

A Study of the Omega Meson Decaying to Three Pions

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Abstract

The Omega Meson decay is studied from photoproduction data obtained by the CLAS experiment at Jefferson Lab. The reaction $\gamma p \rightarrow p\omega \rightarrow p(\pi^+\pi^-\pi^0)$ is study in the beam energy range of 3.6 to 5.3 GeV. The final states $p\pi^+\pi^-$ and beam energies are measured, the π^0 is reconstructed by missing mass. A Extended Likelihood fitting is used to compare data to a Dipersive Analysis of the decay by I. Dinilkin et al.

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1 Data

2 Likelihood Fit

$$\mathcal{L} = Prob(N) \prod_{i=1}^N p(\vec{x}_i, \vec{a}). \quad (1)$$

Assuming a Poisson distribution for the probability of observing N events, with an expected value of \mathcal{N}

$$Prob(N) = \frac{\mathcal{N}^N}{N!} e^{-\mathcal{N}} \quad (2)$$

the extended likelihood is then

$$\mathcal{L} = \left[\frac{\mathcal{N}^N}{N!} e^{-\mathcal{N}} \right] \prod_{i=1}^N p(\vec{x}_i, \vec{a}) \quad (3)$$

We renormalize $p(\vec{x}_i, \vec{a})$ by a new function $I(\vec{x}_i, \vec{a})$ such that

$$\eta(\vec{x}_i)I(\vec{x}_i, \vec{a}) = \mathcal{N}p(\vec{x}_i, \vec{a}) \quad (4)$$

The function $I(\vec{x}_i, \vec{a})$ is "nature's" production probability, it will be provided by the user. $\eta(\vec{x}_i)$ represents the detection acceptance (detector resolution is not yet taken into account in this version), therefore $\eta = 1$ if the an event is observed and $\eta = 0$ if the event is not observed. Values of η are also provided by the user after a Monte Carlo simulation of the detection process (i.e. Geant MC). Therefore

$$\int_{\Delta x^n} \eta(\vec{x})I(\vec{x}, \vec{a})d^n \vec{x} = \mathcal{N}. \quad (5)$$

For the observed data, therefore

$$I(\vec{x}_i, \vec{a}) = \mathcal{N}p(\vec{x}_i, \vec{a}) \quad (6)$$

The normalization \mathcal{N} represents *the expected number of events to be observed in the full phase-space* of Δx^n . The value of \mathcal{N} can be obtained from our model and knowing the measurement procedure (normally by Monte Carlo simulation, see below).

Then

$$\mathcal{L} = \left[\frac{\mathcal{N}^N}{N!} e^{-\mathcal{N}} \right] \prod_{i=1}^N \frac{I(\vec{x}_i, \vec{a})}{\mathcal{N}}. \quad (7)$$

Therefore

$$\mathcal{L} = \left[\frac{1}{N!} e^{-\mathcal{N}} \right] \prod_{i=1}^N I(\vec{x}_i, \vec{a}) \quad (8)$$

and taking the log on both sides

$$\ln \mathcal{L} = -\ln [N!] - \mathcal{N} + \sum_{i=1}^N \ln [I(\vec{x}_i, \vec{a})]. \quad (9)$$

Then, substituting equation (5) and removing the constant term (unimportant in optimization)

$$\ln \mathcal{L} = - \int_{\Omega} \eta(\vec{x})I(\vec{x}, \vec{a})d^n \vec{x} + \sum_{i=1}^N \ln [I(\vec{x}_i, \vec{a})]. \quad (10)$$

and taking the negative values

$$-\ln \mathcal{L} = - \sum_{i=1}^N \ln [I(\vec{x}_i, \vec{a})] + \int_{\Omega} \eta(\vec{x})I(\vec{x}, \vec{a})d^n \vec{x} \quad (11)$$

Since the expected number of events is normally estimated by numerical Monte Carlo methods, we can use

$$\int_{\Omega} \eta(\vec{x})I(\vec{x}, \vec{a})d^n \vec{x} = \frac{1}{N_g} \sum_{i=1}^{N_g} \eta(\vec{x}_i)I(\vec{x}_i, \vec{a}) = \frac{1}{N_g} \sum_{i=1}^{N_a} I(\vec{x}_i, \vec{a}) \quad (12)$$

where N_g is the number of events generated and N_a is the number of events accepted in the Monte Carlo. We introduce the possibility of using a Q_i -factor (i.e., [?, ?]), that for each data event, gives the probability for being a signal event (signal/(signal+background)). We weight each log term on the data by this factor.

Therefore

$$-ln\mathcal{L} = -\sum_{i=1}^N Q_i ln [I(\vec{x}_i, \vec{a})] + \frac{1}{N_g} \sum_{i=1}^{N_a} I(\vec{x}_i, \vec{a}) \quad (13)$$

We will find the best parameters \vec{a} for our model minimizing the function $-ln\mathcal{L}$. The minimization of this function is performed by the python version (iMinuit) [?] of the CERN package MINUIT .

The user will provide the form of the function $I(\vec{x}_i, \vec{a})$, where \vec{x}_i are the data variables (i.e. normally particles four-momenta or Mandelstham variables...) and \vec{a} the set of parameters form the model to be fitted.

The errors in the parameters are given by the square root of their variances. It can be shown the the errors [?, ?, ?] are

$$\sigma_{ij}^2 = E[(a_i - a_i^*)(a_j - a_j^*)] = E[(\vec{a} - \vec{a}^*)^T (\vec{a} - \vec{a}^*)] = [\mathcal{H}]_{ij}^{-1}. \quad (14)$$

The errors of the parameters can be calculated from the inverse of the Hessian matrix evaluated at the minimum. The MINUIT package may use different ways of calculating errors for more general cases (i.e. using the MINUIT package MINOS, see references [?], [?] and [?]).

3 Fitted Function

$$I(sD, tD, uD, \theta, \phi, \mathbf{A1}, \mathbf{A2}, \mathbf{A3}, \mathbf{A4}, \mathbf{A5}) = \mathbf{A1} * W(\theta, \phi, \mathbf{A2}, \mathbf{A3}, \mathbf{A4}) * P(sD, tD, uD) * [F(sD, tD, uD, \mathbf{A5})]^2 \quad (15)$$

whre W is the Schilling et al. spin density matrix (no-polarization):

$$W(\theta, \phi, \mathbf{A2}, \mathbf{A3}, \mathbf{A4}) = \quad (16)$$

$$\frac{3}{4\pi} [0.5 * (1 - \mathbf{A2}) + 0.5 * (3 * \mathbf{A2} - 1) \cos^2(\theta) - \sqrt{2} * \mathbf{A3} * \sin 2(\theta) \cos(\phi) - \mathbf{A4} * \sin^2(\theta) \cos 2(\phi)] \quad (17)$$

and θ, ϕ are Adair's angles. P is a kinematic factor given by:

$$P(sD, tD, uD) = sD * tD * uD - m_\pi^2 [M^2 - m_\pi^2]^2 \quad (18)$$

wherre sD, tD, uD are the Mandelstam variables of the decay such that:

$$sD = (p_X - p_{\pi^+}), sD = (p_X - p_{\pi^-}) \text{ and } sD = (p_X - p_{\pi^0}).$$

and $p_X = p_{\pi^+} + p_{\pi^-} + p_{\pi^0}$, M is the mass of the three pion system and m_π the mass of the pion (plus).

F (sD,tD,uD,A5) is Igor Danilkin et al. amplitude given for a call to his fortran code.

Minimization is done using the *generalFitting* script from the **PyPWA** framework.

Energy bins (all t) -Monte Carlos with t slope of 3

All	A2	A3	A4	A5
3.5-4.0	0.315	-0.016	-0.021	-12.53
4.0-4.5	0.315	-0.016	-0.021	-12.78
4.5-5.0	0.315	-0.016	-0.021	-12.82
5.0-5.5	0.191	0.018	-0.007	-12.8

nonF	A2	A3	A4
3.5-4.0	0.27	-0.018	-0.023
4.0-4.5	0.28	-0.026	-0.068
4.5-5.0	0.29	-0.022	-0.039
5.0-5.5	0.31	0.016	-0.021

F	A2	A3	A4	A5
3.5-4.0	0.194	-0.016	-0.007	0 (fixed)
3.5-4.0	0.299	-0.018	-0.08	1 (fixed)
3.5-4.0	0.191	-0.018	-0.007	-12.8 (fixed normalization)