KAON DISTORTION AND KINEMATICAL EFFECTS IN ELECTROPRODUCTION OF HYPERNUCLEI

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MOTIVATION OF STUDYING ELECTROPRODUCTION OF HYPERNUCLEI IN DISTORTED WAVE IMPULSE APPROXIMATION

$$e + A \rightarrow e' + H + K^+$$

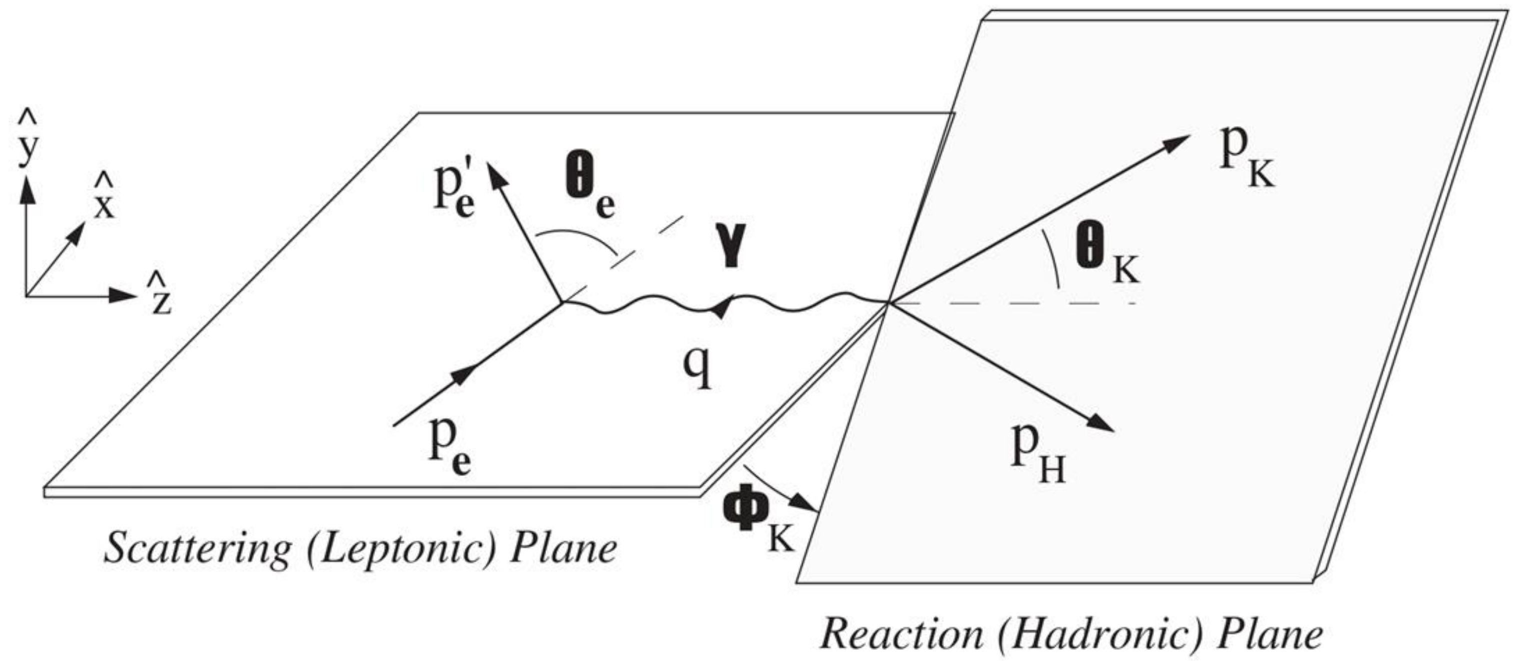
- Studying spectra in electroproduction of hypernuclei provides important information about details of the effective ΛN interaction, mainly on its spin-dependent part. A reasonable analysis of experimental data, however, require a good knowledge of the reaction mechanism. Here we will discuss effects from various choices of the proton momentum in the optimal factorization approximation and a kaon distortion. Assuming non-zero momentum of proton is important extension of the DWIA calculations.

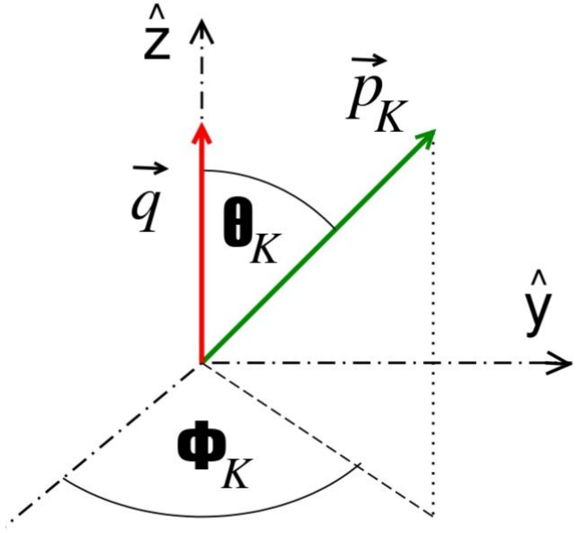
KINEMATICS

Input: $E(e), E(e'), \theta_e, \phi_K, \theta_K$

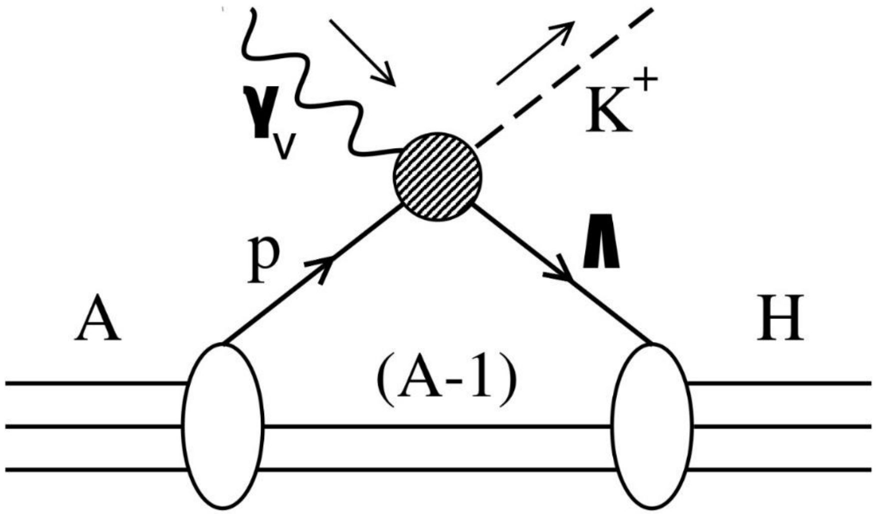
Calculated: $P_\gamma, P_K, P_H \ldots$

Lab reference frame: $\vec{P}_A$=0





MOMENTUM AND ENERGY CONSERVATION IN PWIA



- $\gamma_\nu(P_\gamma) + A(P_A) \rightarrow H(P_H) + K^+(P_K)$
- 3-momentum conservation in each vertex

$$\vec{P}_\gamma + \vec{p}_p = \vec{P}_K + \vec{p}_\Lambda, \ \vec{P}_A = \vec{p}_c + \vec{p}_p,$$

$$\vec{p}_c + \vec{p}_\Lambda = \vec{P}_H \Rightarrow \vec{P}_A + \vec{P}_\gamma = \vec{P}_H + \vec{P}_K$$

- Energy conservation

$$E_A = \sqrt{M_C^2 + (\vec{P}_A - \vec{p}_p)^2} + \sqrt{m_p^2 + \vec{p}_p^2} + \epsilon_p$$

$$E_H = \sqrt{M_C^2 + (\vec{P}_A - \vec{p}_p)^2} + \sqrt{m_\Lambda^2 + \vec{p}_\Lambda^2} + \epsilon_\Lambda$$

- In the frozen-proton approximation ($\vec{p}_p = 0$) and Lab frame: $M_A = M_C + m_p + \varepsilon_p$.

DETERMINATION OF P_K

- Energy conservation in the elementary (two-body) system

$$\mathrm{E}_\gamma + \sqrt{m_p{}^2 + \vec{p}_p^{\,2}} = \sqrt{m_K^2 + \vec{P}_K^2} + \sqrt{m_\Lambda^2 + (\vec{P}_\gamma - \vec{P}_K + \vec{p}_p)^2}$$

- Energy conservation in the many-body system (Lab)

$$\mathrm{E}_\gamma + M_A = \sqrt{m_K^2 + \vec{P}_K^2} + \sqrt{M_{\mathrm{H}}^2 + (\vec{P}_\gamma - \vec{P}_K)^2}$$

- "Optimum on-shell" proton momentum, for which equations would be solvable with a given momentum transfer $\vec{\Delta} = \vec{P}_\gamma - \vec{P}_K$

$$\mathrm{E}_\gamma - \sqrt{m_K^2 + \vec{P}_K^2} = \sqrt{m_\Lambda^2 + \left(\vec{\Delta} + \vec{p}_p\right)^2} - \sqrt{m_p^2 + \vec{p}_p^{\,2}} = \sqrt{M_{\mathrm{H}}^2 + \vec{\Delta}^{\,2}} - M_A$$

- A solution was denoted as $\vec{p}_{opt}$ and use it in presented calculations.

- In optimal factorization approximation the elementary amplitude is calculated at an effective proton momentum $\vec{p}_{eff}$.
- Effects from various values of $\vec{p}_{eff}$ (Fermi motion effects) will be demonstrated on the angle and energy dependent cross sections calculated in the OFA. Three cases are considered in further calculations:
- 1) frozen p: $\vec{p}_{eff} = 0 \Rightarrow \vec{p}_{\Lambda} = \vec{\Delta}$
- 2) frozen Λ: $\vec{p}_{eff} = -\vec{\Delta} \Rightarrow \vec{p}_{\Lambda} = 0$
- 3) optimum: $\vec{p}_{eff}=\vec{p}_{opt}$
- $\vec{\Delta}= \vec{P}_{\gamma}- \vec{P}_{K}$

ELEMENTARY AMPLITUDE

- The invariant amplitude

$$M \cdot \varepsilon = \overline{u_\Lambda}\gamma_5 \left(\sum_{j=1}^{6} M_j \cdot \varepsilon A_j \right) u_p = X_\Lambda^+ (\vec{J} \cdot \vec{\epsilon}) X_p$$

- The elementary amplitude in the spherical coordinates

$$\vec{J} \cdot \vec{\epsilon} = \sum_{\lambda = \mp 1, 0} (-1)^{-\lambda} J_\lambda^{(1)} \epsilon_{-\lambda}^{(1)}$$

- The spherical components of $J^{(1)}$ can be defined via 12 spherical amplitudes $F_{\lambda,\xi}^S$ with $S = 0, 1$ and $\lambda, \xi = \pm 1, 0$

$$J_\lambda^{(1)} = \sum_{\lambda,\xi,S} F_{\lambda,\xi}^S \sigma_\xi^S$$

MANY-PARTICLE MATRIX ELEMENT

- Matrix element with the production amplitude

$$M_\mu = (2\pi)^3 \delta^{(3)}(\vec{P}_A + \vec{P}_\gamma - \vec{P}_K - \vec{P}_H) T_\mu$$

- The laboratory amplitude in the optimal factorization :
- $\mathcal{T}_\lambda^{(1)} = \sum_{Jm} \frac{1}{[J_H]} C_{J_A M_A Jm}^{J_H M_H} A_{Jm}^{\lambda}$
- Where the reduced amplitudes are

$$A_{Jm}^{\lambda} = \frac{1}{[J]} \sum_{S\eta} F_{\lambda\eta}^{S} \sum_{LM} C_{LMS\eta}^{Jm} \sum_{\alpha'\alpha} R_{\alpha'\alpha}^{LM} H_{l'j'lj}^{LSJ} (\Psi_H \| [b_{\alpha'}^{+} \otimes a_\alpha]^J \| \Psi_A)$$

α=[nlj]

- The kaon distortion is included in the radial integral:

$$R_{a'a}^{LM} = \int_0^\infty d\xi \ \xi^2 R_{a'}^{\Lambda}(\xi)^* \ F_{LM} \ (\Delta B\xi) \ R_a^{\mathrm{p}}(\xi)$$

- With F_{LM} $(\Delta B\xi)$ determined from

$$e^{(iB\overrightarrow{\Delta}\vec{\xi})}\mathrm{X}_K^* \left(\overrightarrow{p_{KH}}, B\vec{\xi}\right) = \sum_{LM} F_{LM} \ (\Delta B\xi) \, Y_{LM}(\hat{\xi})$$

- Where X_K^* is the kaon distortion calculated in eikonal approximation assuming the first-order optical approximation.

- The unpolarized triple differential cross section in electroproduction of hypernuclei in the laboratory frame

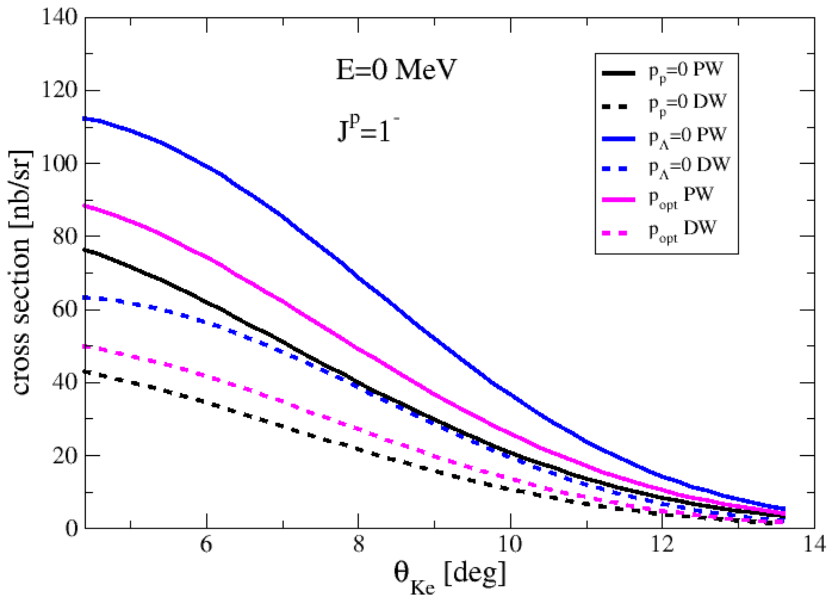
$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma[\frac{d\sigma_T}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \varepsilon \frac{d\sigma_{TT}}{d\Omega_K} + \sqrt{\varepsilon_L(\varepsilon+1)} \frac{d\sigma_{TL}}{d\Omega_K}]$$

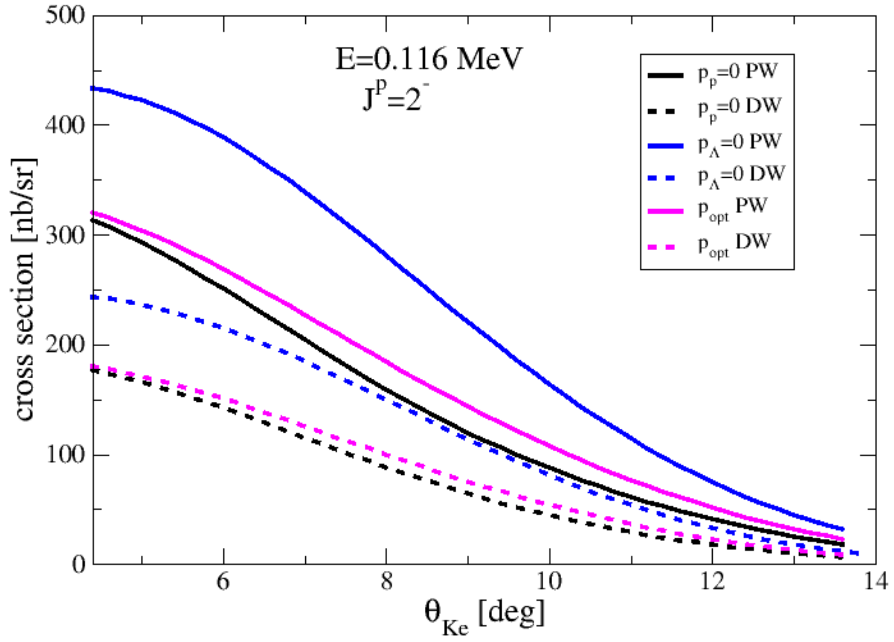
- The transverse and longitudinal cross sections

$$\frac{d\sigma_T}{d\Omega_K} = \frac{\beta}{2(2J_A+1)} \sum_{Jm} \frac{1}{2J+1} \left(\left|A_{Jm}^{+1}\right|^2 + \left|A_{Jm}^{-1}\right|^2\right)$$

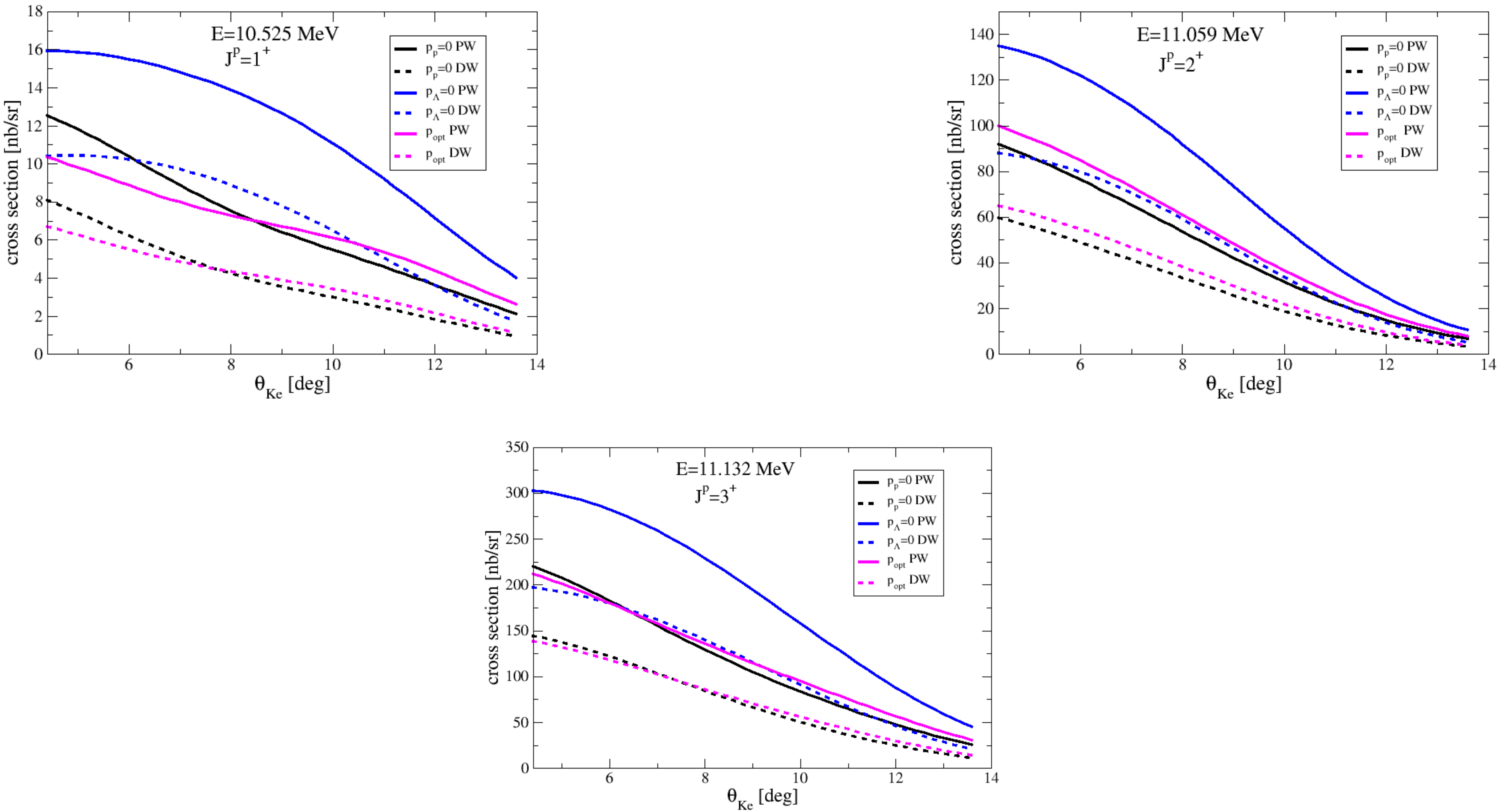
$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1} \sum_{Jm} \frac{1}{2J+1} \left|A_{Jm}^{0}\right|^2$$

CROSS SECTIONS FOR HYPERNUCLEUS ELECTROPRODUCTION IN THE PWIA AND DWIA





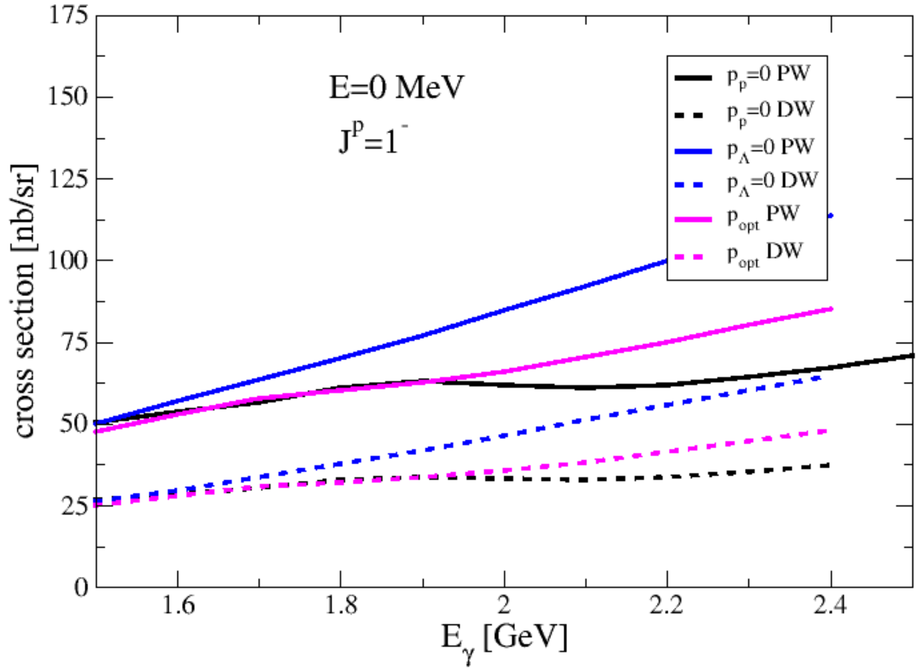
Reaction $C^{12}(e,e'K^+)B^{12}_\Lambda$ E_i =3.77 $E_f{}'$ =1.56 GeV θ_e = 6 $\Phi_K = 180$
Calculations are with elementary amplitude BS3.
The nuclear structure is described in shell-model calculations by John Millener.

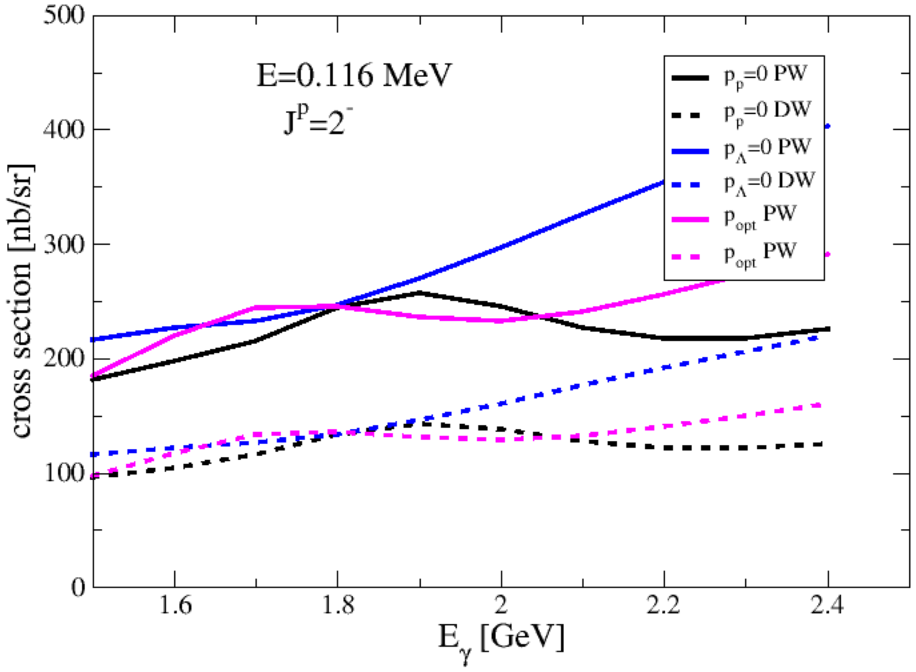


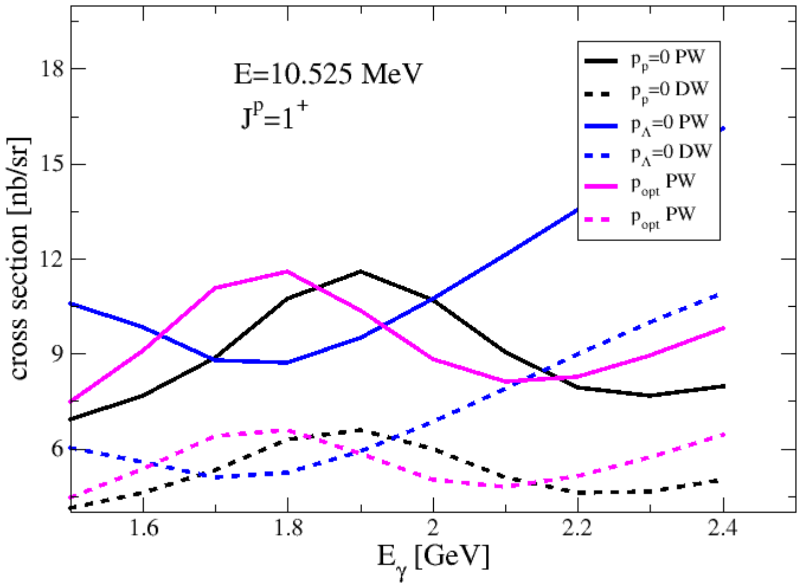
E=10.525 MeV
J^P=1^+
E=11.059 MeV
J^P=2^+
E=11.132 MeV
J^P=3^+
p_p=0 PW
p_p=0 DW
p_Λ=0 PW
p_Λ=0 DW
p_opt PW
p_opt DW
cross section [nb/sr]
θ_Ke [deg]

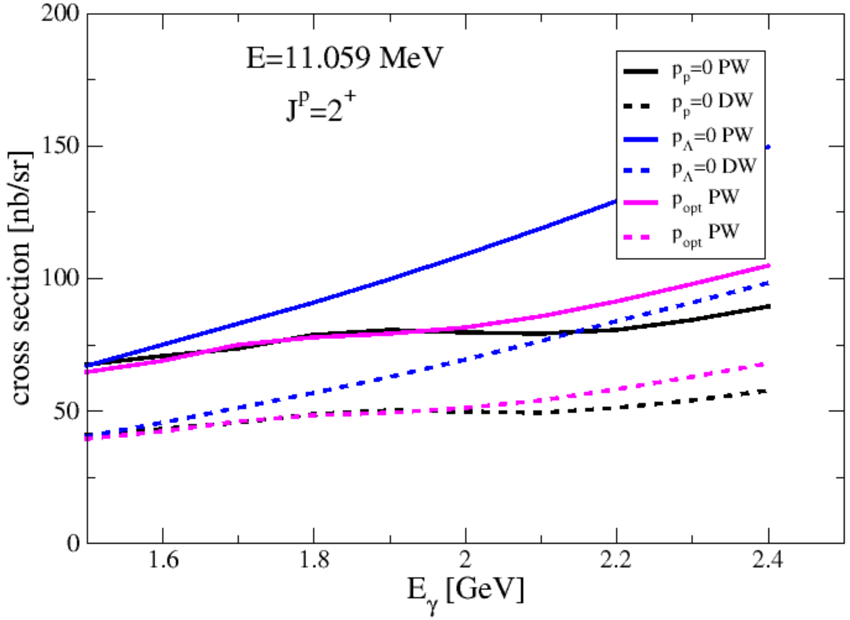
ENERGY DEPENDENCE OF THE CROSS SECTION

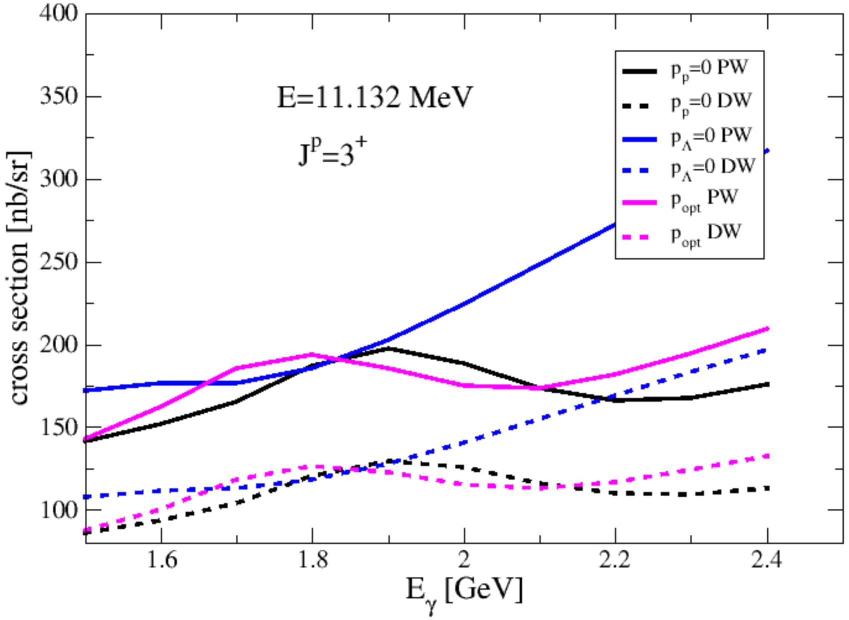
Reaction $C^{12}(e,e'K^+)B_\Lambda^{12}$ $\quad Q^2 = 0.02\ \text{GeV}^2, \quad \varepsilon = 0.7, \quad \theta_{Ke} = 6$






E=10.525 MeV
$J^P=1^+$
p_p=0 PW
p_p=0 DW
p_Λ=0 PW
p_Λ=0 DW
p_{opt} PW
p_{opt} PW
cross section [nb/sr]
E_γ [GeV]
18
15
12
9
6
1.6
1.8
2
2.2
2.4

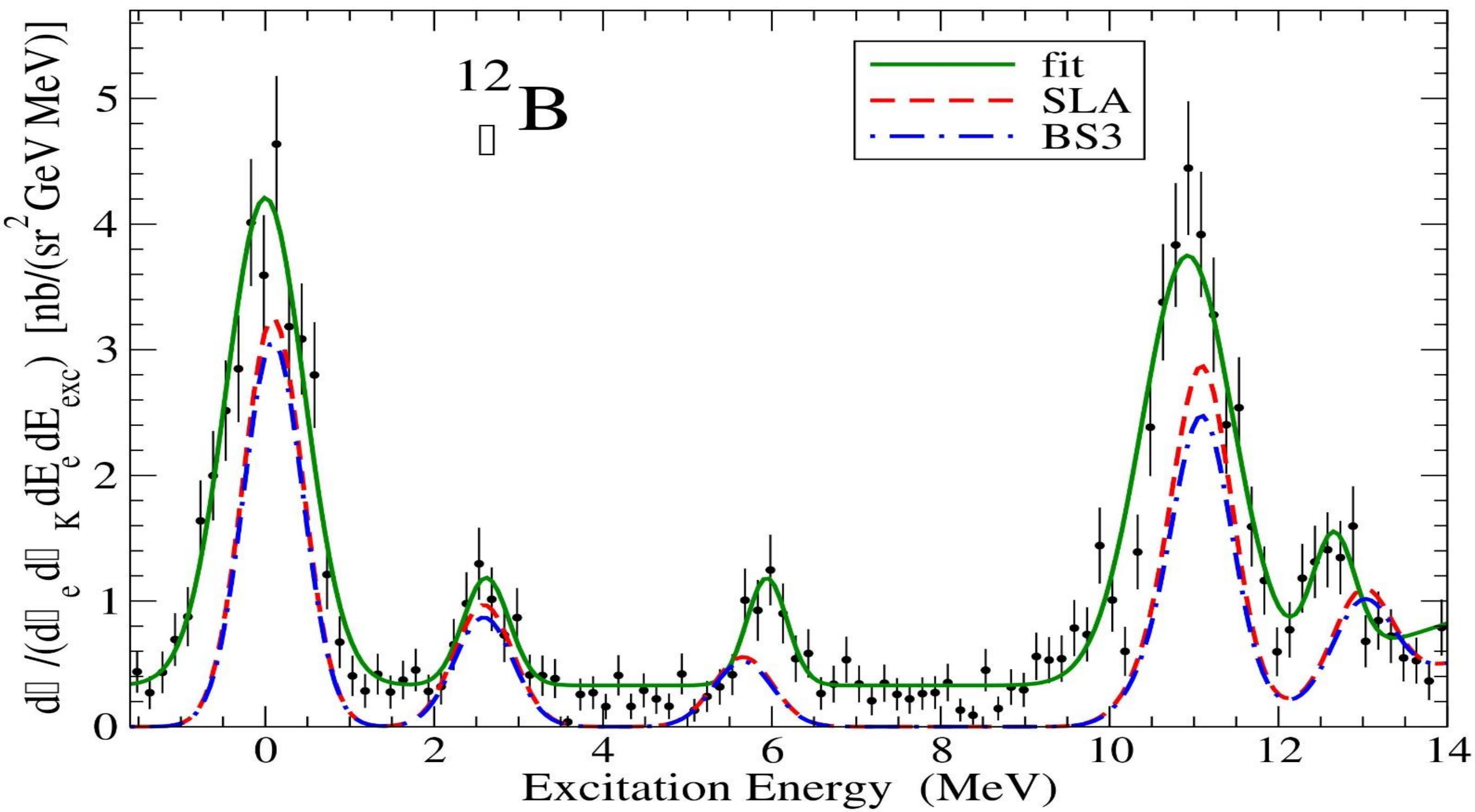

E=11.059 MeV
$J^P=2^+$
p_p=0 PW
p_p=0 DW
p_Λ=0 PW
p_Λ=0 DW
p_{opt} PW
p_{opt} PW
cross section [nb/sr]
E_γ [GeV]
200
150
100
50
0
1.6
1.8
2
2.2
2.4


E=11.132 MeV
$J^P=3^+$
p_p=0 PW
p_p=0 DW
p_Λ=0 PW
p_Λ=0 DW
p_{opt} PW
p_{opt} DW
cross section [nb/sr]
E_γ [GeV]
400
350
300
250
200
150
100
1.6
1.8
2
2.2
2.4

SUMMARY AND OUTLOOK.

- A general two-component form of the amplitude in electroproduction of kaons on the nucleon was constructed. This new form, which contains a dependence on the proton momentum, allows extend our calculations beyond the frozen-proton approximation and shows effects from a proton Fermi motion in the target nucleus. The comparison of PWIA and DWIA calculations demonstrates the 30-40% different magnitude of the cross section. The DWIA and the optimum on-shell approximation are preferable for the further calculations as the results are close to the experimental cross section.
- I will continue studying effects on the cross sections from the kinematics, kaon distortion, various wave functions and elementary amplitudes.

THANK YOU FOR ATTENTION



12 B
fit
SLA
BS3
Excitation Energy (MeV)
0
1
2
3
4
5
6
8
10
12
14