Re-fitting an isobar model's parameters for $K^+\Lambda$ photoproduction using cross-validation

D. Petrellis, D. Skoupil

Nuclear Physics Institute, Czech Academy of Sciences

Outline

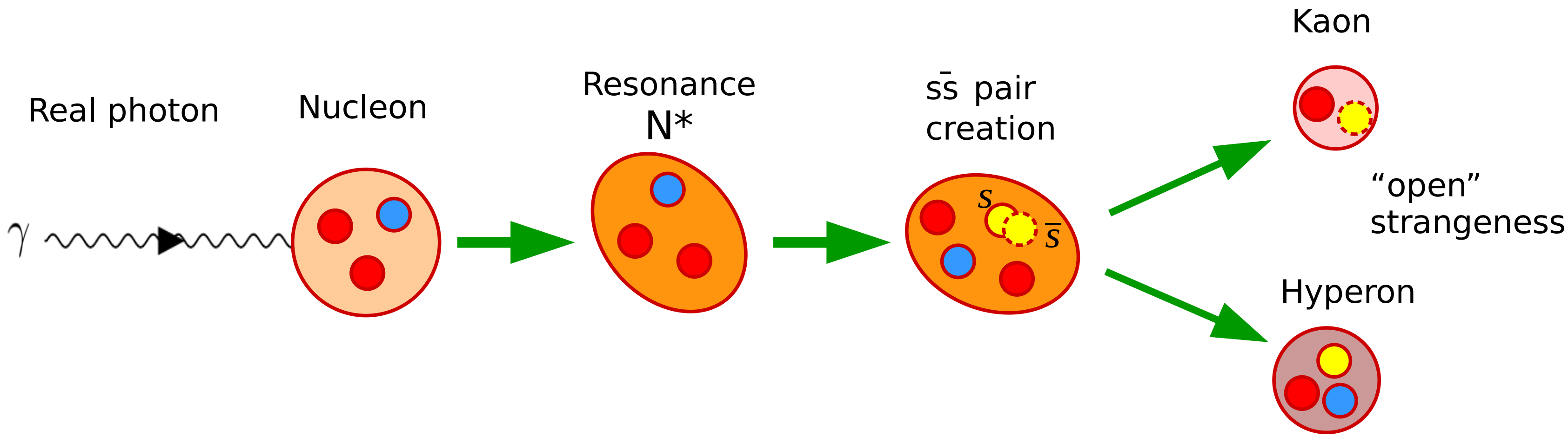
1. Motivation

2. The Isobar model

3. The fitting procedure

4. Numerical results

Photoproduction of kaons and hyperons off nucleons



$$\gamma + p \rightarrow K^{+} + \Lambda$$

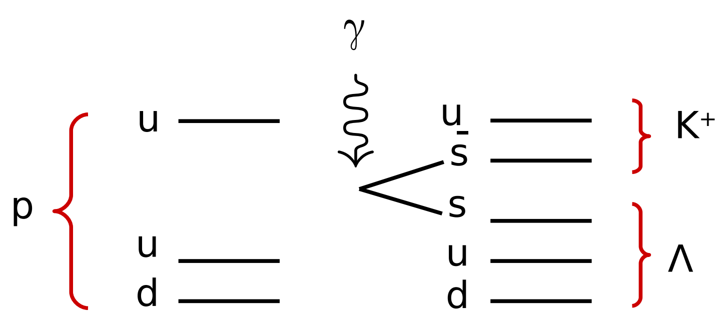
$$\gamma + p \rightarrow K^{+} + \Sigma^{0}$$

$$\gamma + p \rightarrow K^{0} + \Sigma^{+}$$

$$\gamma + n \rightarrow K^{0} + \Lambda$$

$$\gamma + n \rightarrow K^{0} + \Sigma^{0}$$

$$\gamma + n \rightarrow K^{+} + \Sigma^{-}$$



1. Motivation

Motivation

- Why KΛ photoproduction?
  - "missing" resonances: possible decays to kaon-hyperon channels
  - Information on elementary process → important for predictions on production of Λ hypernuclei
- Why refit?
  - Include recent measurements of *polarization observables*
  - Need to investigate role of *hyperon resonances* in KΛ photoproduction
  - Large values of hyperon *couplings* → **Ridge** regression to suppress them during the fitting process

2. The Isobar model

General features of isobar models

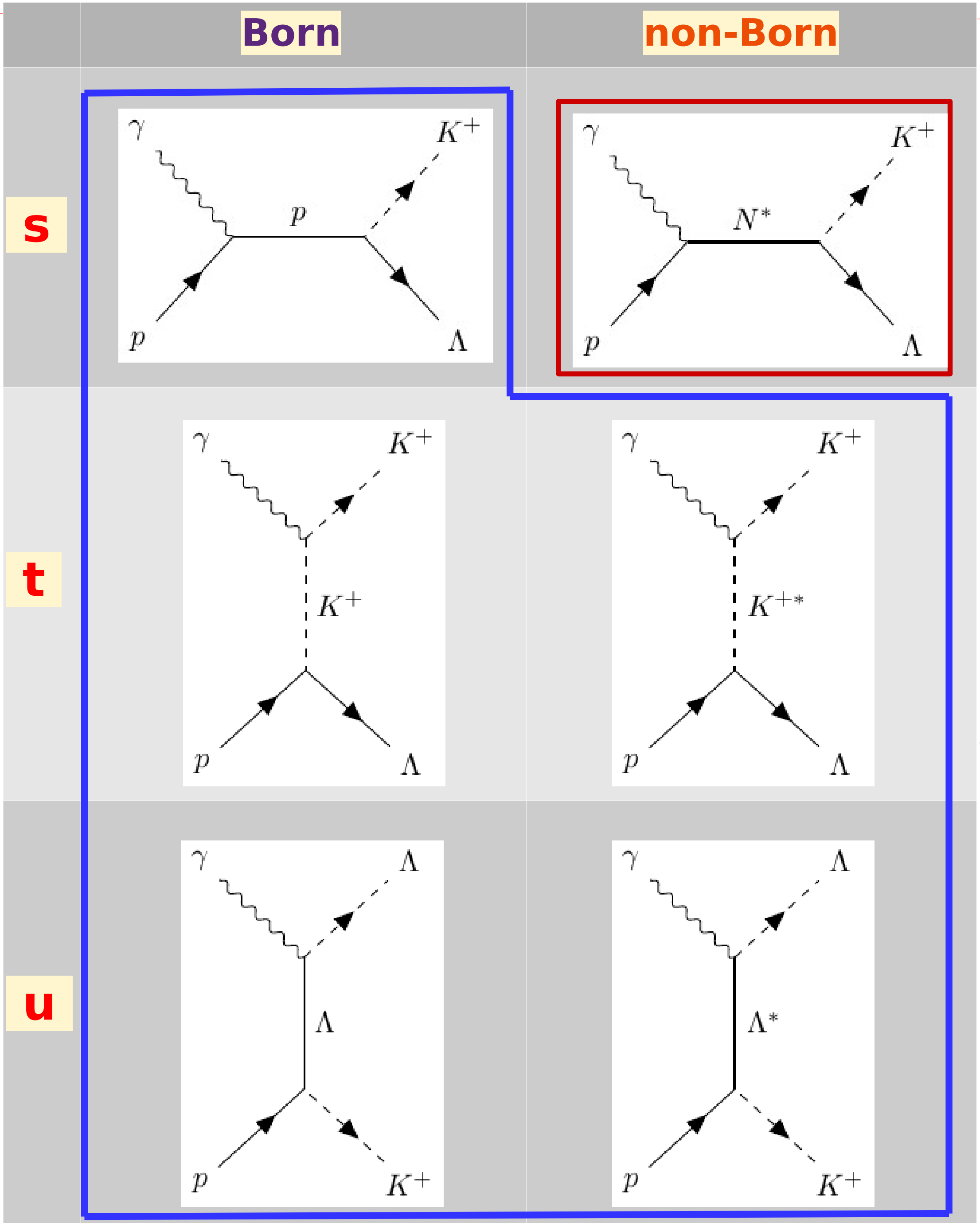
- interactions described by means of effective Lagrangians
  - effective degrees of freedom: hadrons

- amplitude = sum of tree-level Feynman diagrams
  - s-, t-, u- channels: exchange of nucleon, kaon, hyperon
  - intermediate state: ground state hadron (Born), resonance (non-Born)

- single-channel approach: intermediate channels (2^{nd} and higher orders) not taken into account → coupling constants: *effective* values

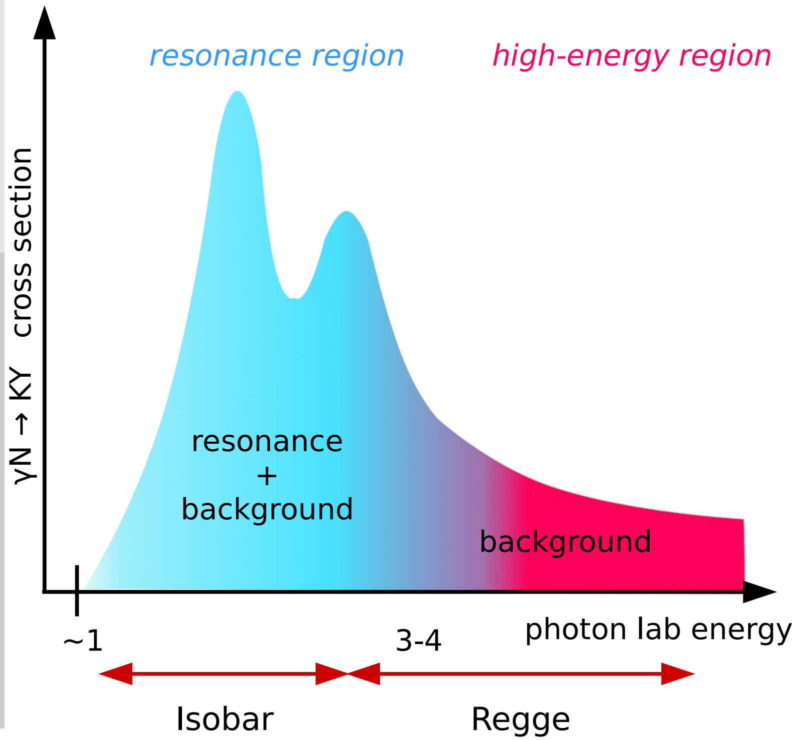
- Saclay-Lyon, MAID & Kaon-MAID, Gent, BS1,2,3 models

Tree-level contributions to p(γ,K^+)Λ



the only resonant diagrams

rest of diagrams
→ background



Specific features of isobar model

Hadronic form factors

- hadron internal structure
- mitigate Born terms' contribution to cross sections

$$F_d = \frac{\Lambda_h^4}{\Lambda_h^4 + \left(x - m_h^2\right)^2}$$

Λ_h cutoff → high energy behaviour of interaction

x 4-momentum^2,

m_h mass,

of intermediate hadron h

Decay widths

- finite lifetime of resonances
- decay widths Γ, introduced by hand in propagators of s-channel particles

$\mathcal{P} \sim \frac{1}{q^2 - m^2}$ $\quad q^2 = s \quad$ $s - m_R^2 \rightarrow s - m_R^2 + i\, m_R \Gamma_R$ $\quad R = N^*, \Delta^*$

Parameters and observables

Resonances

masses, widths: from PDG

Parameters to fit

- $g_{K\Lambda p}$
- coupling constants of resonances
(= products of E/M and strong c.c.)
- hadron form factor cutoffs

3384 data points from: CLAS, LEPS

Observables

differential cross sections $d\sigma/d\Omega$
photon beam asymmetries Σ
target polarization asymmetries T
double polarizations O_x , O_z

Tag	Resonance	Mass [MeV]	Width [MeV]
K*	$K^*(892)$	891.7	50.8
K1	$K_(1272)$	1272	90
N3	$N(1535)\ 1/2^-$	1530	150
N4	$N(1650)\ 1/2^-$	1650	125
P5	$N(1860)\ 5/2^+$	1860	270
N7	$N(1720)\ 3/2^+$	1720	250
P4	$N(1875)\ 3/2^-$	1875	200
P2	$N(1900)\ 3/2^+$	1920	200
P3	$N(2050)\ 5/2^+$	2050	220
N9	$N(1685)\ 5/2^+$	1685	130
N6	$N(1710)\ 1/2^+$	1710	140
L1	$\Lambda(1405)\ 1/2^-$	1405	51
S1	$\Sigma(1660)\ 1/2^+$	1660	100
L4	$\Lambda(1800)\ 1/2^-$	1800	300
S4	$\Sigma(1940)\ 3/2^-$	1940	220

resonances included in the BS2 model

D. Skoupil and P. Bydzovsky, PRC 93, 025204 (2016)

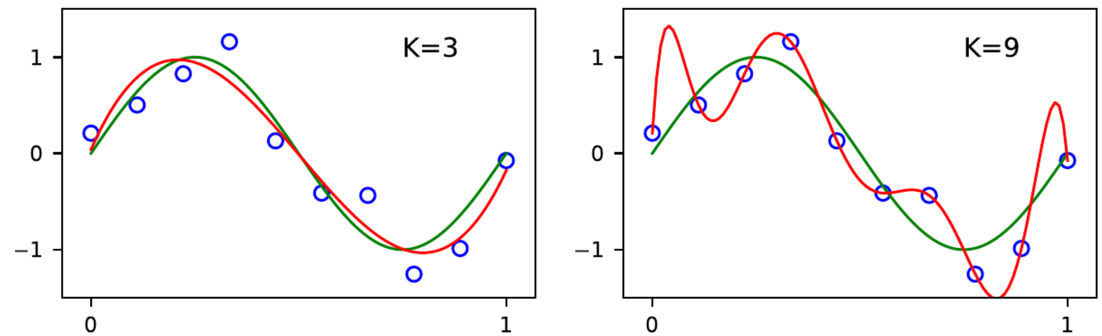
Minimization with: MINUIT Library
Isobar code available at:
http://www.ujf.cas.cz/en/departments/department-of-theoretical-physics/isobar-model.html

3. Fitting procedure

Regularization: a remedy for overfitting

Overfitting

e.g. K-th order polynomial fitting



Regularization

Error function

penalty term

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{y_i - f(x_i, \mathbf{w})}{\sigma_i} \right]^2 \Rightarrow \chi^2_T(\lambda) = \chi^2 + \lambda \sum_{j=1}^{K} |w_j|^q$$

λ: regularization parameter

w_j: model parameters

Constrained error minimization

→ reduces parameter values
↑ λ → ↑ reduction

LASSO:

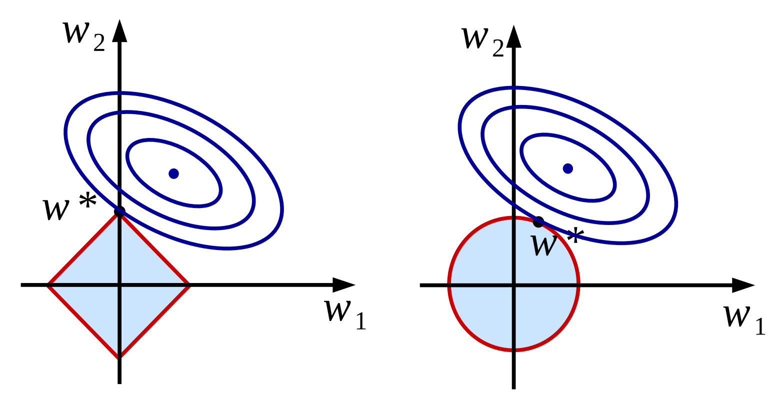
- some parameters become zero ($w^*_1 = 0$) → model selection

Ridge:

- parameter values reduced, but *not* to zero

LASSO

Ridge

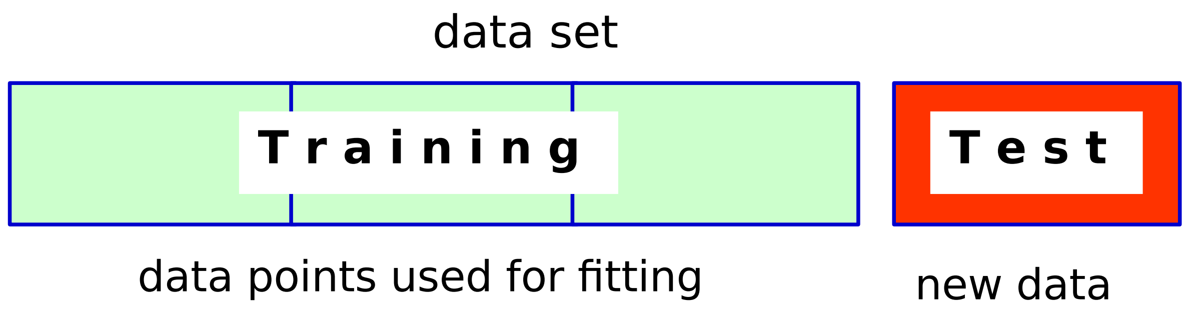


(q=1)

(q=2)

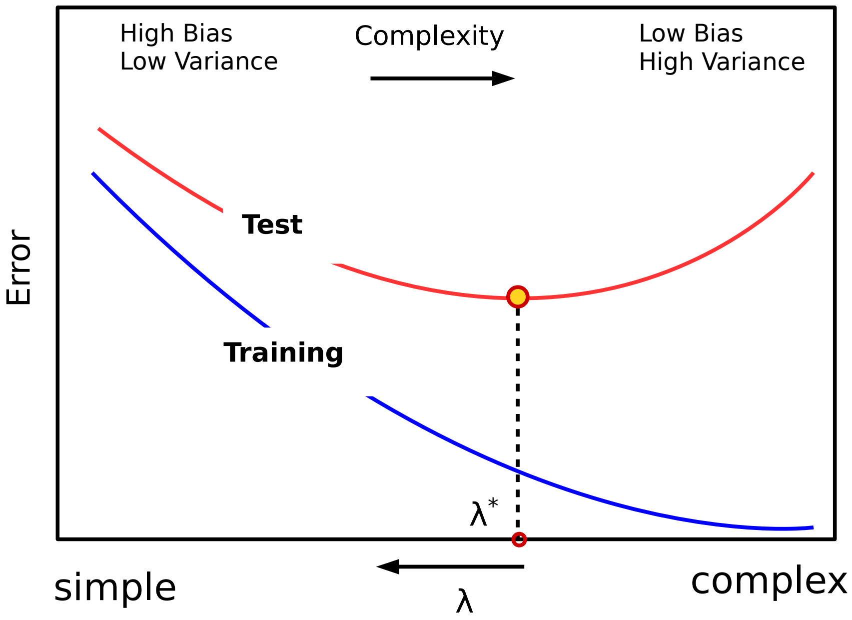
⇨ what is the “right” λ?

Validation: Training & Test set errors



- For each λ within a range $\lambda_{max},\ldots,\lambda_{min}$
- Fit model on the training set → Training Error
- Test the fitted model on the test set → Test Error
- Repeat, using the fitted parameter values of the last run as starting values, while decreasing λ (increasing complexity = Forward selection)
- Test Error_{min} → λ^*, best model

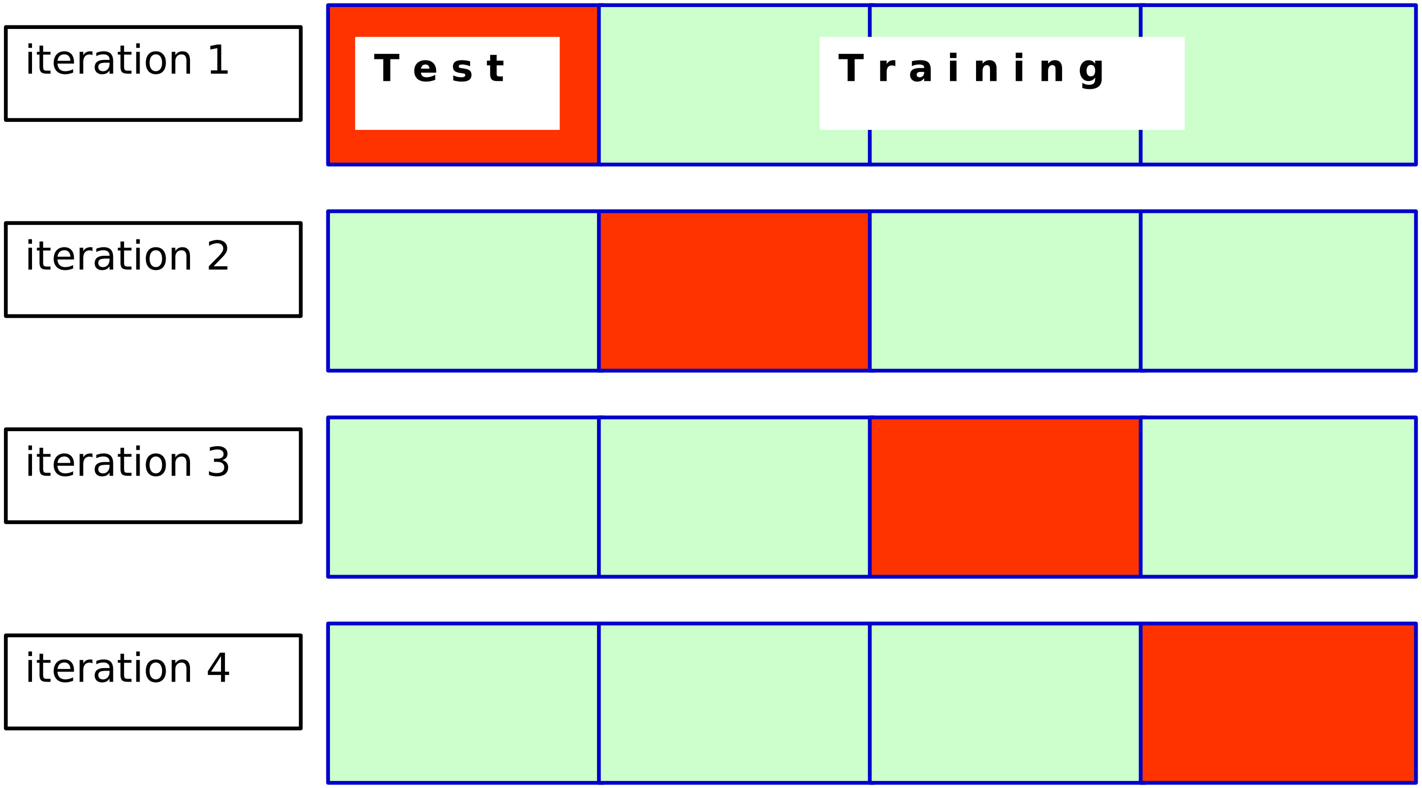
Bias-Variance trade-off



Cross-validation

- to avoid selection bias in the choice of Training / Test sets

e.g. 4-fold
cross-validation



in general: **n**-fold cross-validation → *average* over **n** runs

4. Numerical results

Cross-validation and the "1-σ" rule

Ridge regularization

$$\chi^2_T(\lambda) = \chi^2 + \lambda^4 \sum_{j=1}^{K} |w_j|^2$$

K → no. of hyperon res. couplings

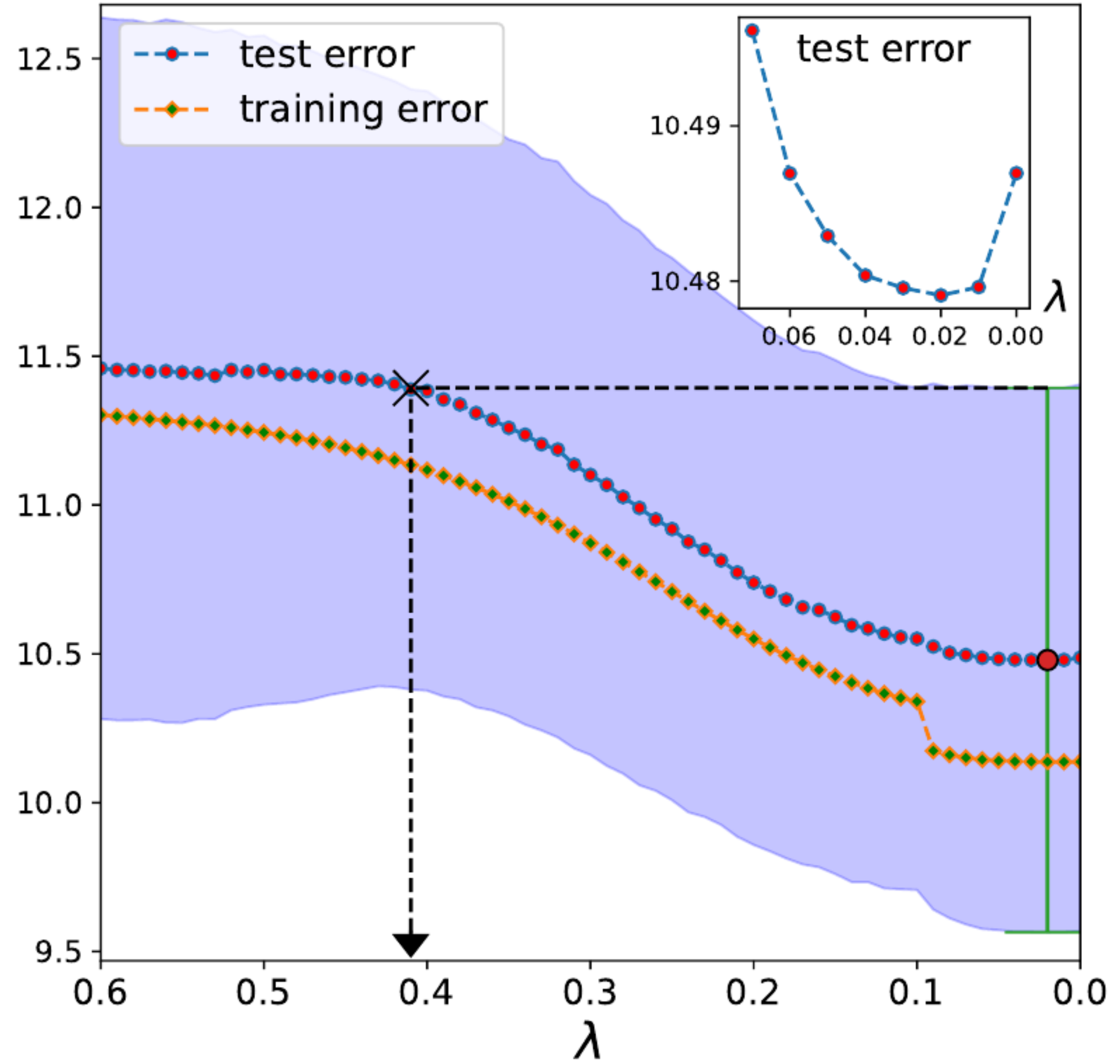
3-fold cross validation

"1-σ" rule

"...choose the most parsimonious model whose error is no more than one standard error above the error of the best model."

T. Hastie, R. Tibshirani, and J. Friedman,

The Elements of Statistical Learning (2009)



although test error minimum at $\lambda^* = 0.02$, "1-σ" rule → $\lambda_0 = 0.41$

Effect on the couplings of hyperon resonances

BS2

Tag	Resonance	g_1	g_2
L1	$\Lambda(1405)$ 1/2	9.67	–
S1	$\Sigma(1660)$ $1/2^+$	-8.09	–
L4	$\Lambda(1800)$ $1/2^-$	-11.55	–
S4	$\Sigma(1940)$ $3/2^-$	-0.86	0.18

BS2 with Ridge

Tag	Resonance	Mass [MeV]	Width [MeV]	g_1	g_2
K*	$K^*(892)$	891.7	50.8	-0.176	0.011
K1	$K_(1272)$	1272	90	0.321	-1.136
N3	$N(1535)$ $1/2^-$	1530	150	-0.012	–
N4	$N(1650)$ $1/2^-$	1650	125	-0.075	–
P5	$N(1860)$ $5/2^+$	1860	270	-0.019	0.009
N7	$N(1720)$ $3/2^+$	1720	250	0.157	0.009
P4	$N(1875)$ $3/2^-$	1875	200	0.141	0.135
P2	$N(1900)$ $3/2^+$	1920	200	-0.045	-0.010
P3	$N(2050)$ $5/2^+$	2050	220	-0.012	0.013
N9	$N(1685)$ $5/2^+$	1685	130	0.048	-0.041
N6	$N(1710)$ $1/2^+$	1710	140	-0.172	–
L1	$\Lambda(1405)$ $1/2^-$	1405	51	1.308	–
S1	$\Sigma(1660)$ $1/2^+$	1660	100	-1.938	–
L4	$\Lambda(1800)$ $1/2^-$	1800	300	-0.342	–
S4	$\Sigma(1940)$ $3/2^-$	1940	220	-0.567	-0.025

Polarization observables

Single polarization asymmetry

$$A = \frac{d\sigma^{\lambda_X=+s} - d\sigma^{\lambda_X=-s}}{d\sigma^{\lambda_X=+s} + d\sigma^{\lambda_X=-s}}$$

Double polarization asymmetry

$$\frac{d\sigma^{(++)} + d\sigma^{(--)} - d\sigma^{(+-)} - d\sigma^{(-+)}}{d\sigma^{(++)} + d\sigma^{(--)} + d\sigma^{(+-)} + d\sigma^{(-+)}}$$

$$(\text{+-}) \rightarrow (\lambda_A = +s_A, \lambda_B = -s_B)$$

λ_X : polarization of particle *X*

for *X* = *N*, *Y* → λ_X : *spin projection* of nucleon, hyperon on the y(=y') axis

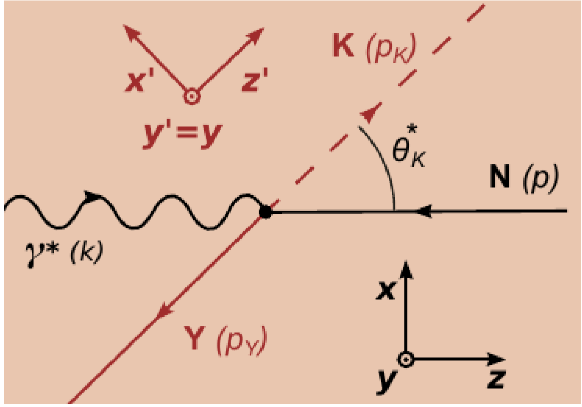
- Target asymmetry: *X* = *N* → **T**
- Recoil asymmetry: *X* = *Y*
- Photon beam asymmetry → **Σ**

- Beam-recoil: *A*,*B* = γ, *Y* → $\mathbf{O_{x'}}$ and $\mathbf{O_{z'}}$
- Beam-target: *A*,*B* = γ, *N*
- Target-recoil: *A*,*B* = *N*, *Y*

$$\Sigma^{\text{lin.}} = \frac{d\sigma^{\perp} - d\sigma^{\parallel}}{2d\sigma^{\text{unpol}}}$$

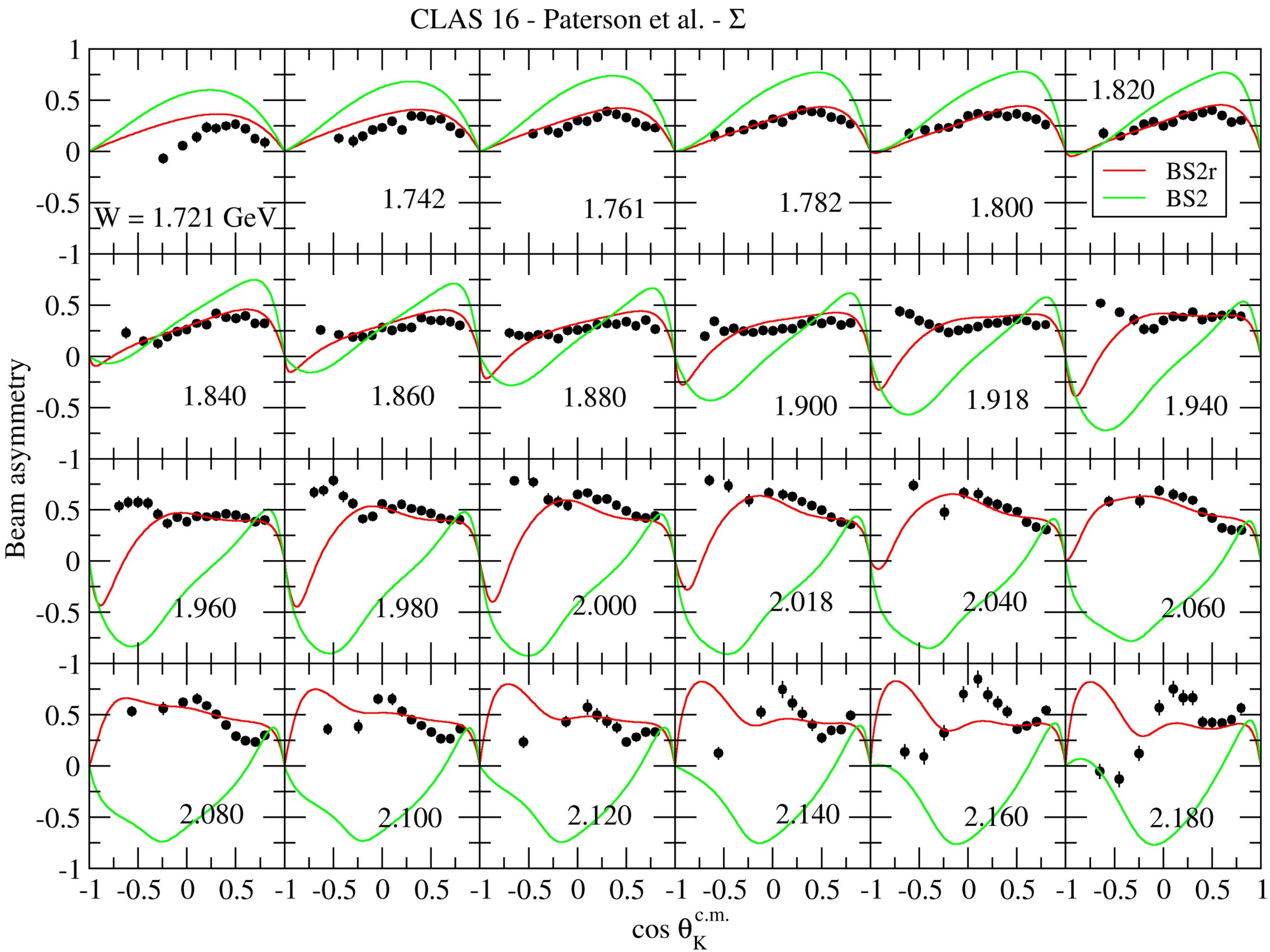
linearly polarized photons

$$\varepsilon^{\lambda=x} \equiv \varepsilon^{\parallel} = (0,1,0,0)$$
$$\varepsilon^{\lambda=y} \equiv \varepsilon^{\perp} = (0,1,0,0)$$

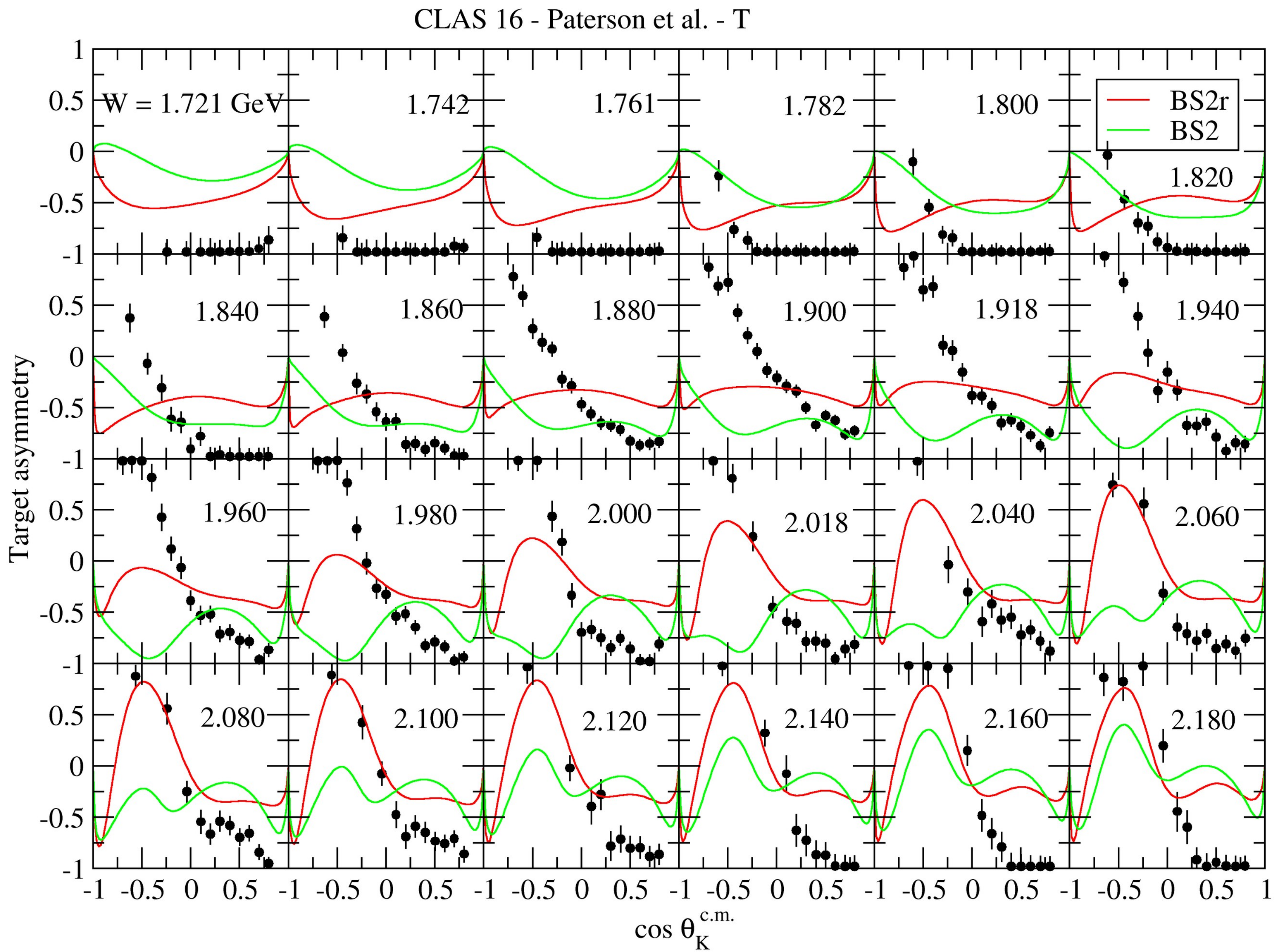


from
L. De Cruz, Bayesian model selection for electromagnetic kaon production in the Regge-plus-resonance framework, PhD Thesis, Ghent University (2012)

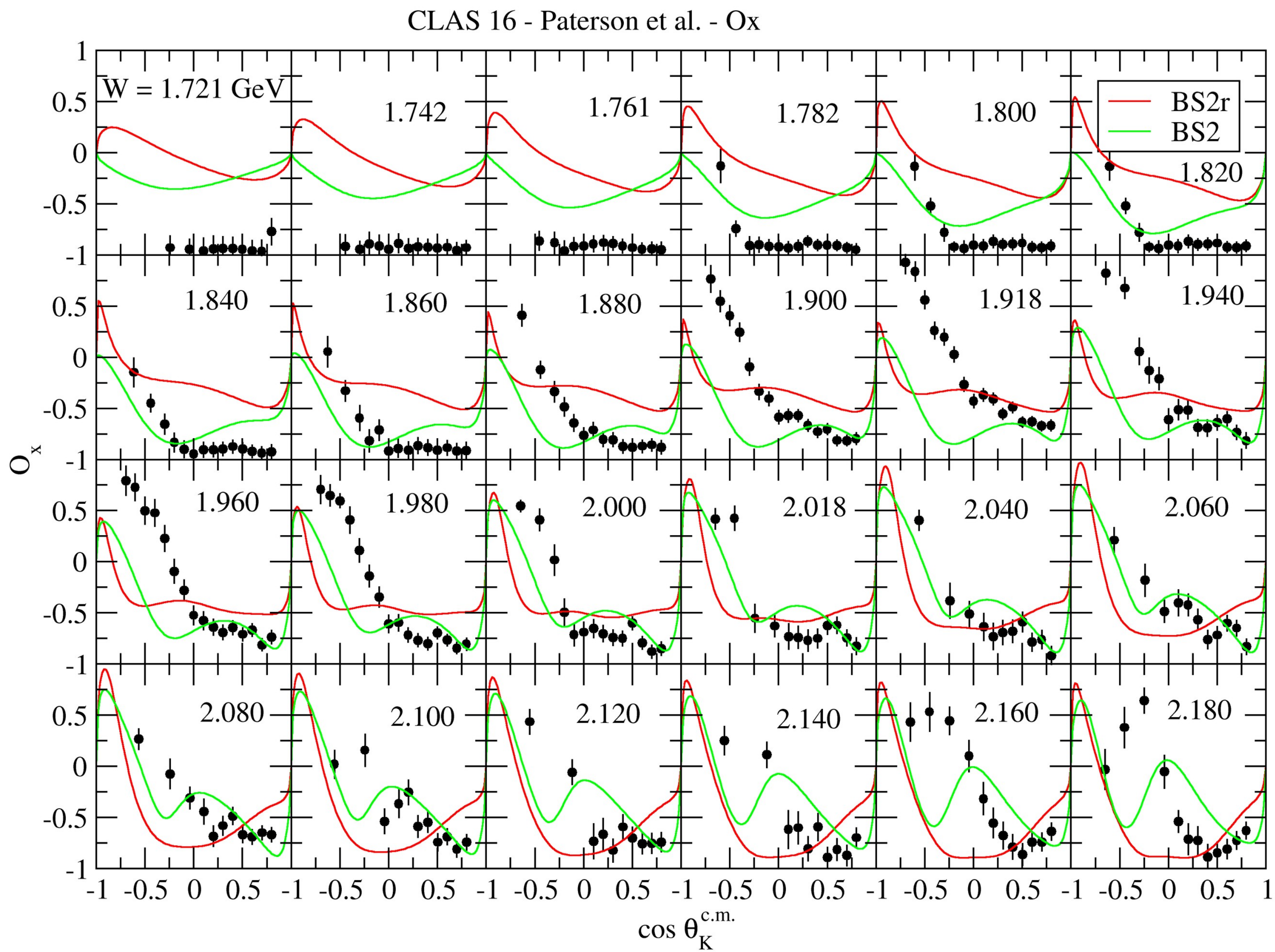
Photon beam asymmetry Σ



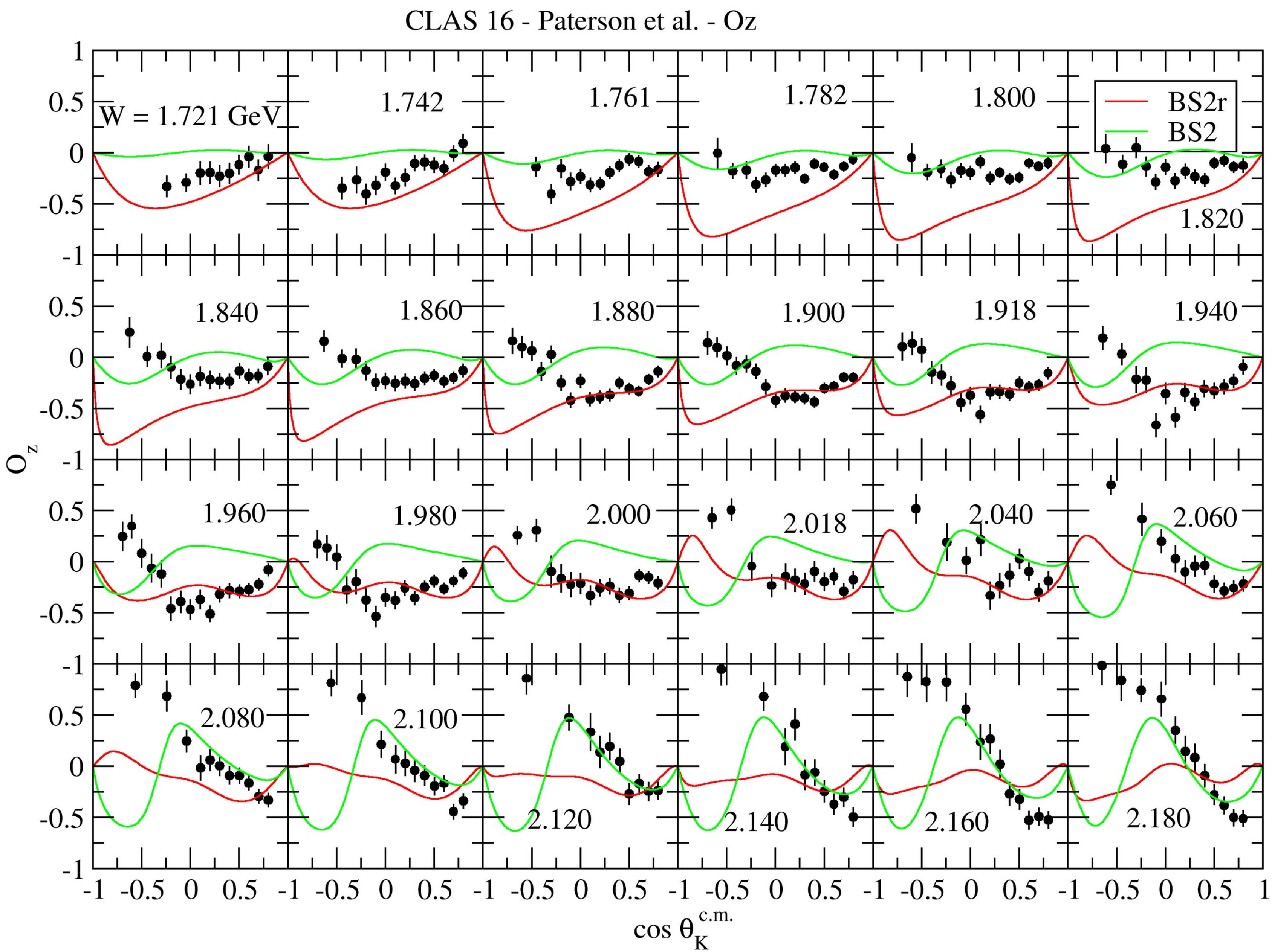
Target asymmetry T



Beam-recoil asymmetry $O_{x'}$



Beam-recoil asymmetry $O_{z'}$



Summary and outlook

- We fitted data for $K^+\Lambda$ photoproduction using the BS2 isobar model and *Ridge* regularization.
- With Ridge regularization, the couplings of the hyperon resonances are substantially decreased.
- Recent photon beam asymmetry (Σ) data are, in general, better described under Ridge. This is not the case for the other asymmetries (T, $O_{x'}$, $O_{z'}$), except in certain energy bins.
- Plan to further investigate the role of the hyperon resonances and their couplings especially in relation to polarization observables.
  (Work in progress)

Thank you!