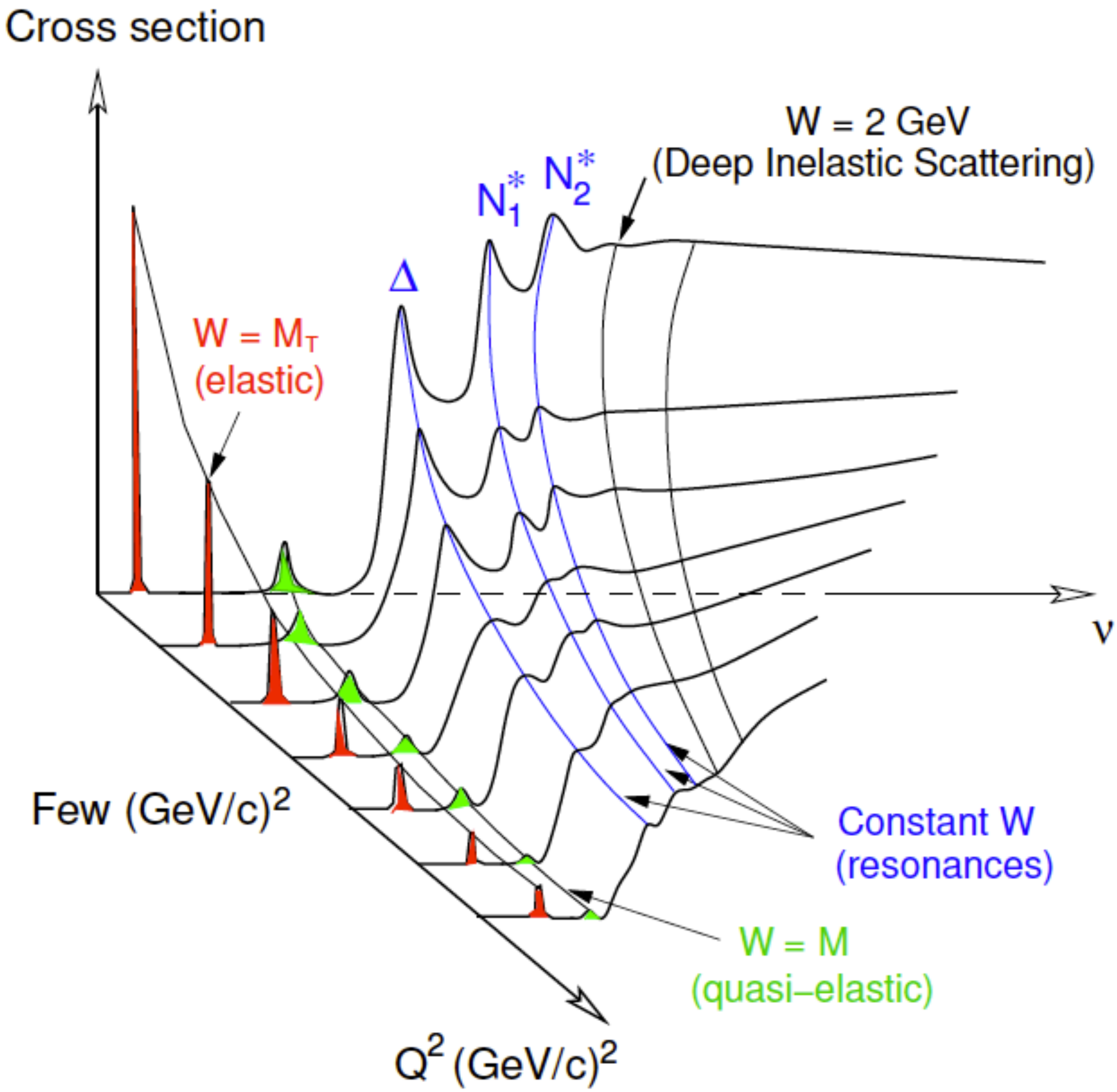
- Differential cross section vs. Q^2, with 1.1 GeV data (preliminary).
- Statistical uncertainty at this stage: ~0.2% per point.
- Systematic uncertainties at current stage: 0.3% ~0.6% (shown as shadow area).



Elastic ep→ep Cross Sections, 2.2 GeV (Preliminary)

- Differential cross section vs. Q^2, with 2.2 GeV data.
- Statistical uncertainty at this stage: ~0.18% , per point.
- Systematic uncertainties at current stage: 0.3% ~ 1.3% (shown as shadow area).




Cross section
W = 2 GeV
(Deep Inelastic Scattering)
N_1^*
N_2^*
Δ
W = M_T
(elastic)
ν
Few (GeV/c)^2
Constant W
(resonances)
W = M
(quasi–elastic)
Q^2 (GeV/c)^2