Effects from Baryon-Baryon Interaction in Electroproduction of Hypernuclei

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Baryon Interaction Study from Hypernuclear Reaction and Structure via Electroproduction Method 2023 (BISHOP 2023),

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Motivation

Work on development of **many-body method**(s) suitable for description of **nuclear and hypernuclear structure** which aim to describe wide range of hypernuclei including medium-size & heavy systems.

Two methods – **Nucleon-Lambda Tamm Dancoff Approximation** (**TD_Λ**) and **Nucleon-Lambda Equation of Motion Phonon Method** (**$EMPM_\Lambda$**) – can be used in calculations of **hypernuclear production** – especially in the hypernuclei whose production is planned by experimentalists in close future (**$^{40}_{\Lambda}K$, $^{48}_{\Lambda}K$, $^{208}_{\Lambda}Tl$**).

Outline:

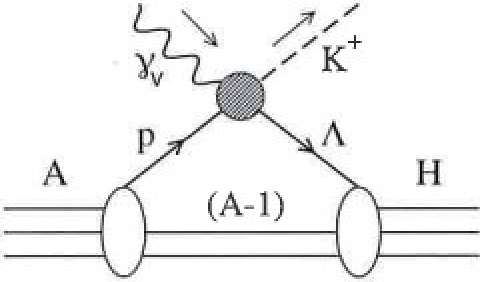
- **Electroproduction** of Hypernuclei.
- Applications of **TD_Λ** and **$EMPM_\Lambda$** in the Calculations of **Hypernuclei.**
- **Results** – cross sections of the production of **$^{12}_{\Lambda}B$, $^{16}_{\Lambda}N$, $^{28}_{\Lambda}Al$, $^{40,48}_{\Lambda}K$.**
- **Summary** and Possible Extensions.

Electroproduction of Hypernuclei

Hypernuclear production:

- Elementary process of electroproduction: **p (e,e‘ K$^+$) Λ**
- **Kinematics** of the reaction
- Information about (many-body) **nuclear & hypernuclear structure**
- Information about the **ΛN(N) interactions**

$$\gamma_v(P_\gamma) + \mathrm{A}(P_\mathrm{A}) \longrightarrow \mathrm{H}(P_\mathrm{H}) + \mathrm{K}^+(P_\mathrm{K})$$



All “ingredients“ important for description of hypernuclear production.

$$\frac{d^3\sigma}{dE_e' d\Omega_e' d\Omega_K} = \Gamma\left[\frac{d\sigma_U}{d\Omega_K} + \varepsilon \frac{d\sigma_P}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \sqrt{\varepsilon_L(\varepsilon+1)}\frac{d\sigma_I}{d\Omega_K}\right]$$

(one-photon exchange approx.)

$$\frac{d\sigma_U}{d\Omega_K} = \frac{\beta}{2(2J_A+1)}\sum_{jm}\frac{1}{2j+1}\{|A_{jm}^{+1}|^2 + |A_{jm}^{-1}|^2\},$$

$$\frac{d\sigma_P}{d\Omega_K} = -\frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}\mathrm{Re}\{A_{jm}^{+1} A_{jm}^{-1*}\},$$

$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}|A_{jm}^{0}|^2,$$

$$\frac{d\sigma_I}{d\Omega_K} = \frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}\mathrm{Re}\{A_{jm}^{0*}[A_{jm}^{+1} - A_{jm}^{-1}]\}$$

Transition amplitude

$$T_\lambda^{(1)} = \frac{Z}{[J_H]}\sum_{S\eta}\mathcal{F}_{\lambda\eta}^{S}\sum_{LM}\sum_{Jm} C_{LMS\eta}^{Jm}\, C_{J_A M_A Jm}^{J_H M_H}\,(\,J_H \| F_{LM}[Y_L \otimes \sigma^S]^J \| J_A\,)$$

Operator can be expressed in the second quantized form.

We evaluate $(\,\Phi_H \,\|\, [b_{\alpha'}^{+} \otimes a_\alpha]^J \,\|\, \Phi_A\,)$

nucleus

hypernucleus

annihilation **p**

creation Λ

P. Bydžovský et al., **Phys. Rev. C 106, 044609 (2022).**

Electroproduction of Hypernuclei

Hypernuclear production:

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma\left[\frac{d\sigma_U}{d\Omega_K} + \epsilon\frac{d\sigma_P}{d\Omega_K} + \epsilon_L\frac{d\sigma_L}{d\Omega_K} + \sqrt{\epsilon_L(\epsilon+1)}\frac{d\sigma_I}{d\Omega_K}\right]$$

$$\mathrm{M}_\mu = \langle\Psi_\mathrm{H}|\langle\chi_\mathrm{K}|\sum_{\mathrm{j}=1}^{\mathrm{Z}}\hat{\mathrm{J}}_\mu(\mathrm{j})\,|\chi_\gamma\rangle|\Psi_\mathrm{A}\rangle \qquad (\,\Phi_H\,||\,[b^+_{\alpha'}\otimes a_\alpha]^J\,||\,\Phi_A\,)$$

$$\gamma_\mathrm{v}(P_\gamma) + \mathrm{A}(P_\mathrm{A}) \longrightarrow \mathrm{H}(P_\mathrm{H}) + \mathrm{K}^+(P_\mathrm{K})$$

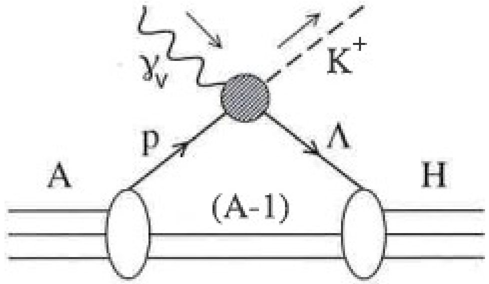
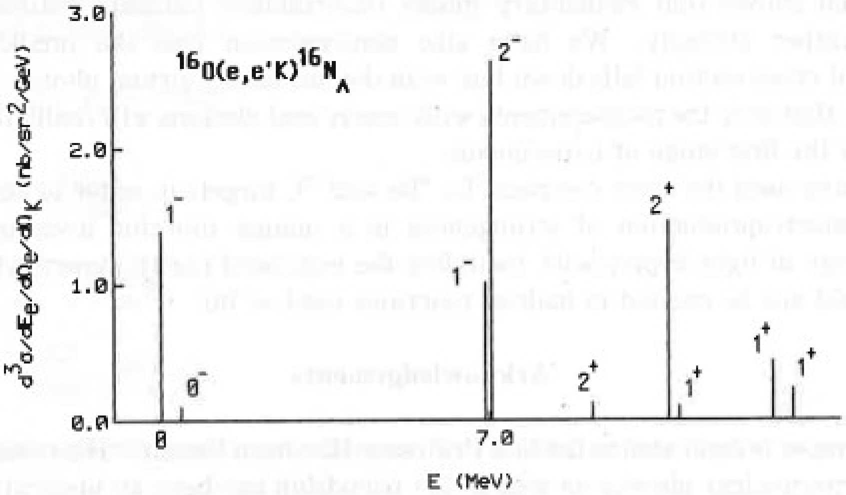
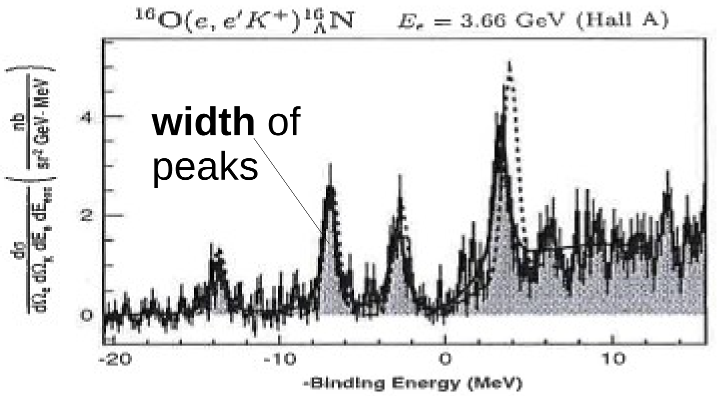


Fig. from M. **Sotona**, S. **Frullani**,
Prog. Theor. Phys. Suppl. 117, 151 (1994)

Study of the **effects** of the **ΛN(N) interactions** & the used **many-body model**...





width of peaks

Fig. 17. Experimental spectrum of the ^{16}O$(e, e'K^+)^{16}_\Lambda$N reaction obtained at JLab Hall A. Taken from Ref. 7).

7) F. Cusanno et al., Phys. Rev. Lett. **103** (2009), 202501.

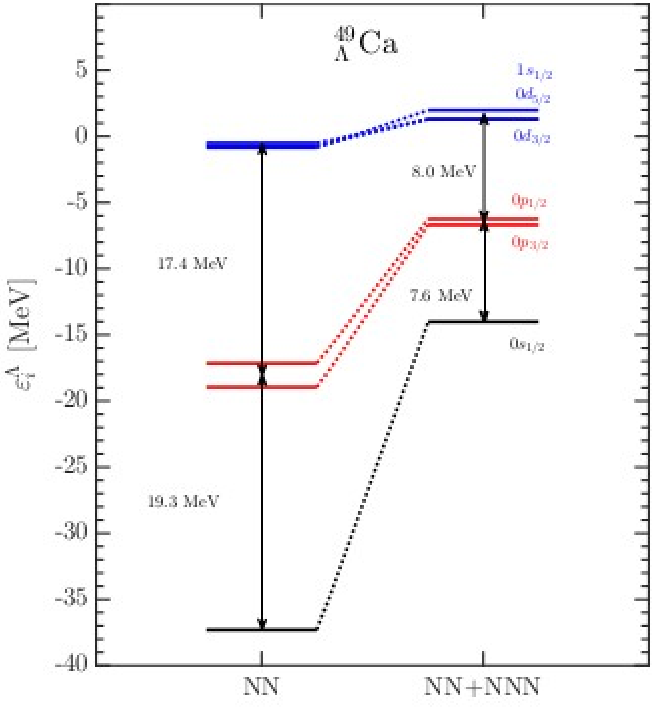
p-n-Λ Hartree-Fock Method

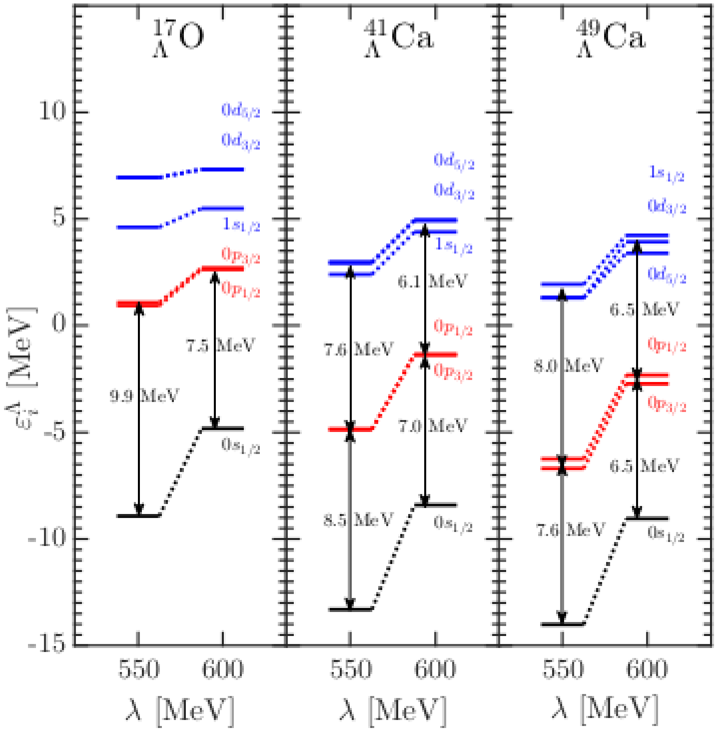
p-n-Λ HF = Hartree-Fock method in the proton-neutron-Λ formalism

- diploma thesis of **J. Pokorný** “*Three-body Interactions in Mean-Field Model of Nuclei and Hypernuclei*“, **Czech Technical University**, (2018)
- **Phys. Scr. 94, 014006, (2019)**; **Acta Phys. Pol. B Proc. Suppl. 12, 657, (2019)**

We obtain: - **single-particle levels** of protons, neutrons and Λ

Single-particle Λ energies:





realistic chiral **NN+NNN** potential **NNLO$_{\text{sat}}$**

realistic chiral LO **YN** potential (ΛN-ΛN channel) with different **regulator cut-off λ**

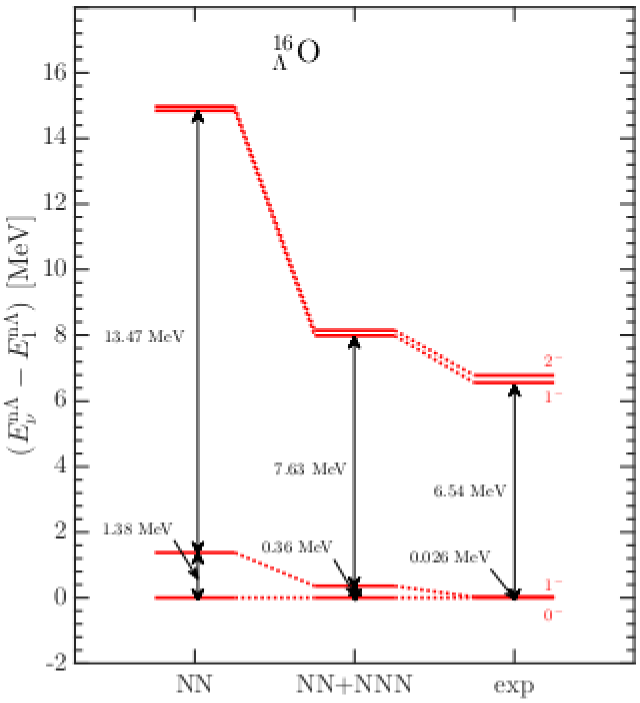
NΛ Tamm-Dancoff

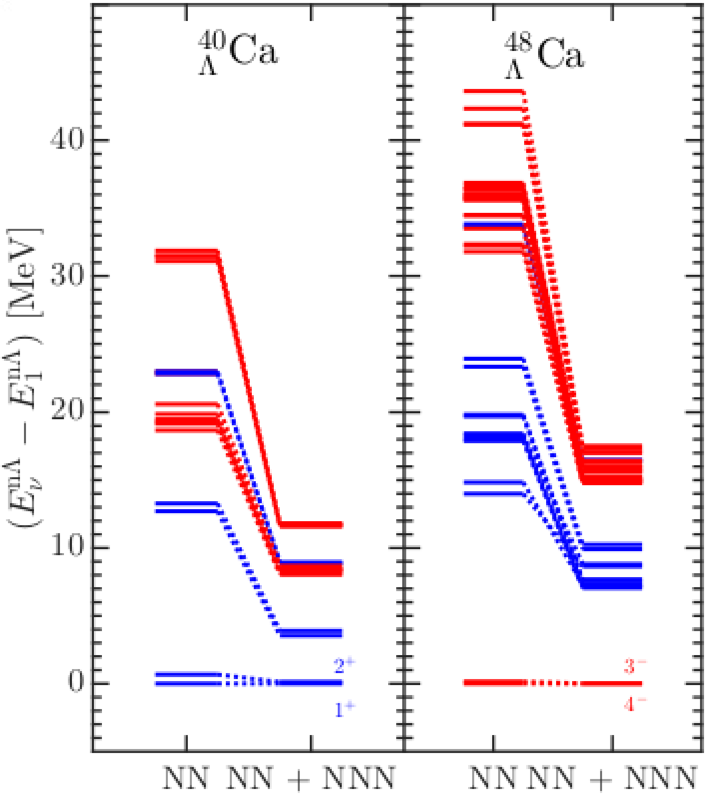
NΛ TDA = Nucleon-Λ Tamm-Dancoff Approximation (TD$_\Lambda$)

- diploma thesis of **J. Pokorný** “*Three-body Interactions in Mean-Field Model of Nuclei and Hypernuclei*“, **Czech Technical University**, (2018)
- **Phys. Scr. 94, 014006, (2019)**; **Acta Phys. Pol. B Proc. Suppl. 12, 657, (2019)**

Suitable for **hypernuclei** with Λ in **even-odd nuclear cores**

NΛ TDA Phonon $R^\dagger_\nu|\mathrm{HF}> = \sum_{\mathrm{ph}} \mathrm{r}^\nu_{\mathrm{ph}} \mathrm{c}^\dagger_{\mathrm{p}} \mathrm{a}_{\mathrm{h}} |\mathrm{HF}>$





realistic
chiral
NN+NNN
potential
NNLO$_{\text{sat}}$

realistic chiral LO
YN potential
(ΛN-ΛN channel)
with different
regulator cut-off λ

EMPM$_\Lambda$ for Hypernuclei

EMPM extended on single-Λ hypernuclei

hypernuclei with Λ in **even-odd nuclear cores**

$$\hat{H} = \hat{T}^{N} + \hat{T}^{\Lambda} + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \hat{V}^{\Lambda NN} - \hat{T}_{CM}.$$

$$\mathcal{H} = \boxed{\mathcal{H}_0 \oplus \mathcal{H}_1} \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

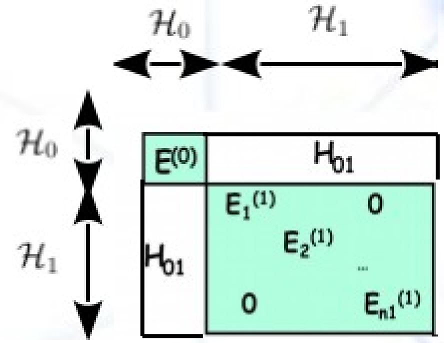
$$\boxed{\begin{aligned} \mathcal{H}_0 &= \{R_\nu^\dagger|\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{R_\nu^\dagger O_{\mu_1}^\dagger|\mathrm{HF}>\} \end{aligned}}$$
$$\mathcal{H}_2 = \{R_\nu^\dagger O_{\mu_1}^\dagger O_{\nu_1}^\dagger|\mathrm{HF}>\}$$

It is more important to study such hypernuclei from the point of view of experiment (production of hypernuclei **$^{4}_{\Lambda}$H, $^{16}_{\Lambda}$O, $^{16}_{\Lambda}$N, $^{40}_{\Lambda}$K, $^{48}_{\Lambda}$K,...**)

Our theoretical **formalism**:

$$|\nu> = R_\nu^\dagger|\mathrm{HF}>$$

$$|\beta> = \sum_{\nu\mu} X^{\beta}_{\nu\mu} R_\nu^\dagger Q_\mu^\dagger|\mathrm{HF}>$$



We construct the **Hamiltonian** matrix:

1) The diagonal block $\mathcal{H}_0 \times \mathcal{H}_0$ = **NΛ TDA** energies
2) The diagonal block $\mathcal{H}_1 \times \mathcal{H}_1$ = **Equation of Motion**
3) The nondiagonal block $\mathcal{H}_0 \times \mathcal{H}_1$ => not difficult to calculate

Equation of Motion:

$$\mathcal{A}X = EX$$

A-matrix $\mathcal{A} =< \beta|[\hat{H}, R_\nu^\dagger]|\mu> + E_\mu < \beta|R_\nu^\dagger|\mu>$

Eigen-value problem in an **overcomplete non-orthogonal basis**…

$$\overline{\mathcal{A}D}C = E\overline{D}C$$

Eigen-value problem in the **reduced space** (linearly independent subset of states)

$-B_\Lambda = E_i + \varepsilon_F^N$

D-matrix = overlap matrix of the basis states
(A.D) – must be hermitian

Hypernuclear Hamiltonian

$$\hat{H} = \hat{T}^{N} + \hat{T}^{\Lambda} + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \cancel{\hat{V}^{\Lambda NN}} - \hat{T}_{CM}$$

- realistic chiral **NN+NNN** potential **NNLO$_{\mathbf{sat}}$**: Phys. Rev. **C91**, 051301(R), (2015)

- **G-matrix** effective **ΛN** potential derived from **Nijmegen-F YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)
  Gaussian-like form – easy to implement, interaction is effective (ΛN force already effectively includes Λ-Σ coupling) but dependent on a parameter **k$_F$**

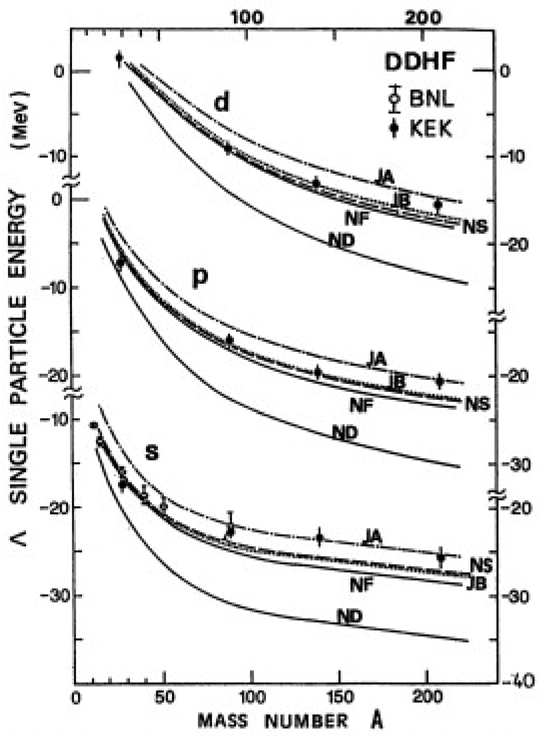


Fig. 2. The Λ single-particle energies $\epsilon_\Lambda(nl; A)$, nl $=0s$, $0p$ and $0d$, calculated in DDHF with five effective interactions. The experimental data

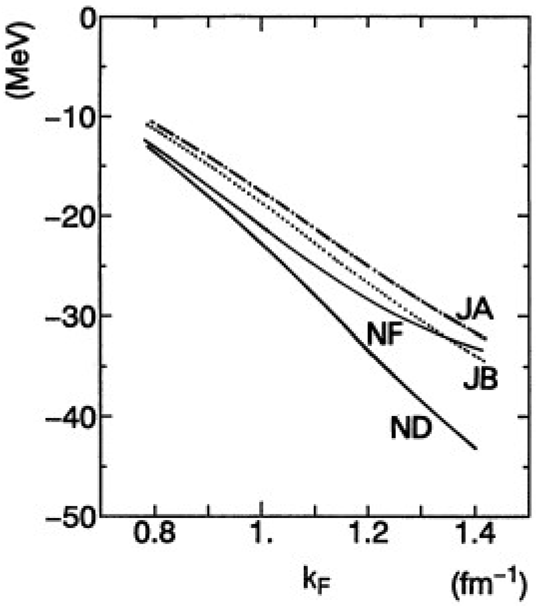


Fig. 1. The Λ single-particle potentials calculated as a function of k_F. The different interaction

$$V_{\Lambda N}(r) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\exp[-r^2/\beta_i^2]$$

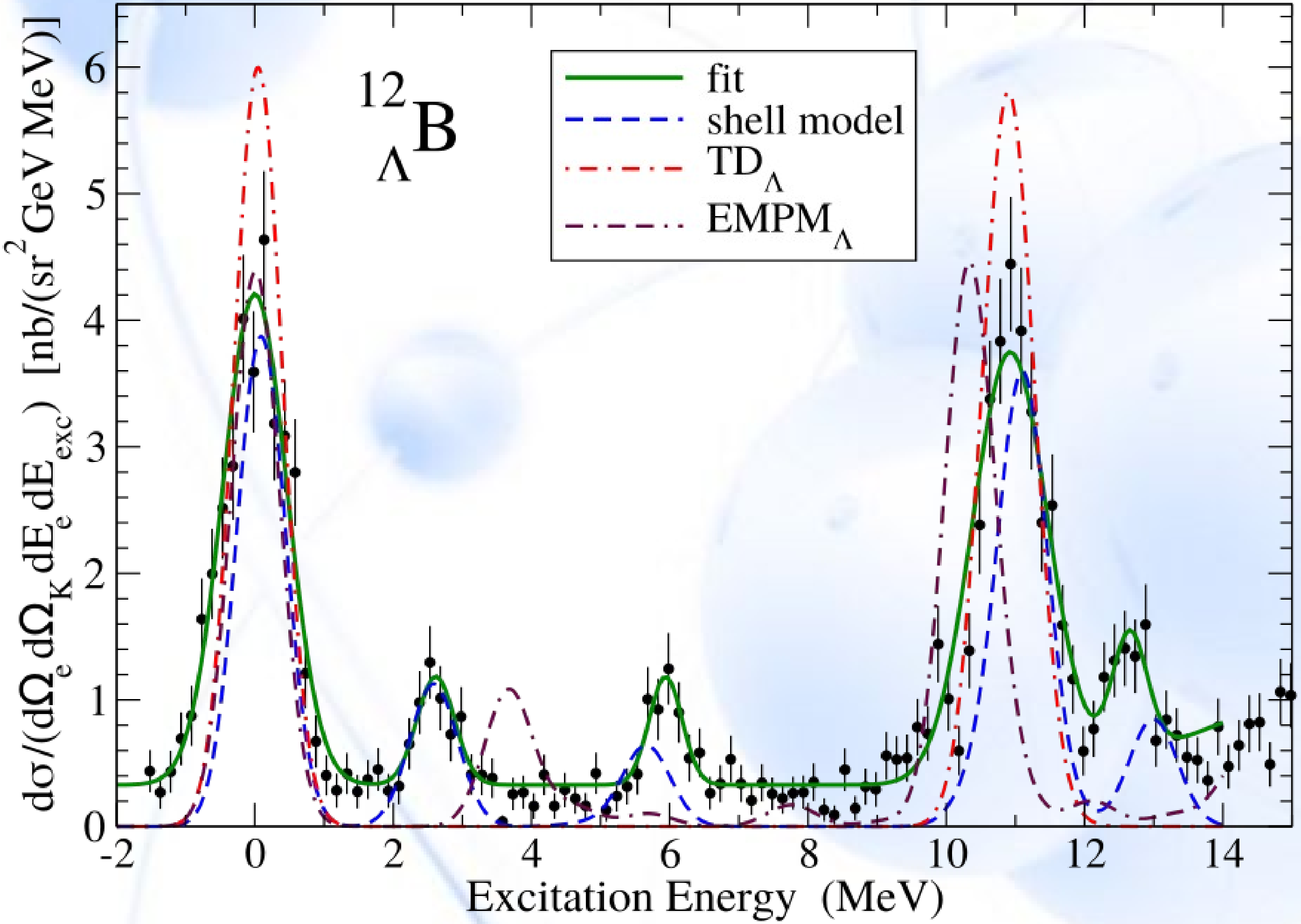
Dependence of the **Λ single-particle** energies on **k$_F$**

k$_F$ as a parameter to tune the proper effective ΛN interaction.

Prog. Theor. Phys. Suppl. **117**, 361 (1994)

Results - $^{12}{}_{\Lambda}B$

Fit to **experimental data**.
Shell model calculation by John Millener - **Nucl. Phys. A** 804, 84 (2008).
TD$_\Lambda$ & **EMPM$_\Lambda$** calculated with Nijmegen-F **k_F = 1.1 fm^{-1}** (tuned to describe ($0s_\Lambda$ – $0p_\Lambda$) gap).



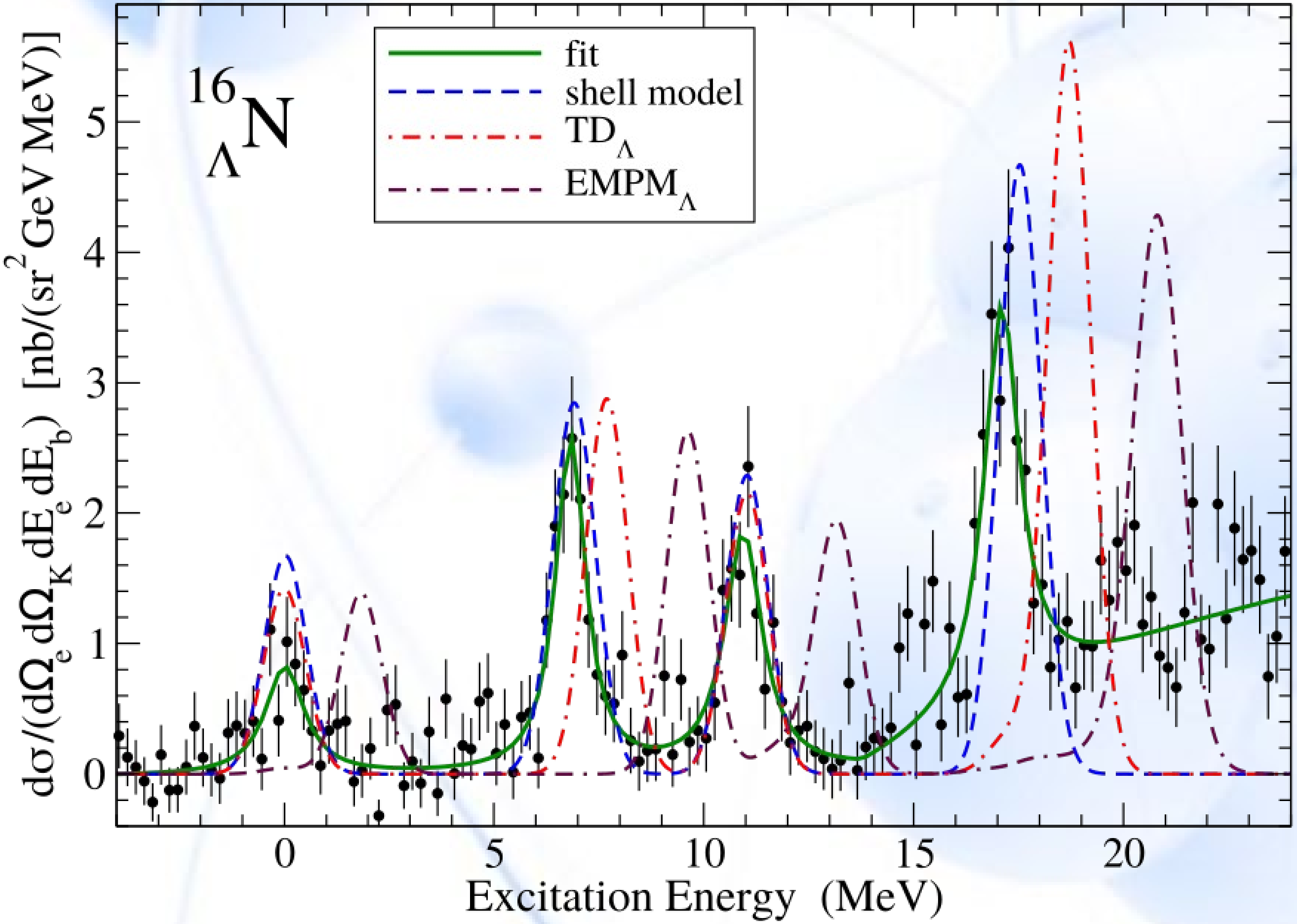
TD$_\Lambda$ describes well two major peaks (both dominantly Lambda-particle nucleon-hole configuration).
B_Λ = 10.37 MeV

EMPM$_\Lambda$ describes qualitatively the presence of smaller configurations (effect of coupling to nuclear core).
For better quantitative agreement 2p-2h or higher terms necessary.
B_Λ = 12.88 MeV

Experiment B_Λ = 11.37 +/- 0.06 MeV

Results - $^{16}_{\Lambda}N$

Fit to **experimental data**.
Shell model calculation by John Millener - **Nucl. Phys. A** 804, 84 (2008).
TD$_\Lambda$ & **EMPM$_\Lambda$** calculated with Nijmegen-F **k_F = 1.1 fm^{-1}** (tuned to describe ($0s_\Lambda$ – $0p_\Lambda$) gap).

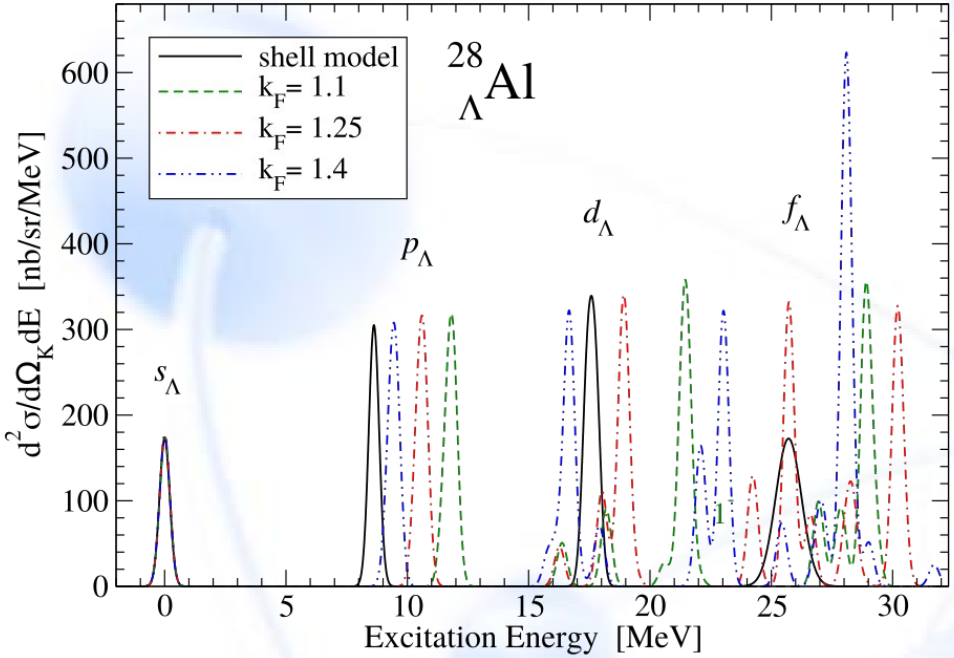


Experiment B_Λ = 13.76 +/- 0.16 MeV

TD$_\Lambda$ describes quite well four major peaks (all dominantly Lambda-particle nucleon-hole configuration).
B_Λ = 12.93 MeV

EMPM$_\Lambda$ shifts binding down in energy. Relative position of 4 main peaks remain good. Only unnatural splitting of **ground state (0^-)** from remaining states. (Coupling to **2p-2h** states still missing)..
Also **spin-dependence** of ΛN force might be not well determined.
$B_\Lambda(0^-_1)$ = 17.19 MeV
$B_\Lambda(1^-_1)$ = 15.36 MeV

Results - $^{28}_{\Lambda}$Al



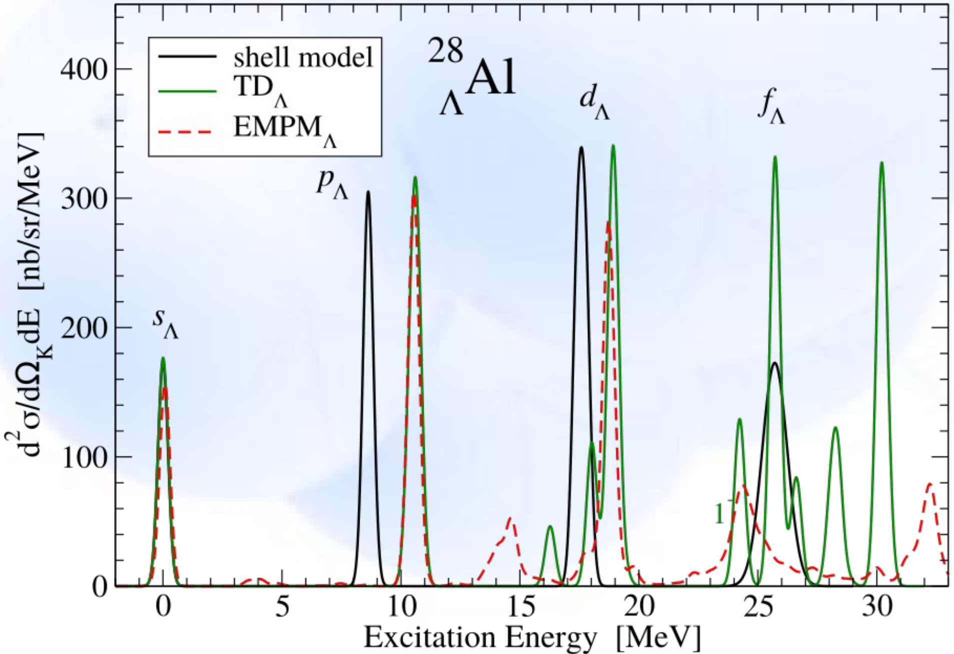
TD$_\Lambda$ calculation with various $\mathbf{k_F}$ - range of possible predictions..

Elementary amplitude **SLA**, frozen proton. kinematics E_i=1.8, E_f=0.5 GeV, θ_e=5.4°, θ_{Ke}=5.1°, Φ_K=180°.

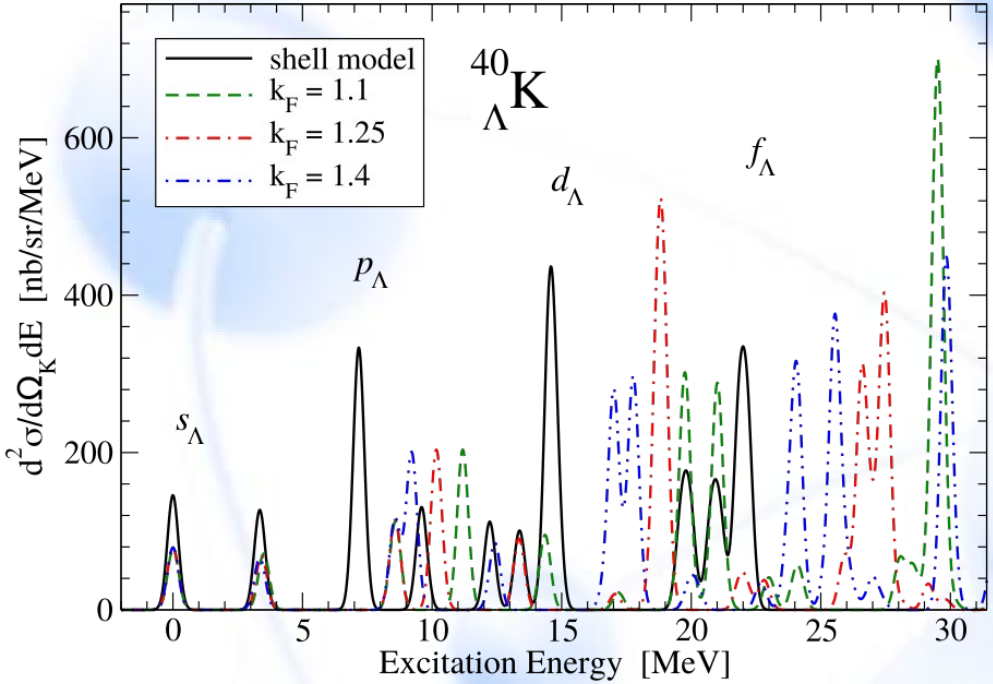
Compared to the shell model calculation by Motoba – **Prog. Theor. Phys. Suppl.** 185, 224 (2010).

Estimate due to the **Average Density Approximation**: k_F = 1.24 fm^{-1}.

EMPM$_\Lambda$ – similar to **TD**$_\Lambda$ for **s-, p-, d- orbitals** (k_F = 1.25 fm^{-1}).
Then some additional structures (result of the coupling to the excitations of nuclear core).
Strength around **f- orbital** suppressed due to too small number of configurations taken into account in EMPM$_\Lambda$.



Results - ${}^{40}_{\Lambda}$K



TD$_\Lambda$ calculation with various **k**$_F$ - range of possible predictions..

Elementary amplitude **SLA**, frozen proton. kinematics E_i=1.8, E_f=0.5 GeV, θ_e=5.4°, θ_{Ke}=5.1°, Φ_K=180°.

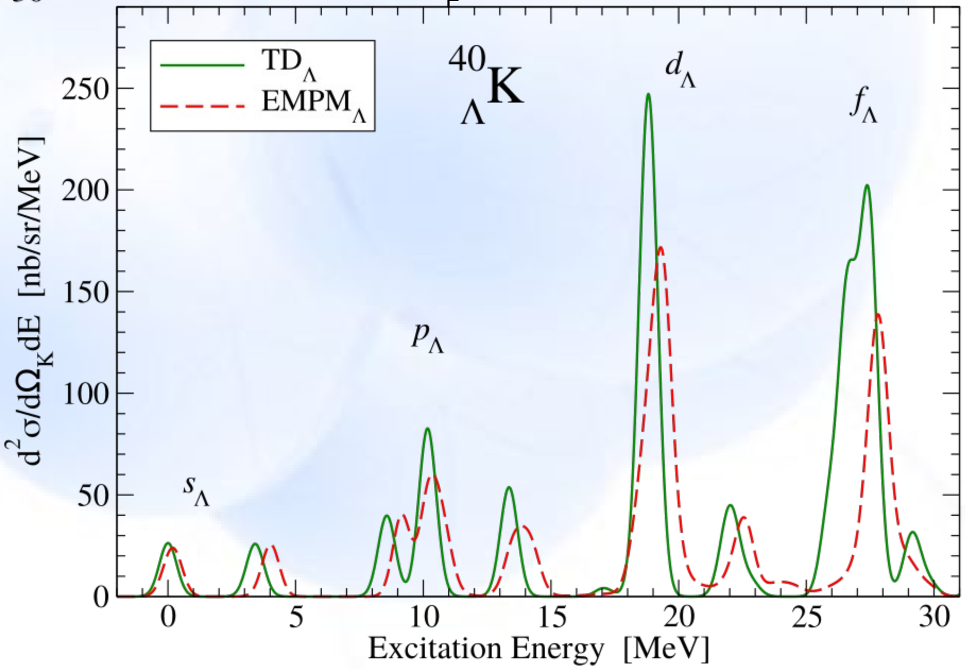
Compared to the shell model calculation by Motoba – **Prog. Theor. Phys. Suppl.** 185, 224 (2010).

Estimate due to the **Average Density Approximation**: $k_F = 1.24$ fm^{-1}.

TD$_\Lambda$ calculation compared to **EMPM**$_\Lambda$ for $k_F = 1.25$ fm^{-1}.

Elem. amplitude **BS3**, optimum on-shell. kinematics E_i=2.24, E_f=0.74 GeV, θ_e=8°, θ_{Ke}=11°, Φ_K=180°.

Effect of **coupling** to **excitations of nuclear core** – more states (**fragmentation**), decreased amplitude of peaks.



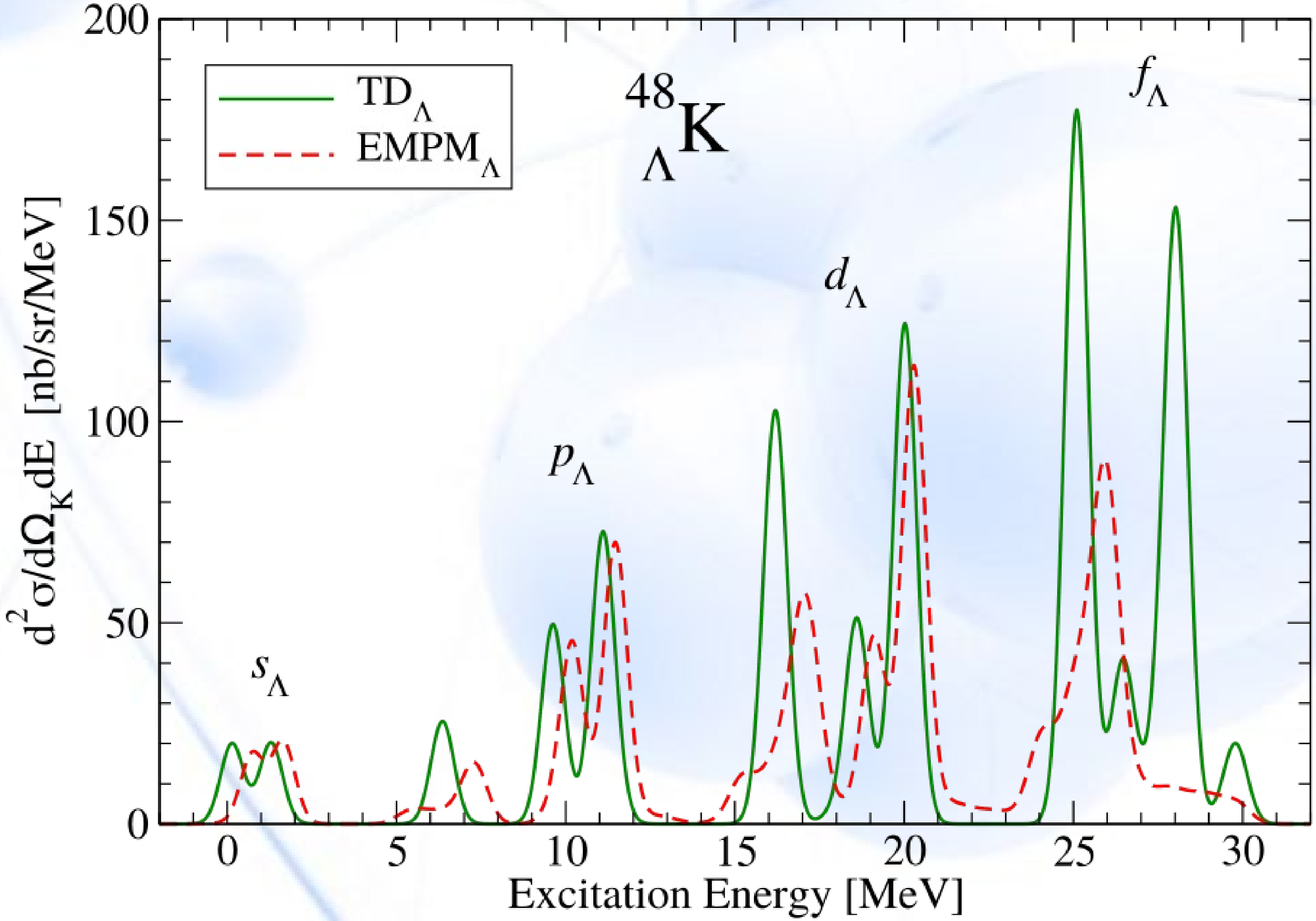
Results - ${}^{48}_{\Lambda}K$

TD$_\Lambda$ calculation compared to **EMPM**$_\Lambda$ for k_F = 1.25 fm^{-1}.

Effect of **coupling** to **excitations of nuclear core** – more states (**fragmentation**), decreased amplitude of peaks.

Elem. amplitude **BS3**, **optimum on-shell**.

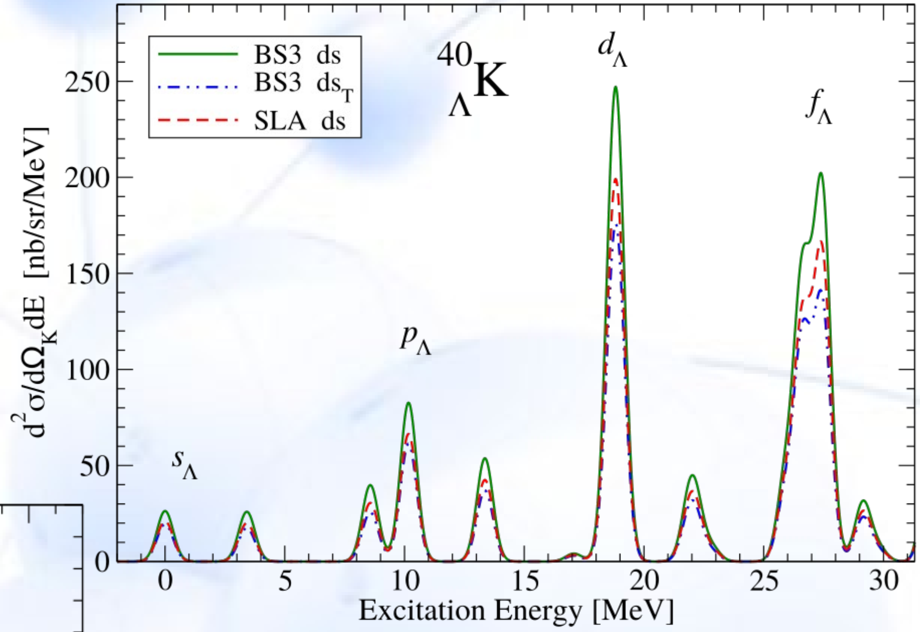
kinematics
E_i=2.24 GeV,
E_f=0.74 GeV,
θ_e=8^o,
θ_{Ke}=11^o,
Φ_K=180^o.

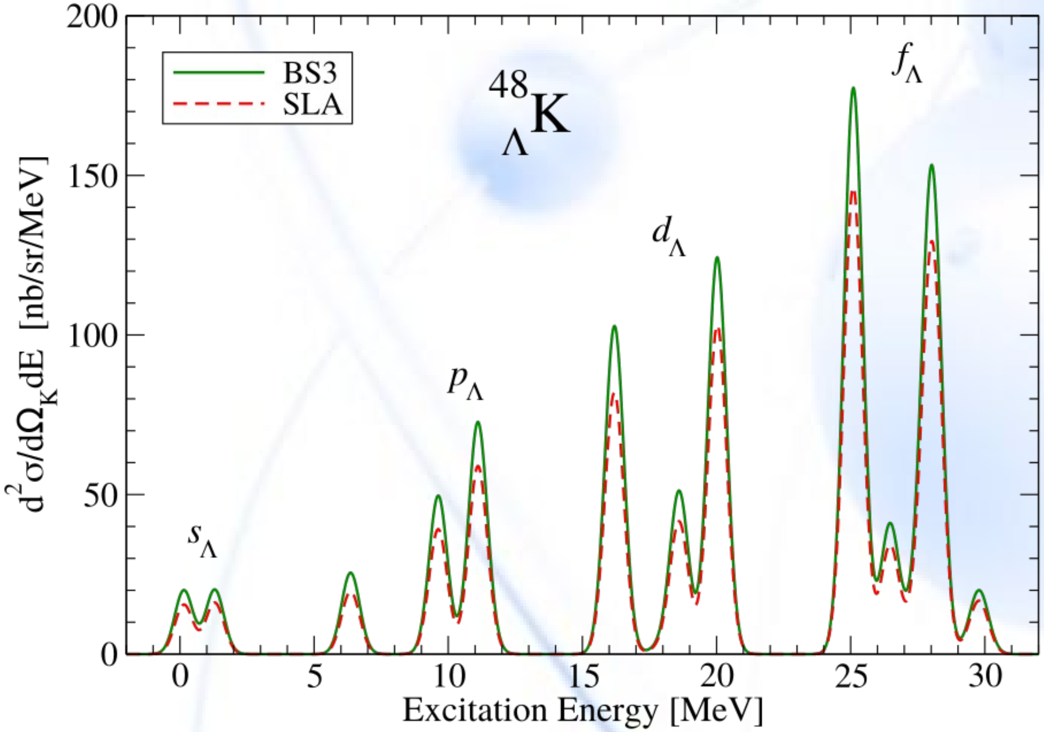


Results – $^{40}_{\Lambda}K$, $^{48}_{\Lambda}K$

Comparison of the effect of two different **elementary amplitudes** (**BS3** & **SLA**).

Electro- & **Photo- production**.





These are predictions (**TD**$_\Lambda$, k_F = 1.25 fm^{-1}) for the experiment **E12-15-008**

Elem. amplitude **BS3**, **optimum on-shell**.

kinematics E_i=2.24 GeV, E_f=0.74 GeV, θ_e=8°, θ_{Ke}=11°, Φ_K=180°.

Summary

- Presented formalism of **electroproduction** of single-Λ **hypernuclei.**

- Presented formalism of **TD$_\Lambda$, EMPM$_\Lambda$** methods.

- Presentation of results of the cross section of the electroproduction of **$^{12}_\Lambda$B, $^{16}_\Lambda$N, $^{28}_\Lambda$Al, $^{40,48}_\Lambda$K.**

- We get reasonable description of **p-shell** hypernuclei **$^{12}_\Lambda$B, $^{16}_\Lambda$N.** In **$^{12}_\Lambda$B** two main peaks described by **TD$_\Lambda$**, smaller peaks require coupling to excitations of nuclear core (**EMPM$_\Lambda$** provides qualitative description). In **$^{16}_\Lambda$N** four main peaks described reasonably by **TD$_\Lambda$.** In **EMPM$_\Lambda$** four main peaks shifted by ~ 1.8 MeV from the ground state 0^-.

- We provide predictions of electroproduction cross sections of **sd-shell** hypernuclei **$^{28}_\Lambda$Al, $^{40,48}_\Lambda$K.** In **$^{28}_\Lambda$Al** reasonable agreement with previous calculations by shell model.

- The cross sections of **electro- & photo- production** – for kinematics relevant in the experiment E12-15-008 (**$^{40,48}_\Lambda$K**) are quite different.

- **Tasks to be potentially addressed**:

  - formalism to study isospin dependence of **ΛNN** interaction (**$^{40}_\Lambda$K & $^{48}_\Lambda$K**)

  - further **development of EMPM$_\Lambda$** itself (coupling to 2-phonon states)

  - study of **structure Ξ hypernuclei**

Thank you for attention!!!