Model selection in kaon photoproduction

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Motivation for the work on kaon photo/electroproduction

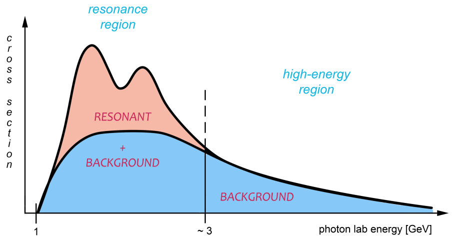
- We aim at understanding the baryon spectrum and production dynamics of particles with strangeness at low energies.
- Constituent Quark Model predicts a lot more N^* states than was observed in pion production experiments $\rightarrow$ "missing" resonance problem.
- Models for the description of elementary hyperon electroproduction are a suitable tool for hypernuclear physics calculations.
- New good-quality photoproduction data from LEPS, GRAAL, MAMI and (particularly) CLAS collaborations allow us to tune free parameters of the models.
- As the α_s increases with decreasing energy, we cannot use perturbative QCD at low energies $\rightarrow$ need for introducing effective theories and models.

Introduction

Photoproduction process

$$p(p) + \gamma(k) \to K^+(p_K) + \Lambda(p_\Lambda)$$

- Photoproduction: a special case of electroproduction with $Q^2 = 0$, $\varphi_K = 0 \Rightarrow \sigma = \sigma_T$.
- Threshold: $E_\gamma^{lab} = 0.911$ GeV, $W = 1.609$ GeV; $p(\gamma, K^+)\Lambda$ occurs on the hadronic plane.
- In the lowest order, the reaction is described by the exchange of hadrons.
    - *The 3rd nucleon-resonance region:*
      many resonant states and no dominant one in the $K^+\Lambda$ production
      $\to$ need to assume a large number of nucleon resonances with mass < 2.5 GeV



- Resonance region:
  resonance contributions dominate (N^*)
- Background:
    - IM: a plenty of nonresonant contributions (p, K, Λ; K^* and Y^*)
    - RPR: exchange of kaon trajectories

Introduction

An overview of assumed resonances and their occurrence in models used for $p(\gamma, K^+)\Lambda$ description

	mass [MeV]	width [MeV]	spin	isospin	parity	*Kaon-MAID*	*Saclay-Lyon*	*Gent IM*	*BS1*	*BS2*	*BS3*	*RPR-2011*	*RPR fit*	*RPR-BS*	*RPR-BSpv*
$K^*(892)$	892	50	1	1/2	−	✓	✓	✓	✓	✓	✓				
$K_1(1270)$	1270	90	1	1/2	+	✓	✓	✓	✓	✓	✓				
$P_{11}(1440)$	1440	300	1/2	1/2	+		✓								
$S_{11}(1535)$	1535	150	1/2	1/2	−				✓	✓	✓	✓	✓	✓	✓
$S_{11}(1650)$	1655	150	1/2	1/2	−	✓		✓	✓	✓	✓	✓	✓	✓	✓
$D_{15}(1675)$	1675	150	5/2	1/2	−		✓						✓	✓	
$F_{15}(1680)$	1685	130	5/2	1/2	+				✓	✓	✓	✓	✓	✓	✓
$D_{13}(1700)$	1700	150	3/2	1/2	−								✓	✓	✓
$P_{11}(1710)$	1710	100	1/2	1/2	+	✓		✓	✓		✓				
$P_{13}(1720)$	1720	250	3/2	1/2	+	✓	✓	✓	✓	✓	✓	✓		✓	✓
$F_{15}(1860)$	1860	270	5/2	1/2	+				✓	✓	✓		✓	✓	✓
$D_{13}(1875)$	1875	220	3/2	1/2	−	✓		✓	✓	✓	✓	✓	✓	✓	✓
$P_{11}(1880)$	1870	235	1/2	1/2	+								✓		✓
$P_{11}(1900)$	1895	200	1/2	1/2	+							✓			
$P_{13}(1900)$	1900	250	3/2	1/2	+						✓	✓	✓	✓	✓
$F_{15}(2000)$	2000	140	5/2	1/2	+				✓	✓	✓	✓		✓	✓
$D_{13}(2120)$	2120	330	3/2	1/2	−						✓		✓		✓
$D_{15}(2570)$	2570	250	5/2	1/2	−									✓	
$\Lambda(1405)$	1405	50	1/2	0	−		✓			✓	✓				
$\Lambda(1520)$	1520	16	3/2	0	−				✓						
$\Lambda(1600)$	1600	150	1/2	0	+						✓				
$\Lambda(1800)$	1800	300	1/2	0	−				✓	✓					
$\Lambda(1810)$	1810	150	1/2	0	+		✓								
$\Lambda(1890)$	1890	100	3/2	0	+				✓		✓				
$\Sigma(1660)$	1660	100	1/2	1	+		✓		✓	✓					
$\Sigma(1670)$	1670	60	3/2	1	−						✓				
$\Sigma(1750)$	1750	90	1/2	1	−				✓						
$\Sigma(1940)$	1940	220	3/2	1	−				✓	✓					

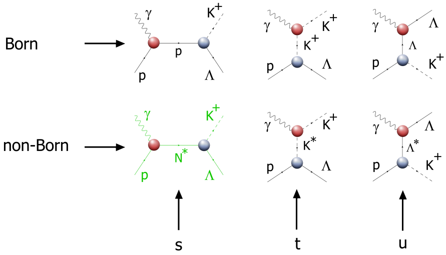
Isobar model

Single-channel approximation

- higher-order contributions (rescattering, FSI) included, to some extent, by means of effective values of the coupling constants

Use of effective hadron Lagrangian

- hadrons either in their ground or excited states
- amplitude constructed as a sum of tree-level Feynman diagrams
  - background and resonant part



Free parameters adjusted to experimental data

Satisfactory agreement with the data in the energy range $W = 1.6 - 2.5\,\mathrm{GeV}$

Isobar model

Calculation procedure

- Reaction amplitude: sum of s-, t-, and u-channel (non) Born amplitudes

$$\mathbb{M} = \sum_x \mathbb{M}_x, \text{ where } x \equiv s,\ t,\ u,\ N^*,\ K^*,\ Y^*$$

- Each contribution can be rewritten in a compact form

$$\mathbb{M}(p, p_\Lambda, k) = \bar{u}(p_\Lambda)\gamma_5 \left(\sum_{j=1}^{6} \mathcal{A}_j(s,t,u)\mathcal{M}_j \right) u(p),$$

where $\mathcal{A}_j$ are scalar amplitudes and $\mathcal{M}_j$ are gauge-invariant operators, *i.e.* $k_\mu \mathcal{M}_j^\mu = 0$,

$$\begin{aligned}
\mathcal{M}_1 &= \tfrac{1}{2}\left[\not{k}\not{\varepsilon} - \not{\varepsilon}\not{k}\right], & \mathcal{M}_2 &= (p\cdot\varepsilon) - (k\cdot p)\tfrac{(k\cdot\varepsilon)}{k^2}, \\
\mathcal{M}_3 &= (p_\Lambda\cdot\varepsilon) - (k\cdot p_\Lambda)\tfrac{(k\cdot\varepsilon)}{k^2}, & \mathcal{M}_4 &= \not{\varepsilon}(k\cdot p) - \not{k}(p\cdot\varepsilon), \\
\mathcal{M}_5 &= \not{\varepsilon}(k\cdot p_\Lambda) - \not{k}(p_\Lambda\cdot\varepsilon), & \mathcal{M}_6 &= \not{k}(k\cdot\varepsilon) - \not{\varepsilon}k^2.
\end{aligned}$$

Isobar model
Calculation procedure

- CGLN amplitudes $f_i(k^2, s, t)$

$$\mathbb{M} = \chi_\Lambda^\dagger \mathcal{F} \chi_p; \quad \mathcal{F} = f_1(\vec{\sigma}\cdot\vec{\varepsilon}) - \mathrm{i} f_2(\vec{\sigma}\cdot\hat{p}_K)[\vec{\sigma}\cdot(\hat{k}\times\vec{\varepsilon})] + f_3(\vec{\sigma}\cdot\hat{k})(\hat{p}_K\cdot\vec{\varepsilon}) + f_4(\vec{\sigma}\cdot\hat{p}_K)(\hat{p}_K\cdot\vec{\varepsilon}) + f_5(\vec{\sigma}\cdot\hat{k})(\hat{k}\cdot\vec{\varepsilon}) + f_6(\vec{\sigma}\cdot\hat{p}_K)(\hat{k}\cdot\vec{\varepsilon})$$

where *e.g.*

$$f_1 = N^*[-(W - m_p)\mathcal{A}_1 + (k\cdot p)\mathcal{A}_4 + (k\cdot p_\Lambda)\mathcal{A}_5 - k^2\mathcal{A}_6]$$

- Response functions, *e.g.* transverse cross section

$$\frac{d\sigma}{d\Omega} = \sigma_T = C\left\{|f_1|^2 + |f_2|^2 - 2\,\mathrm{Re}\, f_1 f_2^* \cos\theta_K + \sin^2\theta_K\left[\frac{1}{2}(|f_3|^2 + |f_4|^2) + \mathrm{Re}\,(f_1 f_4^* + f_2 f_3^* + f_3 f_4^* \cos\theta_K)\right]\right\},$$

(for other response functions see Z. Phys. A **352** (1995) 327)

Isobar model
Hadronic form factors

Hadrons have inner structure, vertices thus cannot be treated as point-like interactions.

Hadronic form factors:

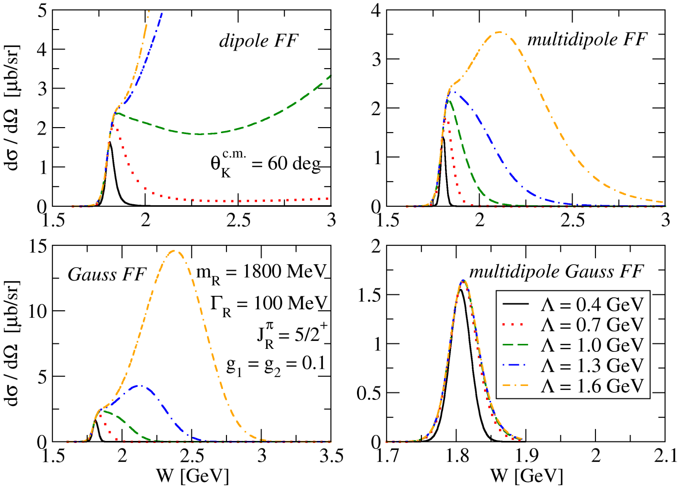
- dipole hff: $F_d(x) = \frac{\Lambda^4}{\Lambda^4 + (x - m_x^2)^2}, \quad x = s, t, u$
- multidipole hff (PR C **93**, 025204 (2016)): $F_{md}(x) = F_d^{J+1/2}(x)$
- Gaussian hff: $F_G(x) = \exp\left(-\frac{(x - m_x^2)^2}{\Lambda^4}\right)$
- multidipole-Gaussian hff (PR C **84**, 045201 (2011)):

$$F_{\text{mdG}}(x) = \left[\frac{m_x^2 \tilde{\Gamma}^2}{(x - m_x^2)^2 + m_x^2 \tilde{\Gamma}^2}\right]^{J-1/2} F_G(x), \quad \tilde{\Gamma}(J) = \frac{\Gamma}{\sqrt{2^{1/2J} - 1}}$$

Hadronic form factor introduces a dependency on value of the cut-off parameter Λ.

Isobar model

Hadronic form factors: dependency on the cut-off parameter Λ



$$\mathbb{M}_{NBs}^{N^*(5/2)} \sim q^4 \frac{\not{q} + m_R}{q^2 - m_R^2 + i m_R \Gamma_R} \mathcal{P}^{(5/2)}_{\mu\nu,\lambda\rho}(q)\, \mathcal{O}^{\mu\nu,\lambda\rho}_{5/2}$$

Isobar model

Novel features of our isobar model

Exchanges of high-spin resonant states

- non physical lower-spin components removed by appropriate choice of $\mathcal{L}_{int}$

$$V_S^{\mu}\, \mathcal{P}^{(1/2)}_{ij,\mu\nu}\, V_{EM}^{\nu} = 0$$

Energy-dependent decay widths of nucleon resonances → restoration of unitarity

$$\Gamma(\vec{q}) = \Gamma_{N^*} \frac{\sqrt{s}}{m_{N^*}} \sum_i x_i \left(\frac{|\vec{q}_i|}{|\vec{q}_i^{\,N^*}|} \right)^{2l+1} \frac{D(|\vec{q}_i|)}{D(|\vec{q}_i^{\,N^*}|)},$$

Extension from photoproduction to electroproduction

- Phenomenological form factors in the electromagnetic vertex
- Longitudinal couplings of N^*'s to γ^* (crucial at small Q^2)

$$V^{EM}(N^*_{1/2} p\gamma) = -i \frac{g_3^{EM}}{(m_R + m_p)^2} \Gamma_{\mp}\, \gamma_\beta\, \mathcal{F}^{\beta},$$

$$V^{EM}_{\mu}(N^*_{3/2} p\gamma) = -i \frac{g_3^{EM}}{m_R (m_R + m_p)^2} \gamma_5 \Gamma_{\mp} \left(\not{q}\, g_{\mu\beta} - q_\beta \gamma_\mu \right) \mathcal{F}^{\beta},$$

$$V^{EM}_{\mu\nu}(N^*_{5/2} p\gamma) = -i \frac{g_3^{EM}}{(2m_p)^5} \Gamma_{\mp} (q_\alpha q_\beta g_{\mu\nu} + q^2 g_{\alpha\mu} g_{\beta\nu} - q_\alpha q_\nu g_{\beta\mu} - q_\beta q_\nu g_{\alpha\mu})\, p^{\alpha} \mathcal{F}^{\beta}.$$

Fitting procedure

Minimization of χ^2/n.d.f. with help of MINUIT code

Resonance selection

- *s* channel: spin-1/2, 3/2, and 5/2 N^* with mass < 2.5 GeV;
- *t* channel: $K^*(892)$, $K_1(1272)$
- *u* channel: $Y^*(1/2)$ and $Y^*(3/2)$

Free parameters ($\approx 30 + 10$):

- SU(3)$_f$: $-4.4 \leq g_{K\Lambda N}/\sqrt{4\pi} \leq -3.0$, $0.8 \leq g_{K\Sigma N}/\sqrt{4\pi} \leq 1.3$
- K^*'s have vector and tensor couplings
- spin-1/2 resonance $\rightarrow$ 1 parameter; spin-3/2 and 5/2 resonance $\rightarrow$ 2 parameters
- 2 cut-off parameters for the hff
- 1 longitudinal coupling for each N^*
- 2 cut-off parameters for the emff of K^* and K_1

Experimental data

3383 $p(\gamma, K^+)\Lambda$ data

- cross section for $W < 2.355$ GeV (CLAS 2005 & 2010; LEPS, Adelseck-Saghai)
- hyperon polarisation for $W < 2.225$ GeV (CLAS 2010)
- beam asymmetry (LEPS)

171 $p(e, e'K^+)\Lambda$ data

- $\sigma_U, \sigma_T, \sigma_L, \sigma_{LT'}, \sigma_K$

Results of the fitting procedure

Solutions: BS1 and BS2, $\chi^2/\text{n.d.f.} = 1.64$ $(p(\gamma, K^+)\Lambda)$

BS3, $\chi^2/\text{n.d.f.} = 1.74$ $[p(\gamma, K^+)\Lambda$ $(\chi^2/\text{n.d.f.} = 1.51)$ and $p(e, e'K^+)\Lambda]$

- χ^2's, fitted parameter values (smallness) and correspondence with data taken into account
- sets of chosen Y^* differ in all BS models $\rightarrow$ different description of background
- electromagnetic form factors of K^* and K_1: crucial for $Q^2 > 1\ (\text{GeV}/c)^2$

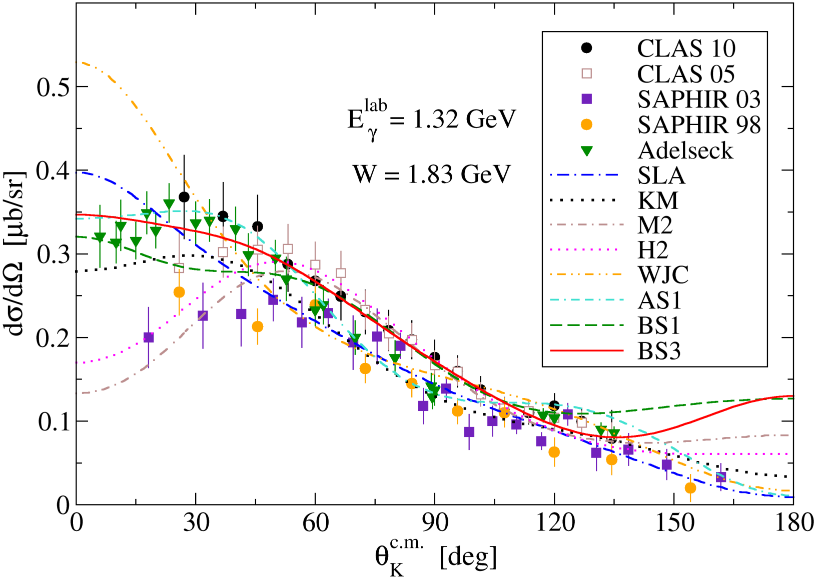
BS1 model

- $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $F_{15}(1860)$, $D_{13}(1875)$, $F_{15}(2000)$;
- $K^*(892)$, $K_1(1272)$;
- $\Lambda(1520)$, $\Lambda(1800)$, $\Lambda(1890)$, $\Sigma(1660)$, $\Sigma(1750)$, $\Sigma(1940)$;
- multidipole form factor: $\Lambda_{bgr} = 1.88$ GeV, $\Lambda_{res} = 2.74$ GeV

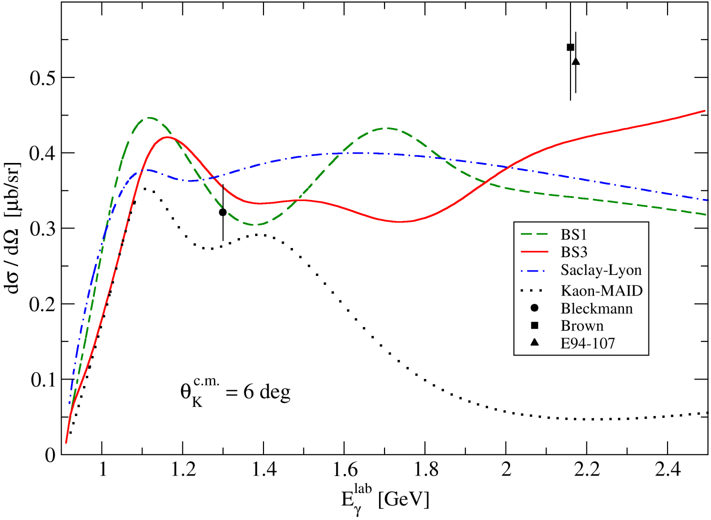
BS3 model

- $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{11}(1710)$, $P_{13}(1720)$, $F_{15}(1860)$, $D_{13}(1875)$, $P_{13}(1900)$, $F_{15}(2000)$, $D_{13}(2120)$;
- $K^*(892)$, $K_1(1272)$;
- $\Lambda(1405)$, $\Lambda(1600)$, $\Lambda(1890)$, $\Sigma(1670)$;
- dipole form factor: $\Lambda_{bgr} = 1.24$ GeV, $\Lambda_{res} = 0.89$ GeV

Angular dependence of the cross section for $p(\gamma, K^+)\Lambda$

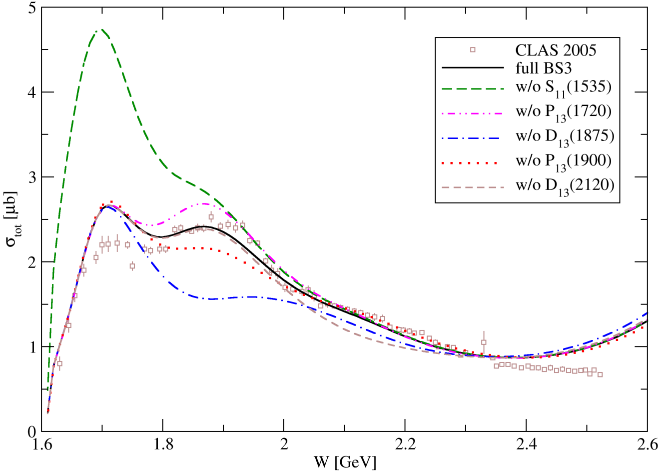


Predictions of $d\sigma/d\Omega$ for $p(\gamma, K^+)\Lambda$ at $\theta_K^{c.m.} = 6^\circ$



- Brown [$Q^2 = 0.18\,(\text{GeV}/c)^2$] & E94-107 [$Q^2 = 0.07\,(\text{GeV}/c)^2$]:
  data for $p(e, e' K^+)\Lambda$ but: $\sigma_L \sim Q^2$, $\sigma_{TT} \sim \sin^2\theta_K^{c.m.}$, and $\sigma_{LT} \sim \sqrt{Q^2}\sin\theta_K^{c.m.} \Rightarrow \sigma \approx \sigma_T$

Total cross section prediction of the BS3 model



	$\Delta\chi^2$ [%]		$\Delta\chi^2$ [%]
$S_{11}(1535)$	331	$P_{13}(1900)$	826
$S_{11}(1650)$	81	$F_{15}(2000)$	30
$P_{11}(1710)$	43	$D_{13}(1875)$	844
$P_{13}(1720)$	188	$F_{15}(1860)$	82
$F_{15}(1680)$	202	$D_{13}(2120)$	125

$$\Delta\chi^2 = \frac{\chi^2_{N*} - \chi^2}{\chi^2} \cdot 100\%,$$

New fits for $K^+\Sigma^-$ channel

χ^2 minimization and overfitting

Fitting procedure with MINUIT library: **minimizing the χ^2**

$$\chi^2 = \sum_{i=1}^{N} \frac{[d_i - p_i(c_1, \ldots, c_n)]^2}{\sigma_{d_i}^2},$$

$(c_1, \ldots, c_n)$ - set of free parameters, $(d_1, \ldots, d_N)$ - set of data points, p_i - theory, σ_{d_i} - error

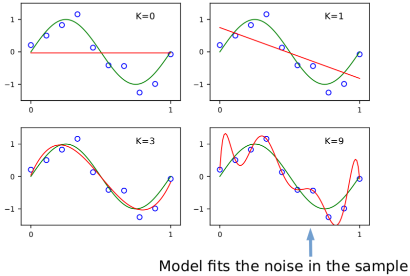
Problem: χ^2 minimization cannot prevent overfitting

Example: polynomial curve fitting

- $f(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \cdots + w_k x^k$
- increasing order of polynomial k fits the data well...

  ...but gives only poor description of the function which generated them...

  ...and may fail to generalize to new data
- **Occam's razor** (law of parsimony): simpler models should be preferred



New fits of $K^+\Sigma^-$ channel

Least Absolute Shrinkage and Selection Operator (LASSO)

Remedy to the overfitting issue: regularization

- introduce a penalty term to the χ^2 $\rightarrow$ penalization of large parameter values
- penalized χ^2_T: $\chi^2_T = \chi^2 + P(\lambda)$
- penalty term: $P(\lambda) = \lambda^4 \sum_{i=1}^{N_{res}} |g_i|$

  λ - regularization parameter, g_i - resonances' couplings
- LASSO forces some of the parameters to zero

  $\rightarrow$ selection of a subset of the fit parameters
- λ controls the strength of the penalty and thus the complexity of the model

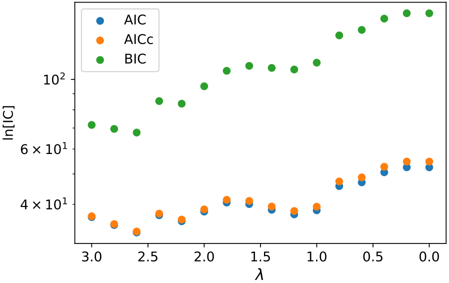
  $\rightarrow$ higher powers of λ allow fine sampling of the region of small λ

New fits of $K^+\Sigma^-$ channel

Information criteria:

- Akaike information criterion
  $\text{AIC} = 2n_i + \chi^2_T$
- Corrected Akaike information criterion
  $\text{AICc} = \text{AIC} + \frac{2n_i(n_i+1)}{N-n_i-1}$
- Bayesian information criterion
  $\text{BIC} = n_i \ln(N) + \chi^2_T$

  n_i - no. of parameters corresponding to λ_i
  N - number of data points



Applying the information criteria – forward selection

1. start with the full model: parameters initialized within $\langle -1; +1 \rangle$; use λ_{max}
2. perform LASSO χ^2_T minimization and compute IC
3. in each run reduce λ and run LASSO with the values of the previous run as starting values
4. repeat until λ_{min} is reached

Optimal λ occurs at the minimum of the IC

New fits of $K^+\Sigma^-$ channel

Fitting procedure

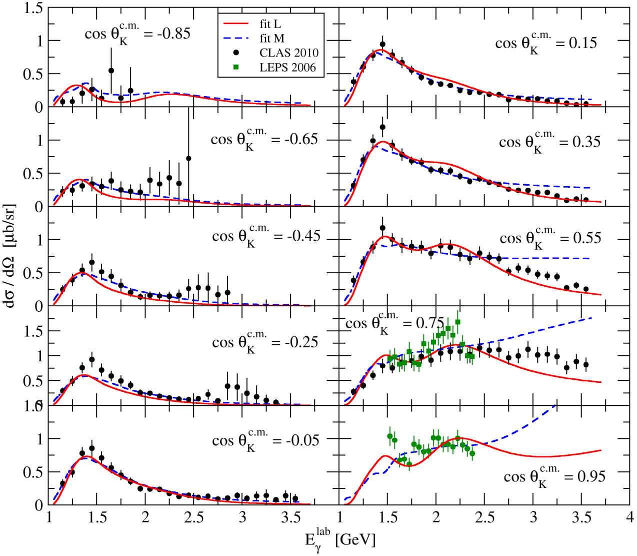
- non resonant part: Born terms and exchanges of K^* and K_1 and Σ^*'s
- resonant part: exchanges of N^*'s and Δ^*'s in the s channel
- around 600 data utilized to fit $\leq$ 25 parameters
- result with the smallest $\chi^2/\text{ndf} = 2.3 \rightarrow$ **fit M** (25 parameters, 14 resonances)
- **LASSO method** used: $\chi^2_T/\text{ndf} = 3.4 \rightarrow$ **fit L** (17 parameters, 9 resonances)

Characteristics of models

- only one Δ resonance introduced
- no hyperon resonances needed for reliable data description
- results in very good agreement with the cross-section and beam-asymmetry data
- fit L is very economical

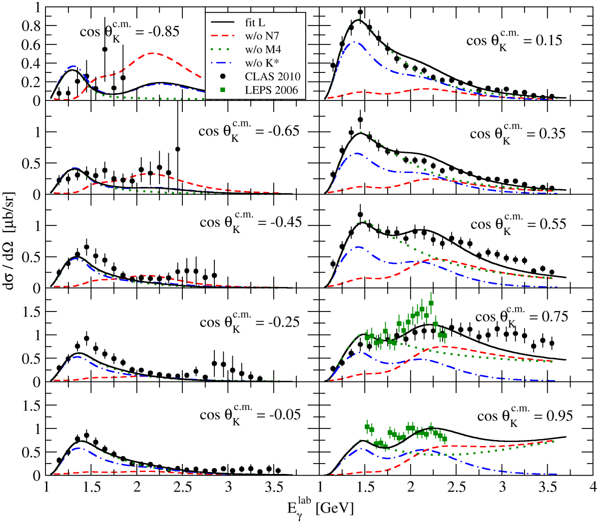
New fits of $K^+\Sigma^-$ channel

Differential cross section in dependence on the photon lab energy



New fits of $K^+\Sigma^-$ channel

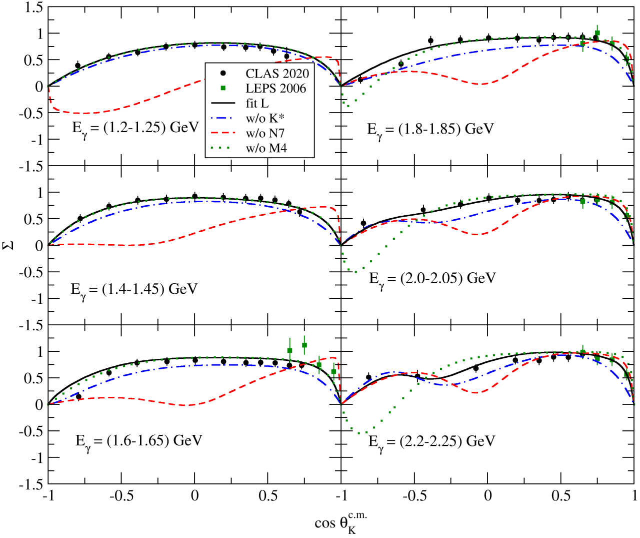
Differential cross section in dependence on the photon lab energy - fit L w/o individual resonances



N7: N(1720)$3/2^+$, M4: N(2060)$5/2^-$

New fits of $K^+\Sigma^-$ channel

Beam asymmetry in dependence on the kaon center-of-mass angle - fit L w/o individual resonances



Refitting the model's parameters in the $K^+\Lambda$ channel

Ridge regression and cross validation for suppressing hyperon couplings

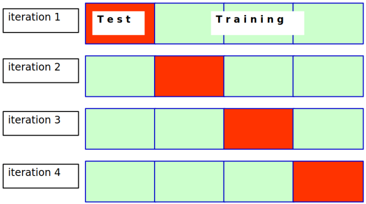
Why refit?

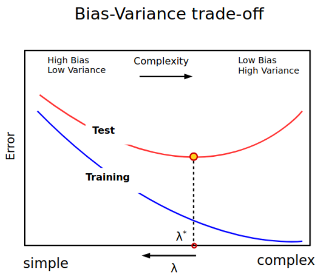
- include recent measurements of polarization observables
- need to investigate more the role of hyperon resonances in *KY* photoproduction
- large values of hyperon couplings: ridge regression to suppress them during the fitting procedure

Ridge regularization

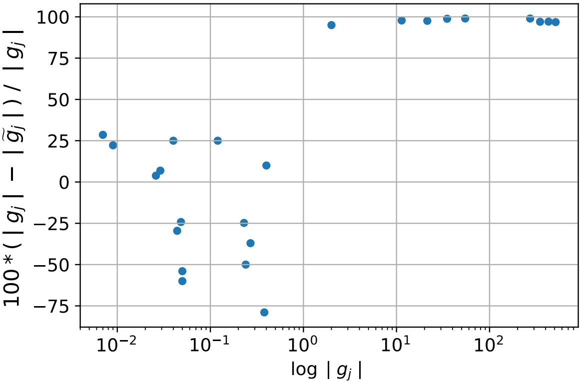
- penalized χ^2_T: $\chi^2_T = \chi^2 + \lambda^4 \sum_{i=1}^{n_\Lambda} g_i^2$, ($n_\Lambda$ = no. of Y couplings)
- parameter values reduced but they are *not* reduced to zero

Cross validation





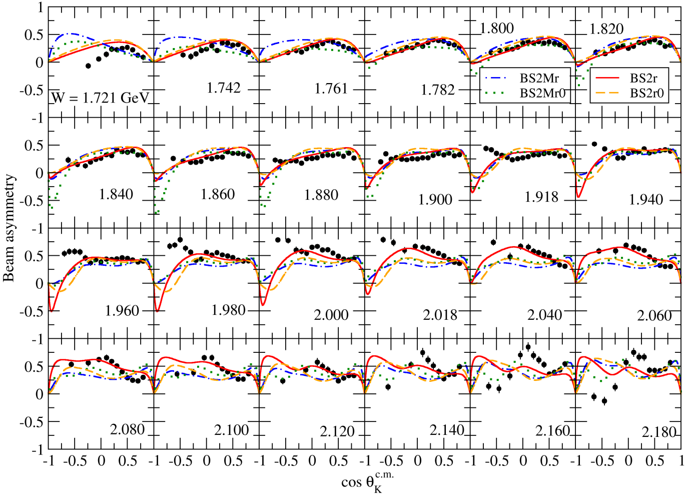
Relative percentage reduction of the resonance couplings



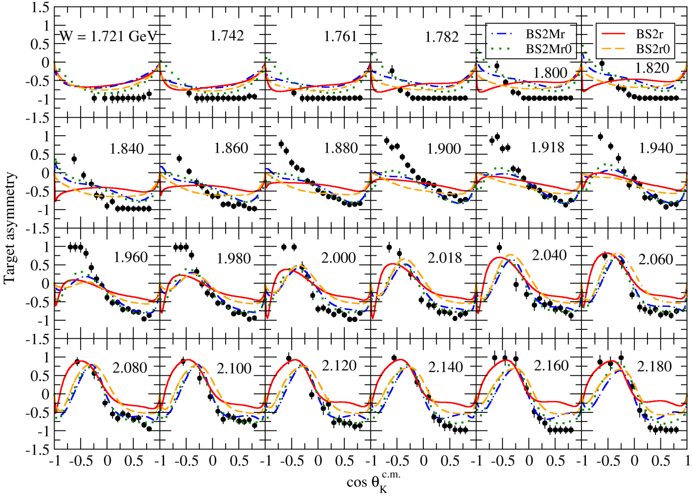
g_j - values from the unregularized fitting

$\tilde{g}_j$ - values after performing Ridge regularization

$K^+\Lambda$ channel: beam asymmetry Σ



$K^+\Lambda$ channel: target asymmetry T



Summary

New version of isobar model and RPR model for the $K^+\Lambda$ channel

- available for calculations online at:

  `http://www.ujf.cas.cz/en/departments/department-of-theoretical-physics/isobar-model.html`

Description extended from the $K^+\Lambda$ channel to the $K^+\Sigma^-$ channel.

Regularization methods (LASSO, ridge) introduced as a remedy for overfitting
(more details will be given in the talk by D. Petrellis).

Outlook

- testing the models in the DWIA calculations for hypernucleus production
- performing a multi-channel analysis of all Σ photoproduction channels
- extending the analysis of electroproduction beyond $Q^2 = 1\,\text{GeV}^2$
- studying the production of Ξ hypernuclei

Thank you for your attention!