Unfolding procedures for radiative corrections

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Event-by-event corrections

- Began thinking about event-by-event corrections
  - Is it feasible?
  - How could uncertainty be determined?

But first, need to consider unfolding
(necessary whether corrections are applied to bins or events)

Unfolding

Experimental cross section σ_{exp} and unradiated Born cross section σ_B are non-trivially related

"Unfolding" procedure necessary for extracting unradiated cross section

- Iterative unfolding
  - Required input: model cross section σ_{mod}
- Smearing unfolding
  - Required input: Monte Carlo

Overview

Mo & Tsai

$$\frac{d\sigma_{exp}}{d\Omega dE_p} = \left(\frac{R\Delta}{E_s}\right)^{T'} \left(\frac{\Delta}{E_p}\right)^{T'} \left(1 - \frac{\xi}{(1-2T')\Delta}\right) \sigma^{eff}(E_s, E_p)$$

$$+ \int_{E_{s\,min}(E_p)}^{E_s - R\Delta} \sigma^{eff}(E_s', E_p) \left(\frac{E_s - E_s'}{E_p R}\right)^{T'} \left(\frac{E_s - E_s'}{E_s}\right)^{T'}$$

$$\times \left[\frac{T'}{E_s - E_s'} \phi\left(\frac{E_s - E_s'}{E_s}\right) + \frac{\xi}{2(E_s - E_s')^2} \right] dE_s'$$

$$+ \int_{E_p + \Delta}^{E_{p\,max}} \sigma^{eff}(E_s, E_p') \left(\frac{E_p' - E_p}{E_p'}\right)^{T'} \left(\frac{(E_p' - E_p)R}{E_s}\right)^{T'}$$

$$\times \left[\frac{T'}{E_p' - E_p} \phi\left(\frac{E_p' - E_p}{E_p'}\right) + \frac{\xi}{2(E_p' - E_p)^2} \right] dE_p'$$

1. Choose cross section model σ_{mod}
2. Radiate σ_{mod} to obtain σ_{rad}
3. Compare σ_{rad} to σ_{exp}
4. Adjust σ_{mod} accordingly
5. Repeat 2-4 until $\sigma_{rad} \approx \sigma_{exp}$
6. Scale σ_{exp} by ratio $\sigma_{rad}/\sigma_{mod}$

Implementation example

Iteration example (based on J. Arrington thesis)

For iteration i, scale the model cross section by smooth function of x:

$$\sigma_{mod} \to f_i(x)\sigma_{mod}$$

with $f_0(x) = 1$.

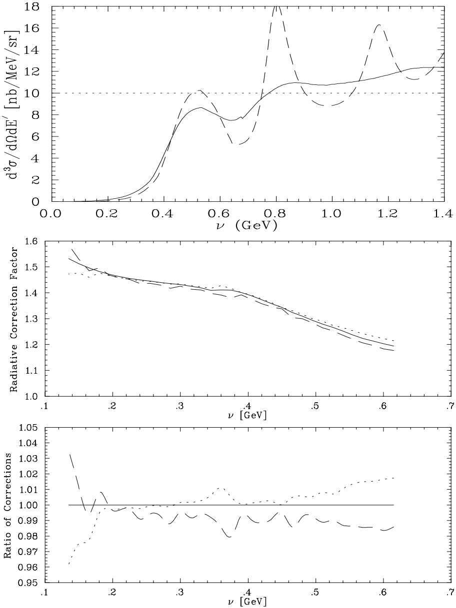
Radiate model and update $f(x)$ at data points n:

$$f_i(x_n) = f_{i-1}(x_n) \times \left(\frac{\sigma_{exp}(x_n)}{\sigma_{rad}(x_n)} \right)$$

Repeat until

$$\chi^2 \equiv \sum_n \frac{(\sigma_{exp}(x_n)/\sigma_{rad}(x_n)) - 1}{(\delta\sigma_{rad}(x_n)/\sigma_{rad}(x_n))^2} \leq 1$$

Model dependence example (from J. Arrington thesis)



Cross sections

Smearing example (based on S. Joosten thesis)

Kinematic point:

$$X = (x,\ Q^2)$$

Experimental cross section in bin i:

$$\frac{d\sigma_{exp}}{dX} \quad \rightarrow \quad \sigma_{exp}(i) = \int_i dX\ \frac{d\sigma_{exp}}{dX}$$

Born cross section in bin i:

$$\frac{d\sigma_B}{dX} \quad \rightarrow \quad \sigma_B(i) = \int_i dX\ \frac{d\sigma_B}{dX}$$

Relating σ_{exp} to σ_B

$X = (x,\ Q^2)$	actual kinematic point
$\overline{X} = (\overline{x}, \overline{Q^2})$	observed kinematic point
$P(\overline{X}\|X)$	probability of observing $\overline{X}$ given X (radiative effects)
$A(\overline{X})$	spectrometer acceptance function
$R(Y\|\overline{X})$	probability of observing Y given $\overline{X}$ (detector resolution)

Experimental cross section in bin i related to Born cross section in bin j by:

$$\sigma_{exp}(i) = \int_i dY \sum_j \left(\int d\overline{X} \int_j dX\ R(Y|\overline{X})\ A(\overline{X})\ P(\overline{X}|X)\ \frac{d\sigma_B}{dX} \right)$$

Smearing matrix

Multiplying previous expression by $\sigma_B(j)/\sigma_B(j) = 1$:

$$\sigma_{exp}(i) = \sum_j \left(\frac{\int_i dY \int d\overline{X} \int_j dX \; R(Y|\overline{X}) \; A(\overline{X}) \; P(\overline{X}|X) \; \frac{d\sigma_B}{dX}}{\int_j dX \; \frac{d\sigma_B}{dX}} \right) \sigma_B(j)$$

$$= \sum_j S(i,\ j) \; \sigma_B(j)$$

If bins are chosen with relatively constant $d\sigma_B/dX$:

$$\sigma_B(j) = \sum_i S^{-1}(i,\ j)\sigma_{exp}(i)$$

$S(i,\ j)$ is *model-independent* but must be determined by *Monte Carlo*

Comments

Event-by-event corrections

- Both iterative and smearing approaches require *binned cross sections*
  - Event-by-event corrections ruled out?

Iterative unfolding

- Model-dependence minimal at intermediate x, possibly large at high x?
- Iteration tunes model to *raw* experimental cross section...could the model possibly be tuned to a cross section ratio?
  - Requires same model for both nuclei in ratio
  - Correlated errors?

Smearing unfolding

- Independent of cross section model
- Requires Monte Carlo simulation of target