Revised decay correction and uncertainty analysis

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Definitions

Use the following variables:

Y_{raw}	raw yield
Y_T (Y_H)	tritium (helium) yield with deadtime correction
T (H)	number of good electrons from tritium (helium) nuclei
n_T (n_H)	number density of tritium (helium) nuclei in target
Q	charge on target
$n = n_T + n_H$	total number of nuclei in target (constant)
$c = n_H/n$	contamination factor (fraction of helium nuclei in target)

Revised correction

Previous correction (neglecting run dependence of contamination factor):

$$Y_{raw} = \frac{T + H}{Q(n_T + n_H)} \qquad \rightarrow \qquad \frac{Y_T}{Y_H} = \frac{Y_{raw}}{Y_H}\left(\frac{1}{1-c}\right) - \left(\frac{c}{1-c}\right)$$

Revised correction:

$$Y_{raw} = \frac{\sum_i (T_i + H_i)}{\sum_i Q_i(n_{T_i} + n_{H_i})} \qquad \rightarrow \qquad \frac{Y_T}{Y_H} = \frac{Y_{raw}}{Y_H}\left(\frac{Q_{tot}}{Q_{tot} - \langle c \rangle}\right) - \left(\frac{\langle c \rangle}{Q_{tot} - \langle c \rangle}\right)$$

where

$$Q_{tot} \equiv \sum_i Q_i \qquad \text{and} \qquad \langle c \rangle \equiv \sum_i Q_i c_i$$

Contamination uncertainty

The contamination factor is defined as:

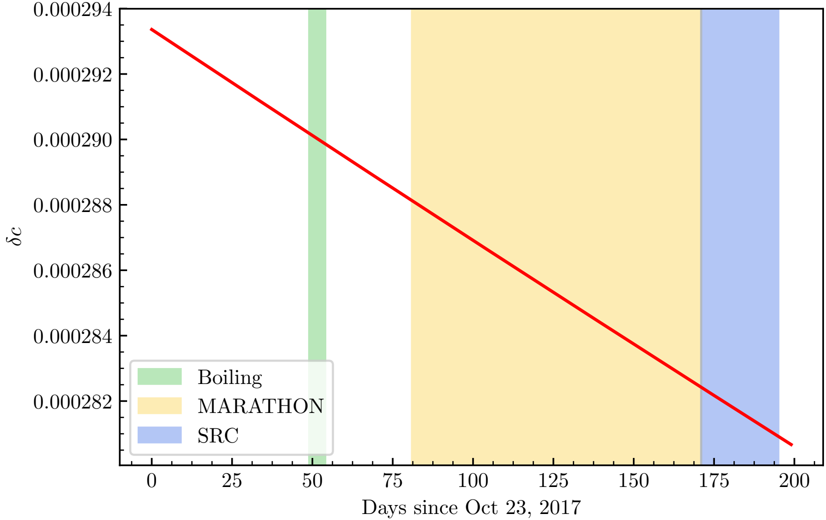
$$c = \frac{n_H}{n_T + n_H} = \frac{n_H}{n_{tot}}$$

The uncertainty in the contamination factor is:

$$\begin{aligned}
\delta c^2 &= \left(\frac{\partial c}{\partial n_T^0} \delta n_T^0 \right)^2 + \left(\frac{\partial c}{\partial n_H^0} \delta n_H^0 \right)^2 \\
&= \left[\left(\frac{1 - e^{-t/\tau}}{n_{tot}} - \frac{n_H}{(n_{tot})^2} \right) \delta n_T^0 \right]^2 + \left[\left(\frac{1}{n_{tot}} - \frac{n_H}{(n_{tot})^2} \right) \delta n_H^0 \right]^2
\end{aligned}$$

$$\delta c \to 0 \text{ as } t \to \infty,\ n_H \to n_{tot}$$

Contamination uncertainty over run period



Cumulative contamination uncertainty

Uncertainty in yield ratio

$$\frac{Y_T}{Y_H} = \frac{Y_{raw}}{Y_H}\left(\frac{Q_{tot}}{Q_{tot} - \langle c \rangle}\right) - \left(\frac{\langle c \rangle}{Q_{tot} - \langle c \rangle}\right)$$

will include terms proportional to $\delta\langle c \rangle$, which can be found by:

$$\delta\langle c \rangle = \sqrt{\sum_j \left(\frac{\partial\langle c \rangle}{\partial c_j}\delta c_j\right)^2}$$

$$= \sqrt{\sum_j \left(\left(\frac{\partial}{\partial c_j}\sum_i Q_i c_i\right)\delta c_j\right)^2}$$

$$= \sqrt{\sum_j (Q_j\ \delta c_j)^2}$$