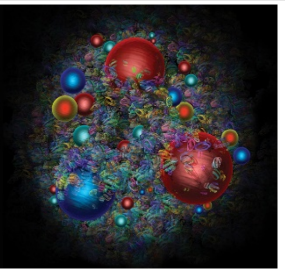
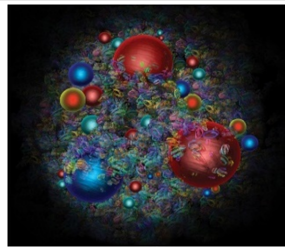
Studies of Radiative Corrections for the PRad-II Experiment

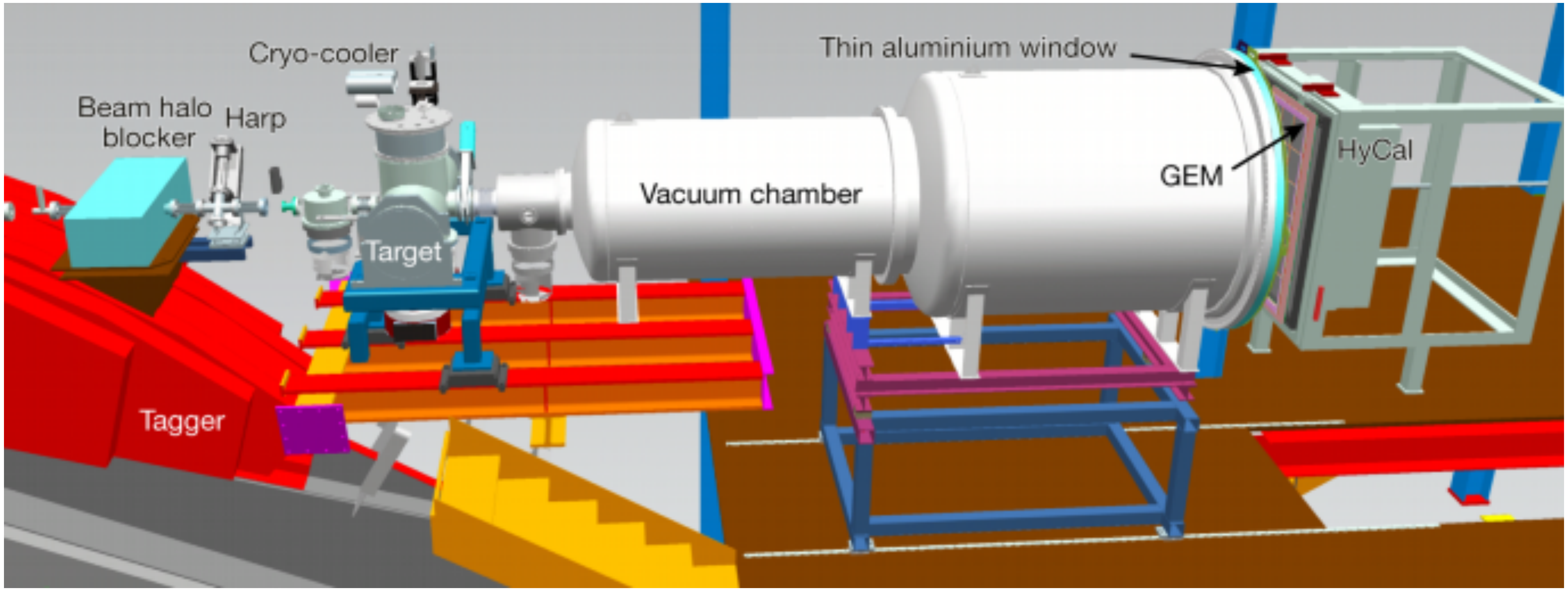


Haiyan Gao
Duke University
For the PRad Collaboration

**PRad-II C1
Review**

March 12, 2021



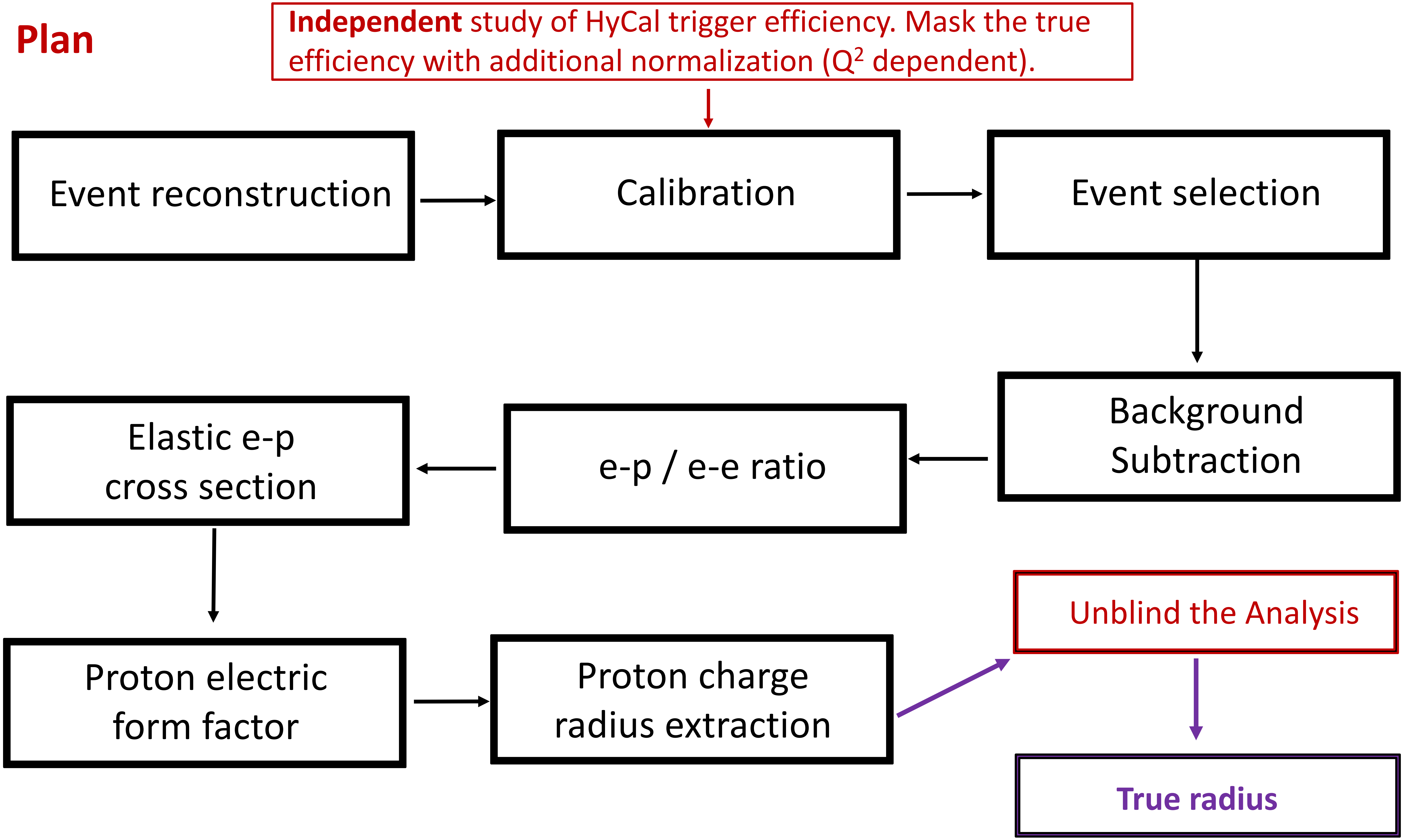


Outline

- Plan for blind analysis to extract the proton radius (r_p) for PRad-II
- Radiative correction (RC) studies for PRad
  - PRad's estimation of the RC systematic uncertainty of r_p
  - Independent study of the RC systematic uncertainty of r_p
- RC studies for PRad-II
  - Integrated Møller method
  - Plans for the next-to-next leading order (NNLO) calculations
  - Improvement from PRad to PRad-II
  - Partial testing of calculations of radiative effects
- Summary

Plan: Blind analysis for extraction of r_p for PRad-II

Plan



Independent study of HyCal trigger efficiency. Mask the true efficiency with additional normalization (Q^2 dependent).

Event reconstruction

Calibration

Event selection

Background Subtraction

e-p / e-e ratio

Elastic e-p cross section

Proton electric form factor

Proton charge radius extraction

Unblind the Analysis

True radius

PRad's estimation of the RC syst. uncertainty of r_p

- Measured radius: *Nature 575, 147 (2019)*
  - $r_p = (0.831 \pm 0.007_{stat} \pm 0.012_{syst})$ fm
- r_p uncertainties for PRad shown in the table
  - Uncertainties estimated using the rational (1,1) function

    *X. Yan et al. PRC **98**, 025204 (2018)*

Using rational (1,1)

$$f(Q^2) = \frac{1 + p_1 Q^2}{1 + p_2 Q^2}$$

Item	PRad δr_p [fm]
Stat. uncertainty	0.0075
GEM efficiency	0.0042
Acceptance	0.0026
Beam energy related	0.0022
Event selection	0.0070
HyCal response	0.0029
Beam background	0.0039
Radiative correction	0.0069
Inelastic *ep*	0.0009
G_M^p parameterization	0.0006
Total syst. uncertainty	0.0115
Total uncertainty	0.0137

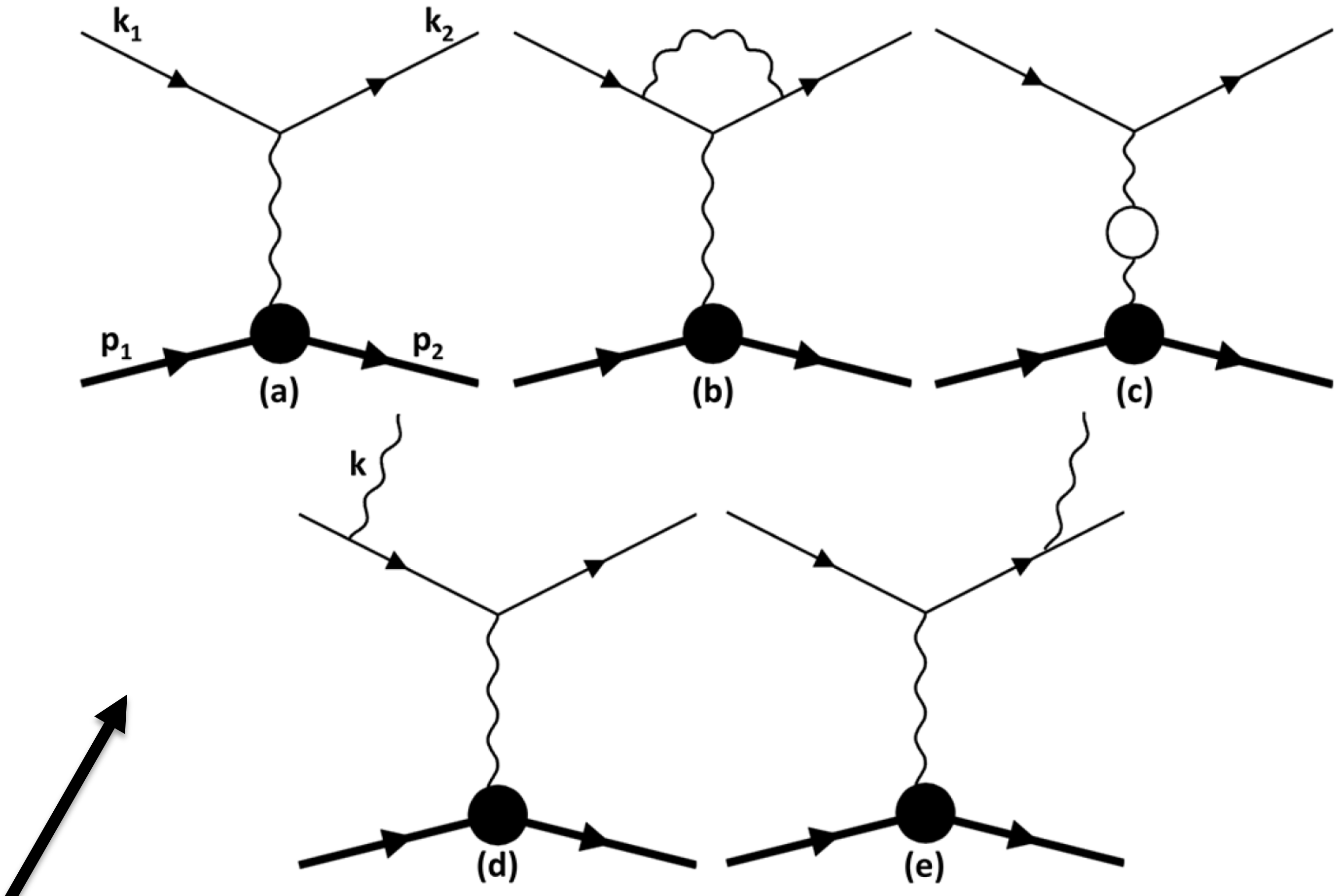
PRad's estimation of the RC syst. uncertainty of r_p

- RCs one of the largest syst. uncertainty sources of r_p for PRad

  - RCs studied for both e-p and Møller scatterings
  - Event generator used, made using the results from *I. Akushevich et al, EPJA **51**, 1 (2015)*
    - Used analytical calculations for one-loop e-p and Møller RC diagrams
    - Calculated within covariant formalism and beyond ultra-relativistic limit
    - Infrared divergence extracted and cancelled by Bardin-Shumeiko approach
  - PRad RC syst. uncertainty on r_p estimated
    - using the first-order RC results from *EPJA **51**, 1 (2015)*
    - using a method from *A. Arbuzov and T. Kopylova, EPJC **75**, 603 (2015)* for estimation of the contribution stemming from higher order RCs
  - Estimated syst. uncertainties correlated and Q^2-dependent

PRad's estimation of the RC syst. uncertainty of r_p

- e-p and Møller NLO diagrams used for cross section calculations in *EPJA* ***51****, 1 (2015)*

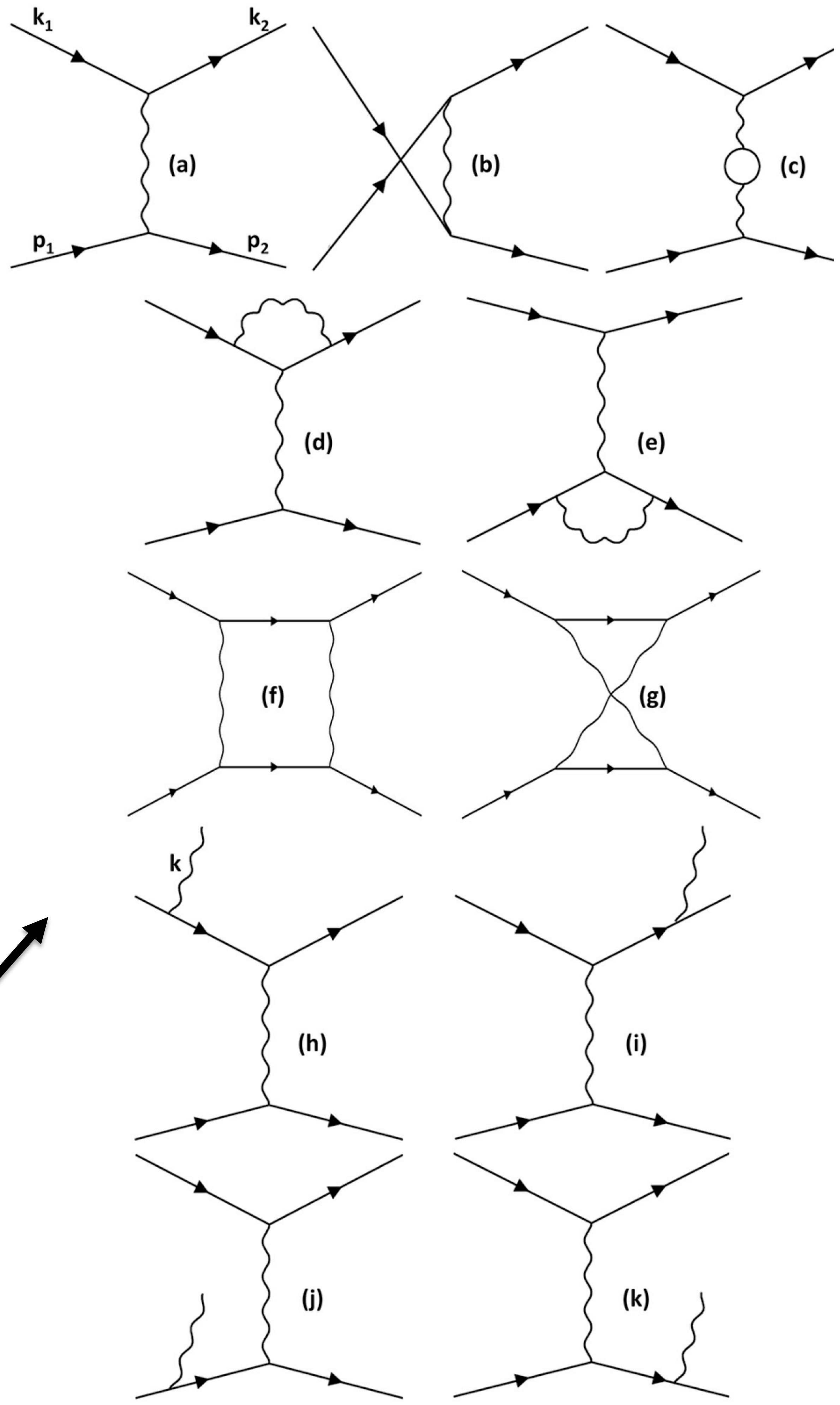


- Feynman diagrams contributing to the Born and RC cross sections in e+p elastic scattering:

  (a) The Born process; (b) Vertex correction;
  (c) Vacuum polarization; (d)-(e) Bremsstrahlung.

- Feynman diagrams contributing to the Born (a)-(b) and RC cross sections for Møller scattering:

  (c)-(e) Vacuum polarization and vertex correction;
  (f)-(g) Box contribution; (h)-(k) Bremsstrahlung.



PRad's estimation of the RC syst. uncertainty of r_p

- Two methods for forming e-p to e-e differential cross section ratio (luminosity cancellation)
  - Bin-by-bin method
    - Forms the ratio using the e-p and e-e counts from the same angular bin
    - Cancels out the energy-independent part of acceptance and GEM efficiency
    - Q^2 -dependent syst. uncertainties from the e-e process introduced
  - Integrated Møller method
    - Uses e-e counts from a selected angular range
    - Gives a common normalization factor for all e-p Q^2 bins; no effect on extracted r_p
    - Not applied to all Q^2 bins in PRad, since the GEM efficiency not precisely determined in all those bins
- Q^2-dependence much larger for Møller RC in PRad
  - Affects the cross-section results via the use of the bin-by-bin method
  - For e-p RC → $\delta r_p = 0.0020\ fm$; for Møller RC → $\delta r_p = 0.0065\ fm$
  - For total RC → $\delta r_p = 0.0069\ fm$

Independent study of the RC syst. uncertainty of r_p

- Independent study performed for the second-order RC effect on r_p
  - Followed the approach of *A. Aleksejevs et al, Physics of Atomic Nuclei,* ***76****, 888 (2013)*
    - Paper calculated two-loop radiative effects in the MOLLER experiment
  - Based on its mathematical framework and for PRad kinematics
    - Contribution from NNLO diagrams on the Born cross section estimated
    - For any reasonable photon energy cut for the PRad experiment
    - Q^2-dependent syst. uncertainties smaller than that estimated in the first approach
    - The largest RC syst. uncertainty computed: $\delta r_p = 0.0047\ fm$
  - However, approximated methods and restricted number of diagrams used
    - Improved and exact NNLO calculations are much desired

Integrated Møller method

- Limitations of GEM efficiency determination in PRad
  - Contributed indirectly to the total syst. uncertainty
  - Should be improved
- Aiming at a significantly better precision in PRad-II compared with PRad
  - Employ two planes of coordinate tracking detectors
  - Achieve a precise measurement of tracking detector efficiency (~ 0.1% level)
  - Reduce various backgrounds
  - Use the integrated Møller method for all angular bins
  - Suppress the Q^2-dependent syst. uncertainties
  - Turn all the Møller syst. uncertainties into cross section normalization uncertainties
  - δr_p from RCs will be reduced from *0.0069 fm* to *0.0015 fm*

Plans for the NNLO calculations

- To achieve the PRad-II goal of total syst. uncertainty of *0.0032 fm*
  - Very necessary also to perform improved NNLO RC calculations
  - Plans in place by the PRad Collaboration’s theory colleagues
  - Leading investigator Dr. Stanislav Srednyak in close collaboration with Drs. Igor Akushevich and Alexander Ilyichev
  - Contacts/potential collaborations with the PSI and Mainz groups on the subject matter established
- Advantages and disadvantages of the original paper *EPJA **51**, 1 (2015)*
  - Advantages
    - Both e-p and e-e treated in the same approach
    - First-order diagrams calculated analytically
    - Dependence on the electron mass kept, accurate in $\mathbb{O}(\alpha)$
  - Disadvantages (*indicated by Andrej Arbuzov at First TPC Collaboration Meeting in Mainz*)
    - Improper treatment of higher-order effects
    - No two-photon exchange, no hadronic vacuum polarization (PRad simulation included TPE effect)
    - No radiation off proton and up-down interference,

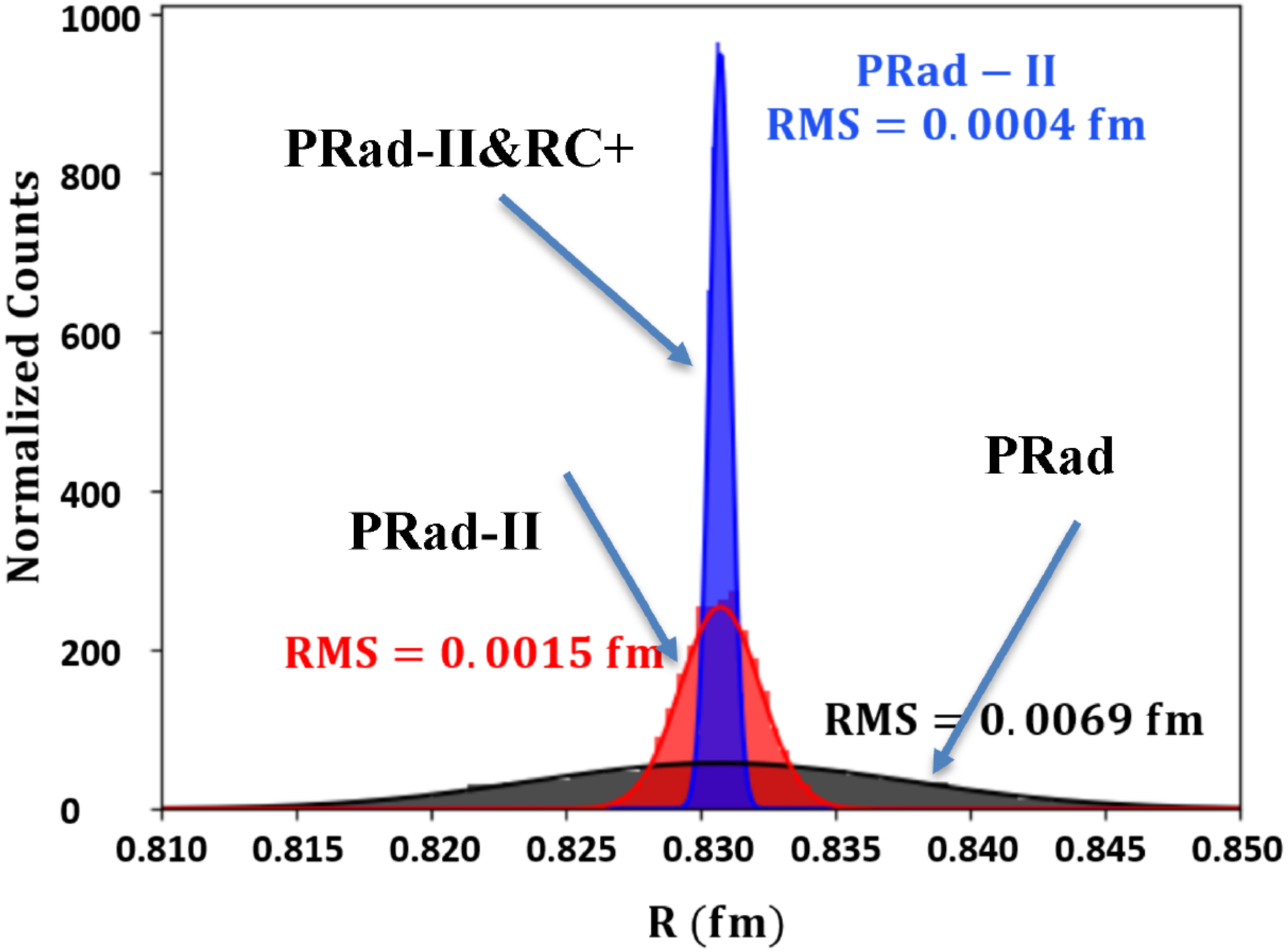
Plans for the NNLO calculations

- Submitted proposal to DOE by Akushevich (PI) and Gao (co-PI) on NNLO RC calculations for PRad-II (under review)

- Need to accomplish the following tasks
  - Calculation of NNLO contributions to e-p and Møller scattering diagrams by the end of 2022 (proposed plan in the DOE proposal)
    - Focus on mathematical approach of Gelfand-Kapranov-Zelevinsky
    - Develop the so-called Gamma series method
    - Calculate one- and two-loop integrals with the new method
  - Obtain all necessary results by the end of 2024
    - Evaluate also the two-photon exchange part to hadronic corrections
    - Make a new MC event generator or update the current one (working with PRad)
    - Finalize the project (working with PRad)

Improvement from PRad to PRad-II

- Improvement of the RC associated syst. uncertainty on r_p
  - Black spectrum → RC δr_p for PRad
  - Red spectrum → projected RC δr_p with two planes of coordinate tracking detectors plus current RC calculations
  - Blue spectrum → projected RC δr_p with two planes of coordinate tracking detectors plus improved RC calculations at NNLO



- Outline of the current project presented
  - Whitepaper on Radiative Corrections: *arXiv:2012.09970 [nucl-th]*
  - Synergy with ongoing RC-related studies for the JLab SoLID SIDIS and the planned studies for the proposed DRad experiment

Improvement from PRad to PRad-II

Item	PRad δr_p [fm]	PRad-II δr_p [fm]	Result of
Stat. uncertainty	0.0075	0.0017	**More beam time and higher DAQ rate**
GEM efficiency	0.0042	0.0008	**2nd tracking detector**
Acceptance	0.0026	0.0002	**2nd tracking detector**
Beam energy related	0.0022	0.0002	**2nd tracking detector**
Event selection	0.0070	0.0027	**2nd tracking + HyCal upgrade**
HyCal response	0.0029	Negligible	**HyCal upgrade**
Beam background	0.0039	0.0016	**Better vacuum** **2nd halo blocker vertex res.** **(2nd tracking)**
Radiative correction	0.0069	0.0004	**Improved calc.**
Inelastic ep	0.0009	Negligible	**Upgraded HyCal**
G_M^p **parameterization**	0.0006	0.0005	**---**
Total syst. uncertainty	0.0115	0.0032	
Total uncertainty	**0.0137**	**0.0036**	

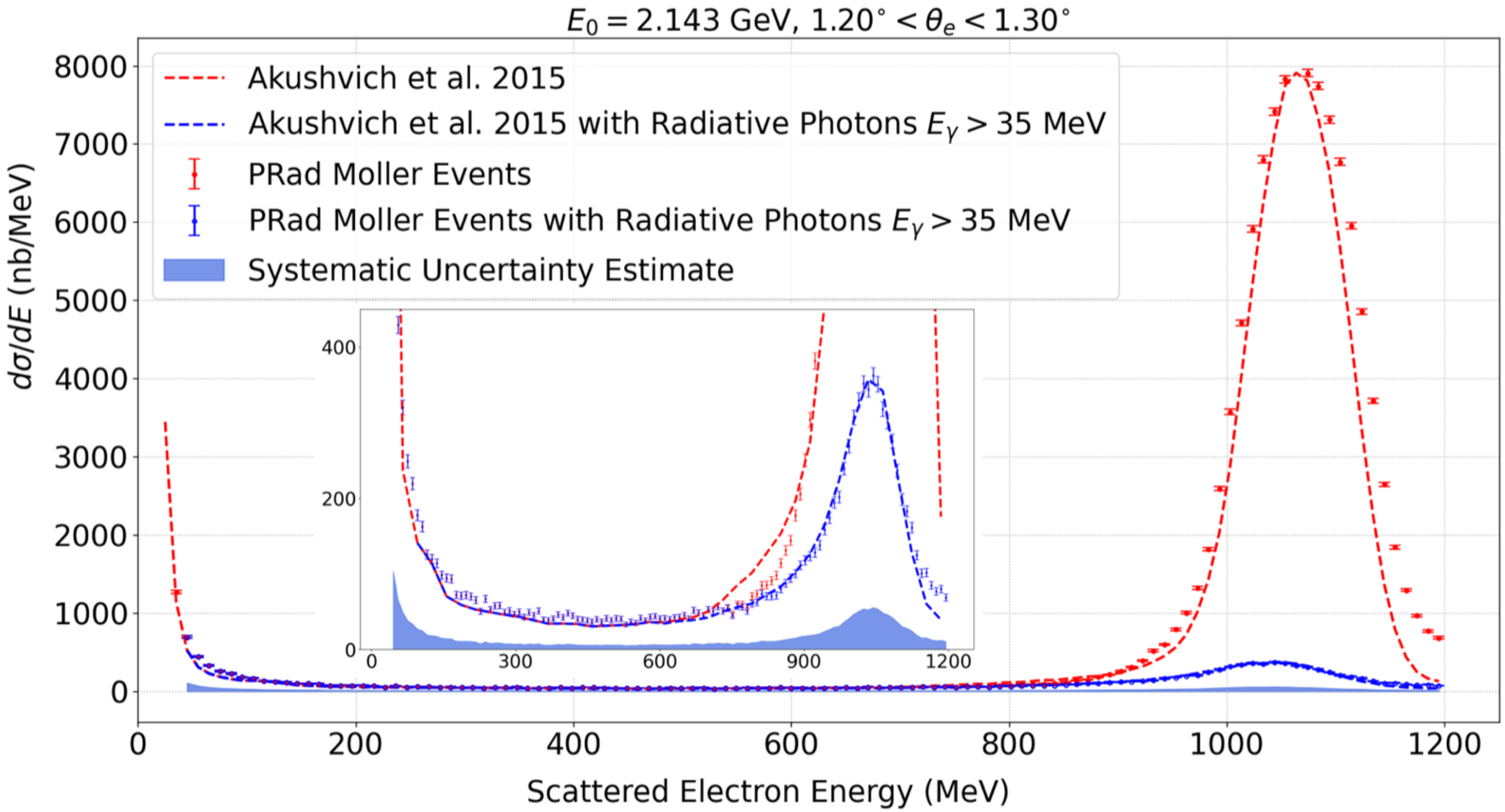
A factor of 3.8 improvement !

Can we test (partially) calculations of radiative effects?

- Validate (partially) calculations of radiative effects by PRad-II data
- PRad-II will be significantly improved compared with PRad
  - Two planes of tracking detectors and upgraded HyCal detector
  - Excellent PID between electrons and photons
  - Simultaneously detection of scattered electrons and "hard" radiative photons
  - Better resolution for low energy photons with all crystal HyCal
  - Higher statistics to check photon distributions vs. their opening angles and energies
- Limitations in PRad
  - GEM efficiency limited knowledge
  - HyCal lead-glass low resolution
  - Inability to determine cross sections with "hard" photon emissions with precision due to poor statistics

Can we test (partially) calculations of radiative effects?

- Study with the PRad Møller data
  - Symmetric single-arm Møller events from the PRad 2.2 GeV data
  - Select “hard” radiative photon with $E_\gamma > 35$ MeV (limited by HyCal resolution)
  - “Hard” radiative process dominating the cross sections at low scattering energy



Summary

- For PRad-II, we plan for blind analyses to reduce possible bias coming from normalization and Q^2-dependence of the form factors
- In PRad, the RCs found as the second largest syst. uncertainty source for r_p
  - Estimated also by an independent study with another method
- Achieve a substantially better precision in PRad-II compared to PRad
  - Employ two planes of coordinate detectors to improve the detector efficiency
  - Apply the integrated Møller method to all angular bins
  - Suppress the Møller Q^2-dependent syst. uncertainties
  - Accomplish improved NNLO calculations for e-p and Møller scatterings
  - Experimental (partial) validation of calculations of radiative effects