A Dalitz Plot Analysis & Extraction of Spin Density Matrix Elements for $\omega \to 3\pi$ Decay

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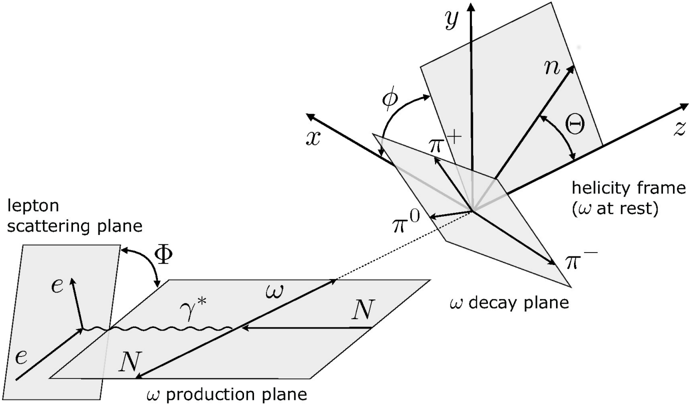
PWA Meeting

02 December 2015

Overview

- Goals of Analysis
- g12 Data for $\omega \to 3\pi$
  - Kinematics
  - Dalitz Plots
- Analysis
  - Fit Function in Brief
  - Decay Amplitude
  - Spin Density Matrix Elements (SDMEs)
  - Preliminary Results and Status
- Next Steps

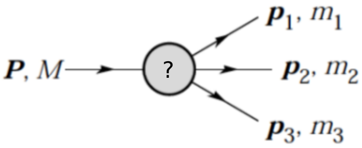
Introduction



$$\gamma\ p \ \rightarrow \ p\ \omega \ \rightarrow \ p\ 3\pi$$

Our Interest: $\omega \rightarrow 3\pi$ Decay

How does the ω resonance decay?



Example

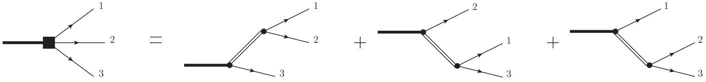


FIG. 1: Isobar decomposition.

$$\omega \rightarrow \rho\pi \rightarrow 3\pi$$

Properties of the Decay

- Spectroscopic Notation for Mesons, $I^G J^{PC}$
- For $\gamma p \to p\omega \to p\pi^+\pi^-\pi^0$,

  γ, $0^-(1^{--})$
  p, $0^-(\frac{1}{2}^+)$
  $\omega(782)$, $0^-(1^{--})$

  $\pi^\pm(139.6)$, $1^-(0^-)$
  $\pi^0(134)$, $1^-(0^{-+})$

- Additional Properties

  Strong Decay
  Decay Width, $\Gamma^{exp}_{\omega\to 3\pi} = 7.57$ MeV (PDG)
  Branching Ratio, $\sim 85\%$

Introduction

Main Goals of Analysis

- Extract SDMEs for the $\omega \to 3\pi$ decay.

- Fit a model for the $\omega \to 3\pi$ decay to data.
  - Working closely with JPAC, Igor Danilkin

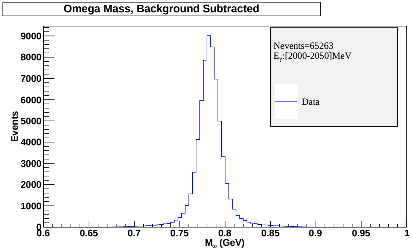
- Fit via event-based, minimum log-likelihood method using AmpTools framework.

- Compare fits with the results of other models

CLAS g12 Data

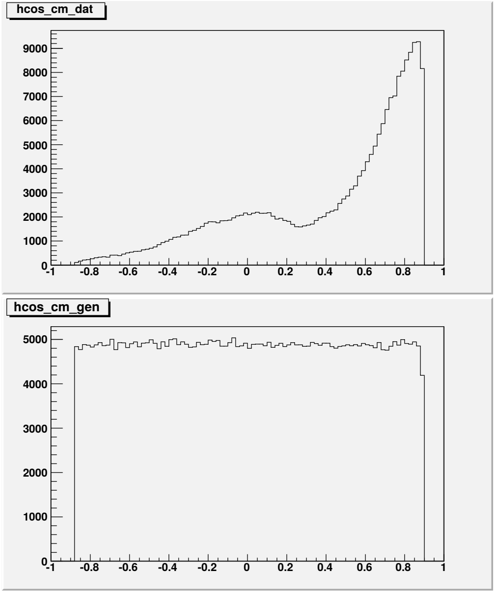
Data	Total Events
-data	8,200,000
-genMC	20,000,000
-accMC	2,000,000

Average Acceptance $\approx$ 0.10

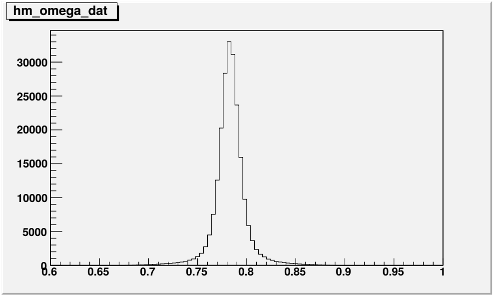


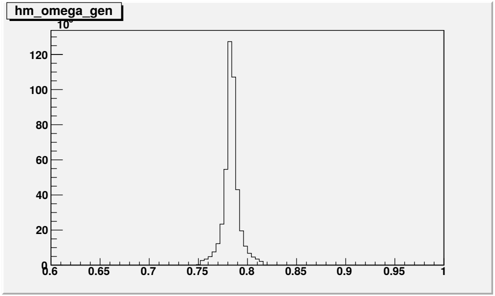
- g12 data covers E_γ:[1108 - 5400]MeV
- Consider E_γ:[1150 - 3800]MeV, W:[1770 - 2830]MeV

Data: Kinematics, Wbin2000-2100

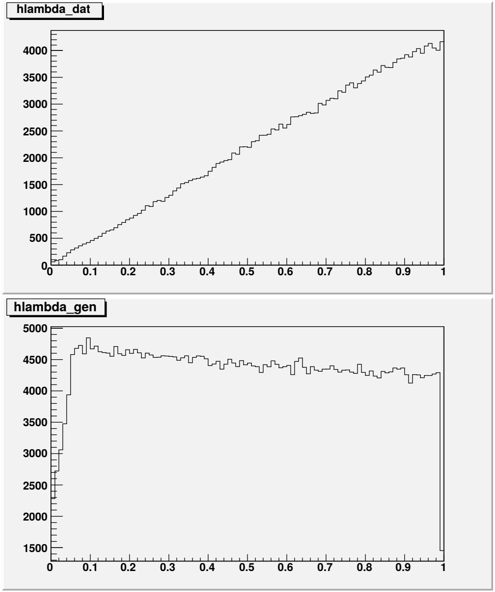


Data: Kinematics, Wbin2000-2100





Data: Kinematics, Wbin2000-2100



Dalitz Plot: Threebody Phase Space

Differential Decay Width

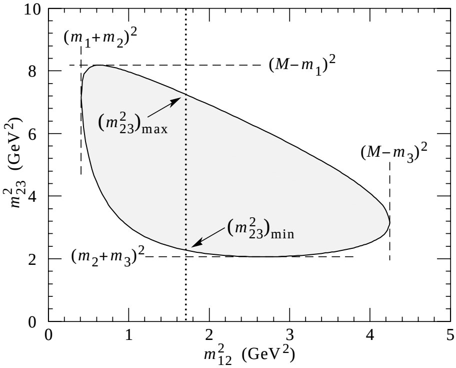
$$d\Gamma = \frac{1}{(2\pi)^3} \, \frac{1}{8M} \, \overline{|\mathcal{M}|^2} \, dE_1 dE_2$$

Defining $p_{ij} \; = \; p_i \; + \; p_j$ and $m_{ij}^2 = p_{ij}^2$

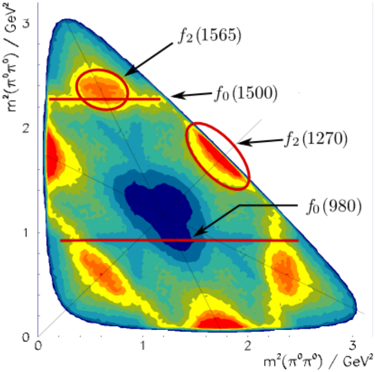
$$\Rightarrow m_{ij}^2 = (P - p_k)^2 = M^2 + m_k^2 - 2ME_k$$

$$d\Gamma = \frac{1}{(2\pi)^3} \, \frac{1}{32M^3} \, \overline{|\mathcal{M}|^2} \, dm_{12}^2 dm_{23}^2$$

The Dalitz Plot



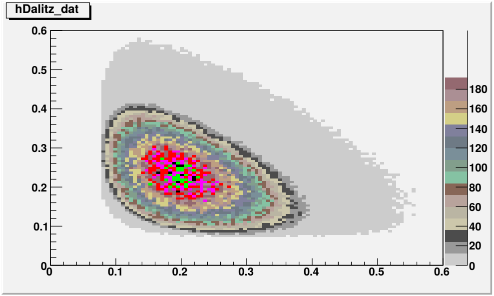
Decay Distribution

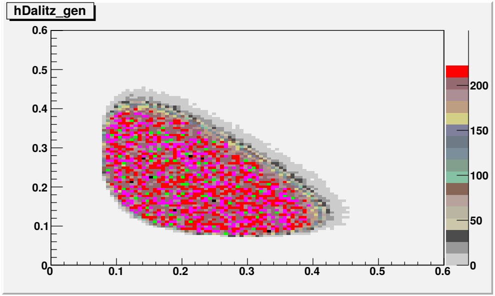


$p\bar{p} \to 3\pi$, Band Structures

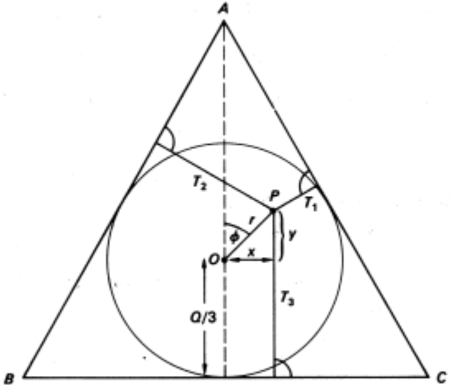
$$(m^2_{23})_{\substack{min \\ max}} = (E^*_2 + E^*_3)^2 - \left(\sqrt{E^{*2}_2 - m^2_2} \pm \sqrt{E^{*2}_3 - m^2_3}\right)^2$$

Data: Kinematics, Wbin2000-2100





Dalitz Plot: X & Y Representation

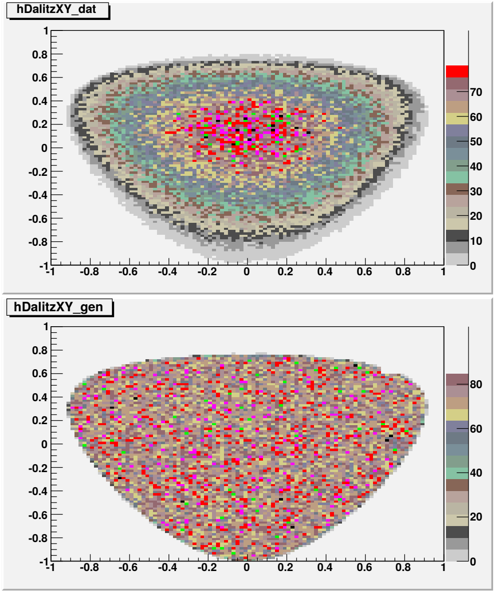


X & Y Variables

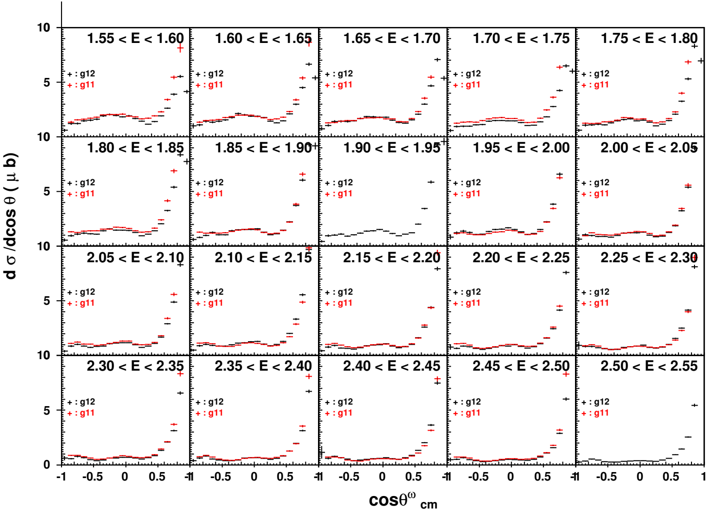
Lorentz Invariant, Dimensionless Variables

$$X = \frac{\sqrt{3}(T_i - T_j)}{Q} \qquad Y = \frac{3T_k}{Q} - 1$$

Data: Kinematics, Wbin2000-2100



g12, g11 Cross-Section Comparison



$E\gamma$:[1550-2550]MeV, Zulkaida Akbar

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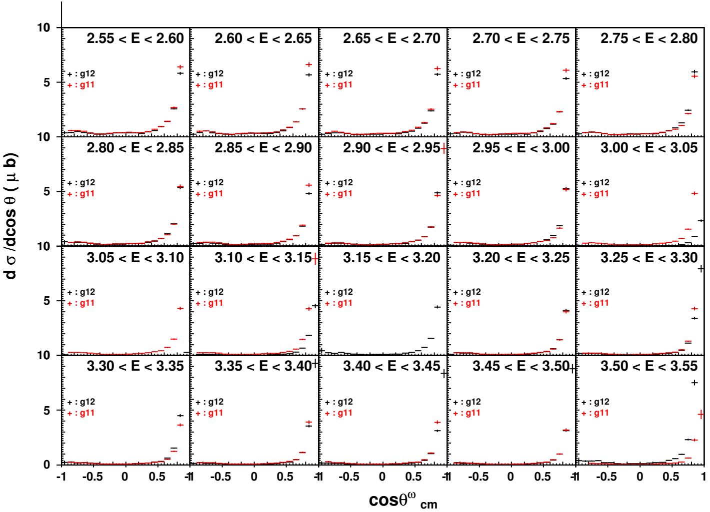

$E\gamma$:[2550-3550]MeV, Zulkaida Akbar

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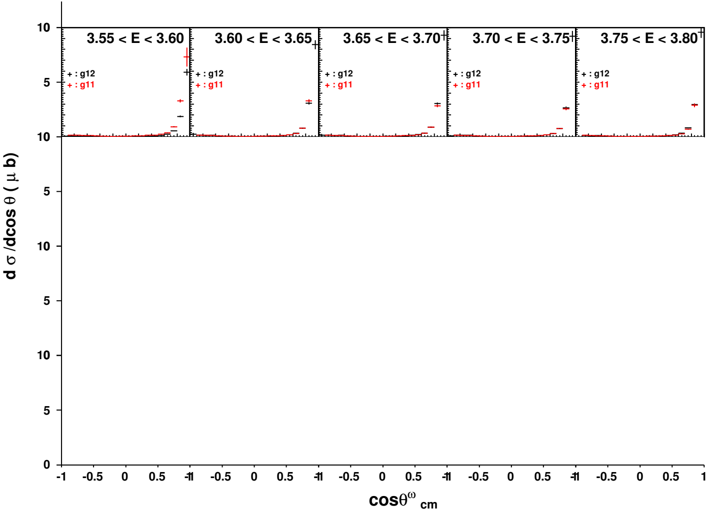


$E\gamma$:[3550-3800]MeV, Zulkaida Akbar

Paper for Model

JLAB-THY-14-1960

Dispersive Analysis of $\omega/\phi \to 3\pi$, $\pi\gamma^*$

I.V. Danilkin,[1,*] C. Fernández-Ramírez,[1] P. Guo,[2,3] V. Mathieu,[2,3] D. Schott,[4] M. Shi,[1,5] and A. P. Szczepaniak[1,2,3]
(Joint Physics Analysis Center)
[1] *Center for Theoretical and Computational Physics, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606*
[2] *Center for Exploration of Energy and Matter, Indiana University, Bloomington, IN 47403*
[3] *Physics Department, Indiana University, Bloomington, IN 47405*
[4] *Department of Physics, The George Washington University, Washington, DC 20052*
[5] *Department of Physics, Peking University, Beijing 100871, China*

(Dated: September 30, 2014)

The decays $\omega/\phi \to 3\pi$ are considered in the dispersive framework that is based on the isobar decomposition and sub-energy unitarity. The inelastic contributions are parametrized by the power series in a suitably chosen conformal variable that properly account for the analytic properties of the amplitude. The Dalitz plot distributions and integrated decay widths are presented. Our results indicate that the final state interactions may be sizable. As a further application of the formalism we also compute the electromagnetic transition form factors of $\omega/\phi \to \pi^0\gamma^*$.

arXiv:1409.7708v1 [hep-ph] (2014)

Illustration

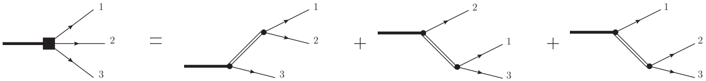


FIG. 1: Isobar decomposition.

Isobar Model

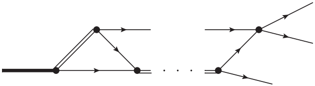


FIG. 2: Crossed channel rescattering effects.

consequence of elastic unitarity
requirement of model

Model

$$F(s) = \Omega(s) \left(\frac{1}{\pi} \int_{s_\pi}^{s_i} ds' \frac{\rho(s')\, t^*(s')}{\Omega^*(s')} \frac{\hat{F}(s')}{s' - s} + \Sigma(s) \right) . \tag{31}$$

The Decay Amplitude
arXiv:1409.7708v1 [hep-ph] (2014)

$$\Sigma(s) = \sum_{i=0}^{\infty} a_i\, \omega^i(s) \tag{29}$$

inelastic contribution
($a' =$ "IgorParameter")

Fit Framework & Function

- PWA Fit Framework: AmpTools
  - Fit Method: Log Likelihood Method
  - Fitter: Parameter Float Method, ROOT Tminuit class based

- Fit Function: $\mathcal{I} = |\mathcal{T}|^2 \; |\mathcal{M}|^2$, where

  $\mathcal{T}$=Production Amp, $\mathcal{M}$=Decay Amp   (J=1, P-Wave)

Factorization of the Fit Function

- Squared Decay-Amp:

$$|\mathcal{M}|^2 = \sum_{\lambda\lambda'} \mathcal{H}^*_\lambda \, \rho^{\lambda'}_\lambda \, \mathcal{H}^\lambda = |\mathcal{F}|^2 \, \mathcal{W}_\rho(\theta, \phi), \quad \text{where}$$

$\mathcal{F}$ = Reduced Decay Amp

$\mathcal{W}$ = Angular Decay Distribution
(Spin Density Distribution)

$\rho_{\lambda\lambda'}$ = Spin Density Matrix Elements

Spin Density Distribution

Angular Decay Distribution

$$\mathcal{W}(\theta, \phi, \rho(\omega)) = \mathcal{M}\rho(\omega)\mathcal{M}^\dagger$$

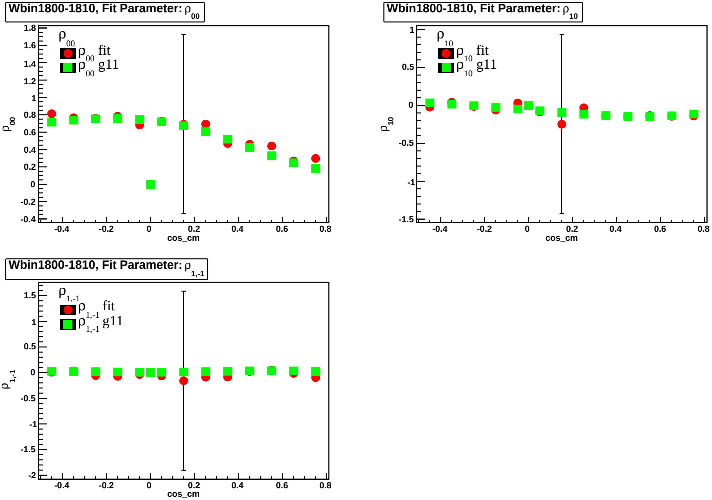
$$\text{where} \quad \rho(\omega) = \mathcal{T}\rho(\gamma)\mathcal{T}^\dagger$$

Unpolarized Beam and Target

$$\begin{aligned}\mathcal{W}^0(\theta, \phi) \equiv & \frac{3}{4\pi}\ [\ \frac{1}{2}(1 - \rho^0_{00})\ +\ \frac{1}{2}(3\rho^0_{00} - 1)\ \cos^2\theta \\ & - \sqrt{2}\ Re\rho^0_{10}\ \sin 2\theta\ \cos\phi\ -\ \rho^0_{1,-1}\ \sin^2\theta\ \cos 2\phi\ ]\end{aligned}$$

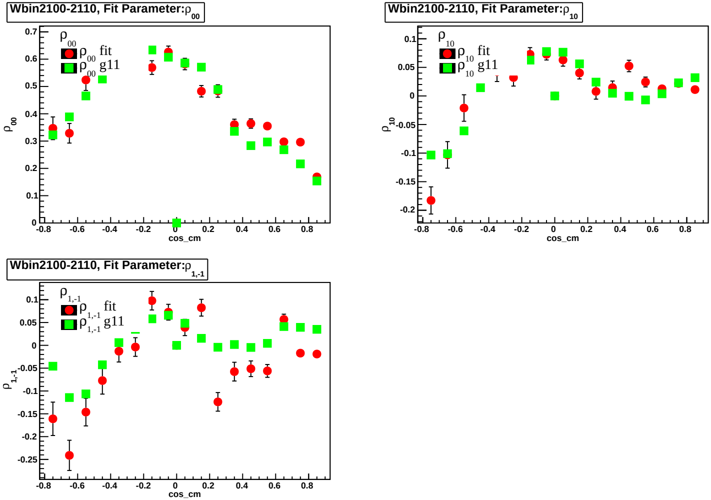
(The Schilling Equation)

Analysis: g12, g11 SDME comparison (unpolarized beam)



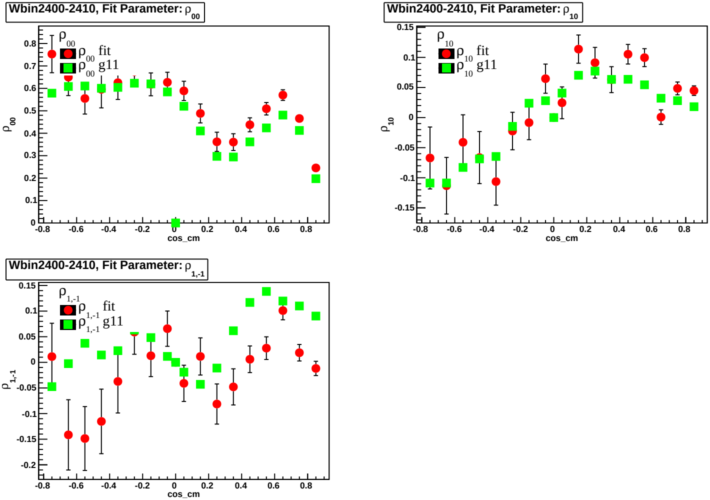
W:[1800 - 1810]MeV, $cos(\theta_{cm})$:[-0.5 - 0.8]GeV^2

g12, g11 SDME comparison (unpolarized beam)



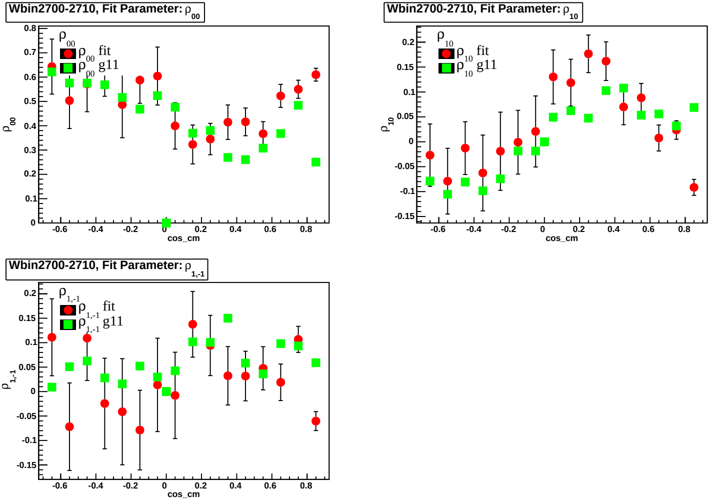
W:[2100 - 2110]MeV, $cos(\theta_{cm})$:[-0.8 - 0.8]GeV^2

g12, g11 SDME comparison (unpolarized beam)



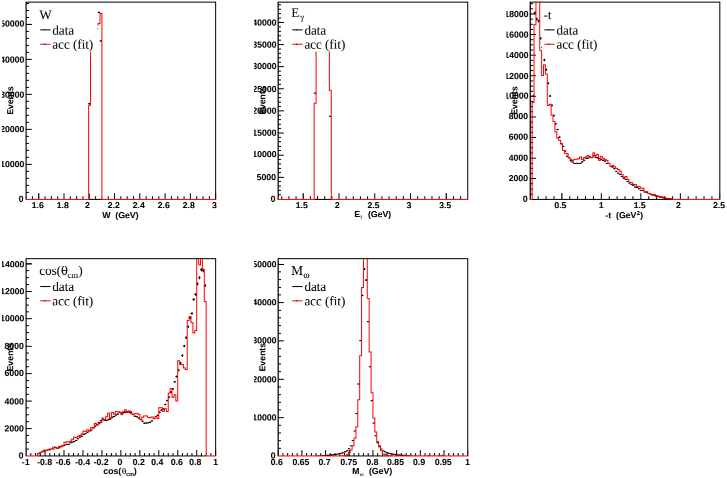
W:[2400 - 2410]MeV, $cos(\theta_{cm})$:[-0.8 - 0.8]GeV^2

g12, g11 SDME comparison (unpolarized beam)

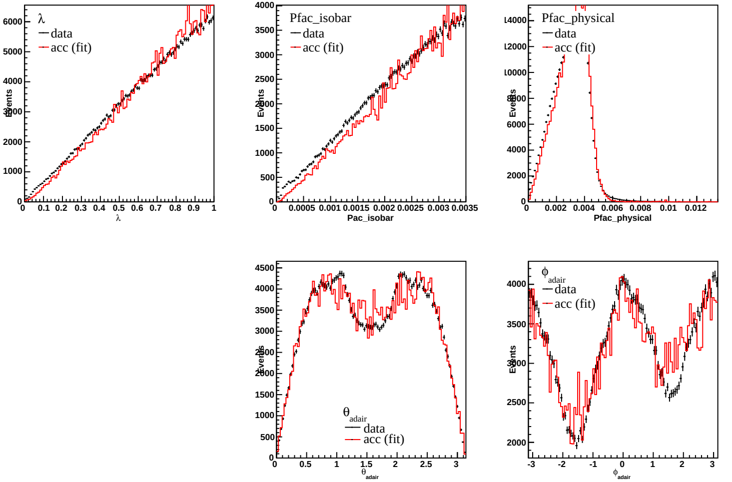


W:[2700 - 2710]MeV, $cos(\theta_{cm})$:[-0.7 - 0.8]GeV^2

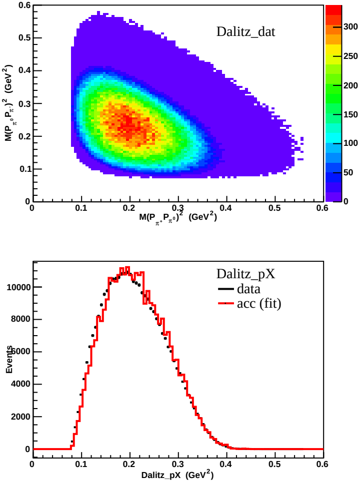
Fit Results: Wbin2000-2100



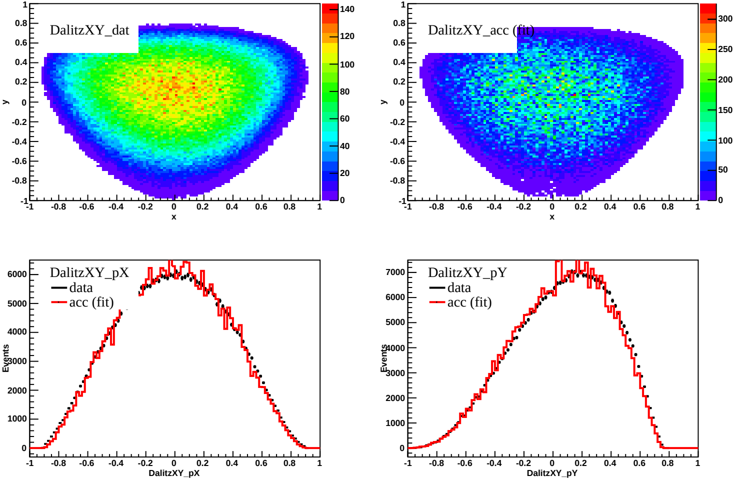
Fit Results: Wbin2000-2100



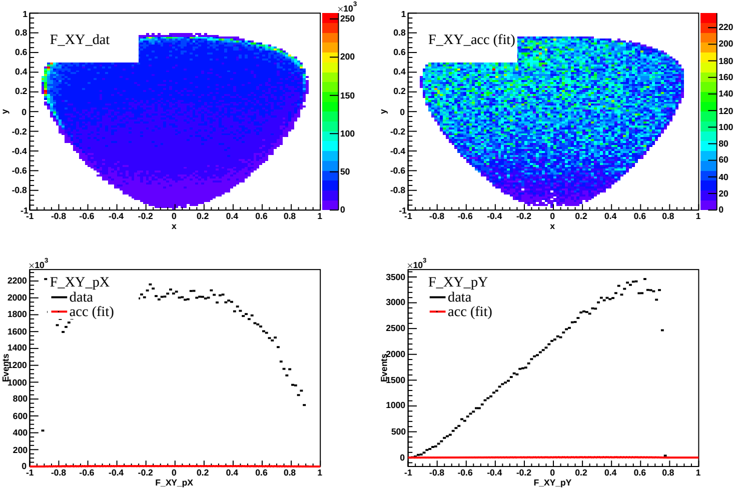
Fit Results: Wbin2000-2100



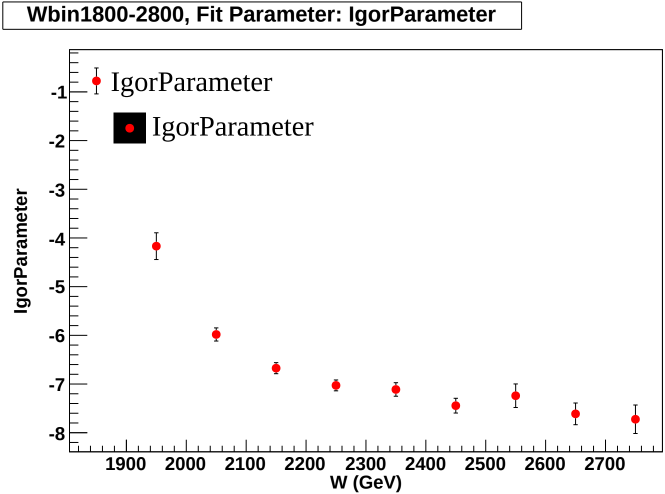
Fit Results: Wbin2000-2100



Fit Results: Wbin2000-2100

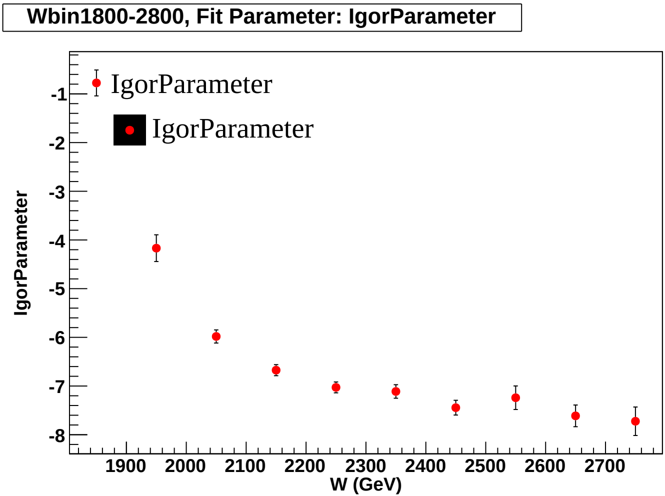


Decay Amplitude Parameter (2Body), Wbin1800-2800



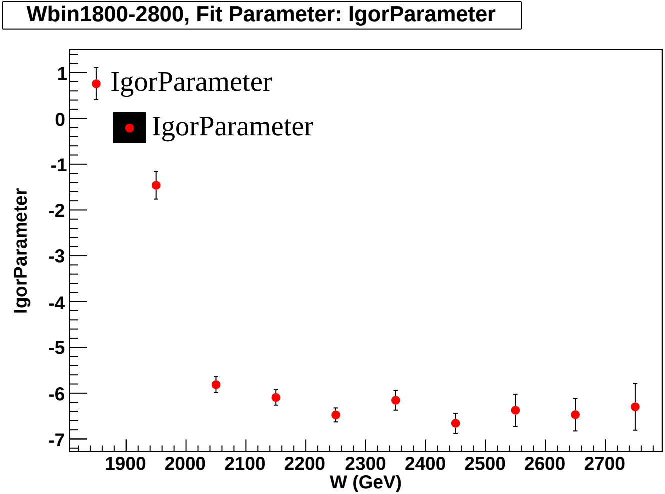
Using $\frac{d\sigma}{d\cos cm}$

Decay Amplitude Parameter (3Body), Wbin1800-2800



Using $\frac{d\sigma}{d \cos cm}$

Decay Amplitude Parameter (3Body), Wbin1800-2800



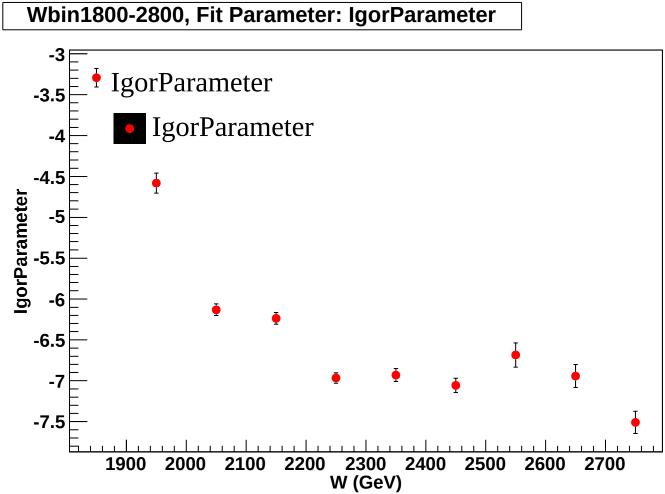
Using $d\sigma/dt$

Analysis Update

PWA Meeting 12-02-15

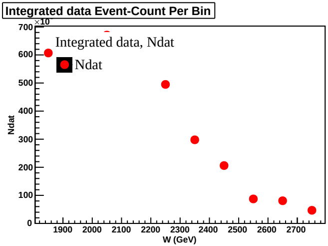
Status Update

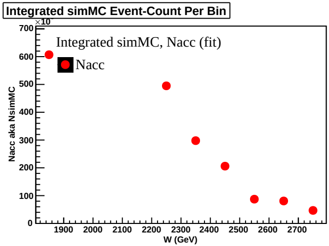
Decay Amplitude Parameter (3Body)

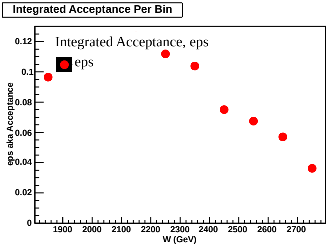


W:[1800 – 2800] MeV, Bin Widths: 100 MeV

Acceptance

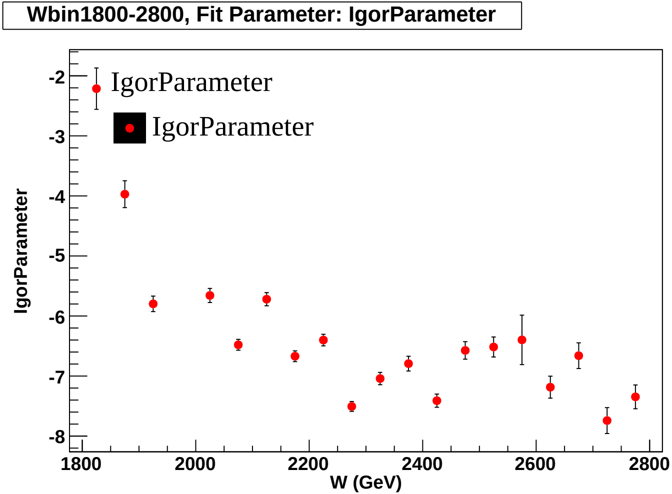






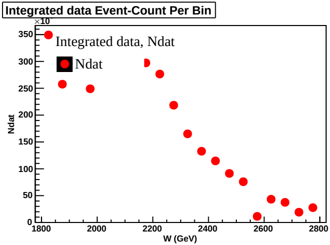
W:[1800 – 2800] MeV, Bin Widths: 100 MeV

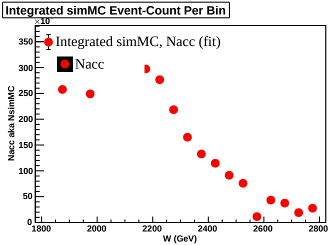
Decay Amplitude Parameter (3Body)

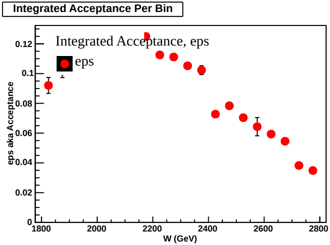


W:[1800 – 2800] MeV, Bin Widths: 50 MeV

Acceptance

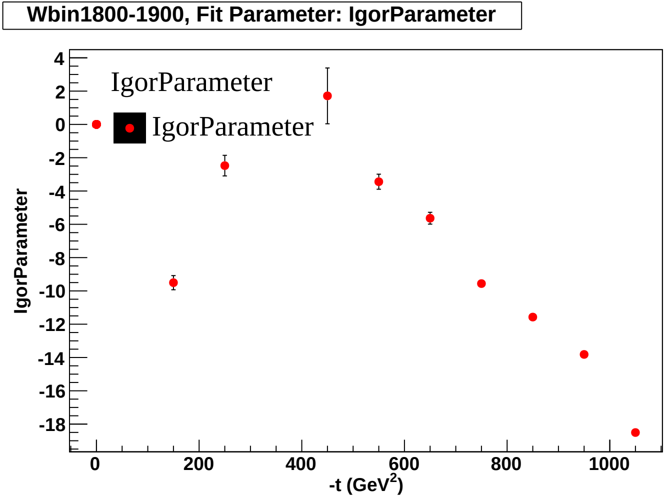






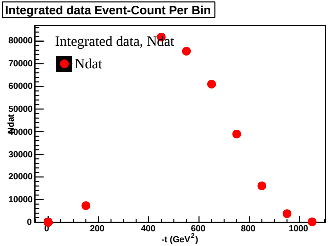
W:[1800 – 2800] MeV, Bin Widths: 50 MeV

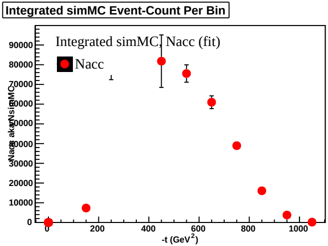
Decay Amplitude Parameter (3Body)

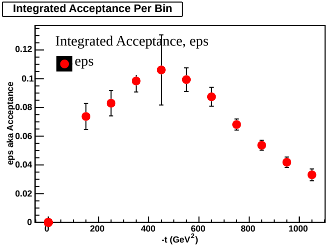


W:[1800 – 1900] MeV, $-$t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

Acceptance

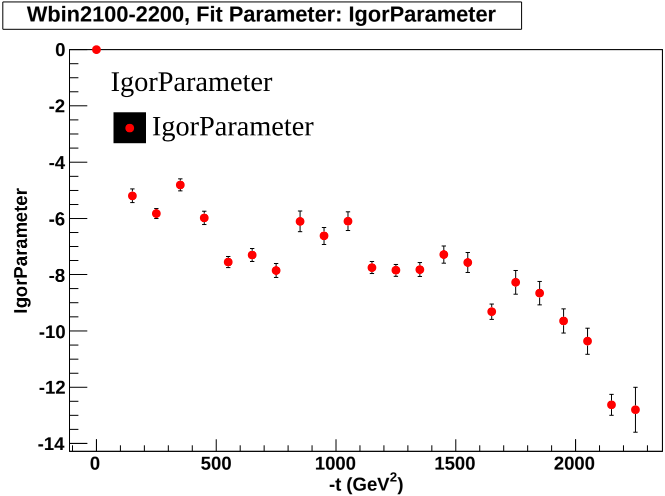






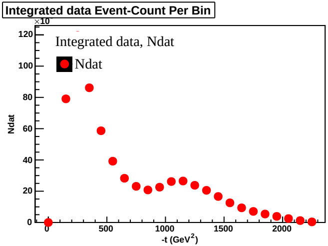
W:[1800 – 1900] MeV, −t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

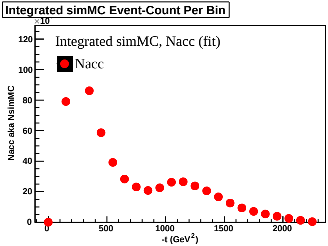
Decay Amplitude Parameter (3Body)

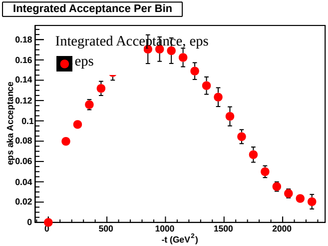


W:[2100 – 2200] MeV, $-$t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

Acceptance

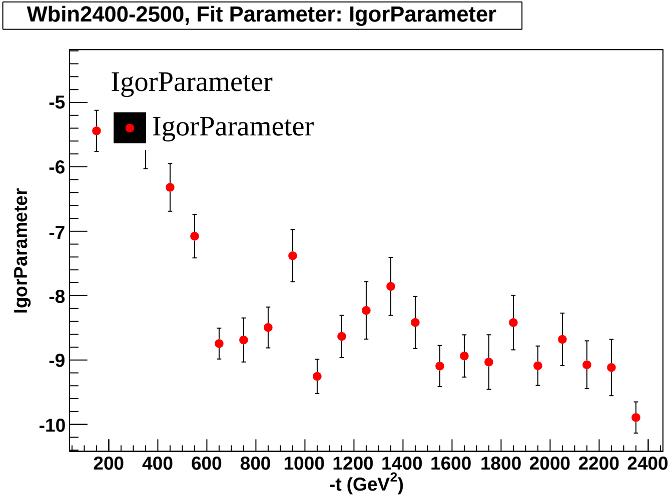






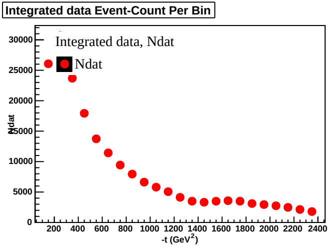
W:[2100 – 2200] MeV, $-$t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

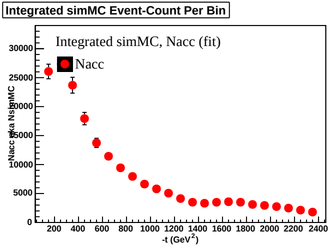
Decay Amplitude Parameter (3Body)

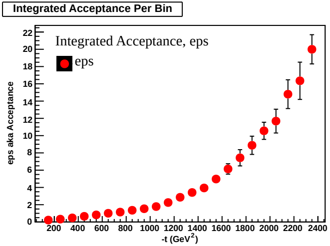


W:[2400 – 2500] MeV, –t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

Acceptance

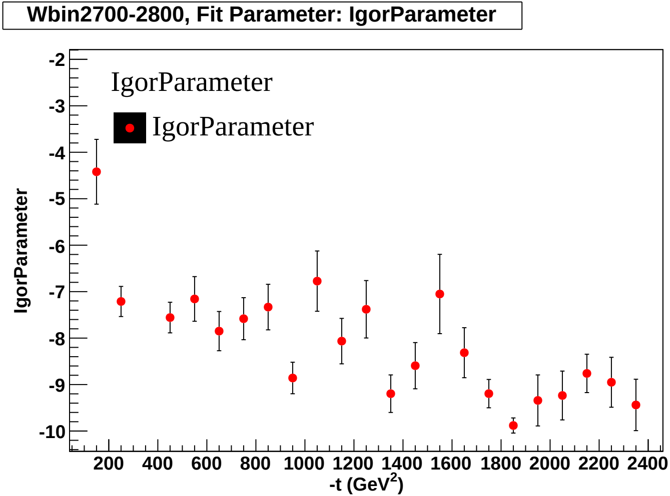






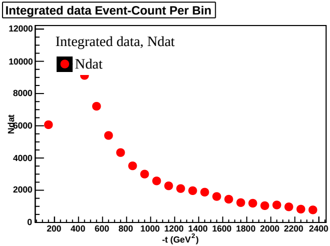
W:[2400 – 2500] MeV, $-$t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

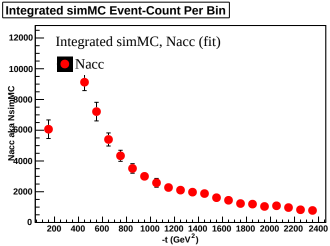
Decay Amplitude Parameter (3Body)

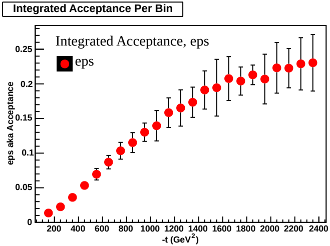


W:[2700 – 2800] MeV, −t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

Acceptance

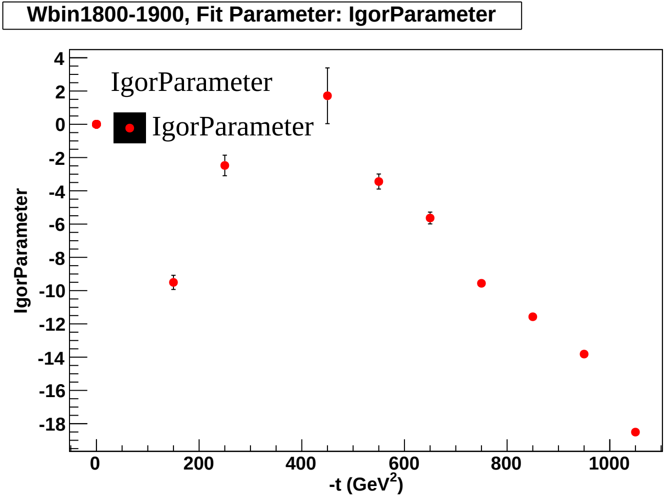






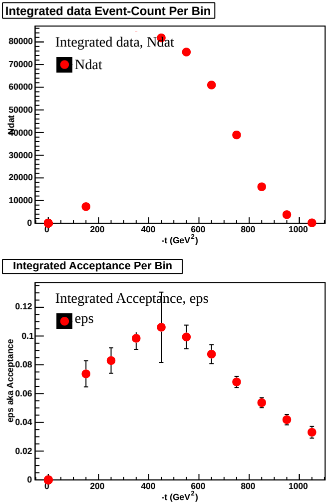
W:[2700 – 2800] MeV, $-$t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

Decay Amplitude Parameter (3Body)



W:[1800 – 1900] MeV, −t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

Acceptance



W:[1800 – 1900] MeV, −t:[100 – 2400] GeV^2, Bin Widths: 100 GeV^2

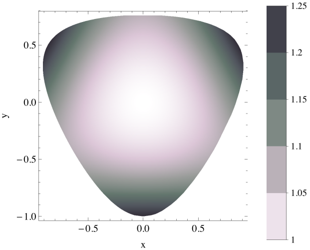
Next Steps

- Try to eliminate energy dependence of IgorParameter
- Statistical and systematic error
  - Refit results which had large statistical error bars
  - Include statistical error bars on g11 SDME results
  - Include systematic error into analysis
- Refit IgorParameter using g12 cross-section and SDME's

Summary

Thank you

Backup Slides



Expected Dalitz Plot,
IgorParameter > 3

$$\begin{aligned} x &= \frac{\sqrt{3}}{Q}(T_1 - T_2) = \frac{\sqrt{3}(t-u)}{2M(M-3\,m_\pi)}, \\ y &= \frac{3\,T_3}{Q} - 1 = \frac{3(s_c - s)}{2M(M-3m_\pi)}. \end{aligned} \tag{38}$$

Lorentz Invariant Variables

$$\begin{aligned} x &= \sqrt{z}\cos\theta \\ y &= \sqrt{z}\sin\theta \end{aligned}$$

$$|F_{par}(z,\vartheta)|^2 = |N|^2\big(1 + 2\,\alpha\,z + 2\,\beta\,z^{3/2}\sin(3\vartheta) + 2\,\gamma\,z^2 + 2\,\delta\,z^{5/2}\sin(3\vartheta) + \mathcal{O}(z^3)\big) \tag{40}$$

Dalitz Plot Amplitude Expansion

Dalitz Plot Parameter Comparison

$$|F_{par}(z,\vartheta)|^2 = |N|^2\big(1 + 2\,\alpha\,z + 2\,\beta\,z^{3/2}\sin(3\vartheta) + 2\,\gamma\,z^2 + 2\,\delta\,z^{5/2}\sin(3\vartheta) + \mathcal{O}(z^3)\big) \quad (40)$$

TABLE I: Dalitz Plot parameters and $\sqrt{\bar{\chi}^2}$ of the polynomial parametrization (40) for $\omega \to 3\pi$. In addition to our results we also show the selected results from Niecknig et al. [37] (dispersive study with incorporated crossed-channel effects) and Terschlusen et al. [19] (Lagrangian based study with the pion-pion rescattering effects).

	$\alpha \times 10^3$	$\beta \times 10^3$	$\gamma \times 10^3$	$\delta \times 10^3$	$\sqrt{\bar{\chi}^2} \times 10^3$
This paper ($\hat{F} = 0$)	136	-	-	-	3.5
This paper (full)	94	-	-	-	3.2
Niecknig et al. [37]	84...96	-	-	-	0.9...1.1
Terschlusen et al. [19]	202	-	-	-	6.6
This paper ($\hat{F} = 0$)	125	30	-	-	0.74
This paper (full)	84	28	-	-	0.35
Niecknig et al. [37]	74...84	24...28	-	-	0.052...0.078
Terschlusen et al. [19]	190	54	-	-	2.1
This paper ($\hat{F} = 0$)	113	27	24	-	0.1
This paper (full)	80	27	8	-	0.24
Niecknig et al. [37]	73...81	24...28	3...6	-	0.038...0.047
Terschlusen et al. [19]	172	43	50	-	0.4
This paper ($\hat{F} = 0$)	114	24	20	6	0.005
This paper (full)	83	22	1	14	0.079
Niecknig et al. [37]	74...83	21...24	0...2	7...8	0.012...0.011
Terschlusen et al. [19]	174	35	43	20	0.1

The Decay Amplitude

$$F(s) = \Omega(s) \left(\frac{1}{\pi} \int_{s_\pi}^{s_i} ds' \frac{\rho(s')\, t^*(s')}{\Omega^*(s')} \frac{\hat{F}(s')}{s' - s} + \Sigma(s) \right) . \tag{31}$$

KT Amplitude

$$\Sigma(s) = \sum_{i=0}^{\infty} a_i \, \omega^i(s) \tag{29}$$

$$\omega(s) = \frac{\sqrt{s_i - s_E} - \sqrt{s_i - s}}{\sqrt{s_i - s_E} + \sqrt{s_i - s}} \tag{30}$$

inelastic contribution

dispersion relation

Comparing JPAC & Dalitz Expansion Amplitude Parameters

$2\alpha \;\propto\; a' = 2\frac{a_1}{a_0} =$ "IgorParameter"