Equation of Motion Phonon Method in Hypernuclei – progress report

Petr Veselý
Nuclear Physics Institute, Czech Academy of Sciences
gemma.ujf.cas.cz/~p.vesely/

Collaboration Japan – Czech Republic,
HIEI2022, March 2022

Motivation

Work on development of **many-body model**(s) suitable for description of **nuclear and hypernuclear structure** which aim to describe wide range of hypernuclei including medium-size & heavy systems.

This model can be used in calculations of **hypernuclear production** – especially the hypernuclei whose production is planned by experimentalists in close future (**$^{40}_{\Lambda}$K, $^{48}_{\Lambda}$K, $^{208}_{\Lambda}$Tl**).

Progress report:

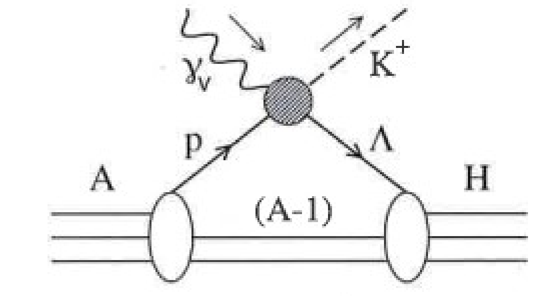
- **Electroproduction** of Hypernuclei
- Applications of **EMPM** for the Calculations of **Hypernuclei**
- **Results** – convergence test of EMPM & preliminary cross sections of the production of **$^{12}_{\Lambda}$B**
- **Summary** and Future Plans

Electroproduction of Hypernuclei

Hypernuclear production:

$$\gamma_v(P_\gamma) + \mathrm{A}(P_\mathrm{A}) \longrightarrow \mathrm{H}(P_\mathrm{H}) + \mathrm{K}^+(P_\mathrm{K})$$

- Elementary process of electroproduction: **p (e,e‘ K$^+$) Λ**
- **Kinematics** of the reaction
- Information about (many-body) **nuclear & hypernuclear structure**
- Information about the **ΛN(N) interactions**



All “ingredients“ important for description of hypernuclear production.

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma\left[\frac{d\sigma_U}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \varepsilon \frac{d\sigma_P}{d\Omega_K}\cos 2\Phi_K + \sqrt{2\varepsilon_L(1+\varepsilon)}\frac{d\sigma_I}{d\Omega_K}\cos\Phi_K\right]$$

$$\frac{d\sigma_U}{d\Omega_K} = \frac{\beta}{2(2J_A+1)}\sum_{jm}\frac{1}{2j+1}\{|A^{+1}_{jm}|^2 + |A^{-1}_{jm}|^2\},$$

$$\frac{d\sigma_P}{d\Omega_K} = -\frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}\mathrm{Re}\{A^{+1}_{jm}A^{-1*}_{jm}\},$$

$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}|A^0_{jm}|^2,$$

$$\frac{d\sigma_I}{d\Omega_K} = \frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}\mathrm{Re}\{A^{0*}_{jm}[A^{+1}_{jm} - A^{-1}_{jm}]\}$$

Transition amplitude

$$T^{(1)}_\lambda = \frac{Z}{[J_H]}\sum_{S\eta}\mathcal{F}^S_{\lambda\eta}\sum_{LM}\sum_{Jm} C^{Jm}_{LMS\eta}\, C^{J_H M_H}_{J_A M_A Jm}\,(\, J_H ||\, F_{LM}[Y_L \otimes \sigma^S]^J \,||\, J_A\,)$$

Operator can be expressed in the second quantized form.

We evaluate $(\,\Phi_H \,||\, [b^+_{\alpha'} \otimes a_\alpha]^J \,||\, \Phi_A\,)$

nucleus

hypernucleus

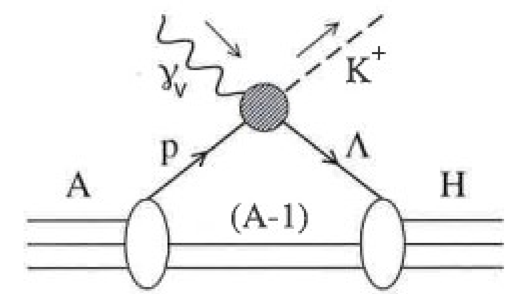
annihilation **p**

creation $\boldsymbol{\Lambda}$

Electroproduction of Hypernuclei

Hypernuclear production:

$$\gamma_v(P_\gamma) + A(P_A) \longrightarrow H(P_H) + K^+(P_K)$$

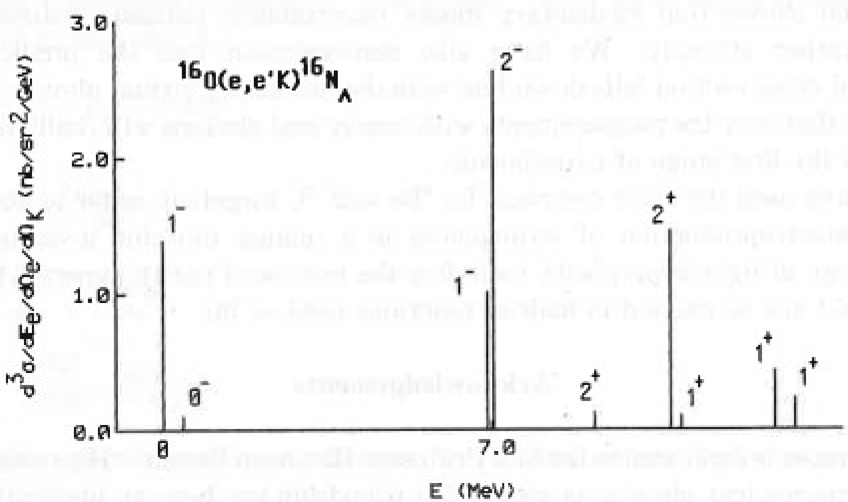


$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma\left[\frac{d\sigma_U}{d\Omega_K} + \epsilon_L \frac{d\sigma_L}{d\Omega_K} + \epsilon\frac{d\sigma_P}{d\Omega_K}\cos 2\Phi_K + \sqrt{2\epsilon_L(1+\epsilon)}\frac{d\sigma_I}{d\Omega_K}\cos\Phi_K\right]$$

$$M_\mu = \langle\Psi_H|\langle\chi_K|\sum_{j=1}^{Z}\hat{J}_\mu(j)\,|\chi_\gamma\rangle|\Psi_A\rangle \qquad (\,\Phi_H\,||\,[b^+_{\alpha'}\otimes a_\alpha]^J\,||\,\Phi_A\,)$$

Fig. from M. **Sotona**, S. **Frullani**,
Prog. Theor. Phys. Suppl. 117, 151 (1994)

Study of the **effects** of the **ΛN(N) interactions** & the used **many-body model**...



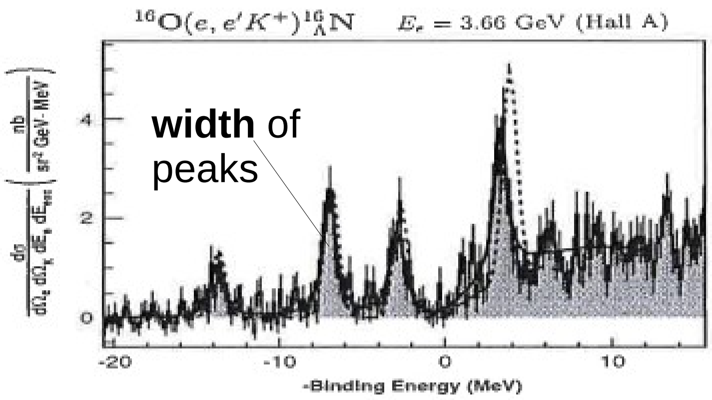


Fig. 17. Experimental spectrum of the $^{16}O(e,e'K^+)^{16}_\Lambda N$ reaction obtained at JLab Hall A. Taken from Ref. 7).

7) F. Cusanno et al., Phys. Rev. Lett. **103** (2009), 202501.

EMPM for Hypernuclei

EMPM extended on single-Λ hypernuclei

hypernuclei with Λ in **even-odd nuclear cores**

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

We **couple ΛN TDA** states with |HF> and **(multi)phonon** excitations of |HF>

$$\begin{aligned} \mathcal{H}_0 &= \{R^\dagger_\nu |\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{R^\dagger_\nu O^\dagger_{\mu_1} |\mathrm{HF}>\} \\ \mathcal{H}_2 &= \{R^\dagger_\nu O^\dagger_{\mu_1} O^\dagger_{\nu_1} |\mathrm{HF}>\} \end{aligned}$$

We can apply this method to calculate structure of $^{12}_{\Lambda}$**B**, $^{12}_{\Lambda}$**C**, $^{16}_{\Lambda}$**N**, $^{16}_{\Lambda}$**O**, $^{40}_{\Lambda}$**K**, $^{40}_{\Lambda}$**Ca**, $^{48}_{\Lambda}$**K**, $^{48}_{\Lambda}$**Ca** etc.

Nuclei:	**HF**	→	**TDA**	→	**EMPM**
Hypernuclei:	**p-n-Λ HF**	→	**NΛ TDA**	→	**ext. EMPM**

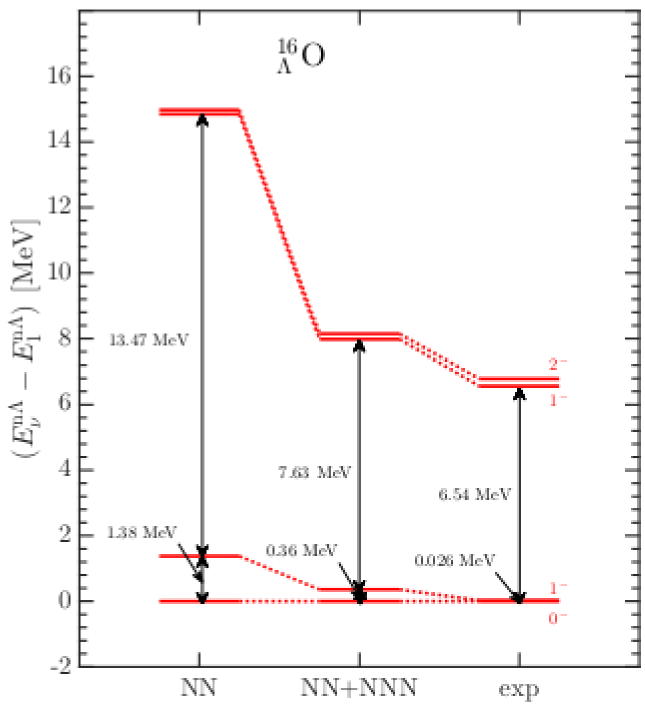
NΛ Tamm-Dancoff

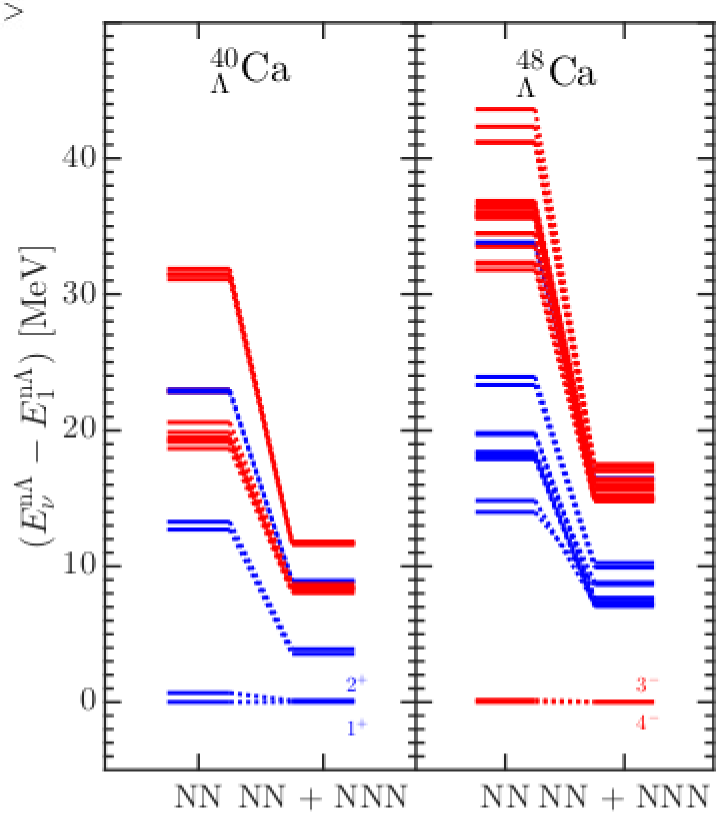
NΛ TDA = Nucleon-Λ Tamm-Dancoff Approximation

- diploma thesis of **J. Pokorný** “*Three-body Interactions in Mean-Field Model of Nuclei and Hypernuclei*“, **Czech Technical University**, (2018)
- **Phys. Scr. 94, 014006, (2019)**; **Acta Phys. Pol. B Proc. Suppl. 12, 657, (2019)**

Suitable for **hypernuclei** with Λ in **even-odd nuclear cores**

NΛ TDA Phonon $R^{\dagger}_{\nu}|\mathrm{HF}> = \sum_{\mathrm{ph}} \mathrm{r}^{\nu}_{\mathrm{ph}} \mathrm{c}^{\dagger}_{\mathrm{p}} \mathrm{a}_{\mathrm{h}}|\mathrm{HF}>$





realistic chiral **NN+NNN** potential **NNLO**$_{\mathbf{sat}}$

realistic chiral LO **YN** potential (ΛN-ΛN channel) with different **regulator cut-off** λ

EMPM for Hypernuclei

EMPM extended on single-Λ hypernuclei

II) hypernuclei with Λ in **even-odd nuclear cores**

$$\hat{H} = \hat{T}^N + \hat{T}^\Lambda + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \hat{V}^{\Lambda NN} - \hat{T}_{CM}$$

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

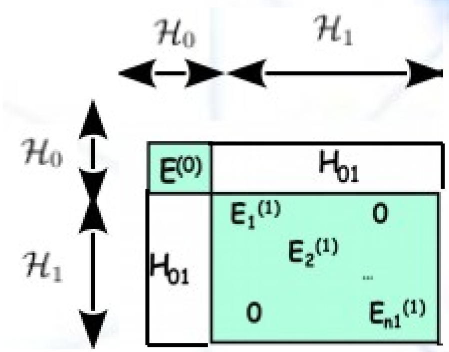
It is more important to study such hypernuclei from the point of view of experiment (production of hypernuclei **$^{12}_{\Lambda}$B, $^{16}_{\Lambda}$O, $^{16}_{\Lambda}$N, $^{40}_{\Lambda}$K, $^{48}_{\Lambda}$K,...**)

$$\begin{aligned} \mathcal{H}_0 &= \{R^\dagger_\nu|\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{R^\dagger_\nu O^\dagger_{\mu_1}|\mathrm{HF}>\} \\ \mathcal{H}_2 &= \{R^\dagger_\nu O^\dagger_{\mu_1} O^\dagger_{\nu_1}|\mathrm{HF}>\} \end{aligned}$$

Our theoretical **formalism**:

$$|\nu>= R^\dagger_\nu|\mathrm{HF}>$$

$$|\beta>= \sum_{\nu\mu} X^\beta_{\nu\mu} R^\dagger_\nu Q^\dagger_\mu|\mathrm{HF}>$$



We construct the **Hamiltonian** matrix:

1) The diagonal block $\mathcal{H}_0 \times \mathcal{H}_0$ = **NΛ TDA** energies
2) The diagonal block $\mathcal{H}_1 \times \mathcal{H}_1$ = **Equation of Motion**
3) The nondiagonal block $\mathcal{H}_0 \times \mathcal{H}_1$ => not difficult to calculate

Equation of Motion:

$AX = EX$ **A-matrix** $A =< \beta|[\hat{H}, R^\dagger_\nu]|\mu> + E_\mu < \beta|R^\dagger_\nu|\mu>$

Eigen-value problem in an **overcomplete non-orthogonal basis**…

$\overline{ADC} = E\overline{DC}$ **Eigen-value problem** in the **reduced space** (linearly independent subset of states)

$-B_\Lambda = E_i + \varepsilon_F^N$

D-matrix = overlap matrix of the basis states
(A.D) – must be hermitian

Description of Hypernuclei

Hypernuclei with **single-Λ** particle:
Hamiltonian

$$\hat{H} = \hat{T}^{N} + \hat{T}^{\Lambda} + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \hat{V}^{\Lambda NN} - \hat{T}_{CM}.$$

Example: NN(+NNN) potential

- phenomenological **NN** potential
  Brink-Boeker
  Nucl. Phys. **A91**, 1, (1967)

- realistic chiral **NN** potential
  NNLO$_{\text{opt}}$
  Phys. Rev. Lett. **110**, 192502, (2013)

- realistic chiral **NN+NNN** potential
  NNLO$_{\text{sat}}$
  Phys. Rev. **C91**, 051301(R), (2015)

ΛN potential

- **G-matrix** effective **ΛN** potential derived from **Juelich & Nijmegen YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)

$$V_{\Lambda N}(r) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\exp[-r^2/\beta_i^2]$$

Juelich-A YN:

	β_i(fm)	1.25	0.70	0.45
	a	−25.82	−389.4	859.0
^1E	b	−12.51	401.2	−303.2
	c	2.437	−136.0	188.8
	a	−45.01	−296.6	1094.
^3E	b	4.620	218.3	−504.6
	c	.7500	−92.50	230.0
	a	−14.54	144.7	734.6
^1O	b	3.615	27.50	76.37
	c	−.8750	−5.000	3.125
	a	−25.91	248.1	615.3
^3O	b	5.410	210.9	−1260.
	c	.5000	−123.1	734.8

Description of Hypernuclei

- **G-matrix** effective **ΛN** potential derived from **Juelich & Nijmegen YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)
  Gaussian-like form – easy to implement, interaction is effective (we can take just ΛN-ΛN part) but dependent on a parameter $\mathbf{k_F}$

$$V_{\Lambda N}(r)=\sum_{i=1}^{3}(a_i+b_ik_F+c_ik_F^2)\exp[-r^2/\beta_i^2]$$

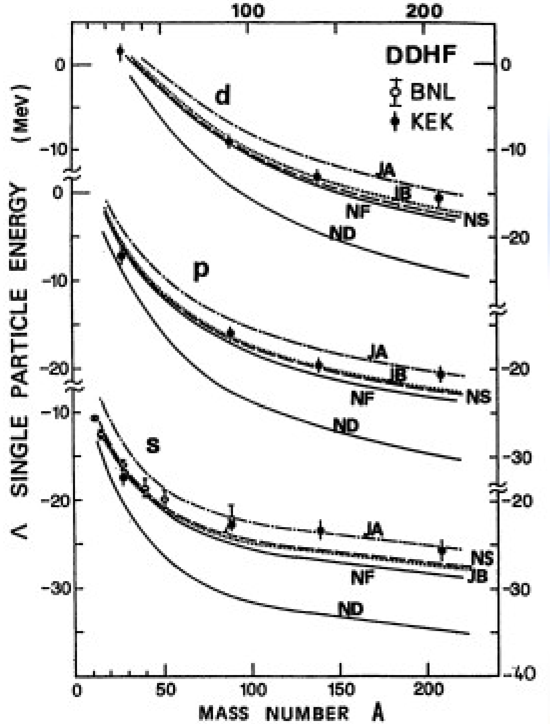


Fig. 2. The Λ single-particle energies $\epsilon_\Lambda(nl;A)$, nl $=0s$, $0p$ and $0d$, calculated in DDHF with five effective interactions. The experimental data

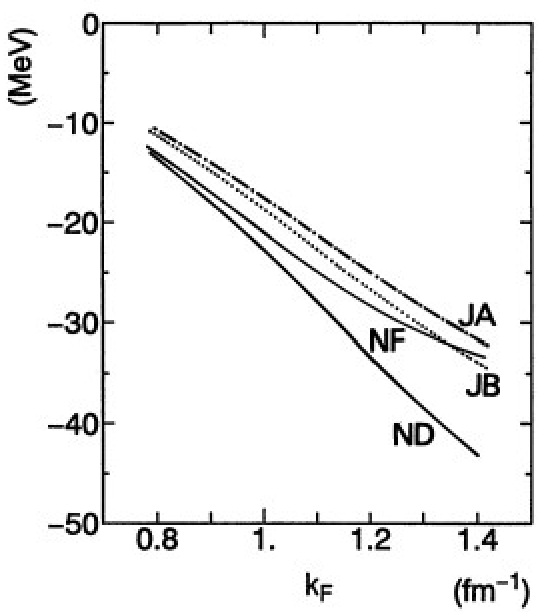


Fig. 1. The Λ single-particle potentials calculated as a function of k_F. The different interaction

Dependence of the **Λ single-particle** energies on $\mathbf{k_F}$

$\mathbf{k_F}$ as a parameter to tune the proper effective ΛN interaction. But tuning should be done at the level of the beyond mean-field calculation.

Prog. Theor. Phys. Suppl. **117**, 361 (1994)

Description of Hypernuclei

- **G-matrix** effective **ΛN** potential derived from **Juelich & Nijmegen YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)

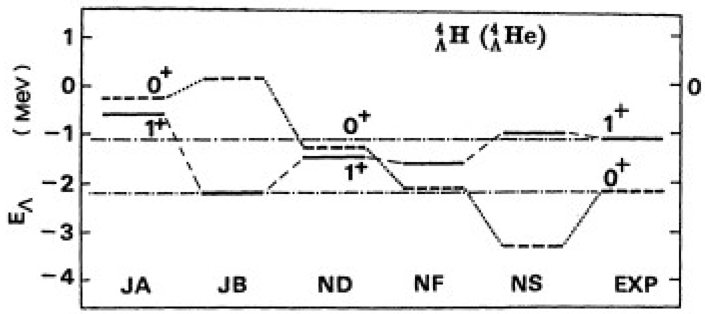
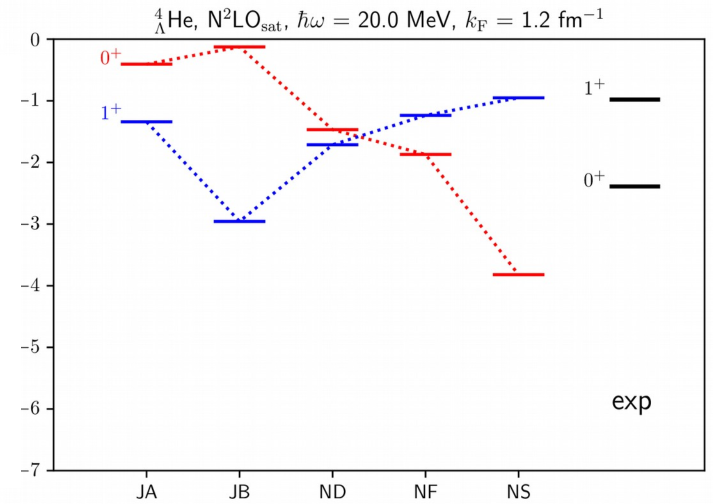


Fig. 3. Energy levels of $^4_\Lambda$H($^4_\Lambda$He), 0^+(dashed) and 1^+(solid). The holizontal dash-dotted lines indicate the experimental positions. The GCM is applied by adopting the size parameter b=0.5 fm with five effective interactions YNG(k_F=0.8 fm^{-1}).



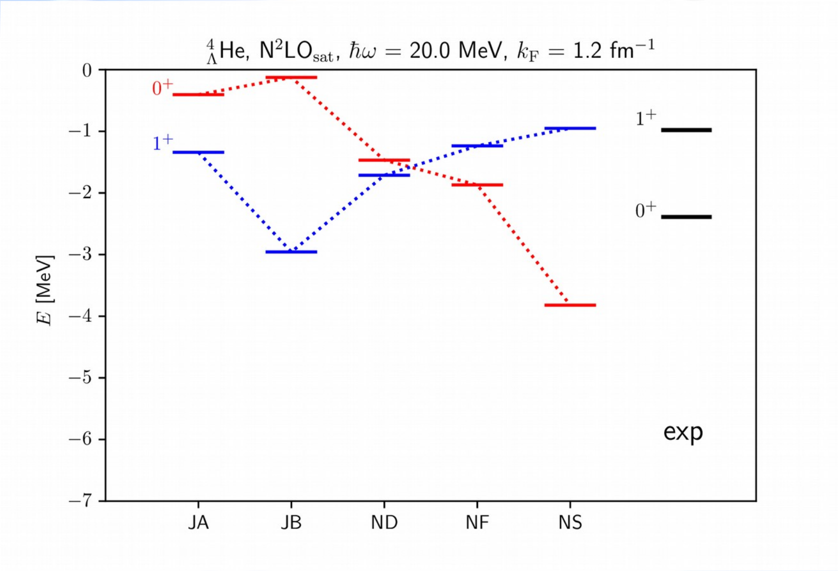
Generator Coordinate Method (GCM)
Prog. Theor. Phys. Suppl. **117**, 361 (1994)

NΛ TDA
(Fig. by J. Pokorny)

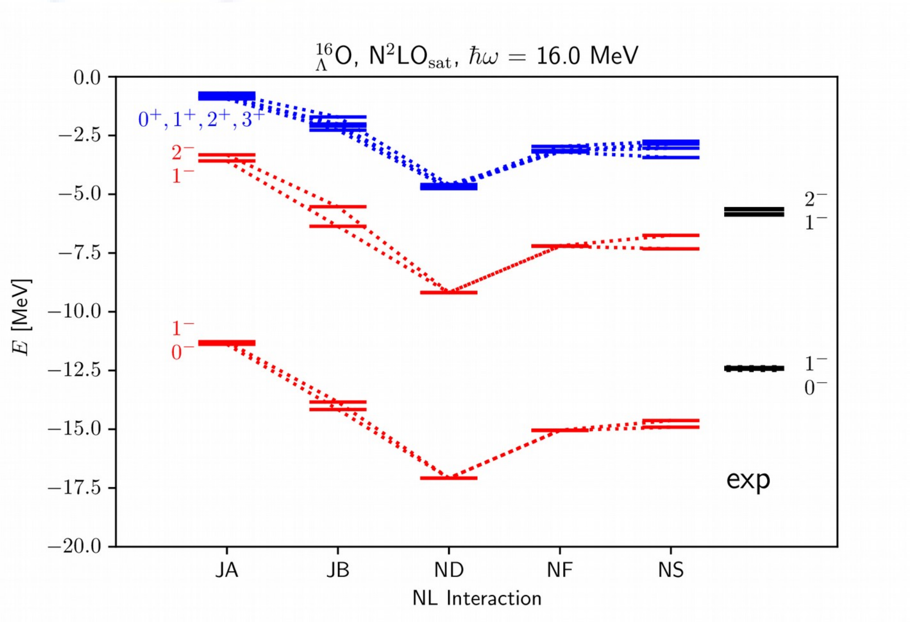
Big variability among different NΛ potentials

Description of Hypernuclei

- **G-matrix** effective **ΛN** potential derived from **Juelich & Nijmegen YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)



NΛ TDA
(Fig. by J. Pokorny)



NΛ TDA
(Fig. by J. Pokorny)

Big variability among different NΛ potentials

NF – seems to have good behavior for **NΛ TDA**
(if $\mathbf{k}_F$ properly tuned)

Convergence of EMPM

- **G-matrix** effective **ΛN** potentials “**JA**“ and “**NF**“ derived from **Juelich** and **Nijmegen YN** Prog. Theor. Phys. Suppl. **117**, 361 (1994)

S.p. basis defined by $\mathbf{N_{max}}$ – number of major oscillator shells (N = 0,1,2,.. N_{max})

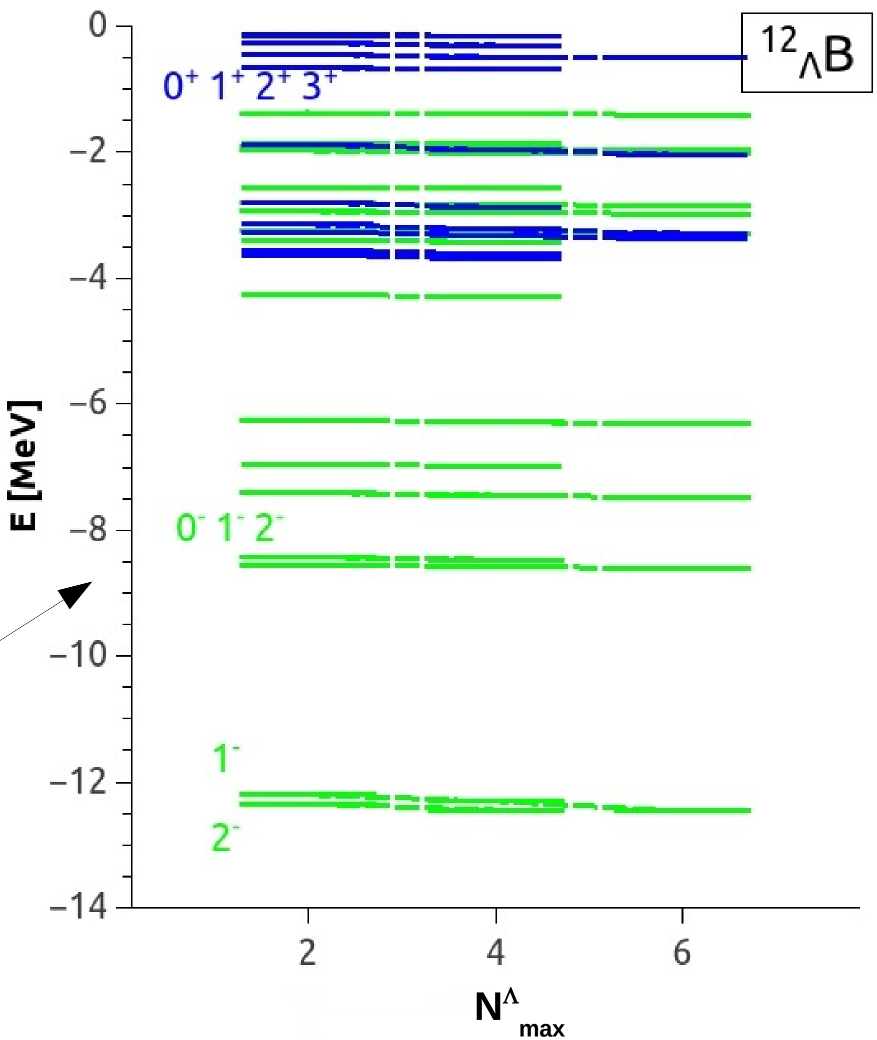
- For the **p-n-Λ Hartree-Fock** we took $\mathbf{N_{max}}$

- For the **TDA** we took $\mathbf{N_{max}}$

- For the **NΛ TDA** we took $\mathbf{N^{\Lambda}_{max}}$
  (in general $\mathbf{N^{\Lambda}_{max}} < \mathbf{N_{max}}$)

It is too computationally costly to apply both **TDA** and **NΛ TDA** in the maximal space…

Calculation for $\mathbf{N_{max}}$ = 10
Potential **JA** with $k_F = 1.2\ fm^{-1}$



Convergence of EMPM

- **G-matrix** effective **ΛN** potentials "**JA**" and "**NF**" derived from **Juelich** and **Nijmegen YN** Prog. Theor. Phys. Suppl. **117**, 361 (1994)

S.p. basis defined by $\mathbf{N_{max}}$ – number of major oscillator shells (N = 0,1,2,.. N_{max})

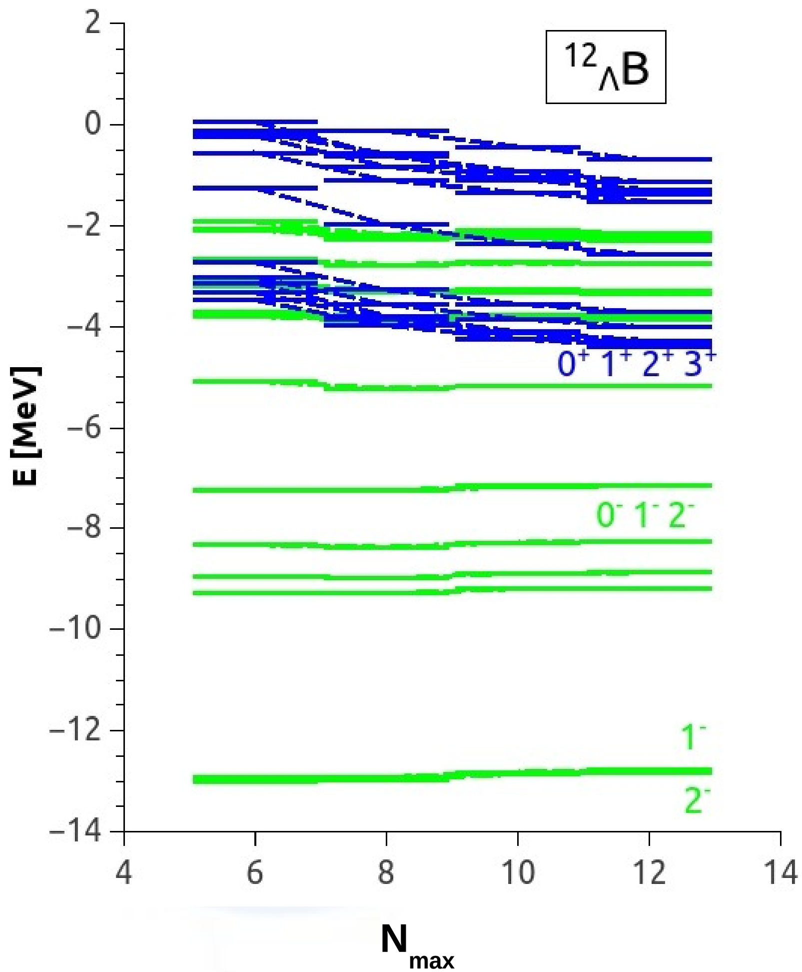
- For the **p-n-Λ Hartree-Fock** we took $\mathbf{N_{max}}$

- For the **TDA** we took $\mathbf{N_{max}}$

- For the **NΛ TDA** we took $\mathbf{N^{\Lambda}_{max}}$
  (in general $\mathbf{N^{\Lambda}_{max}} < \mathbf{N_{max}}$)

It is too computationally costly to apply both **TDA** and **NΛ TDA** in the maximal space…

Calculation for $\mathbf{N^{\Lambda}_{max}} = 4$
Potential **NF** with $k_F = 1.25\ fm^{-1}$

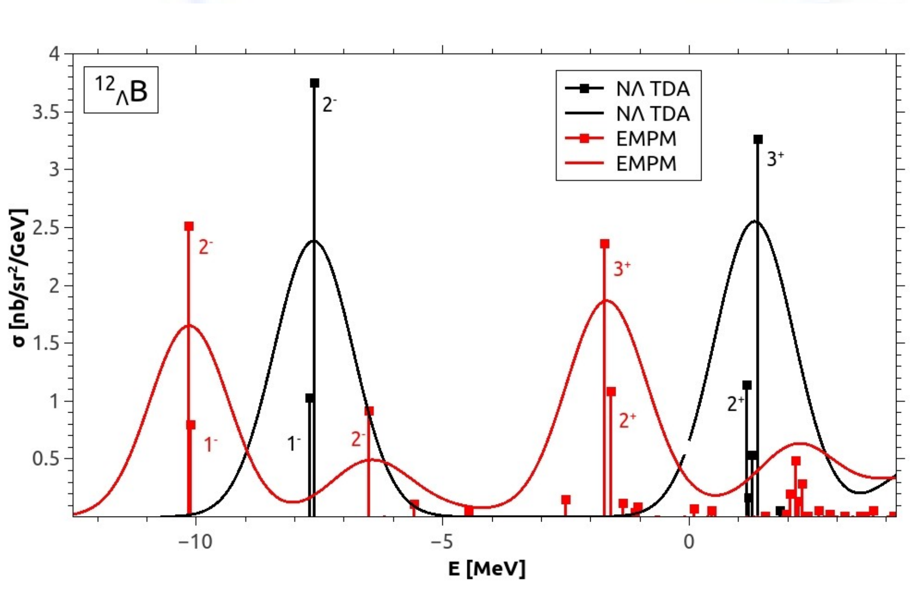


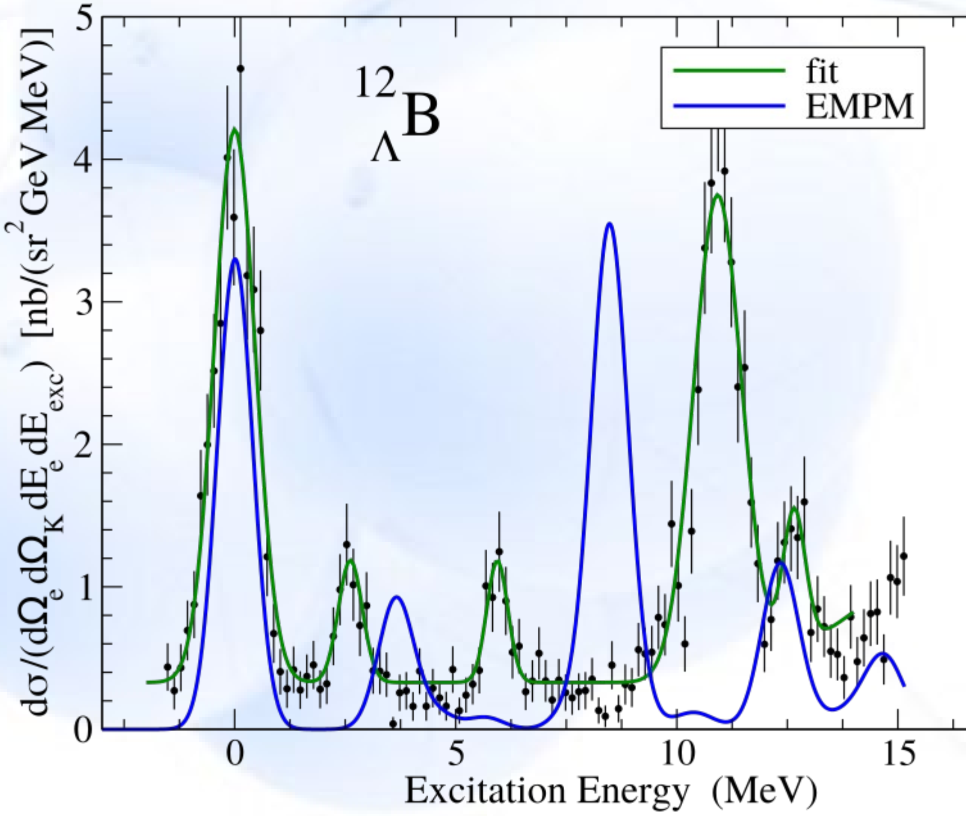
Cross section of electroproduction of $^{12}_{\Lambda}B$

- **G-matrix** effective **ΛN** potentials "**JA**" and "**NF**" derived from **Juelich** and **Nijmegen YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)

Calculation for $\mathbf{N_{max}}$ =12 , $\mathbf{N^{\Lambda}_{max}}$ = 4

Potential **NF** with k_F = 1.25 fm^{-1}





Summary

- **Extensions** of EMPM to calculate single-Λ **hypernuclei**

- Both nuclear & hypernuclear calcs. useful to study **production of hypernuclei**

- Convergence study of **EMPM** (**NΛ TDA + TDA**) for $^{12}_{\Lambda}$**B**

- Preliminary results of the cross section of the electroproduction of $^{12}_{\Lambda}$**B**

- **Tasks to be addressed**:

  - study of $^{40}_{\Lambda}$**K,** $^{48}_{\Lambda}$**K,** $^{208}_{\Lambda}$**Tl** (their exper. measurement is planned in close future)

  - formalism to study isospin dependence of **ΛNN** interaction ($^{40}_{\Lambda}$**K &** $^{48}_{\Lambda}$**K**)

- More **long-term tasks**:

  - further **development of EMPM** itself (coupling to 2-phonon states)

  - formulation of whole method in **deformed HF** basis

Many thanks to all my collaborators!!
P. Bydžovský, G. De Gregorio, D. Denisova, F. Knapp, N. Lo Iudice, D. Petrellis, J. Pokorný, D. Skoupil