Fermi motion effects in electroproduction of hypernuclei

Workshop of Electro- and Photoproduction of Hypernuclei and Related Topics, October 6, 2022

Petr Bydžovský *in collaboration with*

Daria Denisova, Dalibor Skoupil, and Petr Veselý

Nuclear Physics Institute, Řež, Czech Republic

Outline:

- Introduction
- Formalism: optimal factorization, elementary amplitude in CGLN-like form, production amplitude, optimum on-shell approximation
- Results: effects from non zero proton momentum (Fermi motion)
- Summary and outlook

more details can be found in arXiv:2209.0088[nucl-th]

Introduction

- It is quite complicated to study the YN interaction, *e.g.* Λp scattering in a liquid-hydrogen target:

$$\gamma + p \rightarrow K^+ + \Lambda$$
$$\Lambda + p \rightarrow p + \pi^- + p'$$

see J. Rowley etal (CLAS Collaboration), Phys. Rev. Lett. 127, 272303 (2021),
$\rightarrow$ information on the bare YN interaction (Nijmegen model)

- The γ-ray and reaction spectroscopy of Λ hypernuclei can provide information on the spin-dependent part of effective ΛN interaction.

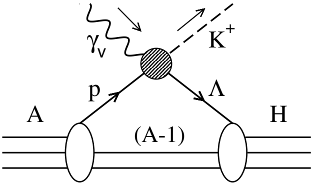
$$\mathrm{V}_{\Lambda \mathrm{N}} = \mathrm{V}_0 + \mathrm{V}_\sigma\, \vec{\mathrm{s}}_\Lambda \cdot \vec{\mathrm{s}}_\mathrm{N} + \mathrm{V}_\Lambda\, \vec{\ell}_{\Lambda \mathrm{N}} \cdot \vec{\mathrm{s}}_\Lambda + \mathrm{V}_\mathrm{N}\, \vec{\ell}_{\Lambda \mathrm{N}} \cdot \vec{\mathrm{s}}_\mathrm{N} + \mathrm{V}_\mathrm{T}\, \mathrm{S}_{12} \,.$$

see, e.g. D. J. Millener, Nucl. Phys. A 804, 84 (2008)

- To obtain reliable information from hypernucleus electroproduction data we need to understand the reaction mechanism well, e.g. to estimate systematic uncertainties due to various approximations.

Introduction – frosen-proton approximation

- In previous DWIA calculations of the cross sections in electroproduction of hypernuclei, e.g. in AIP **2130** (2019) 020014, Phys. Rev. C **99** (2019) 054309, any motion of the initial proton was neglected assuming the frosen-proton approximation ($\vec{p}_p = 0$).



impulse approximation

- In the laboratory frame, $\vec{P}_A = 0$, and with $\vec{p}_p = 0$ one can use a quite simple two-component CGLN form of the elementary-production amplitude $\mathcal{J}_\mu$ in the many-particle matrix element $M_\mu = \langle \Psi_\mathrm{H} | \sum_i \chi_\gamma \chi^*_\mathrm{K} \mathcal{J}^i_\mu | \Psi_\mathrm{A} \rangle$.

- However, in ^{12}C the proton in p orbit moves with the average momentum 179 MeV/c which is comparable to a momentum transfer, $\vec{\Delta} = \vec{p}_\Lambda - \vec{p}_p = \vec{p}_\Lambda$, of about $|\vec{\Delta}| = 250$ MeV/c.

- In the following we will show effects in the cross sections from a non zero proton momentum (the results are taken from arXive:2209.00881).

Formalism – optimal factorization

- The amplitude of production of hypernuclei by virtual photons

$$\gamma_v(\mathrm{q}) + \mathrm{A}(\mathrm{P_A}) \longrightarrow \mathrm{H}(\mathrm{P_H}) + \mathrm{K}^+(\mathrm{p_K})$$

is assumed in optimal factorization approximation replacing $\mathcal{J}_\mu(\vec{p}_K, \vec{q}, \vec{p}_p) \to \mathcal{J}_\mu(\vec{p}_K, \vec{q}, \vec{p}_{eff})$ in the full-folding integral

$$\begin{aligned} \mathcal{T}_\mu &= \mathrm{Z}\,\mathrm{Tr}\Big[\mathcal{J}_\mu(\vec{p}_K, \vec{q}, \vec{p}_{\mathrm{eff}}) \int d^3\xi\, e^{(iB\,\vec{\Delta}\cdot\vec{\xi})}\, \chi^*_K(\vec{p}_{KH}, B\vec{\xi}) \\ &\times \int d^3\xi_1 ... d^3\,\xi_{A-2} \Phi^*_{\mathrm{H}}(\vec{\xi}_1, ... \vec{\xi}_{A-2}, \vec{\xi}) \Phi_{\mathrm{A}}(\vec{\xi}_1, ... \vec{\xi}_{A-2}, \vec{\xi})\Big] \end{aligned}$$

where $\vec{p}_{eff}$ is an effective proton momentum and $B = (A-1)/(A-1+m_\Lambda/m_p)$.

- In previous calculations $\vec{p}_{eff} = 0$. A dependence of the elementary-production amplitude $\mathcal{J}_\mu$ on the proton momentum must be determined. The standard CGLN forms are for the laboratory ($\vec{p}_{eff} = 0$) and c. m. ($\vec{p}_{eff} = -\vec{q}$) frames.

Formalism – elementary amplitude in the CGLN-like form

- The invariant amplitude of K^+ production on a free proton

$$\gamma_v(q,\varepsilon) + p(p_p,\eta_p) \longrightarrow \Lambda(p_\Lambda,\eta_\Lambda) + K^+(p_K)$$

can be expressed via six gauge invariant operators $(\hat{M}_i \cdot \varepsilon)$

$$(\mathcal{M}\cdot\varepsilon) = \bar{u}(p_\Lambda,\eta_\Lambda)\,\gamma_5 \sum_{i=1}^{6} (\hat{M}_i\cdot\varepsilon)\, A_i(q^2,s,t)\, u(p_p,\eta_p)$$

$u(p,\eta)$ is a solution of Dirac equation for on-mass-shell particles

- This form is Lorentz and gauge invariant, depends on the momenta $\vec{p}_p$, $\vec{q}$, and $\vec{p}_K$ but cannot be directly used in the non relativistic expression for the many-particle matrix element $\langle\Psi_{\rm H}|\chi_\gamma\chi^*_{\rm K}(\mathcal{J}\cdot\varepsilon)\,|\Psi_{\rm A}\rangle$ for which a two-componet form is needed.

Formalism – the two-component form

- The two-component form is obtained assuming a new gauge $\epsilon_\mu = \varepsilon_\mu - \varepsilon_0\, q_\mu / q_0 = (\,0,\, \vec{\epsilon}\,)$ and

$$(\mathcal{M} \cdot \varepsilon) = \bar{u}_\Lambda \gamma_5 \sum_{i=1}^{6} (\hat{\mathrm{M}}_i \cdot \varepsilon)\, A_i\, u_p = \mathrm{X}_\Lambda^\dagger\, (\vec{J} \cdot \vec{\epsilon})\;\; \mathrm{X}_p$$

  with Pauli spinors X_p and X_Λ and the amplitude $\mathcal{J}_\mu = (J_0, \vec{J}\,)$.

- This general two-component form has 16 different terms with 16 CGLN amplitudes G_k expressed via the scalar amplitudes A_i

$$\vec{J} \cdot \vec{\epsilon} = G_1\,(\vec{\sigma} \cdot \vec{\epsilon}\,) + G_2\, i(\vec{p}_p \times \vec{q} \cdot \vec{\epsilon}\,) + \ldots\; G_{15}\,(\vec{\sigma} \cdot \vec{p}_K)(\vec{p}_p \cdot \vec{\epsilon}\,) + G_{16}\,(\vec{\sigma} \cdot \vec{p}_K)(\vec{p}_K \cdot \vec{\epsilon}\,)$$

- In a special case, $\vec{p}_p = 0$, (similarly in the c.m. frame, $\vec{p}_p = -\vec{q}$)

$$\begin{aligned} \vec{J}_{LAB} \cdot \vec{\epsilon} \quad = \quad & G_1\,(\vec{\sigma} \cdot \vec{\epsilon}\,) + G_3\, i(\vec{p}_K \times \vec{q} \cdot \vec{\epsilon}\,) + G_8\,(\vec{\sigma} \cdot \vec{q}\,)(\vec{q} \cdot \vec{\epsilon}\,) + \\ & G_{10}\,(\vec{\sigma} \cdot \vec{q}\,)(\vec{p}_K \cdot \vec{\epsilon}\,) + G_{14}\,(\vec{\sigma} \cdot \vec{p}_K)(\vec{q} \cdot \vec{\epsilon}\,) + G_{16}\,(\vec{\sigma} \cdot \vec{p}_K)(\vec{p}_K \cdot \vec{\epsilon}\,) \end{aligned}$$

  is a standard form with only six CGLN amplitudes used in previous calculations

- In the many-particle calculations the spherical form is more suitable

$$\vec{J}\cdot\vec{\epsilon} = -\sqrt{3}\,[\,J^{(1)}\otimes\epsilon^{(1)}\,]^0 = \sum_{\lambda=\pm 1,0} (-1)^{-\lambda} \left(\sum_{S,\eta} \mathcal{F}^S_{\lambda\eta}\,\sigma^S_\eta \right) \epsilon^{(1)}_{-\lambda}$$

- Then the spin-non-flip and spin-flip parts and the transverse and longitudinal parts can be easily separated:

$$\begin{aligned}\vec{J}\cdot\vec{\epsilon} = \; & - \; \epsilon^{\,1}_{\,1}\,(\,\mathcal{F}^{\,0}_{-10} + \sigma^1_1\,\mathcal{F}^{\,1}_{-11} + \sigma^1_0\,\mathcal{F}^{\,1}_{-10} + \sigma^1_{-1}\,\mathcal{F}^{\,1}_{-1-1}\,) + \\ & + \; \epsilon^{\,1}_{\,0}\,(\,\mathcal{F}^0_{00} + \sigma^1_1\,\mathcal{F}^1_{01} + \sigma^1_0\,\mathcal{F}^1_{00} + \sigma^1_{-1}\,\mathcal{F}^1_{0-1}\,) - \\ & - \; \epsilon^{\,1}_{-1}\,(\,\mathcal{F}^0_{10} + \sigma^1_1\,\mathcal{F}^1_{11} + \sigma^1_0\,\mathcal{F}^1_{10} + \sigma^1_{-1}\,\mathcal{F}^1_{1-1}\,)\,,\end{aligned}$$

σ^1_η are the spherical Pauli matrixes.

- The twelve spherical amplitudes $\mathcal{F}^S_{\lambda\eta}$ are expressed via the CGLN-like amplitudes G_k.

- It is suitable to introduce reduced (partial-wave) amplitudes:

$$\mathcal{T}_\lambda^{(1)} = \sum_{Jm} \frac{1}{[J_H]} C^{J_H M_H}_{J_A M_A Jm} \, A^\lambda_{Jm}$$

which read as

$$A^\lambda_{Jm} = \frac{1}{[J]} \sum_{S\eta} \mathcal{F}^S_{\lambda\eta} \sum_{LM} C^{Jm}_{LMS\eta} \sum_{\alpha'\alpha} \mathcal{R}^{LM}_{\alpha'\alpha} \, \mathcal{H}^{LSJ}_{\ell' j' \ell j} \, (\Psi_H || \left[b^+_{\alpha'} \otimes a_\alpha \right]^J || \Psi_A)$$

with radial integrals $\mathcal{R}^{LM}_{\alpha'\alpha} = \int R^*_{\alpha'} F_{LM} R_\alpha$ and OBDME calculated in the Shell model

- The transversal and longitudinal response functions:

$$\frac{d\sigma_T}{d\Omega_K} = \frac{\beta}{2(2J_A+1)} \sum_{Jm} \frac{1}{2J+1} \left(|A^{+1}_{Jm}|^2 + |A^{-1}_{Jm}|^2 \right)$$

$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1} \sum_{Jm} \frac{1}{2J+1} |A^0_{Jm}|^2 \, ,$$

Formalism – optimum on-shell approximation

- $|\vec{p}_K|$ must be determined from energy consertvation in the 2-body ($|\vec{p}_K|_{2b}$) or many-body ($|\vec{p}_K|_{mb}$) systems
- as we requite that the elementary amplitude is on-energy-shell we use $|\vec{p}_K|_{2b}$ for the amplitude but for the other quantities both values can be used: $|\vec{p}_K|_{2b}$ in the "2b" variant and $|\vec{p}_K|_{mb}$ in the "2b-ea" variant of calculation
- in the 2b variant the many-body energy conservation is violated and in the 2b-ea variant we use two different values of $|\vec{p}_K|$
- the optimum proton momentum from

$$E_\gamma + \sqrt{m_p^2 + (\vec{p}_{opt})^2} = \sqrt{m_K^2 + |\vec{p}_K|_{mb}^2} + \sqrt{m_\Lambda^2 + (\vec{\Delta} - \vec{p}_{opt})^2}$$

  then the elementary amplitude is on-shell and we use one value of kaon momentum, $|\vec{p}_K|_{2b} = |\vec{p}_K|_{mb}$; $\vec{\Delta} = \vec{q} - \vec{p}_K$ and $\cos\theta_{\Delta p}$ must be chosen
- this we denote as the optimum on-shell approximation

Results – Fermi motion effects

- We show the cross sections in ^{12}C(e,e'K$^+$)^{12}B$_\Lambda$ calculated with various $\vec{p}_{eff}$:

  a) frosen proton, $\vec{p}_{eff} = 0$ and $\vec{p}_\Lambda = \vec{\Delta} = \vec{q} - \vec{p}_K$ (old calculations)

  b) frosen Λ, $\vec{p}_{eff} = -\vec{\Delta}$ and $\vec{p}_\Lambda = 0$

  c) optimum value, $\vec{p}_{eff} = \vec{p}_{opt}$ with $\theta_{\Delta p} = 180^\circ$ (our choice)

  mean value from the mean kinetic energy, $|\vec{p}_{eff}| = \langle p \rangle = \sqrt{2\mu \langle T_{kin} \rangle}$:

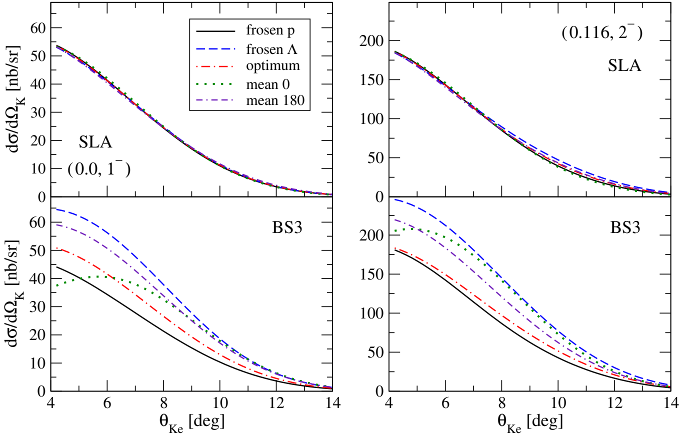
  d) $|\vec{p}_{eff}| = 179$ MeV for ^{12}C and $\theta_{eff} = 0^\circ$, (z-axis $\parallel \vec{q}/|\vec{q}|$)

  e) $|\vec{p}_{eff}| = 179$ MeV and $\theta_{eff} = 180^\circ$

- calculations of the angle and energy dependent cross sections with
  – the Saclay-Lyon and BS3 (Phys.Rev.C**93**,025204(2018)) amplitudes;
  – the hybrid variant 2b-ea;
  – the nucleus-hypernucleus structure (OBDME) by John Millener;
  – kinematics: $Q^2 = 0.06$ (GeV/c)^2, $\varepsilon = 0.7$, $\Phi_K = 180^\circ$;
  angular dependence with $E_\gamma = 2.2$ GeV,
  energy dependence for $\theta_{Ke} = 6^\circ$.

Results – Fermi effects in the angle dependent cross sections

differential cross section for the ground-state doublet of $^{12}B_\Lambda$



the effects depend on

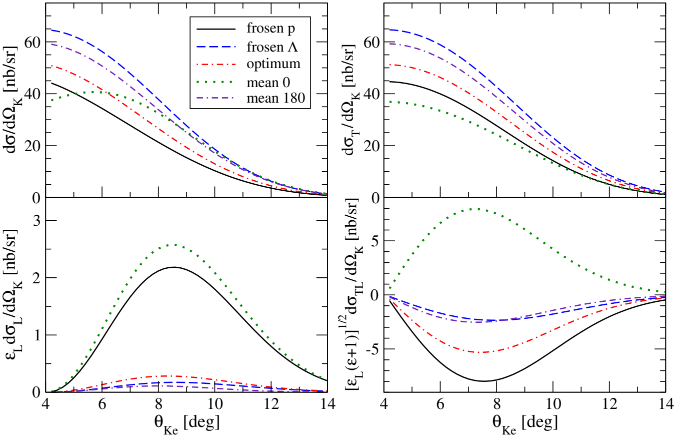
- the elementary amplitude
- the magnitude of $\vec{p}_{eff}$ at $\theta_{Ke} = 6^\circ$: 0; 99; 179; 301 MeV/c
- proton polar angle θ_{eff}

Results – separated contributions to the cross section

the ground-state $J_H^P = 1^-$ of ^{12}B$_\Lambda$ with Λ in s orbit; ^{12}C target g.s. $J_A^P = 0^+ \Rightarrow J = J_H$

$$A^\lambda_{Jm} = \frac{1}{[J]} \sum_{S\eta} \mathcal{F}^S_{\lambda\eta} \sum_{LM} C^{Jm}_{LMS\eta} \sum_{\alpha'\alpha} \mathcal{R}^{LM}_{\alpha'\alpha} \, \mathcal{H}^{LSJ}_{\ell' j' \ell j} \, (\Psi_H || \, [b^+_{\alpha'} \otimes a_\alpha]^J \, || \Psi_A)$$

a "dynamical" selection rule: the contributions with $M = \eta = 0$ dominate



the longitudinal amplitude does not contribute due to the selection rule:

$A^0_{10} \sim C^{J_H 0}_{1010} = C^{10}_{1010} = 0$

$\Rightarrow$ L nad TL contributions are not too much important

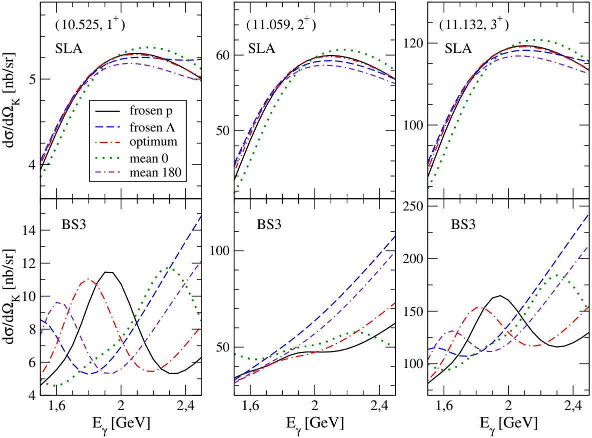
even more,
they mostly cancel each other

in this case the photo- and electroproduction cross sections reveal similar effects

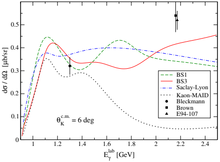
Results – Fermi effects in the energy dependent cross sections

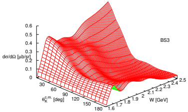
excited states of $^{12}B_\Lambda$ with the Λ in p orbit

the selection rule: $A^0_{Jm} \sim C^{J_H 0}_{2010} \Rightarrow$ the longitudinal contribution only for odd J_H



- the effects depend on the elementary amplitude
- the longitudinal contribution can change a shape of energy dependence and it depends quite strongly on $\vec{p}_{eff}$
- rising cross sections with BS3





$Q^2 = 0.06\ (\mathrm{GeV}/c)^2$, $\varepsilon = 0.7$, $\Phi_K = 180^\circ$, $\theta_{Ke} = 6^\circ$ are fixed

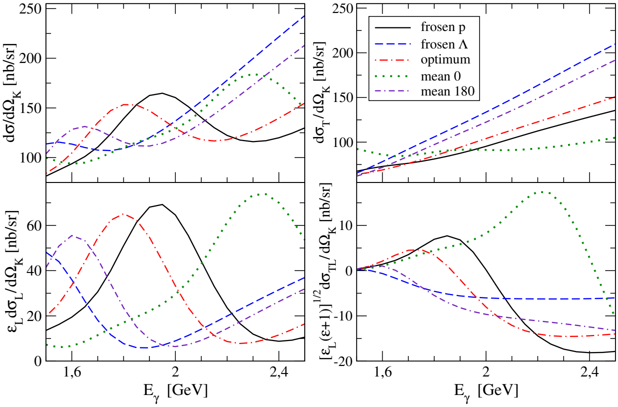
$\Rightarrow E_e$, E'_e, and θ_e change according to E_γ

[D.Skoupil, P.B., Phys. Rev. C 97 (2018) 025202]

Results – separated contributions to the cross section

excited state (11.132, 3^+) of $^{12}\mathrm{B}_\Lambda$

the longitudinal part of the resduced amplitude contributes: $A^0_{30} \sim \mathrm{C}^{30}_{2010} = \sqrt{3/5}$



– the longitudinal contributions $d\sigma_L$ and $d\sigma_{TL}$ are important making the difference between photoproduction ($d\sigma_{\mathrm{T}}$) and electroproduction ($d\sigma$) results even at a small virtuality of the photon, $Q^2 = 0.06$ GeV^2

– Fermi motion effects are important in $d\sigma_{\mathrm{L}}$ and $d\sigma_{\mathrm{TL}}$

– they depend on $|\vec{p}_{eff}|$ and $\theta_{eff} = 0^\circ$; 180°

Results – comparison with experimental data

data (crs in nb/sr^2/GeV) are from

Phys. Rev. C 99 (2019) 054309

at kinematics

E_γ= 2.21 GeV, Q^2= 0.064 (GeV/c)^2,

ε = 0.7, θ_{Ke}= 6°, Φ_K= 180°

calculations are with

the BS3 amplitude, variant 2b-ea, and

OBDME by John Millener

theoretical predictions:

old – from P.R.C 99(2019)054309

NEWa – frozen proton

$\vec{p}_{eff}$ = 0, $\vec{p}_\Lambda$ = 301, $\vec{\Delta}$ = 301 MeV/c

$|\vec{p}_K|_{mb}$= 1964, $|\vec{p}_K|_{2b}$= 1931 MeV/c

NEWb – optimum momentum

$\vec{p}_{eff}$ = 99, $\vec{p}_\Lambda$ = 170, $\vec{\Delta}$ = 269 MeV/c

$|\vec{p}_K|_{mb}$= $|\vec{p}_K|_{2b}$= 1964 MeV/c

comparison old ↔ NEWa shows

improvements in the model

comparison NEWa ↔ NEWb shows

Fermi motion effects

improvement for the main peaks

at 0 and 11 MeV

Data		Theoretical predictions				
E_x (MeV)	crs	E_x (MeV)	J^P	cross sections old	NEWa	NEWb
		0.000	1^-	0.524	0.611	0.741
		0.116	2^-	2.172	2.535	2.677
0.0	**4.51**	sum:		**2.696**	3.145	**3.418**
		2.587	1^-	0.689	0.805	0.956
		2.593	0^-	0.071	0.082	0.027
2.62	0.58	sum:		0.760	0.887	0.983
		4.761	2^-	—	0.022	0.022
		5.642	2^-	0.359	0.422	0.429
		5.717	1^-	0.097	0.113	0.132
5.94	0.51	sum:		0.456	0.558	0.583
		10.480	2^+	0.157	0.175	0.196
		10.525	1^+	0.100	0.111	0.098
		11.059	2^+	0.778	0.870	0.973
		11.132	3^+	1.324	2.169	2.099
		11.674	1^+	0.047	0.085	0.087
10.93	**4.68**	sum:		**2.406**	3.410	**3.453**
		12.967	2^+	0.447	0.504	0.556
		13.074	1^+	0.196	0.219	0.191
		13.383	1^+	—	0.0008	0.0008
12.65	0.63	sum:		0.643	0.724	0.748

Conclusions and outlook

- effects from the proton Fermi motion in electroproduction of ${}^{12}B_\Lambda$ were shown and discussed in a wide kinematical region;
- the effects depend on the elementary amplitude and kinematics and are more apparent in the longitudinal contributions;
- chosing various proton momentum can model resonant structures in the energy dependent cross section;
- a general (proton momentum dependent) two-component (CGLN-like) form of the elementary amplitude was constructed;
- the optimum on-shell approximation was suggested that effectively accounts for the proton motion and avoids uncertainty in determination the kaon momentum;
- the results in the optimum on-shell approximation are, in general, in better agreement with experimental data than our previous results;
- in the next analysis we will study effects from various structure calculations also extending the DWIA calculations for the s-d shell hypernuclei;

Thank you for your attention!