Studies of Radiative Corrections for the PRad-II Experiment

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For the PRad Collaboration

PRad-II C1 Review

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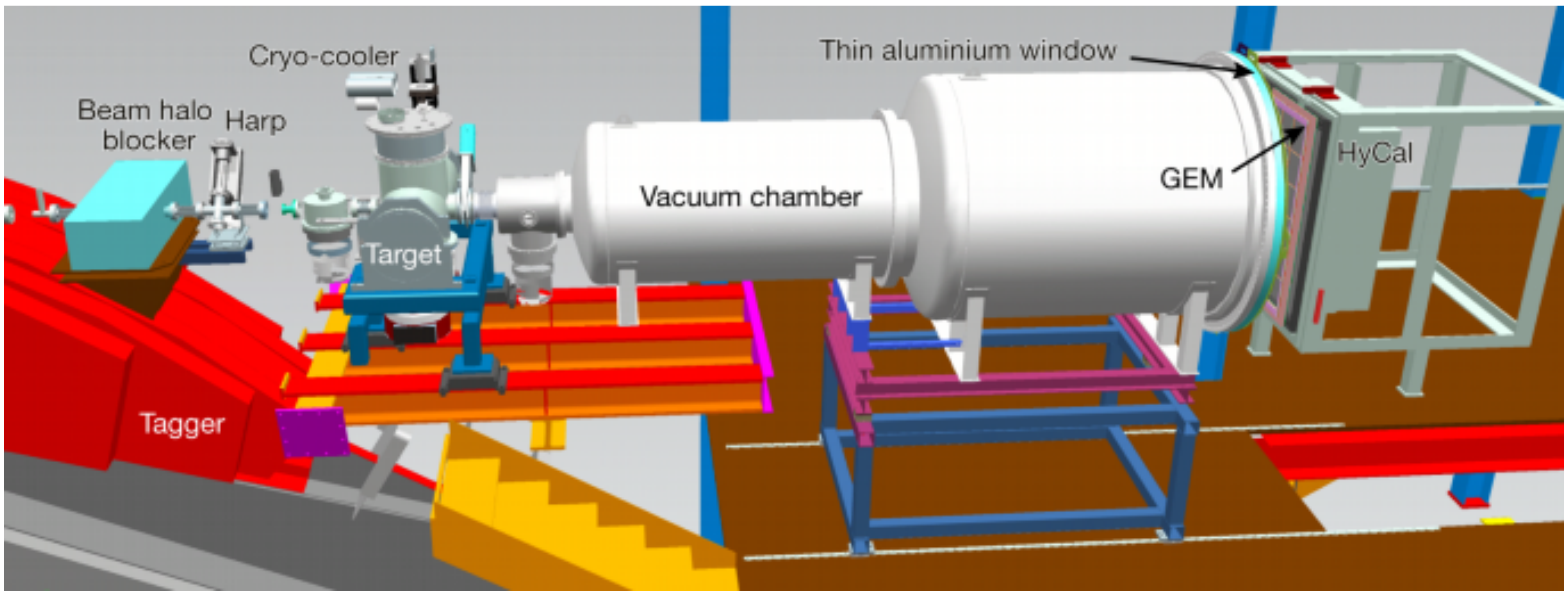


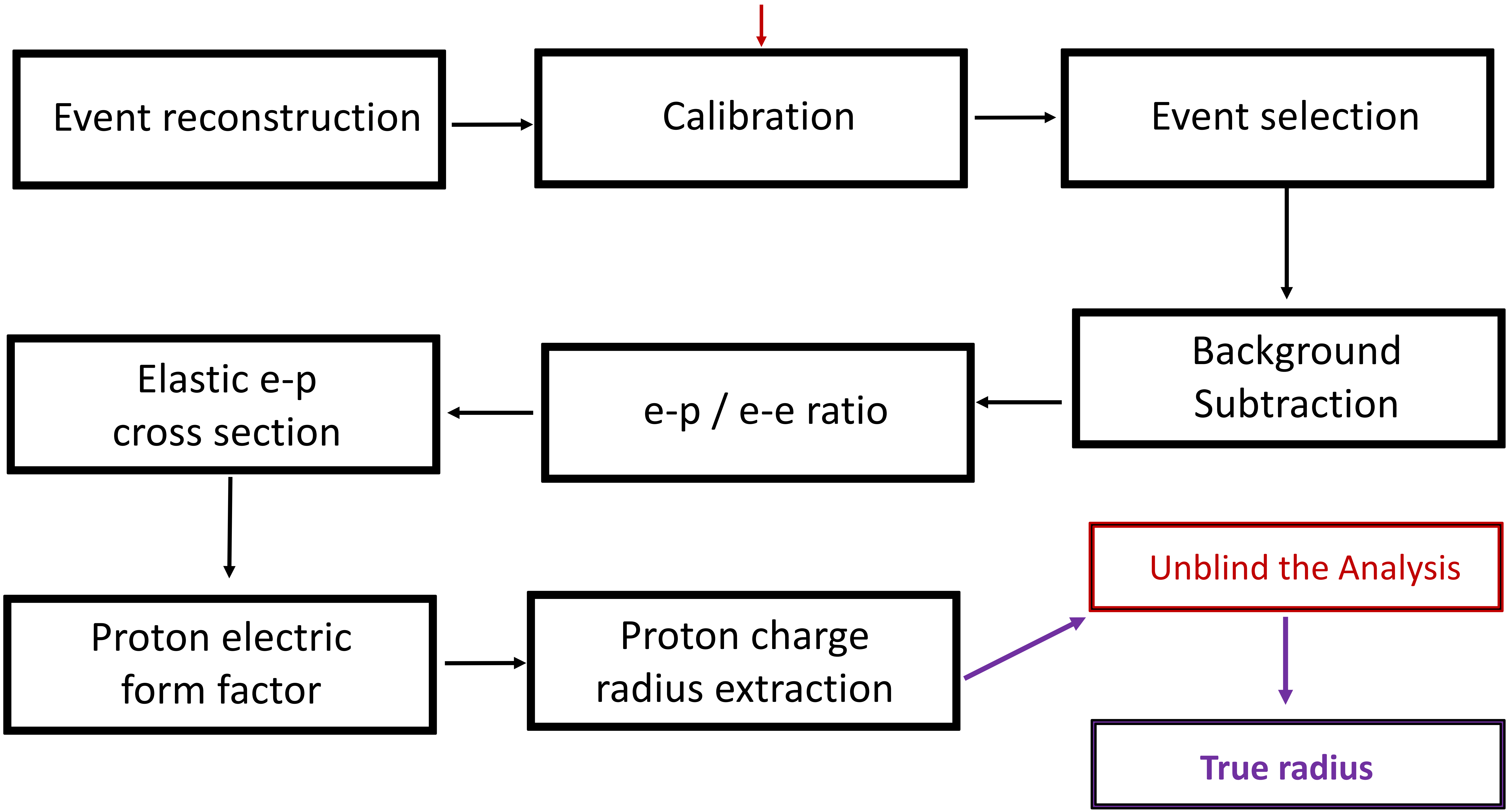
Outline

- Plan for blind analysis to extract the proton radius (r_p) for PRad-II
- Radiative correction (RC) studies for PRad
  - PRad's estimation of the RC systematic uncertainty of r_p
  - Independent study of the RC systematic uncertainty of r_p
- RC studies for PRad-II
  - Integrated Møller method
  - Plans for the next-to-next leading order (NNLO) calculations
  - Improvement from PRad to PRad-II
  - Partial testing of calculations of radiative effects
- Summary

Plan: Blind analysis for extraction of r_p for PRad-II

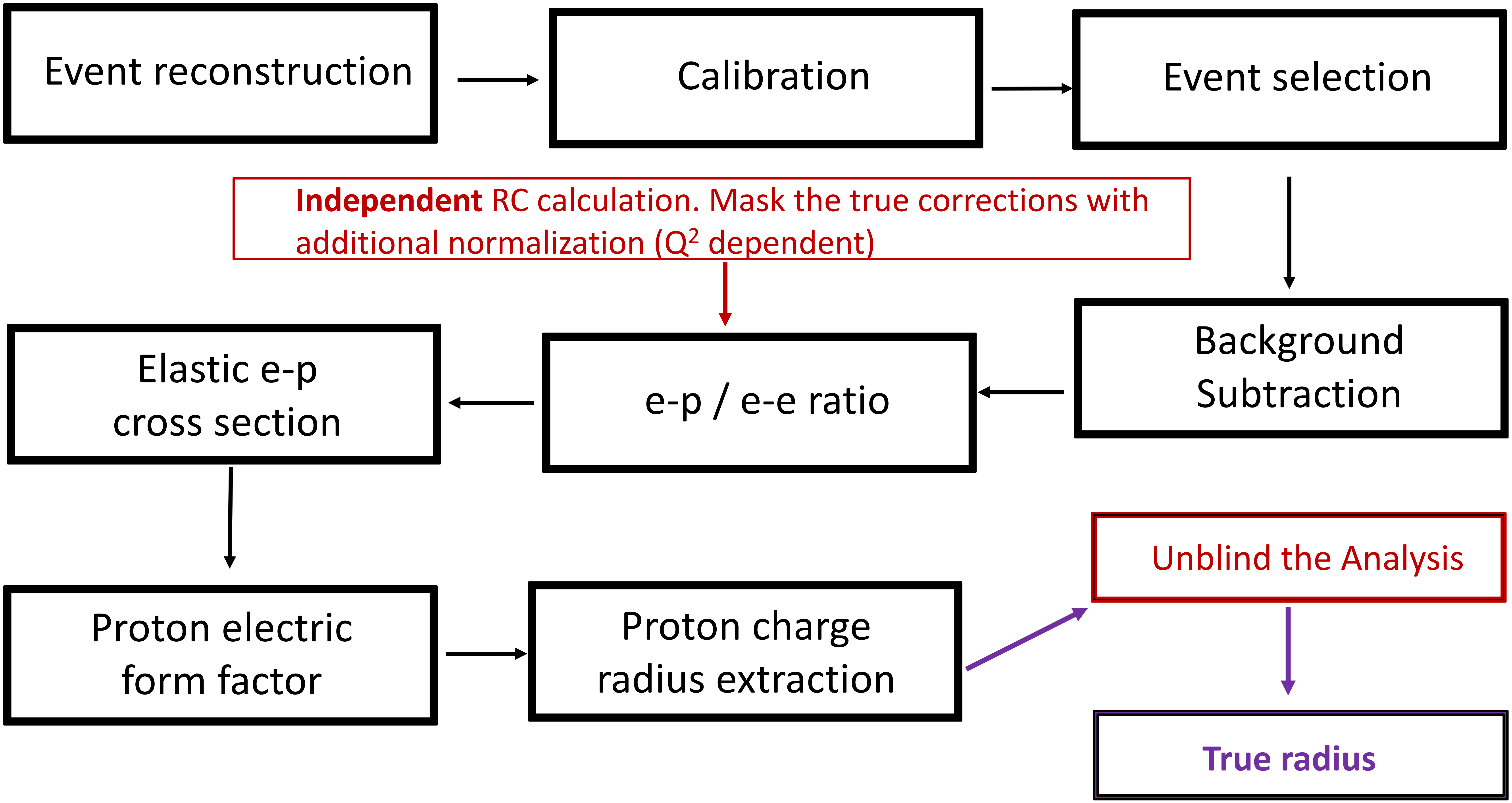
Plan A

Independent study of HyCal trigger efficiency. Mask the true efficiency with additional normalization (Q^2 dependent).



Plan: Blind analysis for extraction of r_p for PRad-II

Plan B



PRad's estimation of the RC syst. uncertainty of r_p

- Measured radius: *Nature 575, 147 (2019)*
  - $r_p = (0.831 \pm 0.007_{stat} \pm 0.012_{syst})$ fm
- r_p uncertainties for PRad shown in the table
  - Uncertainties estimated using the rational (1,1) function

    *X. Yan et al. PRC **98**, 025204 (2018)*

Using rational (1,1)

$$f(Q^2) = \frac{1 + p_1 Q^2}{1 + p_2 Q^2}$$

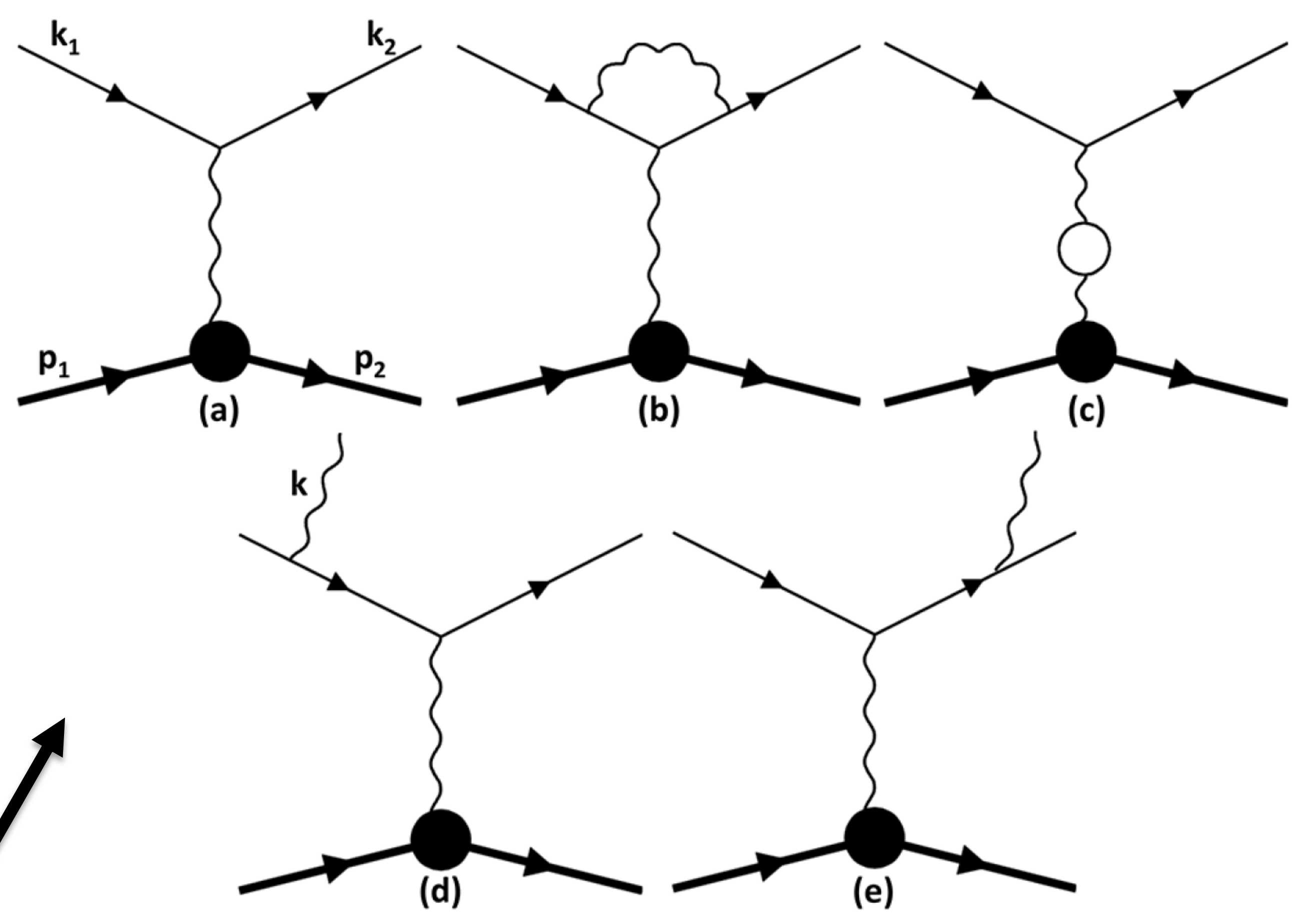
Item	PRad δr_p [fm]
Stat. uncertainty	0.0075
GEM efficiency	0.0042
Acceptance	0.0026
Beam energy related	0.0022
Event selection	0.0070
HyCal response	0.0029
Beam background	0.0039
Radiative correction	0.0069
Inelastic *ep*	0.0009
G_M^p parameterization	0.0006
Total syst. uncertainty	0.0115
Total uncertainty	0.0137

PRad's estimation of the RC syst. uncertainty of r_p

- RCs one of the largest syst. uncertainty sources of r_p for PRad
  - RCs studied for both e-p and Møller scatterings
  - Event generator used, made using the results from *I. Akushevich et al, EPJA **51**, 1 (2015)*
    - Used analytical calculations for one-loop e-p and Møller RC diagrams
    - Calculated within covariant formalism and beyond ultra-relativistic limit
    - Infrared divergence extracted and cancelled by Bardin-Shumeiko approach
  - PRad RC syst. uncertainty on r_p estimated
    - using the first-order RC results from *EPJA **51**, 1 (2015)*
    - using a method from *A. Arbuzov and T. Kopylova, EPJC **75**, 603 (2015)* for estimation of the contribution stemming from higher order RCs
  - Estimated syst. uncertainties correlated and Q^2-dependent

PRad's estimation of the RC syst. uncertainty of r_p

- e-p and Møller NLO diagrams used for cross section calculations in *EPJA* ***51****, 1 (2015)*

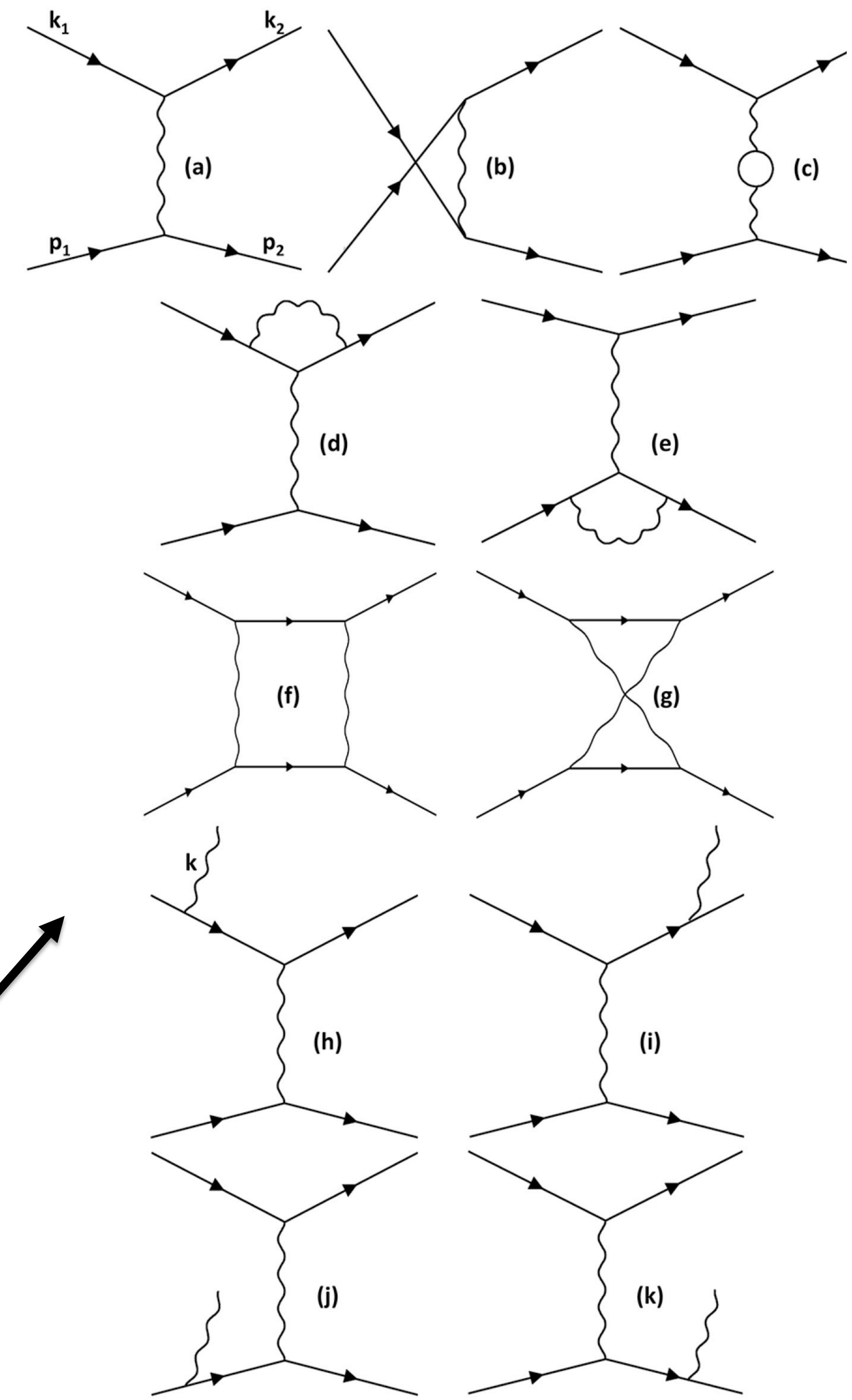


- Feynman diagrams contributing to the Born and RC cross sections in e+p elastic scattering:

  (a) The Born process; (b) Vertex correction;
  (c) Vacuum polarization; (d)-(e) Bremsstrahlung.

- Feynman diagrams contributing to the Born (a)-(b) and RC cross sections for Møller scattering:

  (c)-(e) Vacuum polarization and vertex correction;
  (f)-(g) Box contribution; (h)-(k) Bremsstrahlung.



PRad's estimation of the RC syst. uncertainty of r_p

- Two methods for forming e-p to e-e differential cross section ratio (luminosity cancellation)
  - Bin-by-bin method
    - Forms the ratio using the e-p and e-e counts from the same angular bin
    - Cancels out the energy-independent part of acceptance and GEM efficiency
    - Q^2 -dependent syst. uncertainties from the e-e process introduced
  - Integrated Møller method
    - Uses e-e counts from a selected angular range
    - Gives a common normalization factor for all e-p Q^2 bins; no effect on extracted r_p
    - Not applied to all Q^2 bins in PRad, since the GEM efficiency not precisely determined in all those bins
- Q^2-dependence much larger for Møller RC in PRad
  - Affects the cross-section results via the use of the bin-by-bin method
  - For e-p RC $\rightarrow$ $\delta r_p = 0.0020\ fm$; for Møller RC $\rightarrow$ $\delta r_p = 0.0065\ fm$
  - For total RC $\rightarrow$ $\delta r_p = 0.0069\ fm$

Independent study of the RC syst. uncertainty of r_p

- Independent study performed for the second-order RC effect on r_p
  - Followed the approach of *A. Aleksejevs et al, Physics of Atomic Nuclei, **76**, 888 (2013)*
    - Paper calculated two-loop radiative effects in the MOLLER experiment
  - Based on its mathematical framework and for PRad kinematics
    - Contribution from NNLO diagrams on the Born cross section estimated
    - For any reasonable photon energy cut for the PRad experiment
    - Q^2-dependent syst. uncertainties smaller than that estimated in the first approach
    - The largest RC syst. uncertainty computed: $\delta r_p = 0.0047\ fm$
  - However, approximated methods and restricted number of diagrams used
    - Improved and exact NNLO calculations are much desired

- Limitations of GEM efficiency determination in PRad
  - Contributed indirectly to the total syst. uncertainty
  - Should be improved
- Aiming at a significantly better precision in PRad-II compared with PRad
  - Employ two planes of coordinate tracking detectors
  - Achieve a precise measurement of tracking detector efficiency (~ 0.1% level)
  - Reduce various backgrounds
  - Use the integrated Møller method for all angular bins
  - Suppress the Q^2-dependent syst. uncertainties
  - Turn all the Møller syst. uncertainties into cross section normalization uncertainties
  - δr_p from RCs will be reduced from *0.0069 fm* to *0.0015 fm*

Plans for the NNLO calculations

- To achieve the PRad-II goal of total syst. uncertainty of *0.0032 fm*
  - Very necessary also to perform improved NNLO RC calculations
  - Plans in place by the PRad Collaboration's theory colleagues
  - Leading investigator Dr. Stanislav Srednyak in close collaboration with Drs. Igor Akushevich and Alexander Ilyichev
  - Contacts/potential collaborations with the PSI and Mainz groups on the subject matter established

- Advantages and disadvantages of the original paper *EPJA **51**, 1 (2015)*
  - Advantages
    - Both e-p and e-e treated in the same approach
    - First-order diagrams calculated analytically
    - Dependence on the electron mass kept, accurate in $\mathbb{O}(\alpha)$
  - Disadvantages (*indicated by Andrej Arbuzov at First TPC Collaboration Meeting in Mainz*)
    - Improper treatment of higher-order effects
    - No two-photon exchange, no hadronic vacuum polarization (PRad simulation included TPE effect)
    - No radiation off proton and up-down interference,

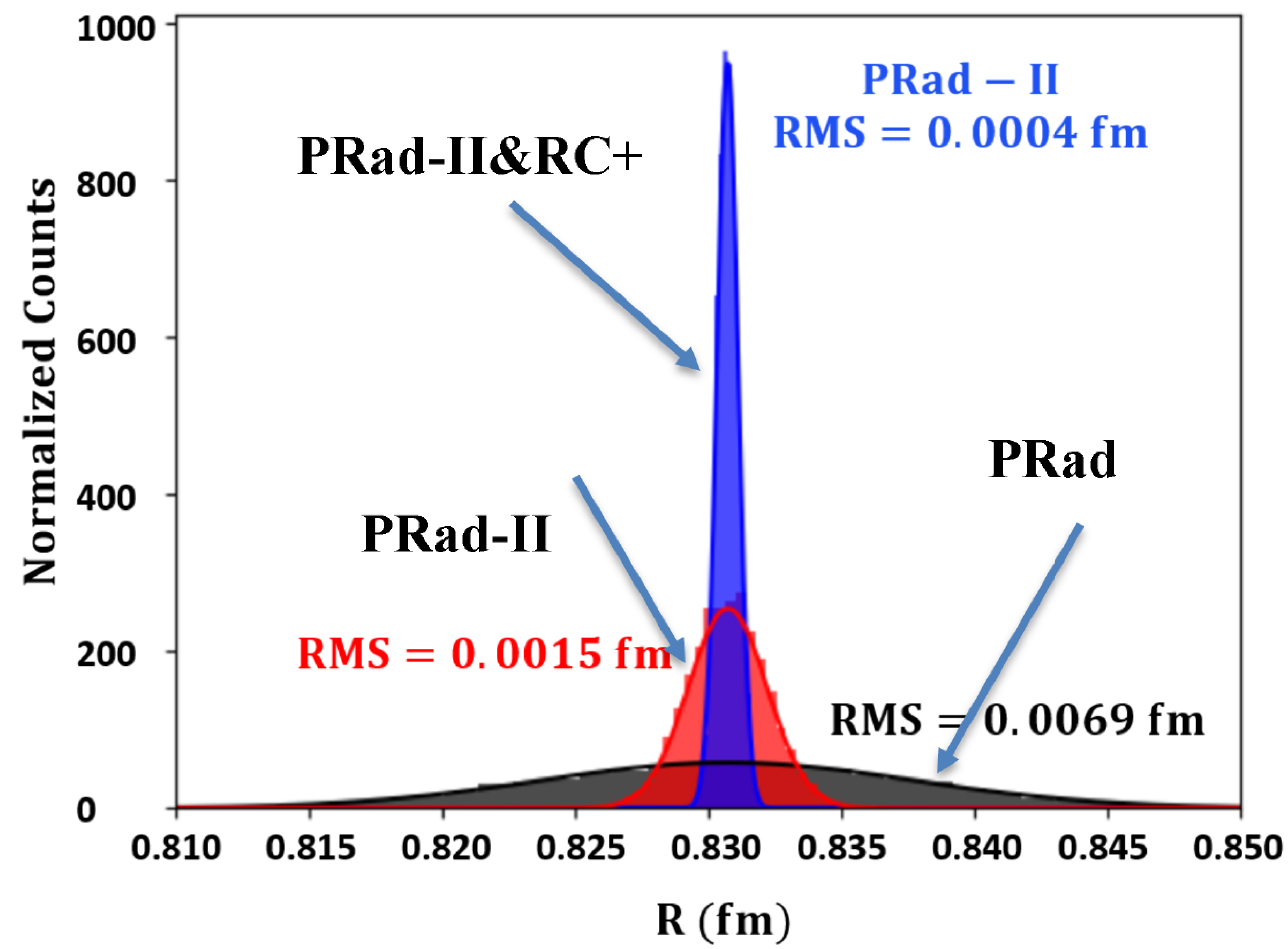
Plans for the NNLO calculations

- Submitted proposal to DOE by Akushevich (PI) and Gao (co-PI) on NNLO RC calculations for PRad-II (under review)

- Need to accomplish the following tasks
  - Calculation of NNLO contributions to e-p and Møller scattering diagrams by the end of 2022 (proposed plan in the DOE proposal)
    - Focus on mathematical approach of Gelfand-Kapranov-Zelevinsky
    - Develop the so-called Gamma series method
    - Calculate one- and two-loop integrals with the new method
  - Obtain all necessary results by the end of 2024
    - Evaluate also the two-photon exchange part to hadronic corrections
    - Make a new MC event generator or update the current one (working with PRad)
    - Finalize the project (working with PRad)

Improvement from PRad to PRad-II

- Improvement of the RC associated syst. uncertainty on r_p
  - Black spectrum $\rightarrow$ RC δr_p for PRad
  - Red spectrum $\rightarrow$ projected RC δr_p with two planes of coordinate tracking detectors plus current RC calculations
  - Blue spectrum $\rightarrow$ projected RC δr_p with two planes of coordinate tracking detectors plus improved RC calculations at NNLO



- Outline of the current project presented
  - Whitepaper on Radiative Corrections: *arXiv:2012.09970 [nucl-th]*
  - Synergy with ongoing RC-related studies for the JLab SoLID SIDIS and the planned studies for the proposed DRad experiment

Improvement from PRad to PRad-II

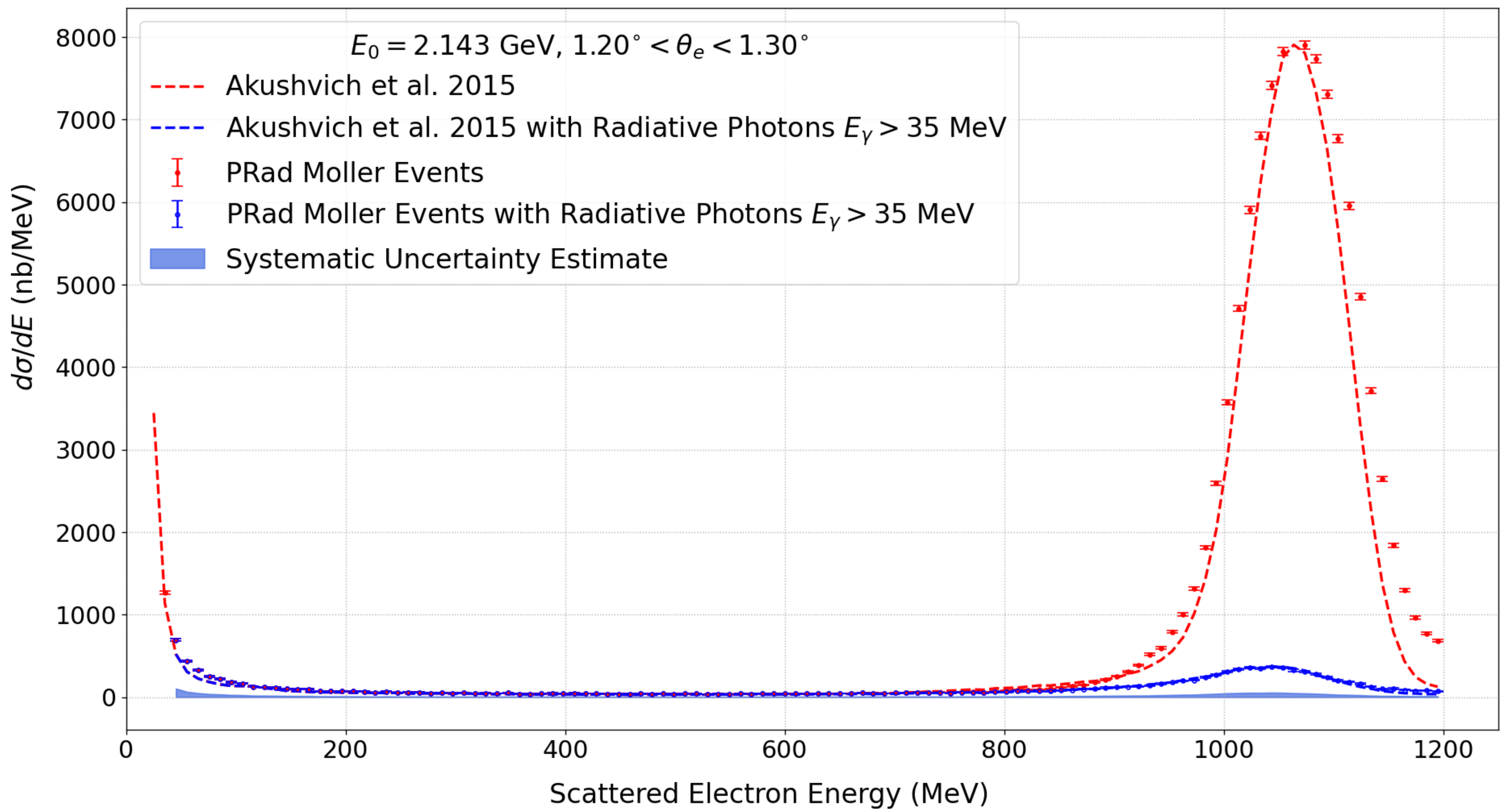
Item	PRad δr_p [fm]	PRad-II δr_p [fm]	Result of
Stat. uncertainty	0.0075	0.0017	More beam time and higher DAQ rate
GEM efficiency	0.0042	0.0008	2nd tracking detector
Acceptance	0.0026	0.0002	2nd tracking detector
Beam energy related	0.0022	0.0002	2nd tracking detector
Event selection	0.0070	0.0027	2nd tracking + HyCal upgrade
HyCal response	0.0029	Negligible	HyCal upgrade
Beam background	0.0039	0.0016	Better vacuum 2nd halo blocker vertex res. (2nd tracking)
Radiative correction	0.0069	0.0004	Improved calc.
Inelastic ep	0.0009	Negligible	Upgraded HyCal
G_M^p parameterization	0.0006	0.0005	---
Total syst. uncertainty	0.0115	0.0032	
Total uncertainty	**0.0137**	**0.0036**	

A factor of 3.8 improvement !

Can we test (partially) calculations of radiative effects?

- Validate (partially) calculations of radiative effects by PRad-II data
- PRad-II will be significantly improved compared with PRad
  - Two planes of tracking detectors and upgraded HyCal detector
  - Excellent PID between electrons and photons
  - Simultaneously detection of scattered electrons and “hard” radiative photons
  - Better resolution for low energy photons with all crystal HyCal
  - Higher statistics to check photon distributions vs. their opening angles and energies
- Limitations in PRad
  - GEM efficiency limited knowledge
  - HyCal lead-glass low resolution
  - Inability to determine cross sections with “hard” photon emissions with precision due to poor statistics

- Study with the PRad Møller data
  - Symmetric single-arm Møller events from the PRad 2.2 GeV data
  - Select "hard" radiative photon with $E_\gamma > 35$ MeV (limited by HyCal resolution)
  - "Hard" radiative process dominating the cross sections at low scattering energy



Summary

- For PRad-II, we plan for blind analyses to reduce possible bias coming from normalization and Q^2-dependence of the form factors
- In PRad, the RCs found as the second largest syst. uncertainty source for r_p
  - Estimated also by an independent study with another method
- Achieve a substantially better precision in PRad-II compared to PRad
  - Employ two planes of coordinate detectors to improve the detector efficiency
  - Apply the integrated Møller method to all angular bins
  - Suppress the Møller Q^2-dependent syst. uncertainties
  - Accomplish improved NNLO calculations for e-p and Møller scatterings
  - Experimental (partial) validation of calculations of radiative effects

Backup slides

- To reduce the systematic uncertainty, the ep cross section is normalized to the Møller cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \left[\frac{N_{\exp}(ep \to ep \text{ in } \theta_i \pm \Delta\theta)}{N_{\exp}(ee \to ee)}_i \cdot \frac{\varepsilon^{ee}_{\text{geom}}}{\varepsilon^{ep}_{\text{geom}}} \cdot \frac{\varepsilon^{ee}_{\text{det}}}{\varepsilon^{ep}_{\text{det}}}\right] \left(\frac{d\sigma}{d\Omega}\right)_{ee}$$

- Method 1: bin-by-bin method – taking e-p/e-e counts from the same angular bin
  - ➢ Cancellation of energy independent part of the efficiency and acceptance
  - ➢ Limited coverage due to double-arm Møller acceptance

- Method 2: integrated Møller method – integrate Møller in a fixed angular range and use it as common normalization for all angular bins
  - ➢ Needs to know the GEM efficiency well

Extraction of ep Elastic Scattering Cross Section

- Luminosity cancelled from both methods

- PRad: Bin-by-bin range: 0.7° to 1.6° for 2.2 GeV, 0.75° to 3.0° for 1.1 GeV. Larger angles use integrated Møller method (3.0° to 7.0° for 1.1 GeV; 1.6° to 7.0° for 2.2 GeV)

- PRad-II: two planes of GEM/μRwell allow for ***integrated Møller method*** for the entire experiment

- Event generators for unpolarized elastic ep and Møller scatterings have been developed based on complete calculations of radiative corrections – ***PRad-II with NNL for RC***
  1. A. V. Gramolin et al., J. Phys. G Nucl. Part. Phys. 41(2014)115001
  2. I. Akushevich et al., Eur. Phys. J. A 51(2015)1 (beyond ultra relativistic approximation)

- A Geant4 simulation package is used to study the radiative effects, and an iterative procedure applied

$$\sigma_{ep}^{Born(exp)} = \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{exp} / \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{sim} \cdot \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{Born(model)} \cdot \sigma_{ee}^{Born(model)}$$

- TPE effect based on calculation of Oleksandr Tomalak
  - O. Tomalak, Few Body Syst. **59**, no. 5, 87 (2018)
  - O. Tomalak and M. Vanderhaeghen, PRD **93** (2016) no.1, 013023
  - O. Tomalak and M. Vanderhaeghen, EPJA **51** (2015) no.2, 24
- Less than 0.2% effect on the cross section within PRad kinematic range