PRad-II Technical Review

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1 Executive Summary

The A rated PRad-II experiment will implement an enhanced apparatus to measure the proton's electric form factor (G_E^p) and root-mean-square (RMS) charge radius (r_p) utilizing a calorimetric technique that was pioneered by the PRad collaboration. The PRad-II experiment will deliver the most precise measurement of G_E^p reaching the lowest ever Q^2 (10^{-5} GeV^2) in lepton scattering experiments, critical for the model-independent extraction of r_p. PRad-II will achieve a total uncertainty of 0.0036 fm on r_p, which is ~ 4 times smaller than PRad and better than the most precise atomic hydrogen (H) spectroscopy result [1] (total uncertainty of 0.0038 fm). The high precision and the unique technique of PRad-II will be critical for resolving the $\sim 6\%$ discrepancy in r_p obtained from modern $e-p$ scattering experiments. This discrepancy is dominated by the data in the high Q^2 region which was accessed only with the lead-glass part of HyCal during the PRad experiment. PRad-II will also address possible systematic differences between the most precise H and μH spectroscopy results and provide independent input for future CODATA recommendations for r_p and the Rydberg constant.

The unprecedented precision of PRad-II will be achieved by a number of critical upgrades to the detectors and DAQ, such as,
i) Adding two new coordinate detectors to enable tracking capability in the experiment and to significantly reduce beam-line associated backgrounds.
ii) Upgrading the hybrid calorimeter (HyCal) to an all $PbWO_4$ setup. The upgraded calorimeter is critical to help eliminate the uncertainty from the model dependent separation of the inelastic background from the elastic signal events in the high Q^2 region accessed by the existing lead-glass portion of the HyCal. The calorimeter upgrade also provides a uniform detector that is needed for realizing the key advantage of the calorimetric method - a large Q^2 coverage spanning two orders of magnitude in a single fixed setting - in the accurate extraction of r_p and achieving the projected precision.
iii) Upgrading the calorimeter DAQ to a flash ADC-based system to improve the DAQ rate and reduce the statistical uncertainty.

The enhanced apparatus, together with improved radiative correction calculations, will open up a new precision frontier in electron scattering allowing for the studies of QCD to be competitive with those of QED in atomic physics. Each of the aforementioned enhancements is essential and synergistic and when combined together achieves the projected highest precision in $e-p$ scattering experiments.

Finally, the upgraded apparatus could serve as a unique large acceptance high precision experimental platform for several planned photo- and electro-production experiments, that require precision cross section measurements at extreme forward directions.

2 PAC48 report

While the PAC48 endorsed the physics of the proposed experiment in the strongest possible terms by awarding the highest and only A rating among all proposals submitted in June 2020, the PAC conditionally (C1) approved the PRad-II experiment with a number of issues raised below. In this document, we are addressing all these issues so that the C1 condition can be removed and the experiment can be fully approved. Below we list the three main issues:

1. The μ-RWell technology has never been used in a running experiment, and its reliability and radiation hardness have not been fully demonstrated. Since the main reduction on the total uncertainty arises from the addition of a second tracking station, the PAC recommends considering a second GEM station instead, further relying on the present GEM technology to reduce the risks of jeopardizing the final physics goal.

2. The upgrade of HyCal implies 1500 additional $PbWO_4$ crystals and a new electronic readout. The cost estimate is about $5M. While it is clear that the new readout based on FADC will strongly increase the rate of data taking (and thus reduce the statistical uncertainty), the PAC could not be convinced on the necessity of the costly replacement of the crystals for reaching the final uncertainty on the extracted proton radius.

3. The PAC strongly suggests the planning of a blind analysis to convincingly reduce possible bias stemming from the normalization and the Q^2-dependence of the form factor. In particular, all radiative correction calculations and their implementation in the Monte Carlo simulation should be fixed before the fit for the proton radius.

In the following section, we will first remind everyone of the proposed PRad-II experimental setup and then address the issues raised by the PAC.

3 PRad-II: Going beyond the state-of-the-art

Based on the PRad experience, we have designed a new experiment (PRad-II) that incorporates an enhanced apparatus. PRad-II was recently approved by the JLab PAC48 with the highest scientific rating for 40 PAC days [2]. The additional beam time compared to the PRad experiment, and the higher DAQ rate made possible by the new flash ADC-based DAQ for the colorimeter, will reduce the statistical uncertainty by a factor of ~ 4. The remaining improvements are from reducing the key systematic uncertainties that dominate the PRad result, namely,

i) the non-uniformity of the detector response due to the ~ 3 times worse resolution of the Pb-glass portion of the HyCal calorimeter leading to larger uncertainty associated with the inelastic background;

ii) improved precision of the efficiency determination of the GEM-based coordinate detector;

iii) an even wider coverage of Q^2 than PRad, by reaching the lowest Q^2 accessed by any lepton scattering experiment;

iv) further suppression of backgrounds associated with the beam-line; and

v) improved radiative correction calculations will further improve the precision of the proton radius determination from the upgraded PRad-II experiment.

Figure 1 shows a schematic layout of the PRad-II experimental setup. A factor of ~ 4 improvement in the total uncertainty of the r_p measurement will be achieved with the following upgrades to the PRad apparatus and the approved beam time.

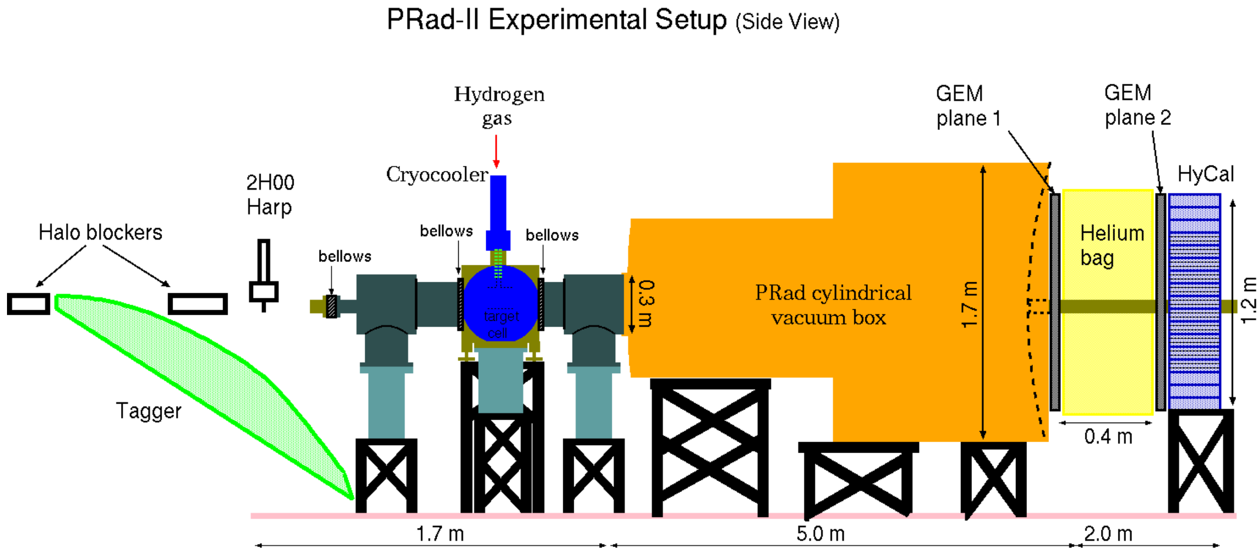

Figure 1: The proposed experimental setup for PRad-II.

3.1 New tracking capabilities

1. The μ-RWell technology has never been used in a running experiment, and its reliability and radiation hardness have not been fully demonstrated. Since the main reduction on the total uncertainty arises from the addition of a second tracking station, the PAC recommends considering a second GEM station instead, further relying on the present GEM technology to reduce the risks of jeopardizing the final physics goal.

We agree with the PAC that large area μ-RWELL detectors have not been used in experiments yet. Given this, we plan to have two layers of GEM detectors for PRad-II. The limitations on the precision of the GEM detector efficiency contributed indirectly to the systematic uncertainty of the PRad experiment. A precise measurement of the GEM detector efficiency (at the level of 0.1%) would allow the integrated Møller method to be used over the entire angular acceptance of the experiment. The uncertainties associated with Møller counts used in this method are normalization type uncertainties and thus, do not contribute to the systematic uncertainty in extracting r_p. However, this method relies on a correction for the inefficiency of the GEM detector. The presence of the spacer grids (which are used to keep the GEM foils apart from each other) in the PRad GEM detectors caused narrow regions of lower efficiency along the spacers. While these efficiencies were measured relative to HyCal and corrected in data analysis, the relatively poor position resolution of the HyCal led to larger uncertainties in the locations of these low efficiency areas of the GEM detectors. This resulted in systematic uncertainties as large as 0.5% in the forward scattering angular region [3, 4]. These larger systematic uncertainties precluded the integrated Møller method from being applied in the forward angle region. Instead, the PRad result relied on the bin-by-bin method for the forward angle region. While the bin-by-bin method is excellent in canceling the effect of the GEM detector inefficiency, it introduces Q^2-dependent systematic uncertainties due to the angular dependence of Møller scattering with contributions from Møller radiative correction, Møller event selection, beam energy and acceptance. Higher precision in the determination of the GEM efficiency would allow for the use of the integrated Møller method over the full experimental acceptance eliminating these Q^2-dependent systematic uncertainties. Therefore, we plan to replace the pair of GEM based coordinate detector used in PRad with 2 new GEM layers with optimized

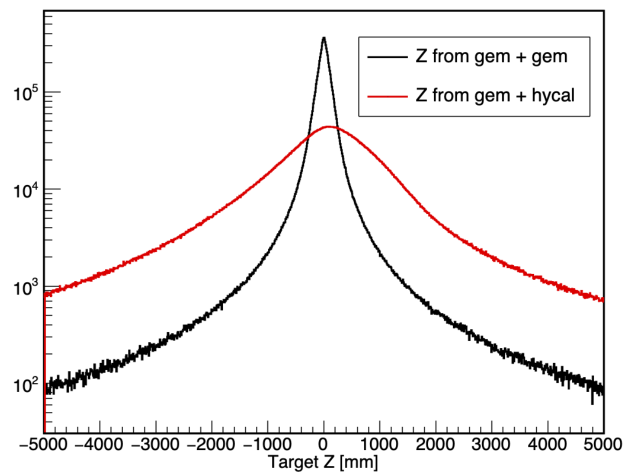

Figure 2: Reconstructed reaction z-vertex when using one GEM plane along with the HyCal vs using two GEM detector planes.

spacers.

A high precision measurement of the GEM detector efficiency profile can be achieved by adding the second GEM detector plane. In this case, each GEM plane can be calibrated with respect to the other GEM plane and the upgraded HyCal. Having a second GEM layer will also help reduce various backgrounds such as cosmic backgrounds and the high-energy photon background that have an impact on the determination of the GEM efficiency [5]. In addition, the tracking capability afforded by the pair of separated GEM detector planes will allow measurements of the interaction $z-$vertex. The improvement in the resolution of the reconstructed z-vertex achievable with two GEM layers is shown in Fig. 2. This improved z-vertex accuracy can be used to eliminate various beam-line backgrounds, such as those generated from the upstream beam halo blocker. The uncertainty due to the subtraction of the beam-line background, at forward angles, was one of the dominant uncertainties of PRad. Therefore, the two new GEM detector planes will reduce the overall systematic uncertainty contributed by the two aforementioned dominant sources.

The tracking capabilities of PRad-II will be enhanced significantly compared to PRad by replacing the original GEM layer with two new GEM layers (4 detectors in all), separated by 40 cm between them. These two new GEM layers will be built by the UVa group. The outer dimensions and readout parameters of these new GEM layers will be similar to the original PRad GEM plane; with an active area of 123 cm $\times$ 110 cm composed of two side-by-side detectors, each with an active area of 123 cm $\times$ 55 cm, arranged so that there is a narrow overlap area in the middle. Having two layers allows for highly accurate determination of efficiency profile for the entire detector area; *i.e.* much smaller inefficiency corrections to make and the inefficiency corrections determined with much higher accuracy.

3.2 An all $PbWO_4$ crystal HyCal Calorimeter

2. The upgrade of HyCal implies 1500 additional PbWO4 crystals and a new electronic readout. The cost estimate is about $5M. While it is clear that the new readout based on FADC will strongly increase the rate of data taking (and thus reduce the statistical uncertainty), the PAC could not be convinced on the necessity of the costly replacement of the crystals for reaching the final uncertainty on the proton radius.

Table 1: *The uncertainty table for r_p from the PRad experiment, and the projected uncertainties for PRad-II as proposed to the PAC48, and without the HyCal upgrade to an all $PbWO_4$ calorimeter and without the use of spacer-free μ-RWELL technology in the two new coordinate detector planes.*

Sources	PRad δr_p [fm]	PRad-II δr_p [fm]	PRad-II δr_p [fm] w/o HyCal upgrade and with 2 new GEM planes
Stat. uncertainty	0.0075	0.0017	0.0017
HyCal non-uniform response	0.0029	0.0001	0.0013
Inelastic *ep*	0.0009	0.0001	0.0009
Event selection	0.0070	0.0027	0.0034
GEM efficiency	0.0042	0.0008	0.0027
Acceptance & beam energy related	0.0034	0.0003	0.0003
Beam background	0.0039	0.0016	0.0016
Radiative correction	0.0069	0.0004	0.0004
G_M^p parameterization	0.0006	0.0005	0.0005
Total systematic	0.0115	0.0032	0.0049
Total uncertainty	0.0137	0.0036	0.0052

The upgrade of HyCal is an integral part of the experimental strategy to achieve the lowest uncertainty possible using the calorimetric technique pioneered by the PRad experiment. As shown in Table 1, three of the major contributions to the suppression of systematic uncertainties are from the non-uniformity of detector response, the separation of the inelastic background in the high Q^2 region and the event selection uncertainty. The total uncertainty that can be achieved without upgrading HyCal to an all $PbWO_4$ calorimeter and using GEM technology [1] is 0.052 fm. This would be an improvement by a factor of 2.6 compared to PRad, significantly lower than the factor of ∼ 4 improvement that was approved by PAC48, and no longer better than the most precise atomic hydrogen result.

In the PRad experiment the Pb-glass portion of the HyCal calorimeter was primarily sensitive to the high Q^2 kinematics covered by the experiment. This high Q^2 region is the main contributor to the ∼ 2% discrepancy in G_E^p between PRad and other modern $e-p$ scattering experiments [6] as can be seen from Fig. 3. The energy resolution of the Pb-glass shower detectors is about ∼ 3 times

[1] Two planes of new coordinate detectors (total 4 detectors) built using the standard GEM technology and optimized spacers

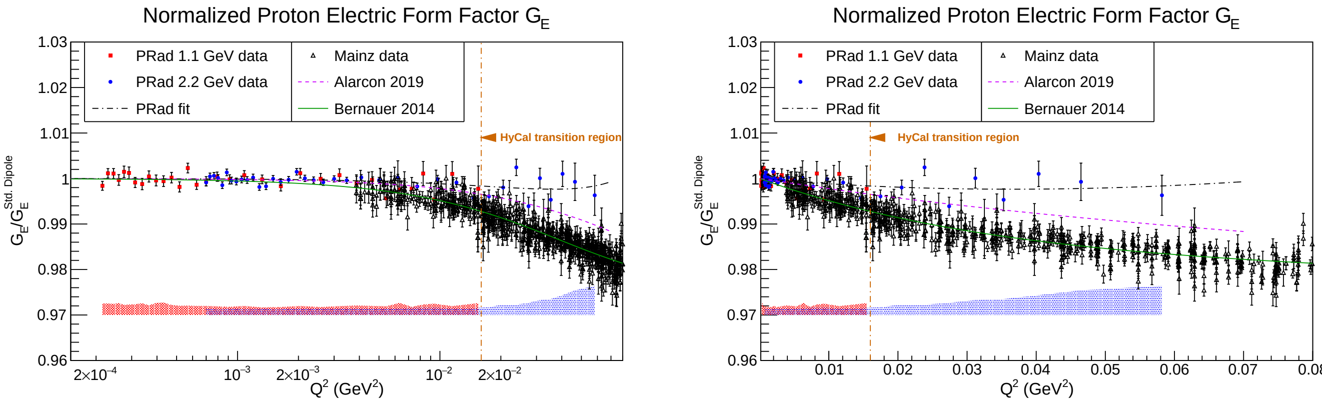

Figure 3: Extracted data and fits for G_E^p, as the ratio to $\mathrm{G}_{std.dipole} = (1+Q^2/0.71)^2$ to compress the range [6]. Shown are the PRad data and fit [3], the Mainz data, polynomial fit and experimental uncertainty [7], and a theoretical calculation by Alarcon *et al.* [8]. Data are normalized according to their respective fits. (*left*) x-axis in log scale and (*right*) x-axis in linear scale. The vertical line indicates the beginning of the Pb-glass region of HyCal for beam energy of 2.2 GeV.

worse than that of the $PbWO_4$ detectors, which increases the inelastic $e-p$ contamination in the elastic $e-p$ yield. Even though the contributions of these factors to the r_p systematic uncertainty are not as large as those from the Møller, their contributions to the cross section and G_E^p are much larger and primarily affect the high Q^2 data. Moreover, the separation of the inelastic background contribution from the elastic signal is model dependent.

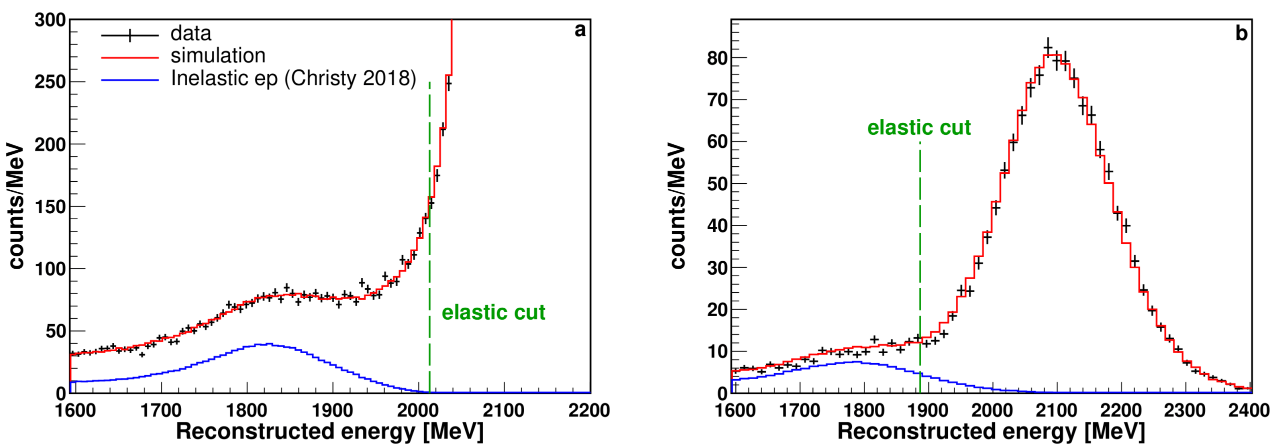

Figure 4: Comparison between reconstructed energy spectrum from the 2.2 GeV data (black) and simulation (red) for: (a) the $PbWO_4$ modules which cover scattering angles from 3.0° to 3.3°, corresponding to Q^2 around 0.014 $(\mathrm{GeV/c})^2$; (b) the Pb-glass modules which cover scattering angles from 6.0° to 7.0°, corresponding to Q^2 around 0.059 $(\mathrm{GeV/c})^2$ (largest Q^2 for PRad). Blue histograms show the inelastic $e-p$ contribution from the simulation. The green dash lines indicate the minimum elastic cut for selecting $e-p$ event for the two different detector modules. Due to the large difference in amplitudes, the elastic $e-p$ peak (amplitude 2800 counts/MeV) is not shown in (a), to display the Δ-resonance peak.

As shown in Fig. 4, in the $PbWO_4$ segment of the calorimeter, there is a clear separation between the elastic and inelastic $e-p$ events. However, for the Pb-glass segment of the calorimeter the contribution from the inelastic tails under the elastic peak were as large as 2% leading to a larger uncertainty. The only way to reduce this uncertainty is to replace the Pb-glass modules by $PbWO_4$ crystals. **This will suppress the inelastic $e-p$ contamination to less than 0.01% for the entire Q^2 range**, compared to the up to a maximum of 2% at the high Q^2 range in PRad. It will also eliminate the model dependence of the inelastic background separation that is displayed in Fig. 5.

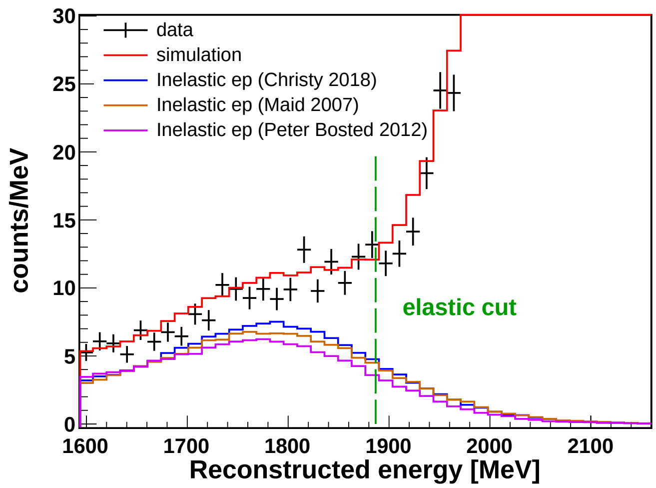

Figure 5: A zoomed in view of the simulated inelastic background under the elastic tail in the Pb-glass region of HyCal. The results from three models are shown to indicate the model dependence of the inelastic background subtraction.

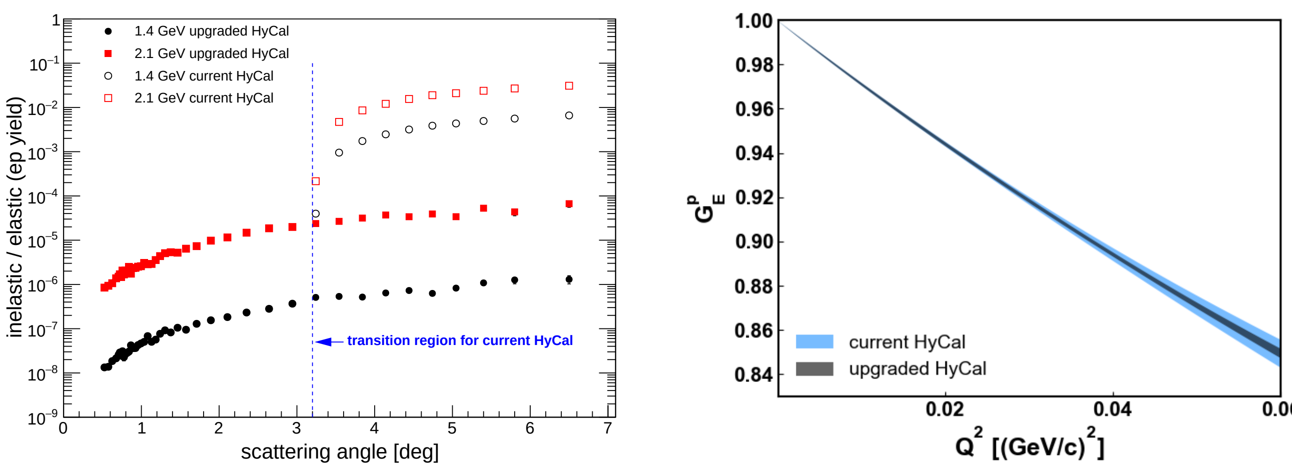

Figure 6: (*left*) The projected ratio of inelastic to elastic yield in the ugraded calorimeter compared to PRad data. (*right*) The one standard deviation systematic uncertainty band in the extracted G_E^p for the current HyCal and the upgraded calorimeter.

Note that the discrepancy in the G_E^p between PRad and the other modern $e-p$ scattering experiments [6] is also about the same magnitude as the inelastic $e-p$ contamination at high Q^2 in the PRad experiment. **An upgraded HyCal reduces the uncertainty due to the inelastic background by a factor $\sim$ 9**. Thus, the upgrade of HyCal to an all $PbWO_4$ calorimeter is essential to be able to address this discrepancy between PRad and the modern $e - p$ scattering experiments.

Furthermore, the differences in the detector properties between the $PbWO_4$ and Pb-glass modules lead to non-uniformity in the detector response. Uniformity of the electron detection is one of the key experimental strategies to reduce the systematic uncertainties of PRad-II relative to PRad. Thus, upgrading the calorimeter to an all $PbWO_4$ device will also suppress the Q^2-dependent systematic uncertainties due to differences in the detector properties between the $PbWO_4$ and Pb-glass modules. **The upgraded calorimeter** and the resultant uniformity of the electron detection over the entire experimental acceptance, **reduces the associated uncertainty by a factor of** $\sim$ **13**. Such uniformity of the detector package allows a large Q^2 coverage spanning two orders of magnitude in a single fixed setting. This is the key advantage of the calorimetric method in the accurate and robust extraction of r_p. The impact of the upgraded HyCal on the uncertainty in event selection and detector response was studied using the PRad comprehensive Monte Carlo simulation. The projected improvement in the ratio of inelastic to elastic yield in the upgraded calorimeter compared to the PRad data is shown in Fig. 6(left). Also Fig. 6(right) shown is the projected improvement in the one standard deviation systematic uncertainty band in the extracted G_E^p.

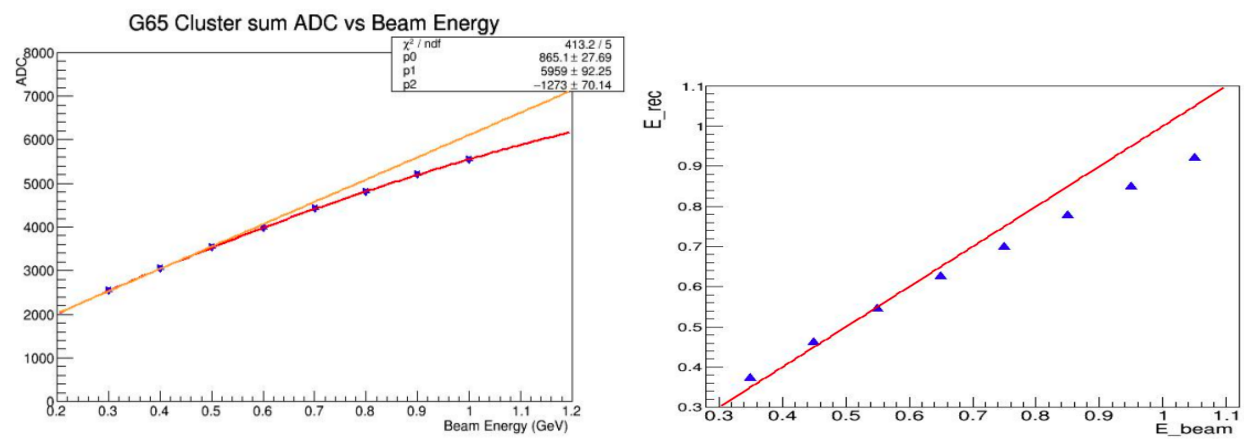

Figure 7: (*left*) The ADC output versus beam energy for a Pb-glass module. The yellow line show the expected linear behavior and the red line is a polynomial fit to the data. (*right*) The reconstructed energy versus beam energy for the same Pb-glass module. The red line shows the expected linear behavior.

Additionally, the PMTs on the Pb-glass modules of HyCal are over 50 years old as they were salvaged from a previous detector. The performance of the PMTs have deteriorated with age and many modules show highly non-linear response, as shown in Fig. 7. Therefore, at the very least the PMTs on the Pb-glass modules would certainly have to be replaced before PRad-II can be run.

Finally, this unique large acceptance high precision experimental setup could be used in several photo- and electro-production experiments requiring precision cross section measurements at extreme forward directions. It will be an ideal setup for the upcoming experimental proposal to measure the neutral pion transition form factor measurement at very low Q^2 values. A proposal is currently being developed by the PrimEx/PRad collaboration based on the PAC48 LOI recommendations. There are also currently active studies underway about using upgraded calorimeter in dark-matter search experiments for new MeV-range particles decaying through both e^+e^- and $\gamma - \gamma$ channels.

3.3 A flash ADC-based readout for the upgraded HyCal

Furthermore, converting the calorimeter readout electronics from a FASTBUS-based system to a flash analog-to-digital converter (ADC)-based setup will dramatically reduce the uncertainty due to detector gain and pedestal stability. The flash ADC readout system provides excellent timing and digital trigger information, allowing the rejection of various accidental events and improved trigger efficiency. The timing information is also critical for being able to use the veto scintilltors to reach the lowest scattering angles and hence the lowest Q^2 of any $e-p$ scattering ecperiment. The all flash ADC-based readout system will provide a seven-fold improvement in the DAQ rate. A faster DAQ will allow us to collect an order of magnitude more data within a reasonable amount of beam time. The projected DAQ rate with the upgraded DAQ is shown in Fig. 8. In addition to the additional beam time the faster DAQ rate compared to PRad is critical for the factor of ~ 4 reduction in the statistical uncertainty.

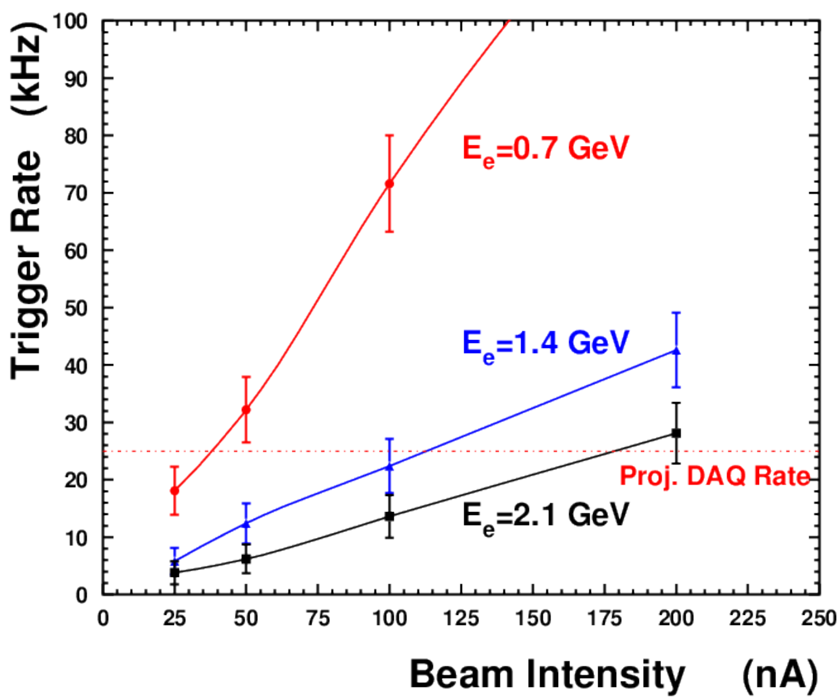

Figure 8: The trigger rate vs beam intensity and the projected DAQ rate.

3.4 Radiative corrections

3. The PAC strongly suggests the planning of a blind analysis to convincingly reduce possible bias stemming from the normalization and the Q^2-dependence of the form factor. In particular, all radiative correction calculations and their implementation in the Monte Carlo simulation should be fixed before the fit for the proton radius.

The total systematic uncertainty of the charge radius of the proton measured by PRad, $r_p = 0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}}$ fm [3, 4, 5], includes also the contribution from radiative corrections (RCs), which is one of the largest systematic uncertainty sources of r_p. PRad has estimated this uncertainty on the extracted r_p based upon the first order RC results from [9], and also using the method of [10] for estimation of the contribution coming from higher order RCs. The estimated systematic uncertainties for both $e-p$ and Møller scatterings are correlated and Q^2-dependent. The Q^2-dependence is much larger for the Møller RC, and it affects the cross section results through the use of the bin-by-bin method [4]. Transforming the cross section uncertainties into the uncertainties of r_p, for $e-p$ we have ~ 0.0020 fm, and for Møller we have ~ 0.0065 fm, and the total systematic uncertainty due to the higher order RCs is estimated to be $\delta r_p = 0.0069$ fm.

In the approved PRad-II experiment [2], the Q^2-dependent systematic uncertainties from the Møller scattering can be suppressed by using the integrated Møller method for all angular bins [4], which will turn all systematic uncertainties from the Møller process into normalization uncertainties for the cross sections. This method can be applied to the PRad-II experiment because we proposed to employ two planes of coordinate detectors, which will allow for the determination of the efficiency of each coordinate detector significantly better compared with the PRad experiment, therefore releasing us from the need of using the bin-by-bin method. Based on the studies for PRad-II, the systematic uncertainty associated with the radiative correction on r_p will be reduced from 0.0069 fm to 0.0015 fm by using two planes of coordinator detectors compared to the PRad result.

The PRad-II data can also test various calculations of the radiative effects. The upgraded GEM and HyCal detectors will provide precise particle identification (PID) between photons and electrons, hence allowing for a simultaneous detection of the scattered electrons and radiative photons from the "hard" radiative process. In this parasitic measurement of the radiative photons, GEM detectors serve as veto detector to discriminate photons from electrons, due to their insensitivity to neutral particles, while HyCal solely determines the position and energy of the photons. These data provide extra latitudes, in the energy and angle of the radiative photons, to benchmark the theoretical calculations of radiative effects within the PRad-II kinematic coverage.

A similar test has already been performed with the PRad data, as shown in Fig. 9. The test results bear large uncertainties, mainly due to three items: a) limited knowledge of GEM efficiency; b) low resolutions from the lead-glass region of HyCal; c) insufficient statistics to precisely determine the differential cross sections with "hard" photon emissions. The above-mentioned upgrade of the detector system in PRad-II will significantly improve these key factors that limited the test of radiative corrections in PRad. Therefore, we expect that PRad-II can provide an experimental validation of the radiative corrections. This kind of experimental validation will also be quite useful and applicable to other RC studies, like those for the deuteron charge radius extraction in unpolarized elastic $e-d$ scattering in the proposed DRad experiment at JLab, as well as for semi-inclusive deep inelastic scattering.

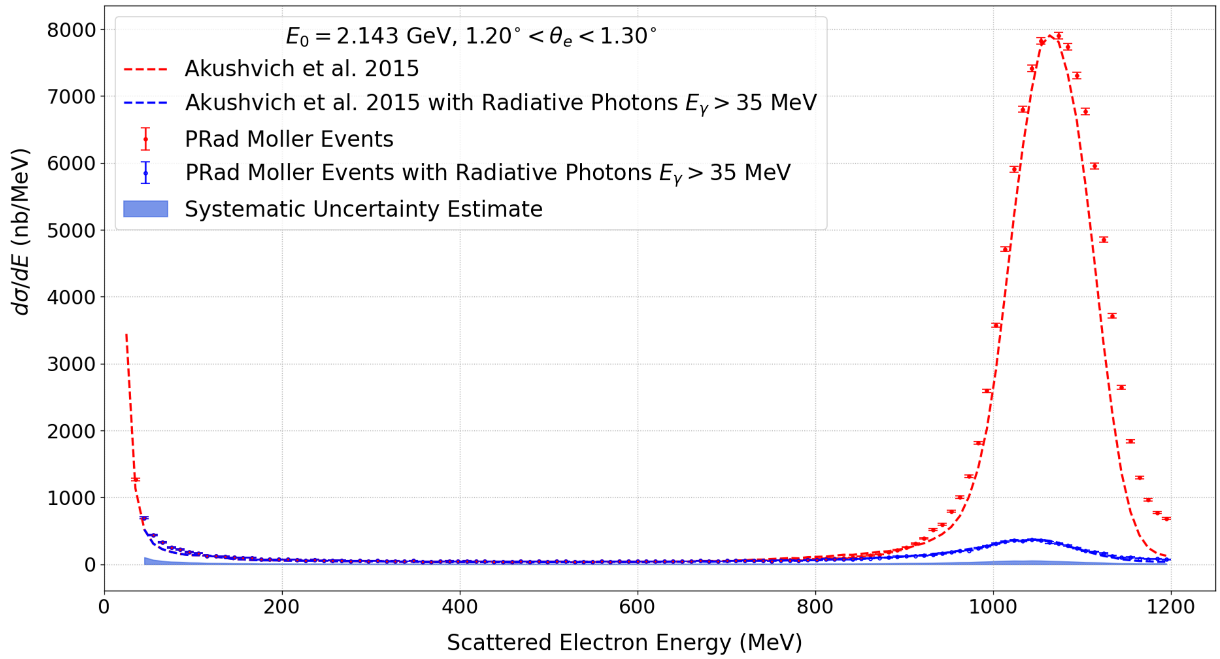

Figure 9: Preliminary results of the radiative effects test with PRad Møller data. This figure shows symmetric Møller events with scattering angle $1.2^\circ < \theta_e < 1.3^\circ$ and beam energy $E_0 = 2.143$ GeV. The "hard" radiative process with photon energy $E_\gamma > 35$ MeV (blue circles) contributes about 5% to the Møller total cross sections (red circles) around the symmetric energy $E_e = 1.122$ GeV. At lower scattering energy $E_e < 0.7$ GeV, the "hard" radiative process dominates the total cross sections. The preliminary results of the systematic uncertainties (blue band) are over 5% of the "hard" radiative cross sections (blue circles). Simulation results with the formalism of Møller radiative corrections from [9] are shown as the red and blue curves.

However, to achieve the PRad-II goal of total systematic uncertainty of 0.0032 fm, it is important and necessary to perform the RC calculations to beyond the next-to-leading order in order to reduce the systematic uncertainty on r_p associated with the RC further. Plans are in place for improved RC calculations to be performed at the next-to-next leading order (NNLO) level for elastic $e-p$ and Møller scatterings beyond ultrarelativistic limit, for the PRad-II kinematics. Currently, we are working with our theory colleagues, especially, with Igor Akushevich, Alexander Ilyichev and Stanislav Srednyak for the purpose of making such improved calculations. Drs. Akushevich and Ilyichev are prominent QED experts on one-loop Feynman diagram computations who have successfully applied the Bardin-Shumeiko [11] infrared divergence subtraction method in their analytic and numerical calculations for many years. Dr. Srednyak is a mathematical physicist, and an expert in QCD and QED calculations, who will have the key role in RC calculations for PRad-II.

The Medium Energy Physics Group at Duke University (one of the member institutions of the PRad collaboration) is already closely collaborating with the aforementioned theorists in addressing the calculations and building corresponding event generators from the RC studies of SIDIS and deuteron charge radius extraction. In September 2020, a proposal on NNLO RC calculations for the approved PRad-II experiment to DOE was submitted by Akushevich (PI) and Gao (co-PI).

The radiative corrections have several sources and accordingly different procedures have to be used for their computations. Thus, we wish to detail our plan on improved RC studies for PRad-II, which can be seen below. We intend to finish the entire project in the next three-four years.

1. During this time period up to the end of 2022, with our theory colleagues we will focus on calculations of two-loop integrals that give the NNLO contribution to the amplitudes of the $e-p$ and Møller scatterings. These are very similar to the two-loop integrals that arise in a variety of physical processes occurring in particle accelerators, ranging from JLab to Fermilab to LHC. There is a very large community of physicists who work on the development of theoretical, computational and numerical methods to deal with these integrals. There are correspondingly several software packages that have been developed for this purpose [12, 13, 14]. Meanwhile, in spite of such tremendous efforts, the problem is not completely solved yet and new computational methods are welcome. In this connection, our approach will evolve in two directions.

    - One of them will be concentrating on the development of new mathematical methods for the evaluation of two-loop integrals. The progress in physics has been paralleled by equally substantial progress in mathematics that is devoted to the same very problem - the study of integrals of power and rational functions. A very large array of methods has been developed in this regard. In particular, the method of Gamma series was proposed about 30 years ago in a series of papers by Gelfand-Kapranov-Zelevinsky. To be applicable for physics problems, this method has to be developed such that it will be possible to restrict systems of differential equations on sub-domains, and also to analytically (numerically) continue the series outside the region of their original definition. We will be developing this method to apply it to the problem of two-loop integration.

    - In the second direction, we will be developing the standard and well-known one-loop approaches. In the past two years, several other groups achieved significant results in the development of computational schemes for one-loop approaches. Eventually, the numerical results of these groups rely on Monte Carlo integration. We will compare these approaches and synthesize our own approach based on them, and apply it to the problem at hand.

2. In 2023 and 2024, we will address the problem of hadronic corrections, and finalize the previously obtained results. A part of these corrections comes from photon emission off the proton. This can be covered by the methods described above. However, there is a part - the so-called Two-Photon Exchange (TPE) part - that is intrinsically dependent on the proton structure. Because of this, it has been studied very extensively in the past 20 years, and there exists very large literature on this topic. With our current understanding of the proton, this part is model-dependent. Therefore, it has the form of an upper estimate. There are various approaches to this problem. In particular, the dispersive approach is believed to be adequate. In this approach a specific form for the hadronic part of the amplitude is written, which has multiple parameters. These parameters have been fitted to data from various experiments at JLab. We will utilize the most recent values of the parameters, and evaluate the TPE part to hadronic corrections. We hope to finish this work by the end of 2023. Afterwards, in 2024 we will accomplish the project along with making a PRad-II event generator for the complete $e-p$ and Møller cross sections (including the NLO and NNLO RCs).

The plan of all these RC-related studies, including that for PRad-II, is presented in a Whitepaper on Radiative Corrections [15]. We have also recently started a collaboration with a group of theoreticians at PSI and Zürich, who have developed good methods for computations of NNLO diagrams in $e-p$ scattering, and are interested in applying their calculations to PRad and PRad-II. Lastly, the group at Mainz, led by Hubert Spiesberger has been performing second-order RC calculations for lepton-proton scattering. In the time frame of the PRad-II experiment and with the efforts from these three groups, we are confident that we will achieve the systematic uncertainty goal of 0.0004 fm associated with the RC corrections for the proposed PRad-II experiment based on our studies (see Fig. 10 for more details).

For PRad-II, we indeed plan to carry out a blind analysis. For PRad, our Geant4 simulation of the experiment with radiative corrections for both the $e-p$ and $e-e$ scattering processes were fully implemented prior to our data analysis. We have also developed and published a method which allowed us to extract the proton charge radius in a robust way prior to our extraction of the proton radius from the PRad data on the proton electric form factor [16]. PRad-II aims at a significantly better precision compared with PRad. Therefore, a blind analysis which helps reduce possible bias stemming from the normalization and the Q^2-dependence of the form factor will be very important.

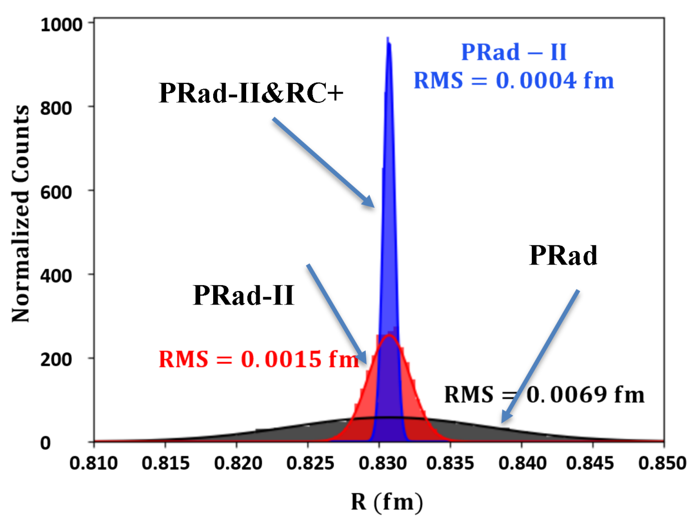

Figure 10: Improvement of the systematic uncertainty associated with the RC on the proton radius. The black spectrum shows the corresponding uncertainty for the PRad result. The red (blue) spectrum shows the projected systematic uncertainty from using two planes of coordinate detectors together with the current (improved) RC calculations.

4 Summary

The Prad-II experiment with upgraded detectors is projected to have a total uncertainty that is about a factor of ~ 4 times smaller than that from PRad. The projected uncertainties for PRad-II are shown in Table. 2. This is the updated table used during the PAC48 proposal defense [17, 18]. It includes new information from extensive simulation studies performed after the submission of the original PAC proposal [19]. The projected result assumes a factor of ~ 16 increase of the total statistics, leading to ~ 4 times reduction of the statistical uncertainty in r_p compared to PRad.

Table 2: *The uncertainty table for r_p from the PRad experiment, and the projected uncertainties for PRad-II. Uncertainties are estimated using the Rational (1,1) function.*

Item	PRad δr_p [fm]	PRad-II δr_p [fm]	Reason
Stat. uncertainty	0.0075	0.0017	more beam time
Inelastic *ep*	0.0009	0.0001	HyCal upgrade
HyCal non-uniform response	0.0029	0.0001	HyCal upgrade
Event selection	0.0070	0.0027	HyCal upgrade + two GEMs
GEM efficiency	0.0042	0.0008	two GEM detectors + HyCal upgrade
Acceptance & beam energy related	0.0034	0.0003	two GEM detectors
Beam background	0.0039	0.0016	better vacuum 2nd halo blocker vertex res.
Radiative correction	0.0069	0.0004	improved calc.
G_M^p parameterization	0.0006	0.0005	-
Total uncertainty	0.0137	0.0036	

The projected r_p from the PRad-II experiment is shown in Fig. 11 along with other measurements and the CODATA values.

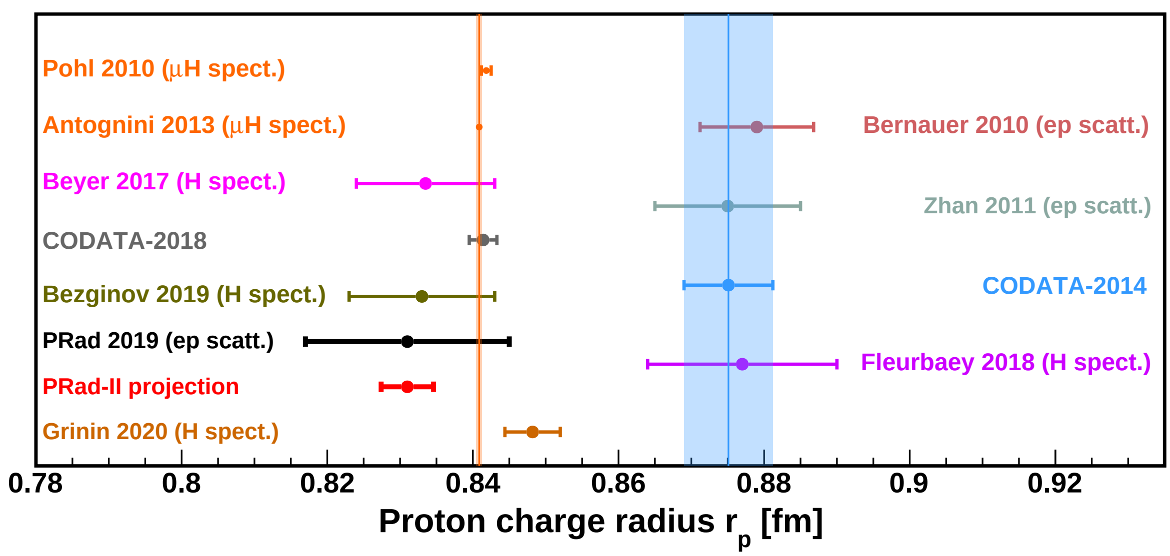

Figure 11: The projected r_p result from PRad-II, shown along with the result from PRad and other measurements.

References

[1] Grinin A. *et al.*, "Two-photon frequency comb specrtoscopy of atomic hydrogen," *Science* **370**, 1061-1066 (2020).

[2] Gasparian, A., Gao, H., Dutta, D., Liyanage, N., Pasyuk, E., Higinbotham D. W., Peng, C., Gnanvo, K. *et al.*, "PRad-II: A New Upgraded High Precision Measurement of the Proton Charge Radius," JLab PAC 48 proposal E12-20-004 (2020); arXiv:2009.10510

[3] Xiong W. *et al.*, "A small proton charge radius from an electron–proton scattering experiment," *Nature* **575**, 147 (2019).

[4] Xiong, W., "A High Precision Measurement of the Proton Charge Radius at JLab," PhD Thesis, Department of Physics, Duke University (2020).

[5] Bai, X., "Jefferson Lab High Precision Proton Radius Measurement Experiment: PRad", PhD Thesis, University of Virginia (2020).

[6] Bernauer, J. C., "The proton radius puzzle – 9 years later", EPJ Web of Conferences **234**, 01001 (2020).

[7] Bernauer, J. C. *et al.*, "High-Precision Determination of the Electric and Magnetic Form Factors of the Proton," *Phys. Rev. Lett.* **105**, 242001 (2010).

[8] Alarcón J. M., Higinbotham D. W., Weiss C. and Ye Z., "Proton charge radius extraction from electron scattering data using dispersively improved chiral effective field theory," *Phys. Rev. C* **99**, 044303 (2019).

[9] I. Akushevich, H. Gao, A. Ilyichev and M. Meziane, Eur. Phys. J. A **51**, 1 (2015).

[10] A. B. Arbuzov and T. V. Kopylova, Eur. Phys. J. C **75**, no.12, 603 (2015).

[11] D. Y. Bardin and N. M. Shumeiko, Nucl. Phys. B **127**, 242-258 (1977).

[12] R. D. Bucoveanu and H. Spiesberger, Eur. Phys. J. A **55**, no.4, 57 (2019).

[13] A new Monte Carlo simulation program for numerical calculations of the second-order leptonic radiative corrections for lepton-proton scattering in Ref. [12], made by R. D. Bucoveanu and H. Spiesberger, will be publicly available in the near future.

[14] C. M. Carloni Calame and Y. Ulrich, "Towards a NNLO Monte Carlo for MUonE," 2nd MUonE Collaboration Meeting at CERN, Feb. 12-13 (2020).

[15] A. Afanasev, J. Ahmed, J. C. Bernauer, P. G. Blunden, A. Bressan, D. Byer, E. Cline, M. Diefenthaler, J. M. Friedrich and H. Gao, *et al.* [arXiv:2012.09970 [nucl-th]]

[16] Yan, X. *et al.*, "Robust extraction of the proton charge radius from electron-proton scattering data," *Phys. Rev. C* **98**, 025204 (2018).

[17] `https://wiki.jlab.org/pcrewiki/images/f/fe/PRadII-PAC48-Presented.pdf`

[18] A. Gasparian *et al.*, "PRad-II: A New Upgraded High Precision Measurement of the Proton Charge Radius," *arXiv:2009.10510*

[19] `https://wiki.jlab.org/pcrewiki/images/3/31/PRad2_Proposal_final.pdf`