Correction for contamination from ^{3}H→^{3}He decay in tritium target

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Initial state of ^{3}H target

From TGT-RPT-17-007 and TGT-CALC-17-020:

- The tritium cell was filled on October 23, 2017 ($t = 0$ for decay)
- The cell contained the following masses of tritium and helium:

$$m_{^3\mathrm{H}} = 0.102 \pm 0.001 \text{ g}$$
$$m_{^3\mathrm{He}} = 3.01 \times 10^{-5} \pm 3.91 \times 10^{-7} \text{ g*}$$

  *No error was provided for initial helium mass. This assumes the same 1% error as in the initial tritium mass.

- Taking into account cell parameters and corresponding uncertainties, this leads to initial target thicknesses (*in the tritium cell*) of:

$$\eta^0_{^3\mathrm{H}} = 0.077 \pm 0.001 \text{ g cm}^{-2}$$
$$\eta^0_{^3\mathrm{He}} = 2.26 \times 10^{-5} \pm 3.69 \times 10^{-7} \text{ g cm}^{-2}$$

^{3}H half-life

According to the *Comprehensive Review and Critical Evaluation of the Half-Life of Tritium*[1], the tritium half-life is:

$$\tau_{^3\mathrm{H}} \equiv \tau = 4500 \pm 8 \text{ days}$$

For exponential decay

$$N(t) = N_0 e^{-t/\tau},$$

the relative error in N arising from $\delta\tau$ is given by:

$$\left(\frac{\delta N}{N}\right)_\tau = \frac{1}{N}\left(\frac{\partial N}{\partial \tau}\right)\delta\tau$$
$$= \left(\frac{t}{\tau^2}\right)\delta\tau$$

With $\delta\tau = 8$ days, it would take 7 years for $(\delta N/N)_\tau$ to reach 0.1%

The uncertainty in the half-life will be neglected

[1] J. Res. Natl. Inst. Stand. Technol. 105, 541 (2000)

Decay

Tritium effective thickness:

$$\eta_{^3\mathrm{H}} = \eta^0_{^3\mathrm{H}}(e^{-t/\tau})$$

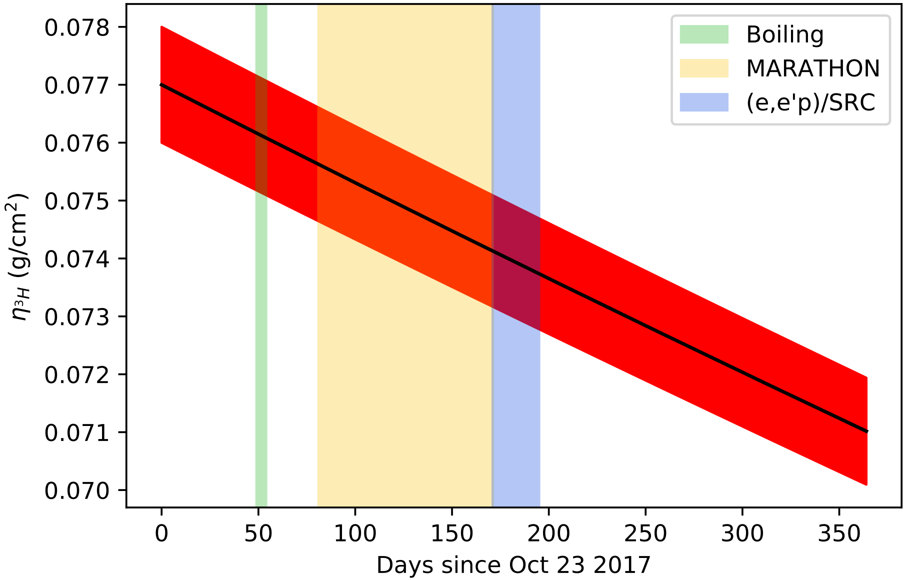
Helium effective thickness (in tritium cell):

$$\eta'_{^3\mathrm{He}} = \eta^0_{^3\mathrm{He}} + \eta^0_{^3\mathrm{H}}(1 - e^{-t/\tau})$$

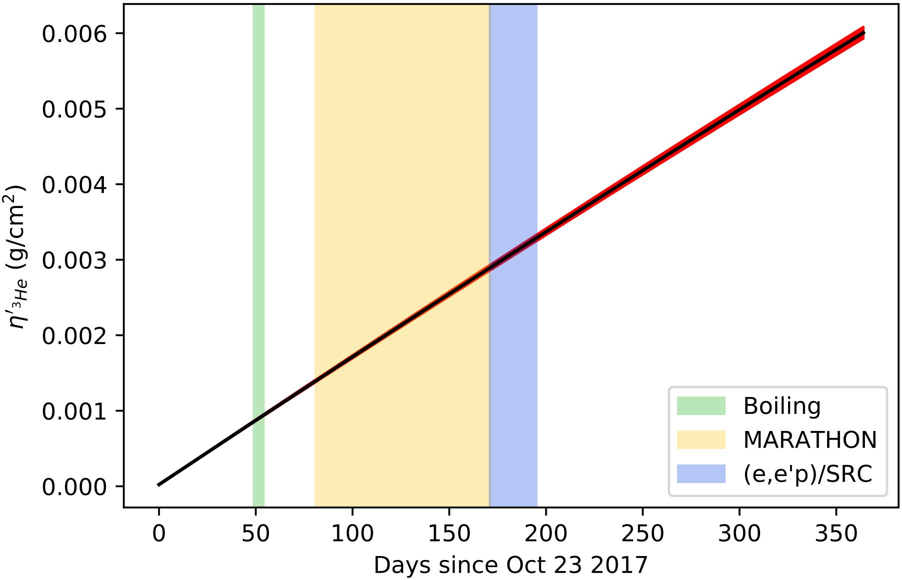
Helium contamination:

$$c \equiv \frac{\eta'_{^3\mathrm{He}}}{\eta_{^3\mathrm{H}}} = \frac{\eta^0_{^3\mathrm{He}} + \eta^0_{^3\mathrm{H}}(1 - e^{-t/\tau})}{\eta^0_{^3\mathrm{H}}(e^{-t/\tau})}$$

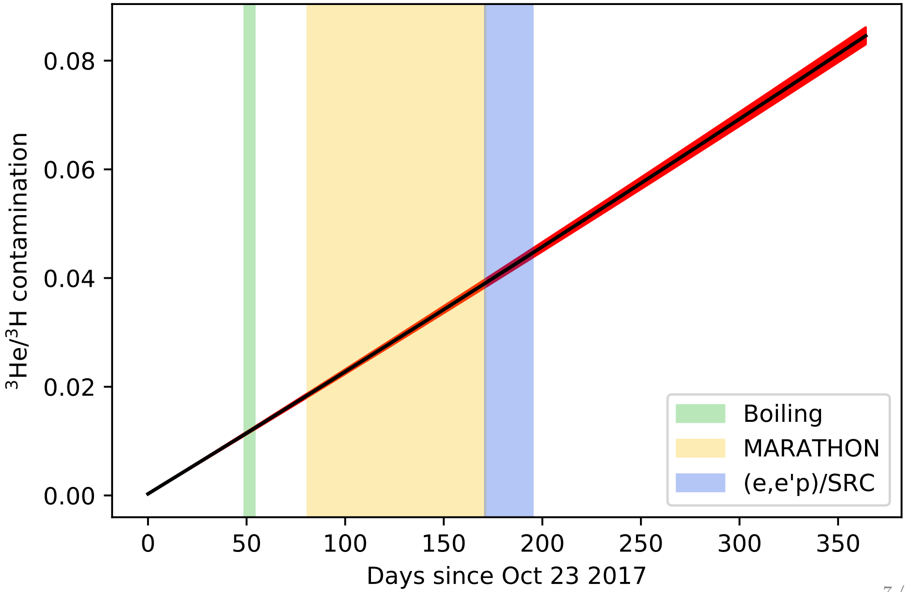
Tritium effective thickness



Helium effective thickness (in tritium cell)



Contamination



Correcting for contamination

Contaminated tritium cross section (what is actually measured):

$$\sigma_{^3\mathrm{H}} \propto \frac{N^c_{^3\mathrm{H}}}{\eta^c_{^3\mathrm{H}}} = \frac{N_{^3\mathrm{H}} + N'_{^3\mathrm{He}}}{\eta_{^3\mathrm{H}} + \eta'_{^3\mathrm{He}}}$$

$$= \frac{N_{^3\mathrm{H}}/\eta_{^3\mathrm{H}} + N'_{^3\mathrm{He}}/\eta_{^3\mathrm{H}}}{1+c}$$

$$\rightarrow \left(\frac{N^c_{^3\mathrm{H}}}{\eta^c_{^3\mathrm{H}}}\right)(1+c) = \frac{N_{^3\mathrm{H}}}{\eta_{^3\mathrm{H}}} + \left(\frac{N'_{^3\mathrm{He}}}{\eta'_{^3\mathrm{He}}}\right) c$$

But $\sigma_{^3\mathrm{He}} \propto N'_{^3\mathrm{He}}/\eta'_{^3\mathrm{He}} \equiv \sigma'_{^3\mathrm{He}}$:

$$\boxed{\frac{N_{^3\mathrm{H}}}{\eta_{^3\mathrm{H}}} = \left(\frac{N^c_{^3\mathrm{H}}}{\eta^c_{^3\mathrm{H}}}\right)(1+c) - c(\sigma'_{^3\mathrm{He}})}$$