A Dalitz Plot Analysis & Extraction of Spin Density Matrix Elements for $\omega \rightarrow 3\pi$ Decay

Chris Zeoli

Florida State University
cpz11@my.fsu.edu

PWA Meeting

02 December 2015
Overview

- Goals of Analysis
- g12 Data for $\omega \rightarrow 3\pi$
  - Kinematics
  - Dalitz Plots
- Analysis
  - Fit Function in Brief
  - Decay Amplitude
  - Spin Density Matrix Elements (SDMEs)
  - Preliminary Results and Status
- Next Steps
Introduction

\[ \gamma p \rightarrow p \omega \rightarrow p 3\pi \]
Our Interest: $\omega \rightarrow 3\pi$ Decay

How does the $\omega$ resonance decay?

Example

$$\omega \rightarrow \rho \pi \rightarrow 3\pi$$

FIG. 1: Isobar decomposition.
Properties of the Decay

- Spectroscopic Notation for Mesons, $I^G J^{PC}$
- For $\gamma p \rightarrow p \omega \rightarrow p\pi^+\pi^-\pi^0$,
  
  \[ \gamma, \ 0^-(1^{--}) \quad \text{and} \quad \pi^\pm(139.6), \ 1^-(0^-) \]
  
  \[ p, \ 0^-(\frac{1}{2}^+) \quad \text{and} \quad \pi^0(134), \ 1^- (0^{-+}) \]
  
  \[ \omega(782), \ 0^-(1^{--}) \]  

- Additional Properties
  
  Strong Decay
  
  Decay Width, $\Gamma_{\omega \rightarrow 3\pi}^{exp} = 7.57$ MeV (PDG)
  
  Branching Ratio, $\sim 85\%$
Introduction

Main Goals of Analysis

- Extract SDMEs for the $\omega \rightarrow 3\pi$ decay.
- Fit a model for the $\omega \rightarrow 3\pi$ decay to data.
  - Working closely with JPAC, Igor Danilkin
- Fit via event-based, minimum log-likelihood method using AmpTools framework.
- Compare fits with the results of other models
CLAS g12 Data

<table>
<thead>
<tr>
<th>Data</th>
<th>Total Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>-data</td>
<td>8,200,000</td>
</tr>
<tr>
<td>-genMC</td>
<td>20,000,000</td>
</tr>
<tr>
<td>-accMC</td>
<td>2,000,000</td>
</tr>
</tbody>
</table>

Average Acceptance $\approx 0.10$

- g12 data covers $E_\gamma: [1108 - 5400] \text{MeV}$
- Consider $E_\gamma: [1150 - 3800] \text{MeV}$, $W: [1770 - 2830] \text{MeV}$
Data: Kinematics, Wbin2000-2100
Data: Kinematics, Wbin2000-2100

\[ \omega \rightarrow 3\pi \] Decay Analysis
Data: Kinematics, Wbin2000-2100

\[ h_{\lambda_{\text{dat}}} \]

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1 \]

\[ 0 \quad 500 \quad 1000 \quad 1500 \quad 2000 \quad 2500 \quad 3000 \quad 3500 \quad 4000 \]

\[ h_{\lambda_{\text{gen}}} \]

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1 \]

\[ 0 \quad 500 \quad 1000 \quad 1500 \quad 2000 \quad 2500 \quad 3000 \quad 3500 \quad 4000 \quad 4500 \quad 5000 \]
Dalitz Plot: Threebody Phase Space

Differential Decay Width

\[ d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8M} |\mathcal{M}|^2 dE_1 dE_2 \]

Defining \( p_{ij} = p_i + p_j \) and \( m_{ij}^2 = p_{ij}^2 \)

\[ \Rightarrow m_{ij}^2 = (P - p_k)^2 = M^2 + m_k^2 - 2ME_k \]

\[ d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2 \]
**The Dalitz Plot**

![Dalitz Plot Diagram]

### 38. Kinematics

#### 5. Kinematic limits

In a three-body decay, the maximum of \( |p_3| \), given by Eq. (38.20), is achieved when \( m_{12} = m_1 + m_2 \), i.e., particles 1 and 2 have the same vector velocity in the rest frame of the decaying particle. If, in addition, \( m_3 > m_1, m_2 \), then \( |p_3|_{\text{max}} > |p_1|_{\text{max}}, |p_2|_{\text{max}} \).

#### 38.4.4. Kinematic limits

Multibody decays: The above results may be generalized to final states containing any number of particles by combining some of the particles into "effective particles" and treating the final states as 2 or 3 "effective particle" states. Thus, if \( p_{ijk...} = p_i + p_j + p_k + ... \), then \( m_{ijk...} = \sqrt{p_{ijk...}^2} \), (38.23) and \( m_{ijk...} \) may be used in place of e.g., \( m_{12} \) in the relations in Sec. 38.4.3 or 38.4.3.1 above.

---

**Symmetries of Dalitz Plots**

Often final-state particles are identical, in which case Dalitz plots will respect exchange symmetries. In this reaction, since all final particles are identical bosons, DP is symmetric with respect to reflection about any of 3 axes.


---

**Decay Distribution**

\[
(m_{23})_{\text{min}} = (E_2^* + E_3^*)^2 - \left( \sqrt{E_2^{*2} - m_2^2} \pm \sqrt{E_3^{*2} - m_3^2} \right)^2
\]

**p\bar{p} \rightarrow 3\pi**, Band Structures

- \( f_2(1565) \)
- \( f_0(1500) \)
- \( f_2(1270) \)
- \( f_0(980) \)

---

Chris Zeoli (FSU)
Data: Kinematics, Wbin2000-2100

ω → 3π Decay Analysis
Dalitz Plot: X & Y Representation

X & Y Variables

Lorentz Invariant, Dimensionless Variables

\[ X = \frac{\sqrt{3}(T_i - T_j)}{Q} \quad Y = \frac{3T_k}{Q} - 1 \]
Data: Kinematics, Wbin2000-2100
$g_{12}$, $g_{11}$ Cross-Section Comparison

$E_\gamma: [1550-2550]$ MeV, Zulkaida Akbar

Chris Zeoli (FSU)  ω → 3π Decay Analysis  02 December 2015  16 / 57
g12, g11 Cross-Section Comparison

$E_\gamma: [2550-3550]\text{MeV, Zulkaida Akbar}$
g12, g11 Cross-Section Comparison

$E_\gamma$: [3550-3800] MeV, Zulkaidaa Akbar
Dispersive Analysis of $\omega/\phi \rightarrow 3\pi, \pi\gamma^*$

I.V. Danilkin, $^1$, *C. Fernández-Ramírez, $^1$ P. Guo, $^{2,3}$ V. Mathieu, $^{2,3}$ D. Schott, $^4$ M. Shi, $^{1,5}$ and A. P. Szczepaniak$^{1,2,3}$

(Joint Physics Analysis Center)

$^1$Center for Theoretical and Computational Physics, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606
$^2$Center for Exploration of Energy and Matter, Indiana University, Bloomington, IN 47403
$^3$Physics Department, Indiana University, Bloomington, IN 47405
$^4$Department of Physics, The George Washington University, Washington, DC 20052
$^5$Department of Physics, Peking University, Beijing 100871, China

(Dated: September 30, 2014)

The decays $\omega/\phi \rightarrow 3\pi$ are considered in the dispersive framework that is based on the isobar decomposition and sub-energy unitarity. The inelastic contributions are parametrized by the power series in a suitably chosen conformal variable that properly account for the analytic properties of the amplitude. The Dalitz plot distributions and integrated decay widths are presented. Our results indicate that the final state interactions may be sizable. As a further application of the formalism we also compute the electromagnetic transition form factors of $\omega/\phi \rightarrow \pi^0\gamma^*$. 

The diagrams representing a system can be truncated such that only low partial waves are significant. Therefore, the infinite partial waves series can be truncated such that only low partial waves are significant. The natural starting point for amplitude construction isobar decomposition. The unconstrained high-energy region, subtracted dispersion integrals should therefore take into account the high-energy contributions to the dispersion integrals. KT amplitudes will couple to other open channels. Any change in the analytical properties of the partial wave equations translates into an arbitrariness in choosing the boundary condition for the solution of an integral equation, which follows from the dispersion relation. It is therefore possible to implement unitarity on a truncated set of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves.

In previous works, in order to suppress sensitivity to additional diagrams, only low partial waves are considered. Any further uncertainty into the KT framework. For example, in previous analyses of the vector meson decays the uncertainty can be suppressed using a Cauchy dispersion relation. Consequently, additional diagrams contribute to the amplitude, which follows from the dispersion relation. It is therefore possible to implement unitarity on a truncated set of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves.

In the KT framework, elastic waves leads to the so-called Khuri-Treiman (KT) amplitudes. As the isobar model, the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves.

The matrix element for the three-pion decay of a vector particle is given in terms of a helicity amplitude $H_{abc}^{\lambda}$, which follows from the dispersion relation. It is therefore possible to implement unitarity on a truncated set of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves, it is intrinsically restricted by the number of partial waves.

FIG. 1: Isobar decomposition.

FIG. 2: Crossed channel rescattering effects.

Consequence of elastic unitarity
Requirement of model
$$F(s) = \Omega(s) \left( \frac{1}{\pi} \int_{s_{\pi}}^{s_i} ds' \frac{\rho(s') t^*(s')}{\Omega^*(s')} \frac{\hat{F}(s')}{s' - s} + \Sigma(s) \right).$$

(31)

The Decay Amplitude


$$\Sigma(s) = \sum_{i=0}^{\infty} a_i \omega^i(s)$$

(29)

inelastic contribution

($a' = "IgorParameter"$)
PWA Fit Framework: AmpTools
- Fit Method: Log Likelihood Method
- Fitter: Parameter Float Method, ROOT Tminuit class based

Fit Function: $I = |T|^2 |M|^2$, where

$T=$Production Amp, $M=$Decay Amp (J=1, P-Wave)
Factorization of the Fit Function

- Squared Decay-Amp:

\[
|\mathcal{M}|^2 = \sum_{\lambda\lambda'} \mathcal{H}_\lambda^* \rho_{\lambda'}^\lambda \mathcal{H}_\lambda = |\mathcal{F}|^2 \mathcal{W}_\rho(\theta, \phi), \quad \text{where}
\]

\[\mathcal{F} = \text{Reduced Decay Amp}\]

\[\mathcal{W} = \text{Angular Decay Distribution} \quad \text{(Spin Density Distribution)}\]

\[\rho_{\lambda\lambda'} = \text{Spin Density Matrix Elements}\]
Spin Density Distribution

Angular Decay Distribution

\[ \mathcal{W}(\theta, \phi, \rho(\omega)) = \mathcal{M}\rho(\omega)\mathcal{M}^\dagger \]

where \( \rho(\omega) = \mathcal{T}\rho(\gamma)\mathcal{T}^\dagger \)

Unpolarized Beam and Target

\[ \mathcal{W}^0(\theta, \phi) \equiv \frac{3}{4\pi} \left[ \frac{1}{2}(1 - \rho_{00}^0) + \frac{1}{2}(3\rho_{00}^0 - 1) \cos^2 \theta 
\right.
\left. - \sqrt{2} \Re\rho_{10}^0 \sin 2\theta \cos \phi - \rho_{1,1}^0 \sin^2 \theta \cos 2\phi \right] \]

(The Schilling Equation)
Analysis: $g_{12}$, $g_{11}$ SDME comparison (unpolarized beam)

$W$: [1800 - 1810] MeV, $\cos(\theta_{cm})$: [−0.5 - 0.8] GeV$^2$
g12, g11 SDME comparison (unpolarized beam)

$W$: [2100 - 2110] MeV, \hspace{1cm} \cos(\theta_{cm}): [-0.8 - 0.8] \text{GeV}^2
g12, g11 SDME comparison (unpolarized beam)

$W$: [2400 - 2410] MeV, $\cos(\theta_{cm})$: [−0.8 - 0.8] GeV$^2$
g12, g11 SDME comparison (unpolarized beam)

$W$: [2700 - 2710] MeV, $\cos(\theta_{cm})$: [−0.7 - 0.8] GeV$^2$
Fit Results: Wbin2000-2100

\[ W \quad (GeV) \]

\[ 1.6 \quad 1.8 \quad 2 \quad 2.2 \quad 2.4 \quad 2.6 \quad 2.8 \quad 3 \]

\[ \text{Events} \]

\[ 0 \quad 10000 \quad 20000 \quad 30000 \quad 40000 \quad 50000 \]

\[ W \quad \text{data} \quad \text{acc (fit)} \]

\[ E_\gamma \]

\[ (GeV) \]

\[ 1.5 \quad 2 \quad 2.5 \]

\[ \text{Events} \]

\[ 0 \quad 5000 \quad 10000 \quad 15000 \quad 20000 \quad 25000 \quad 30000 \quad 35000 \quad 40000 \]

\[ E_\gamma \quad \text{data} \quad \text{acc (fit)} \]

\[ -t \]

\[ (GeV^2) \]

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \]

\[ \text{Events} \]

\[ 0 \quad 2000 \quad 4000 \quad 6000 \quad 8000 \quad 10000 \quad 12000 \quad 14000 \]

\[ -t \quad \text{data} \quad \text{acc (fit)} \]

\[ \cos(\theta_{cm}) \]

\[ 0 \quad 2000 \quad 4000 \quad 6000 \quad 8000 \quad 10000 \quad 12000 \quad 14000 \]

\[ \text{Events} \]

\[ 0 \quad -1 \quad -0.8 \quad -0.6 \quad -0.4 \quad -0.2 \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \]

\[ \cos(\theta_{cm}) \quad \text{data} \quad \text{acc (fit)} \]

\[ M_\omega \]

\[ (GeV) \]

\[ 0 \quad 0.2 \quad 0.3 \quad 0.35 \]

\[ \text{Events} \]

\[ 0 \quad 10000 \quad 20000 \quad 30000 \quad 40000 \quad 50000 \]

\[ M_\omega \quad \text{data} \quad \text{acc (fit)} \]
Fit Results: Wbin2000-2100

\[ \lambda \]
- data
- acc (fit)

\[ \text{Pfac\_isobar} \]
- data
- acc (fit)

\[ \text{Pfac\_physical} \]
- data
- acc (fit)

\[ \theta \]
- data
- acc (fit)

\[ \phi \]
- data
- acc (fit)
Fit Results: Wbin2000-2100

Chris Zeoli (FSU)
Decay Amplitude Parameter (2Body), Wbin1800-2800

Wbin1800-2800, Fit Parameter: IgorParameter

Using $\frac{d\sigma}{d\cos cm}$

Chris Zeoli (FSU)
ω → 3π Decay Analysis
Decay Amplitude Parameter (3Body), Wbin1800-2800

Using $\frac{d\sigma}{d\cos cm}$

Chris Zeoli (FSU)

ω → 3π Decay Analysis
Decay Amplitude Parameter (3Body), Wbin1800-2800

Wbin1800-2800, Fit Parameter: IgorParameter

Using $d\sigma/dt$
PWA Meeting 12-02-15

Status Update
Decay Amplitude Parameter (3Body)

Wbin1800-2800, Fit Parameter: IgorParameter

Acceptance

Integrated data Event-Count Per Bin

Integrated data, Ndat

Integrated simMC Event-Count Per Bin

Integrated simMC, Nacc (fit)

Integrated Acceptance Per Bin

Integrated Acceptance, eps

Decay Amplitude Parameter (3Body)

Wbin1800-2800, Fit Parameter: IgorParameter

$W: [1800 \rightarrow 2800]$ MeV, Bin Widths: 50 MeV
Acceptance

Integrated data Event-Count Per Bin

Integrated data, Ndat

Integrated simMC Event-Count Per Bin

Integrated simMC, Nacc (fit)

Integrated Acceptance Per Bin

Integrated Acceptance, eps

Acceptance

Integrated data Event-Count Per Bin

Integrated simMC Event-Count Per Bin

Integrated Acceptance Per Bin

\[ W: [1800 - 1900] \text{ MeV}, \quad -t: [100 - 2400] \text{ GeV}^2, \quad \text{Bin Widths: 100 GeV}^2 \]
Decay Amplitude Parameter (3Body)

Wbin2100-2200, Fit Parameter: IgorParameter

$W: [2100 - 2200] \text{ MeV, } -t: [100 - 2400] \text{ GeV}^2, \text{ Bin Widths: } 100 \text{ GeV}^2$
Acceptance

Integrated data Event-Count Per Bin

Integrated simMC Event-Count Per Bin

Integrated Acceptance Per Bin

$W$:[2400 − 2500] MeV, $-t$:[100 − 2400] GeV$^2$, Bin Widths: 100 GeV$^2$
Acceptance

Integrated data Event-Count Per Bin

Integrated simMC Event-Count Per Bin

Integrated Acceptance Per Bin

Acceptance

\begin{align*}
W: [2700 - 2800] \text{ MeV}, &\quad -t: [100 - 2400] \text{ GeV}^2, \quad \text{Bin Widths: 100 GeV}^2
\end{align*}
Decay Amplitude Parameter (3Body)

\[ W: [1800 - 1900] \text{ MeV}, \quad -t: [100 - 2400] \text{ GeV}^2, \quad \text{Bin Widths: 100 GeV}^2 \]
$\omega \rightarrow 3\pi$ Decay Analysis

$W: [1800 - 1900] \text{ MeV}, \ -t: [100 - 2400] \text{ GeV}^2, \ \text{Bin Widths: 100 GeV}^2$
Next Steps

- Try to eliminate energy dependence of IgorParameter

- Statistical and systematic error
  - Refit results which had large statistical error bars
  - Include statistical error bars on g11 SDME results
  - Include systematic error into analysis

- Refit IgorParameter using g12 cross-section and SDME’s
Thank you
Backup Slides
Dalitz Plot Expansion

Expected Dalitz Plot, IgorParameter > 3

\[ x = \frac{\sqrt{3}}{Q} (T_1 - T_2) = \frac{\sqrt{3} (t - u)}{2M (M - 3m_\pi)}, \]
\[ y = \frac{3T_3}{Q} - 1 = \frac{3(s_c - s)}{2M (M - 3m_\pi)}. \]  

Lorentz Invariant Variables

\[ x = \sqrt{z} \cos \theta \]
\[ y = \sqrt{z} \sin \theta \]

Dalitz Plot Amplitude Expansion

\[ |F_{par}(z, \vartheta)|^2 = |N|^2 (1 + 2\alpha z + 2\beta z^{3/2} \sin(3\vartheta) + 2\gamma z^2 + 2\delta z^{5/2} \sin(3\vartheta) + \mathcal{O}(z^3)) \]  

\[ (40) \]
\[ |F_{\text{par}}(z, \vartheta)|^2 = |N|^2 \left( 1 + 2 \alpha z + 2 \beta z^{3/2} \sin(3\vartheta) + 2 \gamma z^2 + 2 \delta z^{5/2} \sin(3\vartheta) + \mathcal{O}(z^3) \right) \quad (40) \]

**TABLE I: Dalitz Plot parameters and \( \sqrt{\chi^2} \) of the polynomial parametrization (40) for \( \omega \to 3\pi \).** In addition to our results we also show the selected results from Niecknig et al. [37] (dispersive study with incorporated crossed-channel effects) and Terschlusen et al. [19] (Lagrangian based study with the pion-pion rescattering effects).

<table>
<thead>
<tr>
<th></th>
<th>( \alpha \times 10^3 )</th>
<th>( \beta \times 10^3 )</th>
<th>( \gamma \times 10^3 )</th>
<th>( \delta \times 10^3 )</th>
<th>( \sqrt{\chi^2} \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>This paper (( \hat{F} = 0 ))</td>
<td>136</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.5</td>
</tr>
<tr>
<td>This paper (full)</td>
<td>94</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.2</td>
</tr>
<tr>
<td>Niecknig et al. [37]</td>
<td>84...96</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.9...1.1</td>
</tr>
<tr>
<td>Terschlusen et al. [19]</td>
<td>202</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>6.6</td>
</tr>
<tr>
<td>This paper (( \hat{F} = 0 ))</td>
<td>125</td>
<td>30</td>
<td>-</td>
<td>-</td>
<td>0.74</td>
</tr>
<tr>
<td>This paper (full)</td>
<td>84</td>
<td>28</td>
<td>-</td>
<td>-</td>
<td>0.35</td>
</tr>
<tr>
<td>Niecknig et al. [37]</td>
<td>74...84</td>
<td>24...28</td>
<td>-</td>
<td>-</td>
<td>0.052...0.078</td>
</tr>
<tr>
<td>Terschlusen et al. [19]</td>
<td>190</td>
<td>54</td>
<td>-</td>
<td>-</td>
<td>2.1</td>
</tr>
<tr>
<td>This paper (( \hat{F} = 0 ))</td>
<td>113</td>
<td>27</td>
<td>24</td>
<td>-</td>
<td>0.1</td>
</tr>
<tr>
<td>This paper (full)</td>
<td>80</td>
<td>27</td>
<td>8</td>
<td>-</td>
<td>0.24</td>
</tr>
<tr>
<td>Niecknig et al. [37]</td>
<td>73...81</td>
<td>24...28</td>
<td>3...6</td>
<td>-</td>
<td>0.038...0.047</td>
</tr>
<tr>
<td>Terschlusen et al. [19]</td>
<td>172</td>
<td>43</td>
<td>50</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>This paper (( \hat{F} = 0 ))</td>
<td>114</td>
<td>24</td>
<td>20</td>
<td>6</td>
<td>0.005</td>
</tr>
<tr>
<td>This paper (full)</td>
<td>83</td>
<td>22</td>
<td>1</td>
<td>14</td>
<td>0.079</td>
</tr>
<tr>
<td>Niecknig et al. [37]</td>
<td>74...83</td>
<td>21...24</td>
<td>0...2</td>
<td>7...8</td>
<td>0.012...0.011</td>
</tr>
<tr>
<td>Terschlusen et al. [19]</td>
<td>174</td>
<td>35</td>
<td>43</td>
<td>20</td>
<td>0.1</td>
</tr>
</tbody>
</table>
The Decay Amplitude

\[ F(s) = \Omega(s) \left( \frac{1}{\pi} \int_{s}^{s_i} ds' \frac{\rho(s') t^*(s')}{\Omega^*(s')} \frac{\hat{F}(s')}{{s'} - s} + \Sigma(s) \right). \]  

(31)

KT Amplitude

\[ \Sigma(s) = \sum_{i=0}^{\infty} a_i \omega^i(s) \]  

(29)

\[ \omega(s) = \frac{\sqrt{s_i - s_E} - \sqrt{s_i - s}}{\sqrt{s_i - s_E} + \sqrt{s_i - s}} \]  

(30)

inelastic contribution

\[ \omega \rightarrow 3\pi \text{ Decay Analysis} \]

Comparing JPAC & Dalitz Expansion Amplitude Parameters

\[ 2\alpha \propto a' = 2\frac{a_1}{a_0} = "IgorParameter" \]