Inverstigation the $\gamma P \to K^+K^-P$ process with B_5 model

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1 B4 model(Nov 14 ,2013)

To illustrate the B5 model, we will begin with the G. Veneziano's Narrow resonance model[1], which is also called B_4 model. In 1968, G. Veneziano was the first one wrote down the Regge behavior amplitude for $\pi\omega \to \pi\pi$ with a Beta function,

$$\begin{aligned} T &= \varepsilon_{\mu\nu\rho\sigma}\epsilon^{\mu}_{\omega}p_1^{\nu}p_2^{\rho}p_3^{\sigma}\cdot A(s,t,u), \\ A(s,t,u) &= \frac{\beta}{\pi}[B(1-\alpha(t),1-\alpha(s))+B(1-\alpha(t),1-\alpha(u))+B(1-\alpha(s),1-\alpha(u))], \\ B(x,y) &= \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}, \end{aligned} \tag{1}$$

where β is a constant.

The amplitude can be written,

$$\begin{aligned} A(s,t,u) &= \frac{\beta}{\Gamma(\alpha(t))sin(\pi\alpha(t))}\frac{sin(\pi(\alpha(s)+\alpha(t))}{sin(\pi\alpha(s))}\frac{\Gamma(\alpha(s)+\alpha(t)-1)}{\Gamma(\alpha(s))}+...., \\ &\sim \frac{\beta(t)}{sin(\pi\alpha(t))}\frac{1-cos(\pi(\alpha(t))}{sin(\pi\alpha(s))}[\alpha(s)]^{\alpha(t)-1}, \end{aligned} \tag{2}$$

where $\beta(t)=\frac{\beta}{\Gamma(\alpha(t))}$. When $s\to\infty$, it has $A\propto(\alpha(s))^{\alpha(t)-1}$, which is the Regge behavior.

The Amplitude has following properties:

1, Regge Behavior
2, Crossing symmetries
3, Infinity number of poles
4, Duality
5, Ploynimial residues

6, positive widths (only B4 model was proved)
7, Violation the unitarity.

Consequently, Lovelace[2] applied it to $\pi^+\pi^-$ elastic scattering. The amplitude was written with

$$A(s,t) = -\beta\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} + \gamma\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}, \tag{3}$$

and for the isospin amplitudes

$$\begin{aligned} A^0 &= \frac{3}{2}[A(s,t)+A(s,u)] - \frac{1}{2}A(t,u), \\ A^1 &= A(s,t) - A(s,u), \\ A^2 &= A(t,u). \end{aligned} \tag{4}$$

For ρ trajectory, an imaginary part was considered

$$\alpha_\rho = 0.483 + 0.885s + i0.28\sqrt{s-4m_\pi^2}. \tag{5}$$

The scattering length was given by this amplitude

$$a_0 = 0.395\beta, \;\; a_2 = -0.103\beta. \tag{6}$$

The ratio was comparable with Weinberg's value. For 3-π final states, such as $\bar{p}n \to \pi^\pm\pi^\mp\pi^-$, the amplitude were

$$\begin{aligned} \mathcal{A}(X^\pm \to \pi^\pm\pi^\pm\pi^\mp) &= A(s,t), \\ \mathcal{A}(X^\pm \to \pi^\pm\pi^0\pi^0) &= \mathcal{A}(X^0 \to \pi^+\pi^-\pi^0) = A(t,u) - A(s,t) - A(s,u), \\ \mathcal{A}(X^0 \to 3\pi^0) &= A(s,t) + A(s,u) + A(t,u). \end{aligned} \tag{7}$$

The Dalitz Plot was also fit well with this amplitude. Shapiro[3] had proved all the widthes are positive. In Ref. [4], a dual model with back ground was used,

$$A(s,t) = \sum_{n,m} C_{nm}\frac{\Gamma(n-\alpha_s)\Gamma(m-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}, \tag{8}$$

$n, m = 1, 2$ and an additional term C_{30}. This amplitude can be also consist of the $\bar{p}n \to \pi^+\pi^-\pi^-$ data including the angular distribution of $\pi\pi$.

About the duality–A sum of ploes in one channel diverge to produce poles in overlapping channels, we will not explain it here clearly which can be viewed in the section II. D in the Ref. [5].

2 B_5 model(Nov 15 ,2013)

2.1 Two ways of extending the B_4 to B_5

Several authors has generalized the B_4 model to the B_n model. However, they are mainly based on two different ways. One is from Ruegg[6], and the other is based on Hopkinson[8].

2.1.1 Ruegg's B_5 model

The Ruegg amplitude is a direct generalization of B_4 model to B_n model.

$$B_4(-\alpha(s),-\alpha(t)) = \int_0^1 duu^{-\alpha(s)-1}(1-u)^{-\alpha(t)-1} \rightarrow$$
$$B_5 = \int_0^1\int_0^1 \frac{du_i du_j}{1-u_iu_j} u_1^{-\alpha_{12}-1}u_2^{-\alpha_{23}-1}u_3^{-\alpha_{34}-1}u_4^{-\alpha_{45}-1}u_5^{-\alpha_{51}-1}. \quad (9)$$

The variable u_i is constrained by,

$$u_i = 1-u_{i-1}u_{i+1}, i=1,...,5,\ u_6=u_1, u_0=u_5. \quad (10)$$

Consider the process $1+2\rightarrow 3+4+5$, it is expressed with

$$B_5 = \int_0^1 du_1\int_0^1 du_4 u_1^{-\alpha_{12}-1}u_4^{-\alpha_{45}-1}(1-\frac{u_1}{1-u_1u_4})^{-\alpha_{23}-1}(1-u_1u_4)^{-\alpha_{15}-2}. \quad (11)$$

In the high energy limit($s_{34}, s_{45}\rightarrow\infty$), by changing the variables

$$u_1 = e^{\frac{-xy}{\alpha_{34}\alpha_{45}}},\ u_4 = e^{\frac{y}{\alpha_{45}}}, \quad (12)$$

the amplitude can be rewritten to

$$\begin{aligned} B_5 &= (-\alpha_{34})^{\alpha_{23}}(-\alpha_{45})^{\alpha_{15}} g(K, s_{23}, s_{15}), \\ g(K, s_{23}, s_{15}) &= (a)^{\alpha_{23}+\alpha_{15}} \int_0^\infty dx \int_0^\infty dy x^{-\alpha_{23}-1}y^{-\alpha_{15}-1}e^{-x-y+\frac{-xy}{aK}}, \end{aligned} \quad (13)$$

where a is the slope of the trajectory, $K(x',y') = \frac{1-e^{-x'y'}}{1-e^{-x'y'-y'}}\frac{1}{x'}$ and $x' = -x/\alpha_{23}, y' = -y/\alpha_{45}$. It has exactly the double Regge limit behavior. Based on this double Regge limit amplitude, the authors in Ref. [7] have investigated the $\pi^- p\rightarrow K\bar{K}n$ data. Dalitz Plots are also given by them therein.

2.1.2 Hopkinson's B_5 model

Hopkinson and Plahte start from another way to generate the B_4 model[8],

$$B_N(x) = \sum_{k_{iN-1}=0}^{\infty} \{\prod_{i=2}^{N-3} (-1)^{k_{iN}} \binom{z_{i,N-1}}{k_{i,N-1}}\} B_4(x_{N-2,N-1}, x_{N-1,N+\sum_{i=2}^{N-3} k_{iN-1}}) B_{N-1}(x'), \quad (14)$$

where $z_{ij} = x_{ij} - x_{i+1,j} - x_{i,j-1} + x_{i+1,j-1}$ and $x_{ij} = -\alpha_{ij}(s_{ij})$. For $N = 5$, the amplitude is

$$\begin{aligned} B_5(x) &= \sum_{k=0}^{\infty} (-1)^k \binom{z_{24}}{k} B_4(x_{34}, x_{45}+k) B_4(x_{12}+k, x_{23}) \quad (15) \\ &= B_4(x_{34}, x_{45}) B_4(x_{12}, x_{23}) {}_3F_2(-x_{12}+x_{34}+x_{45}, x_{13}, x_{25}; x_{45}+x_{25}, x_{13}+x_{34}; 1), \end{aligned}$$

where

$$3F_2(a,b,c;d,e;z) = \sum_{k=0}^{\infty} \frac{\Gamma(a+k)\Gamma(b+k)\Gamma(c+k)\Gamma(d)\Gamma(e)}{\Gamma(a)\Gamma(b)\Gamma(e)\Gamma(d+k)\Gamma(e+k)} \frac{z^k}{k!}.$$

Bialas and Pokorski[9] investigated the double Regge limie and the single reggelimit from Eq. (15). The $a + b \to 1 + 2 + 3$ is used by them, and the amplitude is

$$B_5 = B_4(12, A1) B_4(23, B3) {}_3F_2(-AB + 12 + 23, A1, B3; 12 + A1, 23 + B3), \quad (16)$$

where $A1 = -\alpha_{A1}$, and so on.

When s_{12} and s_{23} is large, the amplitude shows a double Regge limit,

$$B_5 = B_4(AB, A1) B_4(23, B3 - A1) + B_4(AB, B3) B_4(12, A1 - B3), \quad (17)$$

and in the very high energy region of s_{12} and s_{23}, the amplitude is approximate to

$$B_5 = B_4(12, A1) B_4(23, B3). \quad (18)$$

The single Regge limit is given when s_{12} or s_{23} goes large, for example when s_{23} is fixed, the amplitude is

$$\begin{aligned} B_5 &= B_4(AB, A1) R_{23}, \quad (19) \\ R_{23} &= \sum_{m=0}^{\infty} \frac{1}{23+m} \frac{(-1)^m}{m!} \frac{\Gamma(B3)}{\Gamma(B3-m)} F(A1, -m; B3-m; \frac{13}{AB}). \end{aligned}$$

Jones and Wyld[10] have examined the single Regge limit with the above equation to $\bar{p}n \to 3\pi$. However, the result shows that the amplitude did not match the data very well.

2.2 The connection between single regge limit and the double regge limit(Dec 17 ,2013)

Bialas and Pokorski investigated the double Regge limie and the single regge-limit. The $a+b \to 1+2+3$ is used by them, and the double Regge limit amplitude Fig. 7(d) is

$$B_5 = B_4(AB, A1)B_4(23, B3-A1) + B_4(AB, B3)B_4(12, A1-B3), \quad (20)$$

and in the high energy region of s_{12} and s_{23}, the amplitude should be

$$B_5 \sim B_4(12, A1)B_4(23, B3) \sim (12)^{-A1}(23)^{-B3}\Gamma(A1)\Gamma(B3), \quad (21)$$

where $A1 = -\alpha_{A1}$, and $B3 = -\alpha_{B3}$.

The single Regge limit is given when s_{12} or s_{23} goes to large energy, for example when s_{23} is fixed Fig. 7(b), the amplitude is

$$\begin{aligned} B_5 &= B_4(AB, A1)B_4(23, B3-A1)F(A1, 23; 1+A1-B3; \frac{12}{AB}) \quad (22)\\ &\quad + B_4(AB, B3)B_4(12, A1-B3)F(B3, 23+B3-A1; 1+B3-A1; \frac{12}{AB}),\\ &= B_4(AB, A1)B_4(23, B3)F(A1, 23; B3+23; \frac{13}{AB}). \quad (23) \end{aligned}$$

We can prove that when s12 and s23 go to large area, this single regge limit can become double regge limit.

$$\begin{aligned} B_5 &\to B_4(AB, A1)B_4(23, B3-A1)F(A1; 1+A1-B3; \frac{12\cdot 23}{AB}) \quad (24)\\ &\quad + B_4(AB, B3)B_4(12, A1-B3)F(B3; 1+B3-A1; \frac{12\cdot 23}{AB})\\ &\to \Gamma(A1)(AB)^{-A1}\Gamma(B3-A1)(23)^{A1-B3}\frac{\Gamma(1+A1-B3)}{\Gamma(1-B3)}(k)^{-A1}\\ &\quad + \Gamma(B3)(AB)^{-B3}\Gamma(A1-B3)(12)^{B3-A1}\frac{\Gamma(1+B3-A1)}{\Gamma(1-A1)}(k)^{-B3}\\ &\sim \Gamma(A1)\Gamma(B3)(12)^{-A1}(23)^{-B3}, \end{aligned}$$

where $k = \frac{12\cdot 23}{AB}$ and $F(a, c, z) \overset{z\to -\infty}{\to} \frac{\Gamma(c)}{\Gamma(c-a)}(-z)^{-A}$ are used.

2.3 The phase space of five particles interaction(Nov 27 ,2013)

For $a+b \to 1+2+3$ system, the distribution of s_{12} and s_{23} is given by[7]:

$$\frac{d^2\sigma}{ds_{12}ds_{23}} = \frac{1}{(4\pi)^2\lambda(s, m_a^2, m_b^2)}\int\int \frac{dt_{a1}dt_{b3}}{\sqrt{-\Delta_4}}|A(s_{12}, s_{23}, t_{a1}, t_{b3})|^2, \quad (25)$$

where $\lambda(s, m_a^2, m_b^2)$ is

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx, \tag{26}$$

and Δ_4 is the symmetric determinant with

$$\Delta_4 = \begin{vmatrix} m_a^2 & \frac{1}{2}(s - m_a^2 - m_b^2) & \frac{1}{2}(-t_{a1} + m_a^2 + m_1^2) & \frac{1}{2}(s - s_{12} + t_{b3} - m_b^2) \\ \frac{1}{2}(s - m_a^2 - m_b^2) & m_b^2 & \frac{1}{2}(s - s_{23} + t_{a1} - m_a^2) & \frac{1}{2}(-t_{b3} + m_a^2 + m_3^2) \\ \frac{1}{2}(-t_{a1} + m_a^2 + m_1^2) & \frac{1}{2}(s - s_{23} + t_{a1} - m_a^2) & m_1^2 & \frac{1}{2}(s - s_{12} - s_{23} + m_2^2) \\ \frac{1}{2}(s - s_{12} + t_{b3} - m_b^2) & \frac{1}{2}(-t_{b3} + m_a^2 + m_3^2) & \frac{1}{2}(s - s_{12} - s_{23} + m_2^2) & m_3^2 \end{vmatrix}, \tag{27}$$

which give the boundary of the s_{12}, s_{23}, t_{a1} and t_{b3}[11]. With fixed t_{a1} and t_{b3}, we can have the Dalitz plot of s_{12} and s_{23}.

2.4 Dalitz Plots of the B_5 model(Nov 22 ,2013)

Here we try to use the two different amplitudes from the above section to investigate the single and double Regge limits of the $\gamma P \to K^+K^-P$ process. One is the Hopkinson's amplitude, and the other one is Chan Hong-Mo's amplitude in Ref. [7] which is a modified Ruegg's amplitude.

The intermediated states are chosen to beρ, K^* and Σ, and the masses of them are $m_p = 0.983$GeV, $m_K = 0.493$GeV, $m_\gamma = 0$GeV. The trajectories are chhosen to be

$$\begin{aligned} \alpha_{t_{b3}} &= 0.483 + 0.885s + 0.1i,\ \alpha_{t_{a1}} = 0.3 + 1.0s + 0.12i, \\ \alpha_{s_{ab}} &= 0.5 + 0.9si, \alpha_{s_{12}} = 0.5 + 0.9s + 0.05i, \alpha_{s_{23}} = -0.2 + 0.9s + 0.23i \end{aligned} \tag{28}$$

s_{ab} is fixed to be $20.1GeV^2$. The Dalitz Plots are showen in Fig. 1.

2.5 Generalized B5 model with kinematic factors(Nov 27 ,2013)

When the interaction includes baryons, there always has a kinematic factors. The amplitude of five-point function with $\pi^- P \to K^0 \bar{K}^- P$ in Fig. 6 generally has the following structure[12, 13, 14], when the intermediate states are spin-1 mesons or P-wave baryons.

$$A = c \cdot \epsilon_{\mu\nu\rho\sigma} p_\pi^\mu p_{\bar{K}}^\nu p_K^\rho p_{\bar{p}}^\sigma \times B_5(1 - \alpha_{K^*}, 1 - \alpha_A, \frac{3}{2} - \alpha_{Y^*}, 1 - \alpha_\omega, \frac{3}{2} - \alpha_\Delta), \tag{29}$$

where c is a normalization constant.

For a photoproduction case in Fig. 7(a), the amplitude should be:

$$\mathcal{A} = c \cdot \epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu p_{K^+}^\nu p_{K^-}^\rho p_{\bar{p}}^\sigma \times B_5(1 - \alpha_{K^*}, 1 - \alpha_A, \frac{3}{2} - \alpha_{Y^*}, 1 - \alpha_\omega, \frac{3}{2} - \alpha_{N^*}), \tag{30}$$

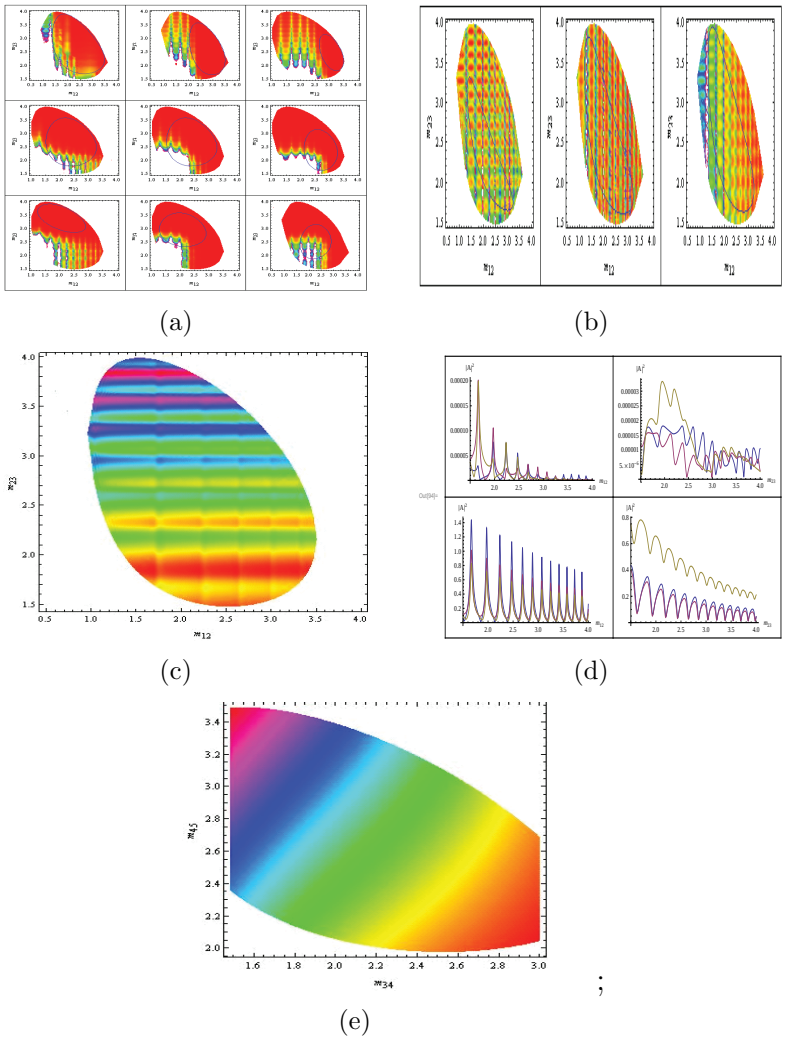
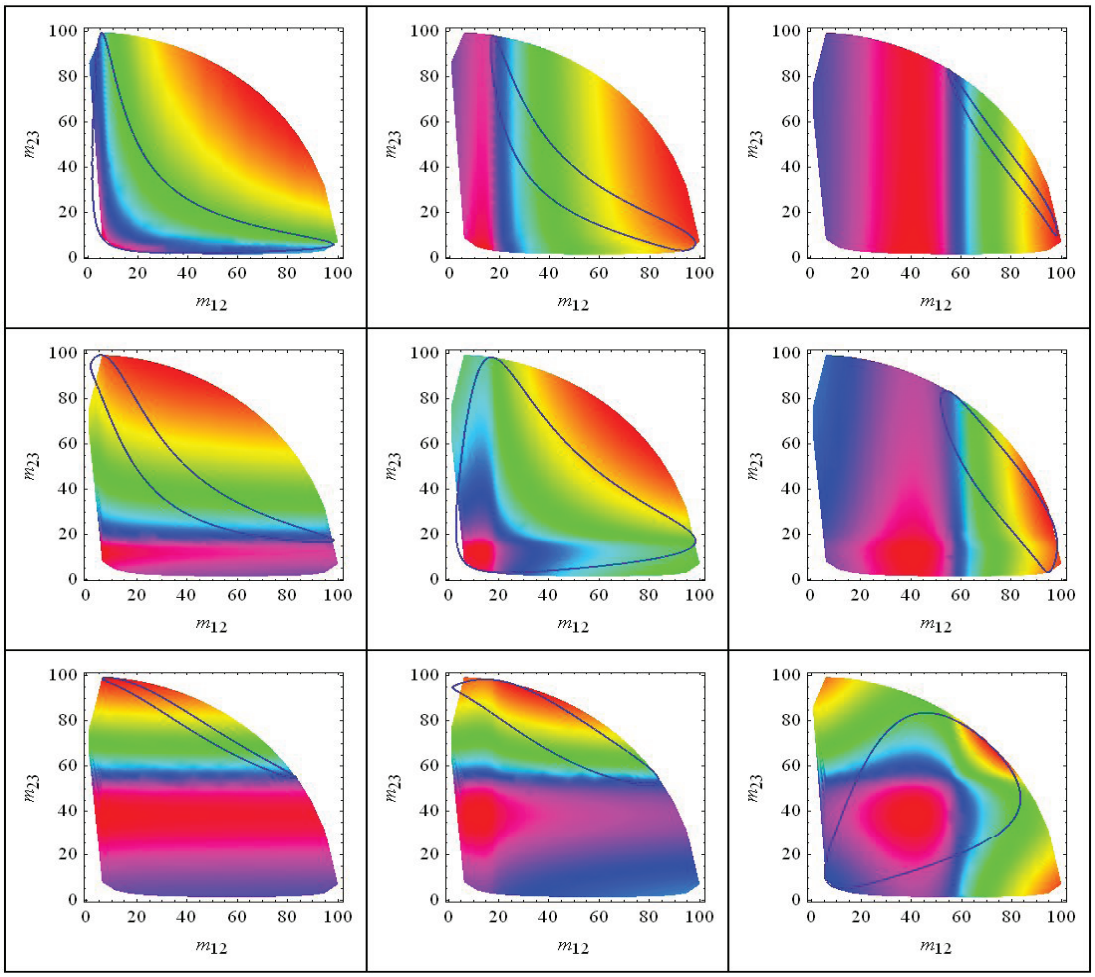
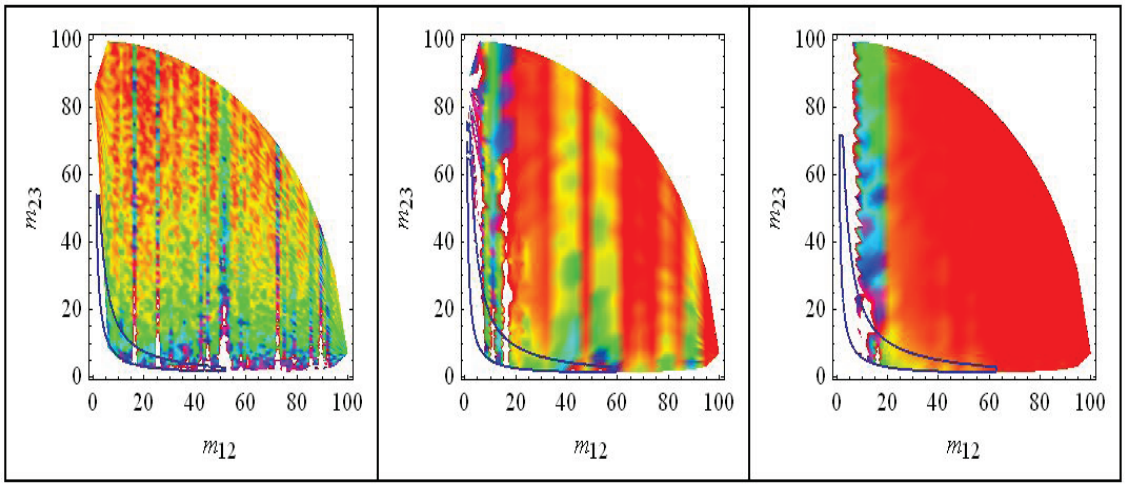


Figure 1: (a), (b)the double Regge limits of Eq. 20, (a) with $t1 = \{-3,-6,-9\}, t2 = \{-3,-6,-9\}$, (b) small $t1, t2$, $t1 = -0.8$ and $t2 = -1.4$. The blue circle in the plot is the real boundary of m_{12} and m_{23}. (c)the double Regge limits of Eq. 18($t1 = t2 = -6$) (d), the single Regge limit of Eq. 22 with fixed s_{23}(left) and fixed s_{12}(right), and $t1 = -6 = t2 = -6$(up); $t1 = -0.3, -0.8, -1.4$, $t2 = -0.8$(down); (e), the double Regge limits from the Ref.[7](Logarithmic Plot).

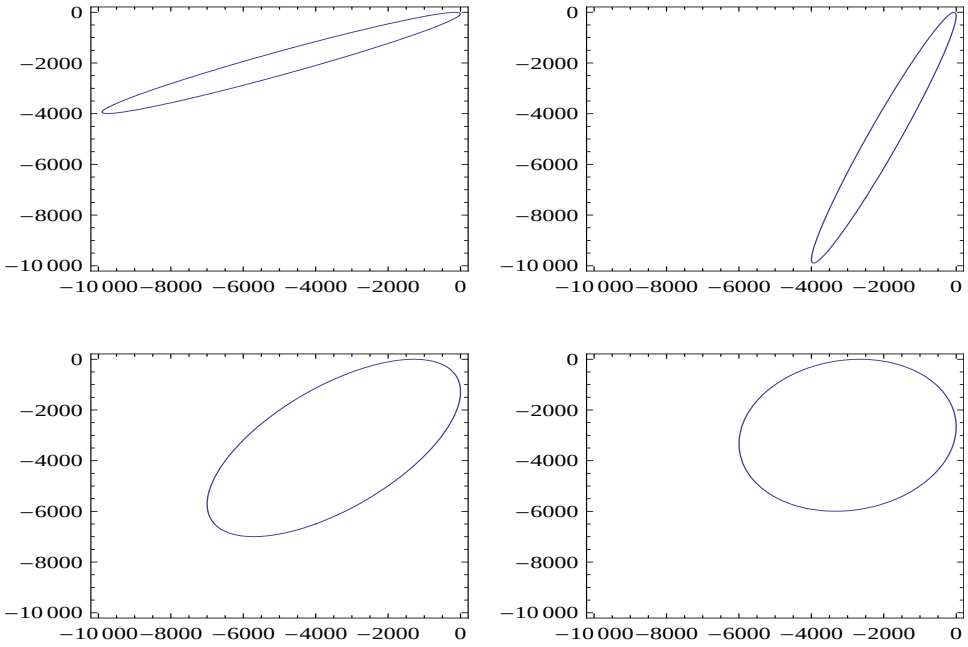


(a)

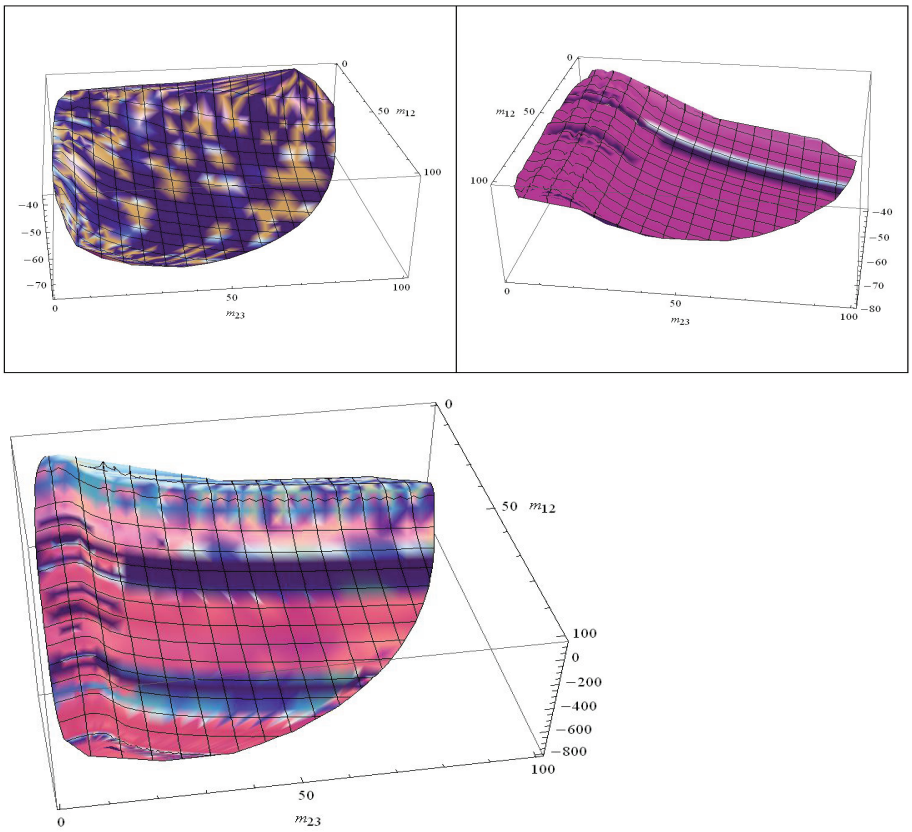


(b)

Figure 2: a, the double Regge limit with larger region, $sab = 10000$, $t1 = -30, -300, -3000$, $t2 = -30, -300, -3000$; b)the double Regge limits with larger region, $sab = 10000, t1 = -0.3, -0.8, -1.4, t2 = -0.8$.



(a)



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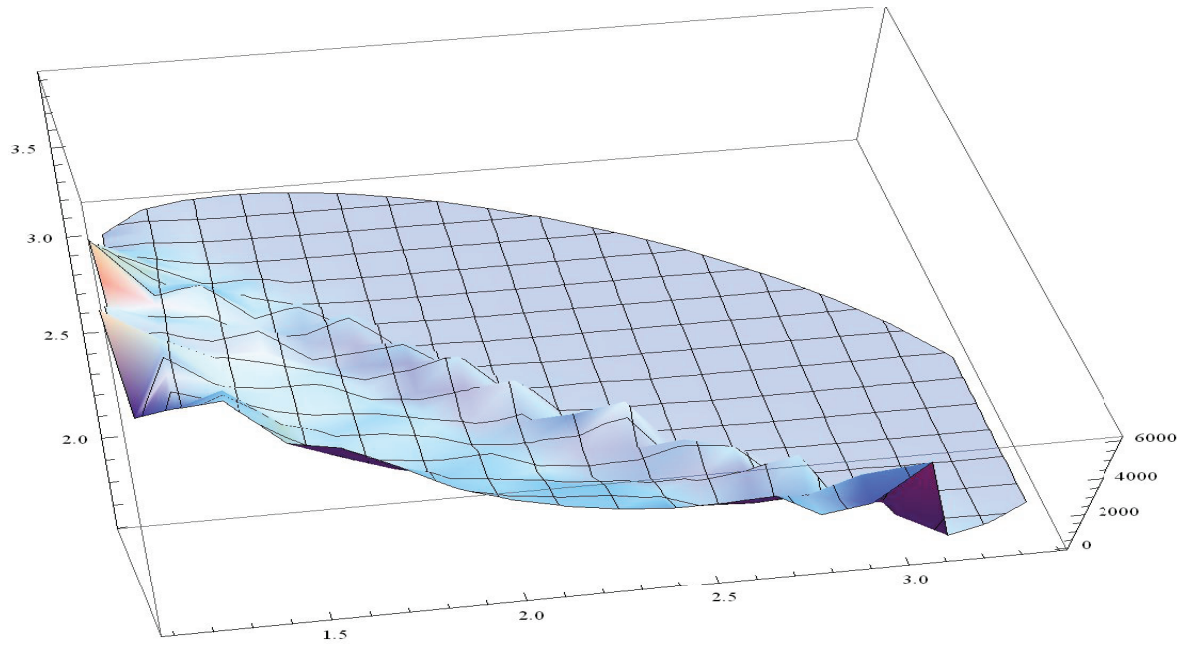
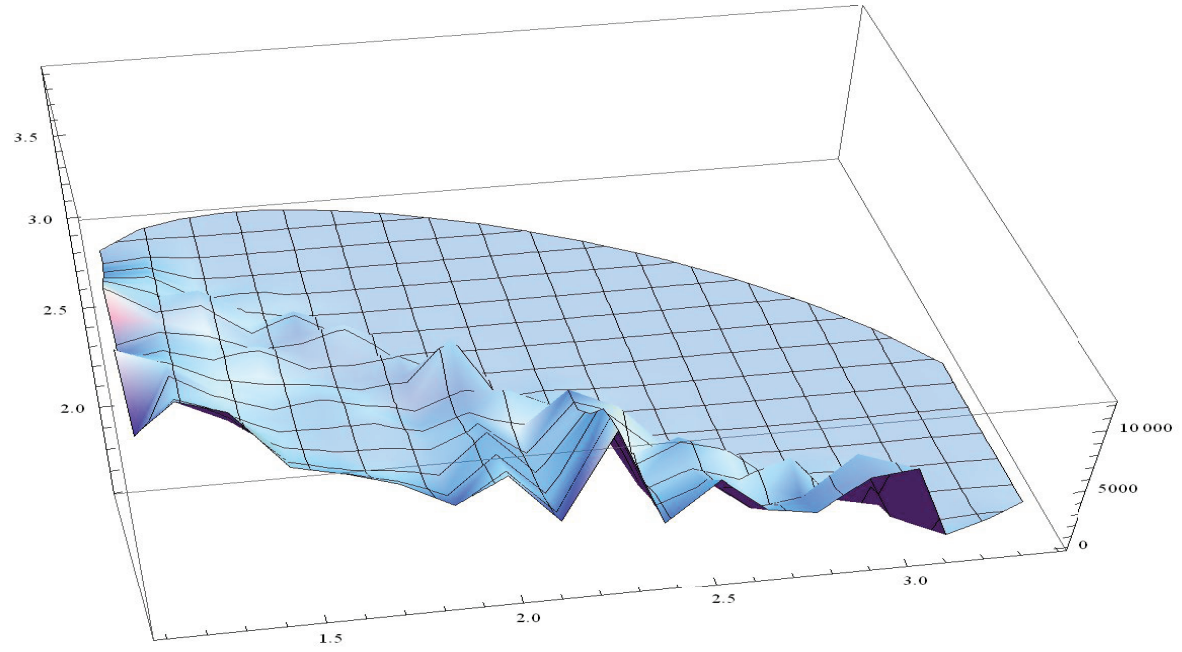
(b)

Figure 3: a), Boundary of t_{b3} and t_{a1}, $t1 = 6000, 10, 3000, 4000, t2 = 10, 6000, 3000, 4000$; b), The behavior of single regge and double regge limits: they have the same behavior at the central region.



(a)



(b)

Figure 4: The Dalitz Plots by Fortran code: a),the single regge limit with fixed s_{23}; b)the single regge limit with fixed s_{12}.

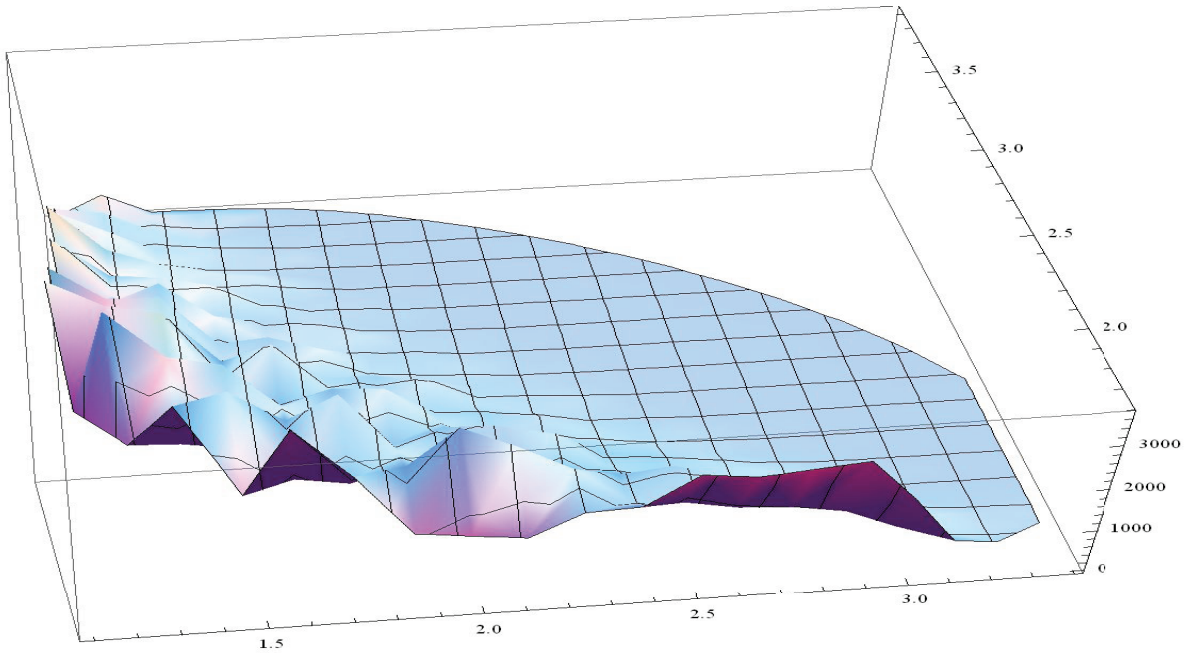


Figure 5: The Dalitz Plots by Fortran code: a), the double Regge limit

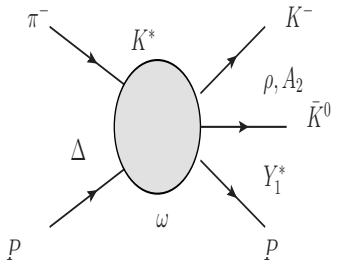


Figure 6: $\pi P \to K^- \bar{K}^0 P$

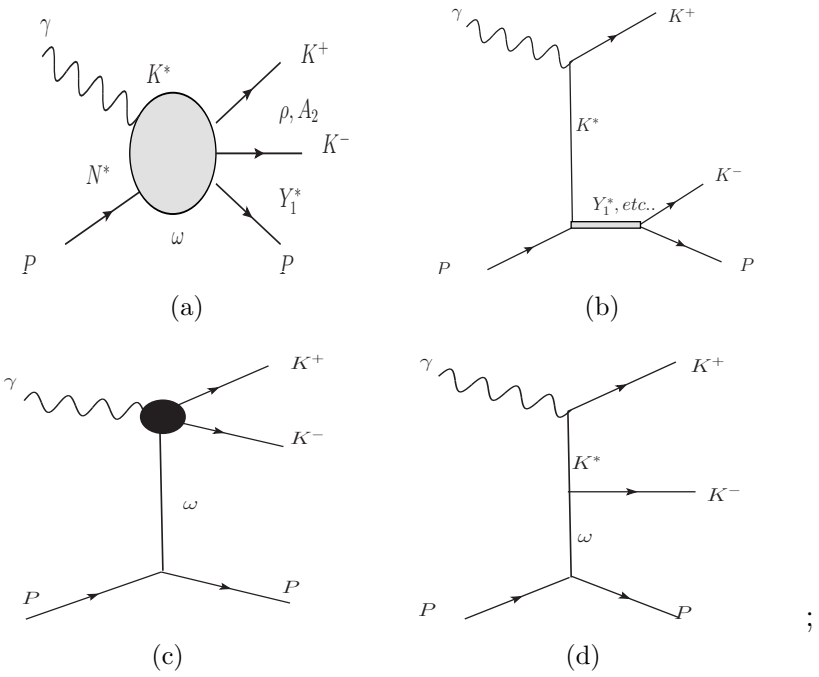


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Figure 7: a, a general diagram of $\gamma P \rightarrow K^+K^-P$; b,c,d, including the particular intermediate states in each diagram.

where ε^μ is the polarization vector of the photon. One only needs to change the π^μ to the photon ploarization vector ε^μ since the intermediate states are same as π production.

The case of Fig. 7(b), Fig. 7(c) and Fig. 7(d) are the single Regge limits and the double Reggge limit, which correspond different energy regions of the Dilatz Plot. The relation between them are still being researched.

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