

分子動力学法を使った計算から解明する ハイパー核の面白い側面って？

井坂 政裕(法政大)

Grand challenges of hypernuclear physics

Interaction: “baryon-baryon interaction”

● 2 body interaction between baryons (Y: hyperon, N: nucleon)

- hyperon-nucleon (YN)
 - hyperon-hyperon (YY)
- } Major issues in hypernuclear physics

Structure: “many-body system of nucleons and hyperon”

● Addition of hyperon as an impurity in (hyper)nuclei

- No Pauli exclusion between N and Y
 - YN interaction is different from NN
- } → “Impurity effects”

Today: ハイパー核構造。Λが核に加わると、何が起こるか？

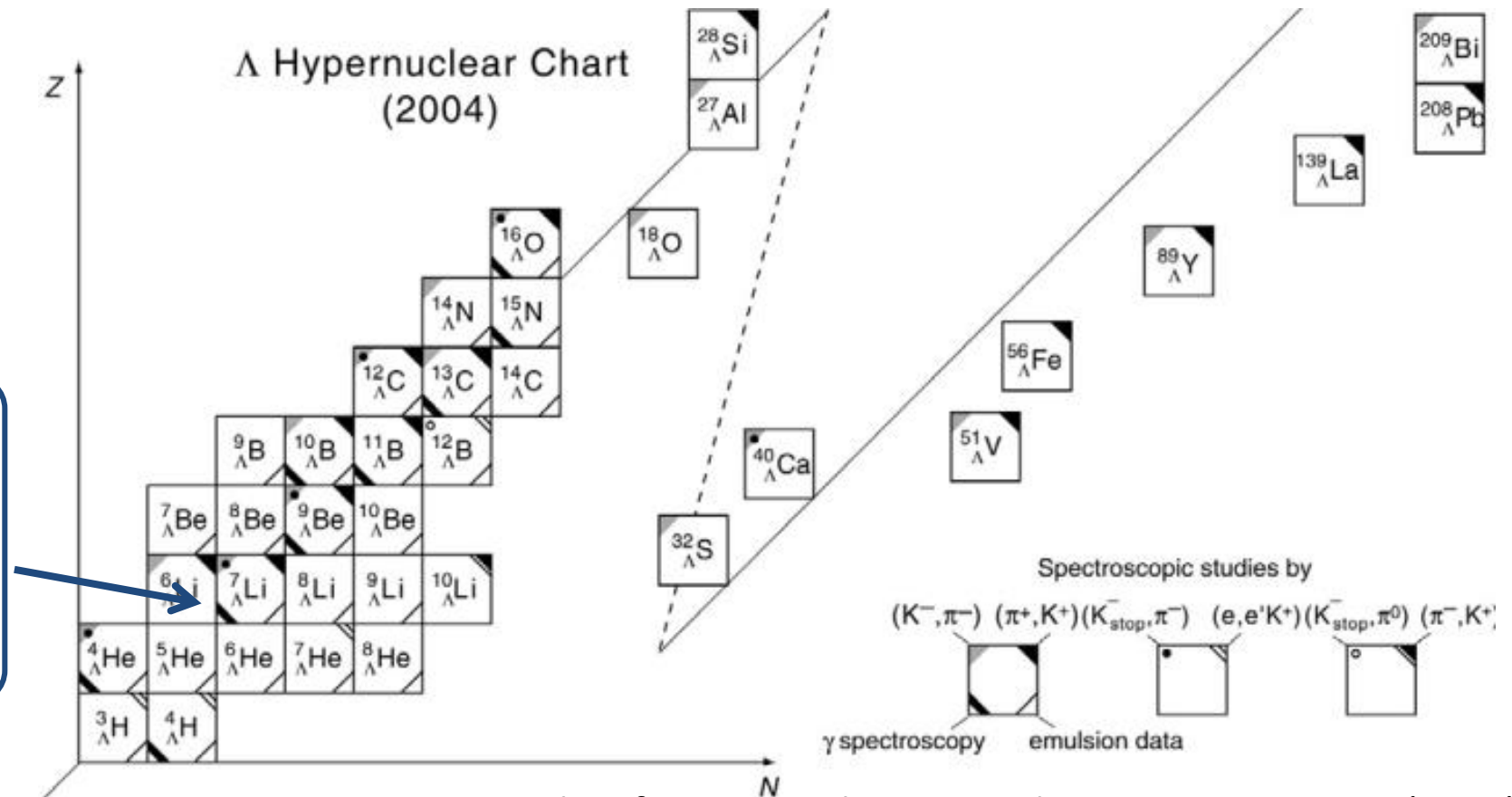
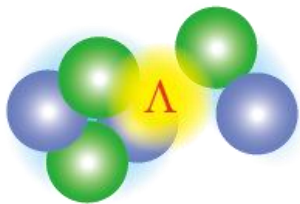
Structure of Λ hypernuclei

◆ Λ hypernuclei observed so far

- Concentrated in light Λ hypernuclei
- Most have well-developed cluster structure

Light Λ hypernuclei

Developed cluster

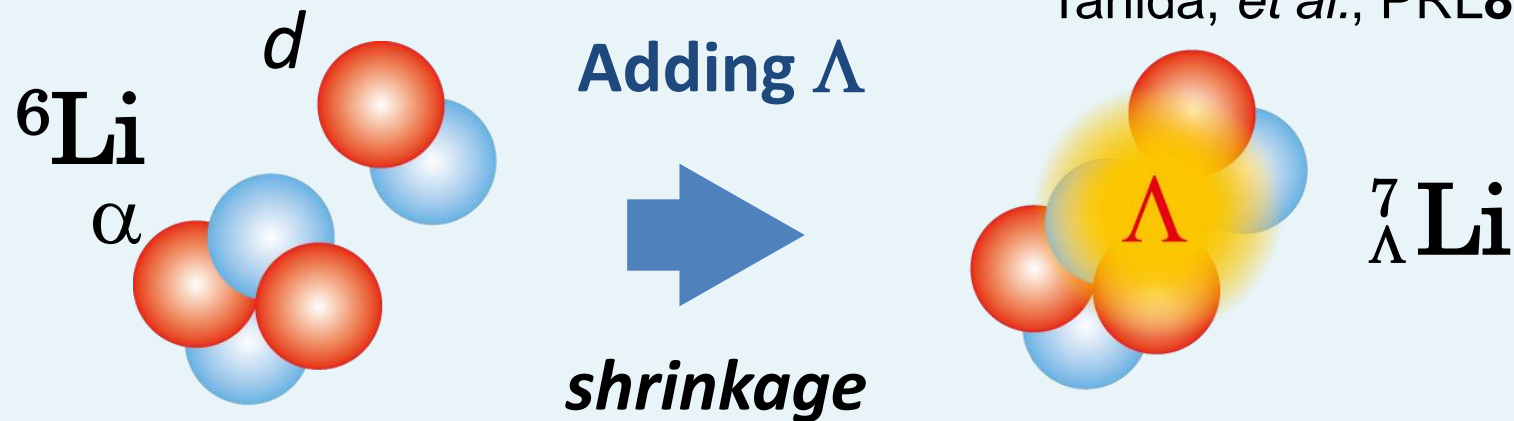


Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Cluster structure in light hypernuclei

◆ Famous example of “impurity effect”

Example: ${}^7_{\Lambda}\text{Li}$



Motoba, *et al.*, PTP**70**,189 (1983)
Hiyama, *et al.*, PRC**59** (1999), 2351.
Tanida, *et al.*, PRL**86** (2001), 1982.

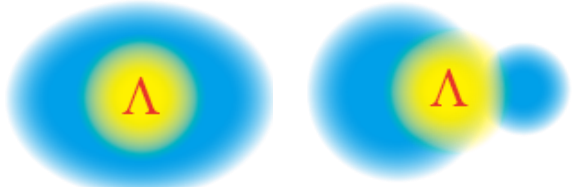
- Λ reduces inter-cluster distance b/w $\alpha + d$ of the core nucleus ${}^6\text{Li}$
- Confirmed through $B(E2)$ reduction
- Λ 粒子によって、どのようなimpurity effectが起こるのか？
- 他の核でも同様のshrinkageが起こるのか？別の変化が起こるのか？

Toward heavier and exotic Λ hypernuclei

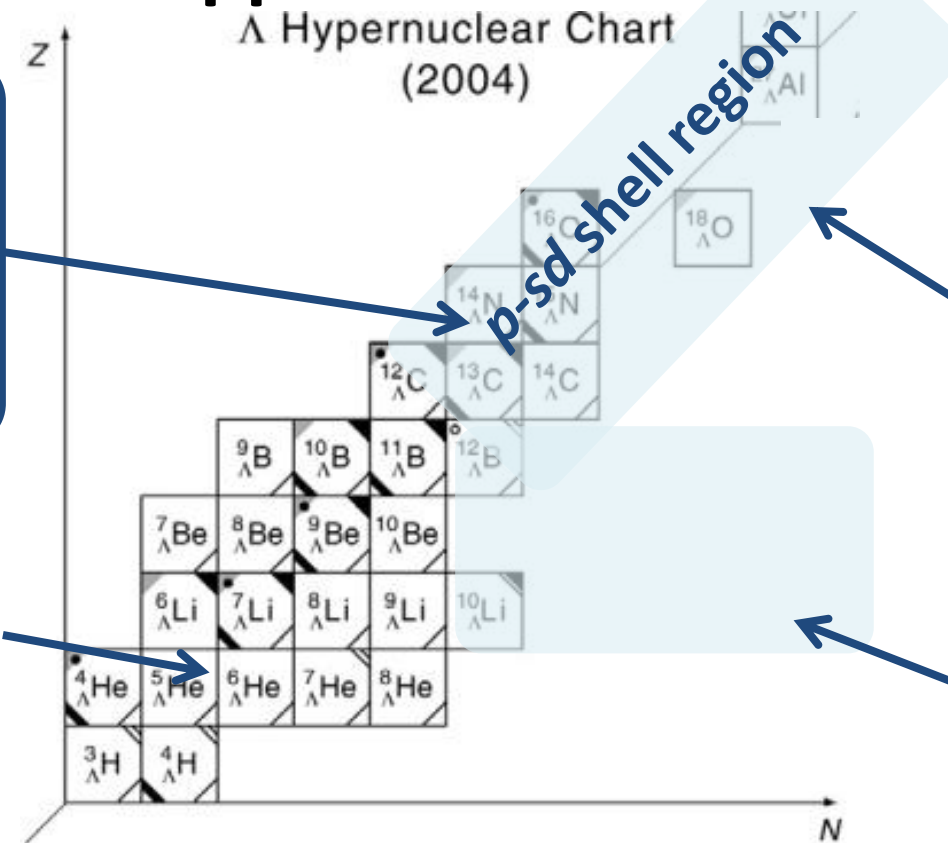
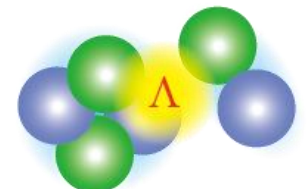
◆ Experiments at J-PARC, JLab, etc.

- Heavier (*sd*-shell and more) hypernuclei can be produced
- Various structures will appear

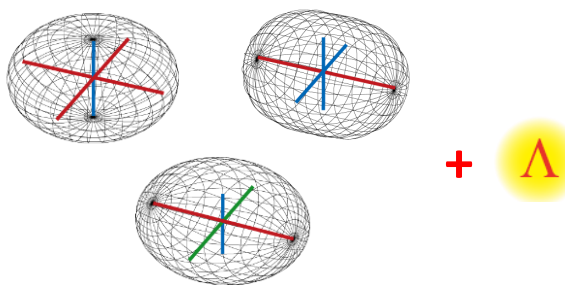
Coexistence of structures



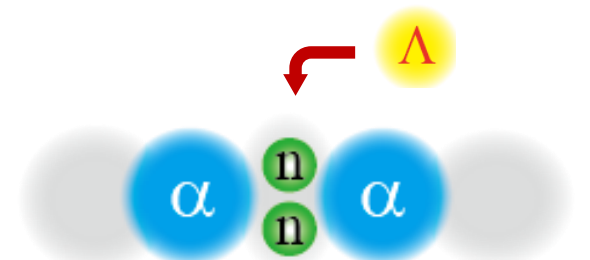
**Light Λ hypernuclei
Developed cluster**



Various deformations

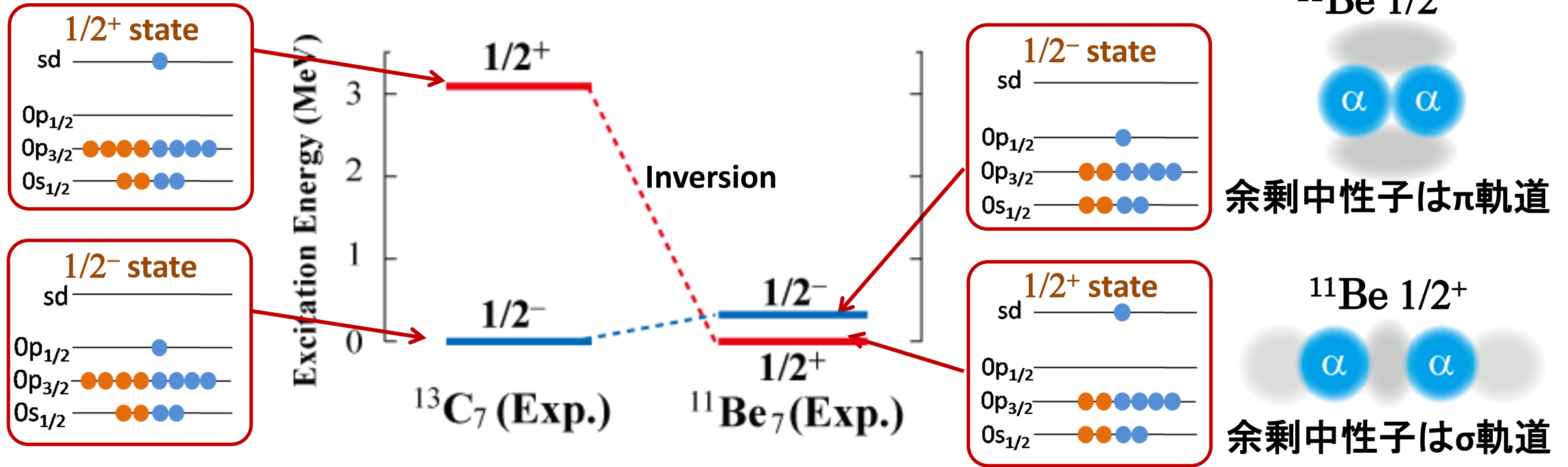


Exotic structure



Exotic structure of neutron-rich nuclei

◆ Example : Parity inversion of the ^{11}Be g.s. → 魔法数N=8の消失



Y. Kanada-En'yo and H. Horiuchi,
PRC 66 (2002), 024305.

Λ粒子が加わると何が起こる？

- 各状態の構造をどれほど変えるのか？
- エネルギー準位(励起スペクトル)の変化は？

Structure of *sd*-shell nuclei

◆*sd*-shell核の構造の特徴

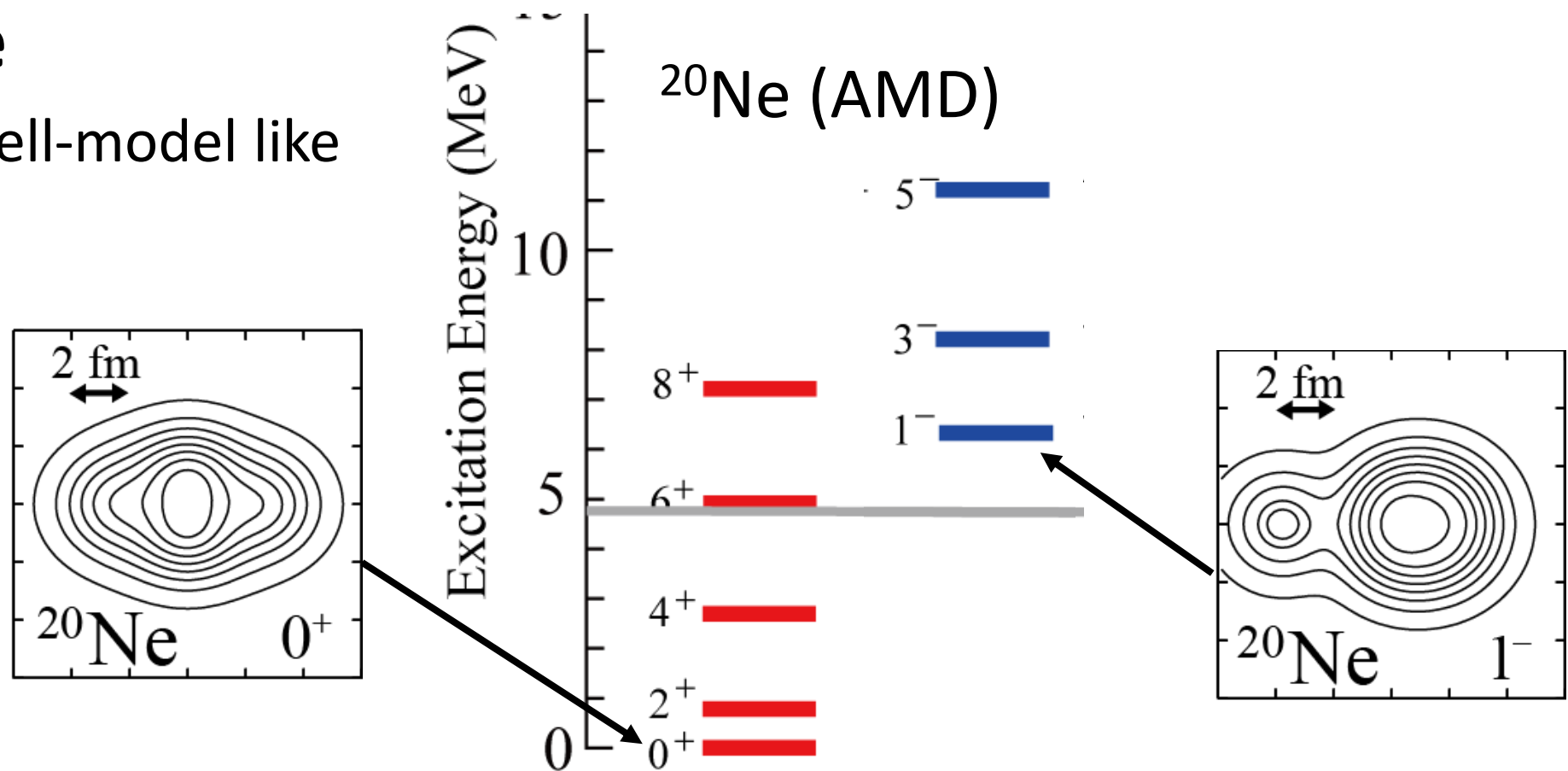
- 平均場構造とクラスター構造が低励起状態に共存
- 様々な変形: プロレート、オブレート、三軸非対称、超変形、etc.

Structure of sd -shell nuclei

◆ Coexistence of mean-field like & cluster structures

Example: ^{20}Ne

- (Deformed) shell-model like
- $\alpha + ^{16}\text{O}$ cluster



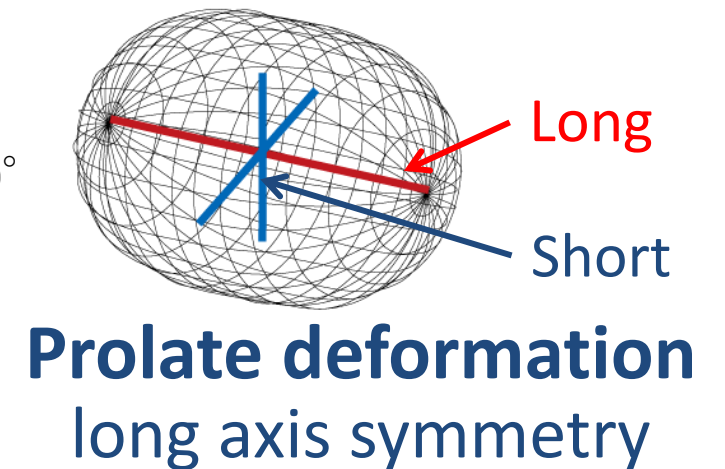
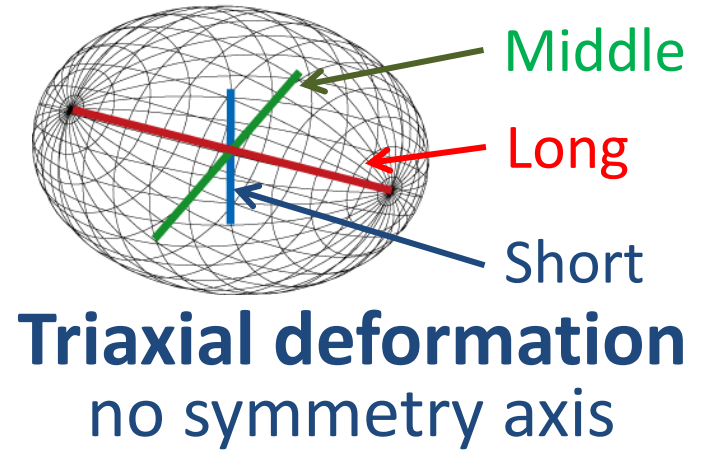
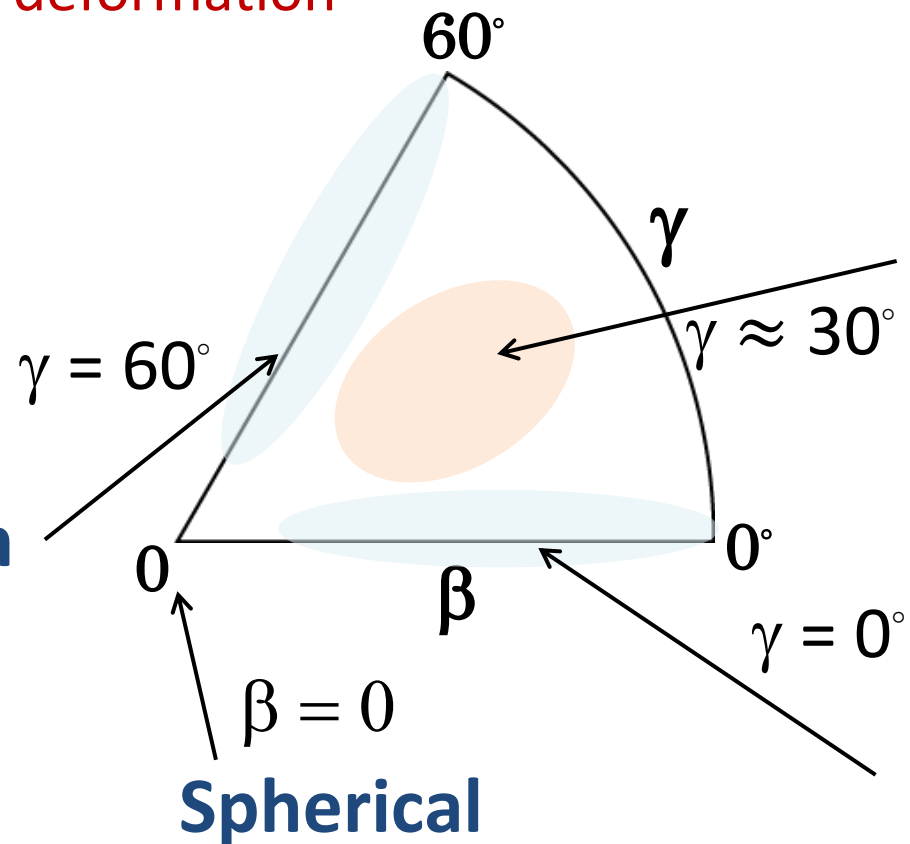
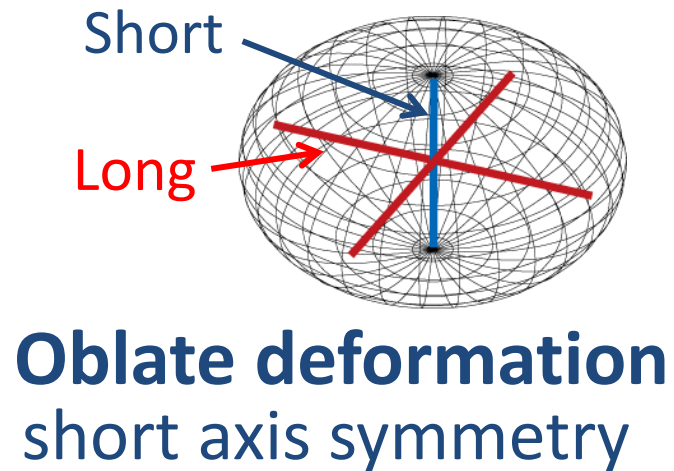
- 構造が異なる状態では、impurity effectsに違いはあるか？
- shrinkageは起こるのか？

Various deformation of nuclei

◆ Most of nuclei are deformed except for magic nuclei

● Nuclear quadrupole deformation (β, γ)

- β : degree of quadrupole deformation
- γ : (tri)axiality

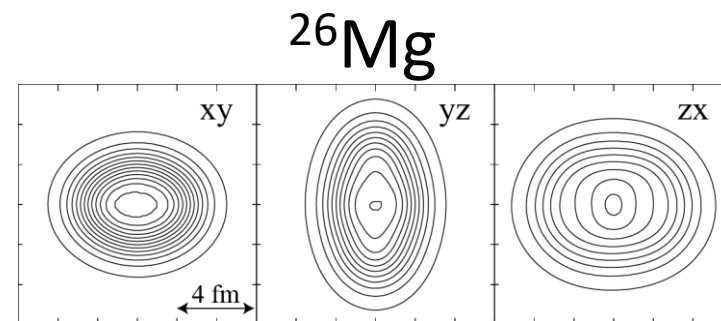
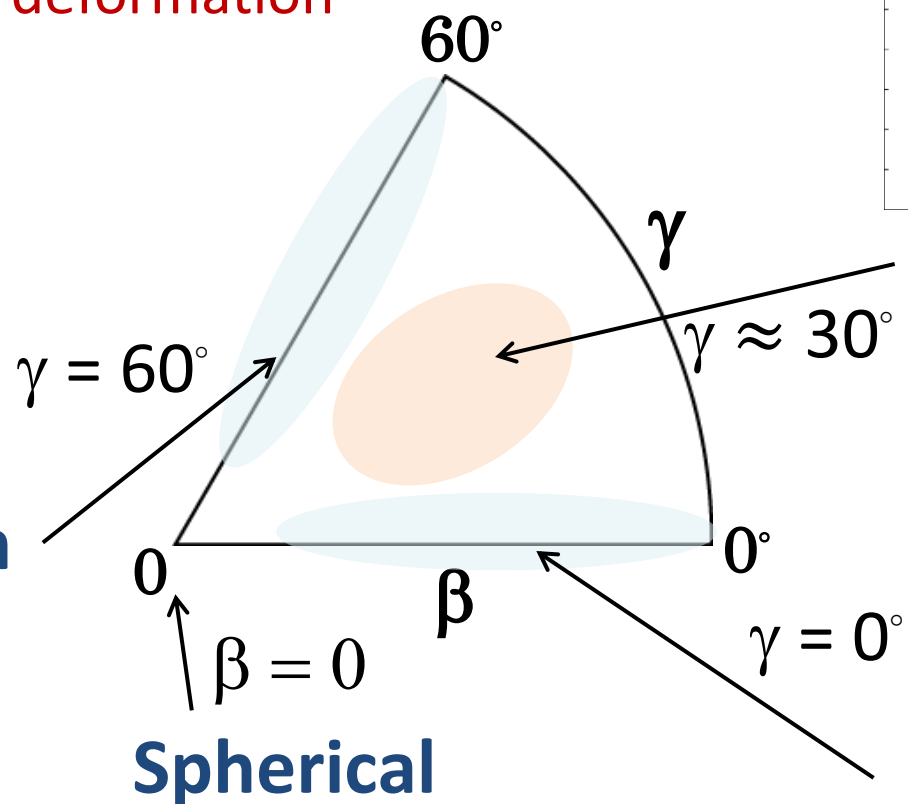
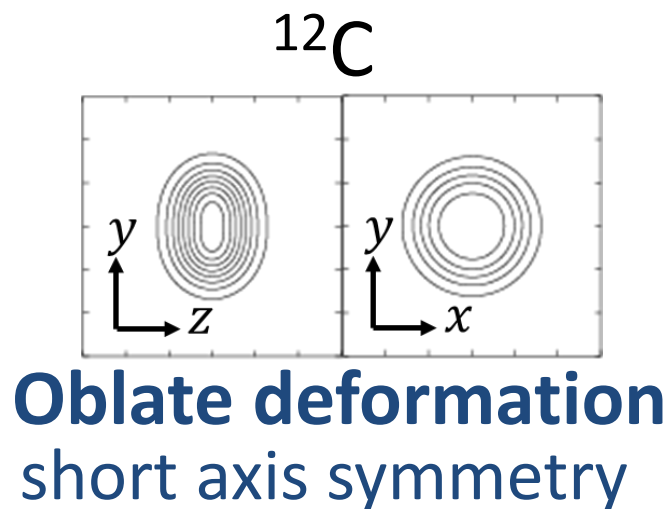


Various deformation of nuclei

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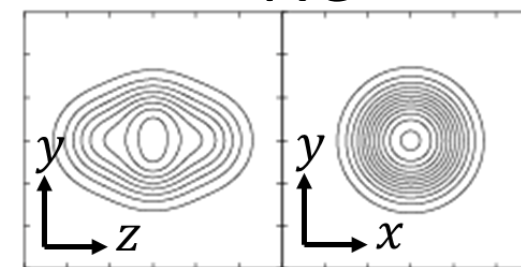
● Nuclear quadrupole deformation (β, γ)

- β : degree of quadrupole deformation
- γ : (tri)axiality



Triaxial deformation
no symmetry axis

^{20}Ne



Prolate deformation
long axis symmetry

変形の変化は？

Λ粒子を加えたときの変形による違いは？

HyperAMD: Antisymmetrized Molecular Dynamics for hypernuclei

◆ Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g$$

NN: Gogny D1S
 Λ N: YNG interaction

◆ Wave function

● Nucleon part: Slater determinant

Spatial part of s.-p. w.f. is described as Gaussian packets

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i$$

$$\chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

● Single-particle w.f. of Λ hyperon:

Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu \nu_\sigma (r - z_m)_\sigma^2\right] \chi_m$$

$$\chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

● Total w.f.:

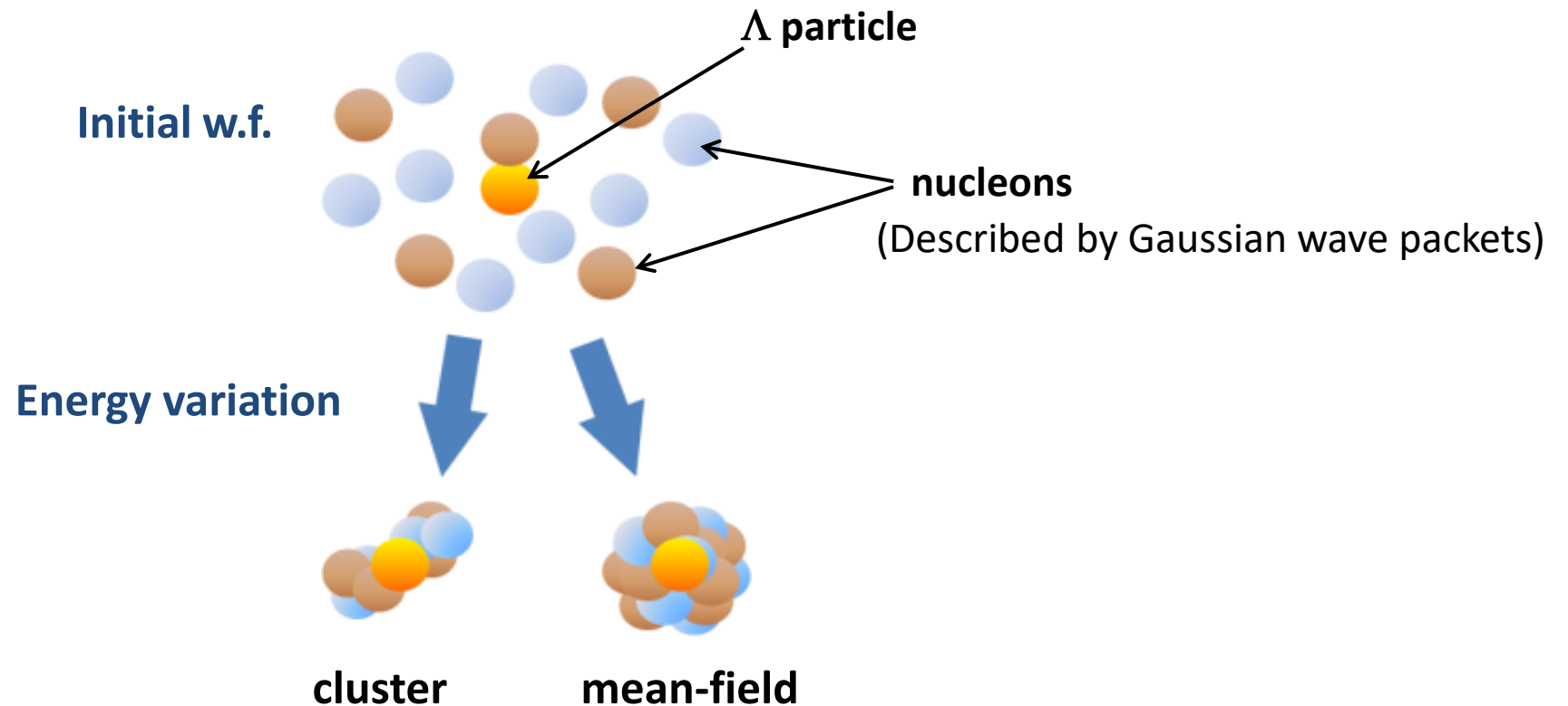
$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)]$$

Theoretical framework: HyperAMD

◆ Procedure of the calculation

Variation

- Imaginary time development method:
$$\frac{dX_i}{dt} = \frac{\kappa}{\hbar} \frac{\partial H^\pm}{\partial X_i^*} \quad \kappa < 0$$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, v_i, c_i$



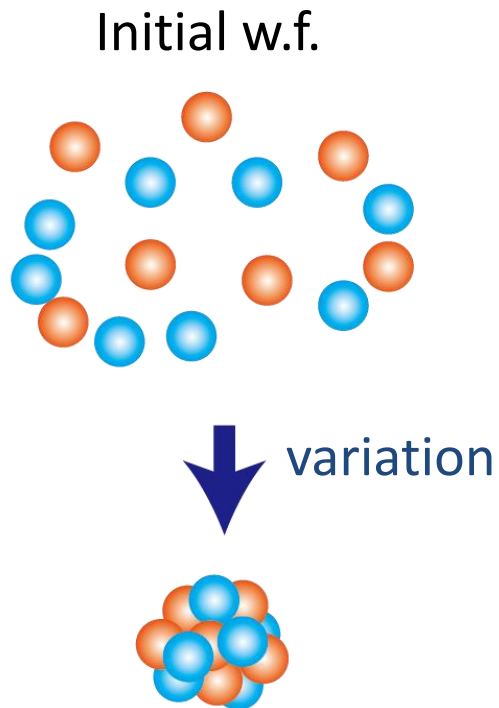
Theoretical framework: HyperAMD

◆ Procedure of the calculation

- Energy variation with constraint on nuclear quadrupole deformation

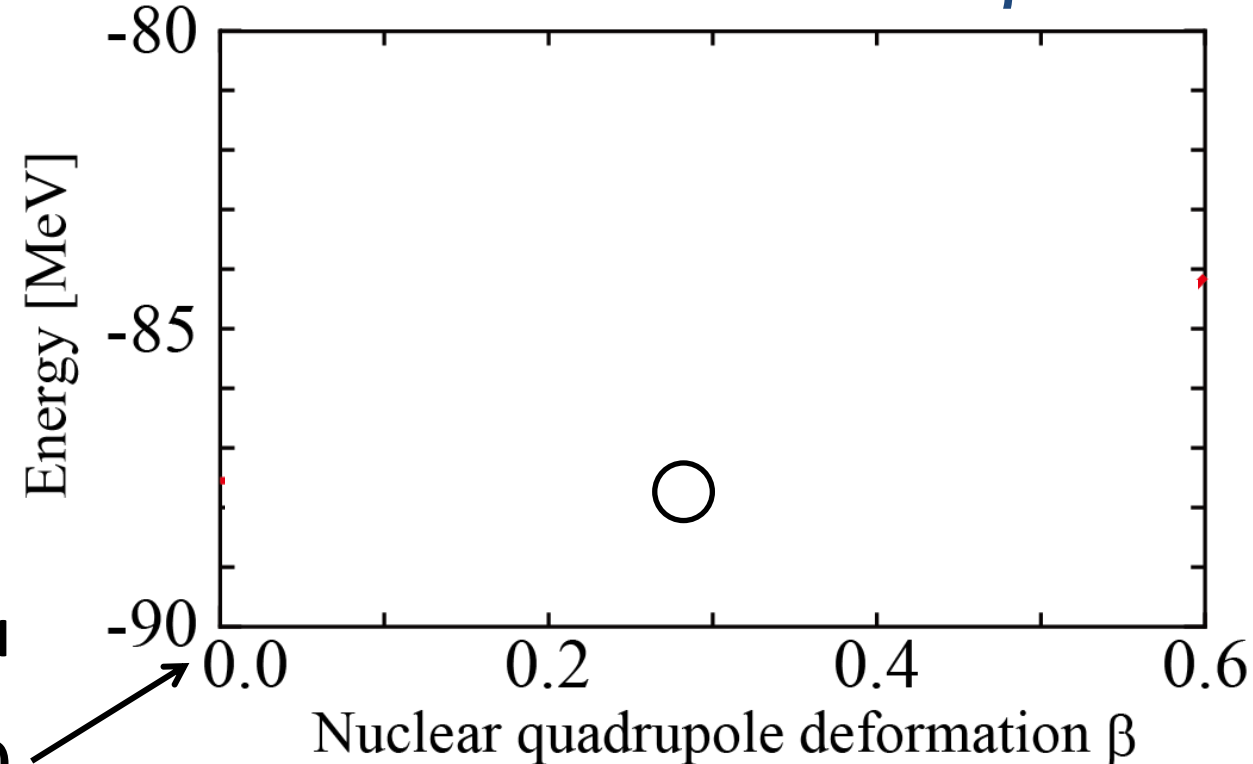
Described by (β, γ)

e.g.) ^{12}C



Spherical
 $\beta = 0.00$

without constraint on β



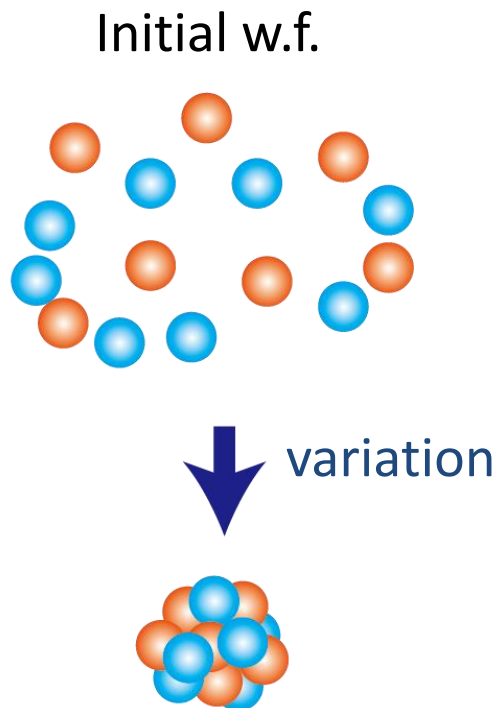
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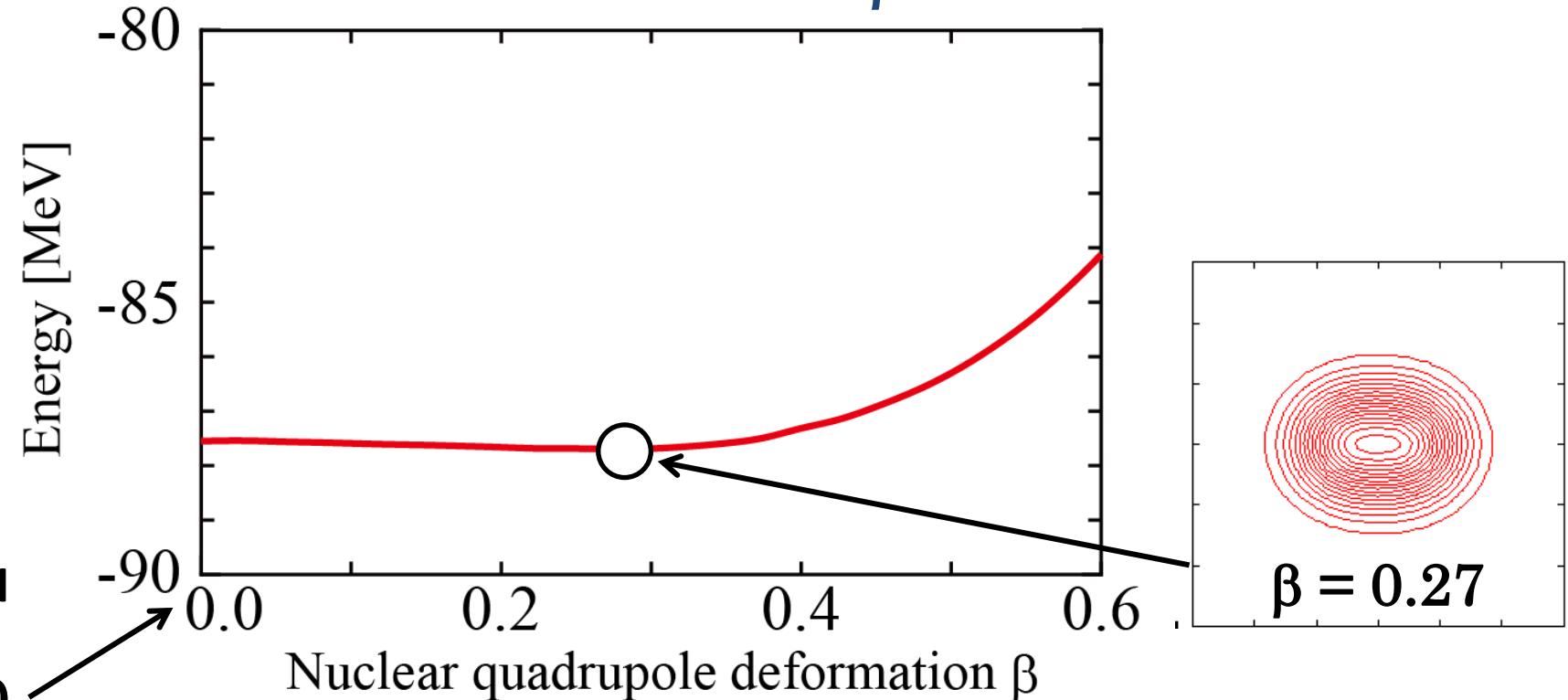
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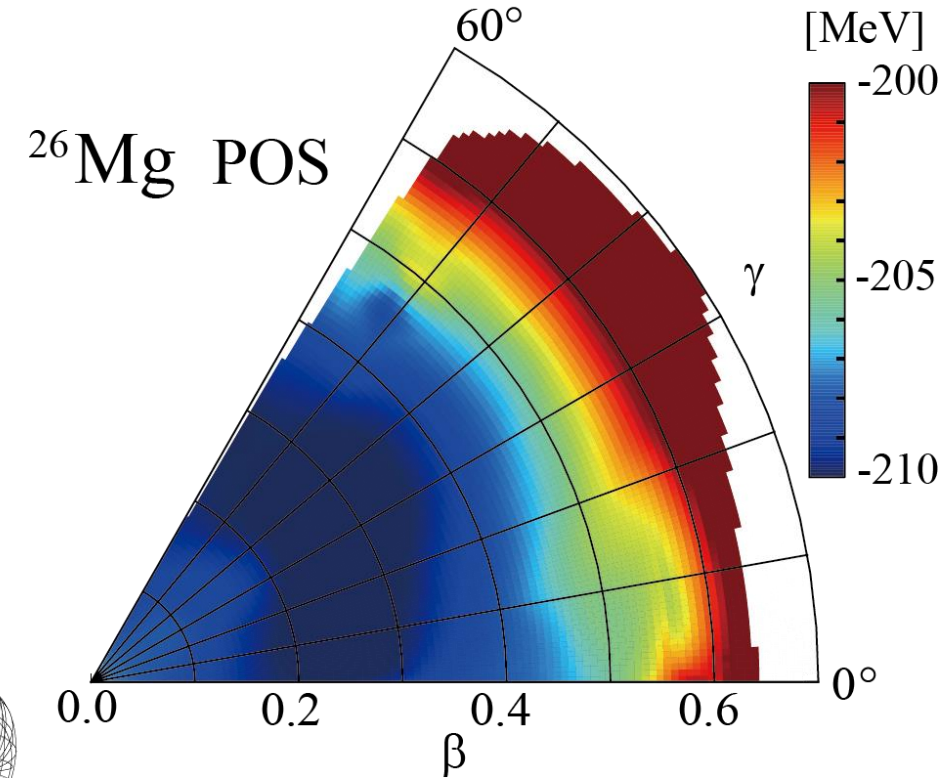
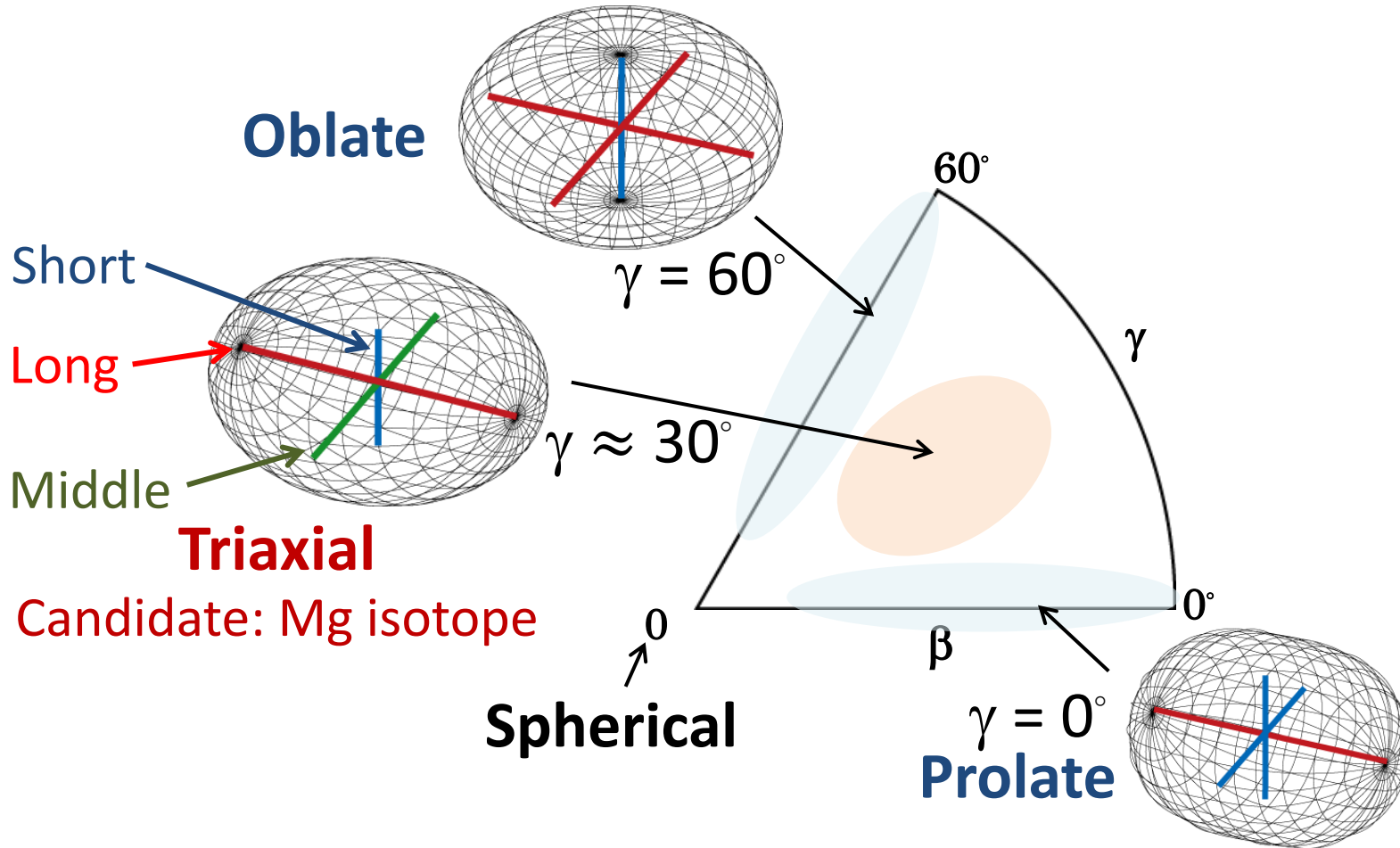


Spherical
 $\beta = 0.00$

with constraint on β



Quadrupole deformation (β , γ)



- Energy variation is performed at each (β, γ)
- p states are obtained by constraint on Λ single particle wf: $V_f = \lambda \sum_f |\varphi_f\rangle\langle\varphi_f|$

◆ Procedure of the numerical calculation

Variation

- Imaginary time development method:

$$\frac{dX_i}{dt} = \frac{\kappa}{\hbar} \frac{\partial H^\pm}{\partial X_i^*} \quad \kappa < 0$$

- Variational parameters:

$$X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, v_i, c_i$$

Angular Momentum Projection

$$|\Phi_K^s; JM\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) |\Phi^{s+}\rangle$$

Generator Coordinate Method (GCM)

- Superposition of intrinsic wave functions with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

$$H_{sK,s'K'}^{J\pm} = \langle \Phi_K^s; J^\pm M | \hat{H} | \Phi_{K'}^{s'}; J^\pm M \rangle$$

$$N_{sK,s'K'}^{J\pm} = \langle \Phi_K^s; J^\pm M | \Phi_{K'}^{s'}; J^\pm M \rangle$$

$$|\Psi^{J\pm M}\rangle = \sum_{sK} g_{sK} |\Phi_K^s; J^\pm M\rangle$$

Theoretical Framework: HyperAMD

◆AMD(HyperAMD)の利点

- クラスター構造も平均場構造も記述できる
- 核構造の動的変化を記述可能
- 変形の軸対称性を仮定しない
- 偶偶核・奇奇核・偶奇核いずれも可

◆適用範囲

- 質量数が大きすぎると適用できない(A=59までは適用例あり)
- 模型計算: bareな核力ではなく、有効核力(バリオン間力)が必要

Recent development based on AMD

To describe short-range & tensor correlations with bare nuclear force

- **TOAMD: adding correlation functions multiplying AMD wf**

T. Myo, H. Toki, K. Ikeda, H. Horiuchi, T. Suhara, PTEP2015,073D02

- **HMAMD: high-momentum components is included in AMD wf**

T. Myo, H. Toki, K. Ikeda, H. Horiuchi, T. Suhara, M. Lyu, M. Isaka, T. Yamada, PTEP2017, 111D01 (2017)

Example: applications of HMAMD to ^3H

AMD wf.

$$\Phi_{\text{AMD}} = \frac{1}{\sqrt{A!}} \det \{ \Pi_{i=1}^A \varphi_i(\vec{r}_i) \},$$

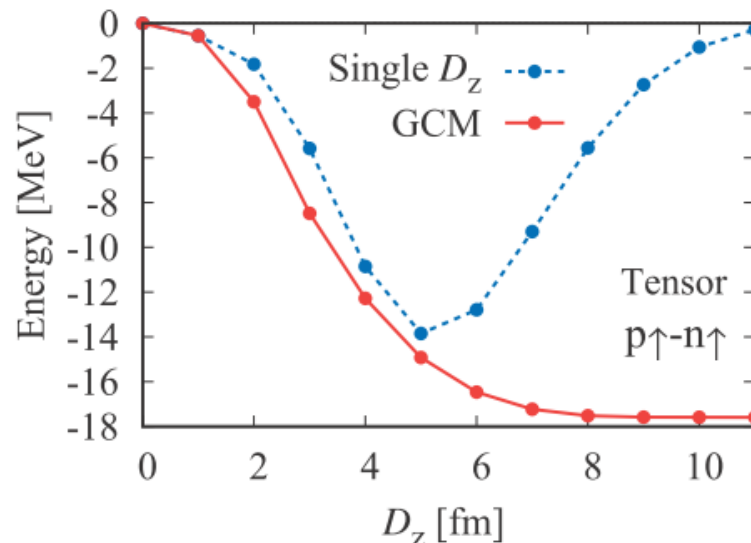
$$\varphi(\vec{r}) = \phi(\vec{r}) \chi_\sigma \chi_\tau,$$

$$\phi(\vec{r}) = \exp\{-\nu(\vec{r} - \vec{Z})^2\},$$

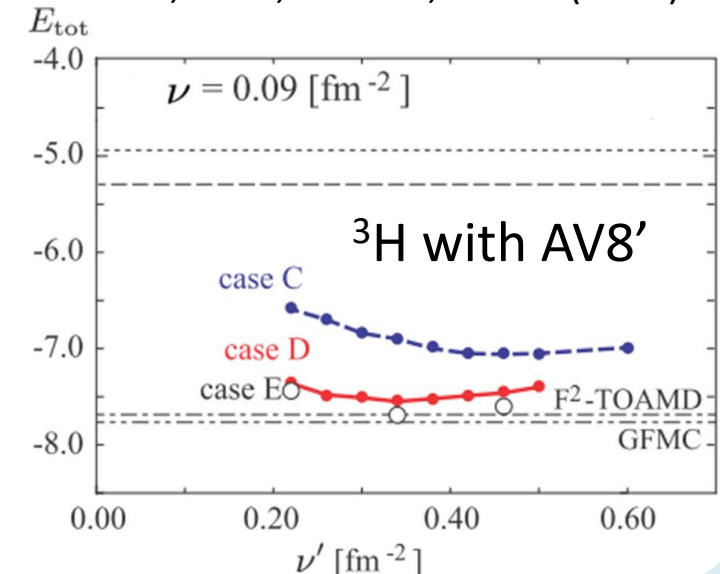
$$\text{Imp. relation: } \frac{\langle \phi | \vec{p} | \phi \rangle}{\langle \phi | \phi \rangle} = 2\nu \text{Im}(\vec{Z}),$$

$$\text{HM-pair: } \vec{Z}_1 = i\vec{D}, \quad \vec{Z}_2 = -i\vec{D}$$

Myo, et al., PTEP2017, 111D01 (2017)



Isaka, et al., PRC106,044310(2022)



Λ ハイパー核の構造として期待できること

- glue-like role: Λ 粒子により、非束縛状態が束縛する
- 構造の動的変化: 変形の変化、核半径の収縮
- 変形・構造の違いに対するエネルギー(B_{Λ})の違い
- Λ 粒子が変形状態に結合することで現れる新奇的な状態

Λ ハイパー核の構造として期待できること

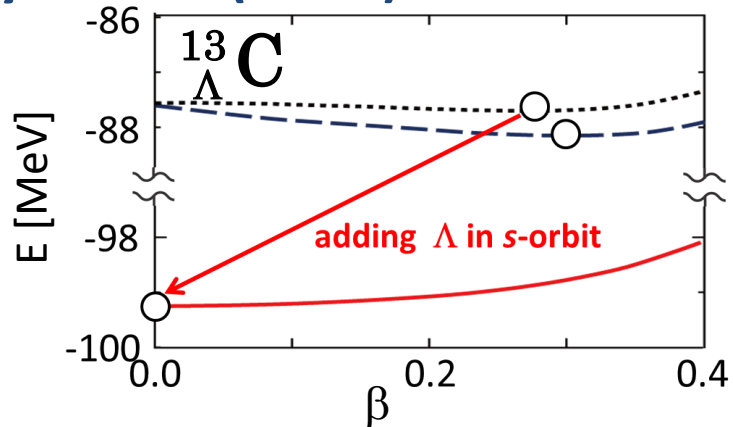
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Deformation change by Λ particle

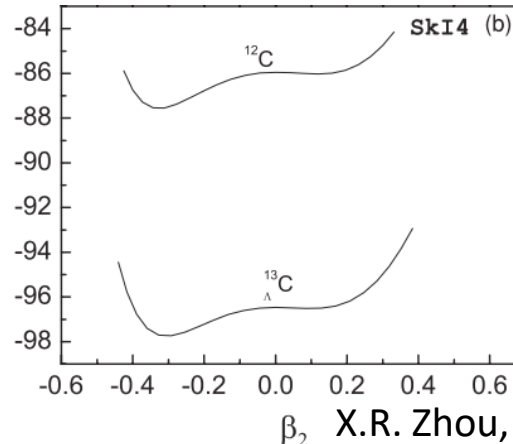
Many authors predict that Λ in s-orbit reduces nuclear deformation

Antisymmetrized molecular

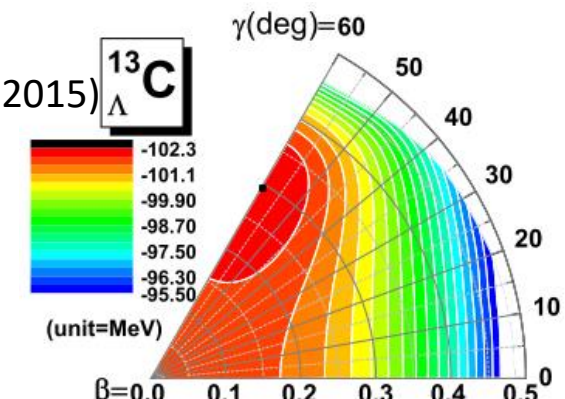
Dynamics (AMD) M.I. et al., PRC83, 044323(2011)



Skyrme-Hartree-Fock (SHF)



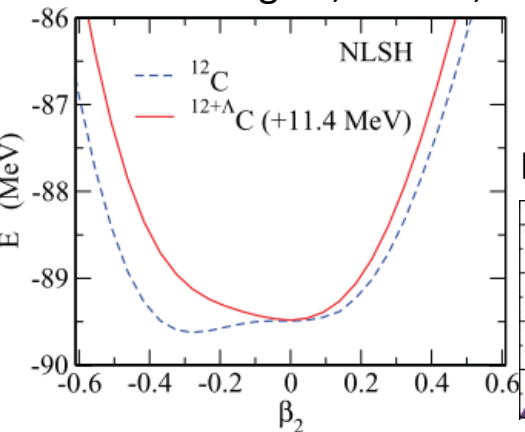
J.W. Cui, et al,
PRC91,054306(2015)



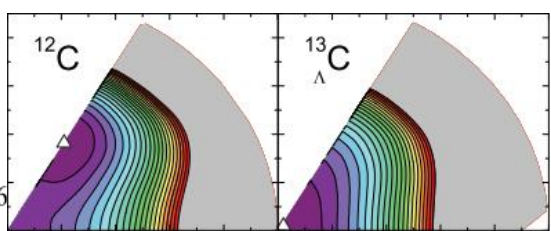
X.R. Zhou, et al., PRC76, 034312(2007)

Relativistic mean-field (RMF)

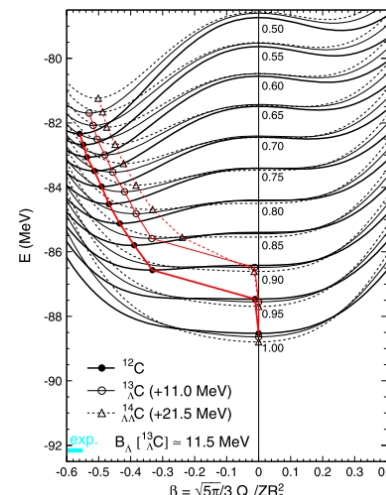
Win and Hagino, PR C78, 054311(2008)



B.N. Lu, et al., PRC84, 014328 (2011)



RMF & SHF



H. J. Schulze, et al.,
PTP123, 569('10)

Deformations/level structure

with beyond-mean-field

J.W. Cui, X.R. Zhou, H.J. Schulze,
PRC91,054306('15)

H. Mei, K. Hagino, J.M. Yao, T. Motoba,
PRC91, 064305(2015); 97, 064318(2018)

... and so on

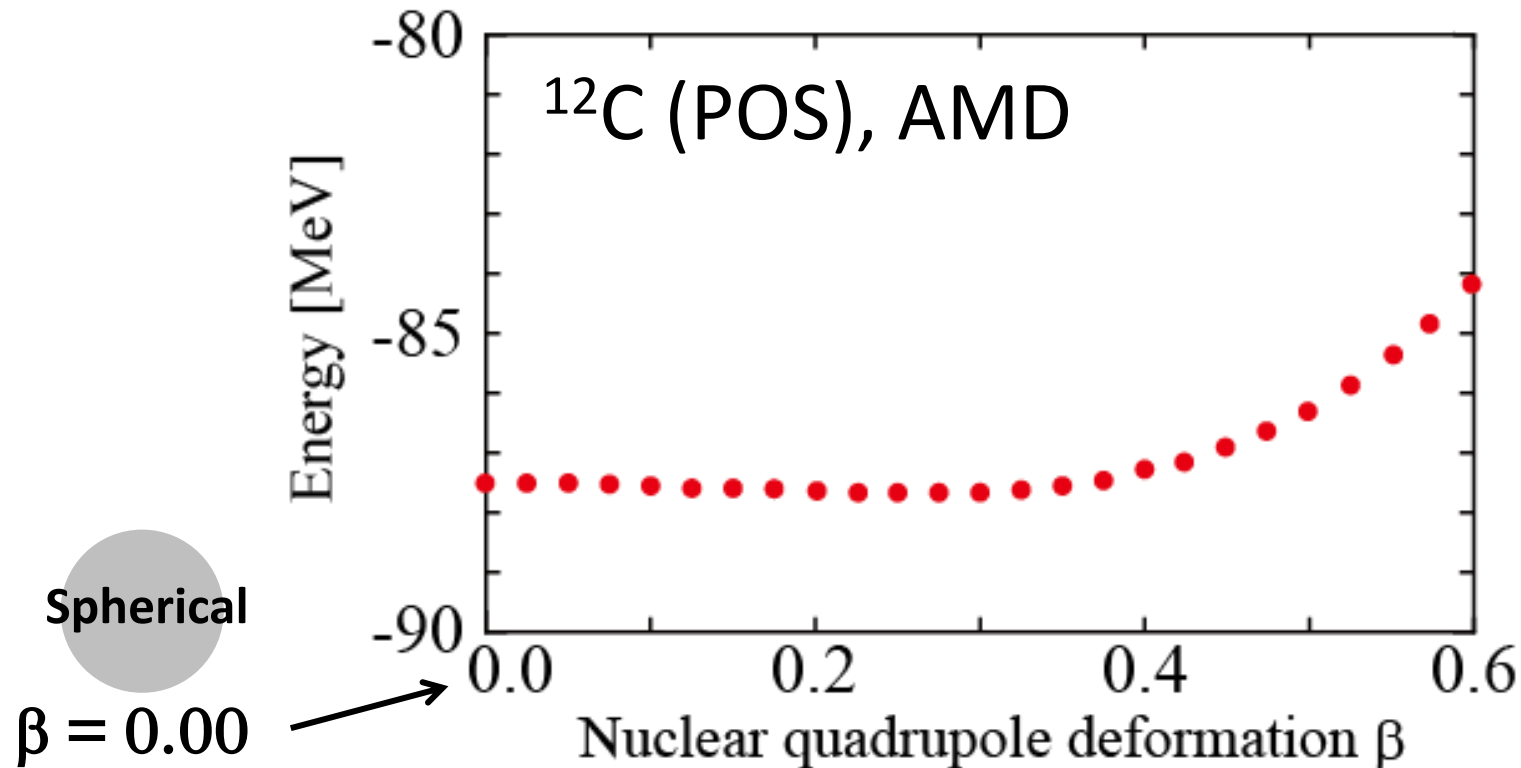
Deformation change by Λ particle

◆ How to analyze from energy surface

Example: ^{12}C with AMD(antisymmetrized molecular dynamics)

- Energy variation at each β (and γ) \rightarrow energy curve as a function of β
- Energy minimum at (β, γ)

M.I, et al., PRC83, 044323(2011)



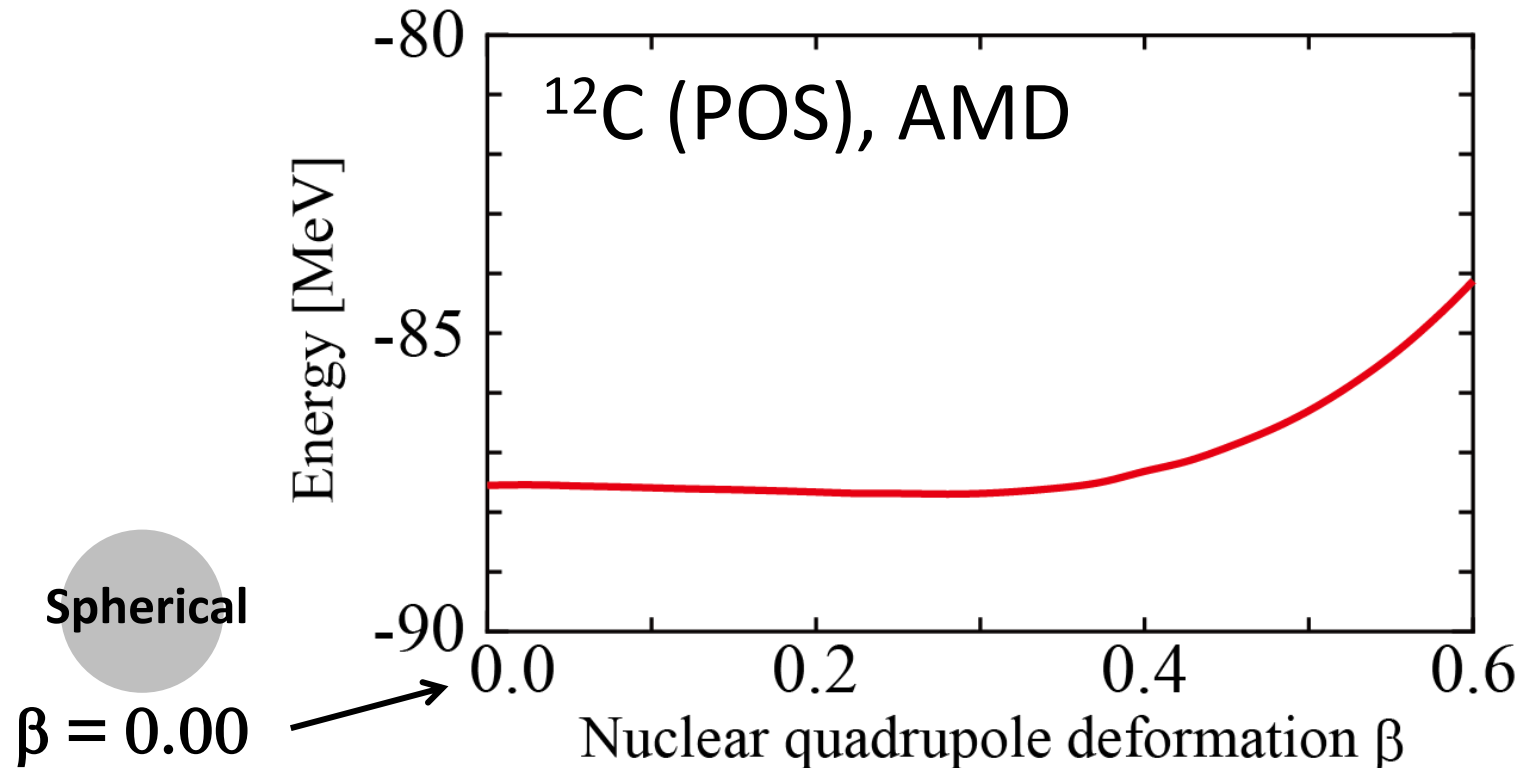
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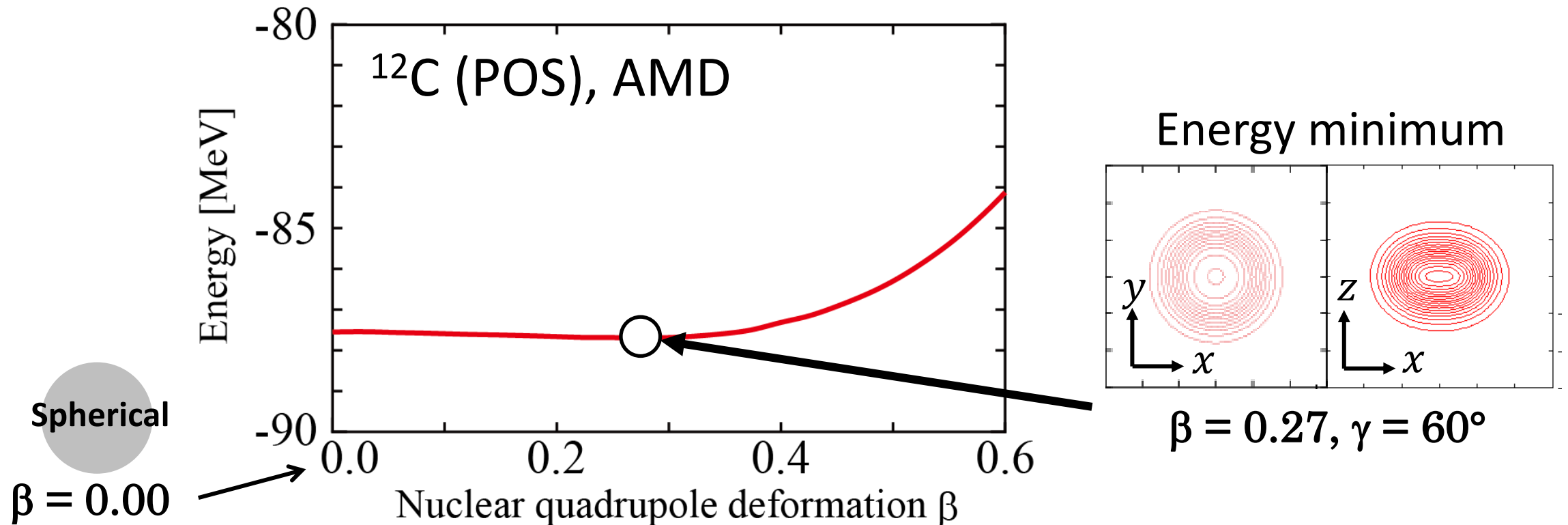
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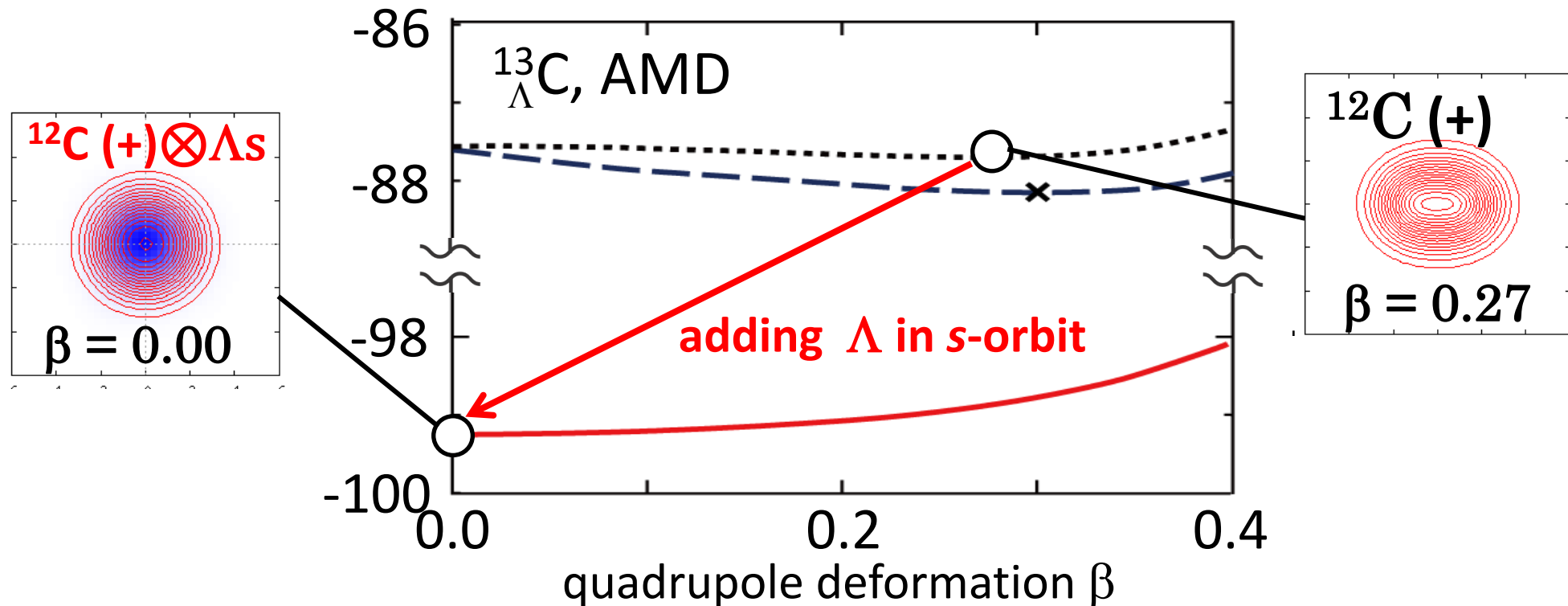
Deformation change by Λ in s-orbit

◆ How to analyze from energy surface

M.I, et al., PRC83, 044323(2011)

Example: ^{12}C with AMD(antisymmetrized molecular dynamics)

- Energy variation of hypernucleus
→ energy minimum moves to smaller β with Λ in s-orbit

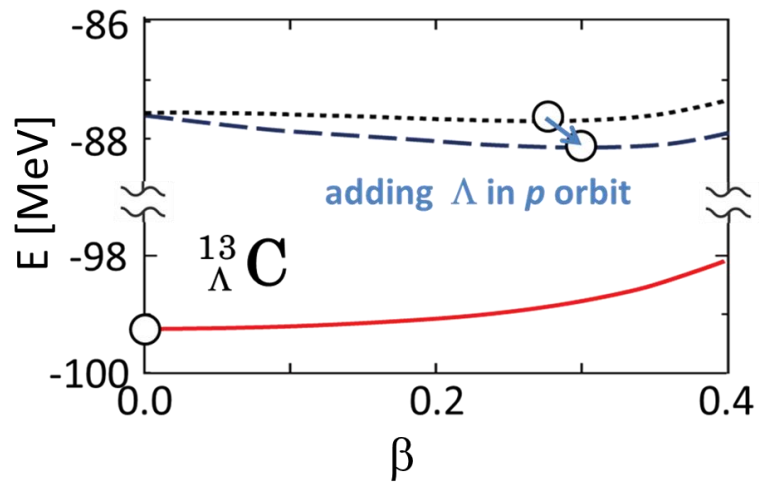


Deformation change by Λ particle

Λ in p -orbit enhances nuclear deformation

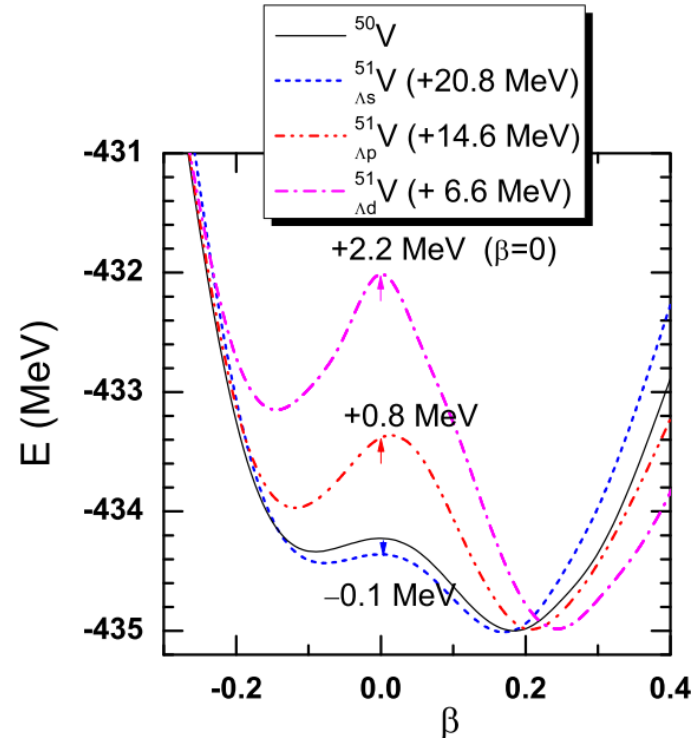
Antisymmetrized molecular dynamics (AMD)

M.I., et al., PRC83, 044323(2011)



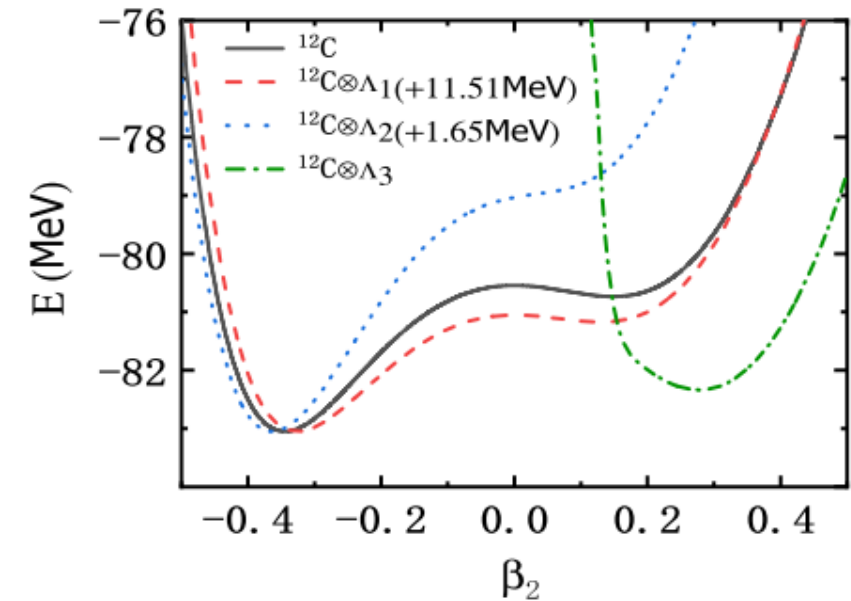
Triaxially deformed relativistic mean-field

W. X. Xue, et al., PRC91, 024327(2015)



Deformed Skyrme-Hartree-Fock (DSHF)

Bi-Cheng Fang et al., EPJA56,11(2020)

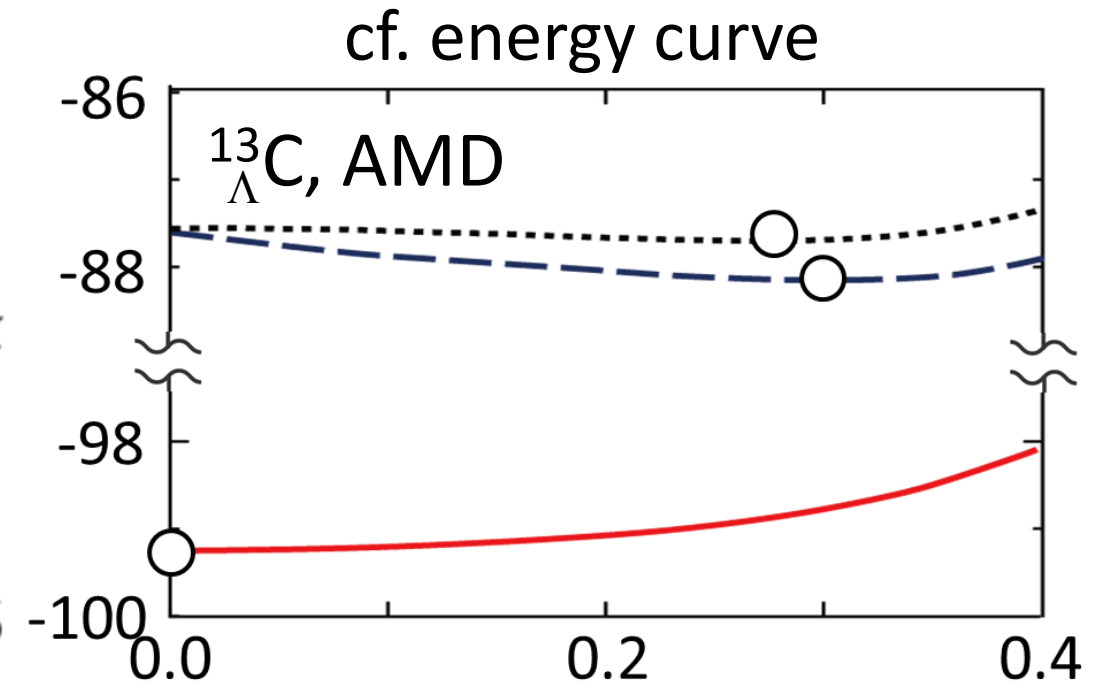
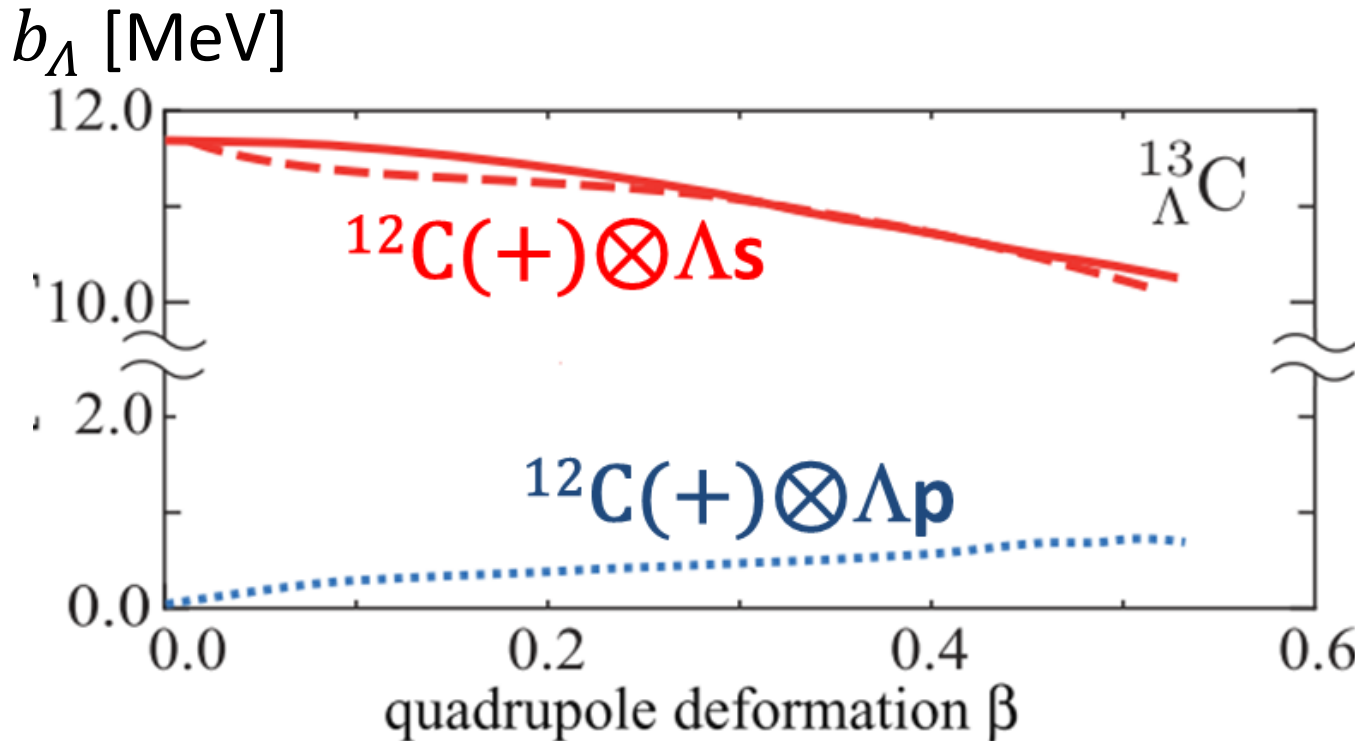


Deformation change of Λ in higher orbits such as d -orbit is also predicted by several papers: W. X. Xue, et al., PRC91, 024327(2015), X. Y. Wu, et al., PRC95, 034309(2017)

Why does Λ change nuclear deformation?

- Λ in s orbit is deeply bound at small β , while Λ in p orbit prefers deformation
- Competition b/w Λ binding energy and energy surface of core nucleus

$$\Lambda \text{ binding energy: } b_{\Lambda}(\beta) = E_{core}(\beta) - E_{\Lambda}(\beta)$$

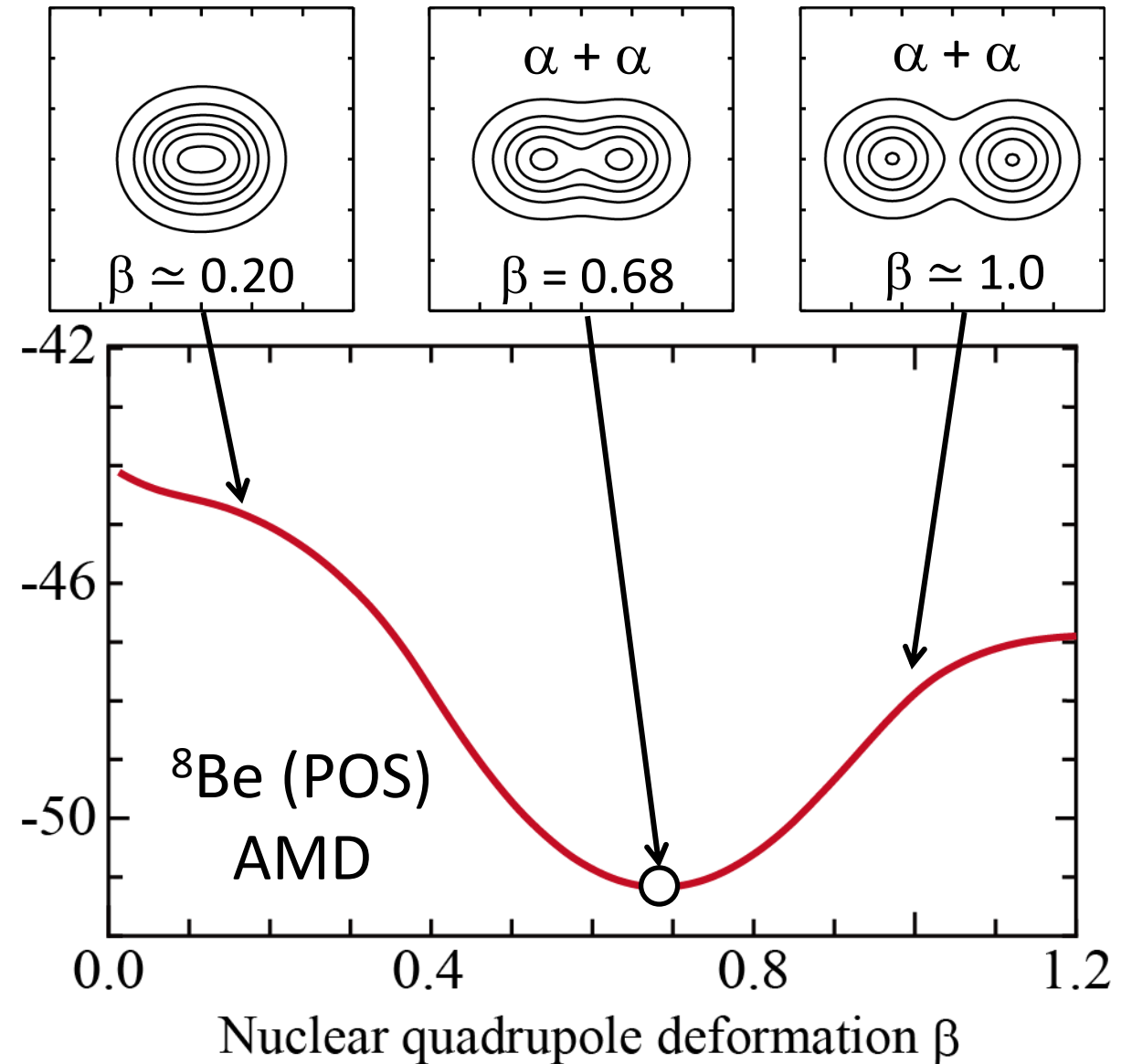
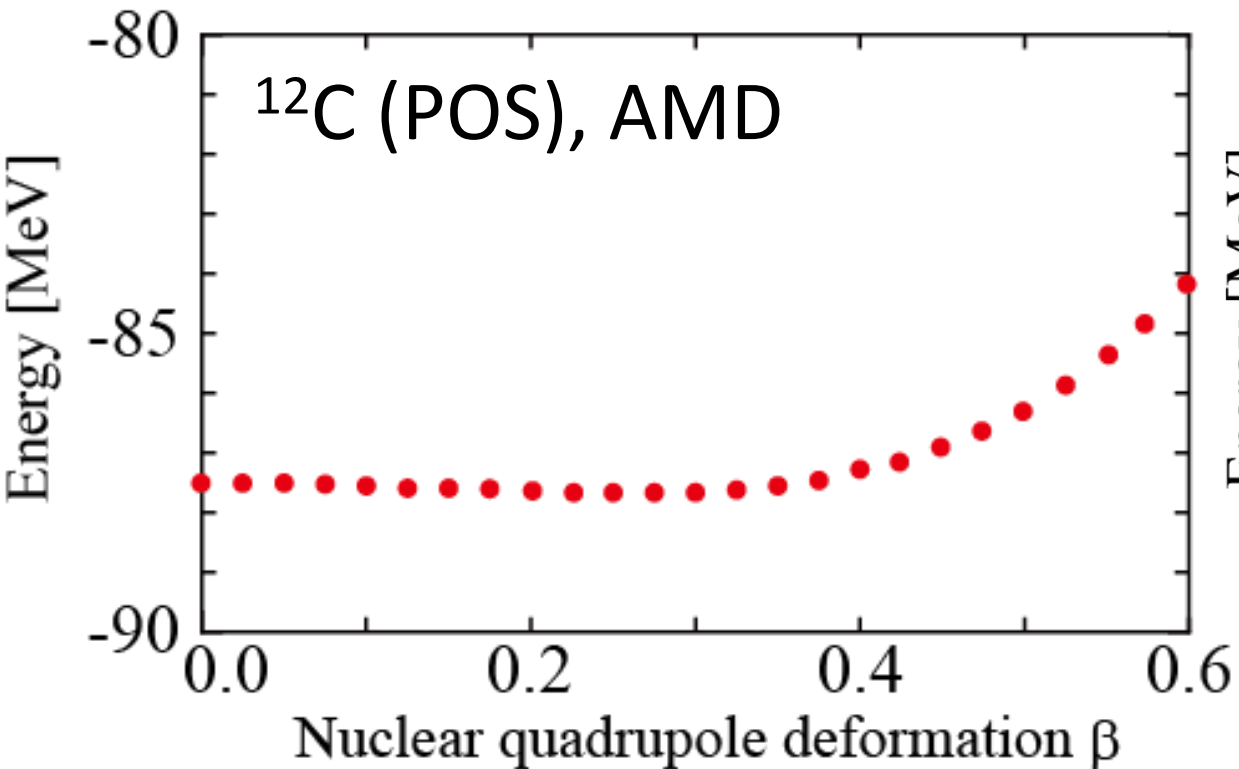


M.I., et al., PRC83, 044323(2011)

“binding energy of Λ ” vs. “energy surface of the core nuclei”

Energy surface of core nuclei

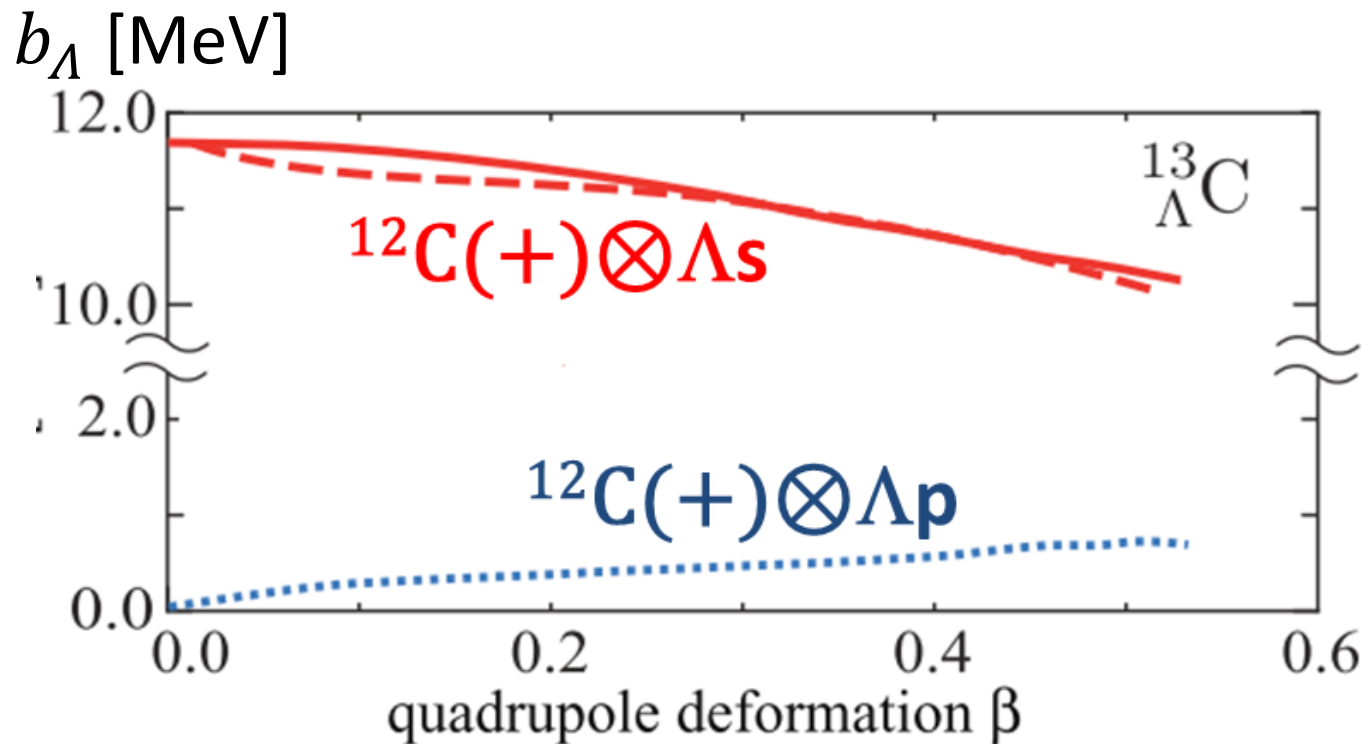
Energy surface (energy curve)は核構造を反映している

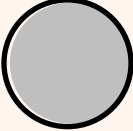
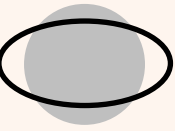
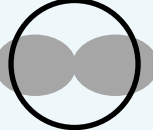



Deformation change by Λ particle

“Overlap between Λ and N” is the key!

Λ in s -orbit (p -orbit) is deeply bound with smaller β (larger β) due to larger overlap between Λ and nucleons



Λ in s orbit		
	Small β	Large β
Overlap b/w Λ & N	Large	Small
ΛN attraction	Large	Small
Λ in p orbit		
	Small β	Large β
Overlap b/w Λ & N	Small	Large
ΛN attraction	Small	Large

Deformation change by Λ particle

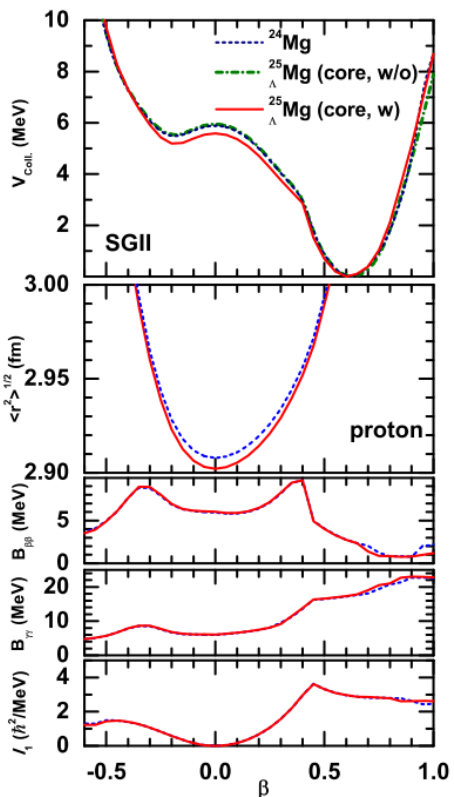
◆ Deformation changes on (β, γ) plane: triaxiality

non-relativistic energy density functional theory

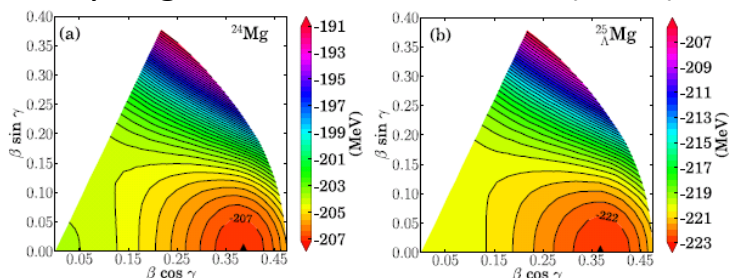
Skyrme-Hartree-Fock

Relativistic mean-field

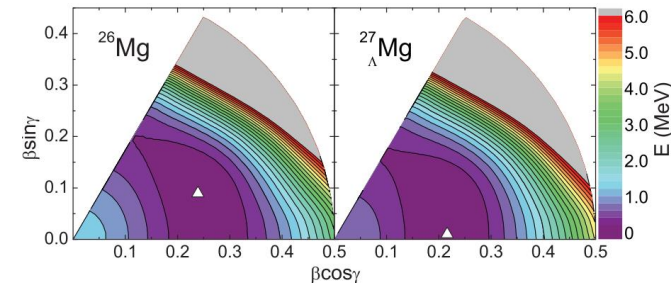
J.M. Yao, et al, NPA868 (2011)



Myaing Thi Win, et al., PRC83 (2011)

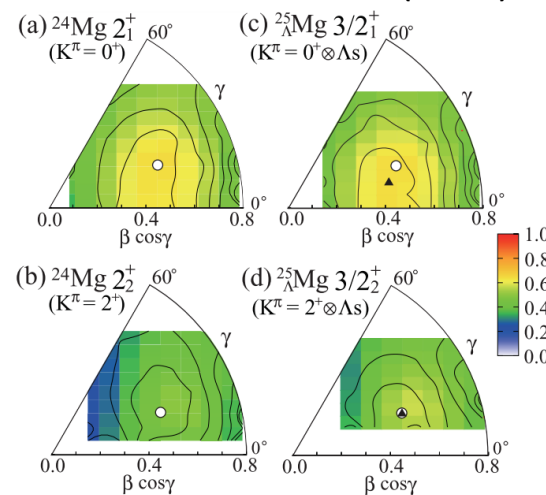


B.N. Lu, et al., PRC84 (2011)

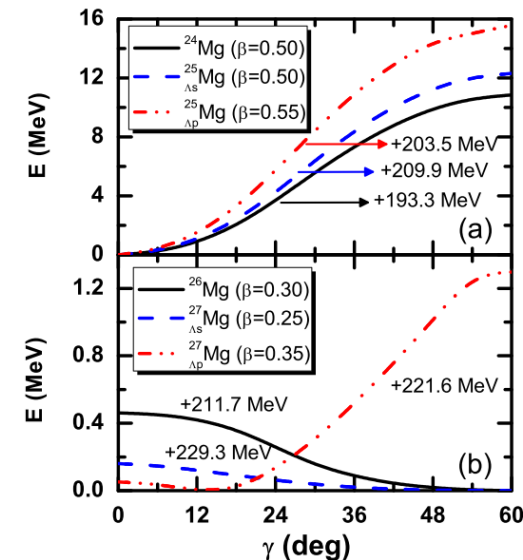
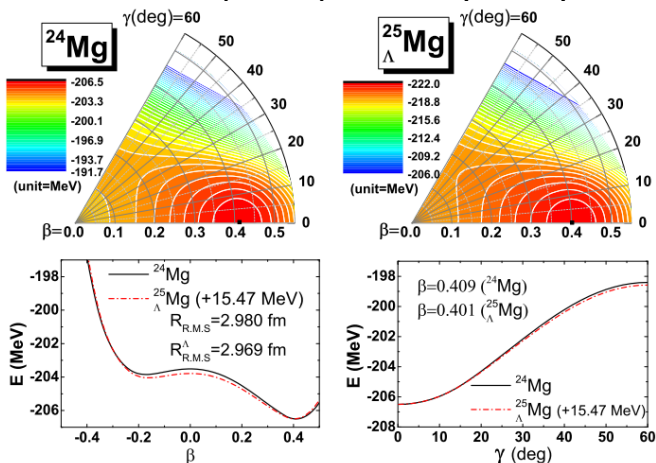


AMD

M.I, et al., PRC85 (2012)



J.W. Cui, et al, PRC91 (2015)



W. X. Xue, et al., PRC91(2015)

Changes of triaxially by Λ is also discussed by several authors

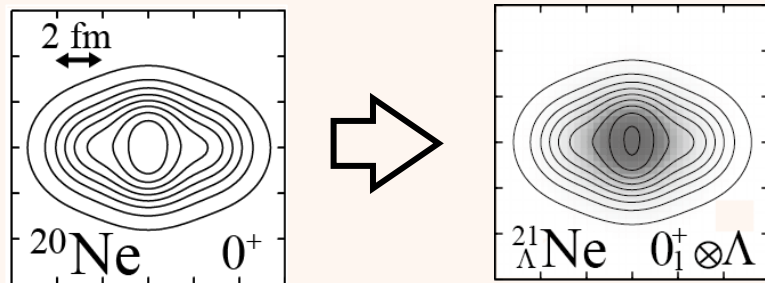
Structure dependence of “impurity effects”

◆ Example: $^{21}_{\Lambda}\text{Ne}$

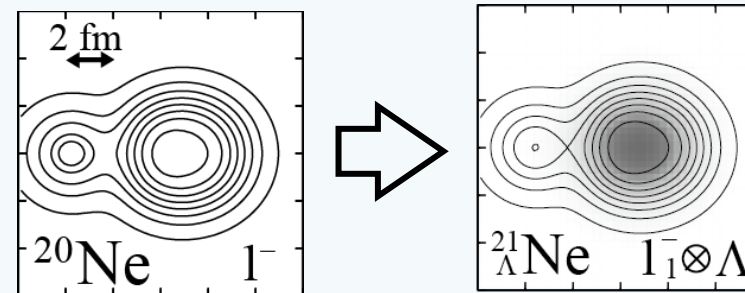
M. Isaka, et al., PRC83, 054304(2011)

- Shrinkage/deformation change are larger in $\alpha + ^{16}\text{O} + \Lambda$ cluster states, which appears as difference in intra-band B(E2) reduction

Ground band

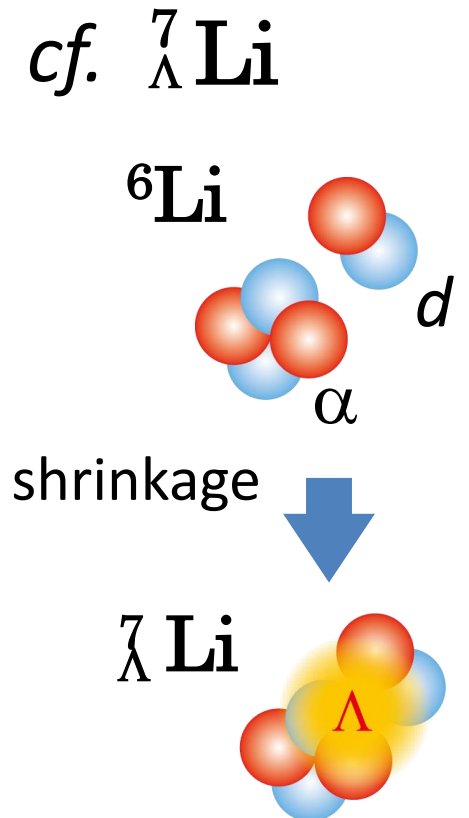


$K^\pi = 0^- (\alpha + ^{16}\text{O})$ band



^{20}Ne		$^{21}_{\Lambda}\text{Ne}$		
$K^\pi=0^+$	$r_{\text{RMS}}(\text{fm})$	$0^+ \otimes \Lambda s$	$r_{\text{RMS}}(\text{fm})$	$\Delta r_{\text{RMS}}(\text{fm})$
0+	2.97	(1/2)+	2.92	-0.05
2+	2.96	(3/2)+ (5/2)+	2.91	-0.05
4+	2.93	(7/2)+ (9/2)+	2.87	-0.06
6+	2.87	(11/2)+ (13/2)+	2.81	-0.05

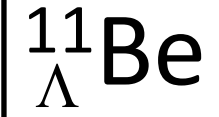
^{20}Ne		$^{21}_{\Lambda}\text{Ne}$		
$K^\pi=0^-$	$r_{\text{RMS}}(\text{fm})$	$0^- \otimes \Lambda s$	$r_{\text{RMS}}(\text{fm})$	$\Delta r_{\text{RMS}}(\text{fm})$
1-	3.27	(1/2)-	3.15	-0.11
3-	3.24	(3/2)- (5/2)- (7/2)-	3.13	-0.11
5-	3.23	(9/2)- (11/2)-	3.11	-0.12
7-	3.23	(13/2)- (15/2)-	3.06	-0.17



Λ ハイパー核の構造として期待できること

- glue-like role: Λ 粒子により、非束縛状態が束縛する
- 構造の動的変化: 変形の変化、核半径の収縮
- 変形・構造の違いに対するエネルギー(B_{Λ})の違い
- Λ 粒子が変形状態に結合することで現れる新奇的な状態

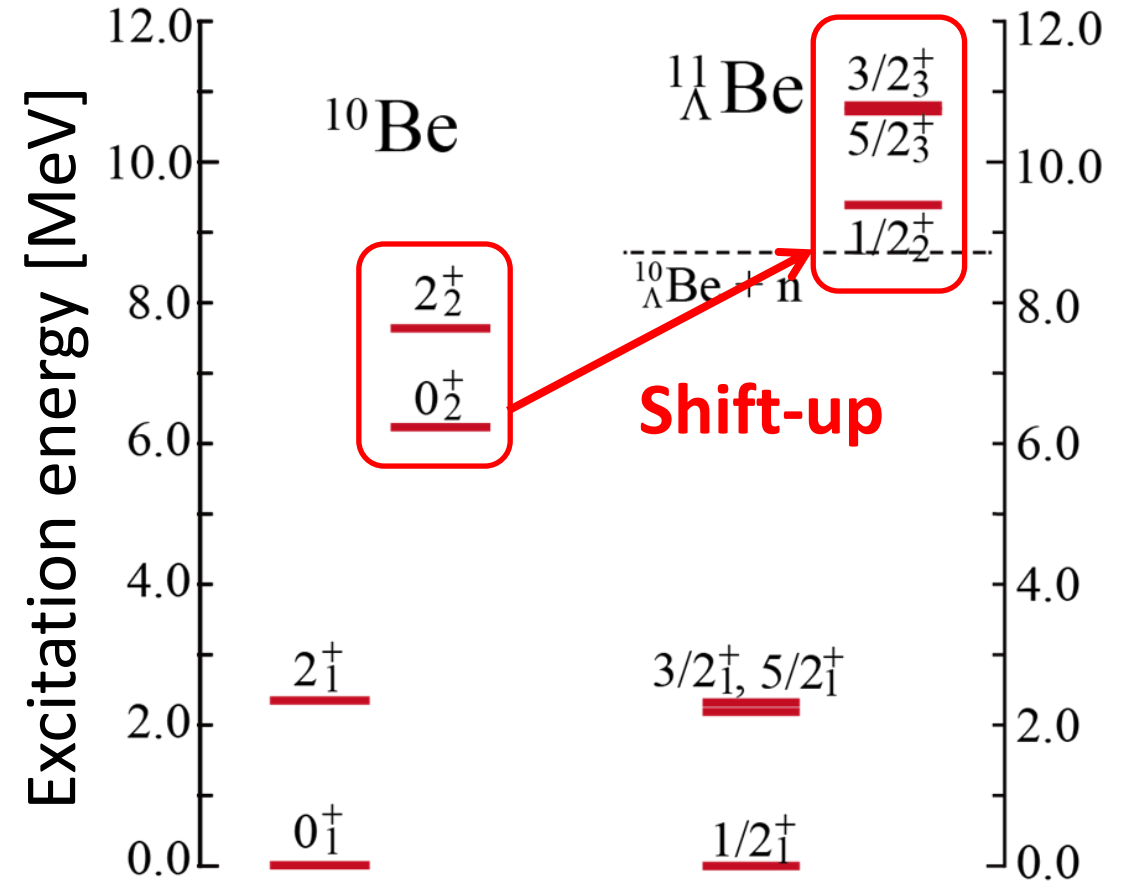
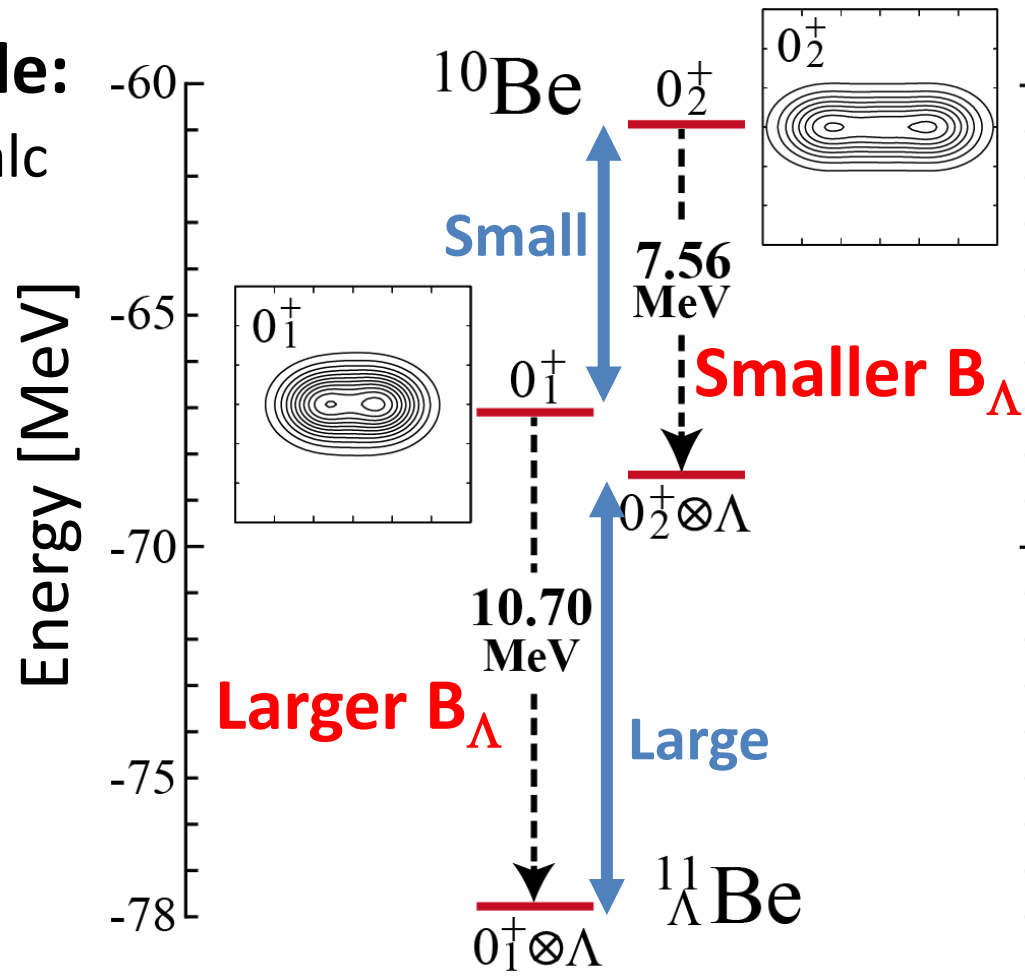
Difference of B_Λ depending on deformation



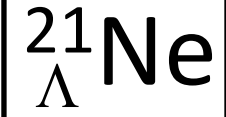
- Different deformations coexist near the ground state
- Λ in s orbit reduces it, but the difference remains
- B_Λ is different due to overlap \rightarrow If different deformation coexist, shift-up/down in excitation spectra

M.I. and M. Kimura,
PRC92, 044326(2015)

Example:
AMD calc



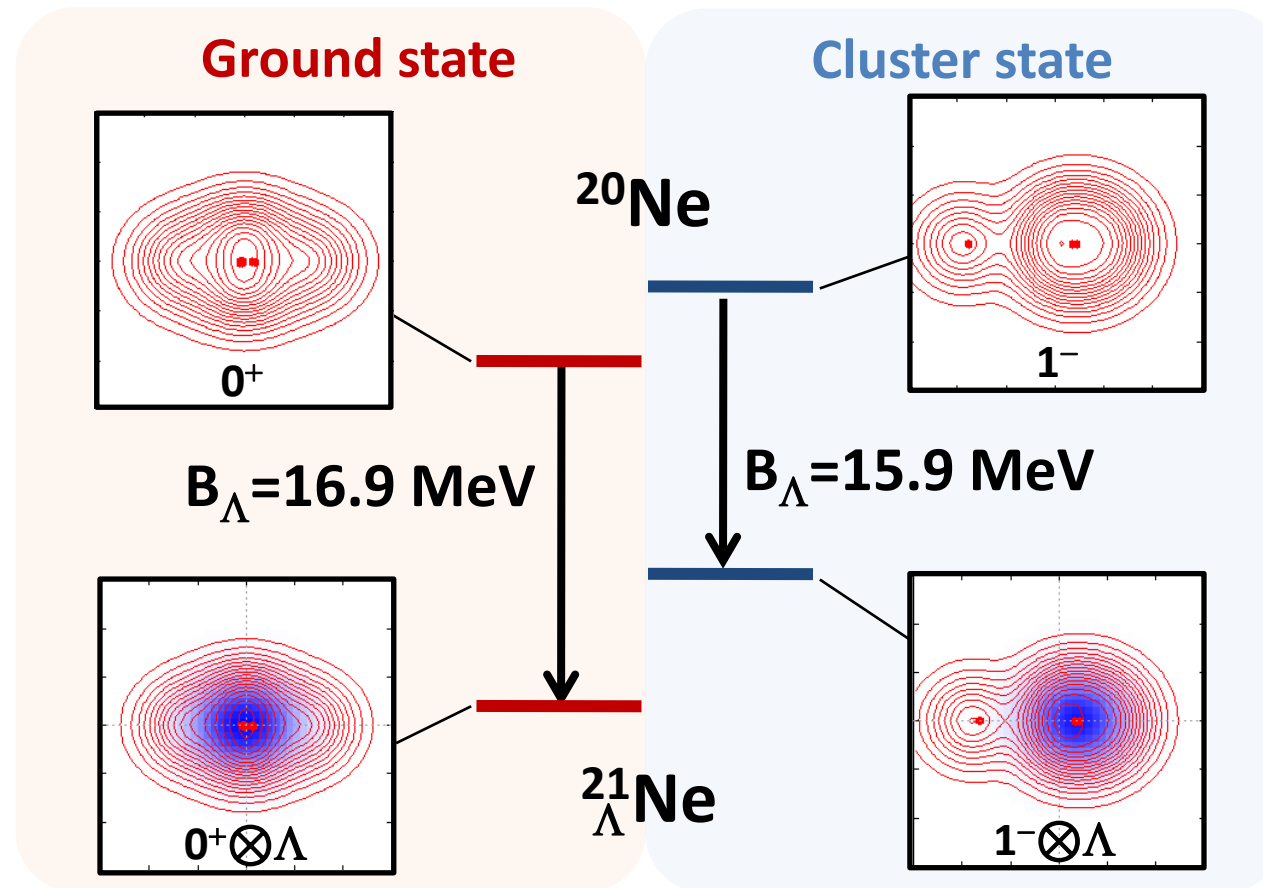
Difference of B_Λ depending on deformation



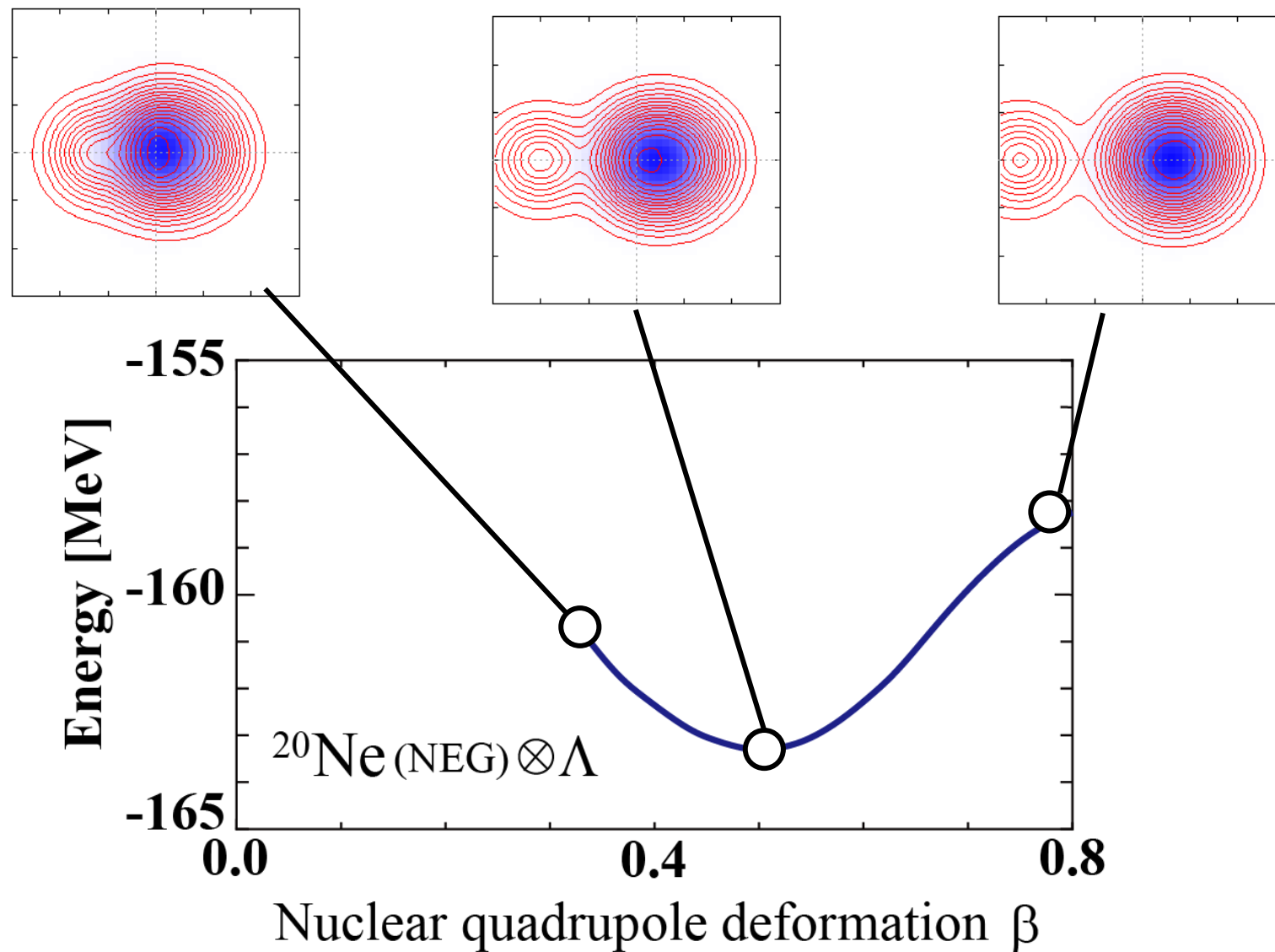
- Different structures coexist near the ground state
- Λ in s orbit changes them, but the difference remains
- Λ is localized around ^{16}O in $\alpha + ^{16}\text{O} + \Lambda$ state \rightarrow difference of B_Λ

M. Isaka, et al.,
PRC83, 054304(2011)

Example:
AMD calc



$\alpha + {}^{16}\text{O} + \Lambda$ states



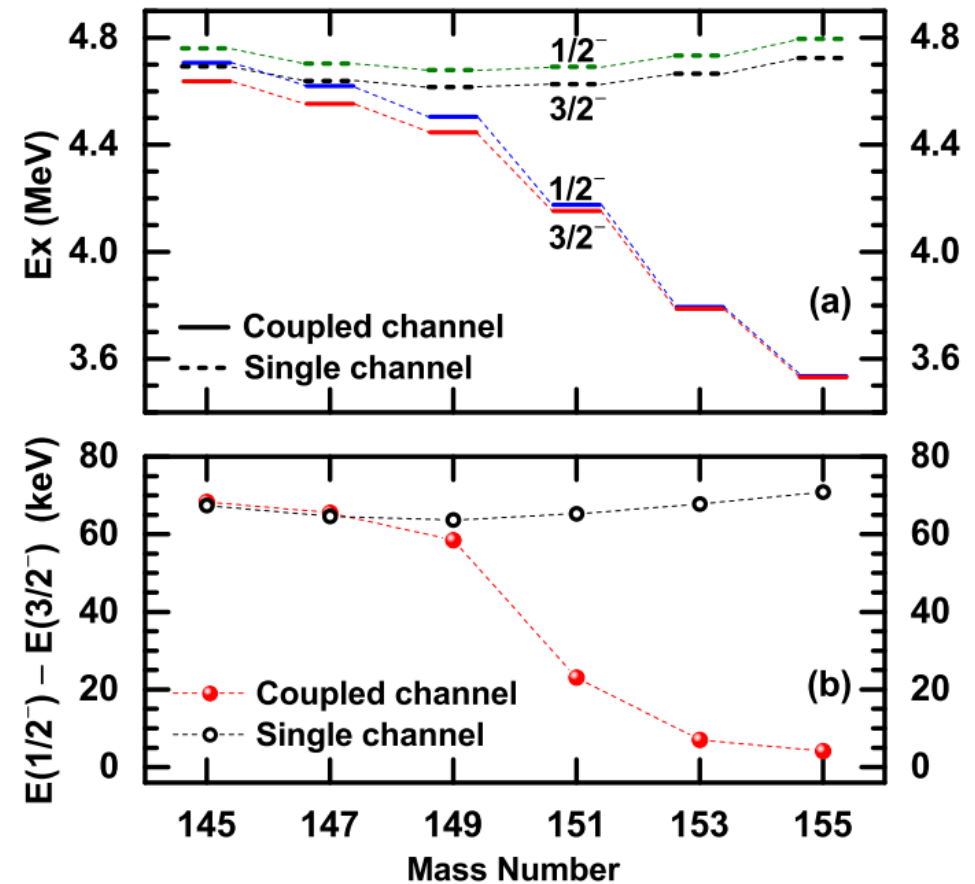
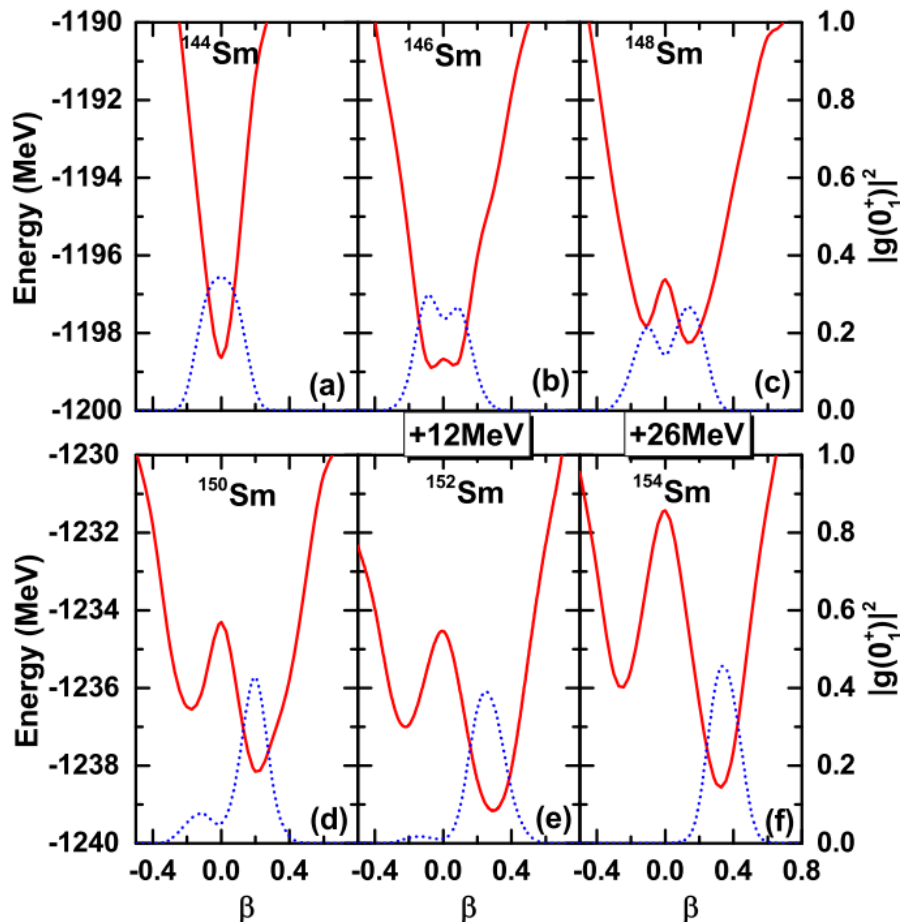
Λ ハイパー核の構造として期待できること

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- Λ 粒子が変形状態に結合することで現れる新奇的な状態

Λ in p orbit coupled to deformed nuclei

H. Mei, K. Hagino, J. M. Yao, T. Motoba, Phys. Rev. C **96**, 014308(2017)

- Nuclear deformation affects excitation energy and energy spacing of p states
- Sm isotopes: vibrational (spherical) to rotational (deformation) character

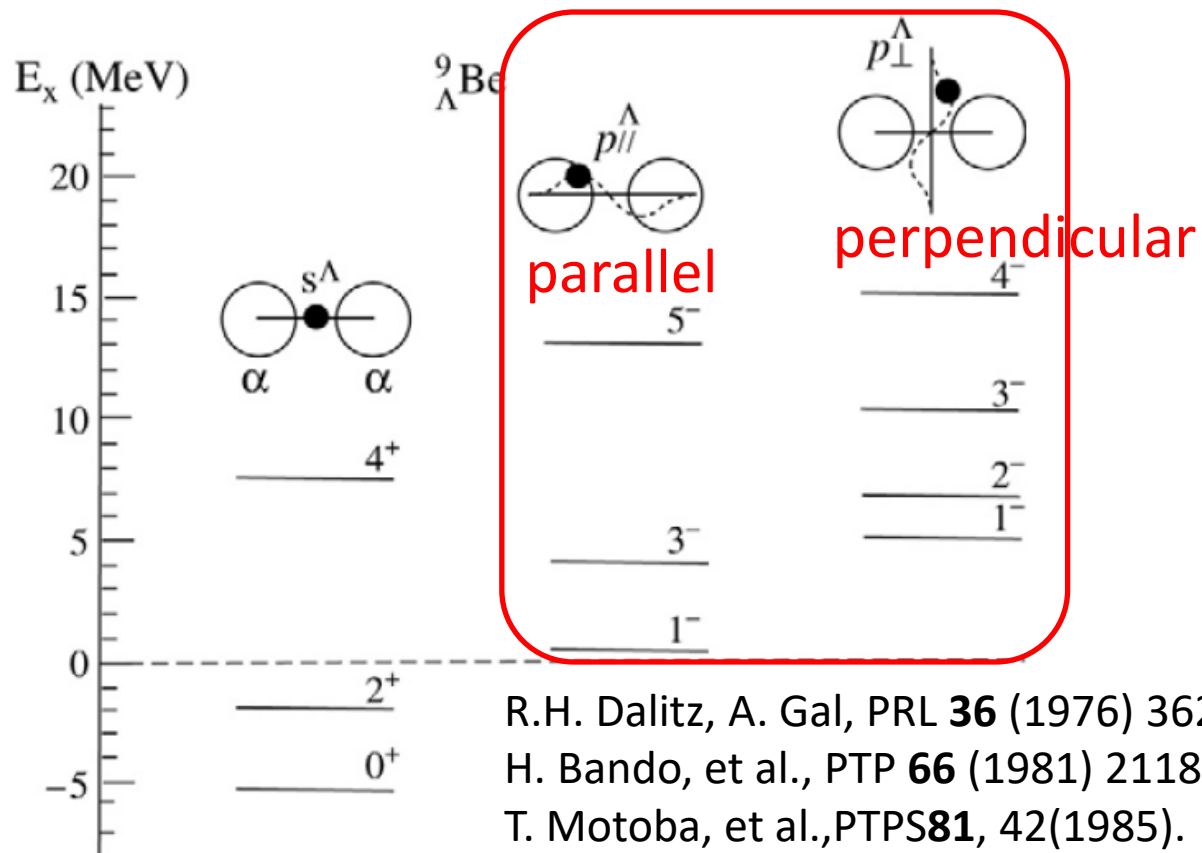


Λ in p orbit coupled to deformed nuclei

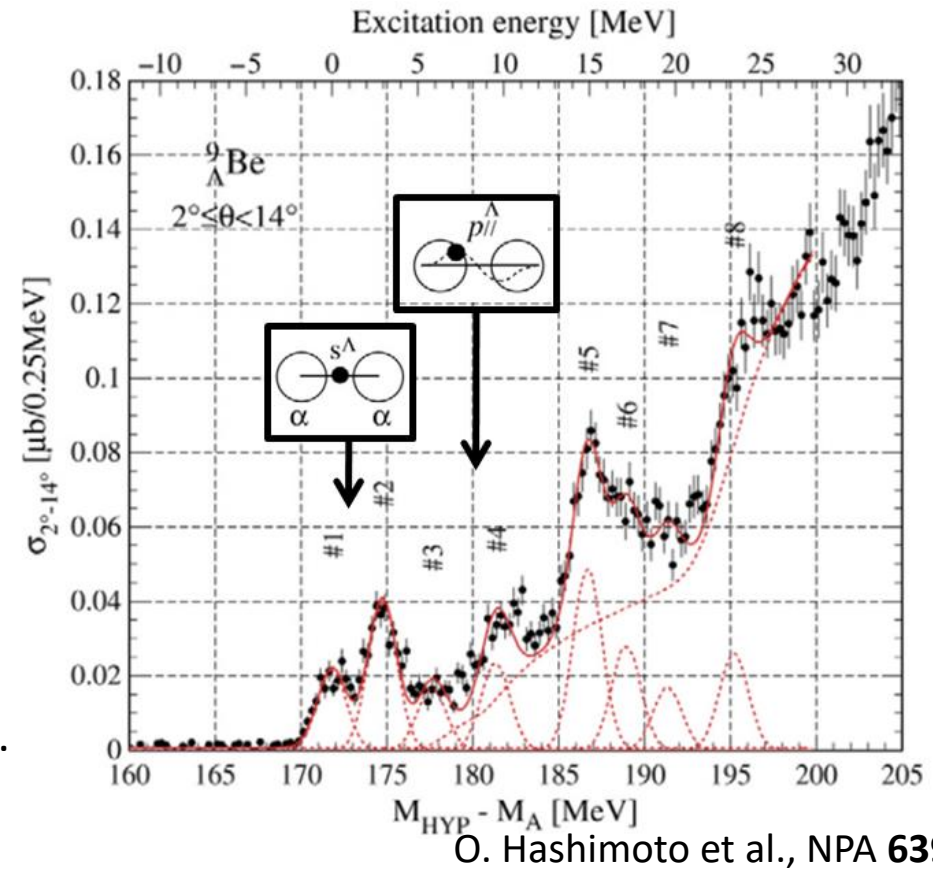
${}^9_{\Lambda}\text{Be}$: axially symmetric 2α clustering

Two rotational bands as p -states $\left\{ \begin{array}{l} \bullet \text{ Anisotropic } p \text{ orbit of } \Lambda \text{ hyperon} \\ \bullet \text{ Axial symmetry of } 2\alpha \text{ clustering} \end{array} \right.$

\rightarrow p -orbit parallel to/perpendicular to the 2α clustering



R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
 H. Bando, et al., PTP **66** (1981) 2118.
 T. Motoba, et al., PTPS**81**, 42(1985).

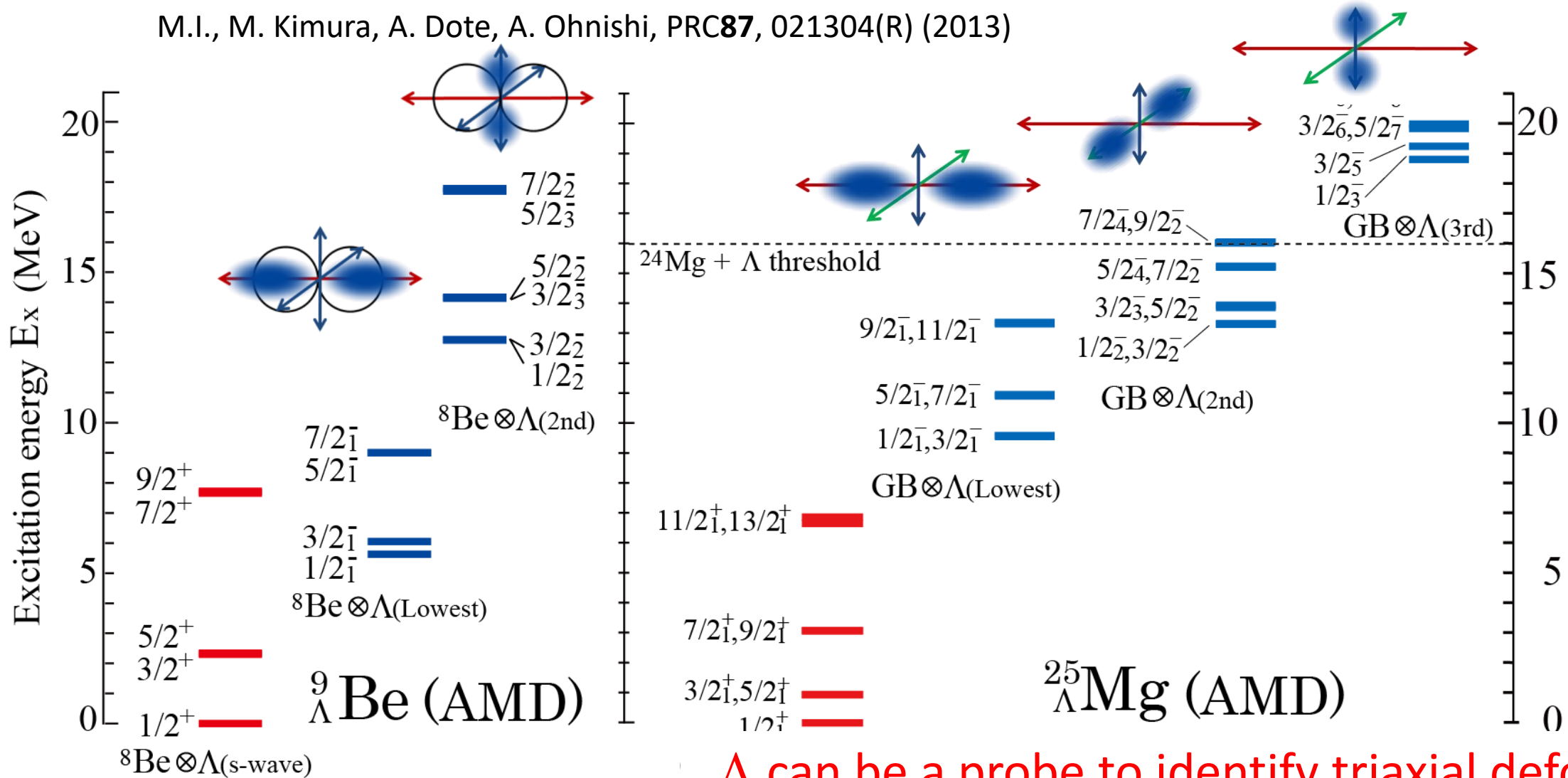


O. Hashimoto et al., NPA **639** (1998) 93c.

Λ in p orbit coupled to deformed nuclei

Splitting of p orbit with **triaxial** deformation \rightarrow 3 different p states

M.I., M. Kimura, A. Dote, A. Ohnishi, PRC87, 021304(R) (2013)

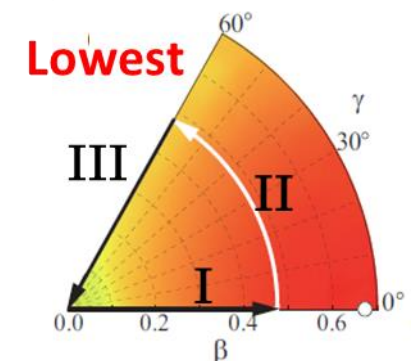
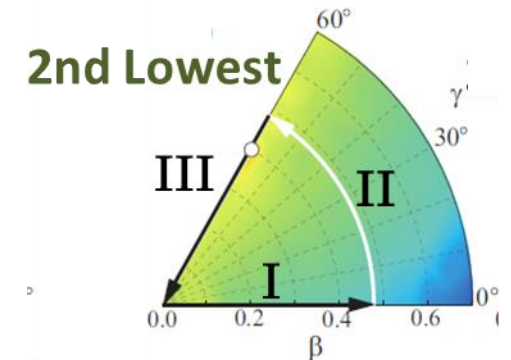
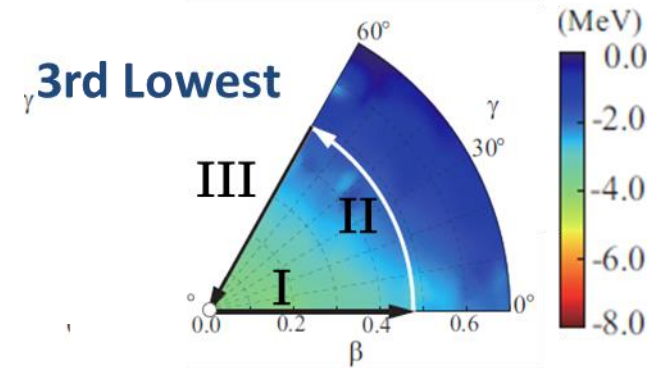
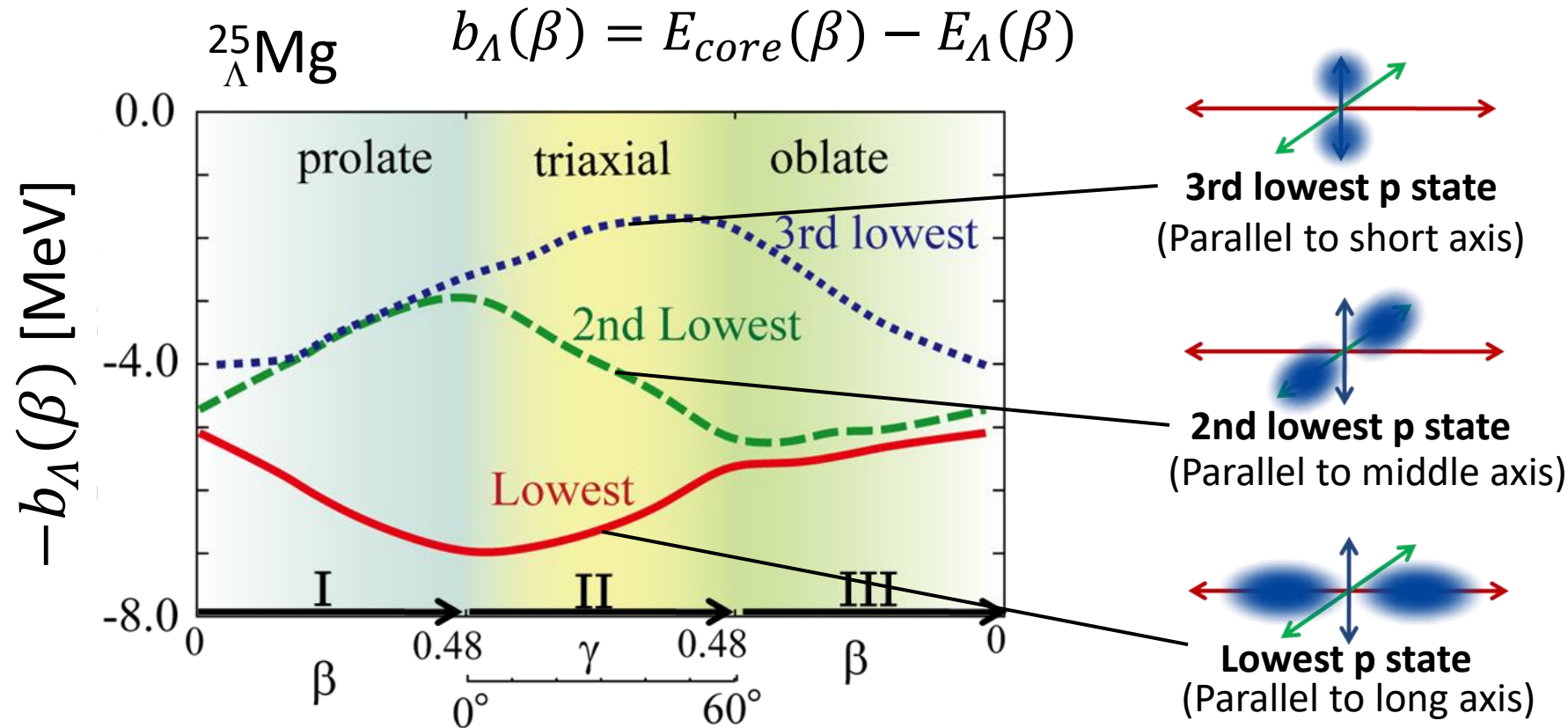


Λ can be a probe to identify triaxial deformation

Λ in p orbit coupled to deformed nuclei

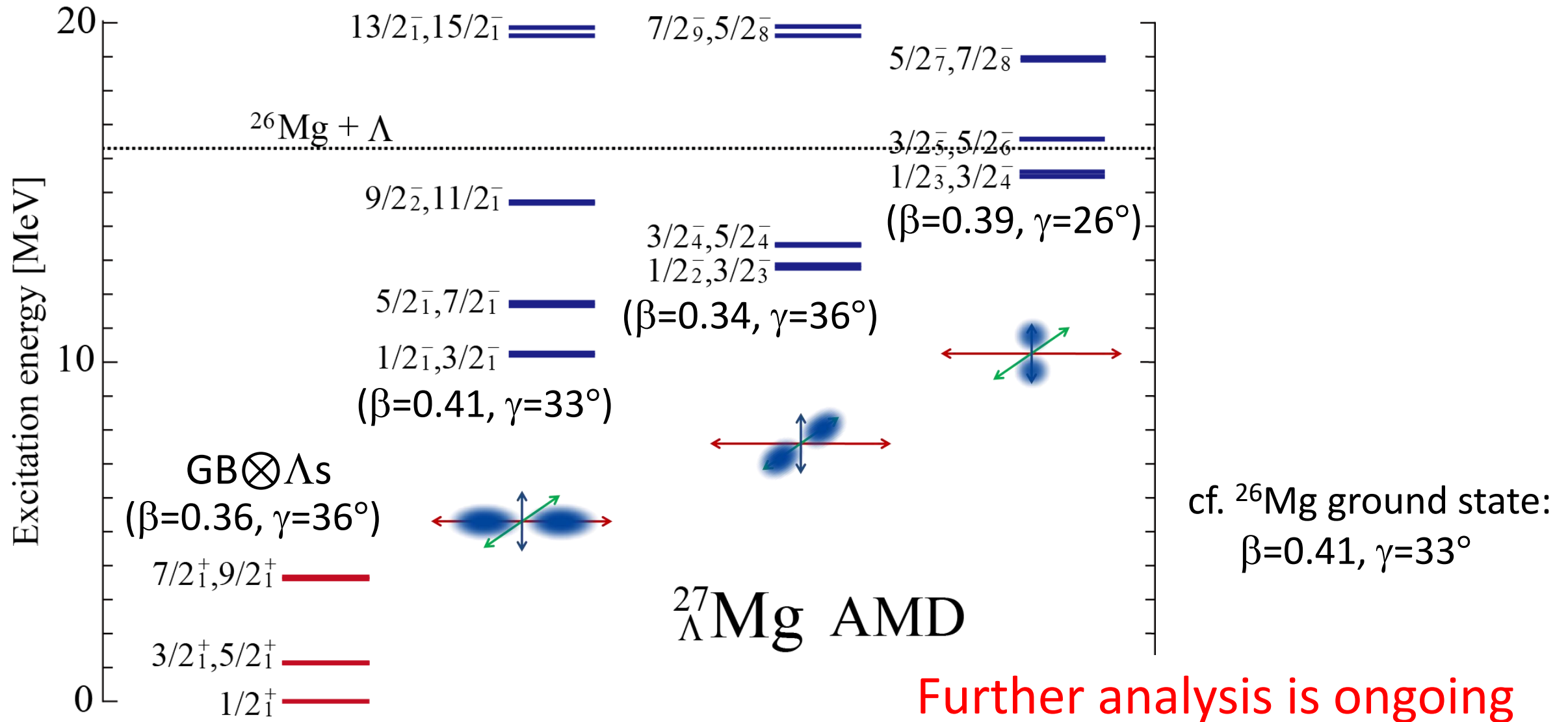
Splitting of p orbit with **triaxial** deformation \rightarrow **3 different p states**

- 3 p -states with different spatial distributions of Λ
- Λ binding energy $b_\Lambda(\beta)$ is different each other due to triaxial deformation



Λ in p orbit coupled to deformed nuclei

● Possibility of $^{27}_{\Lambda}\text{Mg}$ in future JLab experiments



生成断面積：非束縛状態の構造を引き出したい

Production cross section of hypernuclei with AMD wave functions to see effects of various structures

● In future: (γ , K^+) reaction

T. Motoba *et al.*, PTP185, 224(2010)

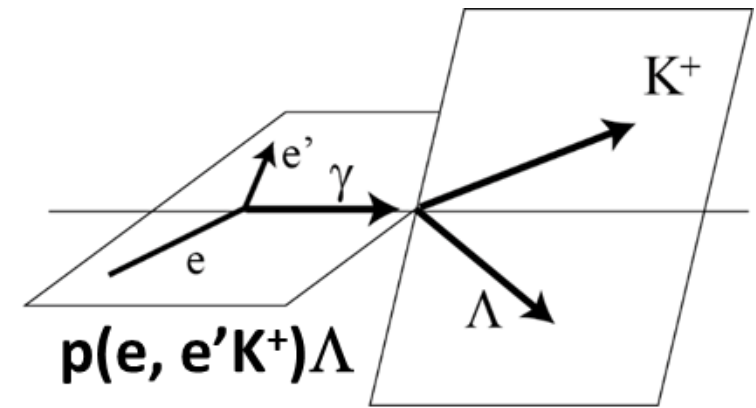
$$\frac{d\sigma}{d\Omega}(\theta_K^{\text{Lab}}) = \frac{sp_K^2 E_K E_H}{p_K (E_H + E_K) - E_\gamma E_K \cos \theta_K^{\text{Lab}}} \sum_{M_f} R(fi; M_f),$$

$$R(fi; M_f) = \frac{1}{2J_i + 1} \sum_{M_i} \Psi_{\text{GCM}}^{J_f \pi M_f} |\langle \Psi_{\text{GCM}}^{J_f \pi M_f} | O | \Psi_{\text{GCM}}^{J_i \pi M_i} \rangle|^2$$

AMD + GCM wave functions
Various structure

$$O = \int d^3r \chi_K^{(-)*}(\mathbf{p}, \xi \mathbf{r}) \chi_K^{(+)}(\mathbf{k}, \mathbf{r}) \sum_{j=1}^A V_-^{(j)} \delta(\mathbf{r} - \eta \mathbf{r}_j) \langle \mathbf{k} - \mathbf{p}, \mathbf{p} | t | \mathbf{k}, 0 \rangle$$

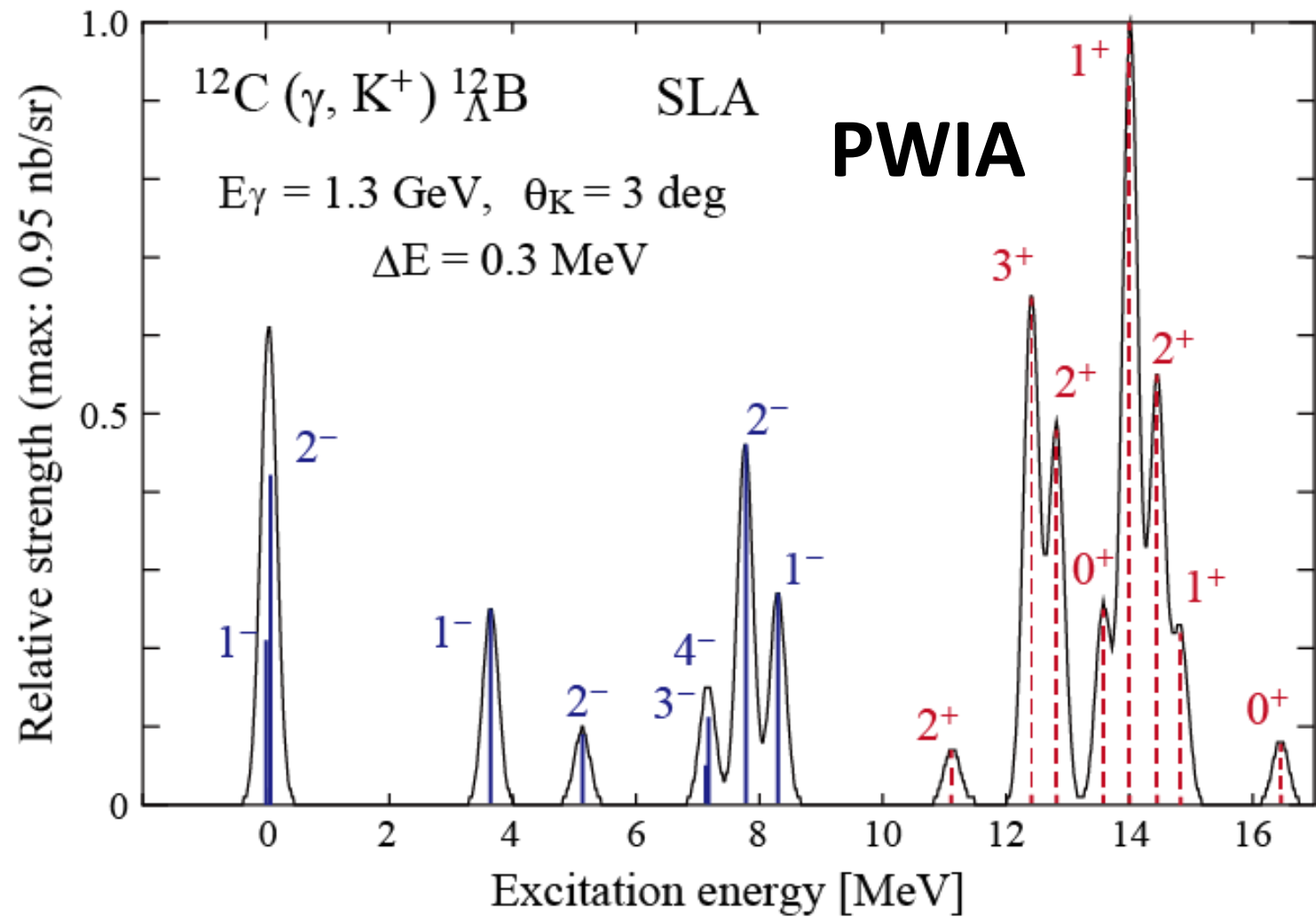
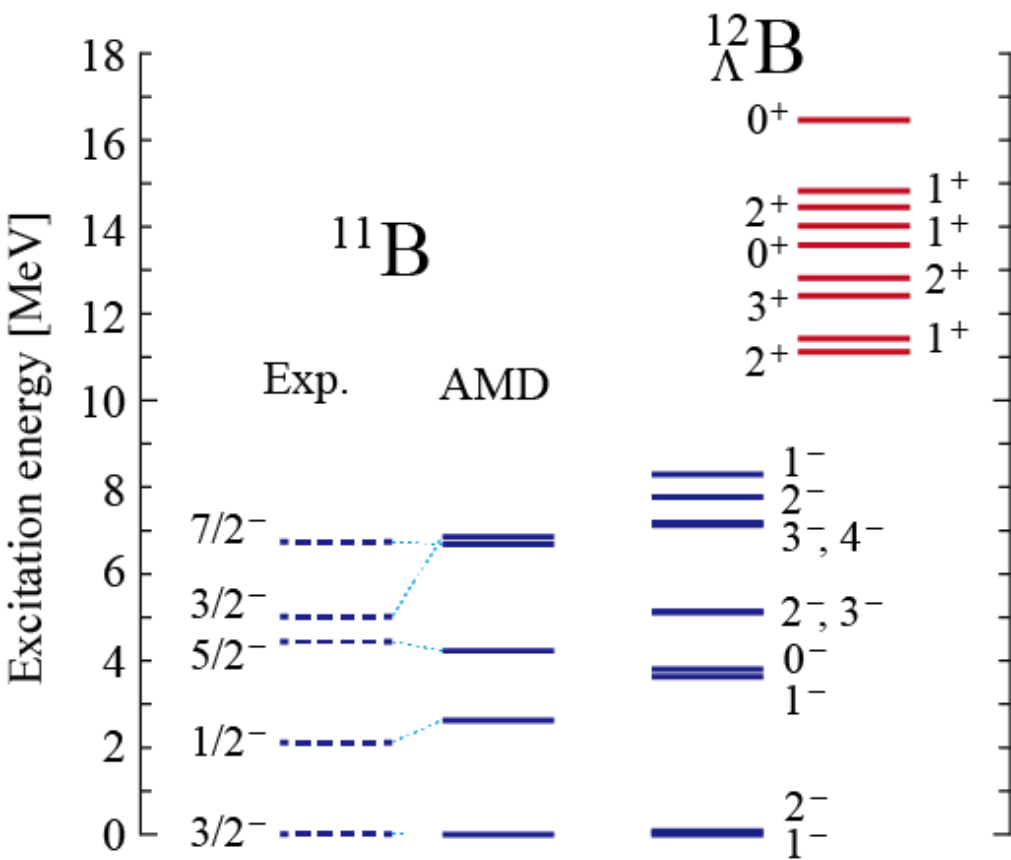
Plane wave
→ Distorted wave
Elementary amplitude



● Current status: PWIA based on effective nucleon number approach

生成断面積：非束縛状態の構造を引き出したい

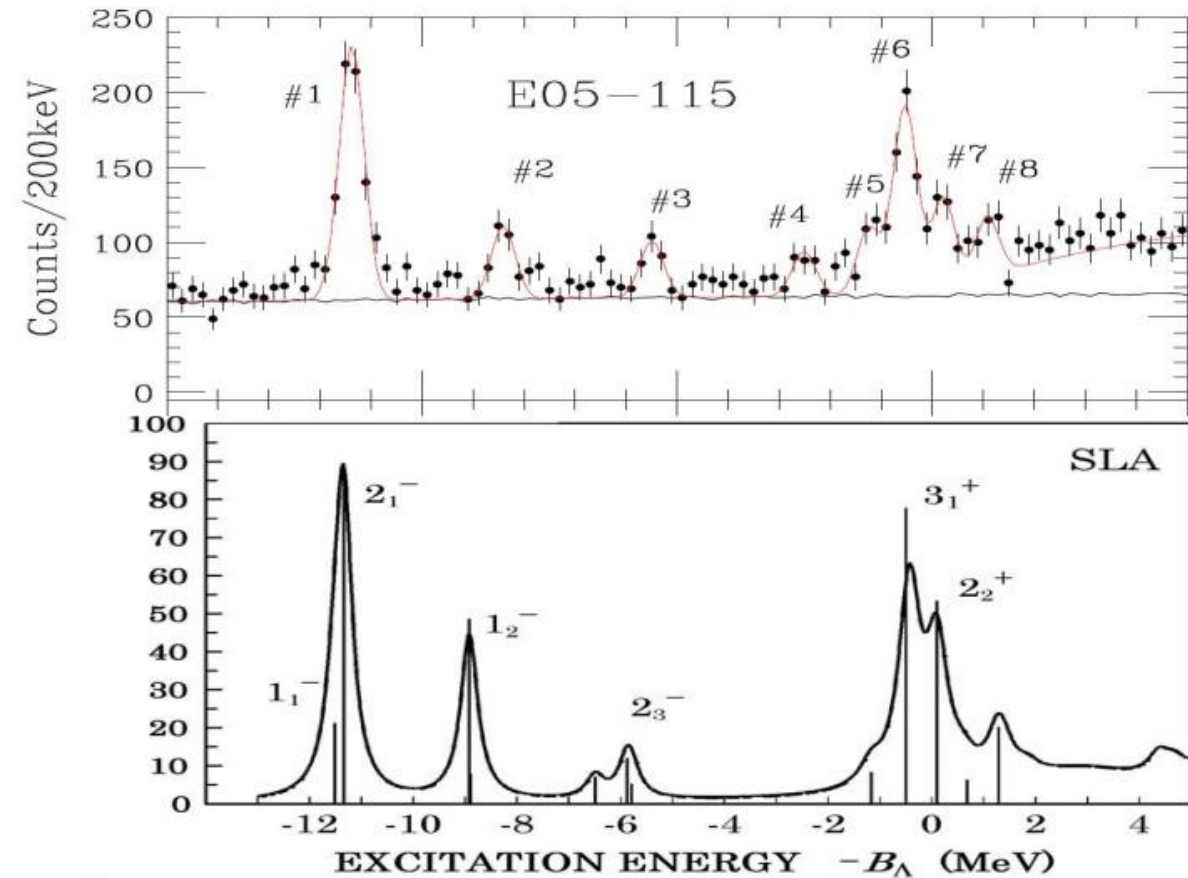
◆ Example: $^{12}\text{C}(\text{g}, \text{K}^+)^{12}_{\Lambda}\text{B}$



生成断面積：非束縛状態の構造を引き出したい

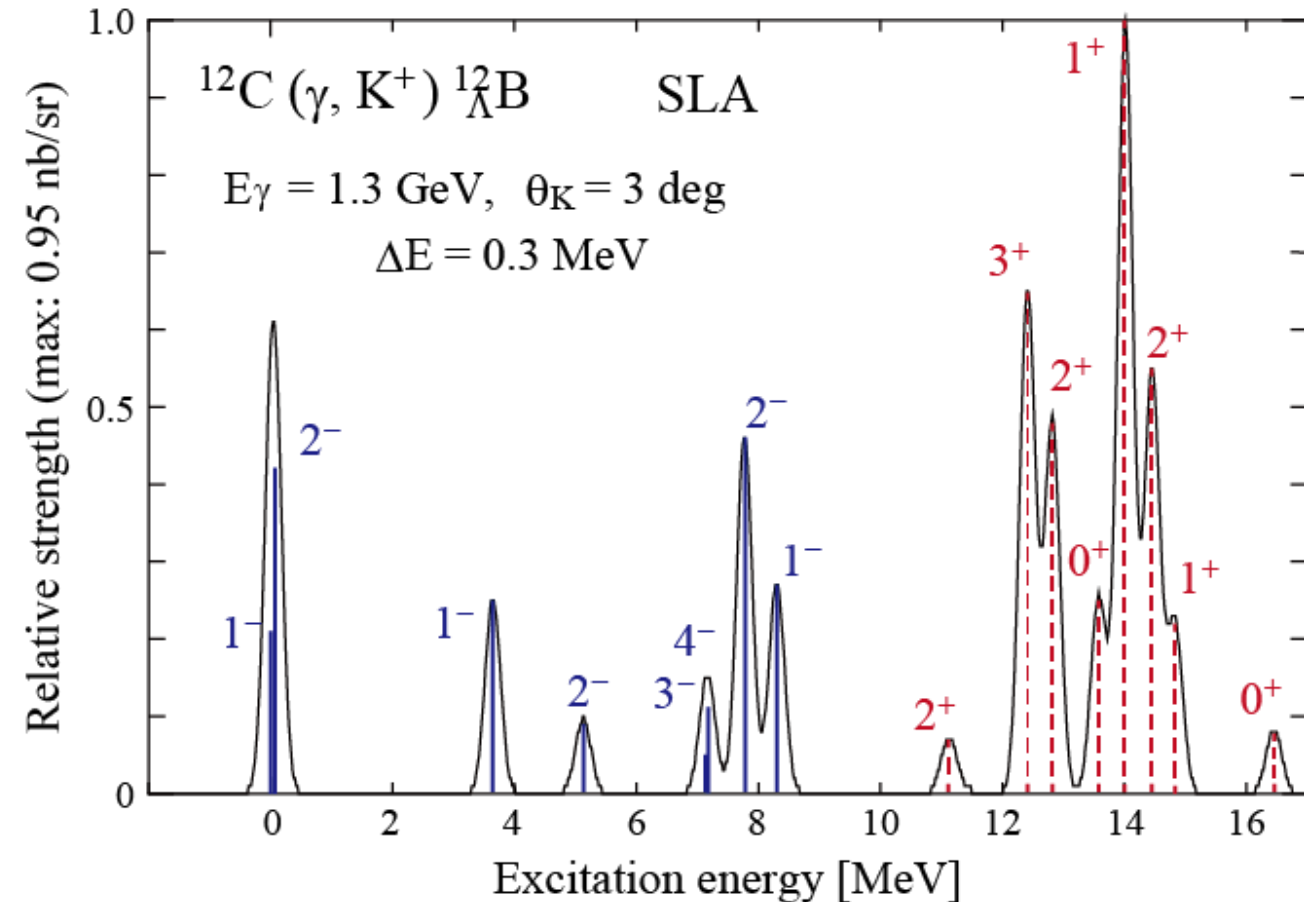
◆ Example: $^{12}\text{C}(\text{g}, \text{K}^+)^{12}_{\Lambda}\text{B}$

L. Tang, et al., PRC90, 034320(2014)



T. Motoba, JPS.Conf.Proc.17,011003(2017)

HyperAMD(present), PWIA



Summary

◆ Λ 粒子が原子核に加わると、何が起こるか？

- glue-like role: Λ 粒子により、非束縛状態が束縛する
- 構造の動的変化: 変形の変化、核半径の収縮
 - “binding energy of Λ ” vs. “energy curve of the core nuclei”
 - Large shrinkage in cluster states
- 変形・構造の違いに対するエネルギー(B_Λ)の違い
 - Shift-up/down in excitation spectra, if different structure coexist
 - 例) $^A_\Lambda\text{Be}$: 2α クラスター構造の違い、 $^{21}_\Lambda\text{Ne}$: 平均場とクラスター
- Λ 粒子が変形状態に結合することで現れる新奇的な状態
 - Deformation can affect excitation spectra/rotational bands of p -states
 - 例) $^{27}_\Lambda\text{Mg}$: Λ をプローブとして核の三軸非対称変形をidentifyできるか