Fermi motion and structure effects in electroproduction of hypernuclei

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Motivation of studying electroproduction of hypernuclei in DWIA

$$e + A \rightarrow e' + H + K^{+}$$

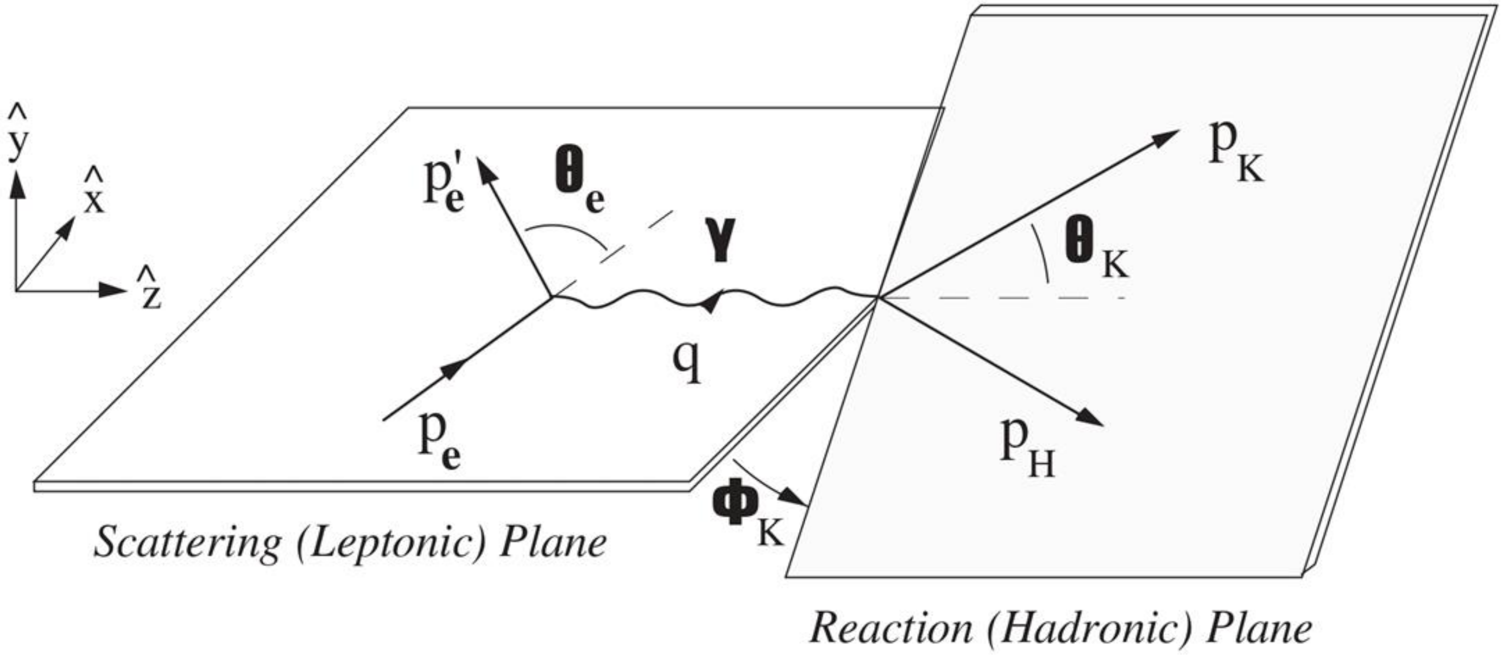
- We obtain information on the spin-dependent part of the hyperon-nucleon interaction.
- The electro-magnetic part of the interaction is well known and can be treated perturbatively which simplifies description of the process.
- The impulse approximation with kaon distortion (DWIA) is well justified in considered kinematics. (P. Bydzovsky. D.Denisova, D.Skoupil, P.Vesely Phys. Rev. C 106, 044609(2022))
- The IA formalism was developed and proved to work well. (F.Garibaldi et al, Phys Rev. C 99, 054309(2019))
- One can achieve a better experimental resolution than in hadron-induced reactions.

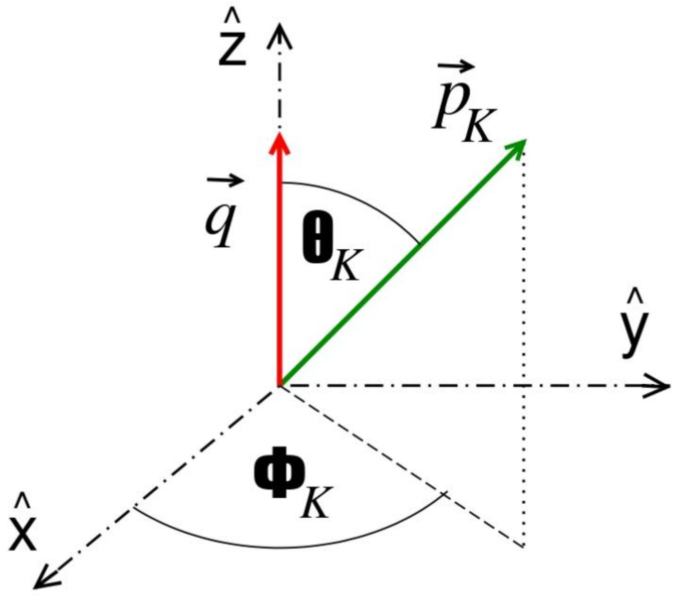
Kinematics

Input: $E(e), E(e'), \theta_e, \phi_K, \theta_K$

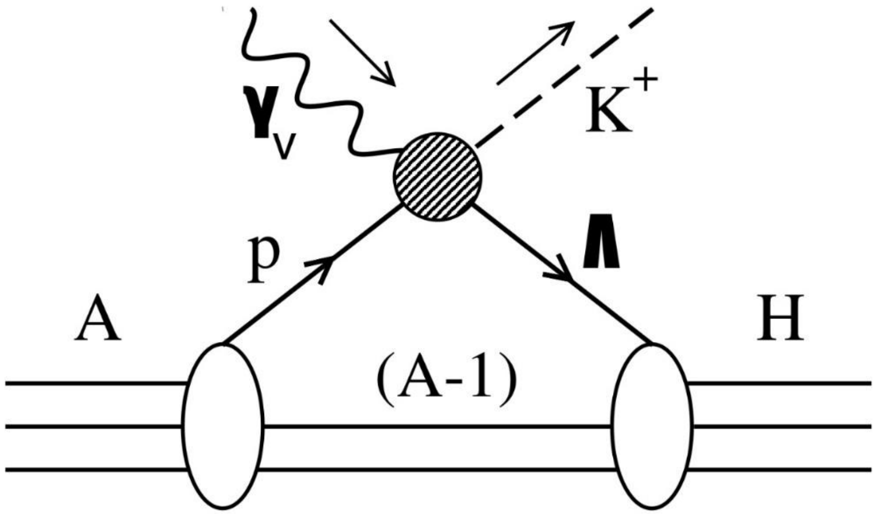
Calculated: $P_\gamma,\ P_K,\ P_H$…

Lab reference frame: $\vec{P}_A = 0$





Impulse approximation



Without Kaon distortion

- $\gamma_\nu(P_\gamma) + A(P_A) \to H(P_H) + K^+(P_K)$
- 3-momentum conservation in each vertex

$$\vec{P}_\gamma + \vec{p}_p = \vec{P}_K + \vec{p}_\Lambda,\ \vec{P}_A = \vec{p}_c + \vec{p}_p,$$

$$\vec{p}_c + \vec{p}_\Lambda = \vec{P}_H \Rightarrow \vec{P}_A + \vec{P}_\gamma = \vec{P}_H + \vec{P}_\mathrm{K}$$

- Nucleus and hypernucleus energy

$$\mathrm{E}_A = \sqrt{M_C^2 + (\vec{P}_A - \vec{p}_p\ )^2} + \sqrt{m_p^2 + \vec{p}_p^{\,2}} + \epsilon_p$$

$$\mathrm{E}_H = \sqrt{M_C^2 + (\vec{P}_A - \vec{p}_p\ )^2} + \sqrt{m_\Lambda^2 + \vec{p}_\Lambda^{\,2}} + \epsilon_\Lambda$$

- In the frozen-proton approximation ($\vec{p}_p = 0$) and Lab frame: $M_A = M_C + m_p + \varepsilon_p$.

Determination of P_K

- $\vec{P}_K$ is calculated from energy conservation in the many-body system (Lab)

$$\mathrm{E}_\gamma + M_A = \sqrt{m_K^2 + \vec{P}_K^2} + \sqrt{M_{\mathrm{H}}^2 + (\vec{P}_\gamma - \vec{P}_K)^2}$$

- Then, the energy conservation in the elementary system (on-shell amplitude)

$$\mathrm{E}_\gamma + \sqrt{m_p{}^2 + \vec{p}_p^2} = \sqrt{m_K^2 + \vec{P}_K^2} + \sqrt{m_\Lambda^2 + (\vec{P}_\gamma - \vec{P}_K + \vec{p}_p)^2}$$

is not satisfied for arbitrary proton momentum.

- Assuming, that $\epsilon_{\mathrm{p}} - \epsilon_\Lambda \approx 0$ and a given momentum transfer $\vec{\Delta} = \vec{P}_\gamma - \vec{P}_K$ the equations can be solved simultaneously for optimum proton momentum $\vec{p}_{opt}$

$$\mathrm{E}_\gamma - \sqrt{m_K^2 + \vec{P}_K^2} = \sqrt{m_\Lambda^2 + \left(\vec{\Delta} + \vec{p}_p\right)^2} - \sqrt{m_p^2 + \vec{p}_p^2} = \sqrt{M_{\mathrm{H}}^2 + \vec{\Delta}^{\,2}} - M_A$$

Many-particle matrix element

- Matrix element with the production amplitude

$$\langle \Psi_H | \langle \chi_K | \sum_{j=1}^{Z} \widehat{J_\mu}(j) | \chi_\gamma \rangle | \Psi_A \rangle = (2\pi)^3 \delta^{(3)}(\vec{P}_A + \vec{\Delta} - \vec{P}_H) T_\mu$$

- The laboratory amplitude in the optimal factorization (OFA)

$$T_\mu = \mathrm{Z} \int d^3\xi e^{\left(iB\overrightarrow{\Delta\xi}\right)} \mathrm{X}_K^*(\vec{\xi}) \langle \Psi_H(\vec{\xi}) | J_\mu \left(\vec{P}_K, \vec{P}_\gamma, \vec{p}_{eff}\right) | \Psi_A(\vec{\xi}) \rangle$$

where $\vec{p}_{eff}$ is an effective proton momentum in the nucleus

- In the spherical coordinates $\mathcal{T}_\lambda^{(1)} = \sum_{Jm} \frac{1}{[J_H]} C_{J_A M_A J m}^{J_H M_H} A_{Jm}^{\lambda}$
- The reduced amplitudes are

$$A_{Jm}^{\lambda} = \frac{1}{[J]} \sum_{S\eta} F_{\lambda\eta}^{S} \sum_{LM} C_{LMS\eta}^{Jm} \sum_{\alpha'\alpha} R_{\alpha'\alpha}^{LM} H_{l'j'lj}^{LSJ} (\Psi_H \| [b_{\alpha'}^{+} \otimes a_\alpha]^J \| \Psi_A)$$

- with the single-particle states denoted as α=[nlj]

Radial integral

- The kaon distortion is included in the radial integral:

$$R_{a'a}^{LM} = \int_0^\infty d\xi \; \xi^2 R_{a'}^{\Lambda}(\xi)^* \; F_{LM} \; (\Delta B \xi) \; R_a^{p}(\xi)$$

- With F_{LM} $(\Delta B\xi)$ determined from

$$e^{(iB\vec{\Delta}\vec{\xi})} X_K^* \left(\overrightarrow{p_K}, B\vec{\xi}\right) = \sum_{LM} F_{LM} \; (\Delta B \xi) \; Y_{LM}(\hat{\xi})$$

where $B = \frac{A-1}{A-1+\frac{m_\Lambda}{m_N}}$

- Where X_K^* is the kaon distortion calculated in eikonal approximation assuming the first-order optical potential.

$$X_K^* = e^{-a \, \sigma_{tot}^{KN} \frac{1-i\alpha}{2} \int_0^\infty dt \, \rho(B\vec{\xi} + \overrightarrow{p_K} t)}$$

OBDME

- The nuclear structure is included via the reduced one-body density matrix elements (OBDME) $(\Psi_H||[b^{+}_{\alpha'} \otimes a_{\alpha}]^J||\Psi_A)$ which are calculated

1) In shell-model calculations (D.J. Millener, Nucl. Phys. A 804, 84 (2008)).

2) In N-Λ TDA (P. Vesely, G. De Gregorio, J. Pokorny, Phys. Scr. 94, 014006, (2019)).

$$\sum_{ph}((e_p^{\Lambda}+ e_h^{\mathrm{N}})\delta_{pp'}\, \delta_{hh'}- V^{\mathrm{N}\Lambda}_{hp'h'p})\, r^{\mu}_{ph}=(E_{\mu}- E_{HF})\, r^{\mu}_{p'h'}$$

where r^{μ}_{ph} are the amplitudes of the Λ-N hole excitations which are related to OBDME, E_{μ} is the energy of N-Λ TDA state, E_{HF} is the Hartree-Fock energy of the nuclear core, e^{Λ} and e^{N} are the single-particle energies for Λ and Nucleons respectively and $V^{N\Lambda}$ is the N-Λ interaction.

Elementary amplitude

- The invariant amplitude M

$$M \cdot \varepsilon = \overline{u_\Lambda} \gamma_5 \left(\sum_{j=1}^{6} M_j \cdot \varepsilon A_j \right) u_p = X_\Lambda^+ (\vec{J} \cdot \vec{\epsilon}) X_p$$

- The elementary amplitude $\vec{J}$ in the spherical coordinates

$$\vec{J} \cdot \vec{\epsilon} = \sum_{\lambda = \mp 1, 0} (-1)^{-\lambda} J_\lambda^{(1)} \epsilon_{-\lambda}^{(1)}$$

- The spherical components of $J^{(1)}$ can be defined via 12 spherical amplitudes $F_{\lambda,\xi}^S$ with S = *0*, *1* and λ, $\xi = \pm 1, 0$

$$J_\lambda^{(1)} = \sum_{\lambda,\xi,S} F_{\lambda,\xi}^S \sigma_\xi^S$$

Fermi motion effect

- In optimal factorization approximation the elementary amplitude is calculated at an effective proton momentum $\vec{p}_{eff}$.
- Effects from various values of $\vec{p}_{eff}$ (Fermi motion effects) will be demonstrated on the angle and energy dependent cross sections in the. Three cases are:

1) frozen p: $\vec{p}_{eff} = 0 \Rightarrow \vec{p}_{\Lambda} = \vec{\Delta}$

2) frozen Λ: $\vec{p}_{eff} = -\vec{\Delta} \Rightarrow \vec{p}_{\Lambda} = 0$

3) optimum: $\vec{p}_{eff}=\vec{p}_{opt}$

$\vec{\Delta}= \vec{P}_{\gamma}- \vec{P}_{K}$

Cross section

- The unpolarized triple differential cross section in electroproduction of hypernuclei in the laboratory frame

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma[\frac{d\sigma_T}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \varepsilon \frac{d\sigma_{TT}}{d\Omega_K} + \sqrt{\varepsilon_L(\varepsilon+1)} \frac{d\sigma_{TL}}{d\Omega_K}]$$

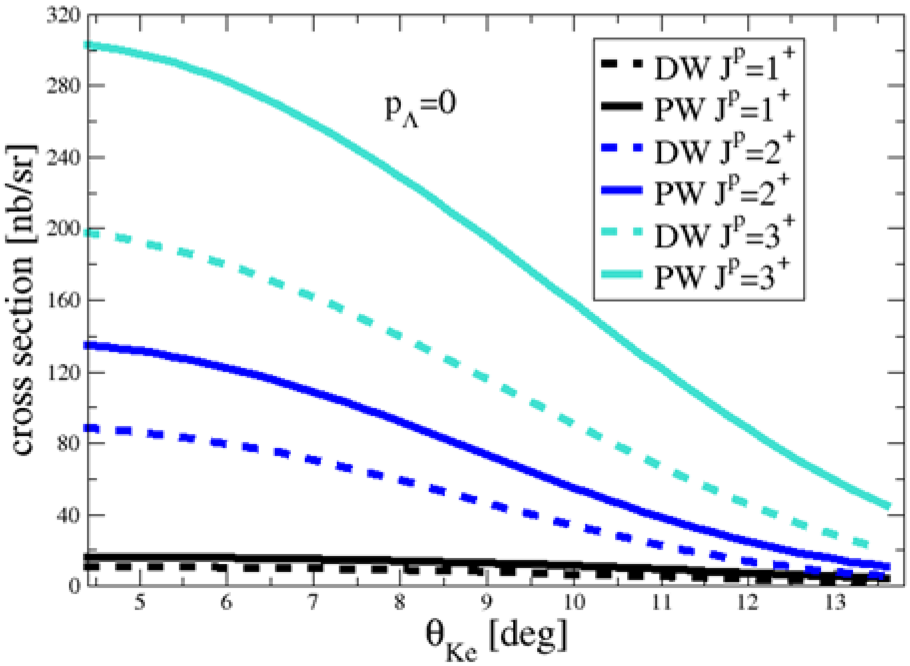
- The transverse and longitudinal cross sections

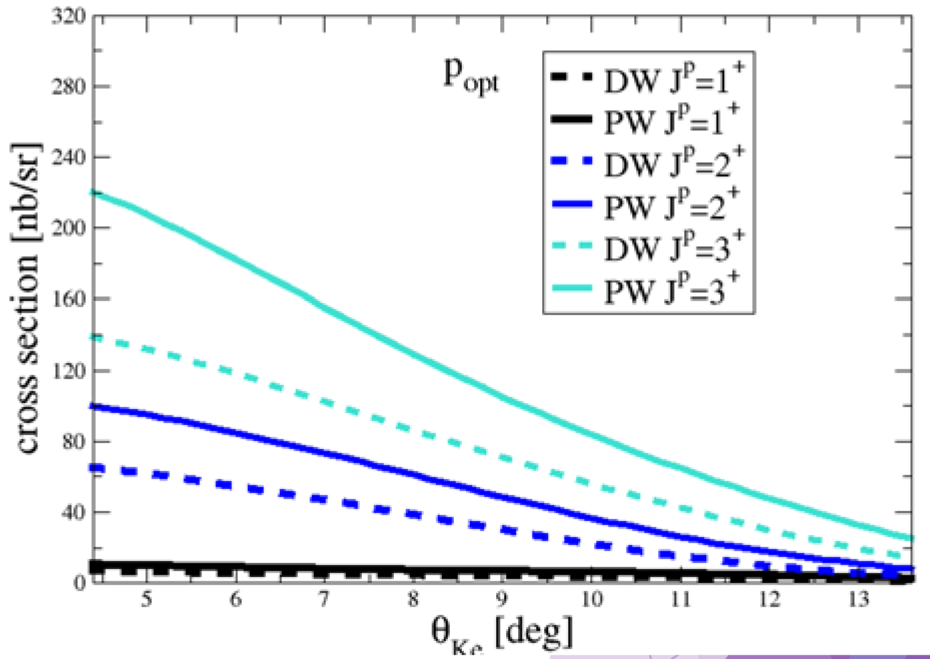
$$\frac{d\sigma_T}{d\Omega_K} = \frac{\beta}{2(2J_A+1)} \sum_{Jm} \frac{1}{2J+1} \left(\left|A_{Jm}^{+1}\right|^2 + \left|A_{Jm}^{-1}\right|^2\right)$$

$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1} \sum_{Jm} \frac{1}{2J+1} \left|A_{Jm}^{0}\right|^2$$

Comparison of the PWIA and DWIA

$E_H \approx$11 MeV





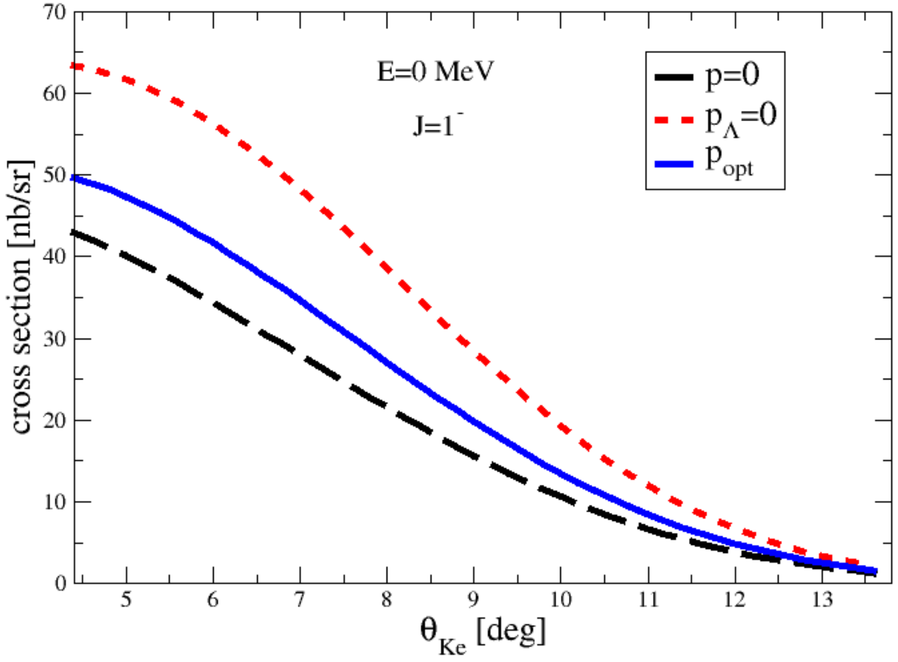
Reaction $^{12}C(e,e'K^+)^{12}_{\Lambda}B$ E_i =3.77 GeV $E_f{}'$ =1.56 GeV $\theta_e = 6°$ $\Phi_K = 180°$

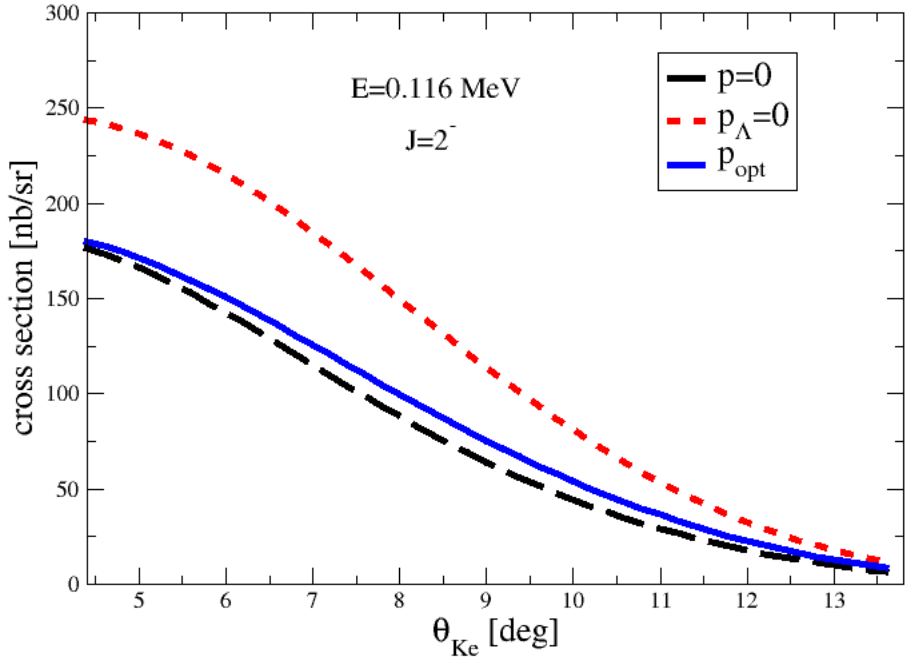
Calculations are with elementary amplitude BS3. (D. S, P. By, Phys. Rev. C 97, 025202(2018).)

The nuclear structure (OBDME) is from shell-model calculations by John Millener. (Nucl. Phys. A 804, 84 (2008))

Fermi effect in DWIA

Experimental value for doublet at θ_{Ke} = 6°: 253.4±38 nb/sr





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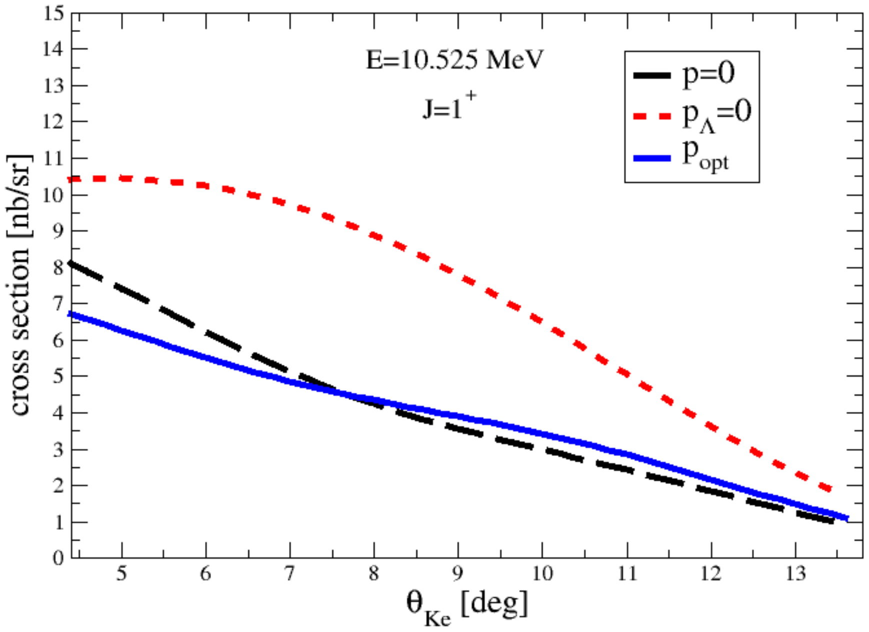
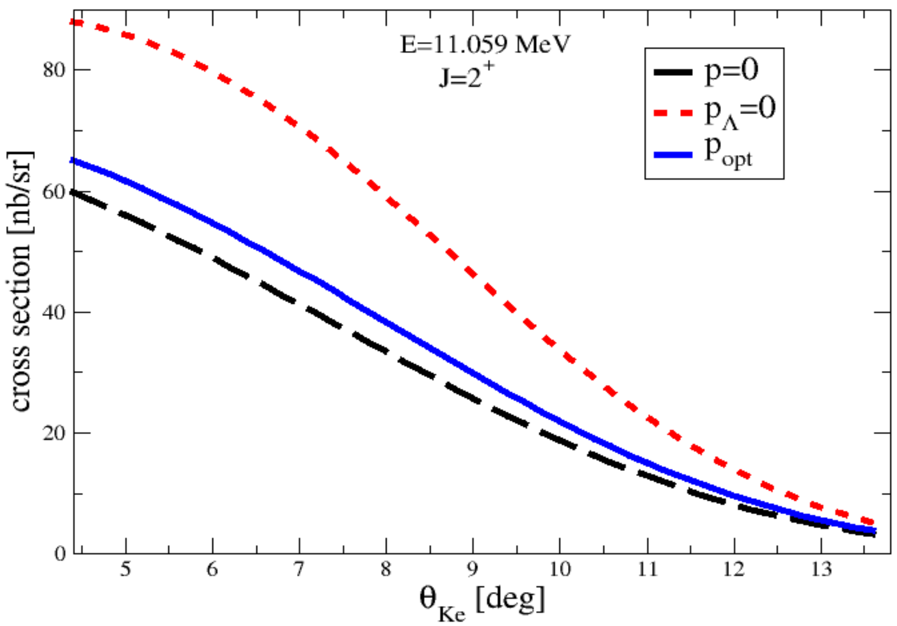


The $d\sigma_L$ depend on the A^0_{Jm} with the largest value for m = 0. The F^1_{00} dominates => the behavior of $d\sigma_L$ is affected by the selection rule for C^{J0}_{L010}.

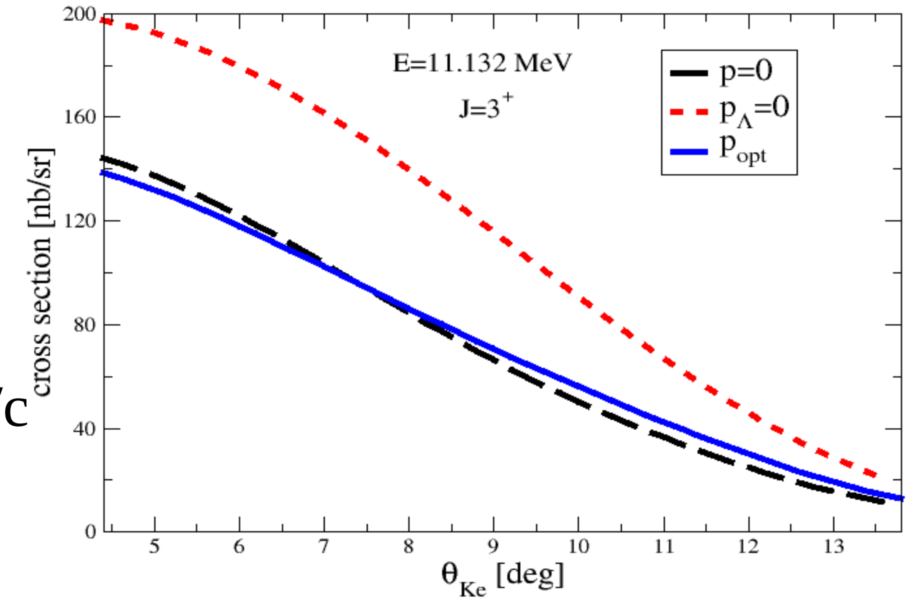


$\theta_{Ke} = 6°$,

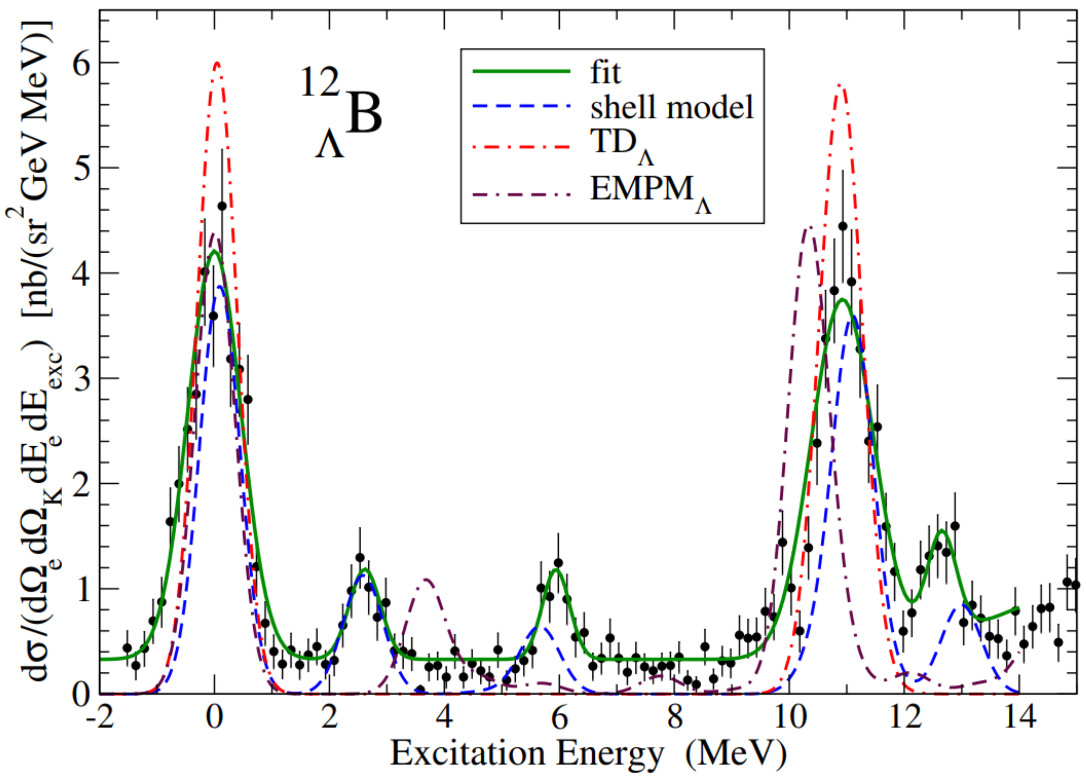
p_{eff} = 0.0 GeV/c, p_Λ = 0.301 GeV/c

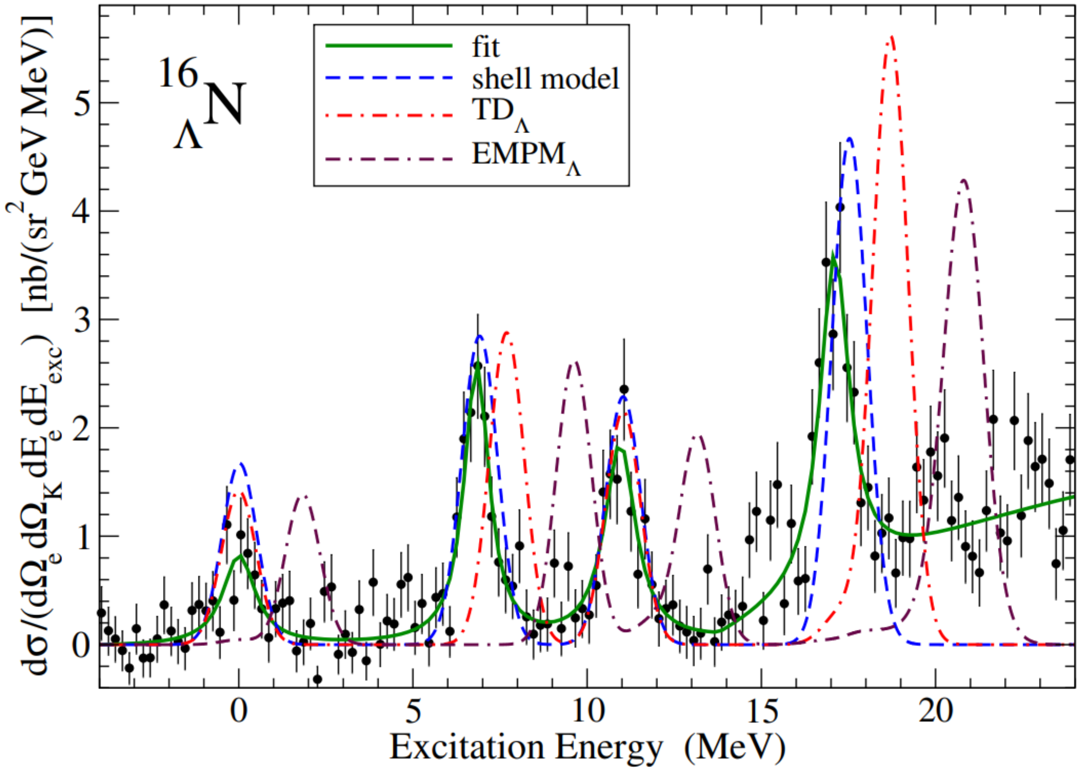
p_{eff} = 0.233GeV/c, p_Λ = 0.0 GeV/c

p_{eff} = 0.099 GeV/c, p_Λ = 0.170 GeV/c



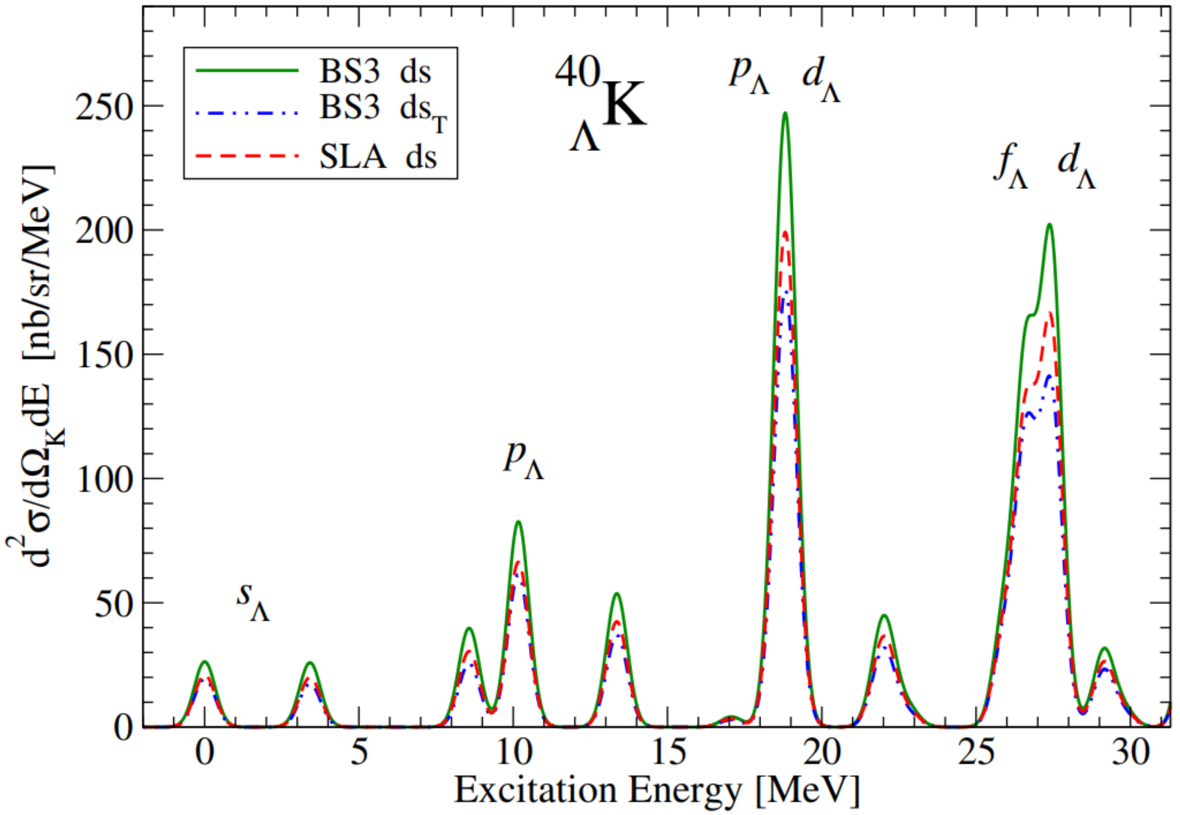
Nuclear structure effects

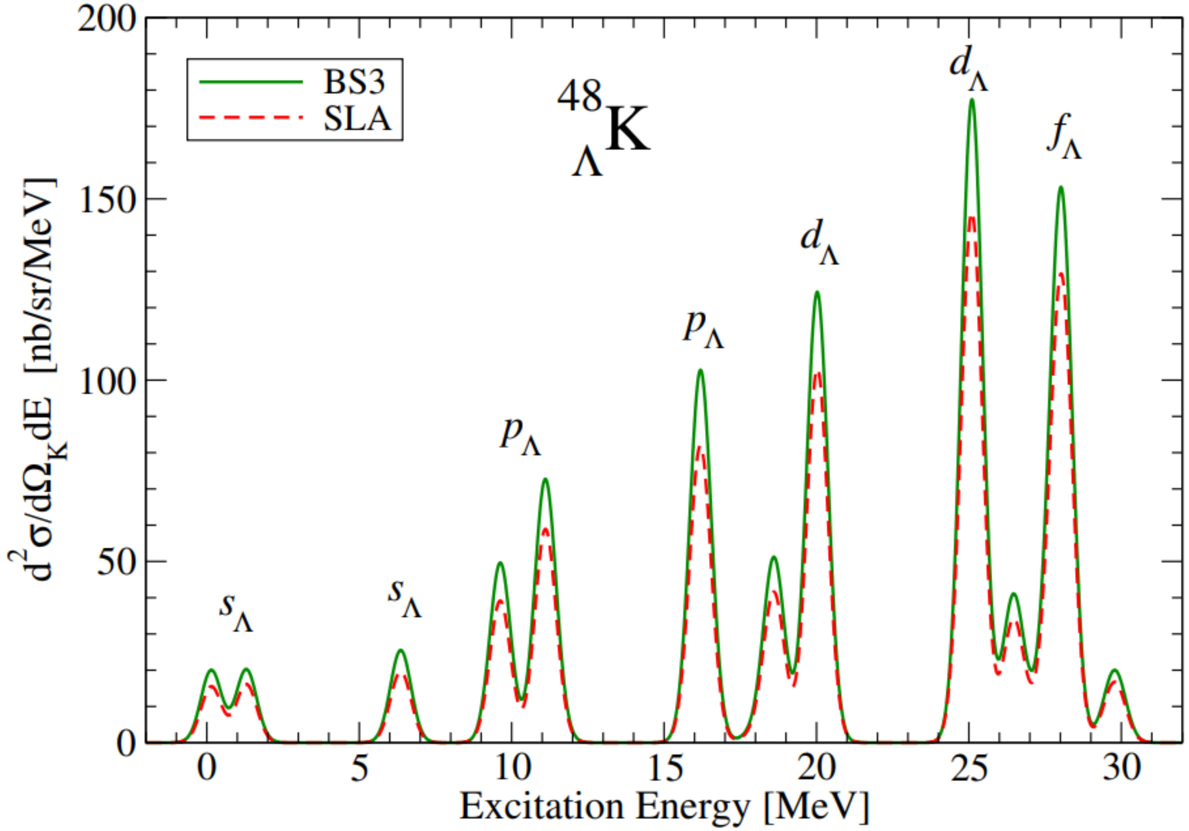




NΛ G-matrix Nijmegen-F interaction was calculated for the $k_F = 1.1 fm^{-1}$.

Prediction calculations





Summary and outlook.

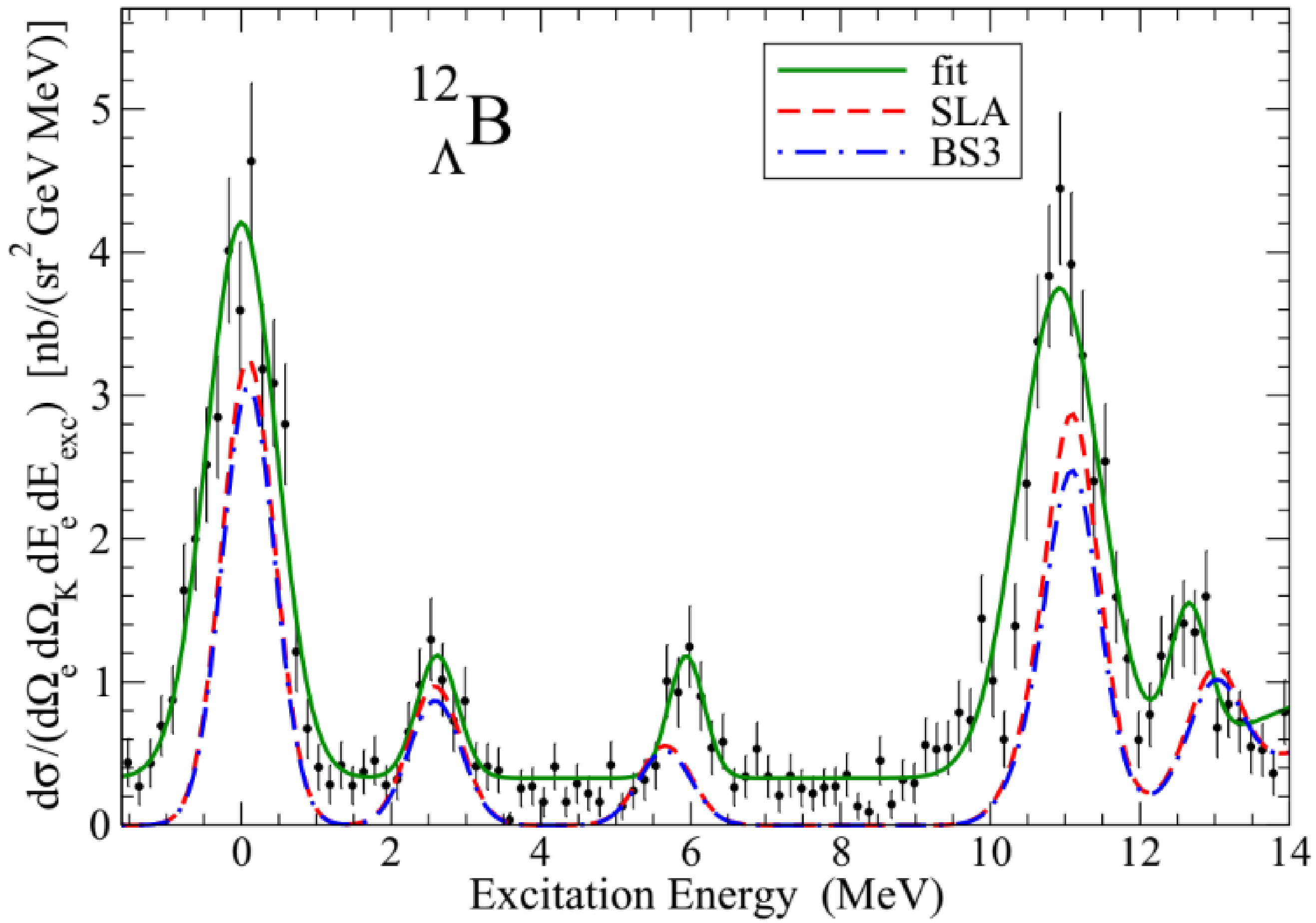
A general two-component form of the elementary amplitude was derived and used to show a dependence on the proton Fermi motion in the target nucleus.

Fermi-motion effect is more important, especially in the longitudinal cross sections, for smaller angles and energies above 2 GeV.

Comparison of PWIA and DWIA cross sections calculated in the optimum on-shell approximation show importance of the kaon distortion (about 30 %). Therefore, the DWIA and the optimum on-shell approximation are preferable for the further calculations as the results are close to the experimental cross section.

We showed that the NΛ TDA approach with the Nijmegen YN and NNLOsat interactions, used to calculate the OBDME and the single-particle wave functions, are appropriate in description of experimental data for the p-shell hypernuclei giving similar results as the shell-model calculations. We can therefore use this formalism to predict the excitation spectra of medium-mass hypernuclei which will be measured in a planned experiment in JLab. We have predicted the spectra for ${}^{40}_{\Lambda}K$ and ${}^{48}_{\Lambda}K$ which will be measured in the planned experiment in JLab.

Thank you for attention



$^{12}_{\Lambda}B$
fit
SLA
BS3
$d\sigma/(d\Omega_e d\Omega_K dE_e dE_{exc})$ [nb/(sr^2 GeV MeV)]
Excitation Energy (MeV)
0
1
2
3
4
5
0
2
4
6
8
10
12
14