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# Impact of the $\pi\Sigma$ photoproduction data on the $\Lambda(1405)$ poles analysis

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based on collaboration with P. C. Bruns, partly also with M. Mai (Bonn)

# Introduction

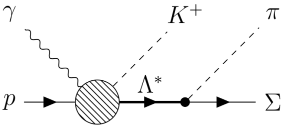
the enigmatic nature of $\Lambda(1405)$ keeps our interest for more than 60 years

- in 1959 Dalitz and Tuan predicted a subthreshold resonance in their K-matrix analysis of $K^- p$ data; confirmed 2 years later in the $\pi\Sigma$ mass spectra in the $K^- p \longrightarrow \pi\pi\pi\Sigma$ reaction
- $\Lambda(1405)\,1/2^-$ is much lighter than $N^*(1535)$ and a potential spin-orbit partner $\Lambda(1520)\,3/2^-$ which is difficult to explain within a standard constituent quark model
- hadronic molecule, a loosely bound $\bar{K}N$ state? a pentaquark?
- most common interpretation - $\bar{K}N$ quasi-bound state submerged in $\pi\Sigma$ continuum, a result of coupled channels $\pi\Sigma - \bar{K}N$ dynamics
- unitary coupled channels approaches based on effective chiral Lagrangian generate two poles related to $\Lambda(1405)$ (Oller, Meißner in 2001)
- model parameters fitted to experimental data available at energies from $K^- p$ threshold up, providing varied theoretical predictions for subthreshold energies and in the isovector sector

# Introduction

- In the $K^- p$ reactions data, the $\Lambda(1405)$ is hidden below the threshold. The resonance can be seen in processes, where $\pi\Sigma$ re-scatter in the final state, e.g.

$$\gamma p \longrightarrow K^+ \pi\Sigma$$




In this two meson protoproduction reaction the $K^+$ meson carries away momentum, enabling a scan in the invariant mass of the $\pi\Sigma$ system down to its production threshold.

- topics related to $\Lambda(1405)$ include $\bar{K}$-nuclei and role of strangeness in dense nuclear matter (e.g. neutron stars)

More in reviews:

T. Hyodo, D. Jido - Prog. Part. Nucl. Phys. 67 (2012) 55

M. Mai - Eur. Phys. J. Spec. Top. 230 (2021) 6, 1593

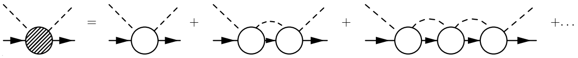
# Chirally motivated $\bar{K}N$ interactions

$\bar{K}N - \pi\Sigma$ system (+ add-ons, mostly more $MB$)

meson octet - baryon octet coupled channels interactions

| | | | | | | |
|---|---|---|---|---|---|---|
| involved channels | $\pi\Lambda$ | $\pi\Sigma$ | $\bar{K}N$ | $\eta\Lambda$ | $\eta\Sigma$ | $K\Xi$ |
| thresholds (MeV) | 1250 | 1330 | 1435 | 1660 | 1740 | 1810 |

- strongly interacting multichannel system with an s-wave resonance, the $\Lambda(1405)$, just below the $K^- p$ threshold
- modern theoretical treatment based on effective chiral Lagrangians
- effective potentials constructed to match the chiral meson-baryon amplitudes up to LO or NLO order
- Lippmann-Schwinger (or Bethe-Salpeter) equation to sum the major part of the perturbation series

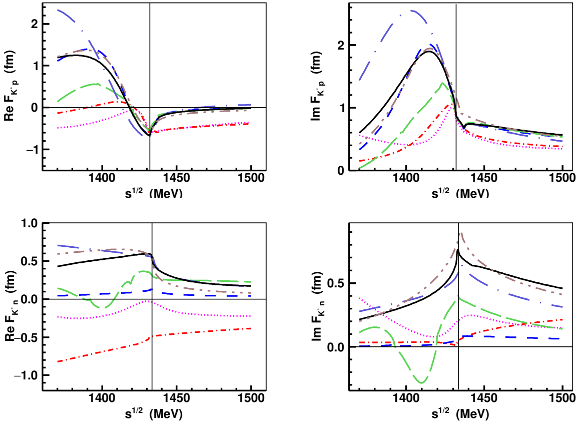


N. Kaiser, P.B. Siegel, W. Weise - Nucl. Phys. A 594 (1995) 325

$K^- p$ data fits: low energy x-sections, threshold BRs, kaonic hydrogen 1s level

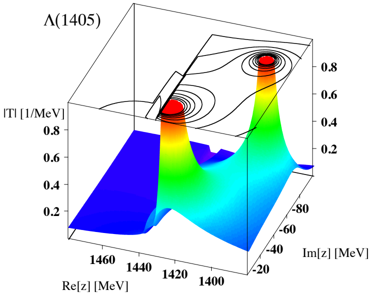
# Model predictions - $K^- N$ amplitudes

$K^- p$ and $K^- n$ elastic amplitudes

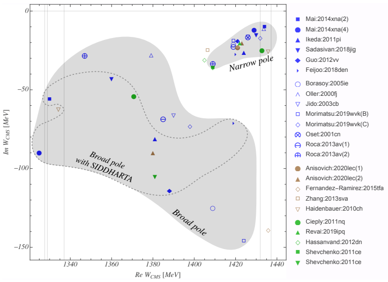



$B_2$ (dotted, purple), $B_4$ (dot-dashed, red), $M_I$ (dashed, blue), $M_{II}$ (long-dashed, green), $P$ (dot-long-dashed, violet), $BCN$ (dot-dot-dashed, brown), $KM$ (continuous, black)

# Model predictions - Λ(1405) resonance




Hyodo, Jido - Prog. Part. Nucl. Phys. 67 (2012) 55




all recent (year $\geq$ 2000) predictions

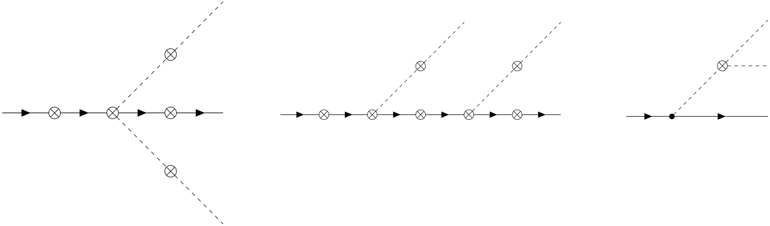
M. Mai - Eur. Phys. J. Spec. Top. 230 (2021) 6, 1593

- the *higher* pole around 1425 MeV couples more strongly to $\bar{K}N$, the *lower* pole is much further from the real axis and has larger coupling to $\pi\Sigma$
- illustrative picture: $\bar{K}N$ bound state submerged in $\pi\Sigma$ continuum (W. Weise)
- all models tend to agree on the position of the $\bar{K}N$ related pole
- the $K^-p$ reactions data are not very sensitive to the position of the $\pi\Sigma$ related pole

# $\pi\Sigma$ photoproduction: Formalism

formalism outlined in: P. C. Bruns - arXiv:2012.11298 [nucl-th] (2020)
application to $\pi\Sigma$ mass spectra predictions:
P. C. Bruns, A. C., M. Mai - Phys. Rev. D 106 (2022) 7, 074017

leading-order BChPT used to derive expressions for the photoproduction amplitude $\mathcal{M}$, constructed from tree level graphs:



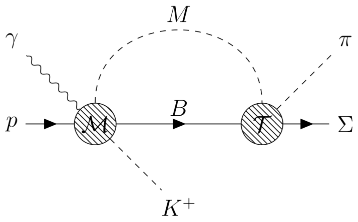
Weinberg-Tomozawa (WT) Born term (BT) - B1, B2 anomalous (AN)

lines: directed - baryons, dashed - pseudoscalar mesons; $\otimes$ symbols - photon insertions

$5 + (2 \times 7) + 1 = 20$ tree graphs, 16 independent $\mathcal{M}_j$ structure functions

# $\pi\Sigma$ photoproduction: Formalism

Final state interaction of the $MB$ pair needs to be accounted for:




$\pi\Sigma - \bar{K}N$ coupled channels models provide the $f^{c',c}_{\ell\pm}(M_{\pi\Sigma})$ amplitudes, that describe the scattering from channel $c$ to channel $c'$ ($c, c' = \pi\Lambda$, $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda, \ldots$)

state-of-the-art approaches based on LO+NLO ChPT, complying with unitarity

$$\mathrm{Im}(f_{\ell\pm}) = (f_{\ell\pm})^{\dagger}(|\vec{p}^{\,*}|)(f_{\ell\pm}) \; ,$$

our aim: implement $f_{0+}$ amplitudes to describe $MB$ s-wave pairs produced in $\gamma p \to K^+ MB$ photoproduction

We need to project the $\mathcal{M}$ amplitude on the $MB_{\ell=0}$ state ...

# $\pi\Sigma$ photoproduction: Formalism

There are four independent structure functions $\mathcal{A}^i_{0+}(s, M^2_{\pi\Sigma}, t_K)$ constructed from $\mathcal{M}_j$, projected on s-wave and satisfying the partial-wave unitarity relation

$$\mathrm{Im}(\mathcal{A}^i_{0+}) = (f_{0+})^\dagger(|\vec{p}^{\,*}|)(\mathcal{A}^i_{0+})\,, \quad i = 1, \ldots, 4\,.$$

Neglecting $\ell > 0$ contributions, we get

$$\frac{d^2\sigma}{d\Omega_K dM_{\pi\Sigma}} = \frac{|\vec{q}_K||\vec{p}^{\,*}_\Sigma|}{(4\pi)^4 s|\vec{k}|}|\mathcal{A}|^2\,,$$

$$\begin{aligned}
4|\mathcal{A}|^2 = {} & (1-z_K)\left|\mathcal{A}^1_{0+} + \mathcal{A}^2_{0+}\right|^2 + (1+z_K)\left|\mathcal{A}^1_{0+} - \mathcal{A}^2_{0+}\right|^2 \\
& + (1-z_K)\left|\mathcal{A}^1_{0+} + \mathcal{A}^2_{0+} + \frac{2|\vec{q}_K|(1+z_K)}{M^2_K - t_K}\left((\sqrt{s}+m_N)\mathcal{A}^3_{0+} + (\sqrt{s}-m_N)\mathcal{A}^4_{0+}\right)\right|^2 \\
& + (1+z_K)\left|\mathcal{A}^1_{0+} - \mathcal{A}^2_{0+} - \frac{2|\vec{q}_K|(1-z_K)}{M^2_K - t_K}\left((\sqrt{s}+m_N)\mathcal{A}^3_{0+} - (\sqrt{s}-m_N)\mathcal{A}^4_{0+}\right)\right|^2,
\end{aligned}$$

with $z_K \equiv \cos\theta_K$, $\theta_K$ being the angle between $\vec{q}_K$ and $\vec{k}$ in the overall c.m. frame.

# $\pi\Sigma$ photoproduction: Formalism

unitarized amplitudes for $\gamma p \to K^+ MB$ will be taken as the coupled-channel vector

$$[\mathcal{A}^i_{0+}] = [\mathcal{A}^{i(\mathrm{tree})}_{0+}] + [f_{0+}]\,[8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})]\,[\mathcal{A}^{i(\mathrm{tree})}_{0+}]$$

The second term represents the final-state $MB$ rescattering and $G(M_{\pi\Sigma})$ is a diagonal channel-space matrix with entries given by regularized loop integrals

$$\mathrm{i}\, G^{c=MB}(M_{\pi\Sigma}) = \int_{\mathrm{reg.}} \frac{d^4 l}{(2\pi)^4} \frac{1}{((p_\Sigma + q_\pi - l)^2 - m_B^2 + \mathrm{i}\epsilon)(l^2 - M_M^2 + \mathrm{i}\epsilon)}$$

two coupled channels approaches for the $f_{0+}$ amplitudes:

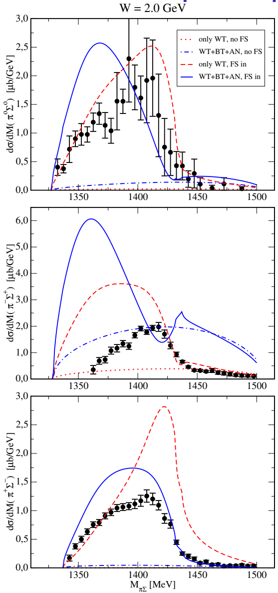
Bonn B2, B4 models - M.Mai, U.-G.Meißner, Eur. Phys. J. A 51 (2015) 30
BW model - D.Sadasivan, M.Mai, M.Döring, Phys. Lett. B 789 (2019) 329–335
dimensional regularization used in $G(M_{\pi\Sigma})$, mass scales $\mu_c$

Prague P model - P.C.Bruns, A.C., Nucl. Phys. A 1019 (2022) 122378
Yamaguchi form factors used in $G(M_{\pi\Sigma})$, inverse ranges $\alpha_c$

CLAS data: K. Moriya et al. - Phys. Rev. C 87 (2013) 035206

Result: different models provide varied predictions of the $\pi\Sigma$ mass spectra

# $\pi\Sigma$ photoproduction: FSI impact




CLAS data (2013) by Moriya et al.

c.m. energy $W = \sqrt{s} = 2.0$ GeV

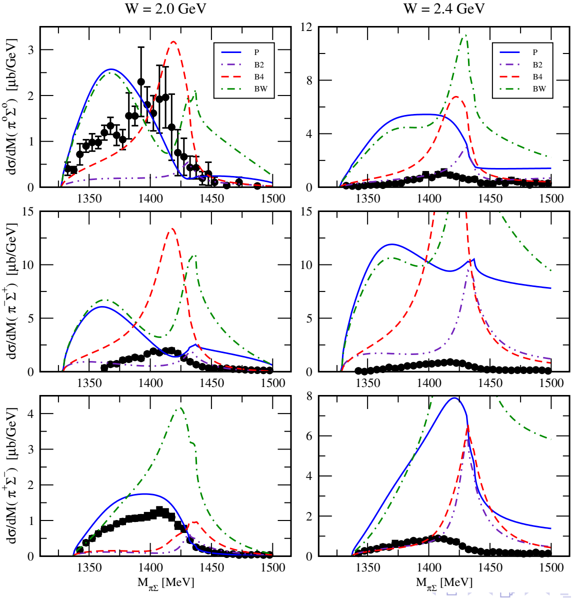
P model used for the *MB* amplitudes

- only WT, no FSI:
  small (or zero for $\pi^+\Sigma^-$) cross sections
- WT+BT+AN, no FSI:
  the cross sections remain flat, the $\pi^-\Sigma^+$ one reaches magnitude comparable with the data
- addition of FSI:
  *MB* rescattering is responsible for the peak structure
  $\pi^0\Sigma^0$ and $\pi^+\Sigma^-$ reproduced rather well
  Born terms move the peak to lower energies

parameter-free predictions!

no adjustment to the $f_{0+}$ amplitudes

# $\pi\Sigma$ mass spectra - model dependence



# Results - FSI not fixed

First time fits to combined $K^- p$ reactions and $\pi\Sigma$ photoproduction data with the FSI no longer fixed at a particular $\pi\Sigma - \bar{K}N$ model parameters setup.
A. C., P. C. Bruns - submitted to NPA, arXiv:2305.06205

- The P model used for the $MB \to \pi\Sigma$ amplitude represented by the $MB$ rescattering $\mathcal{T}$ bubble:
  6 regularization scales $\alpha_c$, 6 NLO couplings ($b_0$, $b_F$, $d_{1\ldots4}$)
- Tree level photoproduction amplitudes representing the $\mathcal{M}$ bubble multiplied by $MB$ and $K^+$ form factors: 6 regularization scales $\beta_c$ (3 fixed), and $\beta_K$
- Yamaguchi forms adopted for all form factors $g_c(k^*) = 1/[1 + (k^*/\alpha_c)^2]$ etc.
- fitted data: kaonic hydrogen characteristics, $K^- p$ threshold branching ratios, $K^- p$ reaction cross sections, $\pi\Sigma$ photoproduction mass distributions at $\sqrt{s} = 2.1$ GeV

| CLAS data | | | $K^- p$ threshold | | $K^- p$ cross sections | | | | | | | | all |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi^0\Sigma^0$ | $\pi^-\Sigma^+$ | $\pi^+\Sigma^-$ | atom | BRs | $\pi^0\Lambda$ | $\pi^0\Sigma^0$ | $\pi^-\Sigma^+$ | $\pi^+\Sigma^-$ | $K^- p$ | $\bar{K}^0 n$ | $\eta\Lambda$ | $\eta\Sigma^0$ | total |
| 34 | 30 | 32 | 2 | 3 | 4 | 4 | 31 | 32 | 27 | 22 | 24 | 7 | 252 |

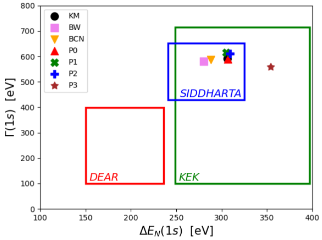
$$\chi^2/\mathrm{dof} = \frac{\sum_i N_i}{N_{obs}(\sum_i N_i - N_{par})} \sum_i \frac{\chi_i^2}{N_i}\,, \quad N_{obs} = 13\,, \quad N_{par} = 16$$

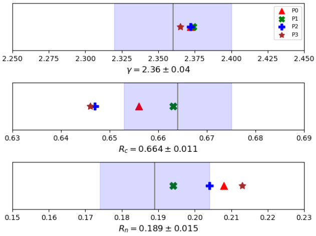
# Results - FSI not fixed

results for 4 selected solutions (local $\chi^2$ minima):

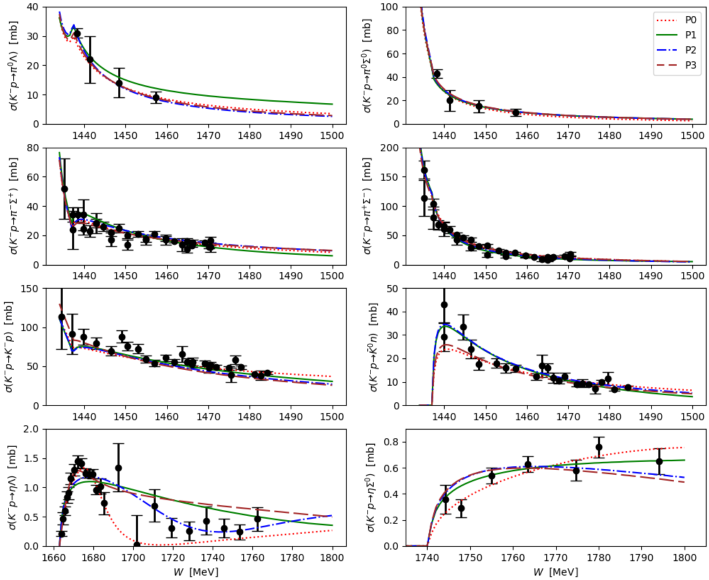
- P0 $\chi^2/\mathrm{dof} \approx 5.40$, *MB* FSI sector fixed to the P model setting, 4 parameters
- P1 $\chi^2/\mathrm{dof} \approx 3.34$, both, FSI and tree level photoproduction sectors varied, 16 par.
- P2 $\chi^2/\mathrm{dof} \approx 4.41$, same as for P1
- P3 $\chi^2/\mathrm{dof} \approx 4.72$, same as for P1

## $K^- p$ threshold data:

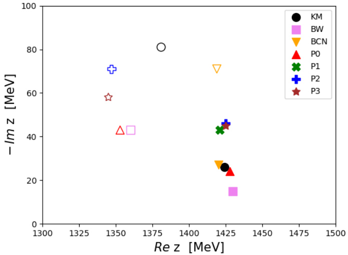


# $K^- p \to MB$ total cross sections



# $\Lambda(1405)$ poles predictions

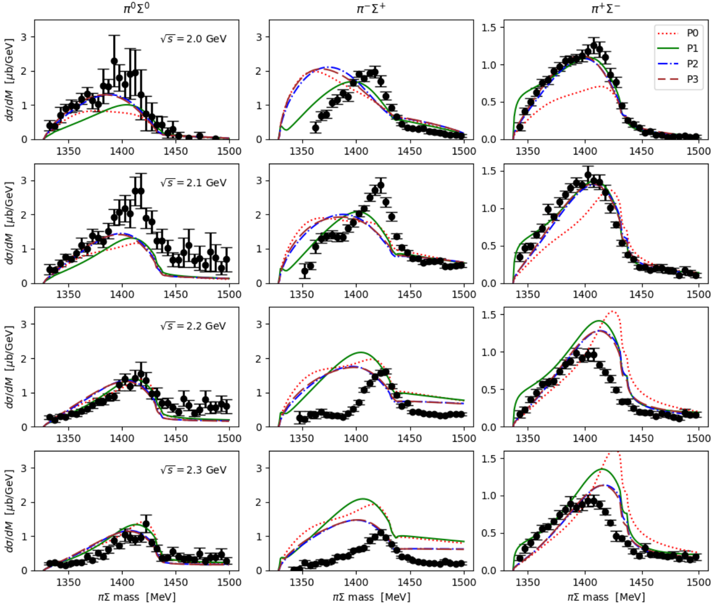



$\Lambda(1405)$ - $z_1$ and $z_2$; $\Lambda(1670)$ - $z_3$

| model | $z_1$ [MeV] | $z_2$ [MeV] | $z_3$ [MeV] |
|---|---|---|---|
| P0 | (1353,-43) | (1428,-24) | (1677,-14) |
| P1 | — | (1421,-43) | — |
| P2 | (1347,-71) | (1425,-46) | (1725,-57) |
| P3 | (1345,-58) | (1425,-45) | (1665,-7.1) |

- The P1 model provides the best $\chi^2$ but was found unphysical due to generating an extremely narrow $I = 1$ resonance close to $\pi\Sigma$ threshold. It is also missing the *lower mass* $\Lambda(1405)$ and the $\Lambda(1670)$ poles.
- Apparently, the $K^- p$ reaction (and to some extend threshold) data are not very sensitive to the pole positions. Similar observations by Revai (with a non-chiral model), Shevchenko (phenomenology), or Anisovich (partial wave analysis), all indicating good reproduction of the $K^- p$ data with one-pole $\Lambda(1405)$ models.
- All our models agree on the position of the *higher mass* $\Lambda(1405)$ pole. The *lower mass* $\Lambda(1405)$ pole seems to be constrained around $z_1 \approx (1350, -60)$ MeV.
- The applied $\pi\Sigma$ photoproduction formalism may not be realistic: $p$-waves, vector meson contributions?

# $\pi\Sigma$ mass spectra predictions



# $\pi\Sigma$ mass spectra predictions

- Despite being rather simple, the model reproduces reasonably well the $\pi^0\Sigma^0$ and $\pi^-\Sigma^+$ mass distributions.
- The energy dependence seems to be under control thanks to introducing the $g_{K^+}$ form-factor in the tree level photoproduction amplitudes.
- Our model fails to reproduce the $\pi^-\Sigma^+$ mass distributions.
- Much better agreement with CLAS photoproduction data can be achieved either by an ad-hoc modelling of the photoproduction dynamics (Roca, Oset, Mai, Meißner) or by employing much more parameters, and keeping some of them energy dependent (Nakamura, Jido).
- Currently, we work on enhancing the photo-kernel by including processes involving vector mesons (in particular $K^*$) and decuplet baryons.

# Summary

- The up-to-date (NLO) chirally motivated $\pi\Sigma - \bar{K}N$ models provide very different predictions for the *MB* amplitudes at energies below $\bar{K}N$ threshold.
- The $\Lambda(1405)$ energy region can be accessed by studying processes involving $\pi\Sigma$ rescattering in the final state. The $\pi\Sigma$ photoproduction on protons represents such a process where the *MB* rescattering plays a crucial role as our results demonstrate.
- Our approach to the two-meson photoproduction implements coupled-channel unitarity, low-energy theorems from ChPT and gauge invariance. We have revealed large variations when different models for the *MB* amplitudes are adopted.
- Our new results of fits that combine the $K^- p$ reactions data with those on the $\pi\Sigma$ photoproduction can achieve a reasonable $\chi^2/\mathrm{dof}$ but are not satisfactory especially for the $\pi^-\Sigma^+$ mass distributions.
- The presented models tend to limit the mass of the *lower mass* $\Lambda(1405)$ pole and yield a larger width of the pole that couples more strongly to $\bar{K}N$. A possible relation to a large absorption width found for the $\bar{K}NN$ bound state at J-PARC?
- We need a more precise experimental data on low energy $K^- p$ reactions as the current ones are not too restrictive on the theoretical models. Our photoproduction model (its kernel) should also be improved.

# EXTRAS: Prague model

$$\mathcal{L}_{MB}^{(1)} = i\langle \bar{B}\gamma_\mu [D^\mu, B]\rangle - M_0 \langle \bar{B}B\rangle + i\frac{w_s}{F_0^2}\eta_0^2 \left(\langle [D^\mu, \bar{B}]\gamma_\mu B\rangle - \langle \bar{B}\gamma_\mu [D^\mu, B]\rangle\right)$$
$$+ \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle + \frac{1}{2}D_s\langle \bar{B}\gamma_\mu\gamma_5 B\rangle\langle u^\mu\rangle$$

two extra terms due to inclusion of the $\eta_0$ field:

- $\eta_0$ - baryon contact term proportional to $w_s$
- $\eta_0$ - baryon axial coupling term proportional to $D_s$

$$\mathcal{L}_{MB}^{(2)} = b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_F\langle \bar{B}[\chi_+, B]\rangle + b_0\langle \bar{B}B\rangle\langle \chi_+\rangle$$
$$+ d_1\langle \bar{B}\{u_\mu, [u^\mu, B]\}\rangle + d_2\langle \bar{B}[u_\mu, [u^\mu, B]]\rangle + d_3\langle \bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle \bar{B}B\rangle\langle u_\mu u^\mu\rangle$$
$$+ (\text{some more } c_{D,F,0} \text{ and } d_{5,6,7} \text{ terms})$$

$$c_{D,F,0} = d_{5,6,7} = 0$$

one-mixing-angle scheme ($\vartheta = -15.5^\circ$) to describe the singlet-octet mixing:

$$\eta_8 = \eta\cos\vartheta + \eta'\sin\vartheta\,, \quad \eta_0 = \eta'\cos\vartheta - \eta\sin\vartheta$$

# EXTRAS: Prague model

$$V_{ij}(k, k'; \sqrt{s}) = g_i(k^2)\, v_{ij}(\sqrt{s})\, g_j(k'^2)$$

$$v_{ij}(\sqrt{s}) = f_{0+,\mathrm{tree}}(s) = \frac{\sqrt{E_i + M_i}}{F_i} \left( \frac{C_{ij}(s)}{8\pi\sqrt{s}} \right) \frac{\sqrt{E_j + M_j}}{F_j}$$

- inter-channel energy dependent couplings $C_{ij}$ determined by the SU(3) chiral Lagrangian
- Yamaguchi form factors $g_j(k) = 1/[1 + (k/\alpha_j)^2]$ used to account naturally for the off-shell effects with inverse ranges $\alpha_j$ introduced as free model parameters

Lippmann-Schwinger equation used to solve exactly the loop series

$$f_{ij}(k, k'; \sqrt{s}) = g_i(k^2) \left[ (1 - v \cdot G(\sqrt{s}))^{-1} \cdot v \right]_{ij} g_j(k'^2)$$

The loop function $G(\sqrt{s})$ is diagonal in the channel space and is regularized by the Yamaguchi form factors.

# EXTRAS: Fitted parameters

| parameter | P0 | P1 | P2 | P3 |
|---|---|---|---|---|
| $b_0$ | $0.525^*$ | -0.833 | -0.596 | -0.387 |
| $b_F$ | $-0.077^*$ | -0.061 | -0.106 | -0.036 |
| $d_1$ | $-0.119^*$ | 0.835 | -0.169 | -0.084 |
| $d_2$ | $0.074^*$ | -0.275 | 0.037 | 0.001 |
| $d_3$ | $0.096^*$ | -0.637 | 0.026 | -0.155 |
| $d_4$ | $0.556^*$ | 0.481 | -0.529 | -0.423 |
| $\alpha_{\pi\Lambda}$ | $400^*$ | 322 | 276 | 301 |
| $\alpha_{\pi\Sigma}$ | $509^*$ | 1118 | 480 | 647 |
| $\alpha_{\bar{K}N}$ | $752^*$ | 467 | 809 | 1109 |
| $\alpha_{\eta\Lambda}$ | $979^*$ | 1217 | 500 | 1500 |
| $\alpha_{\eta\Sigma}$ | $797^*$ | 500 | 500 | 558 |
| $\alpha_{K\Xi}$ | $1079^*$ | 778 | 815 | 1063 |
| $\beta_{\pi\Lambda}$ | 558 | 455 | 1500 | 1500 |
| $\beta_{\pi\Sigma}$ | 205 | 207 | 212 | 217 |
| $\beta_{\bar{K}N}$ | 1393 | 606 | 1500 | 931 |
| $\beta_{K+}$ | 930 | 1250 | 696 | 697 |
| $\chi^2/\mathrm{dof}$ | 5.40 | 3.34 | 4.41 | 4.72 |

# EXTRAS: Comparison with other approaches

our photoproduction amplitude constructed from four $\mathcal{A}^i_{0+}$ amplitudes

$$[\mathcal{A}^i_{0+}(s, M_{\pi\Sigma})] = [\mathcal{A}^{i(\mathrm{tree})}_{0+}(s, M_{\pi\Sigma})] + [f_{0+}(M_{\pi\Sigma})]\,[8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})]\,[\mathcal{A}^{i(\mathrm{tree})}_{0+}(s, M_{\pi\Sigma})]$$

L. Roca, E. Oset - Phys. Rev. C 87 (2013) 055201

M. Mai, U.-G. Meißner - Eur. Phys. J. A 51 (2015) 30

makeshift photoproduction amplitude $[\mathcal{A}] = [f_{0+}]\,[8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})]\,[C(\sqrt{s})]$

S.X. Nakamura, D. Jido - PTEP 2014 (2014) 023D01

similar to our approach with some non-relativistic simplifications, additional contributions from $K^*$ exchange, phenomenological energy dependent contact terms, and adjustments to the first loop function and to the photoproduction vertex

E. Wang et al. - Phys. Rev. C 95 (2017) 015205

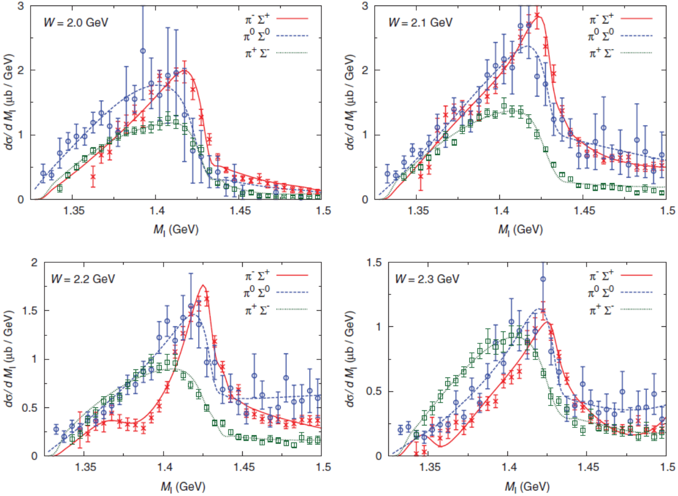
focus on triangle singularity contribution $\gamma p \to N^*(2030) \to K^*\Sigma \to K^+\Lambda(1405)$ combined with $K$, $K^*$ meson exchanges and a contact term

$\mathcal{A} = \mathcal{A}^{\mathrm{tree}} = a\, t_{\mathrm{triangle}} + b\, t_{K\,\mathrm{exchange}} + c\, t_{K^*\,\mathrm{exchange}} + d\, t_{\mathrm{contact}}$

All these fit a good number of model parameters to reproduce the CLAS data.

In contrast, we just demonstrated what can be achieved with a parameter-free approach based on (unitarized) ChPT.

# EXTRAS: Nakamura, Jido




20 subtraction constants, 15 energy dependent complex couplings $\lambda_n^j$, common form factor scale $\beta_c = \beta_{K^+} = \Lambda$